An Implicit Switching Model for Distribution Network Reliability Assessment

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***Abstract*—Modern active distribution networks make use of intelligent switching actions to restore supply to end users after faults. This complicates the reliability analysis of such networks, as the number of possible switching actions grows exponentially with network size. This paper proposes an approximate reliability analysis method where switching actions are modelled implicitly. It can be used graphically as a model reduction method, and simulated using time-sequential or state sampling Monte Carlo methods. The method is illustrated on a simple distribution network, and reliability indices are reported both as averages and distributions. Large speedups result from the use of biased non-sequential Monte Carlo sampling – a method that is hard to combine with explicit switching models.**

***Index Terms*--distribution networks, reliability analysis, network topology, Monte Carlo simulations**

I. Introduction

An understanding of network reliability performance relies significantly on quantitative reliability modelling of distribution networks, because distribution networks are the source of the majority of outages that affect end users [1]. Distribution networks are undergoing significant changes with increased penetration of distributed generation, flexible demand and new monitoring and automation technologies, and their adoption is further affected by changes in transmission networks and market arrangements [2]. Developing realistic future network scenarios thus necessitates rapidly assessing the reliability performance for a large range of parameters and network configurations. This in turn requires the use of reliability assessment methods that are both flexible and efficient.

Reliability modelling of complex distribution systems has been extensively discussed in [3]. The minimal cut-set technique is a common method employed for system simplification, and failure mode effect analysis (FMEA) is well developed for evaluating the impact of specific failure modes. It is pointed out in [2] that FMEA requires the development of a complete table of failure modes with their probability and the corresponding reliability impact. FMEA has been used for network reliability evaluation in [1], [3], [4].

Whereas FMEA and other analytical methods typically take a passive view of the network, realistic distribution networks take a more active approach to fault management. For maximising distribution network reliability, system protection and restoration actions including failure isolation, network rerouting, load shedding and restoration are achieved by coordination of circuit breakers (CBs), sectionalising switches and normally open points (NOPs). The active operation of networks therefore requires real time decision making with the objective to improve reliability for end users.

A particular computational challenge stems from the range of discrete switching actions available to network operators, including control of normally open points, fault isolation and restoration and load shedding. In [5], the (near) optimal post-fault network configuration is first identified by applying load acceptance and load transfer algorithms and then a switching synthesis algorithm is employed to create (near) optimal switching sequences. In general, the optimal allocation and control of switches results in mixed integer optimisation problems (see e.g. [6]), which result in a significant computational burden especially for large and increasingly controllable networks.

In this paper we propose a simplified model for distribution network reliability analysis. Its defining feature is the implicit incorporation of switching actions instead of direct control of switches in the network. This is done by splitting and portioning the network into sets of components that are separated by switches (normally closed or normally open) or circuit breakers. These sets are represented by nodes, connected by links where switchable components are located.

When a fault occurs it propagates to the nearest enclosing circuit breakers or NOPs. After a characteristic switching time it is assumed that network switches are operated to locally isolate the fault, converting the affected node to a non-conducting node. Remaining nodes are assumed to be supplied if a conductive path to a grid supply point exists, even if it passes through a NOP (closing it is implicit).

The resulting reliability model is approximate, but has the following advantageous properties: 1. Complex distribution network composition and topologies can be represented using the graphical method in a simplified fashion whilst switching operations such as failure isolation, network rerouting, restoration can still be (approximately) modelled; 2. The impact of a given state does not depend on its history. This property enables the use of state sampling Monte Carlo schemes and associated variance reduction schemes (e.g. importance sampling); 3. Network flow constraints and load shedding can be embedded using a simple linear optimisation.

II. Distribution Network Reliability Modelling

UK distribution networks are composed of EHV, HV and LV voltage levels. EHV is mainly used for the national transmission network and meshed distribution network, and residential end users are supplied at the LV level. This paper focuses on the intermediate HV level at which most protection and restoration actions take place [1].

An example of a radially operated HV distribution network is shown in Fig. 1. The HV network is connected to an EHV network through a primary substation which is composed of bus-bars, 33-11kV transformers and circuit breakers. At the 11kV level, the substation is connected to feeders equipped with a protection circuit breaker (indicated by a cross). Two feeders (F1 and F2) are connected in this example but more feeders can be supplied by the same substation. Feeder sections can be overhead lines or underground cables depending on local requirements. Sections are equipped with sectionalising switches (diagonal lines) at one or both ends. The LV network is represented as load points (L*x*) in this example, connecting to the HV network via an 11-0.4 kV distribution transformer and a circuit breaker or fuse for protection. Although the network is radially distributed, a normally open point (NOP; a circuit breaker or switch) is deployed for alternative connection when needed.

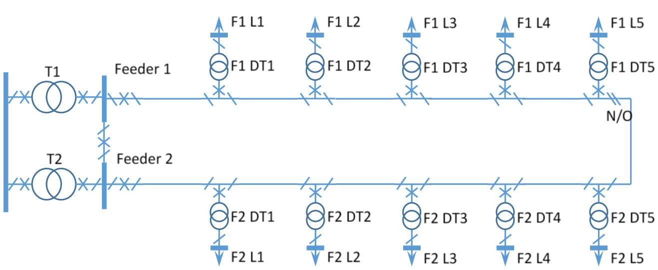

Figure 1 An illustrative HV distribution network

Reliability analysis of such a network must consider a number of possible failure modes. When a 33-11kV transformer (T1 or T2) fails, the circuit breakers/switchgears isolate the transformer so that other parts of the network will not be affected. These transformers usually satisfy the N-1 criterion so that one transformer is adequate to supply the peak demand of network.

When a short circuit fault occurs in HV lines or cables, the corresponding fault clearing device, usually the circuit breaker connecting the substation, will trip the downstream branch instantly without interrupting upstream or other branches. This fault clearing action may disconnect an entire feeder, so a switching action is required to restore the power supply to as many customers as possible. In this example network, the 11kV network is operated as a radial network with a normally open circuit breaker (NOP) that connects different branches for back-feeding during an outage. Furthermore, all network lines/cables are equipped with normally closed switchgears at both sides. When the fault location is identified, a switching action is performed. First, the failed line/cable is isolated by opening the nearest sectionalising switches (upstream *and* downstream). Second, the affected downstream load points can then be resupplied by closing the NOP to the adjacent branch. At the same time, the upstream circuit breaker can be reclosed to supply upstream load points. At the LV level, a circuit breaker or fuse serves to disconnect the load point from the HV network. This way, the HV network is not affected by faults of the LV transformer or LV network.

There are a number of challenges for quantitative reliability analysis of distribution networks. Analytical methods are not well-suited to analyse multi-step processes, such as fault-restoration sequences, or duration-dependent interruption costs. Furthermore, in complex networks the number of possible switching actions grows exponentially, and the optimal sequence of switching actions often depends on historical decisions. Modelling this in detail requires running a simulation with an embedded mixed integer optimisation problem for switching actions, which is computationally very demanding.

III. Implicit Switching Model

A. Features of the proposed model

We propose a simplified reliability analysis model that qualitatively captures the ability to reroute power using switches, but does not require explicit computation of the switching actions. The model is based on the following observations and assumptions:

- Connected components between switches and circuit breakers are always in the same electrical state. We label such an aggregation an ‘electrical node’. It is similar to the concept of a ‘section’ in [6].
- When a short circuit fault occurs within a node, the fault propagates to all connected electrical nodes, until it is stopped by a circuit breaker, NOP or isolated network section. The affected nodes are immediately disconnected from the electricity supply.
- After a fault occurs, there is a characteristic switching time before the fault is diagnosed and switching actions are initiated. These consist of node isolation (opening sectionalising switches) and restoration (closing CBs and NOPs). Switches are operated simultaneously.
- It is assumed that NOPs and sectionalising switches are intelligently controlled so that *if* a node can be supplied then the power to the node will be restored.
- The operation of CBs, NOPs and switches is assumed to be 100% reliable.
- Lack of available capacity due to network constraints does not prevent switching, but results in load curtailment so that the constraint is satisfied.

B. *Network composition and operation modelling*

1) *Node model*

System components and the associated switches are aggregated into electrical nodes. The fault state of each node is modelled as a four state Markov process shown in Fig. 2 and the associated transition rates:

a) "Up state": the component is working

b) "Fault clearing state": the component is faulty; the fault has been cleared by opening the corresponding feeder circuit breaker, and therefore also affects neighbouring nodes.

c) "Repair state": switching action has been taken to isolate the component for repair. This allows neigbouring nodes to be resupplied if possible.

d) "Maintenance state": the component is in scheduled service and it is isolated.

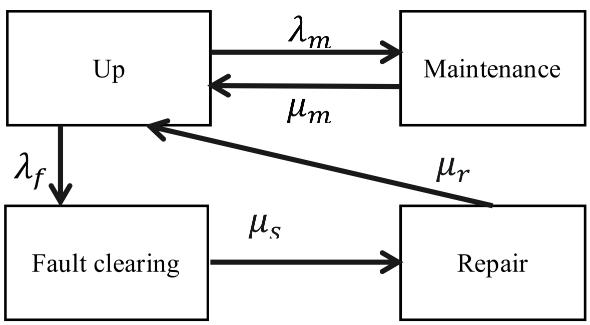

Figure 2 Node state in Markov model

2) *Network model conversion*

In order to use the implicit switching model for reliability analysis, distribution network models must be expressed in a graph representation with four node types:

1. Electrical nodes with fallible components, as described above. Their reliability parameters depend on the physical components they represent. In the case of transformers, it may be convenient to embed circuit breakers in this component, thus effectively skipping the 'fault clearing' state in Fig. 2.
2. Supply nodes that represent the EHV network supply points.
3. Load nodes that represent end users (the LV network).
4. Circuit breaker / NOP nodes that arrest faults on the network.

Fig. 3 depicts the graph representation of the example network in Fig. 1. Different node colours are used to indicate that the underlying components have different reliability parameters. Arcs represent logic linkage of the network topology. The following steps are taken to translate a real network into its corresponding node representation.

a) Network data requisition: The physical network is described in terms of its components (with their attributes, including reliability parameters) and their connections.

b) Construct full node + link network: Convert the component data into a graph, where the nodes are physical components and logical links represent their connections.

c) Merge components into electrical nodes: Identify electrical components that have no intermediary switches/NOPs/CBs and merge them into electrical nodes. For the purpose of the model a node operates as a single component, so its constitutent reliability parameters, i.e. length, failure rate, switching time, repair time, should be aggregated. Load points at this step are not aggregated with other network components.

d) Remove switches: At this step, the graph consists of circuit breakers/NOPs/sectionalising switches, supply nodes, load points and aggregated electrical nodes. Sectionalising switches are then removed because their actions are implicit in the electrical nodes. NOPs are modelled the same as circuit breakers that work as fault clearing devices (which can trip network instantly). Circuit breakers connected to transformers can also be removed since transformers are assumed to be isolated immediately after a fault happens without affecting other parts of the network (i.e. they have no 'fault clearing' state).

e) Complete connections: After removing switches, the physical connections once linking back-to-back switches remain. These may remain as no-action nodes (cannot fail, only serve to connect other nodes) or replaced by links (see for example the triangular motifs in Fig. 3).

3) *Node status modelling*

The electrical status of a node is a dynamic property that is affected by the fault state of the node itself, and that of other nodes. There are four possible states:

a) "Supplied": The node is not faulty and a live route from this node to a power source exists.

b) "Interrupted": The component at this node is affected by an active fault that caused a circuit breaker to interrupt the power supply. This happens if the node itself or a connected node is in the 'fault clearing' state.

c) "Isolated": The component node has experienced a fault and is being repaired. In practice, this usually results from switchgear at the ends of the component being opened. In this state, the node interrupts power flow, but does not otherwise affect flows in the network, thus allowing neighbouring nodes to be reconnected using load transfer via a normally open point if a live route to a power source exists.

d) "Unsupplied": The node has no live route to a power source, and is therefore unsupplied.

The electrical state of a node is determined as follows from the node fault states and the network topology. Network searches are performed using a depth-first network searching algorithm [7].

1. Tag all nodes as *unsupplied*.
2. Tag all nodes that are in the "repair state" or "maintenance state" as *isolated*.

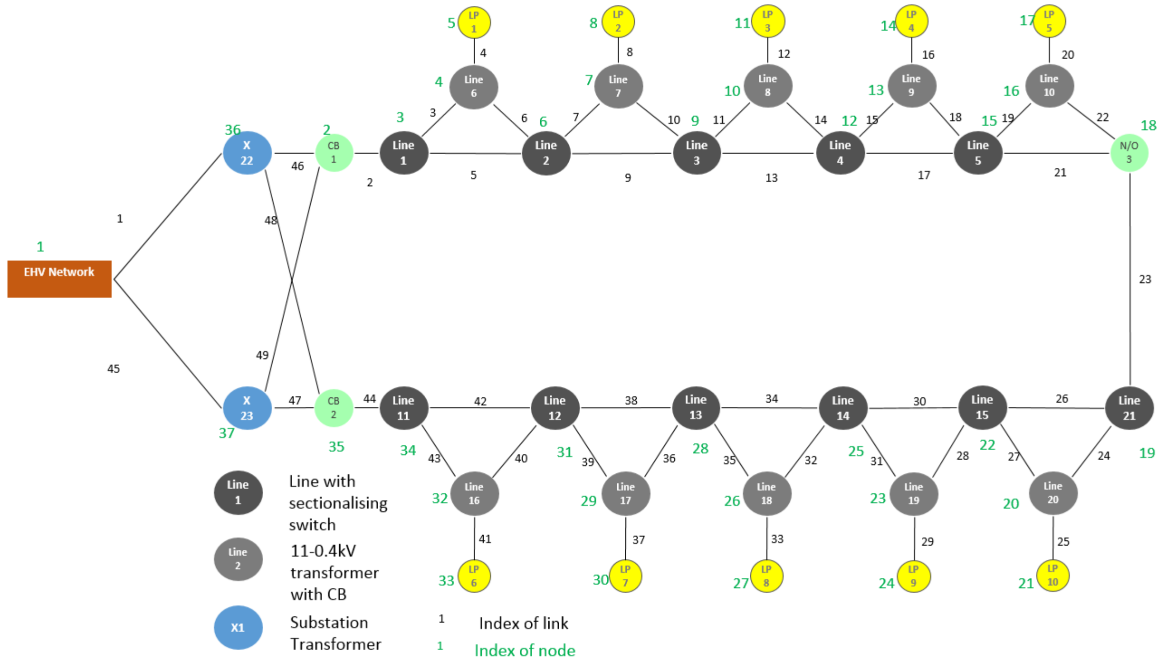

Figure 3 Graph representation of HV network for reliability assessment

3. Tag all nodes that are in the "fault clearing state" as *interrupted*.

4. From each *interrupted* node, iteratively search and tag all connected nodes as *interrupted* until an *isolated* node, or a circuit breaker/NOP node is encountered.

5. From each node power supply node, iteratively search and tag all connected node as *supplied* until an *isolated* or *interrupted* node is encountered.

4) Network contraints

Capacity constraints for lines, circuit breakers and transformers need be considered in planning and operation of the distribution network. When there is a fault, switching actions may happen to restore interrupted customers that could potentially be resupplied by other network power sources. In this situation, the capacity constraints for system components may limit the system restoration ability. In a model where switching actions are explicitly considered, this may result in a decision not to restore power to a section of the network. In our *implicit* switching approach, we instead curtail demand in order to satisfy capacity constraints.

The constraint-driven load curtailment can be formulated as a linear optimisation problem. The input parameters are:

L_i load level at node *i*

f_k^{max} flow constraint between nodes connected by link *k*

π_{ik} directed incidence matrix of node *i* and link *k*: 1 if out from node *i*; -1 if towards node *i*; otherwise 0

The optimisation objective is to minimise load curtailment:

$$\min_{\{c_i, f_k\}} \sum_i c_i \cdot L_i$$

subject to the constraints

$$-f_k^{max} \le f_k \le f_k^{max}, \qquad \forall k$$
$$c_i \cdot L_i - \sum_k \pi_{ik} \cdot f_k = L_i, \qquad \forall i$$

where f_k is the power flow between the nodes connected by link *k*. c_i represents the fraction of curtailed demand at node *i*. In a passive distribution network, load points can be disconnected by opening circuit breaker/switch at the LV transformer when a power shortage happens. In that case, $c_i \in \{0, 1\}$ are binary variables indicating the interruption of load points. With the development of active network technologies, flexible demand control could be used to reduce the load in smaller steps. For those smart networks, c_i is continuous between 0 and 1 – allowing for reduced curtailment and faster computation.

IV. Monte Carlo Simulation

The reliability of the model introduced in the previous section is analysed using Monte Carlo (MC) sampling. We discuss both state based (non-sequential) and time-sequential MC sampling.

1) Non-sequential Monte Carlo Simulation (NSMCS)

With the proposed network switching model, network switching actions are implicit and the electrical status of nodes does not depend on the history of the system. This feature enables the application of non-sequential MC simulation. The reliability indices are estimated as follows:

$$\hat{E}(H) = \frac{1}{N} \sum_{i=1}^{N} H(X_i) \tag{1}$$

where H is the estimation function of a reliability index such as energy not supplied (ENS); N is the number of simulated system states; X_i represents a sampled system state which includes the fault states of all components in the network according to their Markov model and the load profile for each load point.

Network components such as line sections and transformers are usually very reliable. This means that unbiased sampling of states will be very inefficient, as most sampled states will have no components in the fault state – and will therefore not contribute to the result. To improve simulation computational efficiency, one of the variance reduction techniques, Importance Sampling (IS), is applied in company with the proposed implicit switching model for a considerably faster convergence. We assume that no load is shed if all components are in the 'up' state. Therefore, we bias the sampling by forcing at least one component to be in a 'down' (i.e. not-'up') state. For each sample, one component is randomly selected according to its probability to be in the 'down' state. This component is forced to be in the 'fault clearing', 'maintenance' or 'repair' state according to their relative probabilities. All other components are sampled without bias. After reliability indices are quantified for the sampled system state, a weighting factor is used to correct the bias from the adjusted sampling distribution.

The weighting factor is derived as the ratio of the probability of a system state in original distribution and that in the adjusted distribution. For independent components, it can be shown that the relation is:

$$Pr'(X_i) = \frac{N_f(X_i)}{\sum u_k} Pr(X_i) \tag{2}$$

$Pr(X_i)$ is the probability of system state X_i in the original sampling distribution. $Pr'(X_i)$ is the probability of system state X_i in the adjusted sampling distribution. $N_f(X_i)$ is the number of 'down' components in system state X_i. $\sum u_k$ is the sum of unavailabilities (i.e. probability of being in the 'down' state) of all components. If we denote by X'_i sampled states that have been sampled according to the adjusted distribution, reliability indices can be calculated as:

$$\hat{E}(H) = \frac{1}{N} \sum_{i=1}^{N} H(X'_i) \frac{Pr(X'_i)}{Pr'(X'_i)} \tag{3}$$

2) Time sequential Monte Carlo Simulation (TSMCS)

The time-sequential Monte Carlo simulation is a method in which time dependent system operation is reproduced by sampling stochastic sequences and durations of system states. The system states are sampled according to the Markov models of the system components. By randomly sampling durations of component states, a random sequence of system states is produced. The stochastic sampling of system states for period of one year is described below:

a) Step 1: Generate the initial state of each system component according to the steady state probability distribution of its Markov model. The initial load state is generated by randomly sampling a starting time in a year and selecting the corresponding load level from the load profile.

b) Step 2: Sample the transition time from the current state to the next possible state for each component. For those components that have multiple possible transitions, choose the first transition event. The transition time for the load state is obtained by calculating the time to the next half hour boundary.

c) Step 3: List and sort all component transition times in ascending order. The set of all component states is the current system state and its duration is the shortest component transition time T_{min}. Set system simulation time as $T = T_{min}$.

d) Step 4: Identify the status for each node in the system and conduct the capacity constraint optimisation so that, at each load point, reliability indices can be computed for the current system state.

e) Step 5: Deduct the shortest transition time from all component transition times and update the component state as the next sampled state. Sample the time to the next transition for the recently switched component.

f) Step 6: Repeat steps 3-5 until the system simulation time exceeds 1 year. In step 3, set the system simulation time as $T = T + T_{min}$. If step 5 results in load point switching from *supplied* to *unsupplied* status, the counter of customer interruption events is incremented by 1; otherwise it is recognised as a continued interruption. A disconnection priority order is established to prevent spurious rotation of disconnections across load points.

g) Step 7: Evaluate and record the reliability indices of the system for this year.

The expectation value and distribution of reliability indices can be evaluated by repeating the above sampling for N independent years. Using confidence intervals or the coefficient of variation, the convergence of simulation result is monitored, which may be used as a stopping criterion.

V. Case Study

The proposed implicit switching model is applied using non-sequential and sequential MCS in different distribution networks to test its accuracy, efficiency and applicability.

A. The illustrative HV network

The illustrative HV network is shown in Fig. 1, and in reduced form in Fig. 3. The network consists of two branches, each with five load points connected through distribution transformers and line sections. Each line is equipped with sectionalising switches at both ends. An NOP is employed to connect the ends of both branches as an alternative supply route. The network parameters are given in Table I.

Table I Parameters of The Illustrative Network

Parameters	Values
Failure rate for lines	0.2 occ./km.year
Failure rate for transformers	0.006 occ./year
Maintenance rate for primary transformer	0.2occ./year
Switching time	30 min
MTTR for lines	24 hours
MTTR for primary transformers	299 hours
MTTR for distribution transformers	24 hours
Maintenance restoration time for primary transformer	24 hours
Line section length	0.25 km
Loading level	N-1 and N-0

Each load point is assumed to connect 500 customers, with a peak demand of 500kW, or 2.5MW per feeder. A normalised UK load profile with 17520 levels for each half hour is used. For line sections and 33-11kV transformers, a capacity constraint of 5MW and 2.5MW is applied for N-1 (regular utilisation, with redundancy at peak load) and N-0 (full utilisation at peak load), respectively.

With the proposed implicit switching model, the impact of a given state does not depend on the history. This property enables the use of state sampling Monte Carlo schemes and associated variance reduction schemes. In Table II, a comparison study for the illustrative HV network and N-0 loading level is taken for testing the computational efficiency of different simulation methods. It is clear that the computations with discrete load shedding (columns 3-4) is generally slower due to the use of binary variables in the optimisation. The resulting EENS values are also higher than those corresponding to 'smart' systems (continuous c_i). Furthermore, for the same coefficient of variation (CoV) of 1%, applying importance sampling reduces the convergence time to 140s, which is only 0.2% that of conventional NSMCS. TSMCS in this case is still faster than NSMCS since the time sequence sampling also "forces" the next state after an "all good state" to be a state with fault, not the same as the current state. But the convergence speed is significantly restricted by the half-hourly load profile: the simulation must update the load level each half hour.

TABLE II COMPARISON OF COMPUTATIONAL EFFICIENCY FOR DIFFERENT MONTE CARLO SIMULATIONS

Computation time	Continuous c_i		Discrete c_i		CoV
	EENS (MWh/y)	Time (s)	EENS (MWh/y)	Time (s)	
NSMCS	4.112	78978	5.228	108541	1%
NSMCS+IS	4.101	140	5.148	204	1%
TSMCS	4.143	7114	5.239	12139	1%

TABLE III EENS FOR DIFFERENT HV NETWORK LINE FAILURE RATE AND LOADING LEVELS

Network EENS (MWh/y)	Line Failure Rate (occ/y.km)	N-1	N-0
Section length 0.25km	2%	0.04 / 0.04	0.41 / 0.58
	5%	0.11 / 0.10	1.00 / 1.38
	10%	0.22 / 0.22	2.04 / 2.76
	20%	0.44 / 0.44	4.14 / 4.96

TABLE IV EXPECTED CUSTOMER INTERRUPTION (ROUNDED) FOR DIFFERENT HV NETWORK LINE FAILURE RATE AND LOADING LEVELS

Network ECI (occ./100cust./y)	Line Failure Rate (occ/y.km)	N-1	N-0
Section length 0.25km	2%	3 / 3	3 / 3
	5%	7 / 7	7 / 7
	10%	14 / 14	15 / 15
	20%	28 / 28	28 / 28

TABLE V EXPECTED CUSTOMER MINUTE LOST (ROUNDED) FOR DIFFERENT HV NETWORK LINE FAILURE RATE AND LOADING LEVELS

Network ECML (min/y)	Line Failure Rate (occ/y.km)	N-1	N-0
Section length 0.25km	2%	1 / 1	7 / 9
	5%	2 / 2	16 / 22
	10%	4 / 4	33 / 44
	20%	9 / 9	66 / 80

In the UK, distribution network reliability performance is reviewed by regulatory authority OFGEM with three main indices: Energy Not Supplied (ENS) [used implicitly for the P2 distribution reliability standard], Customer Interruption (CI) and Customer Minute Lost (CML). Tables III-V show the expected values of these 3 indices with different failure rate and loading levels using TSMCS (1% coefficient of variation). Results are given in pairs (A/B) for both active (continuous c_i) and passive (discrete c_i) networks. The higher N-0 loading level results in a significant increase in ECML and EENS compared to the N-1 case, but the frequency of interruptions (ECI) is unaffected.

TABLE VI EENS COMPOSITION

Network EENS (MWh/y)	N-1	N-0
Fault clearing	0.43	0.43
Thermal constraint	0.00	3.71
Single failure	0.43	4.13
Double overlapping failure	0.01	0.01

It is worth noting that the implicit switching model also enables the recognition of different types of failures in the network. Table VI shows the EENS composition (for the active network with continuous c_i) for the case with a line failure rate of 0.2occ/year.km. At the N-0 loading level, EENS from fault clearing is 0.43MWh/y, similar to that of N-1, for the outages that occur when a circuit breaker trips the whole feeder. EENS from thermal constraints is the load curtailment after switching actions when the alternative network capacity is not able to fully supply the restored areas. The result shows that, at the N-0 loading level, thermal constraints are the main source of undelivered energy to customers. Table VI also breaks down the contributions caused by single and overlapping failures, for system planners to check the network performance of rare overlapping failures.

The proposed method can be used to obtain probability distributions of network reliability indices, although this does requires the use of a sequential method (TSMCS). We present an example for the case where the network feeder capacity conforms with N-0. Fig. 4 shows the complementary CDF distribution of annual ENS for various cable failure rates.

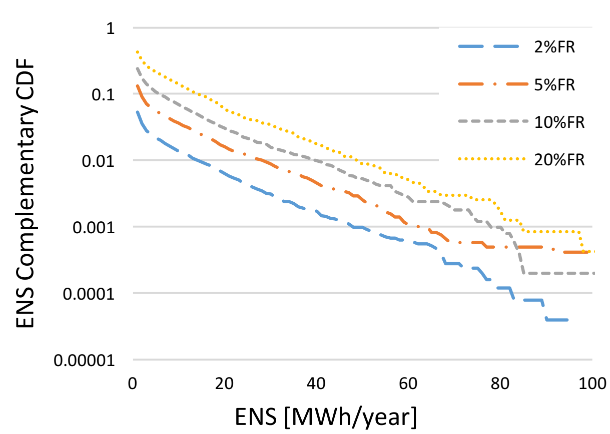

Figure 4 CCDF of annual ENS for failure rate of 2%, 5%, 10%, 20%/km.year

B. *RBTS Bus 4 network*

A second case study was carried out on the well-known distribution network RBTS Bus 4 [8]. Its implicit switching representation is shown in Fig 5. Active load shedding (continuous c_i) has been assumed for all calculations, and computed reliability indices are listed in Table VII for two scenarios, labelled 'N-1' and 'N-0'. The capacity limit of each feeder line is equal to the peak demand of all load points in the associated branch for 'N-0' and double of that for 'N-1'. The half-hourly load profile is applied instead of the average data in [8]. Table VIII compares the time required using different simulation approaches to compute the ENS with a coefficient of variation of 1% for the 'N-0' scenario. Mirroring the results for the smaller network, the importance sampling variant of the non-sequential method is vastly more efficient than both other methods.

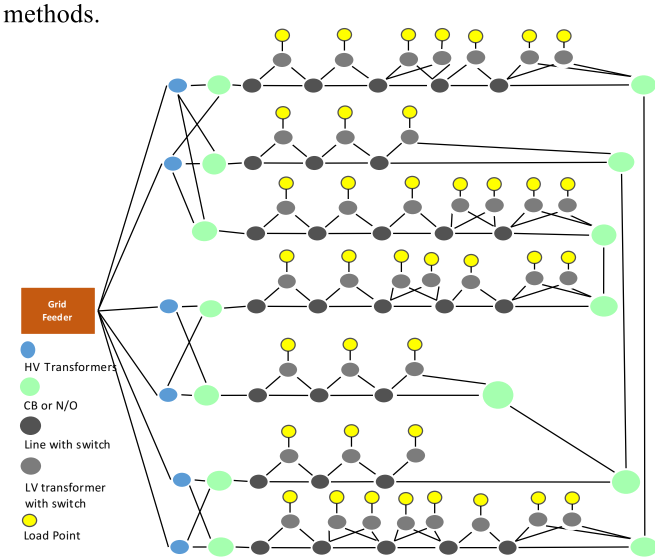

Figure 5 The graphical representation for RBTS bus 4

TABLE VII RELIABILITY PERFORMANCE OF RBTS BUS 4

Reliability indices	N-1	N-0
EENS (MWh/y)	11.5	16.5
ECI (occ./100cust./y)	57.5	56.7
ECML (min/cust./y)	31.2	43.2

TABLE VIII COMPARISON OF COMPUTATIONAL EFFICIENCY FOR DIFFERENT MONTE CARLO SIMULATIONS FOR RBTS BUS 4

Computational Efficiency	EENS (MWh/y)	Time (s)	Coefficient of Variation
NSMCS	16.33	57615	1%
NSMCS+IS	16.52	118	1%
TSMCS	16.29	1900	1%

VI. CONCLUSIONS

We have introduced a simplified model for the reliability analysis of active distribution networks. The model captures the qualitative benefits of restoration by switching, but foregoes explicit computation of switching actions. Instead, a simplified *implicit switching* approach is used to approximate the fault clearing, isolation and restoration processes. In addition, power flow constraints can be assigned to network bottlenecks, potentially limiting restorative power flow adjustments. Although the implicit switching model is based on a number of approximations, these are gradually becoming less artificial as future networks become smarter and deploy technologies such as soft open points and demand response.

The approximations greatly simplify the analysis and – among other things – enable a 'snapshot' analysis of network states that only depends on the current state of network components. This snapshot analysis forms the basis of a non-sequential Monte Carlo technique. In combination with importance sampling approach, very large speedups were obtained, versus sequential simulations and – especially – unbiased non-sequential simulations. This suggests that the implicit switching model may be used for very fast, but approximate, analysis of complex distribution networks. In addition, we demonstrated that the model can be used with time-sequential simulation to obtain distributions of reliability indices.

Thanks to the simplicity of the proposed switching model, various active network technologies can be modelled in an efficient way. Future studies include the extension of the linear optimisation considering distributed generation, storage and responsive demand in the system. Furthermore, in combination with time-sequential Monte Carlo simulations the method can be used to analyse customer interruption costs with non-linear customer damage functions.

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