Impact of Parameter Uncertainty on Power Flow Accuracy in Multi-terminal Systems

A. Beddard, W. Wang, M. Barnes and P. R. Green
University of Manchester
Manchester, UK
mike.barnes@manchester.ac.uk

A. Beddard and T. C. Green
Imperial College London
London, UK
t.green@imperial.ac.uk

Abstract—Accurate power flow in a MT system can be achieved with droop controllers. However, almost all publications have assumed that the DC voltage, DC current and DC cable resistances can be measured with 100% accuracy. In this paper, a novel power flow solver is developed which enables the user to analyse the impact of these parameters on power flow accuracy. The developed Parameter Uncertainty Power Flow Solver (PU-PFS) is shown to be able to accurately calculate the power flow error for hundreds of parameter uncertainty scenarios in less than a second. The PU-PFS is employed to investigate the impact of parameter uncertainty on a potential MT system and the results show that realistic measurement errors (0.2%) can result in significant power flow error (>150MW). Finally, the paper assesses the key factors which influence the power flow accuracy resulting in a number of important conclusions.

Index Terms—Droop control, multi-terminal power flow, DC grid, MTDC, measurement errors.

I. INTRODUCTION

There have been a number of proposals to interconnect offshore generation to a Voltage Source Converter (VSC) High Voltage Direct Current (HVDC) Multi-Terminal (MT) system such as the Atlantic Wind Connection (AWC) [1]. The connection of offshore generation to a common DC grid has the potential to reduce the volume of assets installed offshore, improve operational flexibility and alleviate congestion in the onshore AC network when compared with radial VSC-HVDC systems. The regulation of the DC link voltage for a MT system is however more challenging.

Over the previous five years a large body of work on MT control methods has been built-up and a review of them is given in [2, 3]. Droop control is generally seen as the preferred method of primary control as it enables more than one converter to participate in the regulation of the DC voltage simultaneously. However, in order to obtain accurate power flow in a MT system with droop controllers, the correct operating point on the droop characteristic for each converter must be determined.

This is typically achieved using a Power Flow Solver (PFS) which calculates the set-points for the droop controllers [4, 5], however system representation, controller set-points and measurements were assumed to be ideal. In practice, cable resistances, DC voltages and DC currents cannot be measured with 100% accuracy. The uncertainty of these parameters will therefore have an adverse impact on the accuracy of the power flow in the MT system. This potential issue was acknowledged in [6] and the impact of DC voltage measurement and cable resistance errors were studied for two MT control strategies in [7] using a stochastic approach with fixed error measurements.

The main contributions of this paper are: 1) the development of a PFS which enables the user to rapidly study the impact of relative measurement errors in the DC voltage and DC current as well as cable resistance errors, on the power flow accuracy in any MT system; 2) the factors which influence the power flow accuracy such as controller droop gain and operating point are investigated which leads to a number of key MT control design conclusions.

II. MT CONTROL

Autonomous Converter Control (ACC), is a I-V droop controller proposed by Alstom Grid in [8], Figure 1a. This type of droop controller is often implemented as a V-I droop as shown in Figure 1b. The other main type of droop controller is the V-P droop controller [4], Figure 1c.

The structures of the droop controllers are different, however, they all work on the basic principle of modifying the converter’s active power to regulate the DC voltage in accordance with the droop characteristic, Figure 2. Providing the PFS has correctly calculated the set-points \( (P_{dc}/V_{dc}/I_{dc}) \) for the respective converters, and there are no measurement errors, the operating point on the droop characteristics will correspond to the target power order for the converter. However, incorrect set-points or measurement errors will shift the converter’s operating point resulting in power flow error. It is important to note that error in a single converter can result in power flow error for all of the converters in the MT system.

In a MT system, the user typically sets the target DC voltage for one converter (DC slack bus) and the target DC powers for the remaining converters. The main function of the PFS is to calculate the unknown target DC power for the slack bus and the unknown target DC voltages for the remaining converters. The power injected into, for example, node i from the other nodes can be calculated using (1) [9]. \( V_{dc} \) and \( V_{dcj} \) are the voltages at nodes i and j respectively, \( Y_{ij} \) is the self-admittance of node i and \( Y_{ij} \) is the branch admittance between nodes i and j. In matrix form, the node powers for a system with n nodes can be calculated using (2), where \( \cdot \times \cdot \) denotes element-wise multiplication.

\[
P_{dc} = V_{dc} I_{dc} = V_{dc} \left( \sum_{j=1}^{n} Y_{ij} V_{dj} \right)
\]

\[
[P_{dc}] = [V_{dc}] \times ([Y_{ij}] [V_{dj}])
\]
becomes a DC power controller with a relative error in the DC on the relative error model is given by or a DC volt droop ga measurement systems f have acquisition Extrapolating from this, the IEEE C57.13.6 standard is the 0.15S class highest accuracy class of instr widespread in error in the PFS will therefore have some degree of error, resulting be measured with 100% accuracy; the admittance matrix used by the PFS is therefore assumed to have some degree of error, resulting in error in the set-points and therefore the MT power flow.

III. MODELING PARAMETER UNCERTAINTY

Information on the accuracy of DC transducers for HVDC applications is limited and the lack of standards for DC transducers is acknowledged as a major hindrance in the widespread use of DC side metering [10]. In their absence, the highest accuracy class of instrument transformers according to the IEEE C57.13.6 standard is the 0.15S class [11]. Extrapolating from this, the DC measurement systems (data acquisition system) employed in this study are assumed to have a relative error of ±0.2%.

Relative measurement errors in the voltage and current measurement systems for the V-I droop controller can be modelled as shown in Figure 3. The DC current error, $I_{dc}$, for converter $i$ is given by (4). Noting that the PI controller eliminates steady-state error (i.e. $I_{dc} = I_{dc}(E_i)$), the DC power for converter $i$ is given by (5). By appropriately selecting the droop gain, (5) can be used to describe a DC current controller or a DC voltage controller with a relative measurement error.

The DC power equations for the V-P droop controller based on the relative error model is given by (6). The V-P controller becomes a DC power controller with a relative error in the DC power when the gain is set to zero.

\[
I_{dc} = K_{droop}\cdot I_i (V_{dc}E_{yi} - V_{dc}^*) + I_{dc}^*
\]  

(4)

Relative DC measurement errors for a V-I droop controller, V-P droop controller, $V_{dc}$ controller, $I_{dc}$ controller and a $P_{dc}$ controller can therefore be modelled using (5) and (6).

The series admittance values used by a PFS are typically calculated based on the geometric and material properties of the cable’s core conductor at a specific temperature. The cable’s core resistance cannot be calculated with 100% accuracy since the cable is not perfectly uniform along its entire length and material properties vary with temperature. The maximum continuous conductor temperature for a typical power cable is 90°C [12] and the resistivity of copper at 20°C varies by approximately 0.393% per degree Celsius [13]. The resistance of a cable at 90°C would therefore be approximately 28% higher than the same cable at 20°C.

To improve the accuracy of the admittance values used by the PFS, it has been suggested that the DC voltage and DC current measurements could be used to calculate the resistance of each cable [6]. Employing such a method, however, will most likely decrease the accuracy of the admittance values used by the PFS due to the DC voltage and DC current measurement errors. It is expected that a basic real-time model of the cables would be able to estimate the core conductor resistance within ±10%. A maximum admittance matrix error of ±10% per cable will therefore be assumed for this work.

IV. STEADY-STATE ANALYSIS

The steady-state power flow in a MT system due to a system disturbance (e.g. loss of a converter or a set-point change) can be analysed using a generalised power flow solver [4]. At steady-state in a MT system, the DC power calculated according to (2) is equal to the converters’ respective DC power equations, $P_{conv}^i$, hence:

\[
|P_{dc} - P_{conv}^i| = 0
\]  

(7)

Note that the ‘dc’ subscript used in section III has been replaced with ‘conv’ in this section, to differentiate between, the DC power at node $i$ calculated according to (2), and the DC power at node $i$ calculated according to the converter’s power equation, $P_{conv}^i$.

The objective is to solve (7) so that the DC power and DC voltage values due to measurement errors and admittance errors can be calculated. This can be achieved by using the Newton Raphson method which was described in section II. The target value for (7) is zero; hence the DC power error in (3) is given by (8). Expanding (7) for a V-I droop controller.
with relative measurement errors gives (9) which can be rearranged as in (10).

\[
[P_{\text{err}}] = [P_{\text{nom}}] - [P_{\text{dc}i}]
\]  

(8)

\[
V_{\text{dc}i} \left( \sum_{j \neq i} Y_{ij} V_{\text{dc}j} \right) - V_{\text{dc}i} \left[ \frac{K_{\text{droop-ii}} E_{li}}{E_{li}} (V_{\text{dc}i} E_{Vii} - V_{\text{dc}i}) + \frac{I_{\text{dc}i}^*}{E_{li}} \right] = 0
\]  

(9)

\[
f_i(V) = V_{\text{dc}i} \left( Y_{ii} - \frac{K_{\text{droop-i}} E_{Vi}}{E_{li}} \right) + \sum_{j \neq i} Y_{ij} V_{\text{dc}j}
\]

(10)

The diagonal element of the Jacobian matrix is then given by (11). Following the same procedure, the diagonal Jacobian element for the V-P droop controller is be given by (12). The off-diagonal Jacobian elements are the same irrespective of the type of controller employed, (13).

\[
\frac{\partial f_i}{\partial V_{\text{dc}i}} = 2V_{\text{dc}i} \left( Y_{ii} - \frac{K_{\text{droop-i}} E_{Vi}}{E_{li}} \right) + \left[ \sum_{j \neq i} Y_{ij} V_{\text{dc}j} \right] \frac{K_{\text{droop-ii}} E_{Vii} - I_{\text{dc}i}^*}{E_{li}}
\]

(11)

\[
\frac{\partial f_i}{\partial V_{\text{dc}i}} = 2V_{\text{dc}i} Y_{ii} + \left[ \sum_{j \neq i} Y_{ij} V_{\text{dc}j} \right] - \frac{K_{\text{droop-ii}} E_{Vii}}{E_{li}}
\]

(12)

\[
\frac{\partial f_i}{\partial V_{\text{dc}i}} = V_{\text{dc}i} Y_{ii}
\]

(13)

The key steps for the PU-PFS are shown in Figure 4. The controller set-points (P_{\text{dc}i}^*, V_{\text{dc}i}^*, I_{\text{dc}i}^*) are calculated by the ‘standard’ PFS which assumes that the admittance matrix is 100% accurate and that there are no measurement errors. These set-points, along with the parameter uncertainty data (E_{V}, E_{I} \& E_{Y}), are entered into (8), which is solved in an iterative fashion until the error, (P_{\text{err}}), is within the error tolerance, E_{\text{tol}}. If the droop controller limits have been exceeded, the PU-PFS updates the controller’s settings by setting the droop gain to zero and the controller’s set-point to the limit that has been exceeded. The system is then solved in an iterative fashion until the correct operating condition for each converter is obtained.

V. IMPACT OF PARAMETER UNCERTAINTY ON NAWC

In this section of the paper, the PU-PFS is used to investigate the impact of parameter uncertainty on a potential MT system which is based on the Northern section of the AW (NAWC). The configuration of the NAWC is shown in Figure 5 [14]. All of the VSCs have a power rating of 1GW and a nominal DC voltage of 600kV. The onshore VSCs (MMC 1-3) employ the basic ACC (I-V droop) control and the offshore VSCs (MMC 4-6) control the AC voltage magnitude and frequency of their respective offshore networks. DC measurement errors in the offshore converter stations have little impact on the MT power flow as they are not controlling the DC voltage/current. DC measurement errors in the offshore converter stations will therefore be neglected for the purpose of this study.

The nominal droop gain for all of the onshore converters is set to 4kV/kA. This gain was selected to prevent the DC voltages from exceeding their normal operating range by ±2% [14]. The series resistance of the DC cables is approximately 0.0113Ω/km with a shunt conductance of 10^{-5}S/km [15].

A. DC measurement errors and admittance matrix errors

For the following tests it is assumed that the error in each DC measurement system can be one of three values (i.e. -0.2%, 0% or +0.2%). The target voltage for VSC1 is set to 590kV and the target DC power values for MMC2 and MMC3 are 600MW and 900MW respectively. Windfarms 1 (MMC4), 2 (MMC5) and 3 (MMC6) are ordered to inject 800MW, 900MW and 700MW respectively.

1) DC voltage measurement errors

In this case, the three DC voltage measurement errors (-0.2%, 0%, +0.2%) are applied to the three onshore VSCs, hence there are 27 (3^3) possible scenarios to be studied. The scenarios which produced the maximum individual error for each of the VSCs and the maximum total error are presented.
in Table I. The results show the significant impact that measurement errors in the DC voltage can have on the MT power flow. Of the 27 scenarios, only three resulted in a total power error of less than 100MW. These scenarios were where the onshore converters had no measurement errors or the same measurement error.

**Table I. Maximum power error for each VSC and maximum total power error due to DC voltage measurement errors of ±0.2%**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>VSC1</th>
<th>VSC2</th>
<th>VSC3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error (MW)</td>
<td>-118.50</td>
<td>163.16</td>
<td>-155.22</td>
<td>325.03</td>
</tr>
<tr>
<td>Error (%)</td>
<td>-13.47</td>
<td>27.19</td>
<td>-17.25</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table II. DC measurement errors corresponding to error scenario**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Measurement Error (E_v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VSC1</td>
<td>VSC2</td>
</tr>
<tr>
<td>15</td>
<td>1.002</td>
</tr>
<tr>
<td>23</td>
<td>0.998</td>
</tr>
<tr>
<td>24</td>
<td>0.998</td>
</tr>
</tbody>
</table>

The primary reason that VSC2 experiences the highest absolute power error and VSC1 experiences the lowest absolute power error is because the equivalent resistance between the offshore and onshore VSCs is highest for VSC1 and lowest for VSC2.

2) DC current measurement errors

In this case, the PU-PFS solves the power flow for the 27 DC current error scenarios. Table III shows that errors in the DC current measurement system have significantly less impact on the power flow accuracy than errors in the DC voltage measurement system. The DC measurement errors for scenario 17 are 1.002, 0.998 and 1.002 for VSC1, 2 and 3 respectively.

**Table III. Maximum power error for each VSC and maximum total power error due to DC current measurement errors of ±0.2%**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>VSC1</th>
<th>VSC2</th>
<th>VSC3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error (MW)</td>
<td>1.093</td>
<td>1.378</td>
<td>1.409</td>
<td>2.810</td>
</tr>
<tr>
<td>Error (%)</td>
<td>0.124</td>
<td>0.230</td>
<td>0.157</td>
<td>-</td>
</tr>
<tr>
<td>Scenario</td>
<td>23</td>
<td>17</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

3) Impact of admittance matrix errors

For the following tests it is assumed that the error in the series admittance for each of the five cables can be one of three values (i.e. -10%, 0% or +10%) and that there are no errors in the DC measurements. The PU-PFS is therefore required to solve 243 scenarios (5 cables with three values, 3^5). The results are presented in Table IV.

**Table IV. Maximum power error for each VSC and maximum total power error due to admittance matrix errors of ±10%**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>VSC1</th>
<th>VSC2</th>
<th>VSC3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error (MW)</td>
<td>-43.882</td>
<td>36.124</td>
<td>34.882</td>
<td>87.861</td>
</tr>
<tr>
<td>Error (%)</td>
<td>-7.438</td>
<td>6.064</td>
<td>5.864</td>
<td>-</td>
</tr>
<tr>
<td>Scenario</td>
<td>206</td>
<td>216</td>
<td>233</td>
<td>206</td>
</tr>
</tbody>
</table>

Table V shows that the worst case scenario is when the series admittance for cable 14 is decreased and the series admittance values for cables 25 and 36 are increased. This is expected since cable 14 is the longest cable in the system and therefore a 10% change in admittance for this cable results in a greater absolute change in the series admittance.

**Table V. Admittance matrix errors corresponding to scenarios**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Admittance matrix error (E_v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y14</td>
<td>Y25</td>
</tr>
<tr>
<td>206</td>
<td>0.9</td>
</tr>
<tr>
<td>216</td>
<td>0.9</td>
</tr>
<tr>
<td>233</td>
<td>0.9</td>
</tr>
</tbody>
</table>

4) Worst case combined error for a single VSC

From the maximum individual errors for each converter from the tests conducted in this section (V. A), VSC 2 has the highest combined error of approximately 200.67MW. The DC voltage measurement errors, DC current measurement errors and admittance matrix errors correspond to scenarios 24, 17 and 216 respectively. The power error calculated by the PU-PFS for this scenario was 199.89MW for VSC2. These results show that VSC2 would experience a relative error of approximately 33% and that the superposition of the individual errors gives a reasonable approximation to the converter’s combined power error in this case.

B. Impact of converter power orders

In this section, the tests carried out in section V. A. 1-3 are repeated with the target DC power values for MMC2 and MMC3 equal to 800MW and 300MW respectively while windfarms 1, 2 and 3 are ordered to inject 500MW, 400MW and 650MW respectively.

Comparing Table VI with Table I, 5 and 6 shows that the change of converter power order has had little impact on the absolute power flow error due to DC voltage measurement errors (although a major impact on relative error). However, it has had a significant impact on the power flow error due to admittance matrix errors. This is because changing the converter power orders changes the target DC voltages for the converters. This in turn changes the absolute power flow for a given change in the cable’s admittance.

**Table VI. Absolute power error due to DC measurement errors and admittance matrix errors for new converter power orders**

<table>
<thead>
<tr>
<th>Error Type</th>
<th>VSC1</th>
<th>VSC2</th>
<th>VSC3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC Voltage</td>
<td>-118.70</td>
<td>161.53</td>
<td>154.58</td>
<td>323.32</td>
</tr>
<tr>
<td>DC Current</td>
<td>0.68</td>
<td>1.06</td>
<td>0.85</td>
<td>2.13</td>
</tr>
<tr>
<td>Admittance</td>
<td>-24.35</td>
<td>-24.12</td>
<td>22.03</td>
<td>48.83</td>
</tr>
</tbody>
</table>

C. Impact of droop gain

In this section, the tests carried out in section V. A. 1-3, are repeated with the droop gain for VSC2 increased from 4 to 8kV/kA and the power orders the same as in section V. B. Comparing Table VII with Table VI shows that increasing the droop gain of VSC2 reduces the absolute power error due to the measurement errors in the DC voltage. This is because the droop controller becomes more sensitive to variations in DC current as the droop gain increases. This is also the reason why the power error increases due to DC current measurement errors as the droop gain increases.
A current error function was proposed in [6] which effectively operates the I-V droop controller with a very high droop gain to minimise power flow error. However, in [16] it was recommended not to employ the current error function when one or more inverters are operating in power control. This is due to the potential for multiple characteristic intersections and ambiguous operating conditions.

<table>
<thead>
<tr>
<th>Error Type</th>
<th>VSC1</th>
<th>VSC2</th>
<th>VSC3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC Voltage</td>
<td>-110.83</td>
<td>110.91</td>
<td>-135.75</td>
<td>270.49</td>
</tr>
<tr>
<td>DC Current</td>
<td>0.79</td>
<td>1.23</td>
<td>1.01</td>
<td>2.46</td>
</tr>
<tr>
<td>Admittance</td>
<td>-22.51</td>
<td>-16.66</td>
<td>20.91</td>
<td>45.16</td>
</tr>
</tbody>
</table>

D. Impact of droop gain and DC voltage measurement error

The impact of DC voltage measurement errors and droop gain has shown to be significant in this paper and therefore further investigation is warranted. For this case, the power orders are the same as in section V. A, and the error scenarios are the same as described in section V. A. 4. Figure 6 shows that the impact of the DC voltage measurement systems’ accuracy decreases as the droop gains increase, and that the power flow error is still significant when large droop gains and very accurate DC voltage measurements are employed.

VI. EMT MODEL OF THE NAWC

An EMT model based on the NAWC was implemented in [14]. In this model, the VSCs are represented using average value models based on a half-bridge MMC and the DC cables are represented using the frequency dependent phase cable model. The I-V droop controllers, DC voltage controllers and inner current controllers are modelled for each of the onshore converters, and the AC voltage magnitude and frequency controllers are modelled for the offshore VSCs.

The scenario described in section V. A. 4 will be used for this test. The simulated DC power values for each of the converters are compared with the calculated values in Table VIII. These results verify the accuracy of the PU-PFS for the NAWC with an error of less than 0.04%. This small error is mainly due to the 0.1MW error tolerance used in the PU-PFS.

<table>
<thead>
<tr>
<th>VSC1</th>
<th>VSC2</th>
<th>VSC3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated (MW)</td>
<td>783.19</td>
<td>799.79</td>
</tr>
</tbody>
</table>

In section V. A. 3, the PU-PFS was employed to solve the 243 combinations of admittance matrix error for the NAWC.

The PU-PFS solved the power flow error for all 243 error combinations in just over 200ms on a standard laptop. As a basis of comparison, the NAWC EMT model takes about 275 times longer to run a 5 second simulation on the same laptop.

VII. CONCLUSIONS

In this paper a PU-PFS was developed which enables the user to rapidly study the impact of relative measurement errors and admittance matrix errors on power flow accuracy. The developed PU-PFS was used to investigate the impact of parameter uncertainty on a potential MT system. The key findings and recommendations from this study are as follows:

- Errors in the DC voltage measurements (0.2%) have the greatest impact on power flow accuracy (>150MW)
- Power flow accuracy should be considered when designing and parameterising droop controllers
- Low power VSCs should operate in power control since they are more susceptible to relative power flow errors and their contribution to regulating the DC voltage is limited
- Very high accuracy (0.1%) measurement systems and large droop gains (20kV/kA) cannot eliminate this issue and therefore further research is required.

REFERENCES