Graphene, plasmons and transformation optics.
Supplementary material

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A. Graphene's conductivity

The conductivity of graphene depends on frequency, ω, chemical potential of the sheet, μ, and temperature, T (we use $T = 300$ K throughout this paper). It consists of a sum of intraband and interband contributions, $\sigma = \sigma_{\text{intra}} + \sigma_{\text{inter}}$, which can be written in the random phase approximation [2] as follows,

$$\sigma_{\text{intra}} = \frac{2ie^2t}{\hbar\pi\left[\Omega + i\gamma\right]} \ln\left[2\cosh\left(\frac{1}{2t}\right)\right], \tag{S.1}$$

$$\sigma_{\text{inter}} = \frac{e^2}{4\hbar}\left[\frac{1}{2} + \frac{1}{\pi}\text{arctan}\left(\frac{\Omega - 2}{2t}\right) - \frac{i}{2\pi}\ln\frac{(\Omega + 2)^2}{(\Omega - 2)^2 + (2t)^2}\right]. \tag{S.2}$$

where we have used a normalized frequency, $\Omega = \hbar\omega/\mu$, and temperature $t = k_BT/\mu$. In addition, we have introduced a damping term, $\gamma = \hbar/(\mu\tau)$, where $\tau = m\mu/v_F^2$ is the carriers' scattering time (m is the mobility and v_F the Fermi velocity).

In the main text we have used a mobility $m = 10^6$, which at $\mu = 1$ eV corresponds to a scattering time of $\tau = 10$ ps. Figure S2 shows the absorption peaks for two conductivity gratings of parameters $w_0 = 1.5$ (a) and $w_0 = 2.5$ (b) for $\tau = 10$ ps and for a more realistic value of the scattering losses in graphene, $\tau = 1$ ps. For the latter case the lower order plasmon peaks are still visible despite the broadening of the absorption peaks due to losses. The result for an infinite scattering time is also shown for reference (dashed line).

B. Fourier expansion coefficients of the coordinate transformation

The conformal transformation used in the main text,

$$w = \gamma \log\left(\frac{1}{e^z - iw_0} + iy_o\right), \tag{S.3}$$

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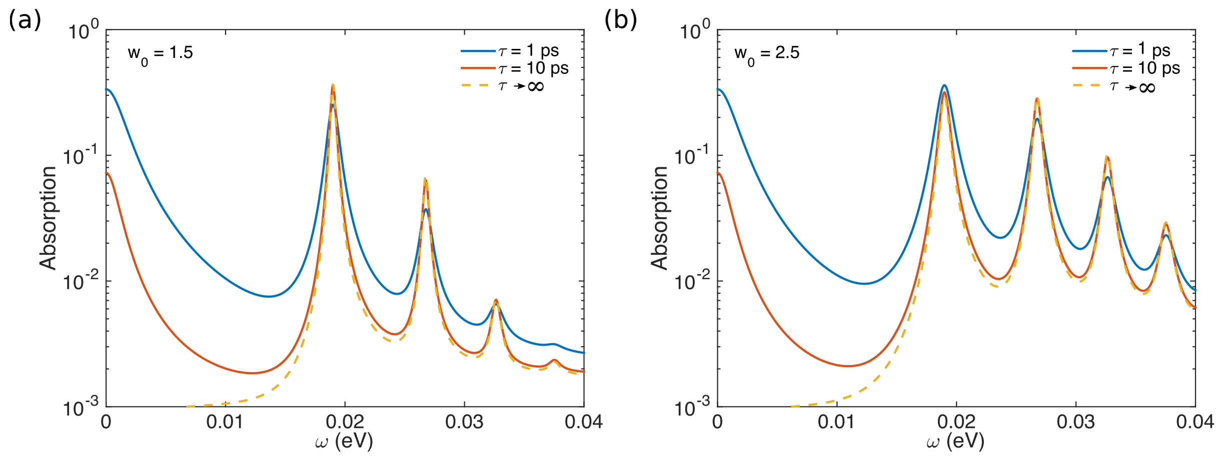

FIG. S1. Absorption spectrum at normal incidence for different grating modulation strengths, (a) $w_0 = 1.5$ and (b) $w_0 = 2.5$, and for different values of the scattering times for the electrons in graphene.

with $z = x + iy$ and $w = u + iv$, can be written as a Fourier series [1],

$$u = \gamma \left[\log(1 + y_0 w_0) - x + \sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} \left(d_g^+ e^{|g|x} + d_g^- e^{-|g|x} \right) e^{igy} \right] \tag{S.4}$$

$$v = \gamma \left[-y + \sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} \left(h_g^+ e^{|g|x} + h_g^- e^{-|g|x} \right) e^{igy} \right], \tag{S.5}$$

with the following expansion coefficients,

$$d_g^+ = \frac{e^{-i\pi g/2}}{2|g|} \frac{-1}{(w_0 + 1/y_0)^{|g|}} \tag{S.6}$$

$$d_g^- = \frac{e^{-i\pi g/2}}{2|g|} w_0^{|g|} \tag{S.7}$$

$$h_g^+ = \frac{e^{-i\pi g/2}}{2|g|} \frac{i\text{sign}(g)}{(w_0 + 1/y_0)^{|g|}} \tag{S.8}$$

$$h_g^- = \frac{e^{-i\pi g/2}}{2|g|} i\text{sign}(g) w_0^{|g|} \tag{S.9}$$

provided that x is between the branch points at $x = \log(w_0)$ and $x = \log(w_0 + 1/y_0)$.

C. Conductivity profile

An infinitely thin graphene sheet of conductivity σ_g can be modelled as a thin layer of thickness δ_0 and permittivity,

$$\epsilon_g = 1 + i\frac{\sigma_g}{\omega\epsilon_0\delta_0} \,. \tag{S.10}$$

Under the transformation (S.3) the conductivity of the thin sheet transforms as,

$$\sigma(v) = \sigma_g\frac{\delta(v)}{\delta_0\gamma} \,, \tag{S.11}$$

where δ_0 is the scaling factor and $\delta(v)$ is the inhomogeneous thickness of the graphene in the grating frame. An analytical expression for $\delta(v)$ and hence for the conductivity profile can be obtained by using the Fourier expansion of the transformation.

Let us assume that the right boundary of the graphene sheet in the slab frame is placed at $x = b$, and its left boundary at $x = b - \delta_0$. The formula for u above allows us to calculate the thickness $\delta(y)$ in the grating frame,

$$\begin{aligned}\delta(y) &= |u(b) - u(b-\delta_0)| \qquad\qquad (\text{S.12})\\ &= \gamma\left|-b + \sum_{\substack{g=-\infty\\g\neq 0}}^{\infty}\left(d_g^+ e^{|g|b} + d_g^- e^{-|g|b}\right)e^{igy} + (b-\delta_0) - \sum_{\substack{g=-\infty\\g\neq 0}}^{\infty}\left(d_g^+ e^{|g|b-\delta_0} + d_g^- e^{-|g|b-\delta_0}\right)e^{igy}\right|\end{aligned}$$

Using only the first order Fourier term,

$$\begin{aligned}\delta(y) &\approx \gamma\left|-\delta_0 + \left(d_1^+e^b + d_1^-e^{-b}\right)e^{iy} + \left(d_{-1}^+e^b + d_{-1}^-e^{-b}\right)e^{-iy}\right.\\ &\quad \left. - \left(d_1^+e^{b-\delta_0} + d_1^-e^{-b+\delta_0}\right)e^{iy} - \left(d_{-1}^+e^{b-\delta_0} + d_{-1}^-e^{-b+\delta_0}\right)e^{-iy}\right|\\ &= \gamma\left|-\delta_0 + \left(d_1^+e^b(1-e^{-\delta_0}) + d_1^-e^{-b}(1-e^{\delta_0})\right)\left(e^{iy} - e^{-iy}\right)\right|\\ &= \gamma\left|-\delta_0 + 2i\left(d_1^+e^b(1-e^{-\delta_0}) + d_1^-e^{-b}(1-e^{\delta_0})\right)\sin(y)\right|\end{aligned} \tag{S.13}$$

where we have used $d_1^+ = \frac{-i}{2}\frac{-1}{(w_0+1/y_0)}$, $d_{-1}^+ = \frac{i}{2}\frac{-1}{(w_0+1/y_0)} = -d_1^+$, $d_1^- = \frac{-i}{2}w_0$ and $d_{-1}^- = \frac{i}{2}w_0 = -d_1^-$. For $\delta_0 \ll 1$,

$$\delta(y) \approx \gamma\delta_0\left|-1 + 2i\left(d_1^+e^b - d_1^-e^{-b}\right)\sin(y)\right| \,. \tag{S.14}$$

Finally, in order to obtain $\delta(v)$ we need an analytical expression for $y(v)$. To zeroth order we have $y = -v/\gamma$ and, up to an additive constant, A, we can write an explicit expression for the sinusoidal dependence of $\delta(v)$

$$\delta(v) \approx \gamma\delta_0\left[A + 2i\left(d_1^+e^b - d_1^-e^{-b}\right)\sin(v/\gamma)\right] \,, \tag{S.15}$$

which is accurate for moderate modulations $w_0 \lesssim 2$. Then from Eq. S.11 we get that the conductivity in the transformed frame is given by

$$\sigma(v) = \sigma_g \left[A + 2i\left(d_1^+ e^b - d_1^- e^{-b}\right)\sin(v/\gamma)\right] \tag{S.16}$$

D. Fourier components of the gratings

Here we provide the numerical Fourier coefficients of the modulation profile of the conductivity grating, as well as of the geometric relief of the dielectric grating. In both cases, the modulation can be expanded in Fourier terms as

$$f(v) = a_0 + \sum_n \left[a_n \sin(\omega v) + b_n \cos(\omega v)\right] \tag{S.17}$$

The relative weight of the coefficients a_n and b_n with respect to a_1 is shown in Fig. S1 for the weak and strong modulations. The top panel corresponds to the conductivity grating and the bottom panel to the dielectric grating.

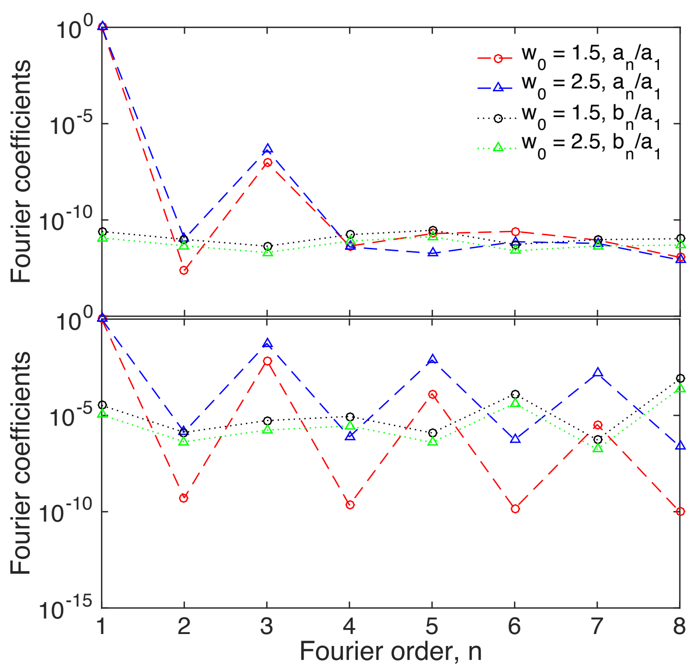

FIG. S2. Relative weight of Fourier coefficients a_n and b_n for Fourier components n ranging from 0 to 8. Top panel: conductivity grating profile. Bottom panel: dielectric grating relief profile.

E. Optical response under plane wave illumination: conductivity grating

Here we summarize the main steps needed for the derivation of the reflection and transmission coefficients. The derivation closely follows that of Ref. [1], where a more detailed account can be found. Here we provide details of the main differences with the metal grating case treated in that reference.

Making use of the Fourier expansion of the coordinate transformation given by Eq. S.4, the incident plane wave can be written in the original frame as

$$\mathbf{H^{sou}} \approx \frac{\omega\epsilon\epsilon_0}{k} E^{sou}(1 - iku)\mathbf{z}$$

$$= -\frac{\omega\epsilon\epsilon_0}{k} E^{sou} \left(1 - ik\gamma \left[\log(1 + y_0 w_0) - x + \sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} \left(d_g^+ e^{|g|x} + d_g^- e^{-|g|x} \right) e^{igy} \right] \right) \mathbf{z} \quad \text{(S.18)}$$

The source electrostatic potential then reads

$$\phi^{sou} = E^{sou}\gamma \left[\sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} i\text{sign}(g) \left(d_g^+ e^{|g|x} - d_g^- e^{-|g|x} \right) e^{igy} + y \right] \quad \text{(S.19)}$$

1. Potential in the original space

We now focus on the conductivity grating [see Fig. 1(a) in the main text]. In the illumination region the electrostatic potential has the form $\phi_L = \phi^{sou} + \phi_L^{near} + \phi_L^{rad}$, with

$$\phi_L^{near} = \sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} b_g^{sca} e^{|g|x} e^{igy} \quad \text{(S.20)}$$

$$\phi_L^{rad} = E^{ref}\gamma \left[\sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} i\text{sign}(g) \left(d_g^+ e^{|g|x} - d_g^- e^{-|g|x} \right) e^{igy} + y \right] \quad \text{(S.21)}$$

In the substrate region, $\phi_R = \phi_R^{near} + \phi_R^{rad}$, with

$$\phi_R^{near} = \sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} e_g^{sca} e^{-|g|x} e^{igy} \quad \text{(S.22)}$$

$$\phi_R^{rad} = E^{tra}\gamma \left[\sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} i\text{sign}(g) \left(d_g^+ e^{|g|x} - d_g^- e^{-|g|x} \right) e^{igy} + y \right] \quad \text{(S.23)}$$

From the potential we derive the fields according to $E_y = -\frac{\partial\phi}{\partial y}$ and $D_x = -\epsilon\epsilon_0\frac{\partial\phi}{\partial x}$, and impose the boundary conditions at $x = b$ in the slab frame, $E_{y,L} = E_{y,R}$ and $D_{x,R} - D_{x,L} = \Sigma$. We find the surface charge density, Σ, from the continuity equation,

$$\partial_t\Sigma + \nabla\mathbf{j} = 0 \;\rightarrow\; -i\omega\Sigma + \nabla\mathbf{j} = 0 \tag{S.24}$$

where the surface current density is $\mathbf{j} = j_y\mathbf{y} = \sigma_g E_y\mathbf{y}$. Hence,

$$\Sigma = \frac{\sigma}{i\omega}\frac{\partial E_{y,L}}{\partial(\gamma y)} \tag{S.25}$$

where the dimension factor, γ, ensures that the surface charge density has the right units.

The application of the boundary conditions yields the following system

$$\begin{aligned}
&E^{sou} + E^{ref} = \\
&= E^{tra}, && \text{(S.26)}\\
&(E^{sou} + E^{ref})\gamma i\mathrm{sign}(g)\left(d_g^+ e^{|g|b} - d_g^- e^{-|g|b}\right) + b_g^{sca}e^{|g|b} = \\
&= E^{tra}\gamma i\mathrm{sign}(g)\left(d_g^+ e^{|g|b} - d_g^- e^{-|g|b}\right) + e_g^{sca}e^{-|g|b}\,, && \text{(S.27)}\\
&-\epsilon_1 E^{tra}\gamma i\mathrm{sign}(g)\left(d_g^+ e^{|g|b} + d_g^- e^{-|g|b}\right) + \epsilon_1 e_g^{sca}e^{-|g|b} + \\
&+\epsilon_2(E^{sou} + E^{ref})\gamma i\mathrm{sign}(g)\left(d_g^+ e^{|g|b} + d_g^- e^{-|g|b}\right) + \epsilon_2 b_g^{sca}e^{|g|b} = \\
&= \frac{\sigma}{i\omega\epsilon_0\gamma}|g|\left[(E^{sou} + E^{ref})\gamma i\mathrm{sign}(g)\left(d_g^+ e^{|g|b} - d_g^- e^{-|g|b}\right) + b_g^{sca}e^{|g|b}\right] && \text{(S.28)}
\end{aligned}$$

which has one equation too few to unambiguously determine all the coefficients.

Taking E^{sou} and E^{ref} as known, we write

$$\begin{pmatrix} e^{|g|b} & -e^{|g|b} \\ \left(\epsilon_d - \frac{\sigma}{i\omega\epsilon_0\gamma}|g|\right)e^{|g|b} & \epsilon_1 e^{-|g|b} \end{pmatrix}\begin{pmatrix} b_g^{sca} \\ e_g^{sca}\end{pmatrix} = \begin{pmatrix} 0 \\ B_g \end{pmatrix} \tag{S.29}$$

where

$$\begin{aligned}
B_g = {} & \gamma i\mathrm{sign}(g)(E^{sou} + E^{ref})\,\cdot \\
& \left\{\frac{\sigma}{i\omega\epsilon_0\gamma}|g|\left(d_g^+ e^{|g|b} - d_g^- e^{-|g|b}\right) + (\epsilon_1 - \epsilon_2)\left(d_g^+ e^{|g|b} + d_g^- e^{-|g|b}\right)\right\}
\end{aligned} \tag{S.30}$$

Solving the system yields

$$b_g^{sca} = \frac{i\gamma\epsilon_0\omega B_g e^{-|g|b}}{i\gamma\epsilon_0\epsilon_1\omega + i\gamma\epsilon_0\epsilon_2\omega - \sigma|g|} \tag{S.31}$$

$$e_g^{sca} = \frac{i\gamma\epsilon_0\omega B_g e^{|g|b}}{i\gamma\epsilon_0\epsilon_1\omega + i\gamma\epsilon_0\epsilon_2\omega - \sigma|g|} \tag{S.32}$$

2. Current

We can obtain the missing boundary condition from the tangential component of the magnetic field in the grating frame. The graphene acts as a thin sheet with surface current density $\mathbf{j}^c$

$$\nabla \times \mathbf{H} = \mathbf{j}^c \tag{S.33}$$

where $\mathbf{j}^c = J_v/(2\pi\gamma^2)$ is the total conduction current density in the transformed frame. Now, we need to determine the total current along the vertical direction,

$$J_v = \int \mathrm{d}u \int \mathrm{d}v \, j_v \tag{S.34}$$

The current transforms as,

$$j_v = \frac{1}{\det(\Lambda)} \left(\frac{\partial v}{\partial x} j_x + \frac{\partial v}{\partial y} j_y \right) \tag{S.35}$$

where

$$\Lambda = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial y} & 0 \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{S.36}$$

So we have

$$J_v = \int \mathrm{d}x \int \mathrm{d}y \, \det(\Lambda) \frac{1}{\det(\Lambda)} \left(\frac{\partial v}{\partial x} j_x + \frac{\partial v}{\partial y} j_y \right) = \int_{-\infty}^{\infty} \mathrm{d}x \int_0^{2\pi} \mathrm{d}y \, \left(\frac{\partial v}{\partial x} j_x + \frac{\partial v}{\partial y} j_y \right) \tag{S.37}$$

where the partial derivatives can be evaluated from the coordinate transformation,

$$\frac{\partial v}{\partial x} = \gamma \left[\sum_{\substack{g=-\infty \\ g \neq 0}}^{\infty} |g| \left(h_g^+ e^{|g|x} - h_g^- e^{-|g|x} \right) e^{igy} \right] \tag{S.38}$$

$$\frac{\partial v}{\partial y} = \gamma \left[-1 + \sum_{\substack{g=-\infty \\ g \neq 0}}^{\infty} ig \left(h_g^+ e^{|g|x} + h_g^- e^{-|g|x} \right) e^{igy} \right] \tag{S.39}$$

and the current flowing along the graphene in the original frame is

$$J_y^c = \sigma_g \delta[x - b] E_y \,. \tag{S.40}$$

Carrying out the integrals in Eq. S.37 and using

$$E_{y,R} = E^{tra}\gamma\left[\sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} |g|\left(d_g^+ e^{|g|x} - d_g^- e^{-|g|x}\right)e^{igy} - 1\right] - \sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} ige_g^{sca}e^{-|g|x}e^{igy} \tag{S.41}$$

yields a total conduction current density given by

$$j^c = \frac{\sigma}{\gamma}\left[E^{tra}\gamma + \sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} |g|^2\left(E^{tra}\gamma i\text{sign}(g)(h_g^+ d_{-g}^+ e^{2|g|b} - h_g^- d_{-g}^- e^{-2|g|b} - h_g^+ d_{-g}^- + h_g^- d_{-g}^+)\right.\right.$$
$$\left.\left. - \left(h_g^+ + h_g^- e^{-2|g|b}\right)e_{-g}^{sca}\right)\right] \tag{S.42}$$

This can be re-written as

$$j^c = \sigma E^{tra} N \tag{S.43}$$

$$N = 1 + \sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} |g|^2\left[i\text{sign}(g)(h_g^+ d_{-g}^+ e^{2|g|b} - h_g^- d_{-g}^- e^{-2|g|b} - h_g^+ d_{-g}^- + h_g^- d_{-g}^+)\right.$$
$$\left. - \left(h_g^+ + h_g^- e^{-2|g|b}\right)e_{2,-g}^{sca}\right] \tag{S.44}$$

where

$$e_{2,g}^{sca} = \frac{e_g^{sca}}{E^{tra}\gamma} \tag{S.45}$$

3. Reflectance and transmittance

Applying the radiation boundary condition with the current calculated above,

$$H_z^{tra} - H_z^{sou} - H_z^{ref} = j^c\,. \tag{S.46}$$

yields

$$\epsilon_0 c\left[\sqrt{\epsilon_2}E^{sou} - \sqrt{\epsilon_2}E^{ref} - \sqrt{\epsilon_1}E^{tra}\right] = j^c = \sigma E^{tra} N \tag{S.47}$$

Using $E^{tra} = E^{ref} + E^{sou}$ we find

$$E^{sou}\left(\sqrt{\epsilon_2} - \sqrt{\epsilon_1} - \frac{\sigma}{\epsilon_0 c}N\right) = E^{ref}\left(\sqrt{\epsilon_2} + \sqrt{\epsilon_1} + \frac{\sigma}{\epsilon_0 c}N\right) \tag{S.48}$$

Hence

$$r = \frac{\sqrt{\epsilon_2} - \sqrt{\epsilon_1} - 2\alpha N}{\sqrt{\epsilon_2} + \sqrt{\epsilon_1} + 2\alpha N} \tag{S.49}$$

$$t = \frac{2\sqrt{\epsilon_2}}{\sqrt{\epsilon_2} + \sqrt{\epsilon_1} + 2\alpha N} \tag{S.50}$$

where we have introduced a normalized conductivity, α, such that

$$\frac{\sigma}{\epsilon_0 c} = 2\alpha \tag{S.51}$$

Finally, transmittance and reflectance are given by

$$R = |r|^2 \tag{S.52}$$
$$T = \frac{\sqrt{\epsilon_1}}{\sqrt{\epsilon_2}}|t|^2 \tag{S.53}$$

Absorption can be obtained from

$$Q = 1 - R - T\,. \tag{S.54}$$

frame

F. Conductivity and dielectric grating

Here we consider the conductivity and dielectric grating depicted in Fig. 1(c) in the main text. For simplicity we take a symmetric environment: $\epsilon_1 = \epsilon_3 = 1$.

1. *Dispersion relation*

The dispersion relation of surface plasmons in a system composed of a graphene sheet and a dielectric slab of permittivity ϵ_d and thickness d can be derived by writing the fields in each region of space and applying the appropriate boundary conditions. This yields

$$\begin{aligned} &e^{2ik_{z,2}d}(\epsilon_3 k_{z,2} - \epsilon_d k_{z,3})(\epsilon_1 k_{z,2} - \epsilon_d k_{z,1} + 2\alpha k_{z,1}k_{z,2}) \\ &\quad = -(\epsilon_d k_{z,3} + \epsilon_3 k_{z,2})(\epsilon_1 k_{z,2} + \epsilon_d k_{z,1} + 2\alpha k_{z,1}k_{z,2}) \end{aligned} \tag{S.55}$$

where $k_{z,j} = \sqrt{\epsilon_j k_0^2 - k^2}$. In the quasistatic limit, $k_{z,j} \approx ik$, and for a symmetric environment it is straightforward to show that the expression above reduces to,

$$e^{2|k|d} = \left(\frac{\epsilon_d - 1}{\epsilon_d + 1}\right)\frac{(\epsilon_d - 1)k_0 - 2i\alpha|k|}{(\epsilon_d + 1)k_0 + 2i\alpha|k|}\,. \tag{S.56}$$

2. Potential in the original space

In the illumination region, the electrostatic potential in the original frame is $\phi_L = \phi^{sou} + \phi_L^{near} + \phi_L^{rad}$, where ϕ^{sou} is given by Eq. S.19 and

$$\phi_L^{near} = \sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} b_g^{sca} e^{|g|x} e^{igy} \tag{S.57}$$

$$\phi_L^{rad} = E^{ref}\gamma \left[\sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} i\text{sign}(g) \left(d_g^{+} e^{|g|x} - d_g^{-} e^{-|g|x} \right) e^{igy} + y \right] \tag{S.58}$$

Within the dielectric slab we have, $\phi_M = \phi_M^{near}$,

$$\phi_M = \sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} \left(c_g^{+} e^{|g|x} + c_g^{-} e^{-|g|x} \right) e^{igy} + E_0^{v}\gamma y \tag{S.59}$$

In the substrate side, $\phi_R = \phi_R^{near} + \phi_R^{rad}$, with

$$\phi_R^{near} = \sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} e_g^{sca} e^{-|g|x} e^{igy} \tag{S.60}$$

$$\phi_R^{rad} = E^{tra}\gamma \left[\sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} i\text{sign}(g) \left(d_g^{+} e^{|g|x} - d_g^{-} e^{-|g|x} \right) e^{igy} + y \right] \tag{S.61}$$

The boundary conditions in the slab frame are: $E_{y,L} = E_{y,M}$ and $D_{x,L} = D_{x,M}$ at $x = x_0$; $E_{y,M} = E_{y,R}$ and $D_{x,R} - D_{x,M} = \Sigma$ at $x = b$, where the surface charge density is derived from the continuity equation. This yields the equation system

$$E^{sou} + E^{ref} = E_0^v, \tag{S.62}$$

$$E_0^v = E^{tra}, \tag{S.63}$$

$$(E^{sou} + E^{ref})\gamma \left[i\text{sign}(g) \left(d_g^+ e^{|g|x_0} - d_g^- e^{-|g|x}\right)\right] + b_g^{sca} e^{|g|x_0} =$$

$$= \left(c_g^+ e^{|g|x_0} + c_g^- e^{-|g|x_0}\right), \tag{S.64}$$

$$(E^{sou} + E^{ref})\gamma i\text{sign}(g) \left(d_g^+ e^{|g|x_0} + d_g^- e^{-|g|x_0}\right) + b_g^{sca} e^{|g|x_0} =$$

$$= \epsilon_d \left(c_g^+ e^{|g|x_0} - c_g^- e^{-|g|x_0}\right), \tag{S.65}$$

$$\left(c_g^+ e^{|g|b} + c_g^- e^{-|g|b}\right) =$$

$$= E^{tra}\gamma i\text{sign}(g) \left(d_g^+ e^{|g|b} - d_g^- e^{-|g|b}\right) + e_g^{sca} e^{-|g|b}, \tag{S.66}$$

$$-\epsilon_d\epsilon_0 E^{tra}\gamma i\text{sign}(g) \left(d_g^+ e^{|g|b} + d_g^- e^{-|g|b}\right) + \epsilon_d\epsilon_0 e_g^{sca} e^{-|g|b} + \epsilon_d\epsilon_0 \left(c_g^+ e^{|g|b} - c_g^- e^{-|g|b}\right)$$

$$= \frac{\sigma}{i\omega\gamma}|g| \left(c_g^+ e^{|g|b} + c_g^- e^{-|g|b}\right) \tag{S.67}$$

Taking E_0^v and E^{ref} as known we write

$$\begin{pmatrix} e^{|g|x_0} & -e^{|g|x_0} & -e^{-|g|x_0} & 0 \\ e^{|g|x_0} & -\epsilon_d e^{|g|x_0} & \epsilon_d e^{-|g|x_0} & 0 \\ 0 & e^{|g|b} & e^{-|g|b} & -e^{-|g|b} \\ 0 & (\epsilon_d - \frac{\sigma}{i\omega\gamma\epsilon_0}|g|)e^{|g|b} & -(\epsilon_d\epsilon_0 + \frac{\sigma}{i\omega\gamma\epsilon_0}|g|)e^{-|g|b} & \epsilon_d e^{-|g|b} \end{pmatrix} \begin{pmatrix} b_g^{sca} \\ c_g^+ \\ c_g^- \\ e_g^{sca} \end{pmatrix} = \begin{pmatrix} A_g \\ B_g \\ C_g \\ D_g \end{pmatrix} \tag{S.68}$$

where

$$A_g = -\gamma(E^{sou} + E^{ref}) \left[i\text{sign}(g) \left(d_g^+ e^{|g|x_0} - d_g^- e^{-|g|x_0}\right)\right] \tag{S.69}$$

$$B_g = -\gamma(E^{sou} + E^{ref}) \left[i\text{sign}(g) \left(d_g^+ e^{|g|x_0} + d_g^- e^{-|g|x_0}\right)\right] \tag{S.70}$$

$$C_g = \gamma E^{tra} \left[i\text{sign}(g) \left(d_g^+ e^{|g|b} - d_g^- e^{-|g|b}\right)\right] \tag{S.71}$$

$$D_g = \gamma E^{tra}\epsilon_d \left[i\text{sign}(g) \left(d_g^+ e^{|g|b} + d_g^- e^{-|g|b}\right)\right] \tag{S.72}$$

Solving the system yields

$$b_g^{sca} = \frac{e^{-|g|x_0}}{(\epsilon_d+1)\left(2\epsilon_d\epsilon_0\omega\gamma + i\sigma|g|\right)e^{2|g|d} + (\epsilon_d-1)i\sigma|g|}\cdot$$
$$\Big[2\omega\gamma\epsilon_0\epsilon_d\left((A_d\epsilon_d+B_g)e^{|g|d} + \epsilon_d C_g + D_g\right)e^{|g|d}$$
$$+\left(A\epsilon_d - B_g + (A_d\epsilon_d + B_g)e^{2|g|d}\right)i\sigma|g|\Big] \tag{S.73}$$

$$c_g^{+} = \frac{e^{-|g|x_0}\left[\omega\gamma\epsilon_0(\epsilon_d+1)(\epsilon_d C_g + D_g)e^{|g|d} + (A_g - B_g)i\sigma|g|\right]}{(\epsilon_d+1)\left(2\epsilon_d\epsilon_0\omega\gamma + i\sigma|g|\right)e^{2|g|d} + (\epsilon_d-1)i\sigma|g|} \tag{S.74}$$

$$c_g^{-} = \frac{e^{|g|(d+x_0)}\left[\omega\gamma\epsilon_0(\epsilon_d-1)(\epsilon_d C_g + D_g) - (A_g - B_g)(2\omega\gamma\epsilon_d\epsilon_0 + i\sigma|g|)e^{|g|d}\right]}{(\epsilon_d+1)\left(2\epsilon_d\epsilon_0\omega\gamma + i\sigma|g|\right)e^{2|g|d} + (\epsilon_d-1)i\sigma|g|} \tag{S.75}$$

$$e_g^{sca} = \frac{e^{|g|b}}{(\epsilon_d+1)\left(2\epsilon_d\epsilon_0\omega\gamma + i\sigma|g|\right)e^{2|g|d} + (\epsilon_d-1)i\sigma|g|}\cdot$$
$$\Big[-2\omega\gamma\epsilon_d\epsilon_0(A_g - B_g)e^{|g|d} + (\epsilon_d+1)\left(\omega\gamma\epsilon_0(D_g - \epsilon_d C_g) - C_g i\sigma|g|\right)e^{2|g|d}$$
$$+(\epsilon_d-1)\left(\omega\gamma\epsilon_0(D_g + \epsilon_d C_g) - C_g i\sigma|g|\right)\Big] \tag{S.76}$$

3. Current

Two contributions to the current need to be taken into account in this case: the conduction current along the graphene sheet and the displacement current within the dielectric slab.

Conduction current. The current flowing along the graphene sheet in the original frame is

$$J_y^c = \sigma_g \delta[x-b]E_y\,. \tag{S.77}$$

Making use of

$$E_y = -\sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} ig\left(c_g^+ e^{|g|x} + c_g^- e^{-|g|x}\right)e^{igy} - E_0^v\gamma \tag{S.78}$$

and performing the integrals in Eq. S.37, we obtain the total conduction current density

$$j^c = \frac{\sigma}{\gamma}\left[E_0^v\gamma - \sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} |g|^2\left(h_g^+ c_{-g}^+ e^{2|g|b} + h_g^- c_{-g}^- e^{-2|g|b} + h_g^+ c_{-g}^- + h_g^- c_{-g}^+\right)\right] \tag{S.79}$$

which can be rewritten as

$$j^c = \sigma(E^{sou} + E^{ref})N^c \tag{S.80}$$

$$N^c = 1 - \sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} |g|^2\left(h_g^+ c_{2,-g}^+ e^{2|g|b} + h_g^- c_{2,-g}^- e^{-2|g|b} + h_g^+ c_{2,-g}^- + h_g^- c_{2,-g}^+\right) \tag{S.81}$$

where

$$c_{2,g}^{\pm} = \frac{c_g^{\pm}}{(E^{sou} + E^{ref})\gamma} \tag{S.82}$$

Displacement current. The displacement current within the dielectric slab in the original fame is

$$\mathbf{J}^D = i\omega(\epsilon_d - 1)\epsilon_0 \mathbf{E}\,. \tag{S.83}$$

We need to integrate the current over the region between the graphene and the dielectric grating,

$$J_v^D = i\omega(\epsilon_d - 1)\epsilon_0 \int_{x_0}^{b} \mathrm{d}x \int_0^{2\pi} \mathrm{d}y \left[\frac{\partial v}{\partial x} E_x + \frac{\partial v}{\partial y} E_y\right]\,. \tag{S.84}$$

Making use of the fields

$$E_{x,M} = -\sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} |g| \left(c_g^+ e^{|g|x} - c_g^- e^{-|g|x}\right) e^{igy} \tag{S.85}$$

$$E_{y,M} = -\left[\sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} ig \left(c_g^+ e^{|g|x} + c_g^- e^{-|g|x}\right) e^{igy} + E_0^v \gamma\right] \tag{S.86}$$

we arrive to the total displacement current density

$$j^D = i\omega(\epsilon_d - 1)\epsilon_0 \gamma E_0^v N^D \tag{S.87}$$

$$N^D = -d + \sum_{\substack{g=-\infty \\ g\neq 0}}^{\infty} |g| \left[h_g^+ c_{2,-g}^+ \left(e^{2|g|b} - e^{2|g|x_0}\right) - h_g^- c_{2,-g}^- \left(e^{-2|g|b} - e^{-2|g|x_0}\right)\right] \tag{S.88}$$

4. Reflectance and transmittance

The radiation boundary condition in this case reads as,

$$H_z^{tra} - H_z^{sou} - H_z^{ref} = j^c + j^D \tag{S.89}$$

$$\epsilon_0 c \left(E^{sou} - E^{ref} - E^{tra}\right) = \sigma(E^{sou} + E^{ref})N^c + i\omega(\epsilon_d - 1)\epsilon_0\gamma(E^{sou} + E^{ref})N^D \tag{S.90}$$

Using $E^{tra} = E^{ref} + E^{sou}$ we find

$$E^{sou}(\sigma N^c + i\omega(\epsilon_d - 1)\epsilon_0\gamma N^D) = -E^{ref}\left(2\epsilon_0 c + \sigma N^c + i\omega(\epsilon_d - 1)\epsilon_0\gamma N^D\right) \tag{S.91}$$

Hence

$$r = -\frac{2\alpha N^c + ik_0(\epsilon_d - 1)\gamma N^D}{2 + 2\alpha N^c + ik_0(\epsilon_d - 1)\gamma N^D}\,, \tag{S.92}$$

$$t = \frac{2}{2 + 2\alpha N^c + ik_0(\epsilon_d - 1)\gamma N^D}\,. \tag{S.93}$$

Finally, transmittance and reflectance are given by

$$R = |r|^2 \tag{S.94}$$

$$T = |t|^2 \tag{S.95}$$

[1] M. Kraft, Y. Luo, S. Maier, and J. Pendry, Physical Review X **5**, 031029 (2015).

[2] B. Wunsch, T. Stauber, F. Sols, and F. Guinea, New Journal of Physics **8**, 318 (2006).