## **Conformal transformation for nanoantennas**

 V. Pacheco-Peña<sup>1</sup>, M. Beruete<sup>1</sup>, A.I. Fernández-Domínguez<sup>2</sup>, Y. Luo<sup>3</sup>, and M. Navarro-Cía<sup>4,5\*</sup>
<sup>1</sup>Antennas Group – TERALAB, Universidad Pública de Navarra, Spain
<sup>2</sup>Departamento de Física Teórica de la Materia Condensada and Condensed Matter Physics Center (IFIMAC), Universidad Autónoma de Madrid, Spain.
<sup>3</sup>School of Electrical & Electronic Engineering, Nanyang Technological University, Singapore
<sup>4</sup>Optical and Semiconductor Devices Group, Imperial College London, United Kingdom
<sup>5</sup>Ultrafast Laser Laboratory, University College London, United Kingdom
\*corresponding author: m.navarro@imperial.ac.uk

**Abstract**-Modeling the response of nanoantennas excited by a localized emitter is extremely computational intensive. Transformation optics can ease this task. Here we show the conformal transformation required to model a plasmonic bow-tie as a parallel-plate plasmonic waveguide. The eigenmodes of a parallel-plate plasmonic waveguide can be calculated analytically, enabling the calculation of the eigenmodes and the absorption cross section of the original bow-tie via the transformation. Full-wave simulations corroborate the approach.

Originally proposed for radio-frequency and microwaves, antennas have been scaled down in size to reach infrared or even optics [1]. At optics, unfortunately, metals can no longer be treated as perfect electric conductors, and thus, modelling nanoantennas is more challenging. Analytical solutions exist for simple nanoantennas such as rods, discs and their dimer counterparts [2]–[5]. However, there is not yet an analytical solution for the widely used bow-tie nanoantennas. This forces researchers working with bow-ties nanoantennas to rely on computationally intensive full-wave methods.

Given the plasmonic response of metals, nanoantennas are deeply subwavelength at the resonance frequency. In this situation, analytical quasi-static approaches [6] can be used to model the response of them. In particular, one can use a specific conformal transformation that converts the bow-tie into an analytically solvable configuration such as the parallel-plate waveguide. Here we follow this methodology to calculate the absorption cross section of a bow-tie nanoantenna.

Let us assume that we have a bow-tie nanoantenna in the *xy* space with arm lengths of 10 nm and a gap between arms of 1 nm. Furthermore, a localized emitter is placed 1 nm away from the center of the dimer. See Fig. 1(a). By applying the following conformal transformation, the bow-tie is converted into a parallel-plate waveguide in the *x*'y' space, with the localized emitter placed at the position x'y' = (0,0):

$$z' = \ln(z) \tag{1}$$

where  $z'_{(x',y')}$  and  $z_{(x,y)}$  correspond to each point in the parallel-plate domain and bowtie antenna, respectively. The energy of the emitter couples to the eigenmodes (surface plasmons) of the parallel-plate waveguide. Given the finite physical length of the parallel-plate waveguide, the SP modes are distributed at discrete frequencies satisfying a Fabry-Perot condition, which coincide with the absorption peaks in the absorption cross section of

the bow-tie nanoantenna.

Figure 1(b) shows the analytically and numerically computed absorption cross section for a bow-tie with angle  $\theta = 20^{\circ}$  for both, vertical and horizontal dipole. Simulation where carried out using the commercial software Comsol Multiphysics. The bow-tie antenna was immersed in a box of air of 300 nm × 300 nm and a customized mesh was applied with a minimum mesh cell of 0.003 nm. Also, a mesh refinement was applied to the antenna to accurately calculate the absorption. An agreement between both approaches is evident. However, while the Comsol simulation takes ~45 minutes to give the result, the analytical method implemented in Mathematica takes less than a minute in the same computer.



Figure 1. (a) Schematic representation of the bowtie antenna (left) and the resulting parallel-plates (right) when the conformal transformation is applied. Analytical (solid lines) and simulation (dotted lines) results of the normalized absorption for a bowtie antenna with  $\theta = 20^{\circ}$  when a vertical (black) and horizontal (green) dipole is used as a local emitter.

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