Investigating the Relationship between Reservoir Quality and Heterogeneity in Carbonate Reservoirs

By

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A report submitted in partial fulfilment of the requirements for the MSc and/or the DIC

September 2013
Declaration of own work

I declare that this thesis:

*Investigating the Relationship between Reservoir Quality and Heterogeneity in Carbonate Reservoirs* is entirely my own work and that where any material could be construed as the work of others, it is fully cited and referenced, and/or with appropriate acknowledgement given.

Signature:-------------------------------------------------------------

Name of Student: Tausif Ahmad

Name of supervisor: Dr. P. Fitch
My gratefulness to Dr. Fitch who directed me throughout the project

I am also thankful to Prof. Gringarten for his guidance during the year

And to my family, for supporting me all along my studies
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Heterogeneity in carbonates is controlled by geological process of sediment deposition and diagenesis. These heterogeneities are reflected in a variety of petrophysical properties, most commonly in permeability and porosity. As such, these geological heterogeneities influence the volume of fluid stored and its ultimate recovery from the reservoir.

In our study of heterogeneity the technique of Lorenz coefficient which determines inequality in a system was adapted to understand the variation in permeability or porosity within a formation. Synthetic data was generated to simulate permeability and porosity variations. It was observed that the Lorenz coefficient (single) depends on the relative difference between the maximum and minimum value in a dataset. In addition the Lorenz coefficient is also controlled by the percentage/frequency of the maximum and minimum values in a dataset.

This method was extended to understand variation of permeability with porosity using the dual Lorenz coefficient. It was observed that variation in permeability with similitude variation in porosity counter each other’s effect on the dual Lorenz coefficient. Furthermore, heterogeneity in porosity was found to have more pronounced effect on dual Lorenz coefficient than permeability. As heterogeneity is dependent on the scale of investigation, the effect of tool resolution and sampling frequency on Lorenz coefficient was studied.

The quality of a reservoir has often been associated with its heterogeneity. For the same average permeability and porosity the relation between heterogeneity measured by Lorenz coefficient and its influence on reservoir quality was studied. It was observed that for a lognormal or randomly distributed permeability and/or porosity the reservoir quality increased with heterogeneity in porosity. Heterogeneity in permeability increases the spread of the reservoir characteristics without having a profound impact on its quality. As the heterogeneity in permeability increases above 50 % deviation about its mean value such as in fractures, the reservoir quality loses its dependence on porosity. This would imply at low levels of heterogeneity in permeability, a reservoir with greater heterogeneity in porosity would lead to higher recovery factor of hydrocarbons than a one with less heterogeneity. Petrophysical property can thus act as a tool in determining the relative recovery factor between two reservoirs with similar characteristics.

Introduction
Carbonate reservoirs comprises of around 60% of world oil and 40% of world gas reserves (Montaron, 2008; Fitch et al., 2012). In certain geographic region such as Middle East the numbers are as high as 70% in Oil and 90% in gas reservoirs (Montaron, 2008). One of the underlying facts with a carbonate reservoir is the associated heterogeneity which could be stratigraphic, sedimentological and diagenetic origins (e.g., Sibley, 1997; Wayne, 2008; Fitch et al., in review). Furthermore heterogeneities exist at all scales of investigation from pore scale to the basic-scale, depending on the depth of investigating tool (Frykman, 2002; MacDonald, 2009). These depths of investigation can vary from seismic scale (Km) to borehole image analysis. A unit which is homogenous under one scale of investigation might be heterogeneous under another scale of investigation, and vice versa.

The heterogeneity impacts the performance of a reservoir in terms of fluid flow (Lake & Jensen, 1989) and can be subdivided into static or dynamic heterogeneity depending on scale and method of investigation. Static heterogeneity does not account for fluid flow directly and petrophysical properties are used as proxies to relate flow of fluid. Static heterogeneity is measured with samples (cores or wireline logs) and is considered true representation of the reservoir. Different inequality measuring tools such as the Dykstra-Parsons coefficient and Lorenz coefficient are used to measure static heterogeneity in a formation. Dynamic heterogeneity usually considers flow behavior as the basis to determine heterogeneity and is controlled by phenomena such as channeling and dispersion of fluids.

As static heterogeneity measures fluid flow through use of petrophysical properties, permeability and porosity are of prime importance. The root ratio of permeability to porosity is termed as reservoir quality index (Amaefule et al., 1993). As porosity of a reservoir represents its storage capacity and permeability its flow capacity, reservoir quality index thus gives a
representation of the ultimate recovery. In other words a reservoir with higher reservoir quality index will have better recovery than the one with less quality index. As both static heterogeneity and reservoir quality index are related to permeability and porosity there might exist a relation (Figure 1). In accordance with the former, the heterogeneity in permeability and porosity was observed to increase the reservoir quality in carbonates (Fitch et al., in review).

In this paper, the parameters governing the shape of single and dual Lorenz coefficient was studied and its application to permeability and porosity has been analysed. In addition the effect of tool resolution and sampling frequency in determining static heterogeneity through Lorenz coefficient has been studied. Furthermore, an attempt has been made to understand how the relationship between reservoir quality and heterogeneity quantified by Lorenz coefficient is governed. This has been done by considering varying levels and distribution patterns in heterogeneity and then determining the relationship in each scenario with reservoir quality.

**Methodology**

**Heterogeneity**

Lorenz coefficient is a technique used to represent inequality in a system (Lake & Jensen, 1989). As Lorenz coefficient can vary only between 0 and 1(Figure 2), representation of heterogeneity by this technique has a deterministic and visual advantage over its counterparts.

Lorenz coefficient represents variation in Variable A w.r.t Variable B and is represented by plot of $F_m$ versus $H_m$ (Lake & Jensen, 1989), these are detailed in equation 1and 2.

\[
F_m = \sum_{i=1}^{n} \text{Variable } A_i \sum_{i=1}^{n} \text{Variable } A_i
\]

\[
H_m = \sum_{i=1}^{n} \text{Variable } B_i \sum_{i=1}^{n} \text{Variable } B_i
\]

**Figure 2: Schematic Illustration of Lorenz Plot**
For a homogenous medium the Lorenz’s curve is a straight line joining the origin with coordinates (1, 1) and is called line of equality. The Line of perfect equality depicts completely homogenous medium.

The steps in calculating Lorenz coefficient is as follows
a) Sorting variable A in descending order.
b) Computing \( F_m \) and \( H_m \) for each data point.
c) Plot of \( F_m \) against \( H_m \) for all data points.
d) Calculate area under the plot of \( F_m \) against \( H_m \).
e) Lorenz coefficient is given by twice the area between the curve and the line of equality.

As variable A is sorted in descending order the Lorenz coefficient increases from \( m=1 \) to \( m=n \). Increase in heterogeneity of a system shifts the curve away from the equality line (Figure 2).

Traditionally in determining heterogeneity, a plot of cumulative permeability /porosity (variable A) against cumulative thickness (variable B) has been used, and is termed as single Lorenz coefficient.

This technique has been extended to represent variation in permeability (variable A) w.r.t to porosity (variable B) and is known as dual Lorenz coefficient (Fitch et al., in review; Lake & Jensen, 1989).

**Reservoir Quality**

To identify reservoirs units with different qualities the technique of hydraulic flow unit was used (Amaefule et al. 1993). As per this method reservoir units of different quality or flow units are identified on a plot of Reservoir quality index against normalized porosity (e.g., Figure 3). Reservoir Quality index (RQI) given by equation 3 and is ratio of permeability to porosity.

\[
RQI = 0.0314 \times \sqrt{\frac{k}{\phi}} \tag{3}
\]

Normalized porosity (phiz) given by equation 4 is the ratio of pre to grain volume.

\[
\phi_z = \frac{\phi_g}{1-\phi_g} \tag{4}
\]

Flow zone indicator (FZI) given by equation 5 is a representative of each hydraulic flow unit in a reservoir.

\[
FZI = \frac{RQI}{\phi_z} \tag{5}
\]

As FZI is the ratio of RQI to phiz, decrease in phiz increases the Flow Zone Indicator value. Hence an increase in the quality of the reservoir is reflected in the shift of the formation towards the left side of the graph (Fitch et al., in review).

**Data**

**Controls on Lorenz coefficient**

Synthetic data was created with variable A and variable B. Three sets of sensitivity analyses was done to understand relationship between variable A, variable B and variable A and B together on Lorenz coefficient.

**Control of Variable A**

Variable A consists of 30 percent of total data points as high value integer ‘3’ and remaining 70 percent of low value integer ‘1’. Variable B was kept at a constant decimal value throughout this analysis (Figure 4).
The effect of variable ‘A’ on Lorenz coefficient was studied by changing both frequency and integer value in A (Figure 5). The results were compared with the initial set after each sensitivity analysis.

The sensitivity analysis done on variable A (Case A1-A6) could be generalised (Table 1) to understand the trend in Lorenz coefficient.

<table>
<thead>
<tr>
<th>Case No</th>
<th>Total No of data</th>
<th>Frequency of “High Value integer”</th>
<th>Value of “High integer number”</th>
<th>Frequency of “Low Value integer”</th>
<th>Value of Low value Integer</th>
<th>Extras</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>-</td>
<td>-</td>
<td>Increased</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>A2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Increased in steps</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>A3</td>
<td>Increased/decreased</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>A4</td>
<td>-</td>
<td>Decreased/increased</td>
<td>-</td>
<td>Decreased /Increased</td>
<td>-</td>
<td>Mid value integers included</td>
</tr>
<tr>
<td>A5</td>
<td>-</td>
<td>Increased in steps</td>
<td>-</td>
<td>Decreased to maintain same number of data points</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>A6</td>
<td>Increased in steps of case A5</td>
<td>Increased in steps and repeated case no A5</td>
<td>Decreased to maintain same number of data points</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1: Sensitivity analysis Variable A
Control of Variable B

Variable B consists of 30 percent of total data points as high value decimal number ‘0.3’ and remaining 70 percent (frequency) of low value decimal number ‘0.1’. Variable A was kept at a constant decimal value throughout this analysis (Figure 6). Sensitivity analysis (case B1-case B6) similar to variable A was carried out on variable B (see Appendix C1). The effect of variable “B” on Lorenz coefficient was studied by changing both frequency and decimal value in B.

Control of Variable A and B

Three discrete set of data, set 1, set 2 and set 3 were used in the analysis. High value decimal numbers in variable B of set 2 are half of set 1. High value integers in variable A of set 3 are twice of set 1.

- Data Set 1
  Variable A consists of 10 percent of total data points as high value integer ‘40’ and remaining 90 percent of low value integer “1”. Variable B has the same percentage of high value decimal number ‘0.5’ as that of high value integer in variable A (Figure 7).

- Data Set 2
  Variable A consists of 10 percent of total data points as high value integer ‘40’ and remaining 90 percent of low value integer “1”. Variable B has the same percentage of high value decimal number ‘0.25’ as that of high value integer in variable A (see appendix C1).

- Data Set 3
  Variable A consists of 10 percent of total data points as high value integer ‘80’ and remaining 90 percent of low value integer ‘1’. Variable B has the same percentage of high value decimal number ‘0.5’ as that of high value integer in variable A (see appendix C1).

The frequency of variable A and B in both data sets was increased and the results were compared (Figure 8) (see Appendix C2 for Data set 2 and 3).
The sensitivity analysis done on variable A and variable B (Case AB1-AB2) could be generalised to understand the trend in Lorenz coefficient (Table 2).

<table>
<thead>
<tr>
<th>Case no</th>
<th>Sensitivity Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB1</td>
<td>Low value integer in set A associated with high value decimal number in set B. Thus frequency of the data sets in variable B interchanged</td>
</tr>
<tr>
<td>AB2</td>
<td>Increase in frequency of High decimal and integer numbers</td>
</tr>
</tbody>
</table>

**Table 2 : Sensitivity analysis Variable A & B**

**Effect of Tool Resolution and sample frequency on Lorenz coefficient**

Synthetic logs of permeability and porosity were extracted from a geological model of Jurassic carbonate reservoir (Fitch et al., 2012). The existing model consists of single constant value of permeability and porosity for individual formation/facies type.

Spread of 10, 20 and 30% SD was considered about mean value to generate 3 sets of data (Figure 9). This spread of data which appears as noise on logging tool would represents a more realistic reservoir condition. The noise was considered to exist at intervals of 0.1 m. Thus a 0.1m tool resolution would encompass all the heterogeneities existing in the reservoir model.
A tool of resolution less than 0.1 m would read values above and below the intended sample point and average them arithmetically (Roberts, 1955). In other words, a tool with lower resolution such as a 0.7 m averaging resolution, will read petrophysical property 0.35m above and 0.35m below the intended sample point and average them arithmetically (Figure 10a). Averaging of 0.3m, 0.5m and 0.7m was used in the study to simulate different tool resolutions (Figure 10b).

The effect of sampling frequency (as in the case of coring on heterogeneity) was studied by considering various sampling frequencies on the three data sets. As the sampling rate was decreased, the total number of data points sampled in the reservoir decreased. The analysis was repeated for same sampling frequency but considering different sampling starting point (see appendix C2).

**Relationship of heterogeneity with Reservoir Quality**

A set of 100 data points of average permeability 400md and porosity of 0.3 was considered in the analysis. The initial spread of the data was considered to be 5% SD around the average values. This represented a very tight clustered homogenous formation with permeability and porosity values tightly distributed around the average mean value (Figure 11a). To determine the effect of heterogeneity on reservoir quality, heterogeneity in porosity and permeability was considered and its effect on reservoir quality was studied. This was achieved by studying the variation in the cross-plot of reservoir quality index (RQI) against Normalised porosity (phiz) plot. A cross plot of flow zone indicator (FZI) against Lorenz coefficient (Lc) was also analysed to determine the relationship. FZI was calculated for the all the data points as well as data points whose values were within 68.27 % (1 sigma) of the mean FZI value (Figure 11 b). This would represent realistic visualisation of the dataset ignoring too large or small FZI valued data points.

The spread in porosity of the initial data set was increased in subsequent steps. Standard deviation of 10%, 20%, 30% and 40% (about the mean value) with a lognormal distribution were considered in porosity (Figure 12a). Permeability values were held constant the analysis (Table 3).
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<table>
<thead>
<tr>
<th>Case No</th>
<th>Average permeability (md)</th>
<th>Average porosity</th>
<th>Standard Deviation about mean (permeability)</th>
<th>Standard deviation about mean (porosity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP1</td>
<td>400</td>
<td>0.3</td>
<td>5%</td>
<td>10%</td>
</tr>
<tr>
<td>HP2</td>
<td>400</td>
<td>0.3</td>
<td>5%</td>
<td>20%</td>
</tr>
<tr>
<td>HP3</td>
<td>400</td>
<td>0.3</td>
<td>5%</td>
<td>30%</td>
</tr>
<tr>
<td>HP4</td>
<td>400</td>
<td>0.3</td>
<td>5%</td>
<td>40%</td>
</tr>
<tr>
<td>HP5</td>
<td>400</td>
<td>0.3</td>
<td>5%</td>
<td>45%</td>
</tr>
</tbody>
</table>

Table 3: Sensitivity heterogeneity in porosity

As a result of heterogeneity, porosity values were spread about the mean value (Figure 12b). To verify the findings, a similar analysis was repeated for a data set having average permeability of 400md and porosity of 0.13 (case HP6-HP10) (see appendix C3).

Relationship of heterogeneity in permeability with reservoir quality

The spread in permeability of the initial data set was increased in subsequent steps. Standard deviation of 10%, 20%, 30% and 40% (about the mean value) with lognormal distribution were considered in permeability (Table 4). Porosity values were held constant in the analysis.

<table>
<thead>
<tr>
<th>Case No</th>
<th>Average permeability (md)</th>
<th>Average porosity</th>
<th>Spread in Data</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Standard Deviation about mean (permeability)</td>
<td>Standard Deviation about mean (porosity)</td>
</tr>
<tr>
<td>HPE1</td>
<td>400</td>
<td>0.3</td>
<td>10%</td>
<td>5%</td>
</tr>
<tr>
<td>HPE2</td>
<td>400</td>
<td>0.3</td>
<td>20%</td>
<td>5%</td>
</tr>
<tr>
<td>HPE3</td>
<td>400</td>
<td>0.3</td>
<td>30%</td>
<td>5%</td>
</tr>
<tr>
<td>HPE4</td>
<td>400</td>
<td>0.3</td>
<td>40%</td>
<td>5%</td>
</tr>
</tbody>
</table>

Table 4: Sensitivity analysis for heterogeneity in permeability

As a result of heterogeneity, permeability values were spread about the mean value (Figure 13).

Relationship of heterogeneity in permeability and porosity with reservoir quality

The spread in porosity and permeability of the initial data set was increased in subsequent steps. Permeability and porosity with lognormal distribution and standard deviation of 10%, 20%, 30%, 40% and 50% (about the mean value) were considered (Table 5).
Investigating the relationship between Reservoir Quality and Heterogeneity in Carbonate Reservoirs

<table>
<thead>
<tr>
<th>Case No</th>
<th>Average permeability (md)</th>
<th>Average porosity</th>
<th>Spread in data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Standard Deviation about mean (permeability)</td>
</tr>
<tr>
<td>HPP1</td>
<td>400</td>
<td>0.3</td>
<td>10%</td>
</tr>
<tr>
<td>HPP2</td>
<td>400</td>
<td>0.3</td>
<td>10%</td>
</tr>
<tr>
<td>HPP3</td>
<td>400</td>
<td>0.3</td>
<td>20%</td>
</tr>
<tr>
<td>HPP4</td>
<td>400</td>
<td>0.3</td>
<td>20%</td>
</tr>
<tr>
<td>HPP5</td>
<td>400</td>
<td>0.3</td>
<td>20%</td>
</tr>
<tr>
<td>HPP6</td>
<td>400</td>
<td>0.3</td>
<td>30%</td>
</tr>
<tr>
<td>HPP7</td>
<td>400</td>
<td>0.3</td>
<td>30%</td>
</tr>
<tr>
<td>HPP8</td>
<td>400</td>
<td>0.3</td>
<td>40%</td>
</tr>
<tr>
<td>HPP9</td>
<td>400</td>
<td>0.3</td>
<td>50%</td>
</tr>
</tbody>
</table>

Table 5: Sensitivity analysis for heterogeneity in permeability and porosity

To verify the findings a similar analysis (HPP 10-17) was repeated for a data set having average permeability of 100md and porosity of 0.05 (appendix C2).

Relationship of stochastically distributed permeability and porosity with reservoir quality

The initial data was divided into sections and unique lognormal distributed spread added to each section. Thus the overall data set acts as one with random/stochastic distribution (Figure 14a). This would signify a formation that has undergone partial transformation/diagenesis. Permeability and porosity with overall deviation of 10%, 15%, 30%, 40% and 50% were considered in the analysis (Table 6).

<table>
<thead>
<tr>
<th>Case No</th>
<th>Average permeability (md)</th>
<th>Average porosity</th>
<th>Overall Spread in Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Standard Deviation about mean (permeability)</td>
</tr>
<tr>
<td>HPI1</td>
<td>400</td>
<td>0.3</td>
<td>15%</td>
</tr>
<tr>
<td>HPI2</td>
<td>400</td>
<td>0.3</td>
<td>15%</td>
</tr>
<tr>
<td>HPI3</td>
<td>400</td>
<td>0.3</td>
<td>30%</td>
</tr>
<tr>
<td>HPI4</td>
<td>400</td>
<td>0.3</td>
<td>30%</td>
</tr>
<tr>
<td>HPI5</td>
<td>400</td>
<td>0.3</td>
<td>40%</td>
</tr>
<tr>
<td>HPI6</td>
<td>400</td>
<td>0.3</td>
<td>40%</td>
</tr>
</tbody>
</table>

Table 6: Sensitivity analysis for stochastically distributed data

Due to different spread in permeability and porosity in different section, the formation behaved as two or more separate units (Figure 14b).

![Figure 14: Schematic of stochastically distributed permeability and porosity. a) Distribution (permeability), b) spread in data](image)

Relationship of extremely high permeability variance with reservoir quality

The spread in permeability with standard deviation more than 50% about mean was considered and its effect on reservoir quality studied (Table 7).

<table>
<thead>
<tr>
<th>Case No</th>
<th>Average permeability (md)</th>
<th>Average porosity</th>
<th>Overall Spread in Data</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Standard Deviation about mean (permeability)</td>
</tr>
<tr>
<td>HPE1</td>
<td>400</td>
<td>0.3</td>
<td>132%</td>
</tr>
<tr>
<td>HPE2</td>
<td>400</td>
<td>0.3</td>
<td>132%</td>
</tr>
<tr>
<td>HPE3</td>
<td>400</td>
<td>0.3</td>
<td>132%</td>
</tr>
<tr>
<td>HPE4</td>
<td>400</td>
<td>0.3</td>
<td>110%</td>
</tr>
<tr>
<td>HPE5</td>
<td>400</td>
<td>0.3</td>
<td>65%</td>
</tr>
<tr>
<td>HPE6</td>
<td>400</td>
<td>0.3</td>
<td>55%</td>
</tr>
<tr>
<td>HPE7</td>
<td>400</td>
<td>0.3</td>
<td>55%</td>
</tr>
</tbody>
</table>

Table 7: Sensitivity analysis for high heterogeneity in permeability
To verify the findings a similar analysis (Case HPE 8-11) was repeated for a data set having average permeability of 800md and porosity of 0.32 (appendix C2).

**Result and Finding**

**Control on Lorenz coefficient**

*Control of variable “A” on Lorenz coefficient*

The increase in value of high value integer (case A1) increased the Lorenz coefficient, whereas the effect was opposite in case of low value integer (Case A2) (Figure 15a, Figure 15b). The Lorenz coefficient remained constant (case A3) if the overall number of data points is increased or decreased maintaining the ratio of different data points same (Figure 15c). It was also observed inclusion of mid values between the two end points reduced the Lorenz coefficient (Figure 15d). As the frequency/percentage of the high value integer (Case A5) was increased, the coefficient increased up to a certain frequency after which it began to decrease (Figure 15e). This frequency at which reversal in trends initiated was dependent on the end point values (case A6). The reversal in trend was observed to be earlier for a dataset with greater difference in end point values (Figure 15f).

![Figure 15: Lorenz Coefficient plot and values for Variable A sensitivity analysis. a) Case A1, b) Case A2, c) Case A3, d) Case A4, e) Case A5, f) Case A6](image)

*Control of variable “B” on Lorenz coefficient*

As variable A is constant in this analysis, the output of the Lorenz coefficient is dependent on the sorting in variable B (Figure 16).

![Figure 16: Illustration of Sorting Pattern in Variable B](image)
If variable B is sorted in ascending order a similar trend is seen as that in variable A (see appendix D1). However, if the sorting in variable B is in descending order the effect is opposite. The increase in value of high value decimal number (case B1) decreased the Lorenz coefficient whereas the effect was opposite in case of low value integer (Case B2) (Figure 17a, Figure 17b). The Lorenz coefficient remained constant (case B3) if the overall no of data points was increased or/decreased maintaining the ratio of different data points same (Figure 17c). It was also observed inclusion of mid values between the two end points increased the Lorenz coefficient (Figure 17d). As the frequency of the high value decimal number was (Case B5) increased the Lorenz coefficient decreased up till a certain frequency, after which it began to increase (Figure 17e). This frequency at which reversal in trend initiated was dependent on the end point values (case B6). The reversal in trend was observed to be earlier for a dataset having higher difference in end point values (Figure 17f).

**Control of variable “A” and “B” on Lorenz coefficient**

The relationship between A and B was observed to dictate the Lorenz coefficient. The Lorenz coefficient was found to be higher in an inversely related variable A and B (Iteration 1) than in a linearly related one (Initial data set) (Figure 18a, Figure 18b, Figure 18c). As the frequency of variable A and B increased the Lorenz coefficient started to decrease (Figure 18d, Figure 18e, Figure 18f). This is because the difference in end values of the dataset is large (1-40), causing reversal in trend to be earlier (Figure 15f, Figure 17f). The Lorenz coefficient for data set 2 (variable B decreased) was observed to be higher than set 3 (variable A increased) for all iteration (Figure 18e, Figure 18f), implying variable B has greater control on Lorenz coefficient than A.

**Figure 17: Lorenz Coefficient plots and values for Variable B sensitivity analysis (sorted in descending order). a) Case B1, b) Case B2, c) Case B3, d) Case B4, e) Case B5, f) Case B6**

**Figure 18: Lorenz Coefficient plot and values for Variable A & B sensitivity analysis. a) Case AB1 (set 1), b) Case AB1 (set 2), c) Case AB1 (set 3), d) Case AB2 (set 1), e) Case AB2 (set 2), f) Case AB2 (set 3)**
Effect of tool resolution and sampling frequency on Lorenz coefficient

A decrease in dual Lorenz coefficient was observed with decrease in tool resolution. Hence a tool with lower resolution (or greater averaging resolution) will quantify less heterogeneity than the actual (Figure 19).

![Figure 19: Effect of Tool resolution on Lorenz curve](image)

No specific trend in Lorenz coefficient with sampling frequency was observed. The heterogeneity quantified by different sampling frequencies and different sampling start depths may yield different pictures of heterogeneity in the formation (see appendix D2).

Relationship of heterogeneity with Reservoir Quality

**Relationship of heterogeneity in porosity with reservoir quality**

The heterogeneity in porosity increased the spread of reservoir quality distribution (Figure 20a). In addition, the average reservoir quality of this dataset as a whole was observed to increase with heterogeneity in porosity (Figure 20b). The result was in agreement with the dataset with average permeability of 400md and porosity of 0.13 (case HP6-HP10) (appendix D3).

![Figure 20: Effect of heterogeneity in porosity. a) Reservoir quality distribution b) FZI against Lc (porosity)](image)

**Relationship of heterogeneity in permeability with reservoir quality**

The heterogeneity in permeability increased the spread of reservoir characteristics (Figure 21a). In contrast, however the average reservoir quality of the dataset was observed to remain constant with increase in heterogeneity in permeability (Figure 21b).
Investigating the relationship between Reservoir Quality and Heterogeneity in Carbonate Reservoirs

**Figure 21**: Effect of heterogeneity in permeability. a) Reservoir quality distribution, b) FZI against Lc

**Relationship of heterogeneity in permeability and porosity with reservoir quality**

The reservoir quality was seen to increase with heterogeneity in porosity (Figure 22). The heterogeneity in permeability has minimal impact on the quality of the reservoir. As the dual Lorenz coefficient represents variation of permeability with porosity no specific trend was observed (appendix D3). Similar results were observed for data set with average permeability 100md and porosity 0.05 (case HP9-HP16) (see appendix D3).

**Figure 22**: Effect of heterogeneity in permeability and porosity

**Relationship of stochastically distributed permeability and porosity with reservoir quality**

Random distributed data can cause the reservoir to split into two or more discrete units. This is because, lognormal distribution was considered in sections so as the data as a whole has stochastic distribution. Each section of this data appears as different cluster on the plot. The one with higher quality is termed as the upper flow unit while the lower quality as Lower flow unit in our illustration (Figure 23a).

**Figure 23**: Effect of stochastically distributed permeability and porosity. a) Reservoir quality distribution b) FZI against Lc
In this case also, it was observed that the reservoir quality increased with increase in heterogeneity in porosity. The heterogeneity in permeability had minimal impact on quality of reservoir (Figure 23b).

**Effect of high permeability variance on reservoir quality**

When heterogeneity in permeability exceeds 50% SD about the mean value the reservoir can be splits into 2 or more discrete quality units as in the previous case (see appendix D2). In addition at such high heterogeneity in permeability the reservoir quality showed no relation to the heterogeneity in porosity/permeability of the formation (Figure 24). The results were confirmed with dataset with permeability 800md and porosity 0.3 (see appendix D3).

![Figure 24: Effect of high heterogeneity on reservoir quality](image)

**Discussion**

Single Lorenz coefficient provides a simple method to determine the inequality and hence heterogeneity in a dataset. The Lorenz coefficient is dictated by maximum/minimum values in the dataset (Figure 15a). Larger the difference between the maximum and minimum values, greater will be the heterogeneity reflected on the Lorenz coefficient (case A1). A larger difference in maximum and minimum value represents greater inequality in the system and hence it is reflection on Lorenz coefficient. Fitch et al. (in review) also observed the same effect of increase in Lorenz coefficient with increase in differences in end values. In addition, Lorenz coefficient is dictated by the frequency of the maximum and minimum value in a dataset. The Lorenz coefficient increases up to a certain frequency of the maximum value, after which it starts to decrease (Figure 15e). An increase in frequency of maximum value would imply that the overall average of the data set has increased. As a result the minimum values are now further away from the mean of the data set implying increased inequality. As the frequency of the maximum value increases further, there is an increased representation of the former in the dataset. This leads to reduction in inequality and therefore a reversal in the trend is seen. This representation is reached earlier for larger differences in maximum and minimum value. Hence, reversal in trend is reached at a lower frequency of the maximum value (Figure 15f) for greater differences. It was also observed that inclusion of mid values reduces the Lorenz coefficient as the large number of data points are near the mean of the set (transformation from the maximum to minimum is more gradual) hence reducing heterogeneity (Figure 15d).

The effect of variable A on single Lorenz coefficient can be extended on a broader scale to determine heterogeneity in permeability or porosity. A formation with greater difference between maximum and minimum permeability/or porosity will be more heterogeneous. In addition at large differences between maximum and minimum values, a slight increase in percentage/frequency of maximum permeability would decrease in heterogeneity. This will be particularly the scenario in case of fractured reservoirs. As the permeability contrast between fracture and matrix is usually high (Olson et al., 2004), a slight increase in fracture width will reduce the heterogeneity in the formation. Furthermore, a formation having a gradual transformation from high to low permeability or porosity will be less heterogeneous than otherwise.

The effect of sorting in variable B (Figure 16) could be attributed to the fact that Lorenz coefficient is representative of variation in variable A relative to B. Comparison of Lorenz coefficient value of initial data for variable A analysis and variable B analysis show that the have the same absolute value (Figure 15a, Figure 17a). The data set in variable A and variable B analysis have same difference between maximum and minimum values (Figure 4, Figure 6) . It implies that if the data set (in variable B analysis) is sorted in descending order of variable B, the analogy is similar as if the dataset (in variable A analysis) is sorted in ascending order of variable A. As Lorenz coefficient is sorted in descending order in variable A (Figure 2), an ascending sorting would result in a negative coefficient. It could be thus inferred variable B and Lorenz coefficient are inversely related. As a result variable A and variable B if inversely related have high Lorenz coefficient (Figure 18a, b, c Iteration 1). In sensitivity of variable A&B analysis, it was observed that decreasing variable B had more pronounced effect than increasing variable A by same proportion (Figure 18e and Figure 18f). This is because data points in
variable B are decimal numbers, hence variation of A w.r.t small decimal changes in B is much more pronounced. This is reflected on Lorenz coefficient. However, for larger values of variable A the Lorenz coefficient will be dictated by the former and loose its dependency on variable B.

This method of dual Lorenz coefficient could be extended to determine variation in permeability with porosity. A formation with linear relationship between permeability and porosity will be less heterogeneous than the one with inverse relationship. As permeability and porosity are usually directly related (e.g., Osisanya et al., 1998), a formation with large difference in permeability and small difference in porosity will be more heterogeneous. In addition as porosity values are fractional, the dual Lorenz coefficient will be more influenced by porosity values. However, at large values of permeability the relationship will be dictated by variation in permeability.

The true representation of heterogeneity depends on the scale of investigation. In instances of wire line logging the heterogeneity in a system decreases with decrease in tool resolution (Figure 19). This is because of averaging of thin beds/layers with high permeability/porosity values leading to decrease in Lorenz coefficient. This would imply that a tool with lower resolution will quantify lower heterogeneity than actual.

When heterogeneity is quantified using sample points such as in coring, increase in sampling frequency might lead to omission of high and/or low permeability/porosity samples. This will cause Lorenz coefficient to decrease or increase, depending on whether high or low are missed in the coring operation. Hence increase in sampling frequency can lead to unrealistic representation of heterogeneity in a system.

The dependence of Reservoir quality on heterogeneity in porosity can be attributed to the fact that FZI is a ratio of reservoir quality to normalized porosity. Substituting eq. 3 and eq. 4 in eq. 5

FZI can be simplified to following equation

\[ FZI = 0.0314 \times \sqrt{\frac{K \times (1-\phi)^2}{\phi^3}} \]  \hspace{1cm} (6)

As heterogeneity in porosity increases, the low and high-end values of porosity move further apart about the same mean value (Figure 12a). This implies a large fraction of the low porosity values will be encountered in the formation. As porosity is cubed in the denominator in eq. 6, these low porosity values will increase FZI substantially. This increases the overall average reservoir quality for the formation (Figure 20b).

Similarly, as heterogeneity in permeability increases the low and high-end values of permeability move further apart about the same mean. A large fraction of the formation will now be encountered with high permeability values. In contrast however as permeability is directly related to FZI as per eq.6, the increase in FZI is not significant enough to increase the overall quality of the formation (Figure 21b).

In case of fractured reservoirs or reservoirs that have undergone partial diagenesis, some of the permeability values might be extremely high than the mode value. As discussed earlier due to small percentage of these maximum values a high level of heterogeneity will be quantified by the Lorenz coefficient. For the part of formation with these high permeability values, the FZI as per eq. 6 will have high values and hence lose its dependence on porosity as discussed earlier.

The implication of the above finding could be applied to carbonate reservoirs. Lee et al. (1993) observed that with increasing heterogeneity there was increase in pseudo pressure drawdown of a well. This in turn would mean increase in the quality of the reservoir with heterogeneity which is in agreement with the observation made in this study. A formation with greater heterogeneity in porosity was observed to have better reservoir quality than a less heterogeneous one (e.g., Fitch et al., in review). The increase in reservoir quality would translate into better ultimate recovery from the field. Thus two similar formations with different heterogeneities and same oil in place will have different ultimate recovery relating to their heterogeneity.

**Conclusion**

- The single Lorenz curve gives a measure of the heterogeneity in petrophysical properties of a reservoir.
- The single Lorenz’s coefficient depends on the frequency and the maximum/minimum values of permeability/porosity.
- The dual Lorenz curve can be used as a measure of variation of one petrophysical property against other in a reservoir.
- Heterogeneity in porosity has pronounced impact on the dual Lorenz coefficient.
- As permeability and porosity directly related, a set with high variations in permeability with less variation in porosity will be more heterogeneous than one with high variations in both.
- The quality of reservoir increases with heterogeneity in porosity. Heterogeneity in permeability increases the variance in reservoir characteristics but does not increase the overall quality.
- With increase in permeability heterogeneity above 50% SD (about the mean value) the reservoir loses its dependence on heterogeneity in porosity.
Recommendation
The analysis done in this paper should be extended to real field data to check the validity. Reservoir models with same STOIIP values and average permeability and porosity but different heterogeneity levels should be tested to verify the observations made in this paper.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>Permeability</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Porosity</td>
</tr>
<tr>
<td>$\phi_n$</td>
<td>Normalised porosity</td>
</tr>
<tr>
<td>FZI</td>
<td>Flow zone indicator</td>
</tr>
<tr>
<td>RQI</td>
<td>Reservoir Quality index</td>
</tr>
<tr>
<td>$L_c$</td>
<td>Lorenz coefficient</td>
</tr>
<tr>
<td>STOIIP</td>
<td>stock tank oil in place</td>
</tr>
<tr>
<td>SD</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>m</td>
<td>meter</td>
</tr>
<tr>
<td>md</td>
<td>millidarcy</td>
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</table>

References


Appendices
## Appendix A: Major milestones

<table>
<thead>
<tr>
<th>SPE Paper No</th>
<th>Year</th>
<th>Title</th>
<th>Authors</th>
<th>Contribution</th>
</tr>
</thead>
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<tr>
<td>0450</td>
<td>1963</td>
<td>“A Method for Predicting the performance of unstable Miscible Displacement in Heterogeneous”</td>
<td>E.J. Koval</td>
<td>Used K factor to predict viscous fingering in Heterogeneous reservoir. Modified the Buckley Leverette equation to account for heterogeneity and viscosity</td>
</tr>
<tr>
<td>20156</td>
<td>1989</td>
<td>“A review of Heterogeneity Measures used in Reservoir Characterisation”</td>
<td>Larry W Lake, Jerry L. Jensen</td>
<td>Described and classified Heterogeneity as static, spatial and dynamic. Summarised different methods to categorise Heterogeneity</td>
</tr>
<tr>
<td>26436</td>
<td>1993</td>
<td>“Enhanced Reservoir Description: using Core and Log Data to Identify Hydraulic (flow ) Units and Predict Permeability in Uncored Intervals/Wells”</td>
<td>Jude O. Amaefule, Mehmet Altunbay</td>
<td>Derived a new approach to identify and characterize hydraulic units in reservoir. With this new technique permeability prediction could also be more accurately determined from Wireline logs</td>
</tr>
<tr>
<td>36988</td>
<td>1996</td>
<td>“Statistical Analysis of Well Productivity in Heterogeneous Reservoirs”</td>
<td>K.S. Lee, M.A. Miller, K. Sepehmoori,</td>
<td>Deduced relationship between productivity of well and reservoir heterogeneity in closed depletion reservoir. Both univariate and Bivariate analysis on pressure drawdown with heterogeneity was done</td>
</tr>
<tr>
<td>127290</td>
<td>2009</td>
<td>“Effect of Carbonate Heterogeneity on Core - Log Integration”</td>
<td>Robin M. MacDonald, David G. Kersey, Tianhua Zhang, Mahmood Akbar, Wail Mousa</td>
<td>Deduced optimum size of core in carbonate formation with Heterogeneities to have core-log integration</td>
</tr>
<tr>
<td>In Review</td>
<td>2013</td>
<td>“Quantifying Petro physical heterogeneity in complex hydrocarbon reservoirs”</td>
<td>Peter Fitch, Sarah Davies, Mike Lovell, Tim Pritchard</td>
<td>Heterogeneity was defined from various aspects was summed together in this paper. In addition methods to determine heterogeneity using various statistical methods was discussed.</td>
</tr>
<tr>
<td>In Review</td>
<td>2013</td>
<td>“The petro physical link between reservoir quality and Heterogeneity: Application of the Lorenz coefficient”</td>
<td>Peter Fitch, Sarah Davies, Mike Lovell, Tim Pritchard</td>
<td>Relationship of Reservoir Heterogeneity with Reservoir quality was undertaken. Also Effect of various petro physical parameters on Lorenz curve was studied.</td>
</tr>
</tbody>
</table>
Appendix B: Critical Literature Review

SPE 0450

Paper title:
A Method for Predicting the Performance of Unstable Miscible Displacement in Heterogeneous media.

Authors:
E.J. Koval

Objective of the paper:
To predict the recovery factor and solvent cut as a function of pore volume in an unstable miscible displacement process as a result of heterogeneity and viscosity difference between the two fluids.

Methodology Used:
Buckley-Leverett equation was modified introducing the concept of heterogeneity H and viscosity ratio E between the oil and the displacing fluid. The viscosity ratio represents the mixing efficiency between the two fluids to account for miscible displacement. Heterogeneity factor was related to Dykstra–Parsons (P) heterogeneity factor, as both represent channeling in the reservoir. As H included additional effect of dispersion a relationship was sought between H and P. Through experimental data it was confirmed that there exists an agreement between H and P as long as dispersion part of heterogeneity remains small.

Conclusion:
The Use of Buckley leveret with inclusion of K factor which can predict the interaction of heterogeneity on viscous fingering. The modified Buckley leveret can then be used to predict recovery and solvent cut for miscible displacement process. In addition experimental data showed that for low dispersion effect heterogeneity could be linked to Dyktra-Parson coefficient static heterogeneity.

Comments:
This paper shows how dynamic heterogeneities could be linked to static heterogeneities for low dispersion effects and using this heterogeneity recovery factor could be calculated.
SPE 20156 (1989)

Paper title:
A review of Heterogeneity Measures used in Reservoir Characterisation

Authors:
Larry W Lake, Jerry L. Jensen

Objective of the paper:
Define and divide heterogeneity into different types based on the scale of investigation. Understand the various techniques for assessing the different types of heterogeneity in permeability.

Methodology Used:
Heterogeneity was categorized into 3 major types namely Static, Static with correlation and Dynamic. In addition the tools/methods employed to quantify each type of heterogeneity was also discussed.

Conclusion
Tools such as Dyktra-Parson and Lorenz curve can provide measure of static heterogeneity in a system. Static heterogeneity with correlation can be quantified with statistical tools such as Polasek and Hutchinson’s Factor and Alpay’s sand index. Dynamic heterogeneity is dependent on factors such as channeling and mixing and is determined by resistance to fluid flow through a system.

Comments:
One of the first papers to give detailed explanation on the types of heterogeneity and the tools/methods employed to quantify them.
SPE 26436 (1993)

Paper title: Enhanced Reservoir Description: Using Core and Log data to identify hydraulic flow units and predict permeability in uncored intervals/wells.

Authors: Jude O. Amaefule, Mehmet Altunbay, Djebar Tiab U, David G. Kersey and Dare K. Keelan

Objective of the paper: To develop a new methodology for identification and characterization of hydraulic flow units. Once this method has been mapped, tool responses from certain logging tool can be used to develop regression model and predict permeability in cored and uncored wells.

Methodology Used: Kozney-Carmen equation and concept of mean hydraulic radius was used as a basis for developing a new methodology for identification of flow units. Kozney-Carmen equation is given by

\[ K = \frac{\phi^3}{(1 - \phi)^2} \left( \frac{1}{F \sigma^2 \nu^2} \right) \]

Where
\[ F \sigma^2 = \text{Kozeny constant} \]
And \( s_{gr} = \text{surface area per unit grain volume} \)
Dividing equation by \( \phi \) and taking square root
Following equation was reached \( \text{FZI} = \text{RQI}/\phi \). The equation was rewritten in logarithmic form to show the plot of reservoir quality against normalised porosity for each hydraulic unit s gave the Flow zone indicator for the same.

The theory was further enhanced to derive a new equation for capillary function based on concept of hydraulic unit.

Conclusion
Permeability and permeability distribution can be predicted in wells which are uncored based on hydraulic unit concept. Different reservoir rocks types could be distinguished based on the concept of Flow zone units.

Comments: This is a simple way of identifying and distinguishing different reservoir units through use of a cross-plot of Reservoir quality against normalised porosity.
SPE 36988 (1993)

Paper title:
Statistical Analysis of Well Productivity in Heterogeneous Reservoirs.

Authors:
K.S Lee, M.A. Miller and K. Sepehmoori.

Objective of the paper:
To investigate the effect of static heterogeneity using correlation on the productivity of a well in a closed depletion drive. Both univariate and bivariate analysis was carried to come to a conclusion.

Methodology Used:
A geo statistical model with certain average permeability heterogeneity measured by Dyktra-Parsons coefficient correlation length and spherical variation variogram was used in the analysis. Performance of the well was studied by varying the properties of this model. This was done by generating permeability in probabilistic sense and varying heterogeneity and correlation length. Two separate analyses were conducted. In univariate analysis the effect of heterogeneity on drawdown pressure was observed. In bivariate analysis the correlation between drawdown pressure and well block permeability for different heterogeneity was studied.

Conclusion:
- For univariate analysis it was seen that the drawdown pressure increased with heterogeneity and correlation length. The dependency was greater on Dyktra-Parsons coefficient than correlation length.
- A log-log regression equation relates well block permeability and drawdown pressure.

Comments
This paper shows the relationship between heterogeneity and drawdown pressure in a reservoir. In addition a mathematical equation has been reached to predict pseudo pressure with permeability.
SPE 127290 (2009)

Paper title:
Effect of Carbonate Heterogeneity on Core - Log Integration.

Authors:
Robin M. MacDonald, David G. Kersey, Tianhua Zhang, Mahmood Akbar, Wail Mousa.

Objective of the paper:
To determine the effective size of cores and logs so they measure rock volumes representing all rock properties.

Methodology Used:
Logging and coring data was obtained for different wells. Porosity and mineralogy was determined from density, neutron and elemental capture spectroscopy (ECS) wireline logs. These values were compared with porosity, XRF (X-ray florescence) and XRD (X-ray diffraction) from coring data of the same wells. Core data consisted of 3 different scales/frequency of investigation.

Conclusion:
Homogenous medium is less affected by coring sample size and normal core plugs correlate well with wireline logs. For reservoirs with heterogeneity length 2-8 in., core plugs with 3-4in sampling rate can be used to represent the reservoir. For higher heterogeneity, finer core samples should be used.

Comments:
This paper shows the effect heterogeneity on core sampling rate.
Paper title:
The Petrophysical link between Reservoir Quality and heterogeneity: application of the Lorenz coefficient

Authors:
Peter Fitch, Sarah Davies, Mike Lovell, and Time Pritchard.

Objective of the paper:
To determine heterogeneity in a petrophysical system using Lorenz coefficient. In addition, the effect of heterogeneity in carbonate and silicate reservoirs on the reservoir quality index has been carried out.

Methodology Used:
Synthetic linear and exponential datasets were constructed to understand the basic controls on both the Lorenz Coefficient and the shape of the Lorenz Curve. The traditional single Lorenz curve was extended to dual property of Lorenz curve. To study the effect of heterogeneity on reservoir quality, proprietary data from three carbonate reservoirs was used. Heterogeneity was measured by using single and dual Lorenz curve. The values of heterogeneity were compared against reservoir quality of each unit and conclusion drawn therefrom.

Conclusion:
- A Lorenz coefficient of 0.1 in porosity and 0.63 in permeability marks the distinction between linear and exponential pattern or alternatively between low and high heterogeneity.
- The reservoir quality in carbonate reservoirs increases with heterogeneity.

Comment:
This paper gives an outline regarding the different tools of investigating heterogeneity and its relationship to reservoir quality.
Paper title:  
Quantifying petrophysical heterogeneity in complex hydrocarbon reservoirs.

Authors:  
Peter Fitch, Sarah Davies, Mike Lovell, and Time Pritchard.

Objective of the paper:  
To understand the various parameters governing heterogeneity and the different statistical tools that can be used to quantify them.

Methodology Used:  
Heterogeneity as perceived by different authors was summarised to define the concept of heterogeneity. The effect of scale of investigation on heterogeneity was then studied. In the subsequent part the technique for characterizing/quantifying heterogeneity was explored. Logging derived porosity from a carbonate reservoir was used for the analysis. Heterogeneity was characterized using tools such as tables and histogram. Heterogeneity was quantified using statistical tools such as Lorenz coefficient and Dyktra-Parson technique.

Conclusion:  
Heterogeneity exists at variety of scales and should be defined in terms of structure and resolution. Various statistical tools such as Lorenz curve, Dyktra-parson and coefficient of variation could be used to quantify heterogeneity.

Comment:  
Through this paper various tools with their methods were outlined to quantify heterogeneity.
Appendix C: Methodology

C1: Controls on Lorenz curve

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<th>Total No of data</th>
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<th>value of “High value decimal”</th>
<th>Frequency of “Low Value decimal number”</th>
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<td>-</td>
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<tr>
<td>B2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Increased</td>
<td>-</td>
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<tr>
<td>B3</td>
<td>Increased</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B4</td>
<td>-</td>
<td>decreased</td>
<td>-</td>
<td>decreased</td>
<td>-</td>
<td>Mid value decimal numbers included to compensate for decrease in overall no of data points</td>
</tr>
<tr>
<td>B5</td>
<td>-</td>
<td>Increased in steps of 5 percent</td>
<td>-</td>
<td>Decreased by same ratio to maintain same number of data points</td>
<td>-</td>
<td>--</td>
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<tr>
<td>B6</td>
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<td>Increased in steps and repeated case no 5.</td>
<td>Decreased by same ratio to maintain same number of data points</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

**Table C1-1** Sensitivity Analysis Variable B

**Figure C1-1:** Sensitivity Analysis variable B
Investigating the relationship between Reservoir Quality and Heterogeneity in Carbonate Reservoirs

Figure C1-2: Initial Data set 2, variable A&B analysis

<table>
<thead>
<tr>
<th>Variable A</th>
<th>Variable B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
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</tr>
<tr>
<td>40</td>
<td>0.25</td>
</tr>
<tr>
<td>40</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Low value integer "1" with frequency 90%
High value integer "40" with frequency 10%
Low value decimal "0.1" with frequency 90%
High value decimal number "0.25" with frequency 10%

Figure C1-3: Initial Data Set 3, variable A&B analysis

<table>
<thead>
<tr>
<th>Variable A</th>
<th>Variable B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
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<tr>
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<td>0.1</td>
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</tr>
<tr>
<td>80</td>
<td>0.5</td>
</tr>
<tr>
<td>80</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Low value integer "1" with frequency 90%
High value integer "80" with frequency 10%
Low value decimal "0.1" with frequency 90%
High value decimal number "0.5" with frequency 10%
Investigating the relationship between Reservoir Quality and Heterogeneity in Carbonate Reservoirs

Figure C1-4: Sensitivity analysis Variable A&B, Data Set 2

Figure C1-5: Sensitivity analysis Variable A&B, Data Set 3
C2: Effect of Tool resolution and Sample frequency on Lorenz coefficient

Figure C2-1: Coring data at different sampling frequency (30% SD data)

Figure C2-2: Coring data at different sampling start depth (30% SD data)

C3: Relationship of heterogeneity on reservoir quality

<table>
<thead>
<tr>
<th>Case No</th>
<th>Average permeability (md)</th>
<th>Average porosity</th>
<th>Variance about mean (permeability)</th>
<th>Variance about mean(porosity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP6</td>
<td>400</td>
<td>0.13</td>
<td>5%</td>
<td>10%</td>
</tr>
<tr>
<td>HP7</td>
<td>400</td>
<td>0.13</td>
<td>5%</td>
<td>20%</td>
</tr>
<tr>
<td>HP8</td>
<td>400</td>
<td>0.13</td>
<td>5%</td>
<td>30%</td>
</tr>
<tr>
<td>HP9</td>
<td>400</td>
<td>0.13</td>
<td>5%</td>
<td>40%</td>
</tr>
</tbody>
</table>

Table C3-1: Sensitivity for heterogeneity in porosity (0.13)
### Table C3- 2: Sensitivity for heterogeneity in permeability (100 md) and porosity (0.13)

<table>
<thead>
<tr>
<th>Case No</th>
<th>Average permeability (md)</th>
<th>Average porosity</th>
<th>Variance about mean(permeability)</th>
<th>Variance about mean(porosity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Data</td>
<td>100</td>
<td>0.05</td>
<td>5%</td>
<td>5%</td>
</tr>
<tr>
<td>HPP9</td>
<td>100</td>
<td>0.05</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>HPP10</td>
<td>100</td>
<td>0.05</td>
<td>10%</td>
<td>40%</td>
</tr>
<tr>
<td>HPP11</td>
<td>100</td>
<td>0.05</td>
<td>20%</td>
<td>20%</td>
</tr>
<tr>
<td>HPP12</td>
<td>100</td>
<td>0.05</td>
<td>20%</td>
<td>30%</td>
</tr>
<tr>
<td>HPP13</td>
<td>100</td>
<td>0.05</td>
<td>20%</td>
<td>40%</td>
</tr>
<tr>
<td>HPP14</td>
<td>100</td>
<td>0.05</td>
<td>30%</td>
<td>20%</td>
</tr>
<tr>
<td>HPP15</td>
<td>100</td>
<td>0.05</td>
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</tr>
<tr>
<td>HPP16</td>
<td>100</td>
<td>0.05</td>
<td>40%</td>
<td>40%</td>
</tr>
</tbody>
</table>

### Table C3- 3: Sensitivity for high heterogeneity in permeability (800 md) and porosity (0.32)

<table>
<thead>
<tr>
<th>Case No</th>
<th>Average permeability (md)</th>
<th>Average porosity</th>
<th>Variance about mean(permeability)</th>
<th>Variance about mean(porosity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPE9</td>
<td>800</td>
<td>0.32</td>
<td>95%</td>
<td>15%</td>
</tr>
<tr>
<td>HPE10</td>
<td>800</td>
<td>0.32</td>
<td>95%</td>
<td>10%</td>
</tr>
<tr>
<td>HPE11</td>
<td>800</td>
<td>0.32</td>
<td>70%</td>
<td>5%</td>
</tr>
<tr>
<td>HPE12</td>
<td>800</td>
<td>0.32</td>
<td>55%</td>
<td>10%</td>
</tr>
</tbody>
</table>
Appendix D: Results

D1: Controls on Lorenz curve

Figure D1-1: Lorenz Coefficient plots and values for Variable B sensitivity analysis (sorted in ascending order). a) Case B1, b) Case B2, c) Case B3, d) Case B4, e) Case B5, f) Case B6

D2: Effect of Tool resolution and Sample frequency on Lorenz coefficient

Figure D2-1: Effect of sampling frequency on Lorenz coefficient

Figure D2-2: Effect of sampling start depth on Lorenz coefficient
D3: Effect of heterogeneity on reservoir quality

Figure D3-1: Increase in reservoir quality with heterogeneity in porosity (Case HP-6-HP10)

Figure D3-2: Increase in reservoir quality with heterogeneity in porosity and permeability (case HPP 9-16)
Investigating the relationship between Reservoir Quality and Heterogeneity in Carbonate Reservoirs

Figure D3-3: Heterogeneity in permeability and porosity Dual Lc against FZI

Figure D3-4: Effect of High Heterogeneity in permeability on reservoir quality (Case HPE1)
Investigating the relationship between Reservoir Quality and Heterogeneity in Carbonate Reservoirs

Figure D3-5: Illustration of Effect of High heterogeneity in permeability on Reservoir Quality (Case HPE2-HPE8)

Figure D3-6: Effect of High heterogeneity in permeability on reservoir quality (Case HPE9-HPE12)

No particular trend between Lorenz coefficient (porosity) and FZI.