Is there anything exceptional about ICT use while travelling?

Time allocation framework for and empirical insights into multitasking patterns and their well-being implications

APPENDIX: MODEL DERIVATION

Jacek Pawlak, Giovanni Circella, John Polak, Patricia Mokhtarian, Aruna Sivakumar

Content:

This document is an accompanying appendix to the paper ‘Is there anything exceptional about ICT use while travelling? Time allocation framework for and empirical insights into multitasking patterns and their well-being implications’. The purpose of the present document is to present details of derivation of time allocation framework set out thereof.

Contact: jacek.pawlak@imperial.ac.uk
The utility maximisation framework in the paper is set out in the following way:

$$\max \ U = \Omega_{WW}^{\theta_W} \left( \prod_{j=1}^{J} \theta_W^{T_{Wj}} \right) \Omega_{WW}^{\theta_W W} \left( \prod_{i=1}^{I} \theta_W^{T_{Wi}} \right) \left( \prod_{i=1}^{I} \prod_{j=1}^{J} \theta_{ij} \right) \left( \prod_{k=1}^{K} \theta_{kk} \right)$$

Subject to:

$$w \left( T_W + \sum_{i=1}^{I} \delta_{iW} T_{Wi} \right) + M - \sum_{k=1}^{K} P_k X_k \geq 0 \hspace{1cm} \rightarrow \lambda \hspace{1cm} (i)$$

$$T_W + \sum_{i=1}^{I} T_i - \tau = 0 \hspace{1cm} \rightarrow \mu \hspace{1cm} (ii)$$

$$T_{WW} + \sum_{j=1}^{J} T_{Wj} - T_W = 0 \hspace{1cm} \rightarrow \mu_W \hspace{1cm} (iii)$$

$$T_W + \sum_{j=1}^{J} T_{ij} - T_i = 0 \hspace{1cm} \forall i \in I \hspace{1cm} \rightarrow \mu_i \hspace{1cm} (iv)$$

$$T_i - T_{iMIN} \geq 0 \hspace{1cm} \forall i \in I \hspace{1cm} \rightarrow \kappa_i \hspace{1cm} (v)$$

Also possible (but let’s ignore this case):  

$$T_{ij} - T_{ijMIN} \geq 0 \hspace{1cm} \forall i \in I, \forall j \in J \hspace{1cm} \rightarrow \kappa_{ij} \hspace{1cm} (vi-a)$$

$$X_k - X_{kMIN} \geq 0 \hspace{1cm} \forall k \in K \hspace{1cm} \rightarrow \psi_i \hspace{1cm} (vii)$$

We can form an appropriate Lagrangian:

$$L = U - \lambda \left[ M_f + w \left( T_W + \sum_{i=1}^{I} \delta_{iW} T_{Wi} \right) - \sum_{k=1}^{K} P_k X_k \right] - \mu \left[ T_W + \sum_{i=1}^{I} T_i - \tau \right] +$$

$$- \mu_W \left[ T_{WW} + \sum_{j=1}^{J} T_{Wj} - T_W \right] - \sum_{i=1}^{I} \mu_i \left[ T_{Wi} + \sum_{j=1}^{J} T_{ij} - T_i \right] - \sum_{i=1}^{I} \kappa_i \left[ T_i - T_{iMIN} \right] +$$

$$- \sum_{k=1}^{K} \psi_k \left[ X_k - X_{kMIN} \right]$$

Now define:

$$\mathcal{F}$$ - set of freely chosen activities

$$\mathcal{R}$$ - set of restricted activities

with the following properties:

$$T_f > T_{fMIN} \hspace{1cm} \forall f \in \mathcal{F} \hspace{1cm} (ix)$$

$$T_r = T_{rMIN} \hspace{1cm} \forall r \in \mathcal{R} \hspace{1cm} (x)$$

$$\mathcal{F} \cup \mathcal{R} = I \hspace{1cm} (xi)$$

$$L$$ - set of freely chosen goods

$$S$$ - set of restricted goods
with the following properties:

\[ X_l > T_{lMIN} \quad \forall l \in L \]  
\[ X_s > T_{sMIN} \quad \forall s \in S \]  
\[ L \cup S = K \]  

Hence define committed, non-work time:

\[ T_R = \sum_{r=1}^{R} T_r = \sum_{j \in R} T_{jMIN} \]  

And committed expenditures:

\[ G_S = \sum_{s=1}^{S} P_s X_s - M = \sum_{k \in R} P_k X_{kMIN} - M \]  

First order conditions:

\[ \frac{dL}{dT_W} = \frac{\theta_W U}{T_W} - \lambda_W - \mu + \mu_W = 0 \]  
\[ \frac{dL}{dT_{WW}} = \frac{\theta_{WW} U}{T_{WW}} - \mu_W = 0 \]  
\[ \frac{dL}{dT_{Wj}} = \frac{\theta_{Wj} U}{T_{Wj}} - \mu_W = 0 \]  
\[ \frac{dL}{dT_{iw}} = \frac{\theta_{iw} U}{T_{iw}} - \lambda_W \delta_{iw} - \mu_i = 0 \quad \forall i \in I \]  
\[ \frac{dL}{dT_i} = \frac{\theta_i U}{T_i} - \mu + \mu_i = 0 \quad \forall i \in F \]  
\[ \frac{dL}{dT_{ij}} = \frac{\theta_{ij} U}{T_{ij}} - \mu_i = 0 \quad \forall i, j \in I, j \in J \]  
\[ \frac{dL}{dX_k} = \frac{\eta_k U}{X_k} + \lambda P_k = 0 \quad \forall k \in L \]  
\[ \frac{dL}{dX_k} = \frac{\eta_k U}{X_k} + \lambda P_k - \psi_k = 0 \quad \forall k \in S \]  
\[ \frac{dL}{d\lambda} = -\left[ M_f + \lambda W \left( T_W + \sum_{i=1}^{i=1} \delta_{iw} T_{iw} \right) - \sum_{k=1}^{k=K} P_k X_k \right] = 0 \]  
\[ \frac{dL}{d\mu} = -\left[ T_W + \sum_{i=1}^{i=1} T_i - \tau \right] = 0 \]  
\[ \frac{dL}{d\mu_W} = -\left[ T_{WW} + \sum_{j=1}^{j=j} T_{Wj} - T_W \right] = 0 \]  
\[ \frac{dL}{d\mu_i} = -\left[ T_{iw} + \sum_{j=1}^{j=j} T_{ij} - T_i \right] = 0 \quad \forall i \in I \]  
\[ \frac{dL}{d\kappa_i} = -[T_i - T_{iMIN}] = 0 \quad \forall i \in I \]
\[
\frac{dL}{d\psi_k} = -[X_k - X_{kMIN}] = 0 \quad \forall k \in K \quad (\text{xxxi})
\]

From (xvi), (xvii) and (xxvi), noting that (xvii) latter applies to all activities j:

\[
\frac{U}{\mu_W} \left( \frac{\theta_{WW} + \sum_{j=1}^{j=j} \theta_{Wj}}{T_{WW} + \sum_{j=1}^{j=j} T_{Wj}} \right) = T_W \quad (\text{xxxiii})
\]

Hence:

\[
\frac{U}{T_W} \left( \frac{\theta_{WW} + \sum_{j=1}^{j=j} \theta_{Wj}}{T_{WW} + \sum_{j=1}^{j=j} T_{Wj}} \right) = \mu_W \quad (\text{xxxiv})
\]

From (20), (21), (23)

\[
\frac{\theta_iU}{T_i} + \mu_i = \mu \quad \forall i \in F \quad (\text{xxxv})
\]

\[
\frac{\theta_{ij}U}{T_{ij}} = \mu_i \quad \forall ij, i \in I, j \in J \quad (\text{xxxvi})
\]

\[
\frac{\theta_{IW}U}{T_{IW}} = \lambda w \delta_{iw} + \mu_i \quad \forall i \in I \quad (\text{xxxvii})
\]

Assuming that \( \delta_{iw} = 0 \) for all activities \( i \), i.e. individual is not paid additional wage for work as a secondary activity enables linking (xxiv), (xxv), and while noting the presence of constraint (xxvii) to yield:

\[
\frac{U}{T_i} \left( \frac{\theta_{IW} + \sum_{j=1}^{j=j} \theta_{ij}}{T_{IW} + \sum_{j=1}^{j=j} T_{ij}} \right) = \mu_i \quad \forall i \in F \quad (\text{xxxviii})
\]

Which can be related to (xxxv):

\[
\frac{\theta_iU}{\mu} + \frac{U}{\mu} \left( \theta_{IW} + \sum_{j=1}^{j=j} \theta_{ij} \right) = T_i \quad \forall i \in F \quad (\text{xxxix})
\]

Since this is valid for all freely chosen activities, it is possible to write:

\[
\frac{U}{\mu} \sum_{i \in F} \left( \theta_i + \theta_{IW} + \sum_{j=1}^{j=j} \theta_{ij} \right) = \tau - T_W - T_R \quad (\text{xli})
\]

Which can be rearranged to yield an expression equivalent to equation 2.60 from Jara-Diaz (2007):

\[
\frac{\sum_{i \in F} (\theta_i + \theta_{IW} + \sum_{j=1}^{j=j} \theta_{ij})}{\tau - T_W - T_R} = \frac{\mu}{U} \quad (\text{xlii})
\]

Similar analysis can be conducted for freely chosen goods, using equations (xxii) and (xxiv), while also recalling the assumption \( \delta_{iw} = 0 \) :

\[
\frac{U}{\tau} \sum_{k \in L} \eta_k = \sum_{k \in L} P_k X_k = wT_W - G_S \quad (\text{xlii})
\]

Which yields the following expression, which is a counterpart of Jara-Diaz (2007) equation 2.61:

\[
\frac{\sum_{k \in L} \eta_k}{wT_W - G_S} = \frac{\lambda}{U} \quad (\text{xliii})
\]
Hence the expression of value of time as a leisure (resource value of time) can be obtained:

\[
\mu = \frac{wT_W - G_S \sum_{i \in F} (\theta_i + \theta_{iW} + \sum_{j=1}^{j=I} \theta_{ij})}{\frac{\tau - T_W - T_R}{\sum_{k \in L} \eta_k}}
\]  

(xlv)

This expression indicates that, ceteris paribus, inclusion of secondary activities in time-use consideration will lead to higher value of time as leisure, since additional utility is now obtained from participation in secondary activities.

More importantly, the derivations lead to expression for the optimal time allocated to a particular secondary activity, given the primary activity which can be obtained by manipulating equations (xxxiv), (xxxvii):

\[
T_{ij}^* = \frac{\theta_{ij}}{\theta_{iW} + \sum_{j=1}^{j=J} \theta_{ij}} T_i \quad \forall ij, i \in I, j \in J
\]  

(xlv)

The expression can be further expanded depending on whether the primary activity is freely chosen:

\[
T_{ij}^* = \frac{\theta_{ij}}{\theta_{iW} + \sum_{j=1}^{j=J} \theta_{ij}} \frac{\theta_i + \theta_{iW} + \sum_{j=1}^{j=I} \theta_{ij}}{\sum_{i \in F} (\theta_i + \theta_{iW} + \sum_{j=1}^{j=I} \theta_{ij})} (\tau - T_W - T_R) \quad \forall ij, i \in I, j \in J
\]  

(xlvi)

Or restricted (committed):

\[
T_{ij}^* = \frac{\theta_{ij}}{\theta_{iW} + \sum_{j=1}^{j=J} \theta_{ij}} T_{iMIN} \quad \forall ij, i \in R, j \in J
\]  

(xlvii)

Note that equations (xlvi) and (xlvii) conform to equations (10) and (11) from the main paper respectively.