

Analysis of pump excited state absorption and its impact on laser efficiency

W R Kerridge-Johns and M J Damzen

Photonics Group, The Blackett Laboratory, Imperial College London, London, SW7 2AZ, UK

E-mail: william.kerridge-johns10@ic.ac.uk

Abstract. Excited state absorption (ESA) is a process that occurs in many laser gain media and can significantly impact their efficiencies of operation. In this work we develop a model to quantify the effect of ESA at the pump wavelength on laser efficiency, threshold and heating. In an analysis based on the common end pumped laser geometry we derive solutions and analytical expressions that model the laser behaviour. From these solutions we discuss the main parameters affecting efficiency, such as the laser cavity loss, pump ESA cross section and stimulated emission cross section. Methodologies are described to minimise the impact of pump ESA, for example by minimising cavity loss. It is also shown that altering the pumping geometry can significantly improve performance by improved distribution of the population inversion. Double end pumping can approximately halve the effect of pump ESA compared to single end pumping, and side pumping also has the potential to arbitrarily reduce its effect.

PACS numbers: 42.55.Ah, 42.55.Xi, 42.60.Lh

Keywords: Pumping, Excited state absorption, Efficiency, Threshold

Submitted to: *Laser Phys. Lett.* and published **12** 125002 (Dec. 2015)

1. Introduction

There are many factors affecting the operational efficiency of a laser: some are intrinsic to the laser material, others to the pumping method, and others due to the cavity design. Considerable discussion and analysis has been made about these factors in the scientific literature (e.g. see Svelto [1]). Excited state absorption (ESA) can significantly affect the efficiency of a laser gain medium, but has received varying levels of investigation. ESA at the laser wavelength is well understood and theoretically derived [2, 3]. The precise effect of ESA at the pump wavelength in optically-pumped laser systems, particularly its mathematical formulation, has received less attention. Some qualitative commentary has been made on pump ESA and its negative impact on laser efficiency [4]. There have also been some more quantitative analyses on dye laser systems [5, 6] and in the rather complex Er-doped [7] and Tm-doped [8] fibre laser systems, where up-conversion and cross-relaxation processes need to be considered alongside ESA. More recent work on Ho-doped fluoride [9] and Er-doped tellurite [10] glass fibre lasers has identified pump ESA as an important process in these systems.

The purpose of this work is to consider in some detail the influence of ESA at the pump wavelength on the operational performance of an optically-pumped laser system. Our key aim at the outset is to derive a pump ESA efficiency factor. This factor elucidates the impact of pump ESA in isolation from other compounding processes and aids in developing strategies for how to diminish its adverse effect on the laser output efficiency. An additional effect of ESA is enhanced heating of the gain medium, which can significantly affect the temperature and thermally-induced lensing of the laser medium. A heating factor is derived to quantify this effect. In this work we perform an analysis of the important case of an optically end-pumped laser under steady-state lasing conditions. This case is of special interest as it is a common laser design, especially in diode end-pumped solid-state laser systems.

2. Theoretical Model

The output power, P_o , of a typical laser using simplifying, but often realistic, assumptions has the form

$$P_o = \eta_s(P - P_{th}), \quad (1)$$

where η_s is the slope efficiency, P is the pump power and P_{th} is the threshold pump power for lasing. The slope efficiency consists of various factors that affect the transfer of energy from pump radiation to laser output. For monochromatic optical pumping with ideal spatial matching to the laser mode, it can be written as

$$\eta_s = \eta_p \eta_a \eta_c \eta_q, \quad (2)$$

where η_p is the pump quantum efficiency, the probability that an absorbed pump photon generates upper laser level population; η_a is the pump absorption efficiency, the fraction of incident pump power that is absorbed; $\eta_c = -\ln R / [-\ln(1-L) - \ln R]$ is the output coupling efficiency, the ratio of output coupling to the total round trip cavity loss, where R is the output mirror reflectivity and L is the round trip loss factor; and $\eta_q = \lambda_p / \lambda_l$ is the Stokes efficiency, where λ_p and λ_l are the pump and laser wavelengths respectively.

Figure 1 shows the energy structure and transitions of the gain medium analysed. The population densities are n_i , where i is the level number. There is ground state

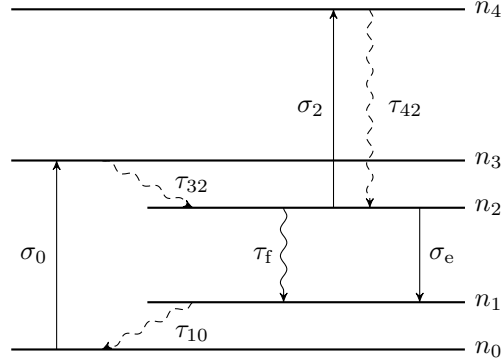


Figure 1. Diagram of the energy level structure of a four level laser, with a fifth level providing ESA at the pump wavelength.

absorption (GSA) at the pump wavelength from level 0 to level 3 with a cross section σ_0 , and stimulated emission at the laser wavelength from level 2 to level 1 with a cross section σ_e . There is also ESA at the pump wavelength, which is absorption from excited population in level 2 to a higher lying level 4 with a cross section σ_2 . Fluorescent decay from level 2 to level 1 occurs at the laser wavelength with a lifetime τ_f . Non-radiative decays are assumed to occur for all other levels with lifetimes τ_{ij} and be near instantaneous, $\tau_{ij} \rightarrow 0$, which results in $n_1 = n_3 = n_4 = 0$.

In this type of system the pump quantum efficiency may not be unity for a number of reasons. One case is where a pump photon is absorbed from the ground state into a higher level pump band, but only a fraction of this level population transfers into the upper laser level due to a finite branching ratio into other states. Another case is pump ESA, where absorption is from already excited ions so does not generate inversion. These effects can be combined as $\eta_p = \eta_{p,ESA} \eta_{p,0}$, where $\eta_{p,ESA}$ is the pump ESA quantum efficiency factor and all other mechanisms are contained in $\eta_{p,0}$. In this analysis only pump ESA will be considered, so $\eta_{p,0} = 1$ and $\eta_p = \eta_{p,ESA}$.

The gain medium is end pumped with the pump beam being non-divergent, collinear with the laser mode and having a constant transverse intensity profile. The effect of spatially varying energy level populations will be considered, which will be one dimensional and along the optical axis z , where $z = 0$ is the input of the pump beam. The coupled population rate equations for this system are

$$\frac{\partial n_2(z,t)}{\partial t} = \frac{1}{h\nu_p} \cdot \sigma_0 n_0(z,t) I(z,t) - \frac{n_2(z,t)}{\tau_f} - c\sigma_e n_2(z,t) \phi(z,t), \quad (3)$$

$$N = n_0(z,t) + n_2(z,t) = \text{const.}, \quad (4)$$

where h is the Planck constant, ν_p is the pump radiation frequency, I is the pump intensity, c is the speed of light in the medium, ϕ is the laser photon density in the medium and N is the total active ion population.

The rate equations are solved in the steady state to find a normalised population inversion distribution, $f(z) = n_2(z)/N$, given by

$$f(z) = [1 - f(z)] \frac{I(z)}{I'_s(z)}, \quad (5)$$

where $I'_s(z) = I_s[1 + \phi(z)/\phi_s]$ is a saturation factor; and $I_s = h\nu_p/(\sigma_0\tau_f)$ and $\phi_s = 1/(\tau_f c\sigma_e)$ are the pump absorption saturation intensity and laser saturation

photon density, respectively. The pump intensity distribution $I(z)$ in the medium can be found using a differential equation and solution

$$\frac{dI}{dz} = -[\sigma_0 n_0(z) + \sigma_2 n_2(z)]I(z), \quad (6)$$

$$I(z) = I(0) e^{-\alpha_0 z} e^{\alpha_0(1-\gamma)F(z)}, \quad (7)$$

respectively, where $\gamma = \sigma_2/\sigma_0$ characterises the strength of pump ESA relative to GSA; $\alpha_0 = \sigma_0 N$ is the pump small signal inverse absorption depth; and an integrated inversion factor is defined as $F(z) = \int_0^z f(z') dz'$.

Upon substitution of (7) into (5), the equation governing $f(z)$ is found to be

$$\frac{df}{dz} = -\alpha_0 f(1-f)[1 - (1-\gamma)f], \quad (8)$$

assuming that $\phi(z)$ (and therefore I'_s) is a constant, an accurate assumption for all but high gain cavities [11, 12]. Integration of (8) then yields the transcendental solution

$$D(z) = D(0)e^{-\alpha_0 z}, \quad (9)$$

$$D(z) = \begin{cases} f(1-f)^{\frac{1}{\gamma-1}} [1 - (1-\gamma)f]^{\frac{1}{\gamma}-1} & \text{for } \gamma > 0 \\ f(1-f)^{-1} \cdot \exp[(1-f)^{-1}] & \text{for } \gamma = 0. \end{cases} \quad (10)$$

The different forms of $D(z)$ arise from differing valid expansions during the derivations, as shown along with the integration method in Appendix A. These relations for the population inversion distribution are equivalent to those found by Hercher [13] and must be solved numerically.

Figure 2 shows the normalised inversion distribution $f(z)$ and the normalised pump intensity distribution $I(z)/I'_s$ for the case with ($\gamma = 1$) and without ($\gamma = 0$) pump ESA. There is stronger absorption in the gain medium with ESA, but this extra absorption does not result in greater population inversion and concentrates it more towards the input of the pump radiation.

To analyse the efficiency of pump absorption the mechanisms of ESA and GSA must be separated. GSA generates population inversion whereas ESA is a loss that

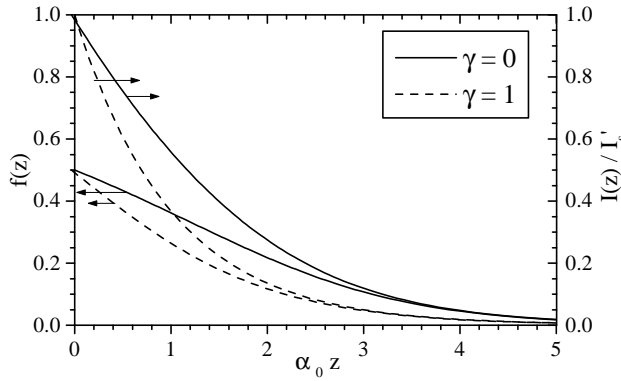


Figure 2. The normalised inversion, $f(z)$, and pump intensity, $I(z)/I'_s$, distributions with (dashed curves) and without (solid curves) pump ESA, for the case $I(0)/I'_s = 1$.

converts pump energy into heat. The total pump power absorbed, P_a , can be found from (7) giving

$$P_a = P_0 \left[1 - T_0 e^{\alpha_0(1-\gamma)F} \right], \quad (11)$$

where $P_0 = AI(0)$ is the incident pump power, A is the cross sectional area of the pump beam, $F = F(l)$, l is the length of the gain medium and $T_0 = \exp(-\alpha_0 l)$ is the small signal pump transmission.

The useful pump power absorbed, P_u , is given by

$$P_u = A \int_0^l \sigma_0 n_0(z) I(z) dz, \quad (12)$$

which comes from (6) by summing pump GSA throughout the gain medium. The integral is calculated using (5), giving $P_u = AI'_s \alpha_0 F$. Then using the absorbed pump powers defined in (11) and (12), the pump absorption fraction η_a and pump quantum efficiency η_p , the ratio of useful to total pump power absorbed, are found to be

$$\eta_a = \frac{P_a}{P_0} = 1 - T_0 e^{\alpha_0(1-\gamma)F} = 1 - T, \quad (13)$$

$$\eta_p = \frac{P_u}{P_a} = \frac{\alpha_0 F}{1 - T} \left(\frac{1}{f_0} - 1 \right), \quad (14)$$

where T is the pump transmission. The unknown quantity in (14) is $f_0 = f(0)$, which is found by calculating the integral relation for F , solved using (8). This also yields the incident pump intensity, $I_0 = I(0)$, through (5). The resulting relations are

$$\eta_p = \frac{1 - T e^{\alpha_2 F}}{1 - T} \cdot \frac{\alpha_2 F}{e^{\alpha_2 F} - 1}, \quad (15)$$

$$\frac{I_0}{I'_s} = \frac{1}{1 - T e^{\alpha_2 F}} \cdot \frac{e^{\alpha_2 F} - 1}{\gamma}, \quad (16)$$

where $\alpha_2 = \sigma_2 N$ and the method used to arrive at these equations is shown in Appendix B. Using the reasonable assumptions of a suitably long gain medium ($T \ll 1$) and not having excessive pumping ($T \exp[\alpha_2 F] \ll 1$), approximating (15) and (16) to first order in $\alpha_2 F$ gives

$$\eta_p \approx 1 - \frac{1}{2} \alpha_2 F, \quad (17)$$

$$\frac{I_0}{I'_s} \approx \alpha_0 F \left(1 + \frac{1}{2} \alpha_2 F \right). \quad (18)$$

The absorbed powers can also be used to find the heating power deposited in the gain medium. Heating comes from the quantum defect fraction of GSA and the total power absorbed from ESA. The fraction of absorbed pump power that is converted to heat, η_H , is then

$$\eta_H = \frac{(P_a - P_u) + \left(1 - \frac{\lambda_p}{\lambda_l}\right) P_u}{P_a} = 1 - \frac{\lambda_p}{\lambda_l} \eta_p. \quad (19)$$

To apply the efficiency and threshold relations to steady state lasing the integrated inversion, F , must be found. This is defined through the round trip threshold gain condition

$$R(1 - L)G_{\text{th}}^2 = 1, \quad (20)$$

where R is the output coupler reflectivity, L is the round trip intracavity loss and G_{th} is the single pass gain at threshold. In general $G = \exp(g)$, where $g = \int_0^l \sigma_e n_2 dz = \alpha_e F$ and $\alpha_e = \sigma_e N$. Rearrangement of (20) then gives

$$F_{\text{th}} = \frac{-\ln(1-L) - \ln R}{2\alpha_e}, \quad (21)$$

where F_{th} is the value of integrated inversion, F , required for laser threshold. This value of F_{th} is then used in (13), (15) and (16) to find the efficiencies (η_a , η_p) and threshold pump intensity ($I_{\text{th}} = I_{0,\text{th}}$) of lasing.

3. Analysis

Figures 3 and 4 show example solutions for the pump quantum efficiency and threshold incident pump intensity ($I'_s = I_s$ at threshold) against output coupling factor ($-\ln R$) from (15) and (16), respectively. These results show what is intuitively expected. A higher pump ESA fraction, γ , leads to a greater ESA loss that causes decreased efficiency and a higher lasing threshold. This is partly due to the increased ESA fraction leading to a correspondingly larger loss, but also due to the concentration of population inversion towards the pump input end, as shown in Figure 2, which results in the majority of pump absorption occurring over a higher population inversion.

Increasing output coupling ($-\ln R$) also increases ESA loss by requiring a higher integrated population inversion for threshold gain (F_{th}). Increasing the intracavity loss L also has this effect, see (21).

To better understand the parameters affecting the laser efficiency and threshold when pump ESA occurs, (21) can be inserted into (17) and (18), giving

$$\eta_p \approx 1 - \frac{\sigma_2}{\sigma_e} \frac{\{-\ln[(1-L)R]\}}{4}, \quad (22)$$

$$I_{\text{th}} \approx \frac{h\nu_p}{\tau_f \sigma_e} \frac{\{-\ln[(1-L)R]\}}{2} \left[1 + \frac{\sigma_2}{\sigma_e} \frac{\{-\ln[(1-L)R]\}}{4} \right], \quad (23)$$

where (18) has been evaluated at threshold conditions. These equations show the general trend of how η_p and I_{th} will be affected when changing the system parameters. Equation (22) is also relevant to pump induced heating, η_H , due to their simple relationship, see (19).

The terms involving the pump ESA cross section σ_2 in (22) and (23) explicitly show the effect of pump ESA on laser threshold and efficiency. The $(-\ln[(1-L)R])$ factor quantifies the total round trip loss of laser radiation. A larger cavity loss requires a higher threshold inversion for lasing, increasing the amount of ESA.

Reducing σ_2 results in decreased pump ESA loss due to lower ESA strength. The effect of increasing the stimulated emission cross section σ_e is also straightforward to understand. A higher gain leads to a lower threshold inversion and a resulting lower ESA loss. These changes both improve η_p and I_{th} .

It should be noted that σ_0 and N are absent from the pump quantum efficiency and threshold pump intensity of (22) and (23). The more exact formulations, (15) and (16), are also independent of σ_0 and N , assuming a suitably long gain medium and not excessive pumping (note I'_s is inversely proportional to σ_0).

In addition to considering the material cross sections and cavity loss factors, the pumping geometry can impact the effect of pump ESA; the formulation in this paper is for single end pumping. By approximating the system as having a constant

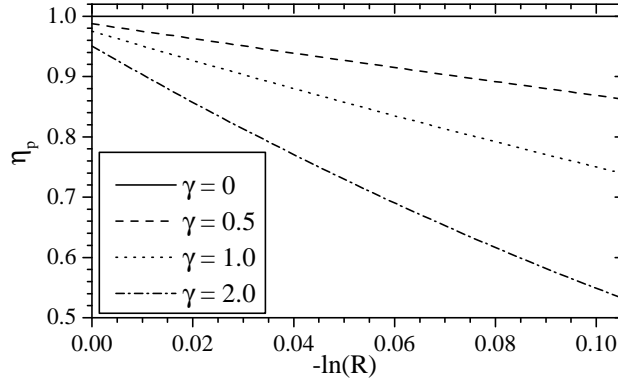


Figure 3. The pump quantum efficiency η_p against output coupling factor ($-\ln R$) for different ratios, γ , of pump ESA to GSA. Parameters: $\alpha_0 = 1000\text{m}^{-1}$, $\alpha_e = 100\text{m}^{-1}$, $L = 1\%$ and $l = 0.01\text{m}$.

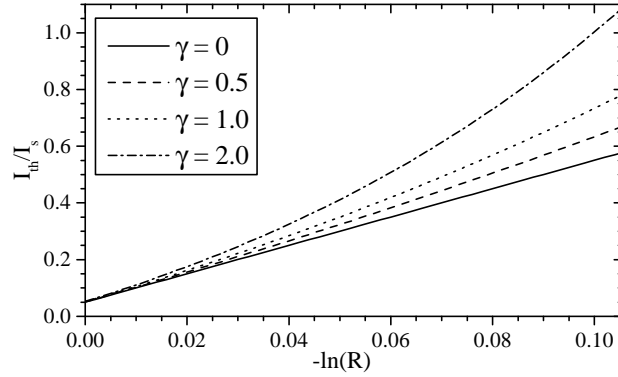


Figure 4. The threshold pump intensity, I_{th} , normalised to saturation intensity, I_s , against output coupling factor ($-\ln R$). Parameters: see Fig. 3.

population inversion fraction \bar{f} to twice the small signal absorption depth over the pumped region, the integrated inversion can be approximated as $F \approx \frac{2}{\alpha_0} \bar{f}$. When substituted into (17) the pump quantum efficiency becomes $\eta_p \approx 1 - \gamma \bar{f}$, in this form the effect of different pumping geometries can be compared. The gain condition of (20) defines the fractional inversion, \bar{f} , needed in each geometry for laser threshold, resulting in

$$\text{Single end pumping: } \eta_p \approx 1 - \gamma \bar{f}_s, \quad (24)$$

$$\text{Double end pumping: } \eta_p \approx 1 - \frac{1}{2} \gamma \bar{f}_s, \quad (25)$$

$$\text{Side pumping: } \eta_p \approx 1 - \frac{2}{\alpha_0 l} \gamma \bar{f}_s, \quad (26)$$

where \bar{f}_s is the average inversion required for single end pumping. These equations show that double end pumping requires approximately half the average inversion of single end pumping and has a corresponding increase in efficiency. Side pumping can also result in an arbitrarily large average decrease of inversion when compared to single end pumping, provided the crystal length l is long compared to the effective single end pumped length ($\approx 2/\alpha_0$).

An important descriptor of laser efficiency is the slope efficiency, η_s . To examine

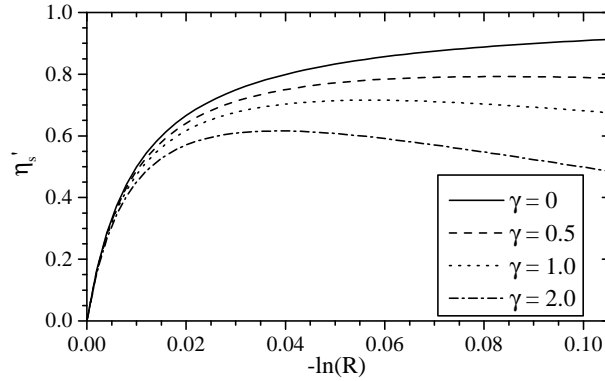


Figure 5. The reduced slope efficiency, $\eta'_s = \eta_p \eta_c$, with output coupling factor ($-\ln R$). Parameters: see Fig. 3.

the effect of output coupler reflectivity on η_s , it is useful to define a reduced slope efficiency of $\eta'_s = \eta_p \eta_c$, where $\eta_c = -\ln R / [-\ln(1-L) - \ln R]$. Figure 5 shows the reduced slope efficiency as a function of output coupling ($-\ln R$) and the impact of changing the strength of pump ESA (γ). With no ESA, $\gamma = 0$ and $\eta_p = 1$, η'_s has the typical trend of a 4-level laser. When ESA is present, $\gamma > 0$ and $\eta_p < 1$, there becomes an optimum R for maximum slope efficiency that increases with increasing γ . The cavity loss L becomes a doubly important design parameter for systems with pump ESA, with regards to slope efficiency, as it reduces both η_p and η_c .

The trend of slope efficiency with output coupling in Figure 5 has been seen in end-pumped Alexandrite lasers [14, 15], which have pump ESA [16]. In these investigations it was suggested that this could be due to energy transfer upconversion - a process that had not been identified for Alexandrite in the literature. However, the presented analysis suggests that this behaviour could be caused by the effect of pump ESA.

4. Conclusion

This work has presented an analytical formulation of the pump ESA loss in a laser gain medium, which quantifies its effects on laser threshold and pump quantum efficiency, applied to an end pumped CW laser. The resulting equations have identified the important material and cavity parameters that impact laser efficiency with pump ESA. Altering the pumping geometry is a proposed mechanism to reduce pump ESA, with the suggested approaches of double end or side pumping, where the efficiency improvement comes from reducing the average inversion in the medium.

Acknowledgments

One of the authors (WKJ) acknowledges support from an Imperial College PhD Scholarship.

Appendix A. Population Inversion Profile

The form of the population inversion distribution is found through the integration of (8),

$$\frac{df}{dz} = -\alpha_0 f(1-f)[1 - (1-\gamma)f], \quad (\text{A.1})$$

$$\int_{f(0)}^{f(z)} \frac{1}{f'(1-f')[1 - (1-\gamma)f']} df' = -\alpha_0 \int_0^z dz', \quad (\text{A.2})$$

where the integral is computed from the input of the pump beam, $z = 0$, up to position z in the medium. The left hand side integral is found through partial fraction decomposition. The function

$$h(f) = \frac{1}{f(1-f)[1 - (1-\gamma)f]} \quad (\text{A.3})$$

is decomposed as

$$h(f) = \begin{cases} \frac{1}{f} + \frac{1}{\gamma(1-f)} - \frac{(1-\gamma)^2}{\gamma[1 - (1-\gamma)f]} & \text{for } \gamma > 1 \\ \frac{1}{f} + \frac{1}{1-f} + \frac{1}{(1-f)^2} & \text{for } \gamma = 0. \end{cases} \quad (\text{A.4})$$

When these results are substituted into (A.2), the integral is simple to find. The different valid expansions of $h(f)$ gives rise to the two forms of $D(z)$ in (9).

Appendix B. Integrated Population Inversion

To eliminate f_0 from (14) the integral

$$F(z) = \int_0^z f(z') dz' \quad (\text{B.1})$$

is calculated. The differential relation for df/dz , (8), is used to change the integration variable to f , giving

$$F(z) = -\frac{1}{\alpha_0} \int_{f_0}^{f(z)} \frac{1}{(1-f')[1 + (\gamma-1)f']} df' \quad (\text{B.2})$$

$$= -\frac{1}{\alpha_0} \int_{f_0}^{f(z)} \left[\frac{1}{\gamma(1-f)} + \frac{\gamma-1}{\gamma[1 + (\gamma-1)f]} \right] df \quad (\text{B.3})$$

$$= -\frac{1}{\alpha_0 \gamma} \left[\ln \left(1 + \frac{\gamma f}{1-f} \right) \right]_{f_0}^{f(z)}, \quad (\text{B.4})$$

which upon rearrangement yields

$$\frac{f_0}{1-f_0} = \frac{1}{\gamma} \left[\left(1 + \frac{\gamma f(z)}{1-f(z)} \right) e^{\alpha_2 F(z)} - 1 \right]. \quad (\text{B.5})$$

Then using the relation of (5),

$$\frac{f(z)}{1-f(z)} = \frac{I(z)}{I_s'}, \quad (\text{B.6})$$

and the transmission of the gain medium up to position z , $T(z)$, given from (7) as

$$T(z) = \frac{I(z)}{I(0)} = e^{-\alpha_0 z} e^{\alpha_0(1-\gamma)F(z)}, \quad (\text{B.7})$$

the $f(z)$ term of (B.5) can be rewritten as

$$\frac{f(z)}{1-f(z)} = T(z) \frac{f_0}{1-f_0}. \quad (\text{B.8})$$

Substituting this into (B.5) and rearranging yields

$$\frac{1}{f_0} - 1 = \frac{\gamma[1 - T(z)e^{\alpha_2 F(z)}]}{e^{\alpha_2 F(z)} - 1}, \quad (\text{B.9})$$

which, when evaluated at $z = l$, gives the inversion of the gain medium at the pump input, f_0 , for a given set of cavity parameters. This directly gives the incident pump intensity and by substituting into (14) also gives the pump quantum efficiency.

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