

# 21-cm signatures of residual HI inside cosmic HII regions during reionization

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## ABSTRACT

We investigate the impact of sinks of ionizing radiation on the reionization-era 21-cm signal, focusing on 1-point statistics. We consider sinks in both the intergalactic medium and inside galaxies. At a fixed filling factor of H II regions, sinks will have two main effects on the 21-cm morphology: (i) as inhomogeneous absorbers of ionizing photons they result in smaller and more widespread cosmic H II patches; and (ii) as reservoirs of neutral gas they contribute a non-zero 21-cm signal in otherwise ionized regions. Both effects damp the contrast between neutral and ionized patches during reionization, making detection of the epoch of reionization with 21-cm interferometry more challenging. Here we systematically investigate these effects using the latest semi-numerical simulations. We find that sinks dramatically suppress the peak in the redshift evolution of the variance, corresponding to the midpoint of reionization. As previously predicted, skewness changes sign at midpoint, but the fluctuations in the residual HI suppress a late-time rise. Furthermore, large levels of residual HI dramatically alter the evolution of the variance, skewness and power spectrum from that seen at lower levels. In general, the evolution of the large-scale modes provides a better, cleaner, higher signal-to-noise probe of reionization.

**Key words:** Key words: dark ages, reionization, first stars – intergalactic medium – methods: statistical – cosmology: theory.

## 1 INTRODUCTION

Reionization, the process in which the first galaxies ionized their surroundings and ultimately the entire Universe, is poorly constrained at present. The 21-cm signal, as produced by a hyperfine transition in neutral hydrogen (HI), provides an excellent tracer of the neutral IGM. It is therefore the hope that future observations of the 21-cm brightness temperature using immense radio arrays, e.g. the LOw Frequency ARray<sup>1</sup> (LOFAR), the Murchison Wide-field Array<sup>2</sup> (MWA), the Precision Array for Probing the Epoch of Reionization<sup>3</sup> (PAPER), the Hydrogen Epoch of Reionization Array<sup>4</sup> (HERA) and the Square Kilometre Array<sup>5</sup> (SKA), will dramatically improve our understanding of reionization.

To make optimal use of these observations it is vital that we build insight into the wide range of physical processes involved in reionization. Two of the most important factors to consider are the star formation history and the degree to which ionizing radi-

ation from galaxies escapes into the intergalactic medium (IGM). These two physical mechanisms have naturally been the focus in the modelling of reionization, with both semi-numerical (e.g. Zahn et al. 2007; Mesinger & Furlanetto 2007; Choudhury et al. 2009; Mesinger et al. 2011; Santos et al. 2010; Thomas & Zaroubi 2011) and numerical simulations (e.g. Trac & Cen 2007; Zahn et al. 2007; Baek et al. 2009; Trac & Gnedin 2011; Iliev et al. 2014) being used to refine our understanding of both. These simulations describe how ionized regions form, grow and ultimately merge to ionize the entire IGM. However, it is not enough to understand the sources alone, we must also consider the sinks of ionizing radiation, i.e. the atoms that once ionized will recombine, wasting ionizing photons that would otherwise contribute to reionization. Sinks will have two main effects on the 21-cm signal:

- (i) as absorbers of ionizing photons they impact on both the timing of reionization and the morphology of cosmic H II patches;
- (ii) as reservoirs of neutral gas they contribute a non-zero 21-cm signal in otherwise ionized regions.

Sinks can reside in both the IGM and the interstellar medium (ISM), i.e. inside and outside of galaxies. In the ionized IGM at reionization redshifts, sinks mostly consist of diffuse structures and partially self-shielded gas clumps called Lyman-limit systems (LLSs, e.g. Furlanetto & Oh 2005,

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<sup>1</sup> <http://www.lofar.org/>

<sup>2</sup> <http://www.mwatelescope.org/>

<sup>3</sup> <http://eor.berkeley.edu/>

<sup>4</sup> <http://reionization.org/>

<sup>5</sup> <http://www.skatelescope.org/>

McQuinn, Oh, & Faucher-Giguère 2011; Muñoz et al. 2014). LLSs are inhomogeneous in both space and time (Crociani et al. 2011; Choudhury et al. 2009); as such, they can dramatically impact the duration and morphology of reionization (effect i above; Sobacchi & Mesinger 2014). Although they are mostly ionized (e.g. McQuinn, Oh, & Faucher-Giguère 2011; Rahmati et al. 2013), LLSs can also contribute to effect (ii). Indeed Sobacchi & Mesinger (2014) recently estimated that LLSs on average contribute a few percent neutral fraction inside cosmic H II patches during reionization.

Sinks are also present inside galaxies where H I can self shield in dense clumps. At lower redshifts (e.g. Wolfe, Gawiser, & Prochaska 2005), sinks inside galaxies [mostly so-called Damped Lyman-alpha systems (DLAs); Prochaska et al. 2005; Noterdaeme et al. 2012] provide a larger reservoir of H I compared to sinks in the IGM. Recent observations suggest that the contribution of DLAs to the cosmological mass density is roughly constant until it increases from  $z = 2.3$  to  $z = 3.5$  by almost a factor of 2 (Noterdaeme et al. 2012). There is currently no consensus in explaining these trends, making extrapolations to reionization redshifts uncertain. The abundance and ionization structure of such galactic sinks are more susceptible to the local environment, including stellar feedback and baryonic cooling. Therefore it is difficult to quantify the contribution of galactic sinks to effect (ii) above. Using a ‘tuning-knob’ approach (similar to the one we use below), Wyithe et al. (2009) predict that by reducing the contrast between cosmic ionized and neutral patches (effect ii), galactic H I could indeed damp the 21-cm power spectrum by 10-20%. Galactic H I also contributes to effect (i) by reducing the efficiency with which ionizing radiation escapes into the IGM, usually quantified with the so-called ionizing photon escape fraction,  $f_{\text{esc}}$ . This clearly has an effect on the timing of reionization. However, reionization studies include the escape fraction and the associated uncertainties in the source term.

In this work we examine the effect that both LLSs (IGM sinks) and differing levels of galactic H I (galactic sinks) have on the statistics of the 21-cm brightness temperature, with emphasis on 1-point statistics. We analyse the simulation presented in Sobacchi & Mesinger (2014), here after SM2014, to isolate the impact of IGM sinks on the 1-point statistics via effects (i) and (ii). We also test how the statistics of this simulation are altered in the presence of galactic sinks due to effect (ii).

The rest of this paper is structured as follows. In Section 2 we describe the simulations that we use to model reionization including sinks in both the IGM and in galaxies; in Section 3 we present our analysis methods and examine the 21-cm maps and their probability-density functions; we then consider the evolution of the 1-point statistics and our ability to constrain them with radio telescopes present and future in Sections 3.4 and 3.5, Section 4 considers how much the power spectrum might be altered in the presence of galactic sinks and finally in Section 5 we summarise our findings. We use the following cosmological parameters  $(\Omega_m, \Omega_\Lambda, \Omega_b, h, \sigma_8, n) = (0.28, 0.72, 0.046, 0.70, 0.82, 0.96)$ , consistent with recent measurements by the Planck satellite (Planck Collaboration et al. 2014).

## 2 THE SIMULATIONS

To investigate the impact of residual H I on 21-cm statistics, we make use of the public simulation tool, 21CMFAST<sup>6</sup>. The density and velocity fields in 21CMFAST are generated with standard perturbation theory (Zel’dovich 1970). To generate ionization fields the excursion-set approach of Furlanetto et al. (2004) is applied to the evolved density fields: to determine whether a region is ionized, this algorithm compares at decreasing radii the number of ionizing photons to the number of baryons (plus recombinations).

In this work, we study how sinks of ionizing photons impact the 21-cm signal, focusing on 1-point statistics. To do this, we run four 21CMFAST realisations of the ionization field during reionization, varying the impact of H I on both the large-scale reionization morphology and the level of residual H I inside cosmic H II regions (effects i and ii discussed in the introduction). We discuss the details below, showing a summary of the four simulations used in Table 1 and Figure 1. All simulations are  $L = 300$  Mpc on a side with a resolution of  $400^3$ . The density and velocity fields are the same in all of the runs.

### 2.1 Homogeneous recombinations (21CMFASTv1)

We call our reference simulation ‘NoLLS’. Sinks have a minimal impact in this simulation. It is generated with the current public release, 21CMFASTv1. The resulting ionization fields are similar to ones obtained with radiative transfer simulations (e.g. Zahn et al. 2011). In 21CMFASTv1, a region is ionized if its (time-integrated) number of ionizing photons exceeds the number baryons plus a global average number of recombinations,  $\bar{n}_{\text{rec}}$ :

$$\zeta f_{\text{coll}}(\mathbf{x}, z, R, M_{\text{min}}) \geq 1 + \bar{n}_{\text{rec}}, \quad (1)$$

where  $f_{\text{coll}}(\mathbf{x}, z, R, M_{\text{min}})$  is the fraction of matter, inside a region of scale  $R$ , which resides in halos with mass greater than  $M_{\text{min}}$  (taken here to correspond to a virial temperature of  $10^4$  K). The ionizing efficiency can be expanded as  $\zeta = f_{\text{esc}} f_* N_{\gamma/b}$ , where  $f_{\text{esc}}$  is the fraction of ionizing photons that escape galaxies,  $f_*$  is the fraction of galactic baryons inside stars, and  $N_{\gamma/b}$  is the number of ionizing photons produced per stellar baryon. This ionization criterion in eq. (1) is checked in an excursion-set fashion (Furlanetto et al. 2004), starting from a maximum radius  $R_{\text{max}}$ <sup>7</sup> down to the cell size. At the last smoothing scale, the unresolved, sub-cell H II fraction is set to  $\zeta(1 + \bar{n}_{\text{rec}})^{-1} f_{\text{coll}}(\mathbf{x}, z, R_{\text{cell}}, M_{\text{min}})$ .

The ‘NoLLS’ run has a different reionization evolution and morphology from the other three runs.<sup>8</sup> In order to facilitate direct morphological comparisons, we vary  $\bar{n}_{\text{rec}}$  in ‘NoLLS’ to match the redshift evolution of the filling factor of H II regions,  $Q_{\text{HII}}$  from the other runs.

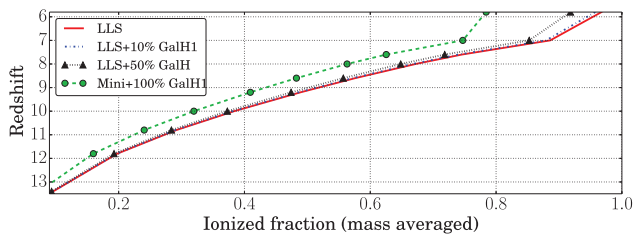
<sup>6</sup> <http://homepage.sns.it/mesinger/Sim.html>

<sup>7</sup>  $R_{\text{max}}$  is a free parameter based on ionized photon mean free path at the redshifts of interest, see Storrie-Lombardi et al. (1994); Miralda-Escude (2003); Choudhury et al. (2008). In our ‘NoLLS’ run it is set to 30 Mpc. However there is no maximum smoothing radius enforced by the simulations of SM2014 since the ionized photon mean free path is determined by the properties of ‘LLS’, which are computed self-consistently.

<sup>8</sup> The mean free path,  $R_{\text{max}}$ , framework does not capture the sub-grid recombination physics of the SM2014 model, discussed in the next section. Nevertheless, these authors find that a choice of  $R_{\text{max}} \sim 10$  Mpc results in a reionization morphology and evolution roughly consistent with their sub-grid model. This ‘effective ionizing photon horizon’ is in fact much smaller than the physical mean free path (defined based on the instantaneous recombination rate and emissivity), and is empirically derived.

**Table 1.** Model summary.

Model	Properties	$\langle x_{\text{HI}} \rangle_M$ in H II regions at $z = 9.20$ , $Q_{\text{HI}} \sim 0.5$
NoLLS	21CMFASTv1; sinks are included as a constant in the ionizing efficiency. Cosmic H II patches are fully ionized.	0.0
LLS	Self-consistent treatment of local recombinations & UVB feedback of SM2014. Small absorbers act as sinks of ionizing radiation to slow the progress of reionization, reduce the size of ionized regions & suppress the 21-cm temperature contrast between H II (over-dense) and H I (under-dense) regions.	0.03
LLS + Gal H I	‘LLS’ with the addition of a fixed percentage of remnant H I in galaxies; its contribution to the 21-cm signal cosmic inside H II regions further suppresses the contrast between ionized and neutral patches.	$0.1 f_{\text{coll}}: 0.04$ $0.5 f_{\text{coll}}: 0.07$
Mini + 100% Gal H I	‘LLS’ with both sterile minihalos (cooled via H <sub>2</sub> ) as well as atomic cooling halos 100% neutral. Describes the most intense impact that remnant H I could have on the 21-cm signal by suppressing the contrast between cosmic ionized and neutral patches.	0.19



**Figure 1.** Evolution of mass-averaged ionized fraction for our 4 main models (by construction, the volume-averaged evolution is the same in all models). To track evolution of the morphology of H II regions, we plot these four models as a function of the H II volume-filling factor hereafter.

## 2.2 Inhomogeneous recombinations in the IGM

As the next step, our ‘LLS’ run includes inhomogeneous recombinations in the IGM using the model of SM2014 (their ‘FULL’ model). Such recombinations, driven by  $\sim$ kpc scale gas clumps can slow the growth of large cosmic H II patches, primarily resulting in reduction of large-scale reionization structure. SM2014 implement these local recombinations by modifying the ionization criterion from eq. (1) to:

$$\zeta f_{\text{coll}}[\mathbf{x}, z, R, \bar{M}_{\text{min}}(\mathbf{x}, z)] \geq 1 + \bar{n}_{\text{rec}}(\mathbf{x}, z). \quad (2)$$

In the model of SM2014, both the number of recombinations and minimum halo mass hosting galaxies depend on the local properties of each  $\sim$ 1Mpc-scale reionization cell. Taking self-shielding into account via the parametrization of Rahmati et al. (2013), and

integrating over the entire density distribution, the recombination rate per baryon in an ionized cell is:

$$\frac{dn_{\text{rec}}}{dt}(\mathbf{x}, z) = \int_0^{+\infty} P_V(\Delta, z) \Delta \bar{n}_{\text{H}\alpha\text{B}} [1 - x_{\text{HI}}(\Delta)]^2 d\Delta, \quad (3)$$

where  $P_V(\Delta, z)$  is the full (non-linear, sub-grid) distribution of overdensities:  $\Delta \equiv n_H/\bar{n}_H$  (Miralda-Escude et al. 2000), and  $x_{\text{HI}}(\Delta)$  is equilibrium neutral fraction computed using the empirical self-shielding prescription of Rahmati et al. (2013). Then the total, time-integrated number of recombinations per baryon, averaged over the local cosmic H II patch can be written as:

$$\bar{n}_{\text{rec}}(\mathbf{x}, z) = \left\langle \int_{z_{\text{IN}}}^z \frac{dn_{\text{rec}}}{dt} \frac{dt}{dz} dz \right\rangle_{\text{HII}}, \quad (4)$$

where  $z_{\text{IN}}$  is the ionization redshift of a simulation cell. Photo-heating feedback from reionization (i.e. the depletion of the gas reservoir available for star-formation), is included in this simulation with a similar approach, in which the minimum halo mass capable of retaining gas is computed using the local (cell’s) photo-ionization history (Sobacchi & Mesinger 2013). Photo-heating feedback has a sub-dominant impact on reionization morphology, compared with inhomogeneous recombinations; however, since the sources and sinks are spatially correlated, the effects are additive in that they both prolong reionization history.

‘LLS’ (corresponding to the ‘FULL’ simulation of SM2014) is our fiducial model for reionization history and morphology. The impact of inhomogeneous, sub-grid H I on the large-scale reionization morphology can be estimated by comparing the ‘NoLLS’ and ‘LLS’ runs (see Figure 2). Our ‘LLS’ run also includes residual H I inside the cosmic H II patches, contributing on average a mass-averaged neutral fraction of a few percent inside H II regions (SM2014; see table 1).

## 2.3 Inhomogeneous recombinations in the IGM + galactic H I

Our remaining runs use the same large-scale reionization morphology and redshift evolution as ‘LLS’ (see Figure 2). However here we explicitly increase the level of residual H I inside the ionized regions. Since the amount of H I inside galaxies is very difficult to predict, we adopt a ‘tuning-knob’ approach similar to Wyithe et al. 2009. Specifically, we compute the average<sup>9</sup> collapsed fraction inside each cell, using the conditional (dependent on the cell’s density) mass fraction (e.g. Bond et al. 1991; Lacey & Cole 1993) normalized to the mean predicted by Sheth-Tormen (e.g. Jenkins et al. (2001); see also the discussion in Barkana & Loeb 2004; Mesinger et al. 2011). We then assign a fixed fraction,  $\alpha$ , of the cell’s baryonic mass to be neutral, so that each cell’s galactic H I number density is  $\alpha \Delta_{\text{cell}} \bar{n}_H f_{\text{coll}}(M_{\text{min}}, z, \Delta_{\text{cell}}, R_{\text{cell}})$ . Keeping with the simplicity of the ‘tuning-knob’ approach, we use a uniform value of  $M_{\text{min}}$  corresponding to a virial temperature of  $10^4$  K. We emphasise that this implementation considers only the additional signal that galactic sinks contribute; their impact on reionization timing is assumed to be accounted for in the ionizing efficiency of sources.

We study different values of  $\alpha$ ; for the sake of brevity, we only present the simulations with 10% and 50% of the galactic mass as neutral hydrogen. To provide a sense of the range of values that  $\alpha$  might take we can extrapolate lower redshift constraints

<sup>9</sup> We ignore the Poisson noise associated with halo discreteness, since this affects scales too small to be observable with the SKA.

from DLAs. The contribution of galactic neutral hydrogen to the critical mass is found to be  $\Omega_{\text{HI}} \sim 10^{-3}$  for  $z \lesssim 4$  (Prochaska et al. 2005; Noterdaeme et al. 2012). This can be related to  $\alpha$  as  $\alpha = \Omega_{\text{HI}}/(\Omega_{\text{b}}f_{\text{coll}})$ , which predicts  $\alpha \sim 0.05$  towards the end of reionization, i.e.  $z \sim 6$ , rising to  $\alpha \sim 1$  at  $z > 11$  as  $f_{\text{coll}}$  decreases with increasing redshift. However at the redshifts for which  $\Omega_{\text{HI}}$  has been constrained,  $\alpha \lesssim 0.03$ ; as such, here after we will make the distinction between ‘reasonable levels of galactic sinks’, ( $\alpha \lesssim 0.1$ ), and ‘large levels’ ( $\alpha \gtrsim 0.5$ ).

It is important to note that the tail of the Miralda-Escude et al. (2000) density distribution should, in principle, include gas inside galaxies. However, the distribution was calibrated to the lower overdensity IGM, and has been shown to be a poor fit to the matter distribution inside lower-redshift galaxies, even neglecting feedback (e.g. Bolton & Becker (2009); McQuinn, Oh, & Faucher-Giguère (2011)). Moreover, the accuracy of the Miralda-Escude et al. (2000) PDF is untested both at the high-redshifts ( $z \sim 10$ ) of interest here, as well as in predicting the conditional (dependent on the local, large-scale overdensity) mass fraction. Hence in our approach we use the conditional, excursion-set approach, which is much better tested in predicting the collapse fraction. However, since the excursion-set galactic HI is added on top of the SM2014 neutral fraction field, our approach is bound to double count some galactic HI, and the values of  $\alpha$  should only be treated as approximate (lower limits). We find that due to the inside-out nature of the epoch of reionization (i.e. biased H II regions), the excursion-set approach generally predicts much higher collapsed fractions inside H II regions than MHR00 (see e.g. table 1); thus we do not expect the double-counting to have a large impact on our conclusions.

#### 2.4 Inhomogeneous recombinations in the IGM + extreme galactic HI

Finally, we consider an extreme model with the maximal amount of residual neutral hydrogen, ‘Mini+100% Gal HI’. Following the same procedure as outlined in the previous section, we assign 100% of the baryonic mass in  $10^3 \leq T_{\text{vir}}$  halos as HI. This model effectively assumes that minihalos with a virial temperature of  $10^3 \leq T_{\text{vir}} < 10^4$  K, i.e. those that have cooled by molecular cooling, have been sterilized by a Lyman-Werner background so that they cannot form stars (e.g. Haiman, Abel, & Rees 2000; Ricotti, Gnedin, & Shull 2001; Mesinger, Bryan, & Haiman 2006). Moreover, this model assumes that the time-scale of minihalo photo-evaporation is much longer than the duration of reionization (e.g. Shapiro et al. 2004; Iliev et al. 2005; Ciardi et al. 2006). Our ‘Mini+100% Gal HI’ run therefore serves to illustrate the most extreme impact that residual galactic HI could have on 21-cm observations.

**Table 2.** Instrumental specifications assumed for noise calculations. LOFAR and SKA parameters are taken from Mellema et al. (2013), HERA from Pober et al. (2014)/ private communications with A. Liu and MWA parameters from Tingay et al. (2013).

Parameter	MWA	LOFAR	HERA	SKA
Number of stations ( $N_{\text{stat}}$ )	128	48	331	450
Effective area ( $A_{\text{eff}}/\text{m}^2$ )	21.5	804	$8.4\text{e}3/N_{\text{stat}}$	$10^6/N_{\text{stat}}$
Maximum baseline ( $D_{\text{max}}/\text{m}$ )	2864	3000	360	$10^4$
Integration time ( $t_{\text{int}}/\text{hours}$ )	1000	1000	1000	1000
Bandwidth ( $B/\text{MHz}$ )	6	6	6	6

### 3 BRIGHTNESS-TEMPERATURE MAPS & MOMENTS

#### 3.1 Computing 1D statistics and noise

Once the ionization boxes have been calculated as described, the brightness temperature ( $\delta T_{\text{b}}$ ) can be computed according to:

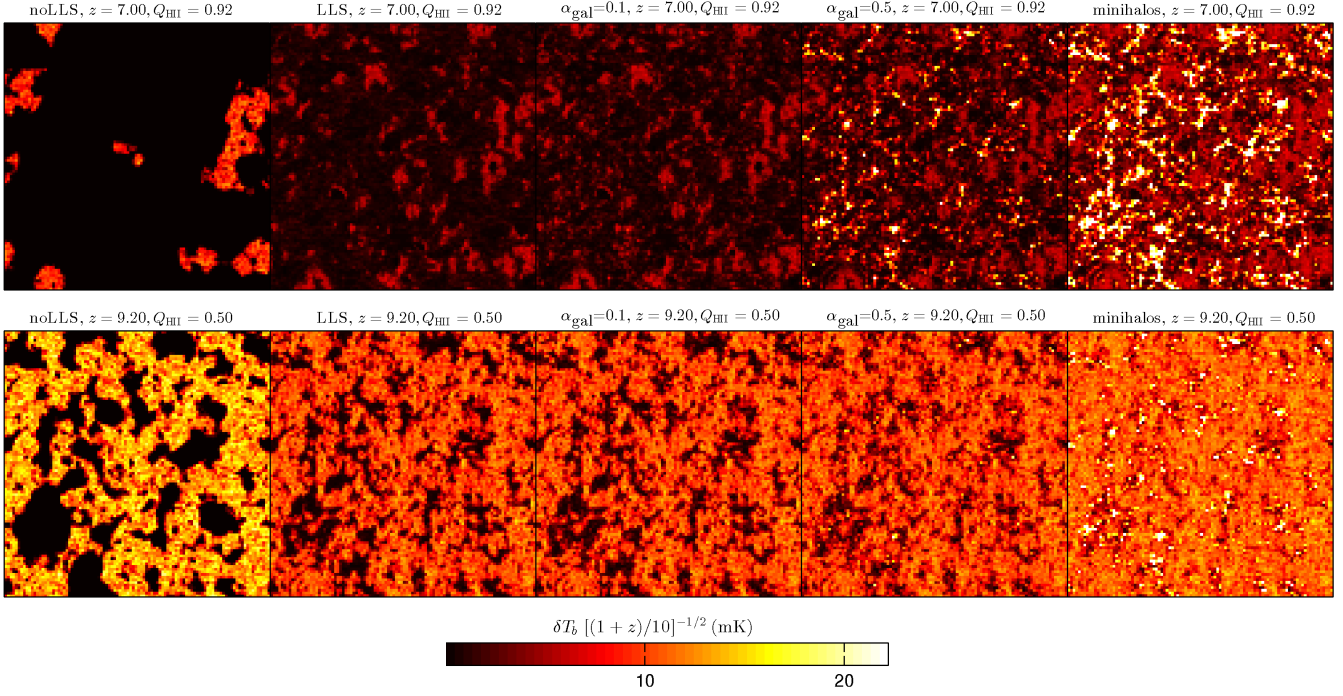
$$\begin{aligned} \delta T_{\text{b}} &= \frac{T_{\text{s}} - T_{\gamma}}{1+z} (1 - e^{-\tau_{\nu_0}}) \\ &\approx 27. \frac{T_{\text{s}} - T_{\gamma}}{T_{\text{s}}} x_{\text{HI}} (1 + \delta) \left[ \frac{H(z)/(1+z)}{dv_{\text{r}}/dr} \right] \\ &\quad \times \left( \frac{1+z}{10} \frac{0.15}{\Omega_{\text{m}} h^2} \right)^{1/2} \left( \frac{\Omega_{\text{b}} h^2}{0.023} \right) \text{mK}, \end{aligned} \quad (5)$$

where  $\delta T_{\text{b}}$  is dependent on the overdensity  $\delta = \rho/\bar{\rho} - 1$ , velocity gradient  $dv_{\text{r}}/dr$  and cosmology. The gas spin temperature  $T_{\text{s}}$  measures the occupation levels of the two hyperfine energy levels involved in the HI 21-cm transition and determines the 21-cm optical depth  $\tau_{\nu_0}$ . It is assumed in this work that  $T_{\text{s}} \gg T_{\gamma}$ , i.e.  $T_{\text{s}}$  is much higher than that of the cosmic microwave background  $T_{\gamma}$ . Whilst this is expected to be a reasonable approximation during reionization due to X-ray heating, the influence of spin temperature fluctuations may not be negligible in the early stages of reionization, i.e.  $Q_{\text{HI}} < 0.2$  (Pritchard & Furlanetto 2007; Mesinger et al. 2013; Ghara et al. 2014).

In our analysis of the simulations described above we use the same approach as Watkinson & Pritchard (2014), hereafter WP2014. We measure the variance  $S_2$  and the skew  $S_3$  of a simulated box according to

$$\begin{aligned} S_2 &= \frac{1}{N_{\text{pix}}} \sum_{i=0}^{N_{\text{pix}}} [\delta T_i - \delta \bar{T}_{\text{b}}]^2, \\ S_3 &= \frac{1}{N_{\text{pix}}} \sum_{i=0}^{N_{\text{pix}}} [\delta T_i - \delta \bar{T}_{\text{b}}]^3, \end{aligned} \quad (6)$$

where  $\delta T_i$  is the brightness temperature of the  $i^{\text{th}}$  pixel, the average brightness temperature in a box is given by  $\delta \bar{T}_{\text{b}} = N_{\text{pix}}^{-1} \sum_{i=0}^{N_{\text{pix}}} \delta T_i$  and  $N_{\text{pix}}$  is the total number of cells in a box. We will consider two normalisations for the skew, the standard skewness  $S_3/(S_2)^{3/2}$  and the dimensional skewness  $S_3/S_2$  which



**Figure 2.** Brightness-temperature maps for 300 Mpc boxes normalised to  $z = 9$ ; maps are resolved to pixels of  $(3 \text{ Mpc})^3$  and presented with the pixel depth flattened into the page. Maps correspond to, from LLS (left), LLS + 10%GalH I (middle-left), LLS + 50%GalH I (middle-right) and minihalos + 100%GalH I (right). The top row is for  $z=7.0$  when the H II volume filling factor is 0.92; the bottom row corresponds to  $z=9.2$  at which time the H II volume filling factor is 0.50. Black regions are those with zero brightness temperature.

was found to be a more natural normalisation for the skew of the neutral-fraction boxes of WP2014.

We make order of magnitude calculations for the error induced by instrumental noise (using the instrument parameters in table 2), but do not consider foreground residuals. We assume the noise is independent on each pixel but identically distributed in a manner well described by Gaussian random noise with zero mean and standard deviation  $\sigma_{\text{noise}}^2$  where

$$\sigma_{\text{noise}}^2 = 2.9 \text{mK} \left( \frac{10^5 \text{m}^2}{A_{\text{tot}}} \right) \left( \frac{10'}{\Delta\theta} \right)^2 \times \left( \frac{1+z}{10.0} \right)^{4.6} \sqrt{\left( \frac{1 \text{MHz} \cdot 100 \text{hours}}{\Delta\nu \cdot t_{\text{int}}} \right)}. \quad (7)$$

This expression is derived with the noise on the brightness temperature described by  $\Delta T^N = T_{\text{sys}}/\eta_f \sqrt{\Delta\nu t_{\text{int}}}$  where the array filling factor is defined as  $\eta_f = A_{\text{tot}}/D_{\text{max}}^2$  in which  $A_{\text{tot}}$  is the total effective area of the array,  $D_{\text{max}} = \lambda/\Delta\theta$ ; the system temperature  $T_{\text{sys}}$  is assumed to be saturated by the sky temperature at the frequencies of interest. Before measuring the moments we smooth and resample the boxes to correspond roughly with the resolution expected from the various instruments; to do so, we use a Gaussian smoothing kernel with width  $R_{\text{pix}} = 6 \text{ Mpc}$  for LOFAR/MWA and  $R_{\text{pix}} = 3 \text{ Mpc}$  for HERA/SKA (although we note that in this work, LOFAR/MWA results are only presented for larger smoothing scales). We also reduce the assumed frequency resolution of each telescope to correspond to  $R_{\text{pix}}$ , i.e.  $\Delta\nu = H_0\nu_0\sqrt{\Omega_m R_{\text{pix}}}/[c\sqrt{(1+z)}]$ .

To estimate how the noise we've so far discussed will propagate onto the skewness and variance we assume that each pixel has a measured signal  $x_i = \delta T_i + n_i$ , where the noise on the pixel  $n_i$

obeys the properties discussed above. We then use a test statistic for the  $m^{\text{th}}$  moments,  $N_{\text{pix}}^{-1} \sum_{i=0}^{N_{\text{pix}}} (x_i - \bar{x}_i)^m$ , to determine unbiased estimators for the moments of Equation 6. We then calculate the noise-induced variance on each unbiased estimator. These derived 1- $\sigma$  errors are then propagated onto the normalised skewness estimators  $\gamma_3 = \hat{S}_3/\hat{S}_2$  and  $\gamma'_3 = \hat{S}_3/(\hat{S}_2)^{3/2}$  to give

$$\begin{aligned} V_{\hat{S}_2} &= \frac{2}{N} (2S_2\sigma_{\text{noise}}^2 + \sigma_{\text{noise}}^4), \\ V_{\gamma_3} &\approx \frac{1}{(S_2)^2} V_{\hat{S}_3} + \frac{(S_3)^2}{(S_2)^4} V_{\hat{S}_2} - 2 \frac{S_3}{(S_2)^3} C_{S_2 S_3}, \\ V_{\gamma'_3} &\approx \frac{1}{(S_2)^3} V_{\hat{S}_3} + \frac{9}{4} \frac{(S_3)^2}{(S_2)^5} V_{\hat{S}_2} - 3 \frac{S_3}{S_2^4} C_{S_2 S_3}, \end{aligned} \quad (8)$$

where

$$V_{\hat{S}_3} = \frac{3}{N_{\text{pix}}} (3\sigma_{\text{noise}}^2 K_4 + 12S_2\sigma_{\text{noise}}^4 + 5\sigma_{\text{noise}}^6), \quad (9)$$

and

$$C_{\hat{S}_2 \hat{S}_3} = \frac{6}{N_{\text{pix}}} S_3 \sigma_{\text{noise}}^2. \quad (10)$$

### 3.2 Brightness-temperature maps

It is useful to take a look at the maps themselves before analysing their statistics. In the maps of Figure 2, ‘NoLLS’, ‘LLS’, ‘LLS + 10%GalH I’, ‘LLS + 50%GalH I’ and ‘minihalos + 100%GalH I’ are represented from left to right; the top strip corresponds to  $z = 7.0$  and  $Q_{\text{HII}} \sim 0.92$  and the bottom strip corresponds to  $z = 9.2$  and  $Q_{\text{HII}} \sim 0.50$ . All maps have been smoothed and resampled to produce pixels of 3 Mpc on a side, the pixel depth has been flattened into the page.

As mentioned above, the impact of inhomogeneous IGM sinks can be seen by comparing the large-scale reionization morphology of the left panel to the other panels. As discussed in SM2014, recombinations in unresolved structures result in a dramatic suppression of large-scale H II regions. Comparing at a fixed  $Q_{\text{HII}}$ , the distribution of H II patch sizes is both narrower and shifted to smaller scales. The impact is dramatic enough that partially ionized cells, hosting unresolved H II bubbles, become important. The ionized fraction of these cells is only  $\lesssim 10\%$ , but since they correspond to the densest (of those not yet reionized)  $\sim \text{Mpc}$ -scale patches, this is sufficient to damp the signal from the neutral cosmic patches (as evidenced by the ‘yellow’ vs ‘orange’ IGM surrounding the H II bubbles in the ‘NoLLS’ and ‘LLS’ panels). This damps the contrast between the fully ionized and fully neutral regions. We caution however that unresolved, sub-grid H II regions are taken into account only in a simplistic fashion in our model, as described above; further work is required to accurately model such sub-grid physics.

The impact of an increasing level of residual H I inside the cosmic H II patches can be seen by comparing the insides of the ‘black’ regions in the panels of the Figure 2 from left to right. The model of SM2014 (our ‘LLS’) only results in a few percent level of H I inside H II regions, and so on this colour scheme the H II regions are still black. However the extreme galactic H I models, ‘LLS + 50%GalH I’ and ‘minihalos + 100%GalH I’, show a notable level of H I inside the cosmic H II regions, with small-scale structure tracing the distribution of galaxies. This further reduces the contrast between neutral and ionized regions, complicating an interferometric detection of reionization. In principle however, resolving this small-scale H I structure with high-resolution 21-cm interferometry could constrain models of galactic H I.

### 3.3 Probability density functions

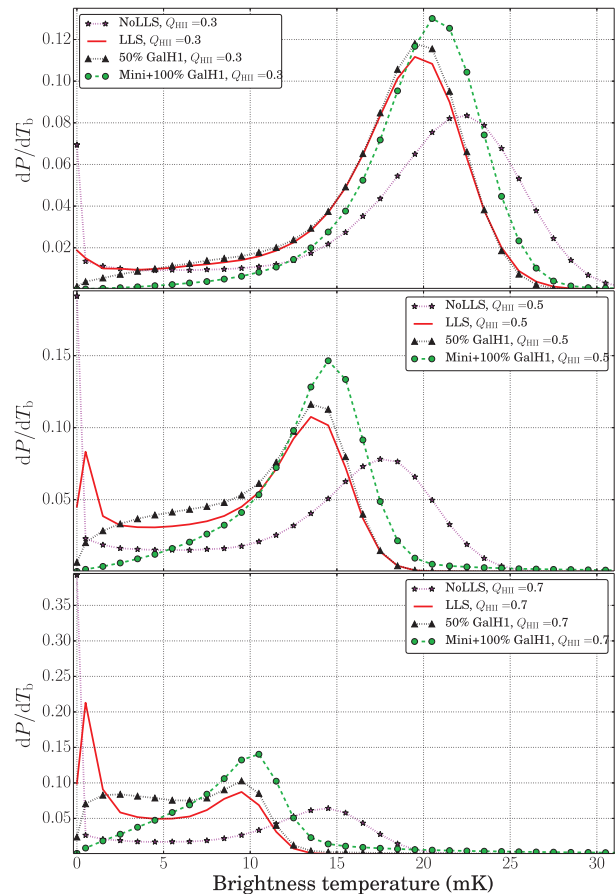
These trends are quantified in Figure 3, where we present the probability density functions (PDFs) from these maps, with lines corresponding to: ‘NoLLS’ (pink dashed w/stars), ‘LLS’ (red solid), ‘LLS + 50% galactic H I’ (black dotted w/triangles) and ‘minihalos + 100% galactic H I’ (green dashed w/circles). Plots correspond to  $Q_{\text{HII}} \approx 0.3, 0.5, 0.7$  ( $z = 10.8, 9.2, 8$ ), top to bottom.

The presence of residual H I inside cosmic H II regions suppresses zero-valued pixels in the PDFs. The bi-modal nature of the PDF is maintained for ‘LLS’ but the sharp peak at  $\delta T_{\text{b}} = 0$  mK is smeared into a wider peak centred on  $\delta T_{\text{b}} \sim 1\text{mK}$  because of a few percent level of residual H I in the ionized IGM (SM2014). It is only with large levels of galactic sinks that the bi-modal nature is entirely suppressed.

Also evident in these PDFs is the impact of sub-cell, unresolved H II regions; these preferentially occur in the highest density pixels hosting newly forming galaxies, which is why there is a suppression of the high  $\delta T_{\text{b}}$  tail in ‘LLS’ when compared to ‘noLLS’. This effect conspires with the loss of a sharp zero peak to further reduce the variance in the maps.

### 3.4 Observing the brightness-temperature moments: Instrumental noise

Figure 4 shows evolution of the variance (top), skewness (middle) and dimensional skewness (bottom) as a function of the H II volume-filling factor. Lines correspond to ‘LLS’ (red solid), ‘LLS + 10% galactic H I’ (blue dot-dashed), ‘LLS + 50% galactic H I’ (black dotted w/triangles), ‘minihalos + 100% galactic H I’ (green



**Figure 3.** Brightness temperature PDFs at  $Q_{\text{HII}} \approx 0.3, 0.5, 0.7$  ( $z = 10.8, 9.2, 8$ ), top to bottom. All maps are smoothed to a co-moving pixel size of 3 Mpc.

dashed w/circles) and ‘NoLLS’ (pink dotted w/ stars); we will use this key for the remains of the paper.

We begin by reviewing what simulations that ignore recombinations in unresolved systems predict for the evolution of the brightness-temperature moments. Studies of statistics from such models identified several important properties that would provide invaluable constraints on the timing of reionization if borne out in reality (e.g. Furlanetto et al. 2004; Harker et al. 2009; Watkinson & Pritchard 2014). It is useful for the discussion that follows to refer to the pink dotted lines w/stars in Figure 4. The amplitude of the variance is maximised at the half-way mark of reionization as the weight of the non-zero distribution of the brightness-temperature’s PDF and its spike at zero are balanced (Figure 4 top). The late-time dominance of the zero-temperature spike induces a rapid increase in skewness at late times (Figure 4 middle). It could be argued that this effect is driven by the variance becoming very small as reionization draws to a close, however the dimensional skewness still exhibits a late-time signature but in the form of a turnover (Figure 4 bottom). At the halfway point both skewness statistics pass from negative to positive. At early times they exhibit a minimum as the PDF transitions from being dominated by the density field to being dominated by the neutral fraction (Lidz et al. 2008); the location of this minimum depends on the overlap of X-ray heating and reionization epochs (Mesinger et al. 2013).

Unresolved H I sinks can dramatically change this picture. Already with ‘LLS’ the magnitude of the variance is suppressed

by up to a factor of two during the mid phases of reionization. Here the additional signal in H II regions, reduces the contrast between ionized and neutral regions. More importantly, smaller H II regions mean that there is more sub-pixel, unresolved ionization structure in ‘LLS’ compared with ‘NoLLS’. This suppression of bi-modality (on resolvable scales) in the ionization structure is further exacerbated by the more disjoint H II regions in ‘LLS’ which get ‘smeared-out’ when the maps are smoothed; i.e. the H II regions in ‘LLS’ are less likely to be resolved by 21-cm interferometers. These effects dramatically suppress the small-scale contrast between ionized and neutral regions; the strength of the turnover associated with the halfway point of reionization is suppressed to the point that even SKA could have difficulty in constraining it. Fortunately, as we will see in the next section, further smoothing of the maps emphasises the turn-over feature, improving our ability to constrain this mid-phase signature. The turn-over in the variance occurs at  $Q_{\text{HII}} \sim 0.4$  in ‘LLS’ rather than nearly exactly halfway through reionization as predicted by ‘NoLLS’. This is because the ionization map is no longer well described as a two-phase field in which the number of pixels in each phase perfectly balance at the mid point of reionization.

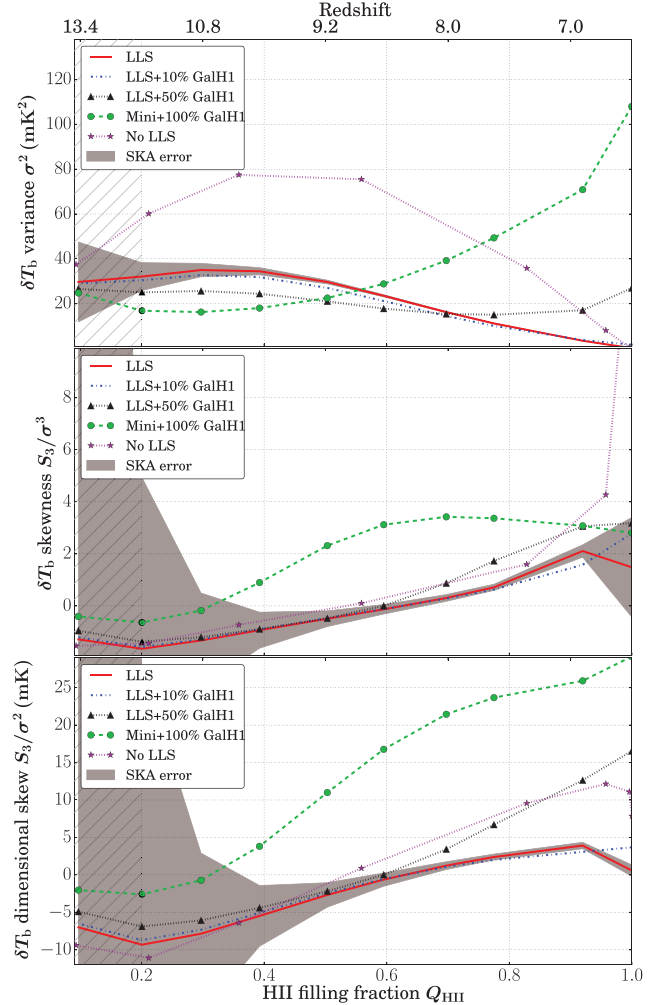
The inclusion of galactic HI reduces the variance even further (see also Wyithe et al. 2009). Reasonable levels of galactic sinks do not dramatically alter the evolution of the variance (as could already be seen from the maps in Figure 2). However, if the galactic HI is tied to a fixed fraction of  $f_{\text{coll}}$  as in our simple illustrative models, the growth of collapsed structure will boost the 21-cm variance at late times. In the extreme minihalo+100%Gal HI model, structure growth totally dominates the signal throughout. SKA could easily detect such a feature.

We see in the middle panel of Figure 4 that the transition from negative to positive skewness including sinks occurs at  $Q_{\text{HII}} \sim 0.6$  (instead of  $Q_{\text{HII}} \sim 0.5$  as predicted by ‘NoLLS’). This transition is reduced to lower  $Q_{\text{HII}}$  in the presence of extreme levels of galactic sinks. For reasonable levels of galactic sinks, this is predominantly due to effect (i), where small H II regions are unresolved. We even see this trend to a lesser degree when smoothing of ‘NoLLS’ on large scales smears-out the H II regions (see Figure 6). For reasonable levels of galactic sinks, this signature and the turn-over in the variance bracket the mid-point and will constrain the timing of reionization more tightly than either statistic on its own.

The late-time rapid increase predicted for the skewness is totally suppressed by the presence of IGM sinks; instead we see a turn over in the skewness of ‘LLS’ as the H II filling factor reaches about 90%. This is because the normalising factor, i.e. the variance, is no longer tending towards zero and we instead see a transition from a skewness driven by the ionization field to one driven by the distribution of IGM sinks. This turn over is wiped out by the presence of even small amounts of galactic sinks. These are important points to be aware of as we begin our efforts to constrain reionization with 21-cm observations. Detecting a turnover in the skewness would imply the presence of IGM sinks (as would reduced variance) and that reionization is in its final phases. The non detection of such a late-time signature in the skewness would not necessarily mean that we are not observing the end of reionization. Its absence combined with evolution in the variance with redshift would instead imply the presence of galactic sinks.

SKA will be sensitive to the details of skewness we have described above, but only in the dimensional skewness, which exhibits more distinct features and better robustness to noise.

The early-time minimum in the skewness statistics is seen to be robust across the models. This is understandable, since the lo-

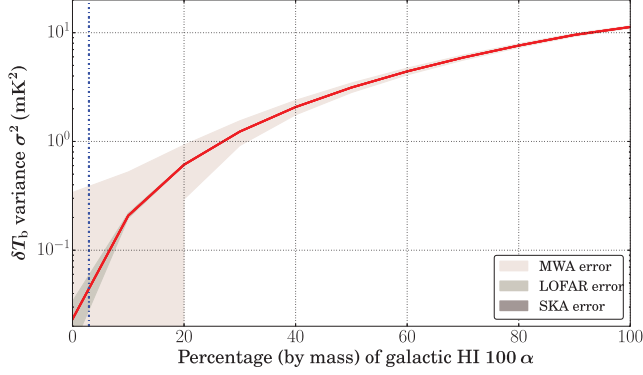


**Figure 4.** Variance (top), skewness (middle) and ‘dimensional skewness’ (bottom) of brightness temperature measured from ‘observed’ maps with a co-moving pixel size of 3 Mpc as a function of volume-filling factor of ionized regions. Beige shadings depict SKA’s 1- $\sigma$  instrumental errors for the ‘LLS’ simulation. Within the hatched region, spin-temperature fluctuations may not be negligible.

cation of this feature is determined by the overlap of X-ray heating and reionization (Mesinger et al. 2013), and here we assume  $T_S \gg T_\gamma$  throughout. Nevertheless, extreme levels of residual HI will almost totally suppress this signature. At its default resolution SKA could struggle to detect this feature, depending on the timings of reionization; however, its non-detection would still provide upper limits on this phase.

Figure 5 shows the variance as a function of the percentage of galactic mass in sinks at the end of reionization, for this simulation  $z = 5.8$ . To reduce noise the maps are smoothed to a radius of  $R_{\text{smooth}} = 10$  Mpc and the beige shadings correspond to, from light to dark, the MWA, LOFAR and SKA 1- $\sigma$  instrumental errors. Note that the signal when measured with SKA-like resolution, i.e. 3 Mpc pixels, is greater by around a factor of ten. The blue dot-dashed line marks the level of residual hydrogen as constrained by DLAs at lower redshift, i.e.  $\alpha = 0.03$  at  $z \sim 4$ .

There are two ways to view this post-reionization signal, one is that it is intrinsic noise to be overcome in our quest to constrain reionization, the other is that it provides a constraint on residual

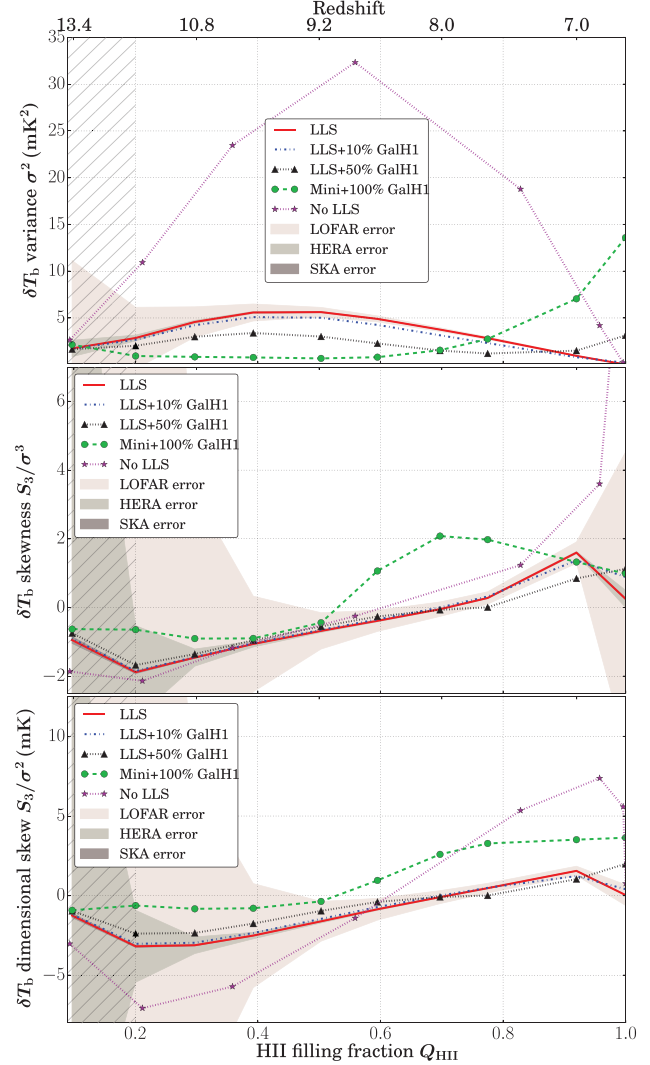


**Figure 5.** Variance as a function of percentage of remnant galactic HI at  $z=5.80$  ( $Q_{\text{HII}} = 1$ ) measured in ‘observed’ maps smoothed to a radius of  $R_{\text{smooth}} = 10$  Mpc to reduce noise, the signal is around a factor of 10 greater with the default pixel size of 3 Mpc. The vertical line corresponds to,  $\alpha = 0.03$ , illustrating the level of residual hydrogen observed in galaxies at lower redshifts. MWA, LOFAR and SKA  $1\text{-}\sigma$  instrumental errors are depicted with beige shadings from lightest to darkest respectively.

HI post-reionization. Clearly the amplitude of the variance at the end of reionization is very sensitive to the level of galactic sinks. This intrinsic noise will be detectable by SKA, which will therefore be well placed to constrain galactic sinks with this statistic. We do not explicitly plot HERA errors but they are indistinguishable from SKA, i.e. negligible, on this plot and so should perform equally well at this task. Before these telescopes come online, both LOFAR and MWA should be able to set some constraints on this quantity, although at the lower and more probable values of  $\alpha$ , MWA’s signal to noise will be too poor to do more than set upper limits on remnant galactic HI. The flip-side of this is that the intrinsic noise from galactic sinks should not be a limiting factor for these first generation instruments.

### 3.5 Observing the brightness-temperature moments: Smoothing to reduce noise

As in Watkinson & Pritchard (2014), we investigate the effects of smoothing the maps to beat down instrumental noise. Figure 6 (top) shows the evolution of the variance when smoothed to  $R_{\text{smooth}} = 10$  Mpc as a function of H II volume-filling factor. LOFAR will be well placed to constrain the presence of IGM sinks through the reduction in the variance, could constrain/ exclude large levels of galactic sinks and will be able to offer timing constraints from the mid-point turnover in the variance (unless the model is extreme and this signature does not exist). We see that smoothing recovers the late-time turnover in both skewness statistics for reasonable levels of galactic sinks. This is fortunate as the late-time maximum in the dimensional skewness will be detectable even by LOFAR for a wide range of galactic HI levels. The mid-point of reionization will be bounded by SKA and HERA measurements of the skewness, which will inform us when  $Q_{\text{HII}} < 0.7$ , and constraints from the variance of  $Q_{\text{HII}} \approx 0.4$ . The early-time minimum should still be detectable in both skewness statistics despite suppression by the presence of IGM sinks. SKA is the only instrument that is likely to actually detect this feature; however LOFAR and HERA should be able to use its absence to set upper-limits on this phase. If residual hydrogen levels are extreme then this signature, like every other, will be suppressed.

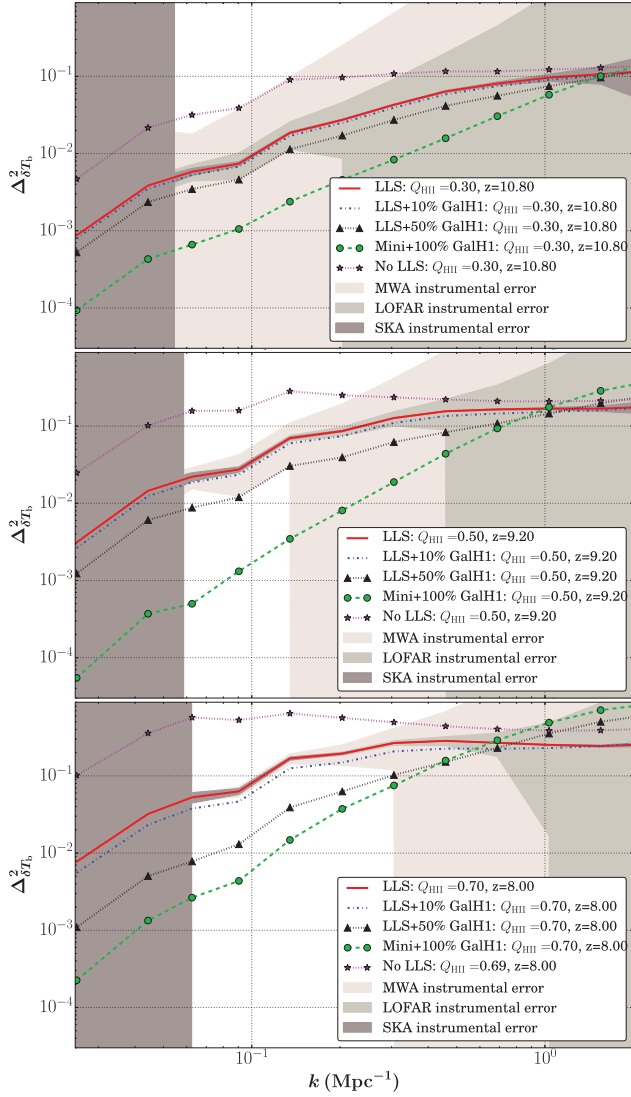


**Figure 6.** Variance (top), skewness (middle) and dimensional skewness (bottom) of brightness temperature as a function of the H II volume-filling factor measured in ‘observed’ maps smoothed to  $R_{\text{smooth}} = 10$  Mpc. Beige shadings depict  $1\text{-}\sigma$  instrumental errors for the ‘LLS’ simulation; tones correspond to MWA, LOFAR and SKA from lightest to darkest. Within the hatched region, spin-temperature fluctuations may not be negligible.

## 4 BRIGHTNESS-TEMPERATURE POWER SPECTRUM

The brightness temperature’s 1-point statistics form the focus of this paper, but the 21-cm power spectrum will be an incredibly important statistic when it comes to constraining reionization. As such it is worth understanding the impact of galactic neutral hydrogen on this statistic. We concentrate on the spherically-averaged dimensionless power spectrum which we describe using  $\Delta_{\delta T_b}^2(k, z) = k^3 / (2\pi^2 V) \langle |\delta_{21}(\mathbf{k}, z)|^2 \rangle_k$  in which  $\delta_{21} = \delta T_b(\mathbf{k}, z) / \bar{\delta T}_b(z) - 1$ ,  $\bar{\delta T}_b(z)$  is the redshift dependent average brightness temperature calculated from the simulation,  $V$  is the volume of the simulated box and the angle brackets denote an average over  $k$ -space. To model instrumental errors on the spherically averaged power spectrum, we take the approach outlined in the appendix of McQuinn et al. (2006) adopting a logarithmic bin width of  $\epsilon = 0.5$ . In calculating the number density of baselines we assume a filled nucleus limited by the minimum separation of stations,  $\sqrt{A_{\text{eff}}}$ , followed by an  $r^{-2}$  drop off in station density. This assumes a smooth density of



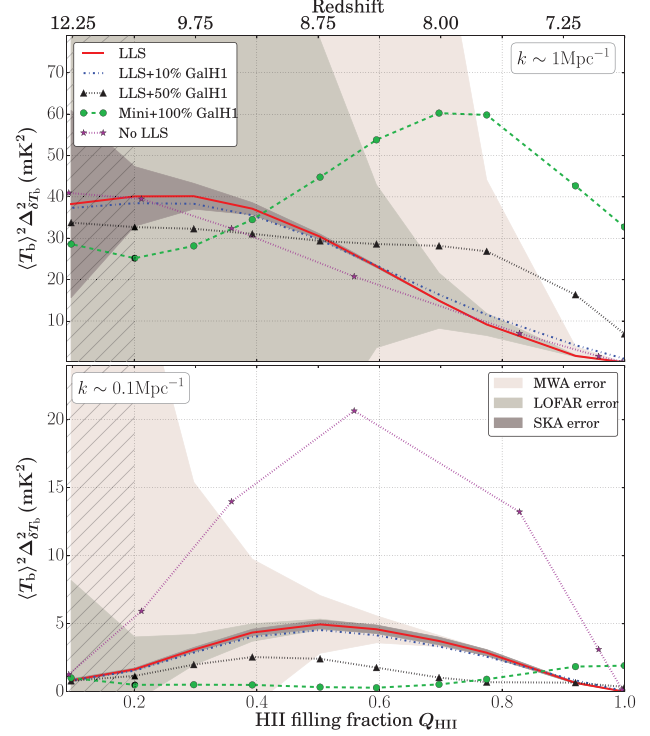


**Figure 7.** Dimensionless power spectrum of the brightness temperature as a function of  $k \text{ Mpc}^{-1}$  for  $z = 10.8$  (top),  $z = 9.2$  (middle), and  $z = 8$  (bottom).

stations, which for LOFAR is a particularly crude approximation. Whilst the exact configuration of SKA is yet to be decided, both LOFAR and MWA have optimized station positioning, as such our power spectrum errors are indicative only.

The dimensionless power spectra for our simulations are presented in Figure 7 for  $Q_{\text{HII}} \approx 0.3, 0.5, 0.7$  ( $z = 10.8, 9.2, 8$ ), from top to bottom respectively. As already discussed in SM2014, IGM sinks can suppress large-scale 21-cm power, resulting in much steeper power spectra throughout reionization. From Figure 7 we see that adding additional galactic HI does not have a large impact on the power spectrum, for reasonable values of  $\alpha$ . More extreme levels of galactic HI result in a further steepening of the power spectrum, since: (i) the contrast between ionized and neutral patches is reduced, and (ii) the distribution of galactic HI contributes additional power on small-scales.

The evolution of the dimensional power spectrum  $\delta T_b^2(\mathbf{k}, z) \Delta_{\delta T_b}^2(k, z)$  as a function of the HII volume filling factor is plotted in Figure 8 for  $k = 1 \text{ Mpc}^{-1}$  (top) and  $k = 0.1 \text{ Mpc}^{-1}$  (bottom). Unlike higher-order moments, the power



**Figure 8.** Dimensional power spectrum of the brightness temperature as a function of the HII volume-filling factor for  $k = 1 \text{ Mpc}^{-1}$  (top) and  $k = 0.1 \text{ Mpc}^{-1}$  (bottom). Within the hatched region, spin-temperature fluctuations may not be negligible.

spectrum at small  $k$  can cleanly pick-up large-scale structure. As such its timing signatures are more robust. For example, the mid-point turnover in the  $k = 0.1 \text{ Mpc}^{-1}$  dimensional power spectrum is robustly at  $Q_{\text{HII}} \sim 0.5$  for the more reasonable models, although it shifts to smaller  $Q_{\text{HII}}$  with large levels of galactic sinks and is completely wiped out in the extreme galactic HI model. We see that whilst reasonable quantities of galactic sinks have little qualitative impact, large quantities do impact on the nature of the power spectrum's evolution with its evolution no longer well described by a simple inverted parabola on large scales. This results from additional small-scale power (evident in the plots of Figure 7 and the top plot of Figure 8) and from the fact that at late times there is still substantial residual HI that drives the power.

## 5 CONCLUDING REMARKS

In this paper we have examined the impact of sinks on the 21-cm statistics of reionization. We consider two classes of sink, IGM sinks and galactic sinks, which have two effects on the 21-cm signal:

- (i) As absorbers of ionizing photons they impact on both the timing of reionization and the morphology of cosmic HII patches.
- (ii) As reservoirs of neutral gas they contribute a non-zero 21-cm signal in otherwise ionized cosmic patches.

We use the simulations of Sobacchi & Mesinger (2014) to investigate the impact of IGM sinks on the 21-cm 1-point statistics due to effects (i) and (ii). In addition we study how the contribution of galactic sinks to (ii) affects the 21-cm statistics. We also consider

an extreme, illustrative model in which gas inside sterile minihaloes and atomically-cooled galaxies is fully neutral.

It is likely that the impact of sub-grid IGM sinks on reionization morphology (Sobacchi & Mesinger 2014; effect i), will suppress the expected ‘rise and fall’ reionization signal of the raw (unfiltered) 21-cm variance. Instead, the large-scale power (or smoothed variance) shows a more pronounced evolution during reionization, since it is less affected by unresolved H II structure. Thus first-generation instruments such as LOFAR should target large-scale modes for cleaner reionization probes.

Similarly, the rapid increase of the skewness previously associated with the end of reionization is also suppressed by inhomogeneous sinks. This is because the variance is non-zero even post-reionization (due to residual H I; effect ii) and so we instead see a turnover as the skewness transitions from being dominated by the ionization field to the residual H I field.

We find that more reasonable levels of galactic sinks, will not change the qualitative evolution of the moments during reionization. It is worth noting that the presence of even low levels ( $< 10\%$ ) of galactic sinks would result in the absence of any late-time skewness signature in SKA maps. However, the late-time turnover in the skewness should be recoverable by smoothing the maps to a lower resolution.

We find that large levels ( $\gtrsim 50\%$ ) of galactic H I have the potential to dramatically alter the qualitative evolution of moments. This provides an exciting opportunity for SKA (and even LOFAR) to constrain levels of galactic H I in the high-redshift Universe.

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