Eighth International Conference on
ADVANCES IN STEEL STRUCTURES
Lisbon, Portugal, July 22-24, 2015

A STUDY OF IMPERFECT CYLINDRICAL STEEL TUBES UNDER GLOBAL BENDING AND VARYING SUPPORT CONDITIONS

Oluwole K. Fajuyitan*, Adam J. Sadowski* and J. Michael Rotter**

* Department of Civil and Environmental Engineering, Imperial College London, UK
e-mails: oluwole.fajuyitan13@imperial.ac.uk, a.sadowski@imperial.ac.uk

** Institute for Infrastructure and Environment, University of Edinburgh, UK
e-mail: m.rotter@ed.ac.uk

Keywords: Ovalisation; Boundary Condition; Imperfection; Linear Buckling Eigenmode; Nonlinear Buckling;

Abstract. *This paper presents the findings of a computational study into the effect of different support conditions and geometric imperfections on the nonlinear elastic stability of tubular members of varying length under global bending. Using the finite element analysis software ABAQUS, the tubular member was modelled as an isotropic thin-walled cylindrical steel shell and subjected to a uniform bending moment distribution. The classical elastic critical buckling moment, the linear bifurcation moment and the critical nonlinear buckling moment were computed and the imperfection sensitivity under the linear buckling eigenmode was examined. The study demonstrates that the support conditions at the edges have a significant effect on the predicted buckling moment for short tubes, but that this influence vanishes for longer tubes. Additionally, the influence of initial geometric imperfections in the form of the critical buckling eigenmode on the nonlinear buckling strength of the tubes appears to be strongly dependent on the length of the tube. A detrimental effect was observed for longer tubes but a neutral or mildly beneficial effect for shorter ones. Overall, tubular members under global bending do not appear to be as imperfection-sensitive as those under uniform axial compression.*

1 INTRODUCTION

Tubular members are widely used in structural applications in which the dominant loading condition is uniform global bending, key examples being chimneys, wind turbine support towers, pipelines and tubular piles. These shell structures exhibit a rich nonlinear behaviour under bending due to the coupling between cross-section ovalisation and local bifurcation buckling [1,2].

Ovalisation is a nonlinear phenomenon where the cross-section of a tubular member undergoes progressive flattening near its midspan and begins to assume an oval shape in response to an increasing meridional curvature [3,4,5,6]. Brazier [7] documented in 1927 that ovalisation reflects a progressive reduction in the bending stiffness as a result of a decreasing second moment of area of the flattening cross-section [8,9], culminating at a limiting-point instability now known as the critical ‘Brazier’ moment (Eq. (1)):

$$M_{Braz} = \frac{2\sqrt{2}}{9}\left(\frac{E\pi rt^2}{\sqrt{1-\nu^2}}\right) = 0.987\left(\frac{Ert^2}{\sqrt{1-\nu^2}}\right) \approx 1.035\,Ert^2 \qquad (1)$$

where r and t are the radius and thickness of the cylindrical tube, and E and $\nu = 0.3$ are the Young’s modulus and is the Poisson’s ratio respectively.

Numerous studies conducted since then on the ovalisation instability behaviour of long thin-walled cylindrical tubes under bending have shown remarkable agreement with the Brazier moment [6,10,11]. However, many of these studies have also shown that this critical moment is usually never reached as another type of instability (local bifurcation buckling) typically precedes the ovalisation limit point [12,13].

Ovalisation had not been detected in moderate length thin-walled cylindrical tubes under global bending, suggesting early on that the phenomenon is length-dependent [14,15,16]. However, it was only recently that Rotter *et al.* [1] documented that the absence of ovalisation in shorter tubes was as a result of the restraining power of the boundary conditions at the ends of the tube. This study also demonstrated that the geometrically nonlinear buckling behaviour of perfect elastic cylindrical tubes under global bending may be classified into four distinct length-dependent domains, termed 'short', 'medium', 'transitional' and 'long', with the clearly-defined length boundaries that depend on the relative influence of end support conditions and ovalisation on the critical buckling behaviour. This is similar to but more complex than the well-known classification of cylinders under uniform axial compression which exhibit only three length domains, namely 'short', 'medium' and 'long' [17]. The additional 'transitional' length domain under bending is a direct consequence of the ovalisation phenomenon which is not present under uniform axial compression. The buckling behaviour of 'short' cylinders was additionally found to be particularly sensitive to the nature of the restraint at the end supports, a phenomenon illustrated in this paper.

The imperfection sensitivity of cylinders under axial compression has been reasonably well documented through numerous analytical and experimental studies e.g. [2,17,18,19,20,21,22,23,24,25], and it is well understood that even minor imperfections lead to steep losses in the load-bearing capacity. By contrast, comparatively little is rigorously known on the corresponding imperfection sensitivity for cylinders under global bending, except what is often inferred without proof from the related but still very distinct load case of uniform axial compression.

The cylinder length is thought particularly likely to change the sensitivity to both the form and amplitude of imperfections under global bending due to the presence of the ovalisation, a phenomenon absent under uniform compression. The form of imperfection that leads to the most detrimental effect is clearly of key interest, and it was traditionally assumed that this was achieved by the linear buckling eigenmode (LBA eigenmode) [26]. Indeed, the European Standard on the Strength and Stability of Metal Shells [27] proposes the LBA eigenmode as the 'default' imperfection form in a computational analysis. This concept, derived from Koiter's original perturbation analysis [18] and valid without reservations for classical column buckling, remains only a guide in shell buckling because the LBA eigenmode does not necessarily always lead to the lowest possible buckling strengths and it does not relate well to more realistic imperfection forms originating from specific manufacturing processes [26,28]. Rotter [29] suggested that the 'worst' imperfection form should depend on the shell geometry and loading, though exactly what form it should take is very much open to discussion.

The aim of this paper is firstly to demonstrate the effect of varying end support conditions on the predicted length-dependent nonlinear elastic buckling strength of thin cylindrical tubes under global bending, assuming the most common 'clamped' and 'simply-supported' cases. Secondly, the study aims to briefly illustrate the possible imperfection sensitivity of such cylinders under the 'traditional' linear buckling eigenmode imperfection form.

2 NUMERICAL MODELLING USING FINITE ELEMENTS

The modelling and analyses of the tubular members was performed using the commercial finite element analysis software ABAQUS v. 6.13.2 (2013).

2.1 Model details

The cylindrical tube was modelled using four-node reduced-integration S4R shell elements, details of which are shown in Figure 1. Only a quarter of the cylinder was modelled to take advantage of its double symmetry and to enhance the computational efficiency of the analyses, acceptable where the shell structure does not undergo torsional deformations [30]. A reference node was created at the centre of the cross-section at the end and connected to the edge surface of the tube using a rigid body kinematic coupling. This reference node was allowed to displace along the meridional z-axis and to rotate about a transverse y-axis of the global coordinate system, with all other degrees of freedom restrained, while also serving as the point of application of the applied global bending moment. Further details of the numerical model may be found in [1,31].

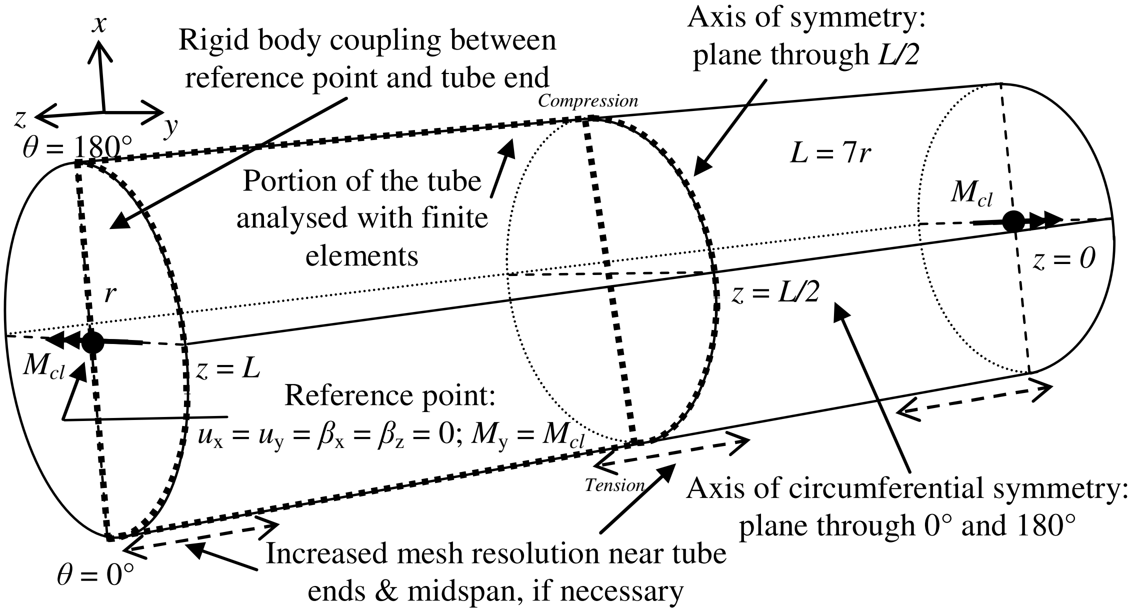

Figure 1: Illustration of the details of the numerical model, after [31]

The magnitude of the bending moment applied through the reference node was related to the classical elastic critical moment M_{cl} (Eq. (2)):

$$M_{cl} = \pi r^2 t \sigma_{cl}(r) = 1.813 \frac{Ert^2}{\sqrt{1-\nu^2}} \tag{2}$$

It should be noted that the above equation is based on the assumption of buckling occurring when the most compressed fibre reaches the classical critical buckling stress for uniform axial compression $\sigma_{cl} = 0.605Et/r$. This 'local buckling hypothesis' is made on the basis that the membrane stresses on the compressed side of the cylinder are approximately uniform over a sufficiently wide zone to support an axial compression buckle [2,15]. This hypothesis is approximately correct under a pre-buckling membrane stress state with little or no local bending deformations or ovalisation. It will be shown that this is no longer the case for either short cylinders which undergo extensive bending deformations due to the influence of the end support conditions or for long cylinders which undergo extensive pre-buckling ovalisation.

Where applicable, the mean meridional curvature φ of the cylinder was deduced from the end rotation β_y (Figure 1) and normalised by the curvature at buckling φ_{cl} as predicted by linear bending theory:

$$\varphi = \frac{2\beta_y}{L} \quad \text{and} \quad \varphi_{cl} = \frac{t}{r^2\sqrt{3\left(1-\nu^2\right)}} \approx 0.605\frac{t}{r^2} \tag{3}$$

2.2 Geometric and material properties

It was assumed that the cylinder was made of steel with a Young's modulus E of 2×10^5 N/mm^2 and a Poisson ratio ν of 0.3. Plastic behaviour was not considered in this study. The radius and thickness were taken as $r = 100$ mm and $t = 1$ mm respectively to maintain a radius to thickness (r/t) ratio of 100, typical of a 'thin' cylindrical shell. The cylinder length L was related to a dimensionless length parameter ω (Eq. (4)) which was varied from $\omega = 1$ to 1600 to cover the full set of different length domains for cylinders under global bending as identified by Rotter *et al.* [1].

$$\omega = \frac{L}{\sqrt{rt}} \tag{4}$$

2.3 End support conditions

Two different sets of conditions for the normal displacement w of the cylinder wall were investigated at the end supports at $z = 0$ and L: simply-supported and clamped. These correspond approximately to the BC1r (Eq. (5)) and BC2f (Eq. (6)) boundary conditions respectively as defined in EN 1993-1-6 [27].

- Clamped boundary condition (BC1r):

$$w = \frac{dw}{dz} = 0 \tag{5}$$

- Simply supported boundary condition (BC2f):

$$w = \frac{d^2w}{dz^2} = 0 \tag{6}$$

The simply-supported condition was modelled by allowing the nodes on the edges of the cylinder to rotate freely about the circumferential axis, while for the clamped boundary condition this rotation was restrained. It was expected that the effect of changing the support conditions would strongest within the 'short' length domain ($\omega \leq 5$). Consequently, the cylinder length was varied at very small increments of $\omega = 0.2$ in this range to better capture this effect.

2.4 Meshing details

Short cylinders of length $L \leq 6\lambda$, where λ is the linear meridional bending half-wavelength from classical shell bending theory (Eq. (7)), were expected to develop extensive bending deformations and high local meridional curvatures spanning the full length of the structure. Consequently, these short cylinders were discretised with a fine mesh of approximately square shell elements of meridional length less than $0.25\sqrt{rt}$ (i.e. approximately one tenth of λ).

Cylinders longer than this maintained the same level fine mesh resolution only within 2λ of the end support and the midspan axis of meridional symmetry where extensive local bending and buckling deformations were anticipated (Figure 1). Between these ranges the cylinder was under membrane action only and thus did not require as high a mesh resolution.

$$\lambda = 2.444\sqrt{rt} \tag{7}$$

2.5 Types of computational analyses

For each length, the perfect cylinder was subject to a linear buckling analysis (LBA) to detect the critical buckling eigenvalue λ and corresponding eigenmode. The critical buckling moment M_{cr} was then computed as (Eq. (8)):

$$M_{cr} = \lambda M_{cl} \tag{8}$$

The computed linear buckling eigenmode was then imported and applied as an imperfection in a subsequent geometrically nonlinear analysis (GNIA). The imperfection amplitudes δ were taken as 0 (perfect – GNA), 0.1, 0.25, 0.5 and 1 wall thickness. The GNA and GNIA analyses were performed using the Riks [32] modified arc length procedure, and terminated at the load proportionality factor *LPF* corresponding to the first reported negative eigenvalues of the global stiffness matrix which denoted either global limit point or local bifurcation buckling. The critical buckling moment M_k was then calculated as (Eq. (9)):

$$M_k = LPF \times M_{cl} \tag{9}$$

3 INFLUENCE OF END SUPPORT CONDITIONS ON THE LENGTH-DEPENDENT BUCKLING BEHAVIOUR OF PERFECT CYLINDERS

The end support conditions were found to play an important role in the linear buckling resistance and also in determining the location of the boundaries between the shorter length domains of the perfect cylindrical tubes under global bending.

3.1 Linear buckling behaviour (LBA analyses)

It is well known that the linear bifurcation moment M_{cr} cannot capture the true strength of the cylinder under bending because the formulation does not permit geometric nonlinearity and hence excludes the possible effects of ovalisation. However, M_{cr} is nonetheless *computed* using a full shell theory formulation and thus includes the pre-buckling bending stresses that arise due to kinematic compatibility requirements with the end support conditions (Eq. (5) or (6)), something that the *calculated* classical elastic critical buckling moment M_{cl} (Eq. (2)) does not. Further, M_{cr} plays an important role in the design of metal shells according to EN 1993-1-6 [27] as the one of the two reference resistances required to determine the physical shell slenderness, the other being the plastic limit strength (in this case the full plastic moment M_p).

The study of Rotter *et al.* [1] was the first to illustrate the length-dependent relationship between the FE-computed M_{cr} and hand-calculated M_{cl} moments for clamped end support conditions (Eq. (5)). It was found that as the cylinder became shorter ($\omega \rightarrow 0$), the restraint offered by the end supports restricts the growth of an axial compression buckle ever more severely which thus requires an ever higher M_{cr} moment in order to form. The variation of the ratio M_{cr}/M_{cl} vs ω is illustrated in Figure 2 and clearly shows that $M_{cr} / M_{cl} \rightarrow \infty$ as $\omega \rightarrow 0$. An alternative way of presenting this relationship is in terms of its inverse (Figure 3), and since now $M_{cl} / M_{cr} \rightarrow 0$ as $\omega \rightarrow 0$ the vertical axis is well-defined between 0 and 1 instead of ill-defined between 1 and ∞ as in Figure 2.

Longer cylinders are not affected by the end restraints or ovalisation, and the 'local buckling hypothesis' that $M_{cr} \approx M_{cl}$ or $M_{cr} / M_{cl} \rightarrow 1$ as $\omega \rightarrow \infty$ becomes a close approximation to the predicted linear buckling behaviour in this length range. It was furthermore proposed by Rotter *et al.* [1] that the relationship between M_{cr} and M_{cl} for the clamped case may be conservatively approximated as:

$$\frac{M_{cr}}{M_{cl}} = 1 + \frac{4}{\omega^2} \quad \text{or} \quad \frac{M_{cl}}{M_{cr}} = \frac{1}{1 + 4/\omega^2} \tag{10}$$

This fit, also illustrated in Figures 2 and 3, captures the correct behaviour qualitatively at either extreme of the length range and has now been incorporated into a new Annex E due to be published in an upcoming update to EN 1993-1-6 [27].

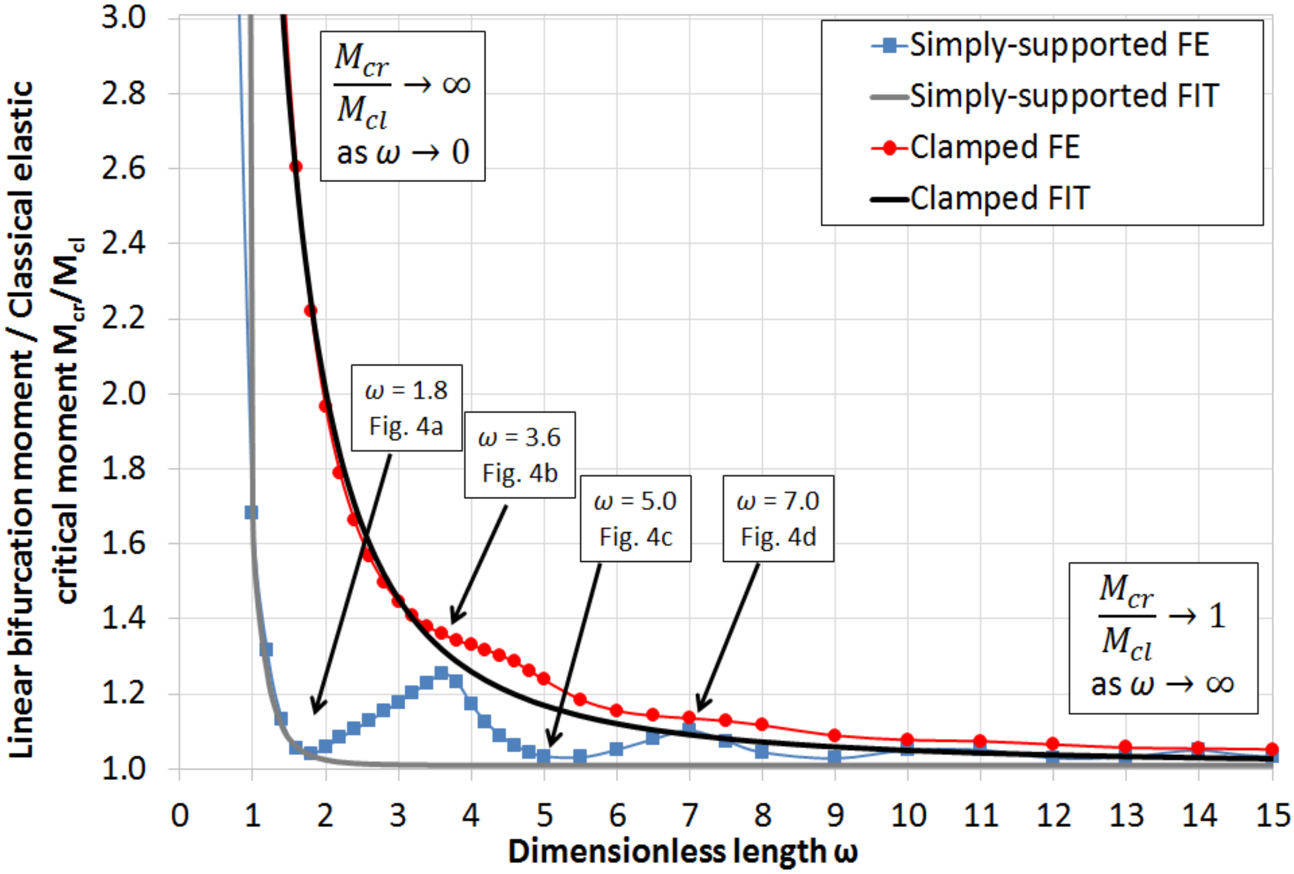

Figure 2: Length-dependent linear buckling behaviour of the perfect tubular members: M_{cr}/M_{cl} vs ω

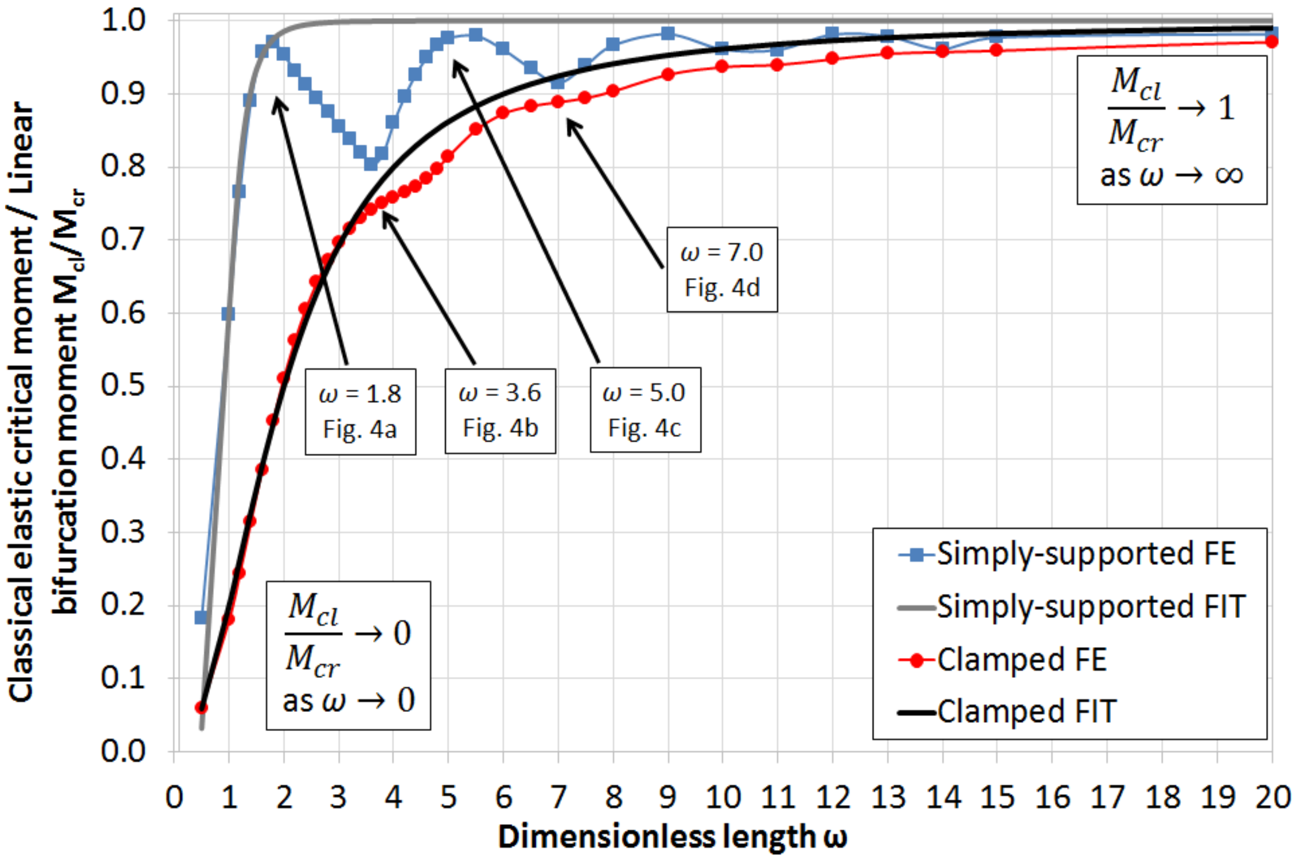

Figure 3: Length-dependent linear buckling behaviour of the perfect cylinders: M_{cl}/M_{cr} vs ω

The calculations have been extended in this study to the case of simply-supported end conditions (Eq. (6)). The rotational restraint offered by a simply-supported boundary is not as strong as for a clamped boundary and only affects very short cylinders (i.e. $\omega < 2$ or $L < 20$ mm). As a result, far shorter cylinders are able to undergo local buckling at a moment close to the classical critical buckling moment M_{cl} than was the case for clamped cylinders, and thus the 'local buckling hypothesis' is valid over a much wider range of cylinder lengths. The moment-length relationship additionally shows a number of distinct 'waves' which relate to the changing critical buckling mode with length, as illustrated in Figure 4. The same effect is also present for clamped end support conditions, although to a much smaller extent.

It is proposed here that the relationship between M_{cr} and M_{cl} for the simply-supported case may be approximated as:

$$\frac{M_{cr}}{M_{cl}} = 1 + \frac{2}{3\omega^{5.5}} \quad \text{or} \quad \frac{M_{cl}}{M_{cr}} = \frac{1.5}{1.5 + 1/\omega^{5.5}} \tag{11}$$

This relationship conservatively ignores the 'waves' on the curve that result from the changing relationship between the critical buckling mode and cylinder length, but still captures the qualitative moment-length relationship at either extreme of the length range. This relationship becomes slightly unconservative for very short cylinders with $\omega < 1$ that anyway fall well outside the range of practical interest.

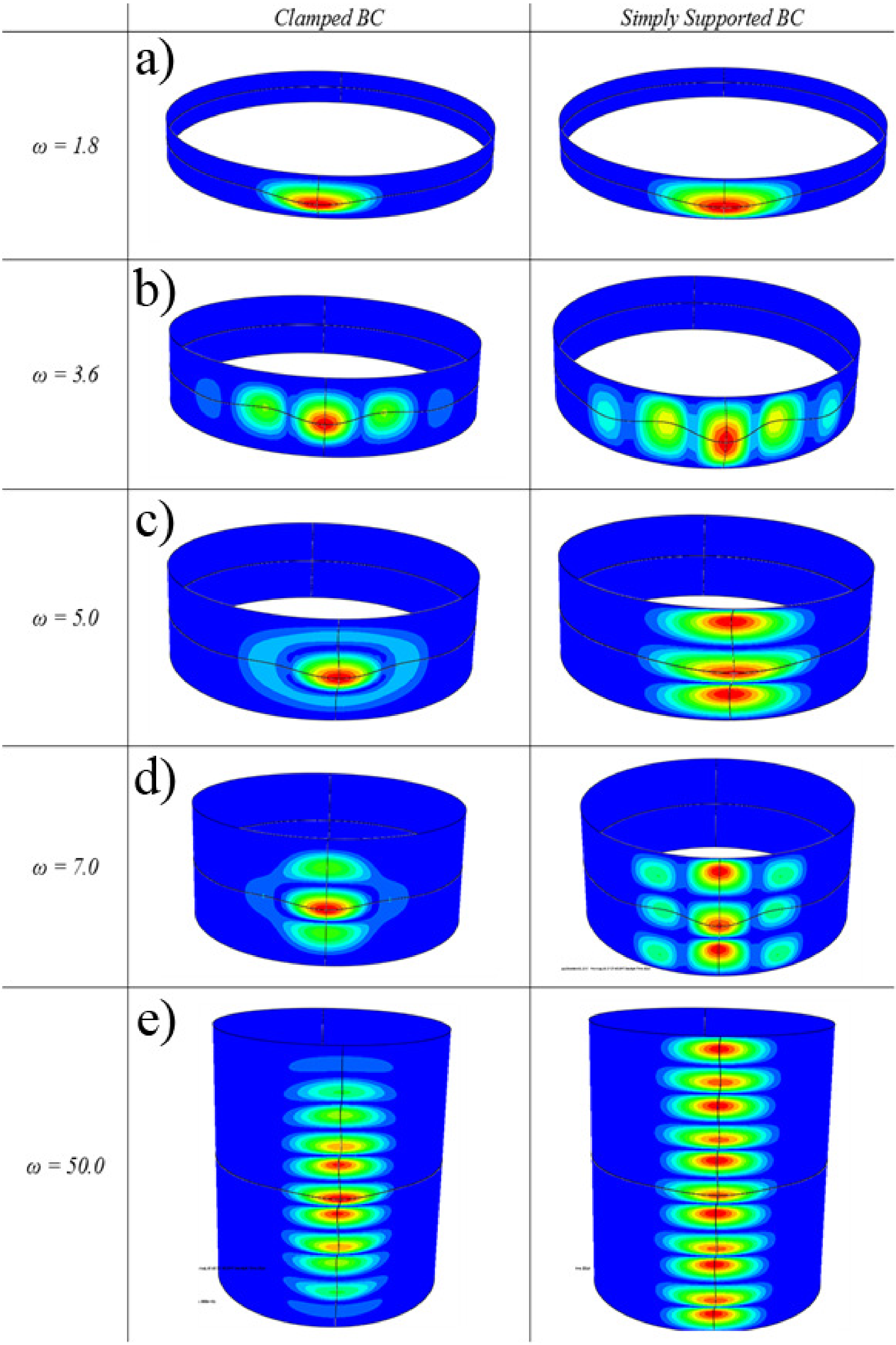

Figure 4: Selected linear buckling eigenmodes from LBA analyses of the perfect cylinders

3.2 Geometrically nonlinear buckling behaviour (GNA analyses)

The study of Rotter *et al.* [1] performed a wide range of nonlinear elastic buckling analyses covering the full practical range of lengths for cylinders under global bending with *clamped* end support conditions. The following four ranges of behaviour were identified and characterised, with the boundaries between them expressed in terms of the dimensionless length ω.

- 'Short' cylinders ($\omega \leq 4.8$) where the end support conditions completely restrain the development of a local axial compression buckle, leading to a significant increase in the critical buckling moment M_k to well above the M_{cl} value.
- 'Medium' length cylinders ($4.8 < \omega \leq 50$) where the end support conditions no longer prevent the development of a local axial compression buckle, but they do prevent ovalisation of the cross-section at midspan. The critical buckling moment M_k is approximately stable in this length range and close to the classical M_{cl} value.
- 'Transitional' cylinders ($50 < \omega \leq 700$) where the restraint of ovalisation provided by the end support conditions begins to diminish. The cross-sectional stiffness begins to drop and there is a decrease in the critical buckling moment with length.
- 'Long' cylinders ($\omega > 700$) which undergo extensive but stable 'Brazier' ovalisation. The buckling moment M_k is at just below the classical 'Brazier' value M_{Braz} (Eq. (1)), which is approximately half of M_{cl}.

The full relationship between the ratio of the nonlinear elastic moment M_k to the computed linear bifurcation moment M_{cr} and the dimensionless length ω is illustrated in Figure 5 using a log scale for the horizontal axis. The dashed lines represent approximate domain boundaries.

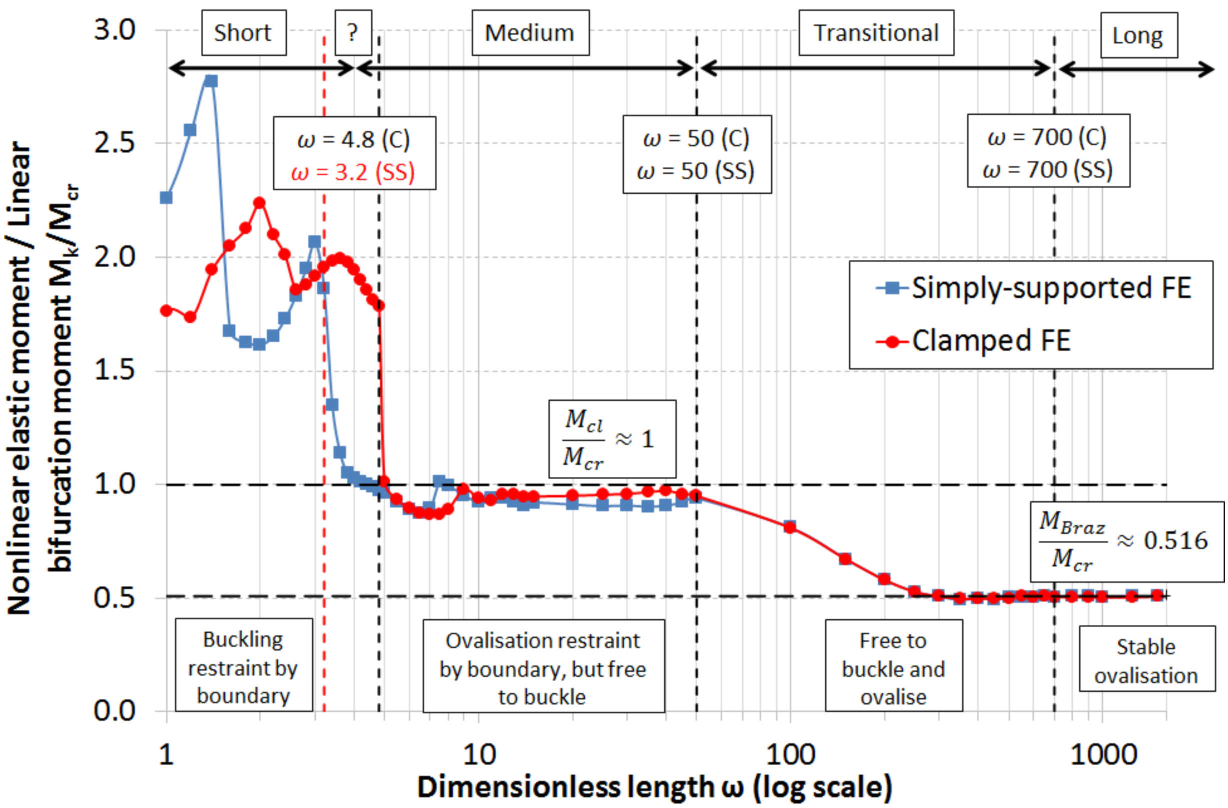

Figure 5: Nonlinear buckling behaviour of the perfect cylinders: M_k/M_{cr} vs ω

These nonlinear calculations were repeated in this study for the case of a cylinder under simply-supported end conditions, also illustrated in Figure 5, and the same length domains may be qualitatively identified as for the clamped case. The main difference is that where the boundary between 'short' and 'medium' length cylinders fell at $\omega \approx 4.8$ for the clamped case, this boundary appears to come rather sooner at $\omega \approx 3.2$ for the simply-supported case. This may be attributed once again to the weaker rotational restraint offered by this boundary condition, allowing shorter cylinders to develop a local axial compression buckle on the compressed side at a moment close to the classical elastic critical M_{cl} value.

The nature of the end support conditions does not appear to have any significant effect on the onset of ovalisation nor on the extent of its influence on the nonlinear critical buckling moment M_k. In both cases the start of the 'transitional' length domain occurs at $\omega \approx 50$, and thereafter both support conditions effectively follow the same relationship.

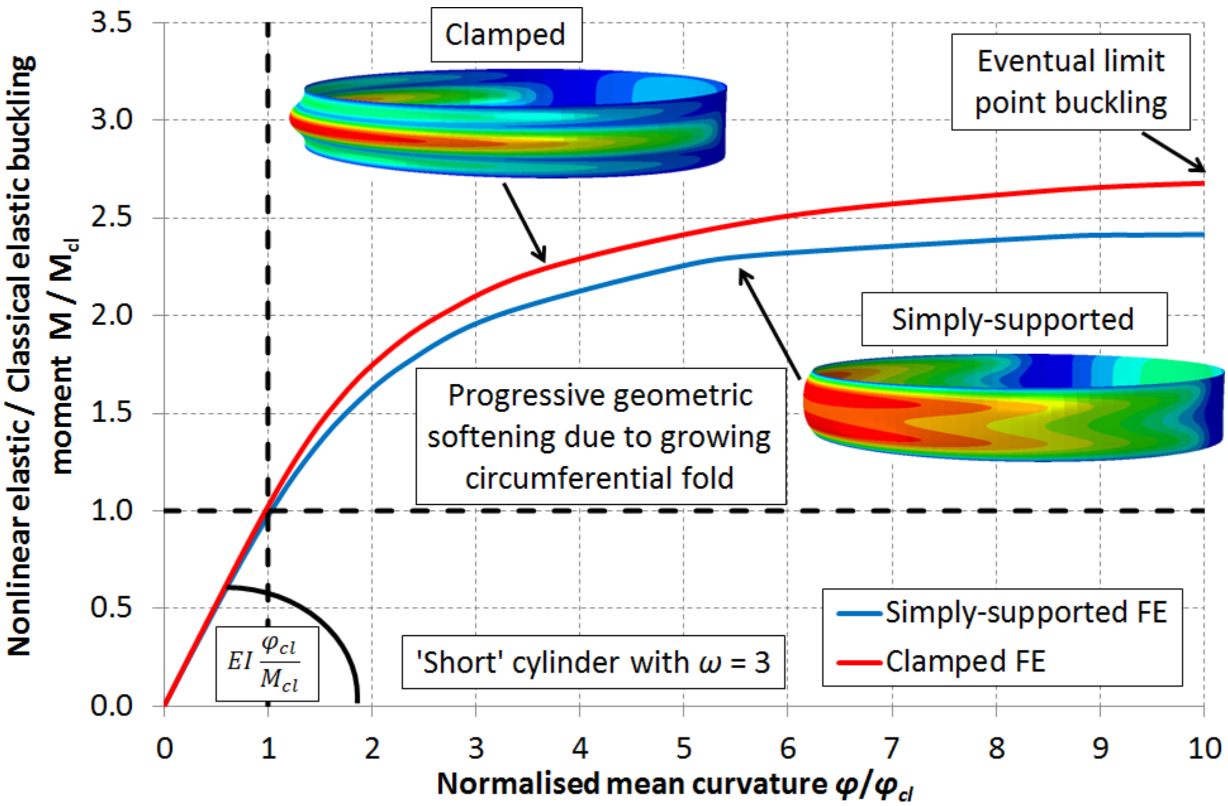

Figure 6: Nonlinear moment-curvature equilibrium paths for short clamped and simply-supported cylinders with $\omega = 3$

The 'short' length domain deserves special mention. Short cylinders under bending exhibit a deep circumferential fold akin to a corrugation developing on the compressed side in a nonlinear analysis, and thus behave very differently to other length domains. The relationship between the applied moment and the mean meridional curvature (Figure 6) shows an initially linear path which begins to undergo significant geometric softening at applied bending moments, significantly higher than M_{cl}. This softening behaviour is not due to ovalisation but is a result of detrimental changes of geometry caused by a progressively deeper circumferential fold (Figure 7a to Figure 7c), a mechanism which does not appear to be well documented in the voluminous shells literature on this topic. Both sets of support conditions exhibit this softening behaviour, although the equilibrium path is noticeably stiffer for the clamped case due to the increased rotational restraint.

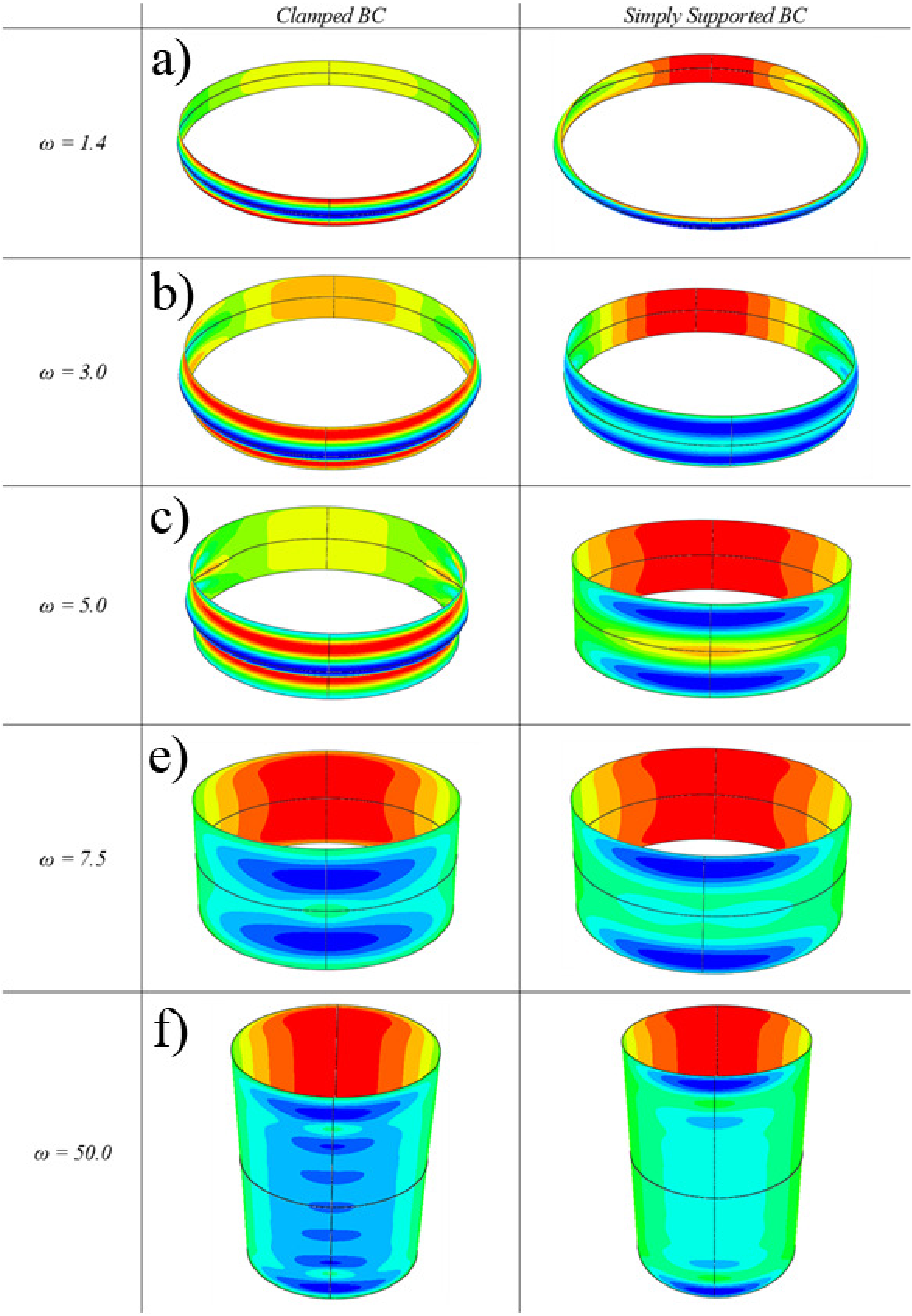

Figure 7: Selected incremental nonlinear buckling modes from GNA analyses of the perfect cylinders

4 INFLUENCE OF INITIAL GEOMETRIC IMPERFECTIONS ON THE NONLINEAR ELASTIC BUCKLING STRENGTH (GNIA ANALYSES)

The computed LBA eigenmodes were introduced as imperfections into the nonlinear buckling analyses in order to obtain a first picture of the potential imperfection sensitivity of cylinders under global bending. This picture is by no means complete, and further research is needed to establish a more complete understanding of this relationship. In particular, other imperfection forms must be taken into account, as it is very possible that one imperfection form will be very detrimental within one length domain but another will be more detrimental within another length domain. This will depend on the physical basis of the imperfection form and how it relates to the mechanics of the structural behaviour within any particular length domain, complicated by the nonlinear effects of the end boundary layer and midspan ovalisation.

The relationship between the computed nonlinear buckling moments M_k and the dimensionless length ω are illustrated in Figure 8 and Figure 9 for the clamped and simply-supported end conditions respectively and for different imperfection amplitudes of the LBA eigenmode: δ/t = 0 (perfect), 0.1, 0.25, 0.5 and 1. A number of observations may be now made.

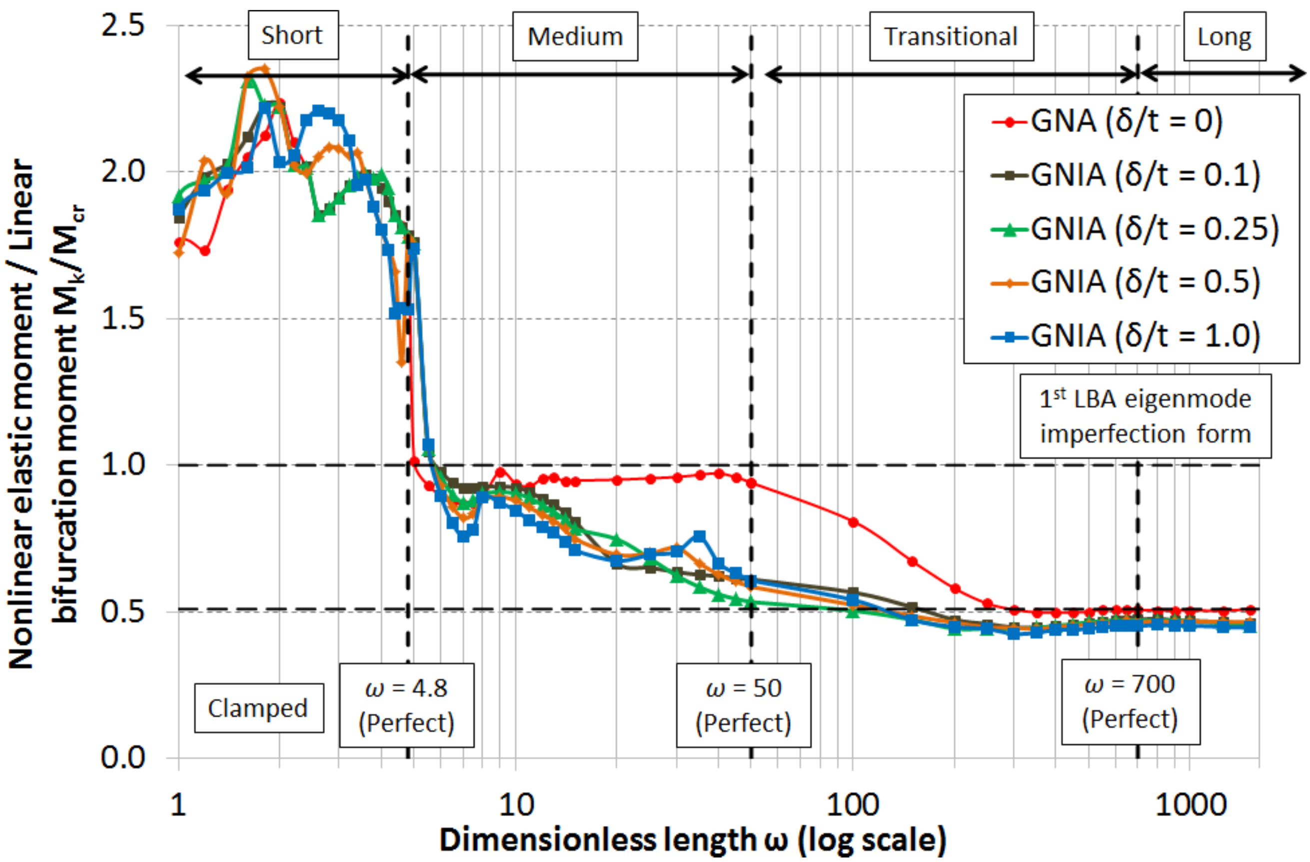

Figure 8: Nonlinear buckling behaviour of imperfect cylinders with clamped end conditions: M_k/M_{cr} vs ω as a function of δ/t

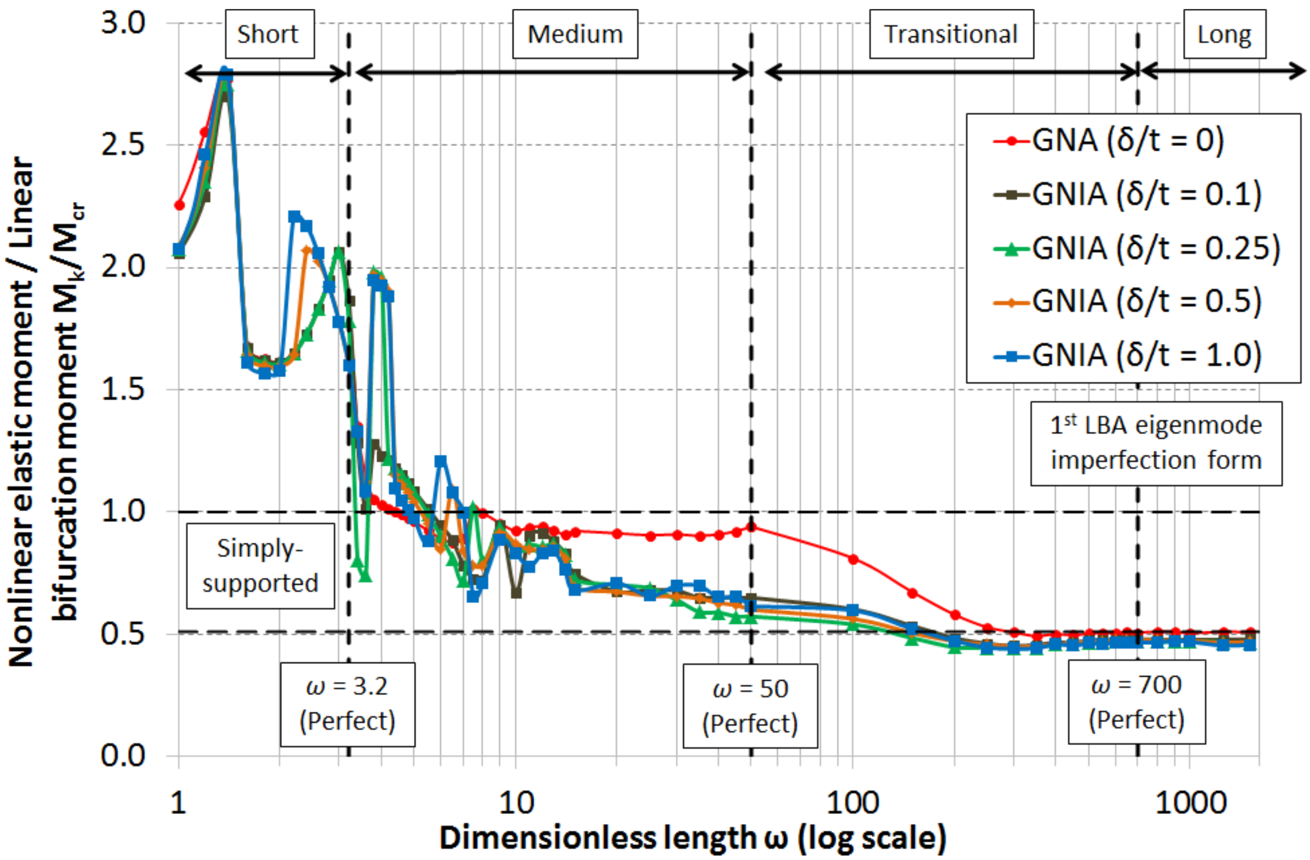

Figure 9: Nonlinear buckling behaviour of imperfect cylinders with simply-supported end conditions: M_k/M_{cr} vs ω as a function of δ/t

First, sensitivity to the LBA eigenmode imperfection appears to be very length dependent. For example, a small imperfection amplitude of δ/t = 0.1 may have either a neutral or even mildly beneficial effect (for 'short' cylinders), a mildly detrimental effect (for cylinders on the boundary between 'short' and 'medium' or 'transitional' and 'long') or a strongly detrimental effect (for cylinders on the boundary between 'medium' and 'transitional'). The reason for this may be attributed to the changing shape of the both the LBA eigenmode (Figure 4) and critical nonlinear buckling mode (Figure 5) with length, and the effect of one on the other.

Second, the decreased stiffness of an even slightly imperfect cylinder appears to permit ovalisation to have a detrimental effect on the buckling strength at significantly shorter lengths than for perfect cylinders. This is apparent in the fact that the boundary between 'medium' and 'transitional' length cylinders no longer falls at a well-defined $\omega \approx 50$ but somewhere closer to $\omega \approx 10$. Since most tubes and cylinders in practice are at least a little imperfect, this suggests that basing a strength assessment on the 'perfect' moment-length relationship may be unconservative. More research is needed to establish whether this is the case for all practical imperfection forms and r/t ratios.

Third, the length boundaries between 'short' and 'medium' or 'transitional' and 'long' cylinders appear unaffected by the presence or the amplitude of the LBA eigenmode imperfection, suggesting that the behaviour at either length extreme is largely insensitive to these imperfections. It was demonstrated in Figure 6 that 'short' cylinders exhibit a distinctive limit point global buckling behaviour without developing any local axial compression buckles, and thus are not likely to be affected detrimentally by the presence of an imperfection in the form of an LBA eigenmode that is similar in shape to a local axial compression buckle. By contrast, an imperfection in the form of localised axial compression buckles (Figure 4e) does not have as great a detrimental effect on 'long' cylinders either due the considerable strength and stiffness reduction that they already suffer due to fully-developed ovalisation.

Given the significant variations in behaviour within each length domain and the uncertainty in the location of the domain boundaries under the influence of imperfections, it is both difficult and unfeasible to devise a single imperfection sensitivity relationship for this load case that would be valid for all lengths. An attempt at formulating a relationship between the strength reduction on the perfect shell α_I (Eq. (12)) and the imperfection amplitude δ/t for each length domain individually, based on an *averaged* value of α_I for that length domain, is shown in Figure 10 and Figure 11 for the clamped and simply-supported end conditions respectively. The scatter in each length domain is represented by error bars showing the standard error defined as $\sigma/\sqrt{n}$, where σ is the standard deviation and n is the number of data points.

$$\alpha_I = \frac{M_{k,GNIA}}{M_{k,GNA}} \tag{12}$$

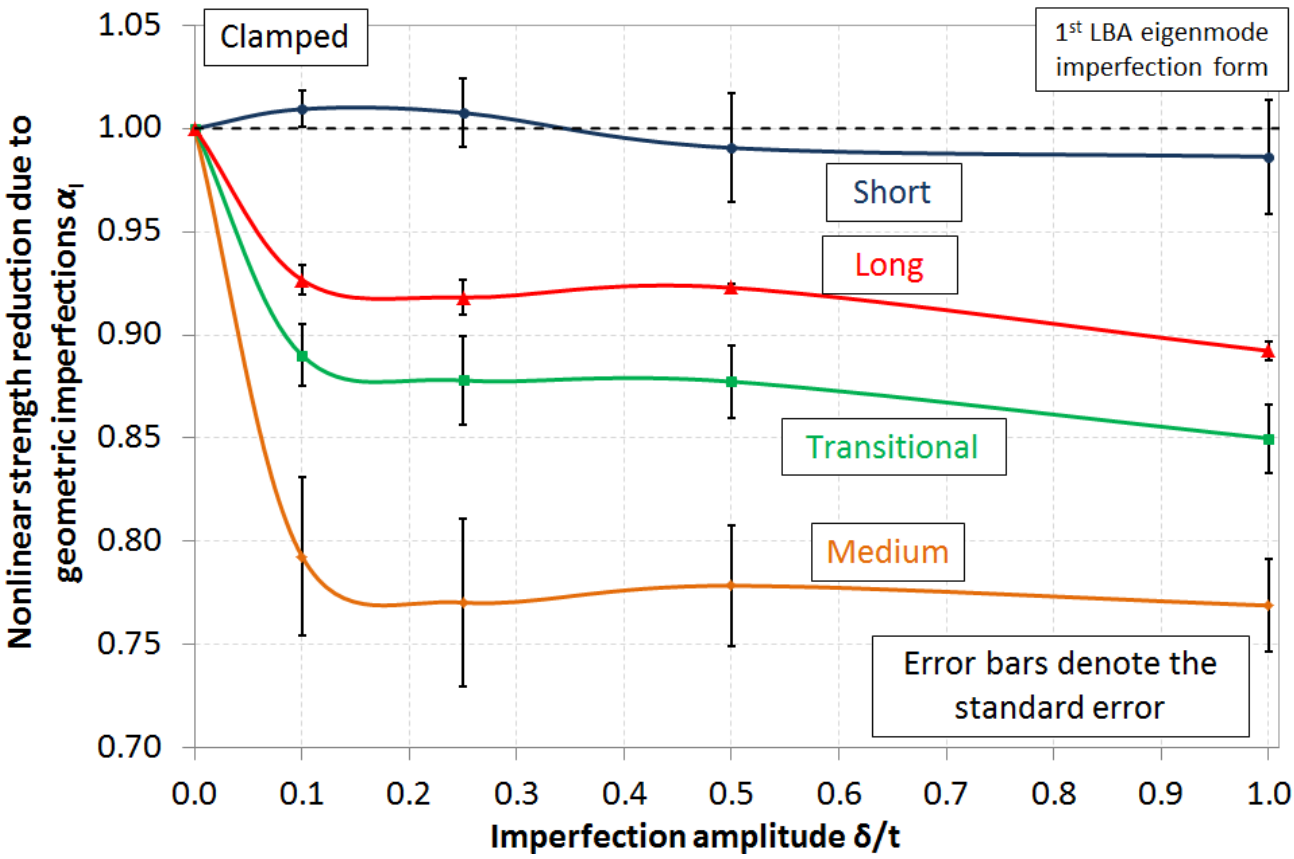

Figure 10: Imperfection sensitivity curves for cylinders with clamped end conditions, averaged on each length domain (as categorised for perfect cylinders) for the LBA eigenmode imperfection form

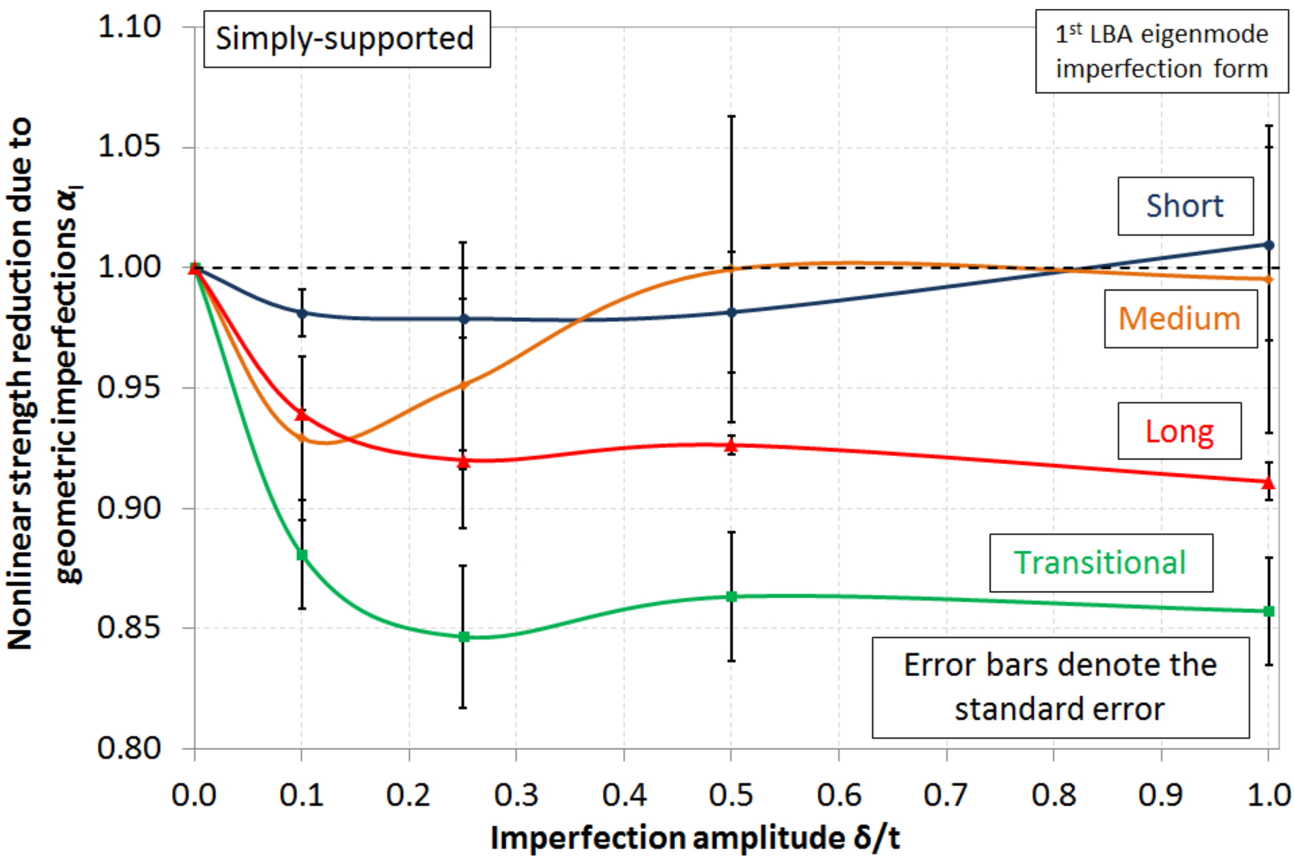

Figure 11: Imperfection sensitivity curves for cylinders with simply-supported end conditions, averaged on each length domain (as categorised for perfect cylinders) for the LBA eigenmode imperfection form

Though approximate, these relationships clearly illustrate the varying imperfection sensitivity and the scatter in predicted strengths for each length domain. The anomalous behaviour of the 'medium' curve for cylinders under simply-supported end conditions (Figure 11) is a reflection of considerable scatter visible in the computed buckling strengths near the start of this length domain (Figure 9). It may be that the definition of the boundary of the 'short' and 'medium' length domains, set at ω = 3.2 for perfect geometries under these support conditions, is no longer appropriate for imperfect ones.

Lastly, it should be stressed that these results are valid only for the LBA eigenmode imperfection form which may or may not be realistic representations of cylinders in practical applications. The authors are actively working to extend the applicability of these relationships to other imperfection forms and r/t ratios, and to characterise them algebraically for practical use.

5 CONCLUSIONS

The following may be deduced from this computational study for cylindrical tubes under global bending:

- The influence of varying the end support conditions on the linear and nonlinear buckling strengths of the elastic tubes is effectively restricted to the 'short' and 'medium' cylinder length domains only. These are the length domains that are still under the influence of by the boundary layer of local bending deformations caused by the end support conditions.

- For cylindrical tubes with simply-supported end conditions, the dimensionless length range for the ‘short’ length domain is suggested to apply to $\omega = L/\sqrt{(rt)} < 3.2$ while the range for the ‘medium’ length domain is then $3.2 \leq \omega < 50$. This is different from the corresponding ranges found in a previous study for cylinders with clamped end conditions, where the ‘short’ length domain is taken as valid on $\omega < 4.8$ while the ‘medium’ length domain is valid on $4.8 \leq \omega < 50$.
- The influence of a linear buckling eigenmode imperfection form on the buckling strength of cylindrical under global bending is strongly length-dependent, with the most detrimental effect visible for the ‘medium’ length domain. Other domains show only a mildly detrimental effect, a neutral effect or even a mildly beneficial effect.
- More research is needed to extend the validity of these conclusions to other imperfection forms and radius to thickness (r/t) ratios as a prelude to a final characterisation of the imperfection sensitivity of cylindrical tubes under global bending for design purposes.

6 ACKNOWLEDGEMENTS

The authors wish to gratefully acknowledge the financial support provided by the Petroleum Technology Development Fund (PTDF).

7 REFERENCES

[1] Rotter J.M., Sadowski A.J. and Chen L., "Nonlinear stability of thin elastic cylinders of different length under global bending", *International Journal of Solids and Structures,* 51(15–16), 2826-2839, 2014.

[2] Calladine C.R., *Theory of Shell Structures,* Cambridge University Press, 1983.

[3] Chen L., Doerich C. and Rotter J.M., "A study of cylindrical shells under global bending in the elastic-plastic range", *Steel Construction,* 1(1), 59-65, 2008.

[4] Chen L., Peng Y.L. and Wan L., "Nonlinear Buckling Behaviour of Imperfect Cylindrical Shells under Global Bending in the Elastic-Plastic Range", *Applied Mechanics and Materials,* 2041045-1052, 2012.

[5] Elchalakani M., Zhao X.L. and Grzebieta R.H., "Plastic mechanism analysis of circular tubes under pure bending", *International Journal of Mechanical Sciences,* 44(6), 1117-1143, 2002.

[6] Li L. and Kettle R., "Nonlinear bending response and buckling of ring-stiffened cylindrical shells under pure bending", *International Journal of Solids and Structures,* 39(3), 765-781, 2002.

[7] Brazier L.G., "On the Flexure of Thin Cylindrical Shells and Other *Thin* Sections", *Proceedings of the Royal Society of London.Series A,* 116(773), 104-114, 1927.

[8] Wadee M.K., Wadee M.A., Bassom A.P. and Aigner A.A., "Longitudinally Inhomogeneous Deformation Patterns in Isotropic Tubes under Pure Bending", *Proceedings: Mathematical, Physical and Engineering Sciences,* 462(2067), 817-838, 2006.

[9] Tatting B., Gürdal Z. and Vasiliev V., "The Brazier effect for finite length composite cylinders under bending", *International Journal of Solids and Structures,* 34(12), 1419-1440, 1997.

[10] Karamanos S.A., "Bending Instabilities of Elastic Tubes", *International Journal of Solids and Structures,* 39(8), 2059-2085, 2002.

[11] Tatting B.F., Gurdal Z. and Vasiliev V.V., "Nonlinear shell theory solution for the bending response of orthotropic finite length cylinders including the Brazier effect", *Proceedings of the 36th Structures, Structural Dynamics and Materials Conference, New Orleans, LA,* 966-976, 1995.

[12] Libai A. and Bert C.W., "A mixed variational principle and its application to the nonlinear bending problem of orthotropic tubes—II. application to nonlinear bending of circular cylindrical tubes", *International Journal of Solids and Structures,* 31(7), 1019-1033, 1994.

[13] Aksel'rad E. and Emmerling F., "Collapse load of elastic tubes under bending", *Israel Journal of Technology,* 2285, 1984.

[14] Mathon C. and Limam A., "Experimental collapse of thin cylindrical shells submitted to internal pressure and pure bending", *Thin-Walled Structures,* 44(1), 39-50, 2006.

[15] Axelrad E., "Refinement of Buckling-Load Analysis for Tube Flexure by way of considering Pre-critical Deformation", *Izvestiya Akademii Nauk SSSR, Otdelenie Tekhnicheskikh Nauk, Mekhanika i Mashinostroenie,* 4133-139, 1965.

[16] Seide P. and Weingarten V.I., "On the Buckling of Circular Cylindrical Shells Under Pure Bending", *Journal of Applied Mechanics,* 28(1), 112-116, 1961.

[17] Yamaki N., *Elastic Stability of Circular Cylindrical Shells,* Elsevier Science, 1984.

[18] Koiter W. *On the Stability of Elastic Equilibrium.* PhD Thesis. Delft University (in Dutch); 1945.

[19] Rotter J.M. and Teng J., "Elastic stability of cylindrical shells with weld depressions", *Journal of Structural Engineering,* 115(5), 1244-1263, 1989.

[20] Donnell L. and Wan C., "Effect of Imperfections on Buckling of Thin Cylinders and Columns under Axial Compression", *Journal of Applied Mechanics - Transactions of the ASME,* 17(1), 73-83, 1950.

[21] Koiter W., "The Effect of Axisymmetric Imperfections on the Buckling of Cylindrical Shells under Axial Compression", *Proc Koninklijke Nederlandse Akademie van Wetenschappen,* 265-279, 1963.

[22] Hutchinson J. and Koiter W., "Postbuckling Theory", *Applied Mechanics Reviews,* 23(12), 1353-1366, 1970.

[23] Cohen G.A., "Computer Analysis of Imperfection Sensitivity of Ring-stiffened Orthotropic Shells of Revolution", *AIAA Journal,* 9(6), 1032-1039, 1971.

[24] Arbocz J. and Sechler E., "On the Buckling of Axially Compressed Imperfect Cylindrical Shells", *Journal of Applied Mechanics,* 41(3), 737-743, 1974.

[25] Singer J., "The Status of Experimental Buckling Investigations of Shells", *Buckling of Shells,* Springer, 501-533, 1982.

[26] Rotter J., "Cylindrical shells under axial compression", *Buckling of thin metal shells,* 42-87, 2004.

[27] EN 1993-1-6. *Eurocode 3: Design of Steel Structures. Part 1-6: Strength and Stability of Shell Structures.* 2007.

[28] Schneider W., Höhn K., Timmel I. and Thiele R., "Quasi-collapse-affine imperfections at slender wind-loaded cylindrical steel shells", *Proceedings of second European Conference on Computational Mechanics-ECCM-2001, Cracow, Poland CD-Rom,* 2001.

[29] Rotter J.M., "The new framework for shell buckling design and the European shell buckling recommendations fifth edition", *Journal of Pressure Vessel Technology,* 133(1), 011203, 2011.

[30] Teng J. and Song C., "Numerical models for nonlinear analysis of elastic shells with eigenmode-affine imperfections", *International Journal of Solids and Structures,* 38(18), 3263-3280, 2001.

[31] Sadowski A.J. and Rotter J.M., "Solid or Shell Finite Elements to model Thick Cylindrical Tubes and Shells under Global Bending", *International Journal of Mechanical Sciences,* 74(0), 143-153, 2013.

[32] Riks E., "An incremental approach to the solution of snapping and buckling problems", *International Journal of Solids and Structures,* 15(7), 529-551, 1979.