ORTHOGONAL MULTIPLEXING TECHNIQUES
FOR SWITCHING IN A
DIGITAL LOCAL TELEPHONE EXCHANGE.

by

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ABSTRACT

The use of digital techniques in local telephone exchanges is very attractive because of the increase in reliability and the saving in space. However, the analogue speech signals which exist in the local network must be converted to digital form, normally pulse code modulation, before these techniques can be applied. The low level of quantisation noise required in such systems demands an encoding process of high accuracy. The economic provision of the encoding process can be achieved by multiplexing a number of analogue channels onto a single high speed encoder in the concentrator stage of the exchange. This project has examined several sets of digital orthogonal functions for the implementation of the multiplexing process. The chief criteria by which the orthogonal functions were assessed have been:

a) the crosstalk between channels on the multiplex system,
b) the noise immunity of the multiplex system,
c) the complexity of the hardware needed for the construction of the system.

Various sets of functions were analysed theoretically and the results of the theoretical analysis were verified by measurements made on an experimental multiplex system. A modification of the original technique improved the performance of the system and several sets of orthogonal functions met the rather stringent performance requirement of crosstalk better than 70dB. Orthogonal functions based on time division multiplexing ideas produced the simplest form of system implementation but the sets of functions based on Hadamard matrices had a much greater immunity to noise added onto the multiplex highway; an important consideration for practical systems.
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INTRODUCTION

1.1 AN HISTORICAL PERSPECTIVE

The development of switching systems and that of transmission systems have been linked since the invention of the telephone, but it would not be true to say that they had advanced equally. In fact, since the 1940s the development of switching systems has lagged behind the advances made in telephone transmission. Consequently the improvement of the switching system and the development of new ideas relating to it, particularly with reference to the local exchange aspects, is currently the most critical problem in wire telecommunications. It is instructive to follow the development of ideas in this field so as to provide a perspective to this study.

1.1.1 The Development of Automatic Telephone Exchanges

When the telephone was invented by A.G. Bell in 1876 (1.1) manual exchanges were already in existence and used for the interconnection of telegraph circuits. It was, therefore, a natural extension for the provision of manual telephone exchanges to follow immediately after the invention of the telephone itself (the first exchange was opened in New Haven, Connecticut in 1878). The possibility of interconnecting telephones automatically was also immediately recognised; the first patent for an automatic telephone switch was taken out by M. Connolly in 1879 (1.2). Other inventions relating to the telephone switch were patented in the 1880s but none proved practically satisfactory until A.B. Strowger invented the two-motion selector in 1889 (1.3). This proved to be the main line of switching development for over thirty years.
Other systems were developed such as the Relay exchange and the Panel switch and these were used for small and very large systems respectively (1.4). However, neither of these achieved the ubiquitous position of the Strowger selector, since this device provided a large switching unit in a comparatively small space. It is suitable for large scale production and the basic switching unit can be combined into exchange systems of different types and sizes.

The next important development was the Crossbar switch patented by Reynolds in 1922 (1.5). The Crossbar switch is a form of multiple relay used to control a switch matrix. Although invented in America, the first Crossbar exchange was opened in Sweden in 1926. Crossbar systems have been used extensively in large exchanges in the U.S.A. and elsewhere from 1938 onwards.

The application of the reed relay was the next major advance when it became the main switching element in the first generation of so-called electronic exchanges. Reed relays were also used as the switching elements in the first series of computer controlled exchanges which began with E.S.S.1 (Electronic Switching Systems) developed in Bell Laboratories and introduced into service in 1964 (1.6).

1.1.2 The Development of Digital Transmission Systems

Attempts were made for many years to use electricity as a means of communication. These resulted in the successful demonstration of the electric telegraph by Henry in 1831 (1.7). A practical system was developed by W. Cooke and C. Wheatstone in 1837 and the telegraph was a well-established means of communication when A.G. Bell invented the telephone and by that time many of the principles of digital transmission were known. The representation of code characters by on-off pulse sequences had been proposed by several independent authors in the 1830's (1.8). The best known of these is S. Morse.
who made the first recorded telegraph message in 1838. Binary and ternary codes were proposed at this time and Lord Kelvin had related the maximum signalling speed on a cable to the product of its resistance and capacitance (1.9). In 1853 H. Farmer had proposed time division multiplexing (t.d.m.), many years before frequency division multiplexing (f.d.m.) was perfected (1.10).

It is interesting to note that when Bell invented the telephone he was working on a 'harmonic telegraph' which in essence was a frequency division multiplex system. Frequency division multiplexing for telephony can be said to date from 1914 when Heising constructed the first practical system (1.11).

There had been experiments in applying the commutator used in telegraph t.d.m. to telephony before W. Miner was successful in 1903. These early experiments used low sampling rates and it was not until Miner used rates between 3,500 and 4,300Hz that the experiments were a reasonable success. He realised that

"It would be necessary to have contacts or closures to correspond as near as possible with the frequency of vibration of the human voice." (1.12)

Carson, in 1920, attempted a mathematical treatment of sampling and realised that it was necessary to sample at slightly more than twice the maximum signal frequency. By this time f.d.m. systems were widespread, as their development had been expedited by the thermionic valve amplifier and the electric wave filter. Since t.d.m. systems were as susceptible to noise as f.d.m. systems, and more liable to crosstalk they were not developed commercially.

In the 1930's there was interest in transmitting speech by means of pulses so that speech signals could have the same immunity from noise as telegraph systems (1.13). This led to the development of
pulse code modulation (p.c.m.) by A.H. Reeves in the Paris laboratories of the International Telephone and Telegraph Company. All the main aspects of a p.c.m. system were explained in Reeve's patent (1.14), but the necessary components were not available for a practical system. The development of p.c.m. continued at the Bell Telephone Laboratories where codes using logarithmic quantisation were developed (1.15). C.E. Shannon theoretically analysed p.c.m. thus laying the foundation of the modern work on information theory (1.16). The development of p.c.m. continued in the 1950's when the advent of the transistor enabled practical systems to be built. The first of these was the Bell T1 system which had 24 speech channels each sampled at 8kHz. This was used to multiplex groups of telephone junctions onto a single pair originally intended for one voice frequency channel. This proved highly successful and more recent advances have resulted in systems suitable for trunk and international routes. The British Post Office has now decided to standardise on 30-channel, 8-bit p.c.m. with A-law companding as a basis for all future digital transmission systems (1.17).

1.1.3 Electronic Switching in the United Kingdom

The fruitful application of electronic techniques to f.d.m. systems in the 1930's and to military projects in the Second World War led to the idea of applying these techniques to the switchblocks and control units of telephone exchanges. It was felt that electronic circuitry would be more reliable and compact than electromechanical Strowger switches. At that time there were no suitable solid-state crosspoints for voice frequency switching and so the British Post Office decided to construct multiplex switching systems. Initially both f.d.m. and t.d.m. switches were constructed but in 1949 research on f.d.m. systems was discontinued in order to concentrate on t.d.m. systems (1.18).
The t.d.m. system was considered capable of satisfactory performance in all aspects except the crosstalk which resulted from distortion of the pulses on interconnecting cables. The first system constructed used thermionic valves in the modulators. Later, efforts were made to improve the crosstalk performance using semiconductors (1.19).

In 1956 the Joint Electronic Research Committee was set up to pool the research effort of the British Post Office and the telecommunications equipment manufacturers in the development of electronic telephone exchanges. The Highgate Wood electronic t.d.m. exchange was then constructed and tested in 1959. This experimental exchange showed the existence of the following severe problems (1.20):

a) the cost was high relative to electromechanical systems;

b) crosstalk performance remained poor at about 60dB;

c) rhythmic noise from the pulse distribution system was audible in the channels;

d) fault location was difficult due to interaction between different parts of the control circuitry;

e) the specified transmission loss across the exchange could not be met without the possibility of instability on tandem connections.

At about the same time an experimental reed relay space-division exchange was built by J. Flood at Associated Electrical Industries, A.E.I. (1.21). While electromechanical crosspoints were used the Reed Electronic Exchange (R.E.X.) was considered suitable for manufacture and the family of T.X.E. (Telephone Exchanges Electronic) was based on this design. Work on t.d.m. systems continued with proposals for a small experimental exchange at Goring-on-Sea, but this was discontinued in 1966 (1.20).
When the first p.c.m. junction systems were being built, W.T. Duerdoth and others at the Post Office Research Station became interested in digitally switching p.c.m. channels. Laboratory studies proved successful and an experimental digital exchange, the Empress Exchange (1.22) was installed in West London, being brought into service in 1966. This exchange was a tandem unit used for switching p.c.m. junctions between local exchanges in the London Director Area.

The success of the Empress Exchange was followed by a detailed study of the application of digital techniques to the trunk network by the Long Range Studies Department of the Post Office. The recommendations of the study group were such that development of long distance, high capacity, digital transmission systems was begun and these systems are now undergoing field trial. Development of digital trunk exchanges is also underway, the first systems for public service in the United Kingdom being scheduled for 1982. There is now renewed interest in electronic local exchanges and there are now several projects to construct digital local exchanges in Britain and abroad (1.23, 1.24).

1.2 INTRODUCTORY REMARKS ON MULTIPLEXING AND SWITCHING

It has long been recognised that multiplex systems can form the basis of a switch for use in telecommunications systems. It is, therefore, worthwhile to consider the common principles used in multiplexing and switching. In particular the orthogonality condition is worthy of investigation because it determines the suitability of functions for use in multiplexing and switching systems. In order to do this a generalised multiplex system is investigated and then considered as a switch. This is applicable to both analogue and digital systems as the underlying principles remain the same.
The system is then modified to show that the multiplex switch can perform a variety of switching operations.

1.2.1 Principles of Multiplexing

Often in telecommunications systems the information capacity of the transmission medium is much greater than the capacity of individual information sources. The prime purpose of a multiplex system is to group together a number of low capacity channels or information sources onto a medium of higher capacity so that the high capacity of the system is fully utilised. In order that this may be achieved the input information on each channel must be modified by the multiplex system so that the information from all the channels can be transmitted together over the high capacity medium and be capable of being separated again at the receivers.

All multiplex systems can be shown to contain the same set of systems components although some systems can be operated with a simplified hardware structure. A generalised multiplex system is shown in Fig. 1.1; it consists of the following:

a) multiplexers comprising a low pass filter, a sample-hold circuit and a multiplier for each low capacity input;

b) a summing amplifier which adds the outputs of the multipliers;

c) the high capacity transmission medium which connects the summing amplifier to each of the demultiplexers;

d) demultiplexers which comprise multipliers, integrators and sample hold circuits.

The system operates as follows: the incoming information is bandlimited to a frequency of $W$ Hz by the low pass filters. The sampled and held information $(k_1, \ldots, k_1, k_j, \ldots, k_n)$ is multiplied by one of a set of functions $f_1(t), \ldots, f_1(t), f_j(t), \ldots, f_n(t)$ to give a product $k_1 f_1$. The functions have a repetition period of $T$ where $T < \frac{1}{2W}$. 
The outputs of all the multipliers are added together in the summing amplifier. The resultant signal is therefore given by

\[ s_0(t) = \sum_{i=1}^{n} k_i f_i(t) \quad \ldots \ldots \ldots \ldots \quad \{1.1\} \]

and this is transmitted over the high-capacity transmission medium (the multiplex highway) to the demultiplexers. Here the signal is multiplied again by one of the set of functions and the resultant is integrated over the period \( T \). The output of the integrator of the \( j \)th demodulator is given by

\[ g_j(t) = \int_{0}^{T} \sum_{i=1}^{n} k_i f_i(t) f_j(t) dt \quad \ldots \quad \{1.2\} \]

Expanding the summation and noting that the signal is constant over the period \( T \) we have

\[ g_j(t) = k_1 \int_{0}^{T} f_1(t) f_j(t) dt + k_2 \int_{0}^{T} f_2(t) f_j(t) dt + \ldots + k_n \int_{0}^{T} f_n(t) f_j(t) dt \]

The desired output of the \( j \)th demodulator is \( k_j \) which is the value of the sampled and held signal at the \( j \)th input channel. For this output to occur we must have in the previous equation

\[ \int_{0}^{T} f_j(t) f_j(t) dt = 1 \quad \ldots \ldots \ldots \ldots \quad \{1.3\} \]

and also either the data in the other channels must be zero or

\[ \int_{0}^{T} f_i(t) f_j(t) dt = 0 \quad i \neq j \quad \ldots \ldots \quad \{1.4\} \]
Since the purpose of the multiplex system is to transmit signals on a number of channels then the data in the other channels cannot be zero. The set of multiplying functions must then have the properties given in equations 1.3 and 1.4, that is

$$\int_0^T f_i(t) f_j(t) dt = 0 \quad i \neq j \ldots \ldots \quad \{1.5\}$$

$$= 1 \quad i = j$$

This is termed the orthogonality condition (1.25 and 1.26). Any set of functions for which this condition holds can be used as a set of carriers in a multiplex system. Also the integral does not need necessarily to be unity for $i = j$ as long as it is some constant and determinable value. If the integral is equal to some constant $c$, this represents a gain through the system of $c$ and can be taken into consideration by introducing a corresponding attenuation of $1/c$ into the system at a suitable point. If the constant is unity the functions are also called orthonormal.

The most common set of functions used to date is the set of sinusoids, which are now considered as an example of the above. A set of sinusoids (Fig. 1.2) are shown to be orthogonal very simply since;

$$g(t) = \int_0^T \sin 2\pi j t \cdot \sin 2\pi k t \ dt \ldots \ldots \{1.6\}$$

$$= \int_0^T \left[ \cos 2\pi (j+k)t - \cos 2\pi (j+k)t \right] dt$$

and since

$$\int_0^T \cos 2\pi nt \ dt = 0 \quad n \neq 0 \ldots \ldots \ldots \ldots \{1.7\}$$

$$= T \quad n = 0$$
equation 1.6 is equal to 0 if \( j \neq k \)

and to \( \frac{T}{2} \) when \( j = k \).

Hence the constant \( c \) is equal to \( \frac{T}{2} \) and an attenuation of \( \frac{2}{T} \) must be introduced into the system to equalise the gain. Multiplex systems using sinusoids as multiplying functions are called frequency division multiplex systems whereby the capacity of the high capacity transmission medium is utilised by translating the baseband channels into different slots in the frequency spectrum. An example of this is the 12 channel f.d.m. system used for cable transmission. Twelve speech channels each occupying the band 300 - 3400 Hz are multiplexed together using sinusoidal carriers spaced at 4kHz intervals from 60 to 104kHz. This and other such systems are described in reference 1.27.

There are other orthogonal sets of continuous functions; examples of these are Bessel functions and Legendre polynomials. However the ease of generation of sets of sinusoids, and their suitability for processing by electric wave filters has led to the exclusive use of sinusoids where continuous functions have been needed for multiplexing.

Sets of non-continuous functions can also be used for multiplexing; an example of these is the set of block pulses used in time division multiplexing (Fig. 1.3). These can readily be shown to be orthogonal since:

\[
\begin{align*}
\int_0^T f_i(t) f_j(t) dt & = 0 \quad i \neq j \quad \ldots \ldots \ldots \quad \{1.8\} \\
& = 1 \quad i = j
\end{align*}
\]

Hence

\[
\int_0^T f_i(t) f_j(t) dt = 0 \quad i \neq j
\]

\[
= T \quad i = j
\]

An important property of this set of functions which can be used to simplify system design is that they are orthogonal with or without
integration and so the integrator in the demultiplexer may be omitted. This set of functions was used as a basis for the electronic exchange system described in Section 1.1.3.

A second set of non-continuous functions are the Walsh functions, some of which are shown in Fig. 1.4. They were first described by J.L. Walsh in 1923 (1.28) although they had been used previously for the transposition of telephone lines to prevent crosstalk (1.29). Their orthogonality can be established as follows: for any pair of Walsh functions the product is defined as (1.30)

\[ \text{wal}(j,t) \cdot \text{wal}(k,t) = \text{wal}(j \oplus k,t) \quad \ldots \ldots \quad (1.9) \]

where \( j \oplus k \) is the sum of \( j \) and \( k \) modulo 2.

In addition

\[ \int_0^T \text{wal}(n,t) = 0 \quad n \neq 0 \quad \ldots \ldots \quad (1.10) \]
\[ = T \quad n = 0 \]

Using these relations and performing the orthogonality integral

\[ \int_0^T \text{wal}(j,t) \cdot \text{wal}(k,t) dt \]

\[ = \int_0^T \text{wal}(j \oplus k,t) dt \]

Noting that \( j \oplus k = 0 \quad j = k \)

\( j \oplus k \neq 0 \quad j = k \)

and applying (1.10) results in

\[ \int_0^T \text{wal}(j \oplus k,t) dt = 0 \quad j \neq k \]
\[ = T \quad j = k \]

This set of functions have not yet been used to a large extent in multiplex systems but have already proved their value in the area
of spectral transformations particularly as the Fast Walsh Fourier Transform (1.31).

1.2.2 Digital Multiplexing

The principles of multiplexing are equally applicable to analogue and digital systems. Each of the components in the generalised multiplex system of Section 1.2.1. can be realised in analogue or digital form; for example the integrator in the demultiplexer could be an operational amplifier integrator in an analogue system or a digital accumulator in a digital system. The differences between the two types of system are due not to differences in the principles of multiplexing but the contrasting natures of the two types of system. Analogue systems have usually used the set of continuous sinusoidal functions. These can be generated easily in analogue form since they appear in the solutions of second-order linear differential equations and can be manipulated by means of electric wave filters which can be readily constructed. The digital system is discrete in both time and amplitude and sinusoids cannot easily be generated. If sinusoids were to be used as the multiplying functions in a digital multiplex system, the functions must be stored as a large number of samples with a large number of levels of quantisation in order that the waveforms are generated accurately. In a digital multiplex system the number of amplitude levels required on the multiplex highway is proportional to the product of the number of amplitude levels in the input channels and the number of amplitude levels in the multiplying functions. Orthogonal functions which have a limited number of amplitude levels and are readily described in sampled data form are therefore preferred to sinusoids in digital systems. Examples of these are the block pulses and Walsh functions described in Section 1.2.1. Multiplication by block pulses can be
performed by a set of NAND gates. The multiplication by 1 and -1 required when using Walsh functions can be implemented by means of an Exclusive-OR gate and using the mapping +1 → 0 and -1 → 1. This makes both these sets of functions suitable for use in digital multiplex systems.

1.2.3 The Multiplex System as a Switch

Multiplex systems were used as switches from the initiation of work on electronic telephone exchanges. How this can be done can be demonstrated by reconsidering the operation of the multiplex system described in Section 1.2.1. There, the signal from the multiplex highway was applied to a set of homogeneous demultiplexers—that is the output from any demultiplexer depended on the waveforms multiplied together in it and not on any intrinsic difference in its construction. The first of these waveforms is the multiplex highway signal from the summing amplifier which is applied to all the demultiplexers. The second waveform is the particular orthogonal function applied to the demultiplexer. Similarly the signal from each of the low capacity channels is applied to one of a set of multiplexers which are homogeneous except for the multiplying orthogonal applied to each one of them. The element which relates the multiplexer and demultiplexer for one of the channels is the orthogonal function which is used for both of them. From equations 1.2 and 1.3

\[ g_j(t) = k_j \int_0^T f_j(t) f_j(t) dt = k_j l = k_j \]

If the multiplying function at the demultiplexer had been interchanged with some other member of the set, for example \( f_m(t) \) previously used at another demultiplexer, then the conditions at the output from the \( j^{th} \)
The output of the \( j \)th demultiplexer is now the information from channel \( m \). Similarly at the \( m \)th demultiplexer

\[
k_m \int_0^T f_m(t) f_m(t) dt = k_m 0 = k_m
\]

but

\[
g_j(t) = k_m \int_0^T f_j(t) f_j(t) dt = k_j 0 = k_j
\]

The output of the \( m \)th demultiplexer is now the information at the input of the \( k \)th multiplexer. The same result could also have been achieved by interchanging the multiplying functions at the multiplexers rather than the demultiplexers. In a telecommunications system a switch is defined as a unit with an arbitrary number of inputs and outputs and the ability to connect any of the inputs to any of the outputs (Fig. 1.5). If the multiplying functions in the multiplex system can be interchanged at either the multiplexers or demultiplexers then any of the inputs can be connected to any of the outputs of the system which thus forms a switch. The operation of the switch depends on the application of any of the chosen set of orthogonal functions to any of the multiplexers or demultiplexers. To do this the functions must either be distributed from some central source or generated at the multiplexers or demultiplexers. The complexity of this task together with the transmission performance achieved by the multiplex system determines the value of the system as a switch,
An example of a multiplex system used as a switch is the SPADE system (Single channel per carrier Pulse code modulation Assignment Demand Equipment) (1.32). The multiplex system consists of multiplexers and demultiplexers contained in satellite ground stations. The multiplex highway comprises the satellite and transmission paths to and from it. When a connection is required between the areas served by two of the ground stations a common channel is allocated between the two stations. The allocation is achieved by using the same sinusoidal orthogonal function at the multiplexer and demultiplexer in the ground station. When the connection is no longer required the function, and hence the channel, can be allocated to another connection not necessarily between the same ground stations. Thus the complete extra-terrestrial relay system forms a highly sophisticated frequency division multiplex switch.

1.2.4 Types of Multiplex Switch

In general a switch can be one of four types. The relative number of inlets (m) to, outlets (n) from and paths (f) through it determine the type of a given switch. These types can be summarised as follows:

a) a distributor switch \( m = n = f \)
b) a concentrator switch \( m > n = f \)
c) an expander switch \( n > m = f \)
d) a bus system \( m = n > f \)

Considering each of these types of switch in turn, the distributor type of switch is usually used to connect channels with a high occupancy so that the paths through the switch are fully utilised. This sort of switch finds application in tandem switching units where junctions between exchanges are interconnected, in trunk switching where the channels are heavily loaded and in the areas of exchanges
where the traffic is heavy; that is, switching between other units or into the control area. In exchange design it is usual to provide a distributor switch to switch between the incoming and outgoing junctions and the control area. In order to do this, switching is required to connect a large number of low traffic lines onto a small number of high traffic inlets to the distributor switch. This is done by means of a concentrator switch which has a large number of inlets and a small number of paths and outlets. A concentrator switch is also used where it is necessary to concentrate a large number of inlets onto a limited number of channels. Examples of this are the multiaccess systems used for time-sharing computer applications. The inverse function to that of concentration is provided by the expander switch where a small number of channels can be connected to any of a large number of outlets.

The last type of switch is the bus system which is often used in place of more complex switching arrangements when the switching system is small. In this system the number of paths is less than the number of inlets or outlets which are about the same. It is therefore suitable for switching between information sources which have low traffic levels. The SPADE system described in Section 1.2.3 can be classed as a bus system.

The multiplex system of Section 1.2.1 can form any of these switch configurations. The multiplexers form the inlets to the switch, the demultiplexers form the outlets and the set of functions determines the number of paths through the switch. Since multiplex switches are unidirectional, two paths are needed to set up a call in an electronic telephone exchange. Thus at first sight the multiplex switch is less efficient than its space-divided counterparts; however the crosspoints in a multiplex switch are shared across a number of channels which invariably results in a significant reduction in switch hardware being required.
FIG. 1.1 A GENERAL MULTIPLEX SYSTEM
FIG. 1.2 A SET OF SINUSOIDS

FIG. 1.3 A SET OF BLOCK PULSES
FIG. 1.4 A SET OF WALSH FUNCTIONS
FIG. 1.5 A SWITCH MATRIX
CHAPTER 2

DIGITAL LOCAL EXCHANGES

So far we have described the theoretical basis of switching and multiplexing. However the suitability of any type of switch depends on the environment in which the switch must perform. In this chapter we describe the exchange in which the switches operate and the environment of the exchange as both of these have important implications for the design of the switch. These are reflected in the number and types of interface which are required between the switch block and the local distribution system. In the light of these observations we assess the suitability of several different types of switch for use in the concentration stage of the digital local exchange.

2.1 A GENERAL LOCAL TELEPHONE EXCHANGE

In this sub-section the general parts and functions that are necessary for a local telephone exchange are examined. Fig. 2.1 shows a block diagram of a general local exchange system which can be divided into two parts:

a) the switchblock,

b) the control unit and its interfaces.

Each part has a different function to perform which we now consider.

2.1.1 The Switchblock

The purpose of the switchblock is to connect any customer on the exchange to any other customer on the exchange or to an outgoing junction to another exchange. The switchblock could take the form of a large square array of switches; then every customer and junction would be
connected to an inlet and outlet on a square switch matrix. This would be a simple but inefficient solution as an extremely large number of switch crosspoints would have to be provided, since the number of crosspoints required in such an array is the square of the sum of the numbers of lines and junctions. Such arrays are only used for the small simple house exchange type of system. For larger systems it is usual to use a combination of the switch types discussed in Section 1.2.

The switches used are as follows:
   a) a concentrator switch,
   b) a distributor switch,
   c) an expander switch.

In a local exchange there is a large number of customer lines, each of which has a low traffic level, i.e. the line is not used most of the time. These lines are given access to the inlets of the distributor switch by means of the concentrator switch, while junctions which carry a high traffic level are directly connected to the distributor switch. The distributor connects the incoming channels from the concentrator and the junctions to the inlets on the expander switch or to outgoing junctions. The expander connects channels from the distributor to a large group of customers' lines which are terminated on its outlets. By using the switch in this manner the total number of crosspoints required is reduced; however, the total number of simultaneous calls the exchange can handle is also reduced, but the exchange can still be designed to give a satisfactory level of service.

2.1.2 The Control Unit and Interfaces

The control unit of an automatic telephone exchange has three main purposes:
a) to signal to and receive signals from the customers and other exchanges;
b) to set up paths through the switchblock;
c) to measure the length of the call and charge at the appropriate rate.

Receiving signals from the customer includes interpreting the digits dialled and recognising the calling, answering and clear down conditions. Signals sent to the customer include ringing the telephone bell and sending tones. This signalling is carried out by means of two interfaces. Each line is provided with a line unit which is permanently connected to it. The line unit detects the removal of the handset from the telephone and the control unit recognises that the customer wishes to make a call. The control unit then sets up a connection through the switchblock to a second interface. This transmits the tones to the customer, intercepts the dialled digits and relays the other customer signals to the control unit. Traditionally the supervisory unit has also applied ringing current to the subscriber's line. In an electronic exchange this is not possible since the switchblock cannot handle the power applied; the ringing current must then be applied via the line unit. Signalling to other exchanges can be performed in two ways. The method used to date is to send the signalling information associated with a call to the remote exchange by using the junction used for speech transmission. The new generations of electronic exchange use a separate signalling channel consisting of a digital data link.

The control unit also has interfaces to the switchblock which enable it to set up the necessary connections. These consist of the control lines which cause the switches to operate, the marker and a second interface which enables the control to sense the paths already connected through the switchblock; this is termed the interrogator.
The control unit is able to calculate the call charges using the number dialled and the length of time the call is held.

2.2 THE LOCAL EXCHANGE ENVIRONMENT

Although the penetration of digital transmission systems into the telecommunications network is growing, the amount of digital equipment now installed is small compared with the total size of the network. Therefore there are problems when introducing digital exchanges into a predominantly analogue environment. Also the length of time an exchange is expected to be in service is long (25 years) compared with the rate of technological progress. Therefore it is germane to consider both the present exchange environment and that which will appertain during the latter part of the service life of the exchange. Figs. 2.2 and 2.3 show the connections to and from a local exchange environment considered probable in the 1990's. Now most of the connections to the exchange carry speech and signals in the form of analogue waveforms with two-wire bidirectional transmission. This situation will change as more digital equipment penetrates the network. It is necessary to distinguish between two parts of the switchblock.

The distributor switch has junctions to other exchanges connected to it, and so this switch will be connected to the digital transmission systems used between exchanges (usually 32 channel p.c.m. systems). Also when digital transmission is used between private branch exchanges (p.b.x.'s) and the local exchange the traffic capacity of the inter-exchange circuits will be sufficiently high to merit direct connection between the p.b.x. and the distributor switch. The tendency will be that the connections to the distributor will be p.c.m. It seems certain that digital local exchanges will have p.c.m. distributor switches as
this will reduce the interfacing circuitry needed between the switch and the transmission systems already discussed. This will also make the switch 'transparent' to the p.c.m. transmission systems and any new services requiring inlets to the distributor switch. The term 'transparency' is used to mean that there is no restriction placed on the content of a message on a channel, so no assumptions are made about the interrelations of any part of the data when it is being transmitted. It is an important feature of a telecommunications system which must handle data of different types from a variety of sources.

The connections to the concentrator expander switches all use bidirectional analogue transmission on individual cable pairs, Unlike the distributor switch this situation will not change. The digital transmission systems introduced into the local distribution system, i.e. from digital private branch exchanges or remote rural concentrator units, will be high capacity links and as such will merit direct connection to the distributor switch. Also any extensive changes to the local distribution network to increase its compatibility with digital exchange techniques would be costly and result in considerable disruption to service. The concentrator and expander switches must then interwork with an analogue local network and a digital p.c.m. distributor. Thus these switches must have interfaces between both the distributor switch and the local network in addition to those between the switchblock and the control unit.

2.3 THE SWITCH INTERFACES

The interfaces used in a digital local exchange are of two types;

a) transmission interfaces,

b) control interfaces.
The transmission interfaces change the format of the incoming analogue signal to pulse code modulation. They include the following:

a) two-to four-wire termination
b) transmission bridge
c) low-pass filter
d) analogue to digital encoder and digital to analogue decoder (referred to collectively as a 'codec').

The control interfaces receive signals from and signal to the customer by means of the telephone instruments; they include:

a) the line unit
b) the direct current signalling detector
c) ringing current generator
d) tone generator.

Fig. 2.4 shows a schematic diagram of a local exchange with electromechanical concentrator and expander and a digital distributor switch. This diagram shows the position of the system's interfaces. Since the switch is an electromechanical 2-wire switch the transmission interfaces occur between the outlets of the concentrator and the inlets of the distributor switch. Also since the switch is capable of transmitting direct current and handling ringing current, the d.c. signalling interface and the ringing generator can also be interposed between the concentrator and distributor.

In a general exchange system the interfaces do not have to be located at this point. With the exceptions of the line unit and the tone generator these interfaces can be positioned at either the inlets or the outlets of the concentrator and expander switches. The choice between these two positions determines the number required of each type of interface. If there are \( l \) lines on the exchange and the concentrator and expander switches have a concentration ratio of \( k \),
the number of a given interface required at the inlet of the concentrator switch would be \( \lambda \), while at the outlet of the concentrator switch it would be \( \lambda/k \). The difference between these two positions is:

\[
\lambda_i - \lambda_o = \lambda(\lambda - \lambda/k) \quad \ldots \quad \ldots \quad \ldots \quad \{2.1\}
\]

where \( \lambda_i \) is the number at the inlets to the switch and \( \lambda_o \) is the number at the outlet of the switch; if the interfaces are placed on the outlets of the switch less are needed. However, it may not be possible to do this because of the design of the switch.

We will now consider how the change from electromechanical switches to a digital switchblock affects the position of each interface.

2.3.1 Two to Four-Wire Termination

Signals enter the exchange on a bidirectional two-wire path but most switches have unidirectional transmission characteristics. Transmission is provided by means of two channels, the 'go' and 'return' channels. The go channel must receive signals from the two-wire input and transmit them through the switch while the return channel transmits the signals from the switch onto the two-wire path. Signals must not pass between the go and return channels or problems with echo and instability can arise. These criteria are met by the two to four-wire termination which is a balanced bridge circuit with a high loss between opposite arms. If the concentrator and expander switches are bidirectional this device can be placed at the outlet of the switches and two-wire switches used. However, if the switches are unidirectional these devices must be placed on the inlet to the switches.
2.3.2 The Transmission Bridge

The transmitter and receiver in the telephone instrument require a direct current between 25 and 100mA for their correct operation. This current is supplied from the exchange by a unit termed the transmission bridge; this unit supplies the required current to the two-wire line while presenting a high impedance to the speech signals. If it is located at the outlets of the concentrator switch, the switch must be capable of carrying the current required. If this is not the case, the transmission bridge must be situated at the inlet of the concentrator switch.

2.3.3 Low-Pass Filter

When using a multiplex system as a switch the incoming signals are sampled and held constant. If aliasing is to be prevented, the signal must be bandlimited to half the sampling frequency before sampling. In the distributor switch the transmission format is 8 k samples/s A-law encoded 8 bit p.c.m. Hence any signal must be bandlimited to 4kHz before entering the distributor switch. This can be done by placing low-pass filters at the inlets or outlets of the concentrator switch. Generally it would seem logical to place the filters at the outlets of the switch since less are needed. However, with multiplex concentrator switches, this is not necessarily the case; if the concentrator is itself a multiplex switch filtering will be required at the inlets of the switch. The amount of filtering required again depends on the sampling rate of the switch, which for other considerations should be kept as low as possible. With this type of system there is a relation between the amount of prefiltering at the inlets of the concentrator switch and the switch speed.
2.3.4 Analogue to Digital Encoders and Decoders

If the transmission through the switchblock is digital the switches can be constructed using one of the standard families of digital integrated circuits (i.c.'s). However, this requires the encoding process at the inlets to the concentrator switches with a corresponding increase in the number of digital encoders and decoders required.

2.3.5 D.C. Signalling Detector

The signalling from the telephone instrument to the exchange presently takes the form of pulses of direct current. This signalling is binary in nature; each state is defined as follows:-

a) loop - a direct current of 25 - 100mA flows through the telephone instrument;

b) disconnect - the line is open circuit and no current flows.

The customer signals to the exchange by a series of looped and disconnected conditions, and this must be sensed and interpreted by the exchange control circuitry. As explained in Section 2.1, this can be done by means of a supervisory unit at the outlets of the concentrator unit if the switch is capable of transmitting the direct current conditions. With a multiplex switch this is not usually possible and so these interfaces also have to be moved to the inlets of the concentrator switch.

2.3.6 The Ringing Current Generator

To operate correctly, the bell in the telephone instrument requires a 25Hz waveform at a voltage of 70 - 100V and a current of 20mA. This power is unsuitable for transmission through a digital switch. Hence in a digital exchange the interface to the ringing generator must be moved to the inlet of the concentrator switch resulting in an increase in the number of interfaces required.
2.4 DIGITAL EXCHANGE SYSTEMS

From the foregoing remarks it is clear that a digital local exchange requires a large increase in the number of interfaces provided. This tends to offset any gains resulting from the use of the smaller, cheaper and more reliable digital switchblock. However, of these interfacing problems it is the provision of low-pass filtering and accurate analogue digital codecs that is the most intractable. The filtering problem has already been discussed (2.1 and 2.2) and is reviewed in Appendix 1. The alternative methods of analogue to digital encoding are now considered. To date three ways of approach this problem have been suggested; these are:

a) a p.c.m. concentrator and expander switch using a single channel codec at each inlet of the concentrator switch.
b) a high speed digital switch using a one-bit-per-sample, (i.e. delta modulation) encoder.
c) a hybrid analogue and digital switch using an analogue multiplexer at each inlet to the concentrator switch and high speed encoding on the multiplex highway.

Each of these approaches together with their merits and drawbacks is now described. In discussing these systems comment is restricted to the techniques used in the concentrator switch, except where the techniques used in the distributor switch differ radically from these, when the reasons for these differences will be given.

2.4.1 A Pulse Code Modulation Switch

Systems capable of transmitting and switching pulse code modulation have been built in several countries. The advantages of this type of switch lie in its small size and low cost. Switches already built contain several stages of time and space switching using time division multiplex techniques. In a concentrator stage, a single stage
switch is sufficient, each customer is provided with an 8 bit codec and has access to a time division multiplex highway constructed from integrated t.t.l. Paths are set up through the switch by allocating a free channel on the t.d.m. highway to a customer. Since the output bit rate of the codec is 64 k bits/s and a t.d.m. switch will operate at 1-2 M bits/s using standard t.t.l., the switch can be constructed with 16 channels. For a standard grade of service (i.e. 1 lost call in 500) the multiplex switch can carry 7.3 erlangs of traffic and each customer generates 0.05 erlangs of traffic; hence, each switch can give service to 7.3/0.05 = 140 customers. The resulting switch (Fig. 2.5) is compact and highly efficient and can easily be interfaced to the control unit which is constructed in a similar technology.

However the cost of the exchange is influenced to a large degree by the cost of the codecs. Traditionally, codecs are produced as hybrid circuits or constructed from discrete components and the cost of these devices is high. Efforts are now being made to integrate the codecs onto single silicon chips so that the cost reductions accruing from Large Scale Integration can be achieved.

Established methods of performing the encoding and decoding process are successive approximation and ramp encoding. Both of these techniques require components of high absolute accuracy and this is difficult to achieve using Large Scale Integration processes. Other techniques using waveform tracking encoders are therefore being investigated. Waveform tracking encoders attempt to minimize the error (or its integral) between the input signal and an internally generated approximation signal by switching a single-bit digital output (2,3). This digital output can be converted to p.c.m. by digital integrating and low pass digital filtering (2,4). The purpose of these filters is to reduce the quantisation to a level within the C.C.I.T.T. p.c.m. specification (2,5).
A number of workers are developing codecs and filters using these techniques, but no codec has been produced to date that is sufficiently cheap for use in a digital local exchange. Yield problems arise when chips of this complexity are required.

2.4.2 A High Speed Digital Switch

Interest has been shown (2.6) in an alternative switch structure which overcomes the problems of p.c.m. encoding at the inlets to the concentrator switch. This consists of placing the waveform tracking encoder at the inlets of the concentrator switch and transmitting its one-bit output to the digital filter which converts to p.c.m. at the inlets of the distributor switch. This technique reduces the number of filters required and hence the cost of the conversion. However the bit-rate required in the switch is now increased. Since the bit-rates on the multiplex highways cannot be increased, the number of channels multiplexed together must be reduced. The overall effect of this is to increase the cost of the switch. There is an inverse relation between the complexity of the encoder and the speed of the output; hence a trade-off exists between the cost of the encoders and the cost of the switch. This approach has been followed by Macdonald, using delta modulation encoders. The output of these high speed encoders (16 M sample/s) is partially filtered to 9 bit/sample, 32 k sample/s differential p.c.m. and then transmitted at a bit rate of 288 k bit/s through the switchblock. Then the signal is encoded into the standard p.c.m. format at the input to the distributor. The use of a type of instantaneously companded delta sigma modulator has also been proposed (2.7). The output of this type of encoder is a 256 k sample/s one bit/sample code. This again is converted to p.c.m. by a digital filter at the inlets of the distributor switch.
An advantage of these approaches using a high sampling rate in
the encoder is that some of the low-pass filtering needed to bandlimit
the analogue signal can be combined with the quantisation noise filter
at the inlets to the distributor switch. This reduces the amount of
filtering required at the inlet to the concentrator switch and reduces
the overall filtering costs.

2.4.3 A Hybrid Multiplex Switch

An alternative method of encoding the analogue signals is to
encode the channels while they are multiplexed together on the highway
of a multiplex transmission system or switch rather than encoding the
channels before multiplexing. The two methods are shown for comparison
in Figs. 2.7 (a) and (b). The digital multiplexers are now replaced by
equivalent analogue devices and the outputs from these devices are summed
onto an analogue multiplex highway and encoded into a digital multiplex
signal (Fig. 2.8). In both the digital and the analogue multiplex
systems the demultiplexing is performed digitally at the outlets of
the concentrator switch. This technique produces a large reduction in
the number of encoders (i.e. by a factor of about 130). However the
speed and dynamic range required in the encoder is now increased since
the signal encoded on the multiplex highway has more degrees of freedom
than the signal at each of the inlets of the switch; in particular
the signal can have n times the number of transitions as those at the
inlets to the switch, and also n times the range of amplitudes of the
input signals, where n is the number of channels through the switch,
here about 15. The multiplexing and summing of the input channels is
now performed in the analogue domain. This is a more complex operation
than performing the same functions on digital signals where the task
can be performed using t.t.l. Also digital multiplex systems almost
always use time division multiplexing as this is easily implemented and gives satisfactory performance in completely digital systems. However, time division multiplexing has been shown to have drawbacks when used in analogue form in an exchange environment (2.8). Hence a different method of multiplexing must be found, which overcomes the problems encountered with t.d.m. and which can be demultiplexed digitally.

2.5 DISCUSSION

The foregoing descriptions of the types of system which may be suitable for an electronic local exchange have brought out the trade-off which exists between the cost of the switch and the cost of the interfaces required. Systems presently under development use electromechanical concentrators with digital distributor switches. This is a logical line of development from the all-electromechanical exchanges such as TXE2 and TXE4. However, choosing the next step along the line to an all-digital system is not obvious. The all-p.c.m. exchange is the best systems solution; it would provide a transparent transmission medium from the local line network through the exchange hierarchy. However, against this must be set the present unavailability of suitable analogue to digital encoders and decoders. The high speed digital switch is under development and a balance between speed and encoder accuracy has been achieved. The hybrid multiplex switch appears well worth consideration, particularly if a set of digital orthogonal functions, which provide a satisfactory transmission performance, can be found.
FIG. 2.1 A GENERAL EXCHANGE SYSTEM
FIG. 2.2 THE PRESENT LOCAL EXCHANGE ENVIRONMENT

FIG. 2.3 THE PROPOSED FUTURE LOCAL EXCHANGE ENVIRONMENT
FIG. 2.4 AN ELECTROMECHANICAL CONCENTRATOR/EXPANDER SWITCH
TO OTHER LINES
CONCENTRATOR MULTIPLEX HIGHWAY (16 CHANNELS 8 bits/CHANNEL)

EXPANDER MULTIPLEX HIGHWAY (16 CHANNEL 8 bits/CHANNEL)

FIG. 2.5 A DIGITAL P.C.M. MULTIPLEX CONCENTRATOR AND EXPANDER SWITCH
FIG. 2.6 A HIGH SPEED DIGITAL SWITCH
FIG. 2.7

(a) DIGITAL MULTIPLEXER

(b) HYBRID MULTIPLEXER
FIG. 2.8 A HYBRID CONCENTRATOR EXPANDER SWITCH
CHAPTER 3

DIGITAL ORTHOGONAL FUNCTIONS

Digital orthogonal functions have been used in communications for many years. An early example of their use was in the time division multiplexing of telegraph signals by M.B. Farmer in 1853 (3.1). This technique was applied to speech by W.M. Miner in 1903. Rademacher functions, shown in Fig. 3.1, were also known by the end of the 19th century (3.2) and the system of functions known as Walsh functions (Fig. 3.2) were found by J.A. Barratt about 1900 (3.3) and used for the transposition of conductors in telegraph lines. These functions were first described by J.L. Walsh in 1923 (3.4). There was little interest in using digital orthogonal function in multiplex systems as frequency division multiplexing (f.d.m.) using sets of sinusoids had then been developed. The availability of digital circuitry in the 1960's awakened new interest in digital orthogonal functions and analogue multiplex systems were constructed by Hübner (3.5), Davidson (3.6), Bagdasarjanz (3.7) and others (3.8). The performance of these systems was generally disappointing because of relatively high levels of crosstalk between channels. Little attempt is made in the available literature to explain the reasons for this poor performance and it is apparent that no complete analysis of the practical problems of implementing such multiplex systems has been carried out.

In this chapter, the theory of digital orthogonal functions is developed for use in analysing the hybrid multiplex switching systems described in Chapter 2 and the analogue multiplex systems constructed by other workers. Sets of digital orthogonal functions are assessed
under three different systems' formats, i.e.

a) an analogue multiplex system (Fig. 3.3a),

b) a hybrid multiplex system with analogue multiplexing and digital demultiplexing (Fig. 3.3b),

c) a hybrid multiplex system with digital multiplexing and analogue demultiplexing (Fig. 3.3c).

By considering the theoretical performance of each of these systems it should be possible to explain the results achieved by other workers and estimate the transmission performance of the hybrid multiplex switch described in Chapter 2. The analogue system (a) is the same type of system as those which have been constructed elsewhere, while the hybrid systems (b) and (c) represent the concentrator and expander switches of the digital local exchange. Particular emphasis is placed in this chapter on the effect of imperfections in the functions and circuit elements in practical multiplex systems. In this way, the advantages of the hybrid system can be examined and compared with the analogue system. Also, the sets of functions which perform well with practical circuitry can be identified. The analysis is carried out mainly in the time domain as the results produced by practical imperfections are more readily illustrated in this manner. In addition use is made, where appropriate, of transform techniques and in particular the Walsh Fourier transform. The greater part of the analysis is concerned with the identification of causes of interchannel crosstalk, but consideration is also given to the performance of sets of functions when the multiplex highway is affected by the types of noise commonly found in telephone exchange systems.

3.1 ORTHOGONAL MATRICES

When working with digital functions, the mathematical manipulation and generation of the functions is simplified by using matrix techniques.
Consider the multiplex system of Fig. 3.3a using a set of orthogonal functions \( f_1, \ldots, f_n \). These functions can be described over a frame period \( T \) by a matrix \( \overline{M} \) provided that the function is invariant and discrete over each sub-interval (timeslot) \( t_s \) of \( T \). Each element of the function \( f_k(t) \) is related to each element of the matrix \( (m_{kj}) \) as follows:

\[
f_k(t) = m_{kj} \text{ for } \frac{j-1}{n} T \leq t < \frac{j}{n} T ;
\]

\[
j, k \in \{1, \ldots, n\} \quad \{3.1\}
\]

where \( n = T/t_s \).

Each row of the matrix represents one of the functions and each row element represents the value of the function in each timeslot.

The input data of the multiplex system is constant throughout the frame period \( T \) and is described by a vector \( \overline{V} \), where the value of each channel at the input is given by each element of the vector \( v_1, \ldots, v_n \). The output of the summing amplifier is the highway vector \( \overline{H} \), given by the product of \( \overline{M} \) and \( \overline{V} \):

\[
\overline{H} = \overline{M} \overline{V} \quad \{3.2\}
\]

The value on the multiplex highway in successive intervals of time is given by the successive elements of the vector \( \overline{h} \):

\[
h(t) = h_j \quad \frac{j-1}{n} T \leq t < \frac{j}{n} T \quad \{3.3\}
\]

At the demultiplexers, the highway vector is premultiplied by \( \overline{M} \) transposed, \( \overline{M}^T \):

\[
\overline{D} = \overline{M}^T \overline{H} \quad \{3.4\}
\]

This operation is equivalent to multiplying the highway vector by each orthogonal function and summing the results over a frame period at
each demultiplexer. Now:

$$\mathbf{D} = \mathbf{H}^T \mathbf{H} = \mathbf{H}^T \mathbf{M} \mathbf{V}$$

If $\mathbf{D} = \mathbf{V}$ then $\mathbf{H}^T \mathbf{H} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix of appropriate order. Since for any square matrix $\mathbf{A} \mathbf{A}^{-1} = \mathbf{A}^{-1} \mathbf{A} = \mathbf{I}$, where $\mathbf{A}^{-1}$ denotes the inverse of $\mathbf{A}$, then it follows that

$$\mathbf{M}^T = \mathbf{M}^{-1} \ldots \ldots$$  \[3.5\]

A matrix whose inverse is itself transposed is termed orthogonal. Larsen (3, 9) has shown that the set of functions derived from such a matrix using the equality of (3.1) is also orthogonal, i.e.

$$\sum_{i=1}^{n} m_{ki} m_{ji} = \begin{cases} n & j = k \\ 0 & j \neq k \end{cases}$$ \[3.6\]

also from equation 3.1, and noting the equivalence of continuous integration and discrete summation, we have;

$$\int_{0}^{T} f_j(t) f_k(t) \, dt = \frac{T}{n} \sum_{i=1}^{n} m_{ki} m_{ji} \ldots \ldots \text{[3.7]}$$

Taking equations 3.6 and 3.7 together we obtain;

$$\int_{0}^{T} f_j(t) f_k(t) \, dt = \begin{cases} n & j = k \\ 0 & j \neq k \end{cases}$$

This expression is recognised as the orthogonality condition discussed in Chapter 1 (Section 1.2).

The simplest matrix to fulfil the orthogonality condition is the identify matrix $\mathbf{I}$ which for order 4 is given by
where the subscript denotes the matrix order.

Applying equation 3.1 to this matrix we derive the set of block pulses used in time division multiplexing. Certain of the sets of Hadamard matrices are orthogonal, for example the matrix generating the first four Walsh functions:

\[
\overline{W}_4 = \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1
\end{bmatrix}
\]

The transpose of this matrix is

\[
\overline{W}_4^T = \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1
\end{bmatrix}
\]

The product of these two matrices is

\[
\overline{W}_4 \overline{W}_4^T = \begin{bmatrix}
4 & 0 & 0 & 0 \\
0 & 4 & 0 & 0 \\
0 & 0 & 4 & 0 \\
0 & 0 & 0 & 4
\end{bmatrix} = 4I
\]

We now consider how these matrices can be generated.

3.2 GENERATION OF ORTHOGONAL MATRICES

There are many sets of discrete orthogonal functions which are multivalued in each of the timeslots. However, the use of digital logic to generate such functions, coupled with the requirement of ease of control of the multiplexers, makes binary-valued orthogonal functions
very useful indeed. It is for this reason that this investigation has been restricted to those orthogonal functions which assume the values of +1, -1 and 0. There are basically three families of matrices which may be constructed using some or all of these values as elements:

a) the identity or binary matrices which have entries 1 and 0,

b) dyadic matrices, including Walsh functions, which have entries +1 and -1,

c) ternary matrices which have all three types of entry, +1, -1 and 0.

As mentioned in the previous section, sets of block pulses used in time division multiplexing are generated by the identity matrices. In these matrices, the order corresponds to the number of channels and timeslots in a t.d.m. system. Square identity matrices of any order can be formed, but the generation of the other orthogonal matrices mentioned above is more involved and is considered below.

3.2.1 Dyadic Matrices

The dyadic matrices are usually based on Hadamard matrices which themselves can be generated by several different methods. These methods have been described by Harmuth (3.10) and Crittenden (3.11) and given a firm mathematical foundation by Larsen (3.9). These methods of generating new orthogonal matrices from other given orthogonal matrices of lower order make extensive use of the Kronecker product of matrices. Given two matrices $\mathbf{A}$ and $\mathbf{B}$, the Kronecker product of the matrices is formed by multiplying $\mathbf{A}$ by each element, $b_{ij}$, of $\mathbf{B}$ and substituting the matrices $b_{ij}\mathbf{A}$ for the elements $b_{ij}$ of $\mathbf{B}$. If the order of $\mathbf{A}$ is $m$ and the order of $\mathbf{B}$ is $n$, the resultant matrix is of order $mn$. For example, the Kronecker product of the Hadamard matrix of order 2 $H_2$ with itself is derived as follows:
\[ H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \]

\[ H_2 \otimes H_2 = \begin{bmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{bmatrix} \]

\[ \otimes \text{ denotes taking the Kronecker product} \]

\[ = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \]

\[ = W_4 \]

This result is the Hadamard matrix of order 4 which generates the set of first 4 Walsh functions.

A second Hadamard matrix of order 4 exists; this is:

\[ H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{bmatrix} \]

Both these matrices can be used to form matrices of order 8 by forming the Kronecker product with matrix \( H_2 \). Matrices of order 16 can be formed in two ways; either by taking Kronecker products of matrices of order 4 or of matrices of order 8 and 2. Figs. 3.4 and 3.5 show two matrices of order 16. The first of these is the set of classical Walsh functions formed by the product \( W_4 \otimes W_4 \); the second uses Harmuth's method of alternating (3.10) to produce a second Hadamard matrix of order 16 \( H_{16} \). Matrices of order other than \( 2^n \), such as the matrix corresponding to the Paley functions, Fig. 3.6, cannot be formed by taking Kronecker products but can be formed by the method described by Crittenden (3.11).
3.2.2 Ternary Functions

Sets of ternary functions can be formed by taking the Kronecker products of the Hadamard matrices and block pulses. Kronecker multiplication is non-commutative and so two products can be obtained from each pair of matrices. For example, using the Hadamard matrix \( H_2 \) and the identity matrix \( I_2 \), the following Kronecker products can be formed:

(a) \[ T_{41} = H_2 \otimes I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \]

and hence

\[
T_{41} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}
\]

(b) \[ T_{42} = I_2 \otimes H_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \]

and therefore

\[
T_{42} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}
\]

The waveforms that correspond to these matrices are shown in Figs. 3.7 and 3.8. In case (a), pairs of orthogonal channels are transmitted alternately while in the second case, pairs of orthogonal channels are interleaved timeslot by timeslot.

It is of course quite apparent that the number of matrices which may be formed in this way grows rapidly as the order of the matrix increases. Table 3.1 lists some of the ternary matrices of order 16 that can be formed by taking Kronecker products of matrices of lower order. Fig. 3.9 shows the matrix of order 16 which is formed by taking the Kronecker product of \( T_{42} \) with itself. Another method of forming sets of
ternary matrices is given by Harmuth (3.10). This consists of taking orthogonal subsets of Hadamard matrices, compressing the subsets into smaller numbers of timeslots and repeating each compressed subset throughout the frame to form several extra functions. This is illustrated in Fig. 3.10 which shows (a) the Haar matrix of order 8 and (b) the functions generated by it. The third and fourth functions are derived from function 2 by the process of compressing and shifting. Functions 5, 6, 7 and 8 are likewise produced by compressing the third function. This type of matrix however is not suitable for use in a switching system, because the gain of a channel depends on the particular function allocated to it. This can be shown if we multiply the Haar matrix of order 4 by its transpose;

$$\overline{\text{Ha}_4} \cdot \overline{\text{Ha}_4}^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hence the gain of the first pair of channels will be twice that of the second pair of channels. This is not serious in a transmission system where the function allocated to a channel is fixed, and the gain can be adjusted on installation, but in a switching system the functions are allocated at random by the control unit, and hence in general it would be necessary to enable the control unit to vary the channel gain when using these functions. This is an unacceptable complication for the system. In discussing ternary functions we will therefore restrict our
interest to those derived from Kronecker products, where this problem does not arise.

3.3 IMPERFECT FUNCTIONS

The mutual orthogonality of functions in a set can be destroyed if either the functions generated or the circuitry used in the multiplex system are non-ideal. These two types of imperfection each have the same final result, namely the presence of crosstalk from one channel to another in the multiplex system. It is important to discover not only how these imperfections affect the performance of the system but also how the imperfections are related to the different types of orthogonal matrix. In Section 3.2 it was shown that there is a large number of each of the three types of matrices even for order as low as 16.

Investigation has of necessity been limited to five sets of functions which are considered representative of the various types. These are the Hadamard matrices \( \overline{W}_{16} \) and \( \overline{H}_{16} \) and the Paley matrix \( \overline{F}_{12} \). These are representative of the dyadic matrices, while the matrix \( \overline{B}_{16} (= \overline{I}_{16}) \) is used for t.d.m. systems. Also selected was a ternary matrix \( \overline{T}_{16} \) generated by taking the Kronecker product of \( \overline{T}_{42} \) with itself.

When discussing the different types of matrix it is useful to distinguish "binary orthogonality" and "dyadic orthogonality". These can be defined as follows.

Two functions are said to be binary orthogonal if one function is zero when the other function is non-zero and hence their product is zero. Two functions are dyadically orthogonal if they simultaneously have non-zero values but are orthogonal over a frame interval.

Hence the t.d.m. pulses are binary orthogonal but Walsh functions are dyadically orthogonal. The ternary functions can have dyadic and binary orthogonality. For example, indicating the function generated by
the n\textsuperscript{th} row of a matrix as F(n), F(0) and F(1) in \(\overline{T}_{16}\) (Fig. 3.9) are binary orthogonal while F(0) and F(2) are dyadically orthogonal. In the following sections we will use pairs of Walsh functions as examples of dyadic orthogonality and pairs of t.d.m. pulses as examples of binary orthogonality, although the results will be generally applicable to matrices of each type.

3.3.1 Timing Errors

Timing errors between the multiplex highway signal and the function driving a demultiplexer can arise when the multiplex highway signal and the function at the demultiplexer experience differing amounts of time delay attributable to their different paths through the system. In transmission systems, lack of timing synchronisation can arise from problems with the clock extraction circuitry. For a digital local exchange, the design and distribution of clock waveforms is carefully controlled; however differential delays of, typically, several tens of nanoseconds can easily arise in various parts of the circuitry.

The effect of this is best illustrated by example: if a signal is present on one channel of a multiplex system and a different function drives the demultiplexer, the output of the demultiplexer will be zero provided that the two signals are synchronised. If the signal is delayed with respect to the function driving the demultiplexer, an output can appear. For example, Fig. 3.11 shows the functions F(7) and F(8) from the matrix \(\overline{W}_{16}\); if F(8) is delayed by an amount equal to the length of a timeslot (t\textsubscript{s}) it becomes indistinguishable from F(7). If the orthogonality integral is performed on the delayed F(8) and F(7) it is found that the result has changed from 0 to 1 and the orthogonality between the two functions has been completely destroyed.
The effect of timing errors between functions in a set can be found by performing the orthogonality integral with the timing of one of the functions changed by a variable amount, \( \tau \), thus producing a measure based on the cross-correlation function whence

\[
R(\tau) = \int f_i(t + \tau)f_j(t)dt 
\]

Hence it can be stated that the degree to which the orthogonality of the functions is maintained under varying amounts of time delay can be found by evaluating the cross-correlation function for every pair of functions in an orthogonal matrix. Fig. 3.12 shows the cyclic cross-correlation function for the set of 16 Walsh functions (taken from Reference 3.10, pp. 196-7). The amounts of time delay experienced in a switching system are small when compared with the length of a timeslot and hence the behaviour of \( R(\tau) \) about the point \( \tau = 0 \) is of practical significance and interest. The effect of small time delays can be found from the slope of \( R(\tau) \) at the point \( \tau = 0 \). This was found by computing \( R(\tau) \) using a simple correlation program (MATRIX A7.1).

For every chosen set of orthogonal functions the cyclic cross-correlation function for every pair of functions in the set was found. The slopes of these functions are displayed in matrix form as shown in Fig. 3.13 for the set of Walsh functions, \( \overline{W}_{16} \). The numerical values in the matrix (\( n \)) indicate the degree of correlation between the functions\( \overline{W}_{16} \). A value \( n = 4 \) indicates that the cross-correlation (\( R(\tau) \)) is unity for a delay \( \tau = t_s \). The other values in the matrix (1, 2 and 3) correspond to \( R(t_s) = \frac{1}{2}, \frac{1}{4} \) and \( \frac{1}{8} \) respectively. When there is no entry in the matrix, the cross-correlation function is zero around the area of interest.

In general, for an arbitrary delay \( t_d \) the crosstalk ratio is given by
since the cross-correlation function is linear about \( \tau = 0 \),

The matrix for any set of functions can be compared with the corresponding matrices for the other sets of functions considered. These are shown in Figs. 3.14-17. If it is assumed that the time delays involved in the practical circuitry occur randomly and, moreover, that all channels should be equally immune to them, the optimum matrix would have equal, low values of the correlation functions. High values of correlation would cause the channels between which they occur to be more sensitive to time delays. Yuen (3.12) has shown that the cross-correlation functions of the sets of Walsh functions can be found from the mathematical forms of the function. He also suggests (3.13) that to minimize the cross-correlation problem, systems should be restricted to using subsets of functions with good cross-correlation properties. However this is extremely wasteful in terms of system bandwidth.

An alternative approach, suggested by Hübner (3.5), is to find sets of functions with cross-correlation matrices that are free from high values. Figs. 3.18 and 3.19 show a matrix of order 8 suggested by Hübner and its cross-correlation matrix. All values in this matrix are low and so its cross-correlation performance can be considered good. However other effects must be considered when a suitable matrix is chosen. These will be discussed in the remainder of this chapter.

3.3.2 Finite Slopes on the Multiplex Highway

So far we have considered the effect of a pure delay on the orthogonality of the functions. For this analysis we assumed that the transitions between \(+V\), \(-V\) and \(0\) are instantaneous. However the outputs of real amplifiers and logic gates take time to change from one state to
another, hence the signal on the multiplex highway can have a finite slope during transitions which can affect the orthogonality of functions in a set. Bagdasarjanz has shown (3.7) that these effects can be quantified by analysing the signal on the multiplex highway (Fig. 3.20a) into an ideal signal (Fig. 3.20b) and an auxiliary function (Fig. 3.20c). The auxiliary function thus generated contains a component of the original signal delayed by a time $t_r/2$ where $t_r$ is the measured risetime of the highway signal, and also other more complex components. However this form of analysis does not show whether the reduction in orthogonality experienced with signals of finite slope is only attributable to the delay in the ideal function or is also caused by the other components in the auxiliary function. Hence it is not possible to say whether correcting for the time delay will restore the orthogonality between the functions. An alternative method of analysis, which provides an answer to this question, has been developed for the purpose of the present study. The signal is again analysed into two components but the first of these is now an ideal function delayed by $t_r/2$ (Fig. 3.20d) and the second is a new auxiliary function (Fig. 3.20e). The effect of the delay in the ideal function can now be assessed by reference to Section 3.3.1. The effect of the auxiliary function can be found by considering a finite slope highway signal applied to a demultiplexer driven by an ideal function. Each of the elements of the auxiliary function is multiplied by +1 or -1 and the function is then integrated. As an example, $F(2)$ of $\tilde{H}_{16}$ shown in Fig. 3.20f multiplies the auxiliary function to give the function shown in Fig. 3.20g, which is then integrated. This new function consists of a series of elements of area

$$A_E = \frac{t_r}{2} \cdot \frac{V}{2} + \frac{t_r}{2} \cdot \frac{-V}{2} = 0 \ldots \ldots \ldots \ldots \{3.10\}$$
Thus, on integration, the auxiliary function produces no output.

An alternative view of this process is to consider the demultiplexer as a selective filter passing only components of the multiplying orthogonal function. This type of circuit is used in this manner for Walsh-Fourier analysis (3.14). The auxiliary function has no component to which the demultiplexer is sensitive and hence no output is produced. It is important to note that these arguments apply if the rise and fall times of the waveform are different, as each of the elements in the auxiliary function will still integrate to zero. Hence, the only effect of this type of waveform would seem to be that caused by the cross-correlation of the functions caused by the time delay. However, considering two examples of the same auxiliary function with two different functions at the demultiplexer will show that, if the time delay is corrected, the orthogonality of the two functions can be affected by the auxiliary function itself.

Fig. 3.21b shows the auxiliary function which is now multiplied by $F(2)$ of $W_{16}$ delayed by $t_r/2$ (Fig. 3.21c) to correct for the timing delay. The result of this multiplication (Fig. 3.21d) is now different, having elements with d.c. components. However, the integral of the function is still zero since the positive and negative going elements cancel in the integrator. A second example will show that cancellation does not always occur. If the same auxiliary function (Fig. 3.21b) is multiplied by $F(3)$ or $W_{16}$ again delayed by $t_r/2$ (Fig. 3.21e), the resulting function input to the integrator is as illustrated in Fig. 3.21f. It is clear that the integral of this function is non-zero. Hence, a voltage dependent on the number and size of the positive elements is produced and the orthogonality of the two functions is affected. The correction of the loss of orthogonality caused by the time delays has resulted in a loss of orthogonality because of other components in the auxiliary function.
To investigate this problem further a computer program (EDGES A7.2) was written to examine every pair of functions in each of the sets for this effect. From the example illustrated in Fig. 3.22, it can be seen that the negative elements occur when the two functions perform the same transition and the positive peaks occur when the functions perform transitions in opposite directions. The program examined each pair of functions for different numbers of the transitions and thus calculated the imbalance in the functions applied to the integrator. The degree of the imbalance was tabulated for each pair of functions as an entry in a matrix referred to as the 'Edge Matrix'.

The results gained for the set of Walsh functions were checked by analysing these types of waveforms by means of a fast Walsh-Fourier transform (FWPT A7.3) given in Reference 3.15. The values computed from the program EDGES for the matrices $H_{16}$, $W_{16}$ and $P_{12}$ are shown in Figs. 3.22, 23 and 24. It was found from this analysis that crosstalk caused by edges always occurs when the cross-correlation matrix is non-zero. For the pairs of functions where these effects occur, it is impossible to correct the crosstalk produced by time-delays in circuit components without crosstalk then occurring from these edge effects.

Fig. 3.25 shows the edge matrix for Hübner's optimum Hadamard matrix. It is evident that the edge effect is a serious problem with this matrix and consequently makes it less attractive for a practical system.

The magnitude of the edge effect can be assessed by noting that the area under each of the triangular peaks is:

$$A_T = \frac{t_x \cdot V}{4}$$

while the area of the whole function is given by
\[ A_F = mVt_s \]
where \( m \) = order of the matrix

\( V \) = nominal peak voltage

\( t_s \) = timeslot width.

From the above, therefore, the crosstalk ratio will be given by

\[ X_E = \frac{bt_s V}{4mVt_s} = \frac{bt_r}{4mt_s} \quad \ldots \ldots \ldots \ldots \quad \{3.11\} \]

where \( b \) is the number of the imbalances between the functions shown in the edge matrix.

The foregoing analysis is suitable only for dyadic functions: for the t.d.m. block pulses and the ternary matrices the effect must be analysed differently. These functions can be analysed by examining the signal present in a timeslot due to the signal in the preceding timeslot. Fig. 3.26 shows the appropriate waveforms. The output voltage, \( V_o \), from a given channel in the system will be

\[ V_o = k \int_0^{t_s} Vdt = kVt_s \]

The integral of the voltage in a timeslot due to the signal in the previous timeslot is for a linear voltage decay given by:

\[ V_E = k \int_0^{t_f} Vdt = \frac{kVt_f}{2} \]

Hence the crosstalk ratio of the binary sets will be

\[ X_{BE} = \frac{t_f}{2t_s} \quad \ldots \ldots \ldots \ldots \ldots \quad \{3.12\} \]

This is directly applicable to the t.d.m. systems. To apply this argument to the chosen ternary matrix we have several facts to consider. The functions in this matrix occupy four timeslots and each function is related to the others in one of three ways, as follows:
a) it occupies the same timeslot but is orthogonal, hence dyadic orthogonality;
b) it is the same member of \( W_4 \) but time shifted, hence binary orthogonality;
c) it is a different member of \( W_4 \) and is time shifted.

For pairs of functions for which (a) holds, the edges have no effects since transitions only occur between ±V and 0. If the pairs of functions for which (b) holds occupy adjacent timeslots the result is the same as for the t,d.m. case. When (c) holds, the shifted functions may remain partially or completely orthogonal to the crosstalk signal. The results calculated for this matrix are shown in Fig. 3.27.

3.3.3 Finite Slopes in the Multipliers

In a system employing the above functions, the multipliers in the multiplexers and demultiplexers are required to change their output voltage, V (which varies linearly as their input voltage) from +V to -V when the control input changes its sign. If the multiplying function applied to the control input has finite slopes during its transitions and the multiplier switches at the midpoint of the control voltages, then switching will be delayed by \( \frac{t_r}{2} \) or \( \frac{t_f}{2} \), these being the time that the waveform at the control input takes to reach the midpoint value. If the risetime equals the falltime (\( t_r = t_f \)) the result is a pure delay in the system as examined in Section 3.3.1; this is illustrated in Fig. 3.28. If however the rise and falltimes are not equal, then a different state of affairs exists as shown in Fig. 3.29. The length of a positive timeslot is determined by its specified length minus the time taken to switch from negative to positive plus the length of time taken to switch from positive to negative. This is

\[
t_p = t_s - \frac{t_r}{2} + \frac{t_f}{2} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (3.13)
\]
Conversely the length of a negative timeslot is

\[ t_n = t_s + \frac{t_f}{2} - \frac{t_r}{2} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \{3.14\} \]

The total length of these two timeslots remains 2t_s and the length of each is t_s if t_r = t_f. However it will be seen from the above that for t_r ≠ t_s they are different by a quantity given by:

\[ t_s = t_s - \frac{t_r}{2} + \frac{t_f}{2} = t_s - \frac{t_r}{2} + \frac{t_f}{2} \]

\[ = t_f - t_r \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \{3.15\} \]

This difference is important when the output of the multiplier is applied to the integrator in the demultiplexer. The voltage at the output of the integrator is given by:

\[ V_o = \sum_{i=1}^{m} \int_{t_{i-1}}^{t_i} v_i dt \ldots \ldots \ldots \ldots \ldots \{3.16\} \]

where

- \( i \) = the number of the timeslot
- \( m \) = the order of the matrix
- \( t_i \) = the effective length of the \( i^{th} \) timeslot
- \( v_i \) = the output voltage of the multiplier during the \( i^{th} \) timeslot.

This can be expanded as

\[ V_o = \sum_{i=1}^{m} k_p \int_{-\frac{t_p}{2}}^{\frac{t_p}{2}} v_i dt - \sum_{i=1}^{m} k_n \int_{-\frac{t_n}{2}}^{\frac{t_n}{2}} v_i dt \]

where

- \( k_p = 1 \)  \( k_n = 0 \)  \( V_n > 0 \)
- \( k_p = 0 \)  \( k_n = 1 \)  \( V_n < 0 \)

When two orthogonal functions are multiplied together these two summations in the expression are equal provided that \( t_p = t_n \),
i.e.,
\[ V_o = \frac{m}{2} \int_0^t p_{vidt} - \frac{m}{2} \int_0^n v_{idt} = 0 \ldots \ldots \{3.17\} \]

However when \( t_p \neq t_n \), substituting 3.13 and 3.14 and noting that there are \( p \) transitions in the functions at the demultiplexer, we obtain the expression
\[
V_o = \frac{m}{2} \int_0^t s v_{idt} - \frac{m}{2} \int_0^n v_{idt} + P \int_0^{t - t_r} v_{idt} - P \int_0^{t - t_f} v_{idt} = 2P \int_0^{t - t_r} v_{idt}
\]

This expression is useful in determining the crosstalk ratio as follows.

When the orthogonal function is multiplied by itself and integrated the output from the integrator will be given by:
\[
V_n = m \int_0^{t_s} v_{idt}
\]

If we note that \( v_i \) is constant over the integration interval, the crosstalk ratio due to the above effect will be given by:
\[
\chi_F = \frac{P v_i (t_r - t_f)}{m v_i t_s} = \frac{p(t_r - t_f)}{m t_s} \ldots \ldots \{3.18\}
\]

The level of crosstalk is hence proportional to the number of transitions in the function at the demultiplexer and will be worse for functions having higher numbers of transitions. This type of problem arises with all the dyadic functions and also with those ternary functions which are dyadically orthogonal. Since the binary orthogonal functions are orthogonal without integration they do not suffer from these effects.
3.4 NON-IDEAL CIRCUITRY

So far we have considered only the effects of non-ideal functions. This investigation has accounted for some imperfections in the circuitry since the functions themselves are generated by practical circuits and imperfections in the functions are caused by the circuits used. The finite slopes and delays in the circuit are caused by the amplifiers and multiplier. Similarly, the difference in the rise and fall-times of the functions is caused by the generation logic. However, other parts of the circuitry can also introduce crosstalk. These, now considered, are assumed to apply to dyadic orthogonal functions except where reference is made to binary orthogonal functions.

3.4.1 Sample and Hold Circuits

The purpose of the sample-hold circuit is to sample the input signal and hold it constant over the frame time $T$. An ideal circuit element would have the following properties:

a) the voltage would change from its old to new value instantaneously, i.e. the acquisition time would be zero;

b) the output voltage would remain constant throughout the frame period, $T$.

Practical circuits take several microseconds to change values and this can affect the orthogonality of the channel functions (3.7). However, a practical multiplex system can be so designed that the acquisition time is outside the frame period and consequently does not represent a problem. The output voltage of the device is affected by leakage currents discharging the holding capacitor. These currents flow through the sampling switch and the input of the sense amplifier (Fig. 3.30). When the signal on the multiplex highway is orthogonal to the function in the demultiplexer and the timeslot time is constant, we have:
\[ V_0 = \sum_{i=1}^{m} v_i t_s = 0 \]

However, if the voltage \( v_i \) varies in a frame period, this result is not achieved. Let the voltage \( v_i \) vary by a small amount \( \delta \) from timeslot to timeslot, i.e.

\[ v_i = v_{i-1}(1 - \delta) \]

This will tend to reduce \( v_i \) as the frame passes so that integration to zero does not occur. The worst case occurs when all the voltages are positive at one end of the frame and negative at the other (Fig. 3.31c), since all the large values of \( v_i \) must be cancelled by all the smaller values. This can be expressed as follows:

\[
\sum_{i=1}^{m} v_i t_s = \sum_{i=1}^{m/2} v_{i-1}(1 - \delta)t_s + \sum_{i=m/2+1}^{m} -v_{i-1}(1 - \delta)t_s
\]

Now the voltage in the \( i^{th} \) timeslot can be approximated to for \( \delta \ll 1 \) as follows:

\[ v_i = v_s (1 - i\delta) \]

where \( v_s \) is the voltage sampled. Hence:

\[
\sum_{i=1}^{m} v_i t_s = v_s t_s \delta \left[ -(1 + 2 + m-1 \cdot \frac{m}{2}) \right] + \frac{m+1}{2} + \ldots + m \cdot \frac{m}{2}
\]

\[ = v_s \delta \left( \frac{m^2}{2} \right) t_s \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \{3.19\} \]

Also the output voltage for a signal in the same channel is

\[ V_o = \sum_{i=1}^{m} v_s t_s = mv_s t_s \]
Therefore the worst case crosstalk from this source will be given by

\[ X_H = \frac{v_s \delta(\frac{m}{2}) t_s}{m \, v_s \, t_s} = \frac{m \delta}{4} \quad \ldots \ldots \ldots \ldots \quad (3.20) \]

3.4.2 The Multiplier

The multipliers used in the multiplexer and demultiplexer can be non-ideal in two ways:

a) the devices take a finite time to switch between the +1 and -1 states;

b) the devices may be asymmetrical, i.e. the positive excursions at the output of the devices may be different in magnitude from the negative excursions for the same input voltage.

The former effect contributes to the finite risetimes on the multiplex highway considered in Section 3.2.2. The latter effect has been analysed by Bagdasarjanz (3.7) by considering the multipliers in the multiplexer and demultiplexer as having gains of +1, -g and +1, -h, which give rise to a crosstalk

\[ X_m = \frac{(1 - g)(1 - h)}{4} \quad \ldots \ldots \ldots \ldots \quad (3.21) \]

between the channels. This approach to the question of multiplier imbalance was formulated for the set of Walsh functions, but, by considering the mechanism by which the imbalance causes crosstalk, the approach can be generalised to other sets of dyadic functions.

The effect of the imbalance at the multiplexers is to cause a voltage:

\[ V_d = \frac{(1 - g)}{2} \cdot V_m \]

to be induced in the d.c. channel. Hence, a demultiplexer multiplying the highway signal by the d.c. function, i.e. \( F(0) \) of \( \bar{W}_{16} \) or \( \bar{P}_{12} \) will detect a crosstalk voltage of level

\[ X_m = \frac{(1 - g)}{2} \]
In a similar manner, multiplying a signal into the d.c. channel at a multiplexer will lead to a crosstalk level

\[ \chi_m = \frac{(1 - h)}{2} \]

at the demultiplexers with this imbalance +1, -1. Other channels with a d.c. component, i.e. F(0), (1), (2) and (3) of H₁₆ will cause these effects in proportion to the level of the d.c. component (a), hence:

\[ \chi_m = \frac{a(1 - s)}{2} \text{ or } \frac{a(1 - h)}{2} \]

depending on the direction in which the crosstalk is measured.

When there is no d.c. component in either channel both imbalances become significant giving the result quoted in Reference 3.21.

3.4.3 The Integrator

An ideal integrator would evaluate the integral of each timeslot exactly:

\[ V_{ts} = \int_{0}^{t_s} v_i dt \]

and, in addition, produce the sum of the integrals of the timeslots over the frame period T:

\[ V_o = \sum_{i=1}^{m} \int_{0}^{t_s} v_i dt \]

The effect of variations in \( t_i \) (Section 3.3.3) and \( v_i \) (Section 3.4.1) have already been considered. The integrator stores the information in the form of charge on the integrator capacitor (Fig. 3.32).

At the end of a frame the result of the integration is transferred to the output sample-hold gate. During successive periods of integration,
the integral of the previous timeslots tends to decay away through the parallel combination of the integrator resistance and the differential input resistance of the operational amplifier.

This effect requires the same form of analysis as that used for the sample-hold gate (Section 3.3.1). Again, the worst case occurs when the integral of one half of the frame period must cancel the integral of the other half since then the maximum amounts of decay will affect inputs to the integrator of one polarity only, resulting in a maximum imbalance between positive and negative integrals. If the voltage in any timeslot decays by $\beta V$ volts, then, from Section 3.3.1, the crosstalk component for the worst case will be

$$\chi^*_i = \frac{m\beta}{4}$$

The value of $\beta$ is determined by the integrator time constant, $\tau_i$, since

$$V_o = v_i \exp\left(-\frac{t}{\tau}\right) = v_i(1 - \frac{t}{\tau})$$

for $t \ll \tau_i$

moreover

$$V_o = v_i - \beta v_i$$

consequently

$$\beta = \frac{t}{\tau_i}$$

and for an analogue operational amplifier integrator

$$\tau_i = \frac{R + Rd}{ARR_d C}$$

(3.16)

and since $R_d >> R$ we have $\beta = \frac{t}{ARC}$. This produces a crosstalk figure for this mechanism

$$\chi^*_i = \frac{m \frac{t}{s}}{4ARC}$$

3.4.4 The Analogue Gates

The foregoing analysis is applicable to the multiplexing and demultiplexing of dyadic orthogonal functions. For systems using
binary orthogonal functions, analogue gates are necessary. These gates are used to switch the appropriate channel onto the multiplex highway in the correct timeslot. Problems can arise through the coupling of signals onto the multiplex highway when the gate is nominally turned 'off'. Metal-oxide-silicon transistor switches developed in recent years have largely overcome this problem. In these devices, the ratio of 'on' resistance to 'off' resistance is very large, typically $10^9$, and the capacitive coupling between the inputs and the outputs of the switches is very small resulting in excellent performance (3.17).

3.5 CIRCUITRY FOR THE HYBRID MULTIPLEX SYSTEM

The conversion of the data on the multiplex highway to digital form in the concentrator switch and from digital form in the expander switch changes the realisation of some of the systems components. Hence the imperfections which affect the analogue systems performance may be absent or appear in a different form in the digital parts of the system. Since the components replaced by their digital counterparts are different for the concentrator and expander switches, each of them will be considered in turn.

3.5.1 The Concentrator Multiplex Switch

In this part of the switch, the analogue demultiplexers are replaced by the equivalent digital circuits which follow the process of analogue to digital conversion (Fig. 3.3b). The input sample-hold gates and the multipliers which form the multiplexers remain unchanged and so the same problems, as mentioned earlier, generated by these circuits are expected to be present. The digital demultiplexers differ in several important respects from their analogue counterparts, in particular in the realisation of the integrator. This circuit is required
to produce the sum of the voltages during each timeslot, which is achieved in the analogue case by integrating the voltage throughout successive timeslots, i.e.

$$V_o = \sum_{i=1}^{m} \int_{t_{i-1}}^{t_i} v_i dt$$

However, in the digital case, the demultiplexer contains an accumulator acting as an integrator which adds together the successive outputs of the multiplier

$$V_o = \sum_{i=1}^{m} v_i$$

Hence the variations in the integration time discussion in Section 2.3.3 will not exist in this digital case. Similarly the digital multipliers in the demultiplexer can also be shown to be ideal. The digital data is represented in the demultiplexers in the form of '2's complement' numbers and multiplication by +1 and -1 can be performed by an Exclusive-OR operation between the signal and the orthogonal functions. Hence there is no possibility of imbalance in the multipliers. Also the signal data is stored in digital latch circuits and therefore it cannot decay unlike the data stored in the analogue integrators. Moreover, the digital gating circuitry used for the binary orthogonal functions is also ideal. However, these observations on the state of the demultiplexer presuppose an accurate representation of the analogue highway signal in digital form. This representation could be inaccurate in two ways:

a) if the converter displays a lack of symmetry about the zero volt level;

b) if the converter itself is affected by the delays and finite slopes on the multiplex highway.
A lack of symmetry in the converters would manifest itself as imbalance in the multipliers in the demultiplexer. However, a correctly adjusted converter will be highly symmetrical and this is not usually a problem. The second problem can be avoided if a converter with a conversion time less than the timeslot time is used. Fig. 3.33 shows the time in which the conversion can then take place in relation to the rise and fall times \((t_r, t_f)\) of the multiplex highway. Unless these times are large the waveform is constant while conversion takes place and an accurate digital representation of the analogue level is achieved. This representation is also unaffected by small variations in delay.

3.5.2 The Expander Multiplex Switch

With the above considerations in mind, the system as shown in Fig. 3.1c is now studied. The multiplexers of the analogue system are now replaced by their digital counterparts. The input sample-hold circuit becomes a set of digital latches and, as mentioned above, the multipliers perform an Exclusive-OR operation on the signal data and the multiplying orthogonal function. Again these digital operations can be regarded as ideal. The demultiplexer remains analogue and it is therefore subject to all the problems which have been considered in previous sections. Consequently it seems reasonable to suppose that the performance of this part of the system will be worse than the performance of the concentrator switch. The digital to analogue converter can be relied on to provide a symmetric highway signal although the output of the converter will have waveforms of finite slope, which may be subject to differential delays on distribution to the various demultiplexers in the switch. It would seem therefore that the performance of the expander switch would be similar to the performances of the various
analogue multiplex systems reported elsewhere (3.5 - 3.8) and less than adequate for the system envisaged. However, the foregoing analysis has shown that many of the causes of reductions in orthogonality are caused by the behaviour of multiplex highway waveform about the time of transition between positive and negative values. Also it was noted that, in the case of the concentrator switch, the degradations should be avoided if the conversion is restricted to times when the multiplex highway is not undergoing transition. In a similar manner, the performance of the expander switch should be improved by only gating the output of the multiplier into the integrator outside the time when the highway waveform is undergoing a transition. In this way, the performance of the systems using analogue demultiplexers, that is the expander multiplex switch or an analogue multiplex system, is expected to be independent of delays and finite slopes on the highway and in the multipliers. While the circuitry will still depend for its performance on the analogue integrators and multipliers, the crosstalk performance of the system should be greatly improved, since any crosstalk components arising out of waveform imperfections will have been eliminated from the system.

3.6 NOISE PERFORMANCE

When analysing the noise performance of a multiplex system it is usual to investigate the effects of burst noise and random noise on the system. However in a digital switching system these types of noise are substantially absent, and two other forms of noise must be considered. These are the quantisation noise implicit in the process of analogue to digital conversion and cyclic noise induced onto the multiplex highway. The cyclic noise is produced by crosstalk from the different waveform and tone generators used in the exchange.
3.6.1 Quantisation Noise

The quantisation noise inherent in any process of analogue to digital conversion also affects the signal on the multiplex highway. The magnitude of this problem is different for binary and dyadic functions.

Consider the set of dyadic functions. With one channel in operation each timeslot voltage will be digitised with an accuracy of \( \frac{\delta_a}{2} \) where \( \delta_a \) is the width of the smallest step in the analogue to digital converter. If the system has \( m \) timeslots, then the signal voltage summed in the digital demultiplexer will be \( mV_n \) and the noise voltage at the same point will be \( \frac{m\delta_a}{2} \). Hence the signal to noise ratio will be proportional to \( \frac{V_n}{\delta_a} \), which is the same as the result for one timeslot for a t.d.m. system or for a non-multiplexed signal.

Now consider the range of the input and multiplexed signals. If the input signals have a range of \( \pm V_m \) then, by the process of addition in the summing amplifier, the multiplexed signal can have a range of \( \pm mV_n \). The maximum values will occur when all the individual channels have their peak values. If this range is to be converted to digital form without any change in signal to noise ratio for each individual channel, then a number of extra encoding digits (equal to \( \log_2 m \)) is required. However, because of the low probability of the occurrence of the peak value, a number less than this may suffice. The exact number of extra levels required is investigated more fully in Chapter 5. With binary orthogonal functions, the range of voltages in any timeslot is the same as that of the input signal, hence no extra levels are required. With ternary matrices the number of extra levels depends on the number of channels added together in any timeslot. This is the same as the order of dyadic orthogonality; that is, the order is the same as that of the dyadic Kronecker factor in the product. With \( T_{16} \) this is 4.
since the highest factor matrix is $W_4$ and hence $\log_2 4 = 2$ extra levels are required. If the extra levels are not provided, then the quantisation noise further degrades the system at a rate approximately equal to $6 \log_2 m$ dB.

3.6.2 Cyclic Noise

In a switching system, there are a large number of waveform generators. These circuits are associated with the control unit switch interfaces and signalling circuitry. Many of these waveforms are distributed through paths near to the transmission highways and this results in the coupling of components of these signals onto the highways, with the possibility of noise on the transmission channels. The immunity which the system displays to this noise depends on the set of functions used. For example, consider an interfering waveform as a square wave with a period of two frame times (2T) which is synchronised to the multiplex highway. We consider this multiplied by a Walsh function, say $F(3)$ of $\overline{W}_{16}$, when the dyadic orthogonal functions are used and by a block pulse, $F(3)$ of $\overline{B}_{16}$, when binary orthogonal functions are used. The Walsh function is orthogonal to the square wave since this appears as a constant value in each frame period. The action of multiplying by $F(3)$ of $\overline{W}_{16}$ and integrating the result causes a zero result at the end of each frame period. On the other hand, the square wave appears as a constant added value in each timeslot when block pulses are used since none of the block pulses are orthogonal to the square wave.

In general, the immunity of the functions to a noise signal of this type will vary inversely as the d.c. component of the channel. The noise is in effect added into the d.c. channel and most of the dyadic functions are orthogonal to this channel, whereas the t.d.m. pulses are not. It seems reasonable, therefore, to expect that the dyadic matrices will have a greater immunity to this type of noise.
FIG. 3.1 RADEMACHER FUNCTIONS $r_n(t)$

FIG. 3.2 WALSH FUNCTIONS
(a) THE ANALOGUE MULTIPLEX SYSTEM

(b) THE HYBRID CONCENTRATOR SYSTEM

(c) THE HYBRID EXPANDER SYSTEM

ANALOGUE CIRCUIT
DIGITAL EQUIVALENT

KEY
SAMPLE HOLD
INTEGRATOR
MULTIPLIER
DIGITAL MULTIPLIER
ACCUMULATOR
ANALOGUE TO DIGITAL CONVERTER
DIGITAL TO ANALOGUE CONVERTER

FIG. 3.3
FIG. 3.4 THE WALSH HADAMARD MATRIX OF ORDER 16($\bar{W}_{16}$)

\[
\begin{bmatrix}
+ & + & + & + & + & + & + & + & + & + & + & + & + & + & + \\
+ & - & + & + & + & + & + & + & + & + & + & + & + & + & + \\
+ & - & + & + & + & + & + & + & + & + & + & + & + & + & + \\
+ & - & + & + & + & + & + & + & + & + & + & + & + & + & + \\
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+ & - & + & + & + & + & + & + & + & + & + & + & + & + & + \\
+ & - & + & + & + & + & + & + & + & + & + & + & + & + & + \\
\end{bmatrix}
\]

\text{+} = +1, \quad \text{-} = -1

FIG. 3.5 THE HADAMARD MATRIX OF ORDER 16($\bar{H}_{16}$)

\[
\begin{bmatrix}
+ & + & + & + & + & + & + & + & + & + & + & + & + & + & + \\
+ & - & + & + & + & + & + & + & + & + & + & + & + & + & + \\
+ & - & + & + & + & + & + & + & + & + & + & + & + & + & + \\
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+ & - & + & + & + & + & + & + & + & + & + & + & + & + & + \\
+ & - & + & + & + & + & + & + & + & + & + & + & + & + & + \\
\end{bmatrix}
\]
FIG. 3.6 THE SET OF PALEY FUNCTIONS ($\bar{P}_{12}$)

FIG. 3.7 THE SET OF FUNCTIONS GENERATED BY $T_{41}$

FIG. 3.8 THE SET OF FUNCTIONS GENERATED BY $T_{42}$
<table>
<thead>
<tr>
<th>DYADIC 2 ⊗ BINARY 8</th>
<th>DYADIC 4 ⊗ BINARY 4</th>
<th>DYADIC 8 ⊗ BINARY 2</th>
<th>TENNARY 4 ⊗ TENNARY 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{T}_8$</td>
<td>$\overline{H}_2$</td>
<td>$\overline{T}_8$</td>
<td>$\overline{T}_{41}$</td>
</tr>
<tr>
<td>$\overline{H}_2$</td>
<td>$\overline{T}_8$</td>
<td>$\overline{T}_4$</td>
<td>$\overline{T}_{41}$</td>
</tr>
<tr>
<td>$\overline{W}_8$</td>
<td>$\overline{T}_2$</td>
<td>$\overline{T}_4$</td>
<td>$\overline{T}_{41}$</td>
</tr>
<tr>
<td>$\overline{T}_2$</td>
<td>$\overline{W}_8$</td>
<td>$\overline{T}_4$</td>
<td>$\overline{T}_{42}$</td>
</tr>
</tbody>
</table>

**TABLE 3.1 CRONECHER PRODUCTS GIVING MATRICES OF ORDER 16**

$$
\begin{bmatrix}
+0 & +0 & 0 & 0 & 0 & 0 & +0 & +0 & 0 & 0 & 0 & 0 & 0
+0 & +0 & 0 & 0 & 0 & 0 & +0 & +0 & 0 & 0 & 0 & 0 & 0
+0 & -0 & 0 & 0 & 0 & 0 & +0 & -0 & 0 & 0 & 0 & 0 & 0
+0 & 0 & -0 & 0 & 0 & 0 & +0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & +0 & 0 & 0 & 0 & 0 & +0 & +0 & 0
0 & 0 & 0 & +0 & -0 & 0 & 0 & 0 & 0 & +0 & 0 & -0 & 0
0 & 0 & 0 & 0 & 0 & +0 & 0 & 0 & +0 & 0 & -0 & 0 & 0
0 & +0 & +0 & 0 & 0 & 0 & +0 & 0 & -0 & 0 & 0 & 0 & 0
+0 & +0 & 0 & 0 & 0 & 0 & -0 & -0 & 0 & 0 & 0 & 0 & 0
0 & +0 & +0 & 0 & 0 & 0 & -0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & +0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & +0 & +0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & +0 & +0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & +0 & +0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & +0 & +0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & +0 & +0 & +0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & +0 & +0 & +0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & +0 & +0 & +0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & +0 & +0 & +0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & +0 & +0 & +0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0

+ \equiv +1
- \equiv -1
0 \equiv 0

**FIG. 3.9 THE CRONECHER PRODUCT OF $\overline{T}_{42}$ WITH ITSELF ($T_{16}$)**
a) THE HAAR MATRIX OF ORDER 8

\[
\begin{bmatrix}
+ & + & + & + & + & + & + & + \\
+ & + & - & - & - & - & - & - \\
+ & + & - & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & + & + & - & - \\
+ & - & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & + & - & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & + & - & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & + & -
\end{bmatrix}
\]

\[+ \equiv +1\]
\[- \equiv -1\]
\[0 \equiv 0\]

b) THE SET OF HAAR FUNCTIONS GENERATED BY 3.10a

FIG. 3.10 HAAR FUNCTIONS
FIG. 3.11

(a) FUNCTION 7 OF $\overline{W}_{16}$

(b) FUNCTION 8 OF $\overline{W}_{16}$
FIG 3.12 CYCLIC CORRELATION OF WALSH FUNCTIONS
FIG. 3.13 THE CYCLIC CROSSCORRELATION MATRIX OF $\overline{W}_{16}$

$$\begin{array}{cccccccccccc}
15 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
14 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
13 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
12 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
11 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
10 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
9 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
8 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
7 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
6 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
5 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
4 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
3 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15
\end{array}$$

FIG. 3.14 THE CYCLIC CROSSCORRELATION MATRIX OF $\overline{H}_{16}$

$$\begin{array}{cccccccccccc}
15 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
14 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
13 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
12 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
11 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
10 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
9 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
8 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
7 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
6 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
5 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
4 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
3 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15
\end{array}$$
FIG. 3.15 THE CYCLIC CROSSCORRELATION MATRIX OF $B_{16}$

FIG. 3.16 THE CYCLIC CROSSCORRELATION MATRIX OF $T_{16}$
FIG. 3.17 THE CYCLIC CROSSCORRELATION MATRIX OF $F_{12}$

\[
\begin{bmatrix}
+++ & ++ & - & ++ & - & + & - \\
+++ & + & + & + & + & - & - \\
+++ & + & - & + & + & - & + \\
+++ & - & - & + & - & + & - \\
++ & - & - & - & - & + & + \\
+- & + & - & - & - & - & - \\
+- & - & + & + & + & + & + \\
\end{bmatrix}
\]

FIG. 3.18 THE OPTIMUM MATRIX ACCORDING TO HUBNER

FIG. 3.19 THE CYCLIC CROSSCORRELATION MATRIX FOR HUBNER'S OPTIMUM MATRIX
FIG. 3.20
(a) HIGHWAY SIGNAL
(b) AUXILLARY FUNCTION
(c) MULTIPLYING FUNCTION AT THE DEMULTIPLEXER
(d) AUXILLARY FUNCTION INPUT TO INTEGRATOR
(e) MULTIPLYING FUNCTION AT THE DEMULTIPLEXER
(f) AUXILLARY FUNCTION INPUT TO INTEGRATOR

FIG. 3.21
FIG. 3.22 THE EDGE MATRIX FOR \( \overline{W}_{16} \)

FIG. 3.23 THE EDGE MATRIX FOR \( \overline{H}_{16} \)
FIG. 3.24 THE EDGE MATRIX FOR $\bar{P}_{12}$

FIG. 3.25 THE EDGE MATRIX FOR HUBNERS OPTIMUM MATRIX
FIG. 3.26 HIGHWAY WAVEFORMS FOR t.d.m

FIG. 3.27 THE EDGE MATRIX FOR T16
DELAY = $\frac{1}{2} = \frac{1}{2}$

(a) IDEAL FUNCTION

(b) CONTROL FUNCTION

(c) MULTIPLIER SWITCHING POINTS

FIG. 3.28

(a) CONTROL FUNCTION (DETAIL)

(b) MULTIPLIER SWITCHING POINTS

FIG. 3.29
FIG. 3.30 A SAMPLE HOLD GATE

(a) HIGHWAY SIGNAL WITH SAMPLE HOLD DECAY

(b) MULTIPLYING FUNCTION AT THE DEMULTIPLEXER

(c) INPUT TO INTEGRATOR

FIG. 3.31 THE EFFECT OF DECAY IN THE SAMPLE HOLD
FIG. 3.32 THE INTEGRATOR

FIG. 3.33 THE ANALOGUE TO DIGITAL CONVERSION TIME
An experimental model of the multiplex system was built to test the validity of the theoretical results derived in Chapter 3 and also to discover the performance capabilities of a practical system. While the hardware constructed is not intended to be representative of that used in a practical exchange system, it will of course contain the same functional units as such a system. Hence the results reported in this section fall into three parts. The measurements of change of crosstalk performance when each of the parameters identified in Chapter 3 is varied, so testing the validity of the analysis carried out in that chapter. Then the measurements on crosstalk performance between all the pairs of functions in the matrices chosen for evaluation is described. These measurements give a useful indication of the likely performance of a practical system. Finally the noise immunity of the sets of functions when random and cyclic noise is added to the multiplex highway is reported.

Therefore after a description of the circuits of the experimental system, each of these sets of measurements will be described in turn.

4.1 THE EXPERIMENTAL SYSTEM

The experimental system (Fig. A4.1) was constructed so that it could either be used as an analogue multiplex system (Fig. 3.3a), referred to as the analogue system; or as a model of the hybrid multiplex switch (Fig. 3.3b), referred to as the hybrid system. Therefore it contains

† The A prefix indicates that the figure is to be found in the relevant Appendix.
two analogue multiplexers, together with a summing amplifier. Also two demultiplexers were constructed, the first of these being an analogue demultiplexer which together with the analogue multiplexers forms the analogue system. The second was a digital demultiplexer which followed an analogue to digital converter. The output of the converter is taken through appropriate circuitry which allows the gain to be normalised for different sets of functions and also compresses the data into A-law p.c.m. format. The data is then expanded into the linear format and reconverted to analogue form: this then forms a model of the hybrid system. The performance of the expander switch (Fig. 3.3c) was not measured separately as this would have required the construction of considerably more hardware and the performance of this circuit can be inferred from the results on the analogue and hybrid systems. These inferences and the limitations of the expander switch are discussed at the end of this chapter.

Each of the multiplexers and demultiplexers in the system is controlled by one of three function generators. The first of these controls an analogue multiplexer and the second both an analogue multiplexer and demultiplexer. The third function generator controls the digital demultiplexer. The interrelation between these units is shown in Fig. A4.1. The whole system is clocked from an internally generated set of waveforms illustrated in Fig. A4.16 and in Plates 24 and 25. The system was constructed to sample the incoming signal at the standard p.c.m. sampling rate, i.e. every 125μs. This period is subdivided into 17 equal timeslots of 7.35μs, 16 of which form a frame period of length 118μs. Thus sets of functions based on matrices of orders up to 16 can be used for multiplexing. This maximum order of matrix was chosen as 16 as it is a suitable number of channels for the concentrator and expander switches. Smaller numbers lead to a more
inefficient use of channel capacity; for example if a 7 channel
system is used the amount of traffic it can handle for the same grade
of service is 2.16 erlangs compared with 7.3 for a 15 channel system.
Larger numbers than 16 lead to a corresponding increase in the speed
required of the converter and also in the dynamic range of the system.
The 17\textsuperscript{th} timeslot was built into the system in order to allow time
to reset integrators between frame periods and to also permit time for
the sampling of the input signals and integrators.

The complete experimental system is shown in Plate 1 and examples
of the circuitry can be seen in Plates 2 and 3. Typical waveforms
associated with the system which illustrate its various modes of
operation can be seen in Plates 4 to 23.

4.1.1 The Function Generators

In order to generate each of the five sets of orthogonal functions
selected in Chapter 3, a function generator of considerable flexibility
is required. The usual method of generating functions such as the
Walsh functions is to use binary division circuits together with
combinational logic (4.1). These generators derive their internal
structure from the innate mathematical relations between functions in
a set. Hence to generate a number of different sets of functions would
require a considerable amount of internal switching. In the experimental
system it is necessary to be able to switch easily between the different
sets of functions chosen and between functions in a given set.
The approach adopted in the model is to generate the orthogonal functions
by electronically reading the positions of a set of switches. By using
a type of two-pole three-way switch it is possible to set the value
of the orthogonal function in each timeslot to +1, -1 or 0 and by doing
this it is possible to generate any set of two- or three-valued digital
orthogonal functions up to order 16. However, to do this it is necessary to control both the sign of the signals and the gating of the signals into the summing amplifier, therefore the control logic has two parallel parts. Each of these, shown in Figs. A4.3 and A4.4, works as follows. At the start of each frame period a pulse is clocked into a 16 bit shift register. The pulse is shifted along by one position in each timeslot. In each position the pulse is gated with the switch setting for that timeslot and the resultant output of all the gates is combined in a NAND gate. The result of this operation is that the positions of the switches in successive timeslots is read sequentially thus generating the function indicated by the switches. Since one half of the circuitry controls the analogue gates and determines whether a function is 0 or 1 in a given timeslot and the second half controls the sign of the function, values +1, -1 and 0 can be set in each timeslot.

4.1.2 The Analogue Multiplexers

These circuits, which are shown schematically in Fig. 4.2 and in detail in Fig. A4.2, consist of three parts:

a) a sample-hold circuit;

b) a multiplier controlled by a function generator;

c) an analogue gate controlled by the same function generator as in (b).

The sample-hold circuit is a simple integrated circuit which contains the sampling gate and buffer amplifiers. The sampling gate is opened during the 17th timeslot only, maintaining the output of the sample-hold gate constant throughout the next 16 timeslots which are used for multiplexing.

Several different types of multiplier were examined before a satisfactory performance was achieved. A typical circuit (Fig. 4.1)
uses a pair of analogue gates to switch between the outputs of two operational amplifiers with gains of +1 and -1. Circuits of this type were generally unsatisfactory since it was difficult to avoid imbalance between +1 and -1 of the type described in Section 3.4. This problem was solved by the use of an integrated modulator/demodulator circuit (4.2, 4.3). In this circuit the balance between +1 and -1 is determined by the matching between two integrated current sources. The balance was tested by measuring the crosstalk between a channel using the d.c. function (e.g. $F(0)$ of $\overline{W_{16}}$) and a second channel using some of the higher functions with a zero d.c. component. The crosstalk measured between these channels is then due to imbalance between the sources in the multiplier. Results indicate that a matching of better than 1 part in $10^{+4}$ is achieved.

The analogue gate used was a complementary metal oxide silicon (C.M.O.S.) switch which was capable of handling signal voltages of 0-14V. The output from this switch is then applied to a buffer amplifier which is connected to the multiplex highway. This amplifier could be removed for some measurements where a less ideal highway waveform was required.

4.1.3 The Analogue Demultiplexer

This circuit consists of a multiplier, analogue gate, integrator and sample-hold gate. These elements are shown in outline form in Fig. 4.3 and more fully in A4.14. Theoretically, the positions of the multiplier and analogue gate can be interchanged. However the gate presents a changing load to the source feeding it as it switches on and off and this can be a problem if the gate is fed directly from the multiplex highway. The changing load can cause the magnitude of the highway signal to change and so degrade the waveform. If however the
multiplier is placed between the highway and the gate, the highway is isolated by the multiplier and is unaffected by the state of the gate. The multipliers, analogue gates and sample-hold gates are of the same type as those used for the multiplexers. The integrator is based on a standard operational-amplifier circuit with an analogue gate in parallel with the capacitor. This analogue gate allows the integrator to be reset during the 17th timeslot. One further circuit was included in the demultiplexer, a digital delay element which allows the multiplying functions to be delayed so that they can be exactly synchronised to the multiplex highway. This device allows not only the effects of time delays to be measured directly, but also allows this parameter to be set to zero so that crosstalk from this source does not mask that from other effects.

The operation of the demultiplexer is as follows: the multiplier multiplies the highway signal by +1 or -1 under the control of the function generator. The analogue gate transmits the multiplied waveform to the integrator also under the control of the function generator. The integrator produces the integral of the signal over the first 16 timeslots and this integral is sampled in the first half of the 17th timeslot before the integrator is reset in the second half in readiness for the next frame of information.

4.1.4 The Summing Amplifier and Analogue to Digital Converter

The summing amplifier, which is shown together with the analogue to digital converter in Fig. A4.5, is a high slew rate operational amplifier used as an analogue adding circuit. The circuit has a small gain in order to scale the multiplex highway signal to match the input range of the analogue to digital converter. The converter (or encoder) is one of the most important elements of a practical system and the implementation of such a system would require a custom-built device with
good gain/level linearity (4.4). However the experimental system was built using a commercially available 12-bit linear converter and so would not meet the C.C.I.T.T. specification. The device used is capable of converting the data from analogue to digital form in 4\(\mu s\) - well within the length of one timeslot (7.35\(\mu s\)). The encoder was set to encode the highway signal in each of the 16 timeslots in the frame. After encoding, the data is sent to the input of the digital demultiplexer in the form of '2s' complement binary numbers.

4.1.5 The Digital Demultiplexer and Gain Adjustment Control

The digital demultiplexer (Fig. A4.6) consists of the following parts:

a) 'true/complement, zero/one' elements;

b) the function generator;

c) the accumulator;

d) the output buffer,

Each of these parts was constructed from standard t.t.l.

In operation the data from the converter is applied to the 'true/complement zero/one' elements. The truth table for this device is given in Fig. 4.4 and shows that the 'A' input controls the polarity of the data while the 'B' input controls the transmission or inhibition of the data. These elements can therefore be used to perform the functions of multiplying by +1 and -1 and by +1 and 0. After multiplication the data is applied to the input of an accumulator which comprises an adder and storage register. The accumulator adds the data from the current timeslot to that accumulated from the previous timeslots which is stored in the register. At the end of the 16\(^{th}\) timeslot the data is transferred to the output buffer and the accumulator is reset by clearing the contents of the storage register.
The accumulation of sixteen 12-bit data words results in an output wordlength of 16 bits, although only the 12 most significant bits of this word are valid since the last 4 bits are the accumulation of quantisation errors from the encoder. Where less than 16 bits are accumulated the bits which are the valid output of the accumulator are different. For example, the ternary matrix $T_{16}$ requires accumulation over four timeslots; hence the two most significant bits of the output of the accumulator never change and only the two least significant bits are invalidated due to quantisation noise. Therefore, in this case, it is the 12 remaining bits which form the valid output, i.e. bits 2 - 13 at the output of the accumulator. If bits 0 - 12 are selected (where bit 0 is the most significant bit) the result at the demodulated output is an apparent reduction in gain of 4 (12dB).

Furthermore, when the t.d.m. matrix is used, only the least significant 12 bits are valid, i.e. the encoded data from one timeslot. A circuit is therefore required to select the significant 12 bits for each matrix. This is the gain adjustment circuit shown in Fig. A4.7. It consists of combinational logic which can select any of 4 groups of 12 bits from the output of the accumulator, and thus equalise the gain between different matrices. Using this, matrices of orders 1, 2, 4 and 16 can be adjusted but not of orders 8 or 12. Order 8 was omitted because no matrix used in the study accumulated data over 8 timeslots and order 12 could not be accommodated by selecting a 12-bit group. Therefore, in the case of the Paley functions, the same setting as that used for matrices or order 16 was used. This led to an attenuation of 2.5dB ($= 20 \log_{10}16/12$) which was corrected for in subsequent measurements.

The gain adjustment circuit can also be seen as a division circuit which divides the output data by the normalisation constant of the matrix,
i.e. where $D_a$ is the accumulated data, $D_o$ is the required output and $n$ is the normalisation constant of the matrix.

$$D_o = \frac{D_a}{n} \quad \ldots \ldots \ldots \ldots \ldots \quad \{4.1\}$$

4.1.6 The Compander

The output of the gain adjustment circuit is still in the form of a 12-bit linear encoded '2s' complement number. This was then converted into an A-law-companded-8-bit-p.c.m. number and then reconverted to the linear form before digital to analogue conversion. A switch included in this circuitry enabled the non-companded or companded signals to be converted to analogue form. In this way the effect of companding on the measurements could be assessed.

The compander circuitry is shown in block diagram form in Fig. A4.8. The circuit consists of the following parts:

- a) '2s' complement to sign and magnitude conversion;
- b) compressor;
- c) buffer;
- d) expander;
- e) sign and magnitude to '2s' complement conversion;
- f) the switch.

In order to convert from '2s' complement to sign and magnitude realisation it is necessary to complement the negative numbers and add one to the result. The same process also performs the conversion in the opposite direction. Hence the same converter circuit (Fig. A4.9) is used for both processes (a) and (e) in the above list.

The conversion to and from A-law p.c.m. is based on techniques described by Kaneko (4.4). These circuits (Figs. A4.10 and A4.11) use a serial algorithm instead of the usual parallel combinational logic.
techniques. The last part of this circuitry is a switch which allows the selection of the unprocessed linear word or the data after companding (Fig. A4.12).

4.1.7 The Digital to Analogue Converter and Filter

The final part of the circuitry comprises a 12-bit digital to analogue converter and an analogue active filter (Fig. A4.13). For strictly accurate conversion a 13-bit converter is needed so that the offset nature of the A-law code can be accurately translated into the analogue step sizes. However for the purposes of the measurements on this experimental system, which did not include investigation of the low level linearity performance, a 12-bit converter was of sufficient accuracy for the measurements undertaken. The operation of the converter is quite straightforward. Data is clocked into the converter in the 17th timeslot of each frame and the converter produces an output which is held constant for the duration of the frame. This output is then fed to a fifth-order elliptic filter with a cut-off frequency of 3.4kHz. This filter meets the C.C.I.T.T. specification for p.c.m. bandlimiting filters. The active filter technique uses the concept of the super-capacitance (4.6) to realise a lossy ladder network. The particular realisation was based on a singly-terminated lossy LCR prototype.

4.2 IMPERFECT FUNCTIONS

In Chapter 3 we showed that the crosstalk ratio between channels of a multiplex system could be attributed to certain imperfections in the modulating functions and the highway signal. In particular the effects of time delay, edges and unequal risetimes and falltimes were considered. In this section we describe the measurements made on the model system in order to verify the theoretical results obtained for these effects.
4.2.1 **Time Delays**

In Chapter 3 the effects of time delay between the multiplex highway signals and the demultiplexing functions were related by the cross-correlation matrices for the different sets of functions. These matrices gave the values of cyclic cross-correlation between pairs of channels; this assumed that the functions in a set repeated periodically without interruption. However, in the model system, there is an interval of one timeslot where the functions are identically zero. The effect of this is to change the cross-correlation in the system from cyclic to non-cyclic. Therefore it was necessary to calculate the differences between the cyclic and the non-cyclic matrices so that the theoretical and experimental results would be comparable. The cross-correlation program, MATRIX A7.1, was therefore re-run for the non-cyclic functions and the experimental results were then compared with the results of these computations. The non-cyclic cross-correlation matrices for the matrices \( \overline{W}_{16}, \overline{H}_{16} \) and \( \overline{P}_{12} \) are shown in Figs. 4.5, 4.6 and 4.7. New results were also calculated for the ternary matrix \((T_{16})\) and are shown in Fig. 4.8. The only difference between the cyclic and non-cyclic correlation matrices for the t.d.m. matrix \( \overline{H}_{16} \) is that the cross-correlation between functions 0 and 15 is reduced to zero. In Chapter 3 the values displayed in the matrices were normalised to a value of 4 as this made all the entries in the matrices integers. The values shown in the non-cyclic cross-correlation matrices are normalised to a value of 16 for the same reason. Hence:

\[
\chi_d = \frac{nt_d}{16t_s}
\]

and comparison between the two sets of results requires those of Chapter 3 to be multiplied by 4. The results were tested experimentally by taking measurements on three pairs of functions selected from the
matrix $\overline{W}_{16}$. Each of these pairs was chosen to have a different cross-correlation coefficient and a zero entry in the edge matrix. This enabled the cross-correlation coefficients to be measured in isolation. The crosstalk between each of these pairs of functions was measured for different values of time delay. The results are shown in graphical form in Graphs 4.1 and 4.2. As can be seen the level of crosstalk varies linearly with time delay in each case, but the slopes are different in each case. The computed and measured values of the crosstalk ratio are also shown on each graph. It is evident that there is sufficient agreement between the computed and measured value to regard the cross-correlation matrices as accurately reflecting the effects due to time delays.

4.2.2 Finite Slopes on the Multiplex Highway

The effect of finite risetimes on the multiplex highway was analysed by selecting three pairs of functions from the edge matrix of $\overline{W}_{16}$. Unlike the cross-correlation matrix, it was not possible to select pairs of functions which are only affected by finite slopes, and not by time delays. Therefore measurements were taken on each of the pairs of functions by adding capacitance to the multiplex highway, which increased both the time delay and the edges, and then by adjusting the demultiplexing function until there was no relative delay between the functions. This point of zero relative delay was assumed to occur when the crosstalk was at a minimum. The level of crosstalk then observed was assumed to be solely due to the finite slopes. The levels for each pair of functions are shown in Graph 4.3. Also shown are the computed and measured values of the matrix coefficients, the measured values being taken from the slope of the lines through the experimental points. It was assumed that the zero on the graph was without significance as no practical measurements could be taken about these points and the position indicated only implies
the point of cancellation of this and other effects. The discrepancies between the measured and computed values are thought to arise from the simple nature of the model on which the computation is based. In particular the analysis of Chapter 3 assumed a linear transition between positive and negative values. In reality this was more nearly approximated to by a single RC time constant.

4.2.3 Finite Slopes in the Multipliers

Using t.t.l. circuitry with a 'totem-pole' output circuit, the risetimes and falltimes of waveforms generated are within a few nanoseconds of each other. This makes these times difficult to measure and the crosstalk associated with this effect can be hidden by that from other sources. In order to investigate this effect the drive to the analogue demultiplexer was modified. The 'totem-pole' output was replaced by an open-collector output with a pull-up resistor. This was then loaded with a variable capacitor and the multiplier (Fig. 4.9). In operation, the capacitor was charged through the pull-up resistor and discharged through the output transistor of the driving gate. The very low effective resistance of the output transistor when switched 'on' resulted in the risetime being longer than the falltime. The degree of asymmetry could be varied by varying the capacitor. The crosstalk was measured under these conditions for a number of different functions from the set \( W_{16} \). The crosstalk level is plotted for different values of \( t_r - t_f \) and the different functions in Graph 4.4. Also shown are the theoretical and measured values of \( P/m \), the proportionality constant for this effect. The close agreement between these values confirms the analysis of this source of crosstalk carried out in Chapter 3.
4.3 MEASURED EFFECTS OF VARIATIONS IN DEVICE PARAMETERS

In Chapter 3 sources of crosstalk caused by both non-ideal waveforms and non-ideal circuit elements in the multiplex system were analysed. So far the levels of crosstalk caused by non-ideal waveforms have been discussed. The effects of decay in the integrator and sample-hold gates, and the measured imbalance in the multiplier are now considered.

4.3.1 The Sample and Hold Circuits

In order to test the effect of decay in the sample-hold gate a variable resistance was connected across the hold capacitor and a variable amount of decay was introduced. The crosstalk between two pairs of functions was measured for varying amounts of decay. The pairs of functions chosen were 14 and 15 and 7 and 11 from the matrix $\overline{W}_{16}$. The first pair of functions give $F(1)$ when multiplied together and the second pair give $F(4)$. As stated in Chapter 3, $F(1)$ has the maximum degree of asymmetry and is hence the worst case for this effect. $F(4)$ is symmetrical about its midpoint and a linear decay should then produce a zero change in the crosstalk level. The graph of crosstalk for various rates of decay is shown in Graph 4.5. Also shown are the measured and calculated values of slope of this graph. The correspondence between the calculated and measured values is very close; the small discrepancy can be accounted for by noting that the analysis assumed a linear decay between the timeslots whereas this is exponential in practice.

4.3.2 The Integrator

An expression for the crosstalk level caused by non-linearity in the integrator was developed in Chapter 3. In this expression
\[ X_I = \frac{t_s}{A R C} \]

so the level of crosstalk depends on the gain of the operational amplifier \((A)\) used. This is variable from device to device, the range of variation being more than one order of magnitude and difficult to measure. In order to test this effect the gain of the circuit had to be known accurately. This was done by introducing a resistance across the integrator capacitance (Fig. 4.10a). The circuit can then be analysed as follows: using the principle of superposition, the circuit is considered without the integrator capacitor (Fig. 4.10b). The circuit is then an inverting operational amplifier with its gain \(A'\) determined by the ratio

\[ A' = \frac{R_1}{R} \]

If we now consider the circuit with the capacitor, the circuit is an integrator but the crosstalk ratio is determined by

\[ X_I = \frac{t_s}{A' R C} \]

hence

\[ X_I = \frac{t_s}{R_1 C} \]

Thus by varying the value of the shunt resistance \((R_1)\) the level of crosstalk can be varied. Graph 4.6 shows the level of crosstalk plotted against the shunt conductance \((1/R_1)\). As can be seen, there is extremely good agreement with the levels predicted from the theory — in particular, the variation between the asymmetric and symmetric products of the functions.
4.3.3 The Multiplier

It is difficult to measure the imbalance in the multipliers directly since this was of a very low level. The method used was to measure the crosstalk between each of the functions 1 - 15 of \( \bar{W}_{16} \) and \( F(0) \). This was done using the digital part of the multiplex system as this prevented deficiencies in the analogue demultiplexer affecting the results. The level of imbalance can then be calculated by rearranging equation 3.22, i.e.

\[
m = 1 - 2X_m
\]

The measurements as described gave an average value of

\[
X_m = 0.00027
\]

hence \( m = 0.9995 \)

Hence, for the remaining pairs of functions, the level of crosstalk due to this cause will be, from equation 3.21

\[
X_m = \frac{(1 - m)(1 - n)}{4}
\]

\[= 143 \text{ dB.}\]

This is sufficiently low to be regarded as insignificant.

A deliberate imbalance was then introduced into the analogue to digital converter which appeared to the system as multiplier imbalance. This was achieved by slightly offsetting the position of the zero which increased the range of the negative quantisation steps while leaving the positive steps unchanged. Hence a voltage \(-V_i\) would be encoded into a negative number different in magnitude from the encoded version of \(+V_i\).

The Walsh functions were again measured in the same way. Under these conditions \( X_m = 0.002 \) and hence \( m = 0.996 \). For the
remaining pairs of functions in this matrix

\[ x_m = \frac{(1 - m)(1 - n)}{4} \]

\[ = 104 \text{ dB}. \]

Random checks on other pairs of functions showed that there was little change in the values measured previously which were of the order of -70 to -80 dB. The system was also checked for the first four functions of \( H_{16} \). For these functions

\[ x_m = a(1 - \frac{m}{2}) \text{ and } a = 1. \]

Using the value of \( m \) just calculated

\[ x_n = 65 \text{ dB} \]

and the averaged measured crosstalk level for these functions was -62 dB.

The agreement between the measurements is again good, as would be expected from the analysis which is free from any simplifying approximations.

4.4 MEASUREMENTS ON SETS OF FUNCTIONS

The measurements described so far have tested the theoretical relations between the crosstalk and some important parameters of the system. The next sets of measurements attempt to investigate the actual levels of crosstalk found on a practical system when each of the five sets of functions chosen for evaluation in Chapter 3 is used. To do this, measurements were made on both the analogue and hybrid systems with the system as described in Section 4.1 and also in modified form. The first modification was to remove the buffer amplifiers at the output of the multiplexers. This has the effect of increasing the
risetimes on the multiplex highway so that the experimental system simulates a system with long runs of wiring between the multiplexers and the encoding circuitry. The second modification is a result of the theory of Chapter 3 and the measurements of Sections 4.2 and 4.3. It was concluded from these results that the degradations were caused mainly by delays and finite slopes on the highways and in multiplying functions. It was therefore decided only to gate the output of the multiplier in the demultiplexer during the stable parts of the waveform, thus excluding the sections of waveform about the points of transition, and to measure the change in crosstalk level this caused. This modification was used with the matrices $\overline{W}_{16}$ and $\overline{B}_{16}$ which were considered representative of the dyadically orthogonal and binary orthogonal matrices.

In order to make a complete set of crosstalk measurements, every function in a set of functions must be used at the demultiplexer while every other function multiplies the incoming signal at the multiplexer. The number of measurements thus required is $n(n-1)$, where $n$ is the order of the matrix, and is thus 240 for matrices of order 16 and 132 for matrices of order 12. Complete sets of measurements were made for the matrix $\overline{W}_{16}$ and for some configurations of the system with other matrices; however the predictability of the results obtained rendered taking complete sets of results in all cases superfluous. Hence in some cases only a single function was set at the demultiplexer and results taken for this function and every other set at the multiplexer. To ensure that the results so obtained were not an atypical subset of the complete set, these were compared with other results from the set obtained by randomly selecting pairs of functions at the multiplexer and demultiplexer. To maintain uniformity of presentation and to aid assimilation of the
results, complete sets of results are not given in this section. Instead, the results for one function at the demultiplexer for each system configuration are shown as described above. The complete sets, where taken, are given in Appendix 2.

4.4.1 The Walsh Functions ($\overline{W}_{16}$)

Table 4.1 shows the measurements taken on $F(3)$ of $\overline{W}_{16}$ and every other function in the set for a variety of configurations of the system. The first row of the results (a) shows the performance of the hybrid system and the second row (b) that of the analogue system. The first point of note is that the levels of crosstalk measured in the analogue system agree with measurements cited by Harmuth (4.7) and others (4.8). From this it is inferred that the experimental system, when used in analogue form, behaves in a manner similar to those studied by other investigators. The second point of note is that the hybrid system performs far better (an average of 29dB) than the analogue system. In Chapter 3 it was suggested that many of the degradations affecting this type of system are related to the behaviour of the waveform around the points of transition between +1 and -1 which is excluded from the time of operation of the analogue to digital converter. This accounts for the differences in the two sets of results so far considered.

The next pairs of results (c) and (d) show the effect of removing the buffer amplifier to simulate the effect of long wiring runs. Without the buffer amplifier the line to the summing amplifier is driven by a source impedance of 1kΩ instead of 40Ω. This increases the delays between the multiplexer and the summing amplifier and increases the risetime of the waveforms from 0.8μs to 1.4μs. The effect of this change can be assessed by comparing Table 4.1(a) and (c) for the hybrid system and (b) and (d) for the analogue system. For the hybrid system there is
some degradation in the performance of those functions with high entries in the non-cyclic cross-correlation matrix (Fig. 4.5), i.e. functions 4, 11 and 12, but the remainder of the results show little change. It would seem that under these conditions the imperfections in the waveforms are beginning to affect the process of analogue to digital conversion. For the analogue system, less change is apparent, probably because of the inferior initial performance. However, in both sets of results, functions 4, 11 and 12 have a substantially worse performance than the others and, in particular, F(11) has the worst performance and it is this function only which has entries in both the cross-correlation matrix and the edge matrix. The final set of results in Table 4.1(e) show the effect of the second modification described at the beginning of this section. The points of transition of the waveform are now excluded from the integration in the analogue demultiplexer and the results show a dramatic improvement over the unmodified loop (b). Initially, the improvement achieved was not quite as great as this, i.e. an average improvement of 25dB rather than 29dB; however attention to the analogue integrator produced better results. In Section 4.3.2 it was shown that the effective gain of the integrator is affected by a resistance appearing in parallel with the capacitor. Investigation showed that only polystyrene capacitors had leakage low enough for the full potential of the system to be achieved.

4.4.2 The Hadamard Functions \((H_{16})\) and the Paley Functions \((F_{12})\)

Measurements were carried out on both these sets of functions using both the hybrid and analogue systems with and without the first modification. The results of these measurements are shown in Tables 4.2 and 4.3 using the same format as that used for the Walsh functions.
Again, where complete sets of results were taken, these are given in Appendix 2.

The level of crosstalk achieved using these two sets of functions is similar to that measured for the Walsh functions under the same conditions. Similar measurements were also made for the optimum matrix suggested by Hübner and these are shown in Table 4.4. These are little different from the values measured for the other matrices considered so far. In fact, for the analogue system using the Hadamard or Paley functions, or the functions of Hübner's matrix, the average level of crosstalk measured was 51dB. In each of these cases the significant difference was that between the hybrid and analogue systems. The second modification was also tried with randomly selected pairs of these functions and the improvement discussed in Section 4.4.1 was again apparent.

4.4.3 The Time Division Multiplexing Functions (B₁₆)

Measurements taken using the system as a time division multiplex system are shown in Table 4.5. This displays the levels of crosstalk in timeslots close in time to the timeslot (t₁) to which the signal is applied and the average level of crosstalk for other timeslots (tₙ). The results achieved can be explained in the following manner:

crosstalk in the timeslots preceding and following t₁ are attributable to finite risetimes in the highway waveform. Crosstalk in timeslot t₁₊₂ is attributable to charge on the highway and the crosstalk in other timeslots enters via the analogue switches. The effect of the first modification is not only to delay the signal and increase the signal risetimes, but also to increase the impedance through which the highway discharges. The added delay causes an improvement to the reading for
the preceding timeslot which, together with the finite risetime, increases the crosstalk in the following timeslot. The increased discharge impedance causes the increased crosstalk in the timeslot \( t_{(i+2)} \) due to the presence of charge during that time. The hybrid system is partially immune to the delays and finite risetimes but is still affected by charge remaining on the highway. The second modification, designed to eliminate transition time effects, was also used with the t.d.m. functions. These results are shown in Table 4.5(e). As can be seen, an improvement is again apparent but the charge on the highway still degrades the performance in timeslot \( t_{(i+1)} \). It would therefore appear that, using this modification, the dyadic orthogonal functions have the superior crosstalk performance.

4.4.4 The Ternary Functions (\( \frac{1}{6} \))

The measurements taken on the ternary functions are shown in Table 4.6. These can be compared with the cross-correlation matrix (Fig. 3.16) and the edge matrix (Fig. 3.27) for the set of functions. Despite some low readings, the agreement between the analogue results and the entries in the edge and cross-correlation matrices is good. In particular, the results for dyadically orthogonal pairs of functions are similar to those measured on the dyadically orthogonal matrices while the results for binary orthogonal pairs of functions are similar to those measured with the t.d.m. matrix. However, despite their greater complexity, they did not show any marked improvement in performance over other sets of function used. In particular, a short test using the second modification gave results which were not as good as those achieved with the dyadically orthogonal functions.
The noise measurements on the experimental system used two forms of noise injected onto the multiplex highway. These were:

a) an unsynchronised square wave;

b) bandlimited white noise (450 - 550Hz).

The first source is an approximation to the control and clock waveforms that are found to be associated with digital logic, while the second approximated to the higher harmonics of the ringing current used for exchange signalling purposes.

The square wave was applied to the highway at frequencies of 450 and 1450Hz and at a level of -40dBm0. The idle-channel noise level (i.e. the received noise level with no signal applied) was measured using the functions of $\overline{W}_{16}$ at the multiplexer and demultiplexer. There was little difference in the received noise level (this being 20dB below that applied) when using different functions in the set except that when using $F(0)$ (the d.c. function) no immunity to the noise was exhibited. Repeating the test at a noise frequency of 7500Hz produced a received noise level 10dB below that applied. The test was repeated at 1450Hz for different noise levels and also using functions selected from the other matrices under test. These results are illustrated in Graph 4.7 for the various sets under investigation. As can be seen from this graph, the higher-order functions of $\overline{W}_{16}$, $\overline{H}_{16}$ and $\overline{F}_{16}$ show the 20dB immunity to the noise voltage already mentioned. This is to be expected since, in any frame, the applied noise will be multiplied by +1 and -1 and integrated. Since the noise will usually be of one polarity throughout the frame, the noise integral will be zero. This will also apply to those members of the ternary matrix whose elements sum to zero over a frame. However, the functions with a d.c. component will show a reduced level
of immunity depending on the magnitude of this component. Hence the t.d.m. pulses, functions 0 of $\overline{W}_{16}$ and $\overline{P}_{12}$ and functions 1, 2, 5 and 6 of $\overline{T}_{16}$ show no immunity while the first four functions of $\overline{H}_{16}$ show an immunity of only about 6dB.

The second test using bandlimited (450 - 550Hz) noise supports these findings. The results of this test (Graph 4.8) show an immunity of 25 - 30dB for the higher order functions of $\overline{W}_{16}$ and $\overline{H}_{16}$. The Paley functions performed less well here due to their asymmetric nature. The other sets of functions showed similar degrees of immunity to those already tested using square waves. These results imply an important distinction between the sets of functions, since immunity to noise is an important property of any multiplex system or switch.

On the basis of these measurements, the best performance was obtained from the set of Walsh functions if the d.c. function is omitted. If a set of eleven functions is required then the Paley functions could also be considered. The Hadamard functions and ternary functions each had four functions with poor performance and the t.d.m. functions gave no immunity to this form of noise. In view of the similarity in crosstalk performance between the sets of functions when the system was modified to exclude transition times, this immunity to highway noise is regarded as a decisive advantage in favour of the dyadically orthogonal functions.
FIG. 4.1 SWITCHING MULTIPLIER

FIG. 4.2 THE MULTIPLEXER
FIG. 4.3 THE DEMULTIPLEXER

FIG. 4.4 TRUTH TABLE FOR A "TRUE COMPLEMENT ZERO–ONE ELEMENT"
FIG. 4.5 NON-CYCLIC CROSSCORRELATION MATRIX FOR $\mathcal{W}_{16}$

```
15 1 1 1 1 2 1 1 1 1 1 1 1 1
14 1 3 3 1 1 3 3 1 1 3 3 3 3
13 1 3 5 1 1 5 3 1 1 3 5 1 1
12 1 1 1 7 7 1 1 1 1 1 7 1 1
11 1 1 1 7 9 1 1 1 1 1 7 1 1
10 1 3 5 1 1 1 1 1 3 1 1 3 1
  9 1 3 3 1 1 3 13 1 1 3 1 3 1
  8 1 1 1 1 1 1 1 1 1 1 1 1 1
  7 1 1 1 1 1 1 1 1 1 1 1 1 1
  6 1 3 3 1 1 3 11 1 1 3 1 3 1
  5 1 3 5 1 1 3 11 1 1 5 3 2
  4 1 1 1 1 1 1 1 1 1 9 7 1 1
  3 1 1 1 1 1 1 1 1 1 7 1 1 1
  2 1 3 1 1 5 1 1 1 5 3 1
  1 1 3 1 1 3 1 1 1 3 3 3 1
   0 3 1 1 1 1 1 1 1 1 1 1 1
   0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
```

FIG. 4.6 NON-CYCLIC CROSSCORRELATION MATRIX FOR $\mathcal{W}_{16}$

(NOTE WHERE $R(r)$ ASYMMETRIC THE LARGEST VALUE IS SHOWN)

```
15 3 3 1 3 3 1 3 3 3 1 3 3 1 3 3 1 3
14 3 5 3 1 1 1 5 3 3 5 3 1 1 3 3 3 3
13 1 3 5 3 3 5 3 1 1 3 5 3 3 3 3 3 3
12 1 1 3 3 3 3 1 3 1 3 3 1 3 1 3 1 3
11 3 1 3 3 13 3 1 3 1 3 3 3 3 3 3 3 3
10 1 3 5 3 3 11 3 1 1 3 3 3 3 3 1 3 3
  9 3 5 3 1 1 3 11 3 3 3 1 3 1 3 3 3
  8 3 3 1 3 3 1 3 13 3 3 3 1 3 3 3 3
  7 3 3 1 3 3 1 3 13 3 1 3 3 3 3 3 3
  6 3 5 3 1 1 3 3 3 11 3 1 3 3 5 3 3
  5 1 3 5 3 3 3 3 1 1 3 11 3 3 5 1 1
  4 3 1 3 3 3 3 1 3 1 3 13 3 3 1 3
  3 3 1 3 3 3 1 3 3 3 3 3 3 1 3 3 3
  2 1 3 3 3 5 3 1 1 3 5 3 3 5 3 3 3
  1 3 3 1 3 3 5 3 3 5 3 1 3 3 3 3 1
  0 3 1 3 3 1 3 3 3 1 3 1 1 3 3 3
   ▲ 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
```
FIG. 4.7 NON-CYCLIC CROSSCORRELATION MATRIX FOR $\overline{P}_{12}$
(NOTE WHERE $R(\tau)$ IS ASYMMETRIC THE LARGEST VALUE IS SHOWN)

FIG. 4.8 NON-CYCLIC CROSSCORRELATION MATRIX FOR $\overline{T}_{16}$
FIG. 4.9 MODIFIED MULTIPLIER DRIVE TO GIVE UNEQUAL RISE AND FALL TIMES

(a) THE MODIFIED INTEGRATOR

(b) THE EQUIVALENT OPERATIONAL AMPLIFIER

FIG. 4.10
### TABLE 4.1 CROSSTALK MEASUREMENT ON WALSH FUNCTION 3

<table>
<thead>
<tr>
<th>SYSTEM CONFIGURATION</th>
<th>FUNCTION No.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>a) HYBRID</td>
<td>76</td>
</tr>
<tr>
<td>b) ANALOGUE</td>
<td>50</td>
</tr>
<tr>
<td>c) HYBRID DEGRADED</td>
<td>70</td>
</tr>
<tr>
<td>d) ANALOGUE DEGRADED</td>
<td>49</td>
</tr>
<tr>
<td>e) MODIFIED ANALOGUE</td>
<td>74</td>
</tr>
</tbody>
</table>

### TABLE 4.2 CROSSTALK MEASUREMENT ON HADAMARD FUNCTION 4

<table>
<thead>
<tr>
<th>SYSTEM CONFIGURATION</th>
<th>FUNCTION No.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>a) HYBRID</td>
<td>82</td>
</tr>
<tr>
<td>b) ANALOGUE</td>
<td>50</td>
</tr>
<tr>
<td>c) HYBRID DEGRADED</td>
<td>82</td>
</tr>
<tr>
<td>d) ANALOGUE DEGRADED</td>
<td>54</td>
</tr>
<tr>
<td>SYSTEM CONFIGURATION</td>
<td>FUNCTION No.</td>
</tr>
<tr>
<td>----------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>0 1 2 3 4 5 6 7</td>
<td>8 2 8 2 8 2</td>
</tr>
<tr>
<td>a) HYBRID</td>
<td>81 82 82 82</td>
</tr>
<tr>
<td>5) ANALOGUE</td>
<td>47 43 49 40</td>
</tr>
<tr>
<td>c) HYBRID DEGRADED</td>
<td>72 76 75 6 5</td>
</tr>
<tr>
<td>d) ANALOGUE DEGRADED</td>
<td>43 38 45 3 3</td>
</tr>
</tbody>
</table>

TABLE 4.3 CROSSTALK MEASUREMENTS FOR PALEY FUNCTION 4

<table>
<thead>
<tr>
<th>SYSTEM CONFIGURATION</th>
<th>FUNCTION No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 1 2 3 4 5 6 7</td>
<td>8 2 8 2 8 2</td>
</tr>
<tr>
<td>a) HYBRID</td>
<td>82 82 8 2</td>
</tr>
<tr>
<td>b) ANALOGUE</td>
<td>45 47 5 2</td>
</tr>
<tr>
<td>c) HYBRID DEGRADED</td>
<td>51 5 7 6 4</td>
</tr>
<tr>
<td>d) ANALOGUE DEGRADED</td>
<td>40 5 6 5 0</td>
</tr>
</tbody>
</table>

TABLE 4.4 CROSSTALK MEASUREMENTS FOR HUBER'S FUNCTION 2
### System Configuration

<table>
<thead>
<tr>
<th>TIME SLOT</th>
<th>( t_{i-1} )</th>
<th>( t_{i+1} )</th>
<th>( t_{i+2} )</th>
<th>( t_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) HYBRID</td>
<td>82</td>
<td>82</td>
<td>82</td>
<td>82</td>
</tr>
<tr>
<td>b) ANALOGUE</td>
<td>40</td>
<td>40</td>
<td>72</td>
<td>75</td>
</tr>
<tr>
<td>c) HYBRID DEGRADED</td>
<td>82</td>
<td>65</td>
<td>67</td>
<td>82</td>
</tr>
<tr>
<td>d) ANALOGUE DEGRADED</td>
<td>82</td>
<td>34</td>
<td>72</td>
<td>75</td>
</tr>
<tr>
<td>e) ANALOGUE MODIFIED</td>
<td>75</td>
<td>67</td>
<td>75</td>
<td>75</td>
</tr>
</tbody>
</table>

**Table 4.5** Crosstalk Measurements for T.D.M

---

### Table 4.6 Crosstalk Measurements for Ternary Function 2

<table>
<thead>
<tr>
<th>SYSTEM CONFIGURATION</th>
<th>FUNCTION No.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>a) HYBRID</td>
<td>62</td>
</tr>
<tr>
<td>b) ANALOGUE</td>
<td>48</td>
</tr>
<tr>
<td>c) HYBRID DEGRADED</td>
<td>78</td>
</tr>
<tr>
<td>d) ANALOGUE DEGRADED</td>
<td>56</td>
</tr>
<tr>
<td>FUNCTIONS</td>
<td>$k_{MEASURED}$</td>
</tr>
<tr>
<td>-----------</td>
<td>---------------</td>
</tr>
<tr>
<td>$W_7 &amp; W_8$</td>
<td>33</td>
</tr>
<tr>
<td>$W_3 &amp; W_4$</td>
<td>15</td>
</tr>
</tbody>
</table>

GRAPH 4.1 CROSSCORRELATION OF WALSH FUNCTIONS
FUNCTIONS | k MEASURED | k COMPUTED
--- | --- | ---
W₁ & W₂ | 5 | 6

GRAPH 4.2 CROSSCORRELATION OF WALSH FUNCTIONS
FUNCTIONS

<table>
<thead>
<tr>
<th>FUNCTIONS</th>
<th>n MEASURED</th>
<th>n COMPUTED</th>
</tr>
</thead>
<tbody>
<tr>
<td>W₂ &amp; W₃</td>
<td>7.0</td>
<td>4</td>
</tr>
<tr>
<td>W₆ &amp; W₁₀</td>
<td>2.6</td>
<td>2</td>
</tr>
<tr>
<td>W₃ &amp; W₁₁</td>
<td>0.2</td>
<td>0</td>
</tr>
</tbody>
</table>

GRAPH 4.3 CROSSTALK AS A FUNCTION OF HIGHWAY RISETIMES
GRAPH 4.4 CROSSTALK AS A FUNCTION OF RISE AND FALL TIMES
### Table: Slope Measurement

<table>
<thead>
<tr>
<th>FUNCTION</th>
<th>SLOPE MEASURED</th>
<th>SLOPE CALCULATED</th>
</tr>
</thead>
<tbody>
<tr>
<td>14-15</td>
<td>0.55</td>
<td>0.50</td>
</tr>
<tr>
<td>3-11</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Graph 4.5** CROSSTALK AS A FUNCTION OF DECAY IN THE SAMPLE HOLD
FUNCTION | SLOPE MEASURED | SLOPE PREDICTED
--- | --- | ---
14 & 15 | 0.125 | 0.0134
7 & 11 | 0.001 | 0.00

GRAPH 4.6 CROSSTALK AS A FUNCTION OF INTEGRATOR NON-LINEARITY
KEY

$H_2 = \text{Hadamard Function 2}$

$H_{11} = \text{Hadamard Function 11}$

$T_4 = \text{Ternary Function 4}$

$P_1 = \text{Paley Function 7}$

$W_4 = \text{Walsh Function 4}$

$X = \text{T. D. M}$
KEY

X = T.D.M  
H₂ = HADAMARD FUNCTION 2  
H₁₅ = HADAMARD FUNCTION 11  
P₇ = PALEY FUNCTION 7  
T₄ = TERNARY FUNCTION 4  
W₄ = WALSH FUNCTION 4  
W₁₅ = WALSH FUNCTION 15

GRAPH 4.8 RANDOM NOISE RESPONSE
CHAPTER 5
THE SYSTEM SIMULATION

In time division multiplexing systems the effects of overloads are restricted to the channels in which they occur. However in systems employing dyadically orthogonal functions, as in frequency division multiplexing systems, an overload in one channel can result in noise being induced into other channels. Hence any system using dyadically orthogonal functions must be designed so that this noise is kept within acceptable limits. In Chapter 3 it was seen that the number of extra bits \( n \) required in the encoding process can be related to the number of channels summed together \( m \) by

\[
n = \log_2 m
\]

However this expression assumes that all channels have a fixed peak voltage and it is necessary to design the system for a simultaneous overload on all channels. However it is difficult to determine a well defined peak value for speech signals and a simultaneous overload on all channels is a very infrequent occurrence. A more fruitful approach therefore is to simulate the speech signals for each of the channels and apply these to a simulated multiplex system. Then a simple heuristic technique can be used. This corresponds to raising the level of overload on the multiplex highway until the noise introduced by the overload mechanism is reduced to a satisfactory level. In this chapter the mechanism of the overload is described and the available information on speech statistics are reviewed. This is followed by a description of the system's simulation and a presentation of the results of the simulation.
5.1 THE OVERLOAD MECHANISM

A simple way of demonstrating the effect of overload is to consider a simplified multiplex system for example the fourth order Walsh function case. The matrix $\bar{W}_4$ is given by

$$\bar{W}_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

The four rows in the above matrix will correspond to the set of orthogonal functions in the multiplex system. Let the inputs to the four channels at a given time (T) be the following vector

$$\bar{D} = \begin{bmatrix} A \\ B \\ C \\ 0 \end{bmatrix}$$

where $A > B > C > 0$. Then the multiplex highway vector will be

$$\bar{H} = \bar{W}_4 \times \bar{D} = \begin{bmatrix} A + B + C \\ A + B - C \\ A - B - C \\ A - B + C \end{bmatrix}$$

The multiplication of this vector by the appropriate functions at the demultiplexer and integrating will produce ideally the original data scaled by the matrix normalisation factor, as follows:

$$I_1 = A + B + C + A + B - C + A - B - C + A - B + C = 4A$$
$$I_2 = A + B + C + A + B - C - A + B + C - A + B - C = 4B$$
$$I_3 = A + B + C - A - B + C - A + B + C + A - B + C = 4C$$
$$I_4 = . A + B + C - A - B + C + A - B - C - A + B - C = 0$$
However when the amplitude of the highway signal is restricted not to exceed $A + B$ for example then the highway vector becomes altered to

$$
\overline{H} \text{ (limited)} = \begin{bmatrix}
A + B \\ A + B - C \\ A - B - C \\ A - B + C
\end{bmatrix}
$$

and the output of each channel will be given by

$$
\begin{align*}
I_1 &= A + B + A + B - C + A - B - C + A - B + C = 4A - C \\
I_2 &= A + B + A + B - C - A + B + C - A + B - C = 4B - C \\
I_3 &= A + B - A - B + C - A + B + C + A - B + C = 3C \\
I_4 &= A + B - A - B + C + A - B - C - A + b - C = -C
\end{align*}
$$

It is apparent from the above that the signal in the third channel has been reduced in amplitude and in addition part of that channel’s signal has appeared in the other channels. This channel leakage is not in general intelligible crosstalk and the possibility of intelligible crosstalk can be prevented by arranging the system so that no single channel can cause the multiplex highway to overload. This itself can be done by restricting the maximum output of any of the multipliers to a value less than the overload point on the multiplex highway. Under this condition any overload must be the result of the addition of levels on two or more channels. Since the levels produced by any channel are statistically independent of the levels produced by any other channel and since the channels contributing to each successive overload will be different, there will be little correlation between the signal on any channel and the overload voltage appearing in another channel. Hence the signal produced by the overload can be treated as an addition to the noise in the channel in which it appears rather than an addition to the intelligible crosstalk. For speech this noise will be most apparent in
the intervals in the conversation when the channel is idle, i.e. when neither party to the conversation is speaking. It seems reasonable therefore to treat the noise caused by overload appearing in other channels as an addition to the idle channel noise and to apply the appropriate specification which is a noise level of -65dBm0. However the overload also reduces the level of signal in the active channels that cause the overload and hence the system must also be designed to meet a second specification concerned with the maximum level of signal required in the active channels. Thus the system must be designed to fulfil these two requirements for satisfactory transmission performance.

The level of noise induced into a channel by the overload mechanism is dependent on the magnitude and frequency of the overloads. These are determined by the statistics of the input signals which are in this case speech. Hence it is necessary to examine the statistics of telephone speech in order to obtain a quantitative answer to the overload problem.

5.2 SPEECH STATISTICS

A comprehensive analysis of speech statistics was produced by Holbrook and Dixon in 1939 (5.1). This work was undertaken in order to calculate the dynamic range requirements for amplifiers used in frequency division multiplex systems. Their analysis identified the following parameters of the telephone speech:–

a) the activity factor
b) the volume distribution
c) the instantaneous amplitude (voltage) distribution.

The significance of these terms is directly appreciated by considering the temporal sequence of events on a typical channel.

The channel is allocated at random to a connection and carries speech from a customer chosen at random from an ensemble. The customers each have a speech volume, i.e. long term mean power which depends on the
physical characteristics of the individual and the losses in the local
distribution network. While talking the customer generates speech
amplitudes from a distribution which is symmetric about zero and has a
root mean square value equal to his speech volume. The customer and
hence the channel is inactive during long silences (i.e. excluding short
pauses between words and syllables) while the second party to the
conversation speaks or while both parties are silent. Finally the
connection is ceased, and the channel is inactive until the channel is
allocated to the next connection.

The activity factor is determined by the conversational pauses
and the average time the channel is active. The pauses due to conversation
on a connected channel give an activity factor of about 0.4. The average
time the channel is in use is termed the loading of the channels.
For very large groups with a 'standard grade of service (0.002)' this is
about 0.6 which gives the overall activity factor of each channel ($\tau$)
as $0.4 \times 0.6 = 0.24$, a value close to 0.25 usually quoted as typical for
this parameter.

In general the channel loading is given by

$$ L = \frac{E}{N} \quad \text{(5.1)} $$

where $E$ is the number of erlangs of traffic carried and $N$ is the number
of channels in a group. $E$ and $N$ are related to the grade of service by
the Erlang B formula (5.2). For a fixed grade of service the amount of
traffic ($E$) carried by each channel falls as the total number of channels
is reduced and is quite low for groups of channels below about 30.
The Erlang B formula is extensively tabulated (5.3) and inspection of
these tables reveal that a group of 15 channels (the value chosen for the
concentrator and expander switches) providing the standard grade of
service can carry 6.58 Erlands. Hence the loading of the concentrator
and expander switches \( L = \frac{6.58}{15} = 0.4 \). Therefore the overall activity factor for the channels on these switches (\( \tau \)) is \( 0.4 \times 0.4 = 0.16 \).

The probability \( (p(n)) \) of a given number of these channels \( (n) \) from a group of size \( N \) being active is given by the Poisson distribution (5.1),

\[
p(n) = \frac{N!}{n!(N-n)!} \tau^n (1-\tau)^{N-n} \ldots \ldots \ldots \{5.2\}
\]

In order to demonstrate the effect of reducing the activity factor of the channels from 0.25 to 0.16, the probability of \( n \) channels active has been computed and is displayed on Graph 5.1 for each of these activity factors. The average number of channels active is given by

\[
\bar{n} = \tau N \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \{5.3\}
\]

As can be seen from Graph 5.1, the probability of a given number of channels being active is large for numbers of channels near the average but this probability falls very rapidly for the higher numbers of channels and is particularly low for the lower of the two activity factors.

The second parameter of interest is the speech volume distribution. The volume of a channel, defined as the long term mean power of the channel while it is active, is a constant for a given connection. The volume scales the selections from the instantaneous voltage distribution to allow for the unique characteristics of different speakers. The probability density function of this parameter is given by Holbrook and Dixon (5.1) as:

\[
p(V)dV = \frac{1}{\sqrt{2\pi\sigma}} \cdot \exp \left(\frac{(V-V_o)^2}{2\sigma^2}\right)dV \ldots \ldots \{5.4\}
\]

where \( V \) is the volume, \( V_o \) the average of the distribution and \( \sigma \) the standard deviation. These parameters are all measured in decibels and so the distribution is lognormal rather than normal. This distribution was measured by Holbrook and Dixon at the microphone and hence the distribution
at the local exchange must be changed by the loss introduced between the telephone and the exchange. This loss is caused by

a) the attenuation caused by the local distribution network,

b) the variation in microphone sensitivity with changing current feed,

c) the action of the regulator in the telephone which tends to compensate for the above.

The average volume levels in the local distribution network range between -10dBm0 and -18dBm0 (5.4). The standard deviation (σ) of the volume distribution has been measured by Berry (5.4) for local exchanges in the United Kingdom reporting values ranging between 4dB and 8dB for this parameter, with the lower figures being measured for exchanges on short lines (i.e. Director exchanges in cities). The system to be simulated will be most likely to overload when the volume levels encountered are high. This is most likely when an exchange is connected by short lines. Therefore for the purpose of the simulation it was assumed that the average volume level (V₀) was -12dBm0 and that this was accompanied by a value of standard deviation of 4.1dB. These figures are typical for a city exchange with short lines, and are more stringent than the conditions for the majority of exchanges in the network.

Measurements on the instantaneous amplitude distribution of speaker were made by Holbrook and Dixon (5.1) and later by Rizonni (5.5) and Richards (5.6). Several theoretical distributions have been proposed as models of the instantaneous speech amplitude distribution; these include the Gaussian (5.5), the exponential (5.7) and the gamma distribution proposed by Richards (5.6). This particular distribution is given by

\[ p(x) = \frac{k}{2\Gamma(\ell)} (kx)^{\ell-1} \exp(-kx) \ldots \ldots \{5.5\} \]

where \( k = \sqrt{\ell(\ell+1)} \), \( x \) is the ratio of instantaneous amplitude to root mean square amplitude and \( \ell \) a parameter to be adjusted to suit the particular
statistics modelled. A value of \( \lambda = 0.2 \) is a close fit to the
distribution of telephone speech while \( \lambda = 0.5 \) fits the distribution
for speech measured using a high quality microphone. Graph 5.2 shows
values generated by this function and also the exponential and the
Gaussian distributions for comparison. For the purpose of the simulation
\( \lambda \) was set to 0.2,

These then are the important statistics which determine the range
of amplitudes of a telephone speech waveform, and hence those at the
input of the multiplex system. They do not completely characterise the
signal as they contain no information about the spectral distribution of
the signal. However the spectral content of the signal has no effect on
the multiplex system since samples of the signal are generated at the
sampling frequency and are treated independently of preceding and following
samples.

5.3  THE METHOD OF SIMULATION

The statistics of telephone speech as reviewed earlier will be
employed in the simulation of the system which is described in this
section. The simulation is in fact divided into two parts:-

a) the generation of the speech signals,
b) the multiplex system.

These parts will be described individually below, together with the
real time operation of the system.

5.3.1 The Simulation of Speech Signals

The generation of the speech signal for each of the multiplex
channels can be divided into three parts:-

a) the decision on whether the channel is active,
b) the generation of an amplitude level,
c) the weighting of the voltage level by the volume selected
   for that channel.
In order to generate these signals a random number generator was required. This was provided by means of a generator using a multiplicative congruence technique. The numbers thus generated have a rectangular distribution between 0 and 1 and the sequence of numbers generated is of length $33,554,431$.

The activity of a channel was decided by comparing a random number with the activity factor. If the random number is greater than the activity factor then the channel is inactive, whereas if it is less than the activity factor it is active. The generation of the instantaneous amplitude levels is a little more involved. It can be shown that a given probability distribution can be obtained by making selections from the inverted cumulative distribution function (c.d.f.) using random numbers with a rectangular distribution between 0 and 1 (5.8). If the c.d.f. can be obtained analytically this is a fairly simple task. However the gamma distribution is not easily integrable and hence numerical techniques must be used to generate the c.d.f. Hence the first part of the simulation performed a numerical integration on the gamma function in order to generate an array corresponding to the c.d.f. This array was then used in conjunction with the random number generator to generate selections from the gamma distribution.

The volumes of the channels could also be obtained in the same way. However a more direct method is possible. The central limit theorem shows that the distribution of the sum of a number of independent random variables tends towards a Gaussian distribution (5.9). Hence if a number of selections are made from a rectangular distribution the sum of the selections can be treated as Gaussian. The values of the Gaussian distribution are given by

$$x = \sigma \left( \sum_{i=1}^{n} R_i - n/2 \right) \sqrt{\frac{12}{n} + \mu} \ldots$$ \quad (5.6)
where \( \sigma \) is the standard deviation, \( n \) is the number of selections, 
\( \mu \) is the mean of the distribution and \( R_i \) are the numbers selected from 
the rectangular distribution. Thus, if \( n = 12, \sigma = 4 \) and \( \mu = 0 \), we have

\[
x = 4 \left( \sum_{i=1}^{n} R_i - 6 \right)
\]

This then can be used to generate samples from the lognormal volume 
distribution. By converting these values which are in decibels to 
linear form, the results can be used to scale the selections from the 
speech amplitude distribution.

In the simulation all amplitude levels were referred to the 
r.m.s. amplitude level of a channel with average volume level (-12dBm0).

Hence we have the following relations between the voltage levels 
in the simulation and measurements in dBm0;

a) The p.c.m. overload voltage = +3dBm0 = 5.6Vrms.
b) The standard reference level = 0dBm0 = 4.0Vrms.
c) The r.m.s. voltage level while active = -12dBm0 = 1.0Vrms.
d) The idle channel noise level = -65dBm0 = 0.0022Vrms.

5.3.2 The Simulation of the Multiplex System

It was decided that the set of orthogonal functions to be used in 
the simulation of the multiplex system would be the set of Walsh functions 
\( \overline{W}_{16} \). These functions were chosen because they are considered a typical set 
of dyadic functions and in addition their matrix is simple to generate.
The generation is carried out by performing the inverse Walsh-Fourier 
transform on a set of impulses in the transform domain. Each of the set 
of impulses generates one of the Walsh functions which can then be stored 
in an array.

The multiplex simulation is exactly equivalent to a hardware multiplex 
system. In each timeslot the signal generated for each channel is multiplied
by the appropriate value taken from the array of functions and then the results from all the channels are added together to form part of a line vector which contains results for all the timeslots. In a similar manner the line vector is processed to form the demultiplexed signals. The r.m.s. value of the demultiplexed signal for each channel is then calculated from the successive outputs of the demultiplexer. The simulation allowed both the input signals to each channel and the multiplex highway signal, to be limited to predetermined values so that the effect of the limiting on the idle channel noise level can be assessed. In this manner the dynamic range required by the multiplex system was determined. The complete simulation program is shown in A7.4.

5.3.3 The Real Time Operation of the Program

This can be described by reference to the flow chart of Figure 5.1. Before the simulation run is started the array of multiplexing functions is formed and the cumulative distribution function of the speech amplitudes is generated. The system then reads in information referring to which channel is idle, and the limit set on the multiplex highway. The length of the simulation is determined by two parameters,

a) the length of each run with the volume levels fixed and
b) the number of times the volume levels are changed.

The program then finds the volume levels for the first run and uses these to scale the amplitudes generated for the first run. The amplitudes are then passed through the multiplex system squared and accumulated, so that at the end of the simulation the root mean square value of the voltages is found. The program then generates a second set of volumes, corresponding to a new set of speakers, and uses these to scale the amplitudes generated on the next run. In this way a long term average level of the signal in each channel is built up. The result for the idle
channel is the idle channel noise power caused by the overload effect. After a simulation with a preselected overload level, the level can be varied for the next simulation until the minimum level which meets the specification is found. In this way the dynamic range required by the system can be determined.

5.4 THE RESULTS OF THE SIMULATION

The results of the simulation are divided into two parts. These parts correspond to the tests which verify that the various parts of the simulated system are operating correctly, and the results of the full simulation which give the required information on the dynamic range needed for the correct operation of a practical system.

5.4.1 The Tests on the Simulated System

The first tests were performed on the random number generator. These were concerned with checking that the distribution produced similar numbers of selections in each interval of the distribution, i.e. the distribution is rectangular. Graph 5.3 shows the occurrence of random numbers within intervals of width 0.1 over the range 0 to 1 and Graph 5.4 shows the occurrence in steps of 0.01 of numbers within one of these intervals. Both histograms are expected ideally to be flat, but for this we would require an infinitely large number of selections which is, of course, impracticable. These results can be considered satisfactory when allowance is made for the lengths of the runs involved. This type of generator has been examined elsewhere (5.8) for correlation between values and this has been found to be negligible.

The next test was to run the program so that it generated unweighted values of the speech amplitude distribution. These values were then collated to show the number of occurrences in each interval of the distribution. This and the theoretical distribution are displayed in
Graph 5.5. As expected the results show good agreement between the generated distribution and the theoretical curve. The speech volume distribution was also checked by taking a large number of samples of the distribution and calculating the mean and standard deviation of the samples. These were found to be very close to the values chosen for the distribution.

The next test was performed on the simulated multiplex system itself. Data representing levels of speech signals on the channels were fed to the system from a teletype terminal and the demultiplexed output of the simulated system was returned to the terminal. The input data and the output returned were checked and were found to be free from discrepancies. The simulation was then run without any limiting on the multiplex highway and the r.m.s. values of the output voltages were measured and compared with the input values. These values were also found to be free from discrepancies and hence the multiplex system was assumed to be operating correctly.

5.4.2 Results and Comments

The first part of the full simulation was to find the overall amplitude distribution on the multiplex highway for the 15 channel system. The simulation was run therefore with the simulation of speech statistics working normally and the multiplex system itself disabled. The outputs of the 15 speech channels were summed together, in place of the multiplex system, to produce signal levels the distribution of which represents the overall amplitude distribution to be found on the multiplex highway. This is shown on Graph 5.6. This part of the simulation was then repeated with limiting of the individual channels before summation. The level of limiting was set to the level required in the British Post Office p.c.m. terminal equipment specification (5.10). This represents more accurately
the practical situation where the incoming speech channels are limited by equipment of finite dynamic range before encoding takes place. The new distribution of amplitude levels is also shown on Graph 5.6. This graph illustrates the concentration of the higher levels around the limiting level when limiting is present. Also of note is the rapid decrease in the occurrence of levels when compared with the unlimited channels. It may be anticipated that the level to which the highway signal may be limited will be found in this area above the limiting level for a single channel.

The final part of the simulation was concerned with the running of the program with the speech statistics generator and the multiplex system fully operating. This was implemented with 14 of the 15 channels operating so that the level of idle channel noise could be measured on the 15th. This part of the simulation was run many times for different levels of overload limit and different channels set to the idle condition. No statistically significant difference in the level of noise could be found between the channels when the overload limit remained the same. However the level of noise was, as expected, strongly dependent on the level set for the overload limit. A graph showing the effect of the limiting level on the level of noise is shown in Graph 5.7. Each point on this graph is the average of a number of runs of the program for a given overload level. The straight line fit would seem a fairly conservative estimate of the way that the noise level falls with increasing overload limit. The maximum allowable level shown is equivalent to an idle channel noise level of -72dBm0. This was chosen to allow a margin over the level specified (-65dBm0) for other mechanisms which contribute to the noise. The intercept of the lines on the graph shows that for a limiting level of 10.2V rms this specified Maximum Allowable Level can be reached.
In the standard p.c.m. terminal equipment the overload level is set to +3dBm0 and with a 12 bit linear encoder (companded into 8 bits) the idle channel noise and quantisation noise specified can be met. With the simulated system the same levels of quantisation noise and idle channel noise must be met. Therefore the extra dynamic range required must be met by additional levels used to encode amplitudes in excess of +3dBm0. The simulation result (a limiting level of 10.2V\textsubscript{rms}) is equivalent to +8.13dBm0, and an encoder with 13 bit definition would give 6dB more range. Hence the encoder would be set to limit at +9dBm0 and the specification could thus be met. This result is obviously dependent on the speech statistics employed, but it should be remembered that the volume statistics were chosen to match the conditions formed in an exchange where high levels were common and also that the Maximum Allowable Level set for the idle channel noise was less than the level specified. For these reasons it can be expected that an encoder with 13-bit definition would meet the required specification.

It is interesting to consider that the result of one extra bit encoding, i.e. doubling of the dynamic range, is less than the simple approximation given by the average number of channels active (\(\bar{n}\)) given by equation (5.3). This is equal to 2.4 for \(\tau = 0.16\) and \(N = 15\). The process of channel summation would seem to perform an averaging process, and would therefore tend to smooth out fluctuations in the load so that the capacity of the multiplex system is used more fully.
START

INPUT LENGTH OF RUN AND NUMBER OF SELECTIONS

CALCULATE SET OF VOLUMES

FIND ACTIVE CHANNELS

SELECT AMPLITUDES FROM GAMMA DISTRIBUTION

APPLY LIMIT AND MULTIPLEX

END OF RUN?

END OF SIMULATION?

CALCULATE ROOT MEAN SQUARE VALUES

OUTPUT ROOT MEAN SQUARE VALUES

STOP

FIG. 5.1 THE FLOW CHART FOR THE SYSTEM SIMULATION
GRAPH 5.1 PROBABILITY OF n CHANNELS OF A GROUP OF 15 ACTIVE, FOR $r = 0.16$ AND $r = 0.25$
Graph 5.2 Some suggested models for the speech amplitude distribution.
GRAPH 5.3 TEST OF RECTANGULAR DISTRIBUTION OVER THE INTERVAL 0 TO 1
GRAPH 5.4 TEST OF RECTANGULAR DISTRIBUTION OVER THE INTERVAL 0.2 TO 0.3
GRAPH 5.5 COMPARISON OF THE OCCURRENCE OF SPEECH SAMPLES AND THE THEORETICAL SPEECH AMPLITUDE DISTRIBUTION ($L=0.2$)
\[ l = 0.2 \]
\[ r = 0.16 \]
\[ \sigma = 4.1 \]

Graph 5.6 Instantaneous Combined Amplitudes for 15 Channels
GRAPH 5.7 IDLE CHANNEL NOISE AS A FUNCTION OF THE LIMIT ON THE MULTIPLEX HIGHWAY
CHAPTER 6

CONCLUSIONS

One solution to the problems of switching and analogue-to-digital conversion in a digital local exchange has been investigated in this research. In Chapter 2 the working environment of a digital local telephone exchange was examined and several interfaces were found to be necessary between the digital exchange and the local distribution network. The position of these interfaces and in particular the analogue-to-digital and digital-to-analogue converters were shown to have a considerable impact on the design of the switchblock in the exchange.

Three techniques of analogue-to-digital conversion were examined and conversion between the analogue and digital domains on the highway of a multiplex switch was thought worthy of consideration. Attention was focused therefore on the problems encountered with this type of system. Previous work carried out by other investigators (6.1) on analogue multiplex switches using time division multiplexing had shown that these switches were susceptible to two transmission problems:

a) a high level of crosstalk between channels on the multiplex highway,

b) poor immunity to noise induced on the multiplex highway.

In order to overcome these problems the main effort of this research was directed towards finding a set of multiplexing functions which would have a performance superior to t.d.m. in these respects. To this end several sets of digital orthogonal functions were investigated theoretically in order to determine the imperfections which degrade their crosstalk performance. Analytical techniques were developed which differentiated between imperfections that limit the performance of binary orthogonal
functions such as the t.d.m. functions and those imperfections which limit the performance of dyadic orthogonal functions such as the Walsh functions. The results of Chapter 3 showed that poor crosstalk performance with the dyadic orthogonal functions is attributable to the behaviour of the functions about the points of transition between +1 and -1 and to the non-ideal behaviour of circuit components such as multipliers or integrators. These results led to the idea that the crosstalk performance could be improved by eliminating from the integration the parts of the signal waveform about the points of transition referred to above. The noise performance of both the binary and dyadic orthogonal functions was discussed and it was shown that functions with a zero d.c. component were substantially immune to noise on the multiplex highway. However this immunity did not extend to the t.d.m. functions or some of the ternary functions, which naturally contain a d.c. component. The quantisation noise and overload performance of the functions were also discussed and it was noted that the dyadic orthogonal functions needed extra levels of encoding for the same quantisation noise performance because of the increased output level after summation onto the multiplex highway.

The experimental system described in Chapter 4 was used to verify the results of Chapter 3. The results showed that there was very good agreement between theory and measurement, and also that the digital demultiplexer gave substantially better results than the analogue demultiplexer due to the elimination of transition time effects. The analogue system was therefore modified so that it could also benefit from this improvement. The modified analogue system was found to have a performance which was equal to that of the digital system and which was in excess of that required by the specification for a digital local exchange (crosstalk level - 70dB) also the results achieved were
considerably better than those published by the other workers investig-
gating multiplex systems using digital orthogonal functions (6.2) (6.3).

The immunity to noise of the sets of functions was also measured
and the results were again in agreement with the theoretical ideas and
it was found that the performance of the dyadic orthogonal functions was
considerably better than that of t.d.m. (20dB).

In Chapter 5 the system was simulated using the set of Walsh
functions for multiplexing. The simulation showed that in order to
achieve a satisfactory level of idle channel noise and overload margin
the dyadic orthogonal functions required one extra bit definition in
the analogue-to-digital and digital-to-analogue encoders.

It would now seem worthwhile to compare the salient features of
systems using t.d.m. and dyadic orthogonal functions. The chief
advantages of t.d.m. are due to the simple circuitry necessary to
implement t.d.m. systems and that the distributor switch in the digital
local exchange uses digital t.d.m. hence simplifying the interface
between the concentrator and the distributor. The t.d.m. systems,
unlike the dyadic systems, need no sample-hold gate in the multiplexer
and no integrator in the demultiplexer. Also with t.d.m. systems the
encoder requires one less level of accuracy and the conversion to and
from A-law companded p.c.m. can take place at the point of encoding.
However against this must be set the superior transmission performance
of the dyadic orthogonal functions. This would be the deciding factor
in many critical applications and the extra complexity of circuitry
required for this type of system could more than offset the more exacting
specification of the highways and other components needed if a t.d.m.
system is to attain the same results. The ternary functions would appear
to combine the disadvantages of both binary and dyadic orthogonal
functions while having the advantages of neither. That is they required
the extra hardware complexity of the dyadic orthogonal functions while
having the poor noise performance of the binary orthogonal functions.
It seems therefore that the choice for any system is between the well
established t.d.m. techniques and the sets of function based on Hadamard
matrices with the ultimate decision being based on the characteristics
of each just considered.

At this point there are two easily identifiable areas of future
work; these are the development of the experimental model into a practical
system and a theoretical investigation of the filtering options available
in the exchange. The development of the model into a practical system
would require the following:-

a) arranging for the control and distribution of the functions so
that the switch can be operated by the control unit;

b) engineering the injection of the signalling between the line
and control unit onto the multiplex highway, and arranging
for extraction at each end, e.g. the 17th timeslot could also
be used for signalling purposes;

c) measuring and improving as necessary other aspects of the
transmission performance such as the system gain-level linearity.
This may require the development of improved types of encoders
and decoders;

d) developing a timeshared demultiplexer so that a complete set
of channels can be converted to t.d.m. within one unit,

The second area for future investigation may be the filtering
problem in this type of system which is described in Appendix 1 and
has been mentioned in Chapter 2 in conjunction with the distribution of
interfaces within the exchange. The requirement for accurate analogue
prefilters with sharp transitions between passband and stopband can be
relaxed if the sampling frequency of the system is increased and some
form of digital filter is added to the system to allow a later reduction
in sampling frequency. Given that the signal on the multiplex highway
is already orthogonally encoded it may be possible to use a finite arithmetic transform and related signal processing to act on the channels in the multiplexed form while maintaining their mutual orthogonality. If this were the case the hardware requirements may well be less than those for alternative filtering techniques and this type of system may prove to have an added advantage over other systems.

The increase in performance of the multiplex system described in the work may also have implications in fields other than that considered here. In particular it has been noted by Harmuth (6.2) that the use of multiplex systems using dyadic orthogonal functions should be restricted to applications where crosstalk levels of 40dB suffice. However this research would indicate that with certain modifications these systems can be reconsidered for far more stringent applications where their superior noise performance may be particularly useful.
APPENDIX 1

FILTERING FOR MULTIPLEXING

The need for filtering of the input signals to prevent aliasing distortion was discussed briefly in Chapter 2. Although the problem is not directly relevant to the problems examined in this work, it nevertheless has a direct bearing on the final system parameters and hence it is discussed here in some detail.

The filter characteristic required (Fig. A1.1) to reduce the aliasing distortion to an acceptable level is specified by the C.C.I.T.T. (A1.1) and this assumes a sampling rate of 8kHz. In the digital local exchange described, this sampling rate is used in the distributor switch when the signals are in the standard p.c.m. format. The signal therefore must be bandlimited in accordance with this characteristic at some point before the distributor switch. There are then two possible ways of performing this required bandlimiting. The first is to filter each signal at the inlets of the concentrator switch so that the specification is met. This allows the concentrator switch to be operated at a sampling rate of 8kHz. This is the approach which was assumed to be used in the construction of the experimental system. The advantages of this approach can be summarised as follows:

i) the multiplex system works at the lowest possible rate, which keeps the speed of the logic and that of the analogue to digital encoders to a minimum;

ii) no further filtering is required.

This approach also has the major disadvantage that a filter which meets the full C.C.I.T.T. specification must be placed at each inlet to the concentrating switch. An alternative approach is to use higher sampling
rates in the concentrator switch which allows the requirement of the analogue prefilter to be relaxed. This is illustrated by Fig. A1.2 which shows the change in filter characteristic required as the sampling rate is increased. Since the transition bandwidth of the filter increases with sampling frequency and moreover the complexity of the filter is inversely proportional to the transition bandwidth, it follows that the complexity of the filter is consequently reduced. This effect is analysed in detail in relation to the same problem applied to the single channel codec switch (Section 2.4.1) in references A1.2 and A1.3. This saving in the filter complexity has however two penalties:

i) the speed of the multiplex system must be increased;

ii) the signals must be filtered at the inlets of the distributor switch so that the C.C.I.T.T. requirement is met when the sampling rate is reduced.

The increased speed of the multiplex system increases the timing problems associated with it. For example the speed of operation of the analogue to digital converters must be increased to enable conversion to take place in the reduced timeslot width. Various techniques exist (A1.4, A1.5, A1.6) to design digital filters to perform the bandlimiting function for rate reduction, but in all cases the complexity of the filter increases as the difference between the two sampling rates grows larger. For any particular system a trade-off between speed of the multiplex system and complexity of the digital filters on the one hand and the complexity of the analogue filters on the other must be made. The exact form that any system will take depends on the relative numbers of analogue and digital filters and the ease with which the speed of the multiplex system can be increased. This latter factor is itself determined by the technology available at the time; the tendency to date has been a continuous increase in the speed of digital circuitry
while the cost of analogue filters has remained static. Thus if this situation continues oversampled systems will clearly become more economic.
FIG. A1.1 CCITT BAND LIMITING FILTER REQUIREMENT

FIG. A1.2 THE EFFECT OF INCREASING THE SAMPLING FREQUENCY ON THE FILTER TRANSITION BANDWIDTH

(HATCHING SHOWS AREA WHERE ALIASING MUST BE AVOIDED)
A2.1 MEASUREMENT TECHNIQUES

As described in Chapter 4, two main forms of measurement were carried out on the experimental system; these were the measurement of interchannel crosstalk and the measurement of the level of idle channel noise. These measurements were both taken using a Marconi Instruments P.C.M. Multiplex Tester, type TF2807. This is a complex instrument which includes a signal generator, attenuators, filters and a true r.m.s. millivoltmeter. Its functions include the measurement of quantisation noise distortion and gain level linearity in addition to those mentioned above.

For clarity the circuit used for crosstalk measurements is shown in simplified form in Fig. A2.1. For this measurement a 325Hz sinusoidal signal is connected to one channel of the multiplexer via a calibrated variable attenuator. The output of the second channel is passed through a second calibrated variable attenuator and filter before being measured with a true r.m.s. voltmeter. The filter has a bandpass response centred about 325Hz and excludes noise components from the crosstalk measurements. Measurements are taken by adjusting the attenuators for a 0dB reading on the meter, and observing the setting of the attenuators.

The circuit used for noise measurements is shown in Fig. A2.2. The internal noise generator and filter of the tester was coupled onto the multiplex highway via one of the variable attenuators. The output of the demultiplexer was passed through a second attenuator and two filters to the voltmeter. The first of these filters limits the received noise bandwidth to the standard range for telephone speech (300 - 3400 Hz)
and the second filter performs a psophometric weighting on the noise in accordance with the C.C.I.T.T. recommendation for telephone circuits (A2.2). The noise level was measured by setting the attenuator in the send side and then changing the attenuator setting on the receive side until a zero reading was obtained on the voltmeter. The two settings were then plotted as a graph of output noise level against input noise level for the particular function used. In order to find the response of the system to cyclic noise the send side of this configuration was replaced by a pulse generator.

A2.2 FURTHER RESULTS

The results given in Chapter 4 are a subset of a much larger number of readings which were taken on the sets of orthogonal functions. While these results accurately reflect the performance of the various functions under the conditions specified, it was necessary to take results on complete matrices of functions to ascertain that this was so.

This was first done for the set of Walsh Functions $\bar{W}_{16}$ and Figs. A2.3-6 show the complete matrices of measured crosstalk for these functions. Each matrix consists of 256 measurements with each function being used at both the multiplexer and demultiplexer. It can be seen that the fourth row of each of these matrices corresponds to the results displayed in Table 4.1 with the exception of 4.1e. Similar matrices of results were also obtained for other sets of functions (Figs. A2.7-11) in some but not all of the systems configuration, since it became apparent that the performance of the sets of functions could be gauged from a comparatively few measurements. The method adopted for the remainder of the measurements was then to measure the crosstalk between one function which previous measurements had shown to be typical of the functions in the set, and each of the other functions. These measurements
were checked by selecting random pairs of functions and repeating the measurements to ensure that these were not substantially different from those for the function selected as typical. Finally the quantisation noise of the experimental system with digital companding was measured. This is illustrated in Fig. A2.12. The level of noise shown is higher than the required p.c.m. specification, as was anticipated in Section 4.1.4. This was thought to be caused by the poor performance of the commercial encoder used and interference from the digital areas of the circuitry.
FIG. A2.1 CROSSTALK MEASUREMENT CIRCUIT

N.B. ALL COMPONENTS EXCEPT MULTIPLEX SYSTEM ARE PART OF TESTER TF 2807

FIG. A2.2 NOISE MEASUREMENT CIRCUIT

N.B. ALL COMPONENTS EXCEPT MULTIPLEX SYSTEM ARE PART OF TESTER TF 2807
FIG. A.2.3 THE MATRIX OF RESULTS FOR THE FUNCTIONS GENERATED BY W16 WITH DIGITAL DEMULTIPLEXING

N.B. VALUES ARE dB BELOW 0 dB CHANNEL REFERENCE

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FIG. A.2.4 AS 2.3 BUT HIGHWAY RISETIMES DEGRADED
FIG. A2.5 THE MATRIX OF RESULTS FOR THE FUNCTIONS GENERATED BY \( W_{16} \) WITH ANALOGUE DEMULTIPLEXING

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FIG. A2.6 AS A 2.5 BUT HIGHWAY RISETIMES DEGRADED
FIG. A.2.7 THE MATRIX OF RESULTS FOR THE FUNCTIONS GENERATED BY $H_{16}$ WITH DIGITAL DEMULTIPLEXING

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**FIG. A2.9** THE MATRIX OF RESULTS FOR THE FUNCTIONS GENERATED BY P₁₂ WITH DIGITAL DEMULTIPLEXING

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**FIG. A2.10** AS A2.9 BUT HIGHWAY RISTIMES DEGRADED
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FIG. 2.11 THE MATRIX OF RESULTS FOR THE FUNCTIONS GENERATED BY $T_{16}$ WITH DIGITAL DEMULTIPLEXING

N.B. OTHER CONDITIONS AS FIG. A 2.3

![Image of graph showing S/N ratio vs signal level](image-url)

**FIG. 2.12 QUANTISATION NOISE OF EXPERIMENTAL SYSTEM USING FUNCTION OF $W_{16}$ AND DIGITAL COMPANDING**
APPENDIX 3

PLATES

PLATE 1

THE EXPERIMENTAL SYSTEM AND TEST EQUIPMENT
PLATE 2

THE ANALOGUE TO DIGITAL CONVERTER

AND DIGITAL DEMULTIPLEXER
PLATE 3

THE ANALOGUE MULTIPLEXER
Plate 4: Walsh Functions

1) Input
2) Sampled Input
3) Sampled Input x Walsh Function 5
4) Output (after Digital Demultiplexer)

Plate 5: Hadamard Functions

1) Input
2) Sampled Input
3) Sampled Input x Hadamard Function 2
4) Output (after Digital Demultiplexer)
Plate 6: Ternary Functions

1) Input
2) Sampled Input
3) Sampled Input x T(16 4)
4) Output (after Digital Demultiplexer)

Plate 7: Time Division Multiplexing

1) Input
2) Sampled Input
3) Sampled Input x T.D.M. Block Pulse 6
4) Output (after Digital Demultiplexer)
Plate 8: Walsh Functions
1) Sampled Input
2) Sampled Input x Walsh Function 5
3) Output of Analogue Integrator
4) Sampled Output

Plate 9: Hadamard Functions
1) Sampled Input
2) Sampled Input x Hanamard Function 2
3) Output of Analogue Integrator
4) Sampled Output
Plate 10: Ternary Functions

1) Sampled Input
2) Sampled Input x T16 4
3) Output of Integrator
4) Sampled Output

Plate 11: Time Division Multiplexing

1) Sampled Input
2) Sampled Input x T.D.M. Block Pulse 6
3) Output of Integrator
4) Sampled Output
Plate 12: Walsh Functions

1) Sampled Input (Channel 1)
2) Multiplex Highway Signal
3) Output Channel 2 (Digital Demultiplexer)
4) Output Channel 1 (Analogue Demultiplexer)

Plate 13: Hadamard Functions

1) Sampled Input (Channel 1)
2) Multiplex Highway Signal
3) Output Channel 2 (Digital Demultiplexer)
4) Output Channel 1 (Analogue Demultiplexer)
Plate 14: Ternary Functions

1) Input Channel 1
2) Multiplex Highway Signal
3) Output Channel 2 (Digital Demultiplexer)
4) Output Channel 1 (Analogue Demultiplexer)

Plate 15: Time Division Multiplexing

1) Input Channel 1
2) Multiplex Highway Signal
3) Output Channel 2 (Digital Demultiplexer)
4) Output Channel 1 (Analogue Demultiplexer)
Plate 16: Walsh Functions
1) Input Channel 1
2) Multiplex Highway Signal
3) Crosstalk – Channel 2 (Digital Demultiplexer)
4) Crosstalk – Channel 2 (Analogue Demultiplexer)

Plate 17: Hadamard Functions
1) Input Channel 1
2) Multiplex Highway Signal
3) Crosstalk – Channel 2 (Digital Demultiplexer)
4) Crosstalk – Channel 2 (Analogue Demultiplexer)
Plate 18: Ternary Functions

1) Input Channel 1
2) Multiplex Highway Signal
3) Crosstalk - Channel 2 (Digital Demultiplexer)
4) Crosstalk - Channel 2 (Analogue Demultiplexer)

Plate 19: Time Division Multiplexing

1) Input Channel 1
2) Multiplex Highway Signal
3) Crosstalk - Channel 2 (Digital Demultiplexer)
4) Crosstalk - Channel 2 (Analogue Demultiplexer)
Plate 20: Frame Waveform

1) Demultiplexing Function
2) Multiplex Highway Waveform

Plate 21: Timeslot Waveform

1) Demultiplexing Function
2) Multiplex Highway Waveform (with unsynchronised data) showing finite risetimes. (1 cm = 1 μS)
Plate 22: Analogue Demultiplexer Waveforms

1) Multiplex Highway Signal
2) Demultiplexing Function
3) Output of Multiplier
4) Output of Integrator

Plate 23: Integrator Waveforms

1) Reset Pulses
2) Output of Integrator with unsynchronised data in the other Channel
Plate 24: Clock Waveforms
1) Read
2) Timeslot Clock
3) Encode
4) Add

Plate 25: Clock Waveforms
1) Timeslot Clock
2) Encode
3) Add
APPENDIX 4

CIRCUIT DIAGRAMS OF THE EXPERIMENTAL SYSTEM
A4 - 1 THE EXPERIMENTAL SYSTEM
A4 - 2 SAMPLE AND HOLD AND MULTIPLEXER
SHIFT REGISTER CLOCK  
RESSET CLOCK  

+5  
1k  

SW 1  

GANGED TO  
SW 1 A4 - 4  

SN 7400  

SN 7400  

SN 7400  

SN 7400  

SN 7400  

SN 7400  

SN 7400  

SN 7400  

SN 7400  

NOTE:-- SWITCHES 2 - 16  
WIRED AS SWITCH 1  

SN 7430  

SN 7430  

SN 7402  

SN 7486  

LOGIC CONTROL A  

FRAME BLANKING CLOCK  

A4 - 3 FUNCTION GENERATOR PART 1. MULTIPLIER CONTROL
SHIFT REGISTER CLOCK
RESET CLOCK

+5
1 k

SW 1

GANGED TO
SW 1 A4 - 3

NOTE:- SWITCHES 2 - 16
WIRED AS SWITCH 1

FRAME BLANKING CLOCK

A4—4 FUNCTION GENERATOR PART 2. TIMESLOT CONTROL
A 4-6 DIGITAL DEMULTIPLEXER

MULTIPLIER CONTROL

TIMESLOT CONTROL

12 BIT INPUT FROM ANALOGUE TO DIGITAL CONVERTER

16 BIT OUTPUT TO GAIN ADJUSTMENT CIRCUIT

RESET ADD READ

KEY: 1, 2, 3 SN 74H87
4, 5, 6, 7 SN 74283
11, 12, 13 SN 74174

12 BIT INPUT FROM ANALOGUE TO DIGITAL CONVERTER

16 BIT OUTPUT TO GAIN ADJUSTMENT CIRCUIT

RESET ADD READ

KEY: 1, 2, 3 SN 74H87
4, 5, 6, 7 SN 74283
11, 12, 13 SN 74174

MULTIPLIER CONTROL

TIMESLOT CONTROL

12 BIT INPUT FROM ANALOGUE TO DIGITAL CONVERTER

16 BIT OUTPUT TO GAIN ADJUSTMENT CIRCUIT

RESET ADD READ

KEY: 1, 2, 3 SN 74H87
4, 5, 6, 7 SN 74283
11, 12, 13 SN 74174

MULTIPLIER CONTROL

TIMESLOT CONTROL

12 BIT INPUT FROM ANALOGUE TO DIGITAL CONVERTER

16 BIT OUTPUT TO GAIN ADJUSTMENT CIRCUIT

RESET ADD READ

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4, 5, 6, 7 SN 74283
11, 12, 13 SN 74174

MULTIPLIER CONTROL

TIMESLOT CONTROL

12 BIT INPUT FROM ANALOGUE TO DIGITAL CONVERTER

16 BIT OUTPUT TO GAIN ADJUSTMENT CIRCUIT

RESET ADD READ

KEY: 1, 2, 3 SN 74H87
4, 5, 6, 7 SN 74283
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MULTIPLIER CONTROL

TIMESLOT CONTROL

12 BIT INPUT FROM ANALOGUE TO DIGITAL CONVERTER

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4, 5, 6, 7 SN 74283
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MULTIPLIER CONTROL

TIMESLOT CONTROL

12 BIT INPUT FROM ANALOGUE TO DIGITAL CONVERTER

16 BIT OUTPUT TO GAIN ADJUSTMENT CIRCUIT

RESET ADD READ

KEY: 1, 2, 3 SN 74H87
4, 5, 6, 7 SN 74283
11, 12, 13 SN 74174
A4 - 7 GAIN ADJUSTMENT CIRCUIT

BIT/s 1 - 12 TO INPUT OF COMPAUNDER

BIT/s 1 - 16 FROM OUTPUT OF DIGITAL DEMULTIPLEXER
A4–8 COMPANDER SCHEMATIC

2’s COMPLIMENT CONVERSION

LINEAR TO A–LAW COMPRESSOR

BUFFER

A–LAW TO LINEAR EXPANDER

BUFFER

2’s COMPLIMENT CONVERSION

COMPANDED/UNCOMPANDED SWITCH

12 BIT/s INPUT/

12 BIT/s

12 BIT/s

8 BIT/s

8 BIT/s

12 BIT/s

12 BIT/s

12 BIT/s

12 BIT/s OUTPUT

SWITCH CONTROL

A4–8 COMPANDER SCHEMATIC
A4–9 2's COMPLEMENT TO AND FROM SIGN AND MAGNITUDE CONVERSION
A4-12 COMPANDED/UNCOMPANDED SWITCH
A 4–13 DIGITAL TO ANALOGUE CONVERTER AND FILTER
APPENDIX 5

PUBLISHED WORK

A SIMULATION OF A HYBRID MULTIPLEX SYSTEM

M.J. Carey* and A.G. Constantinides**

Summary

A multiplex system using digital orthogonal functions derived from Hadamard matrices has been proposed for the concentration and expander stages of a digital local exchange. Limiting the multiplex highway signal can cause noise in channels not in use. The level of the noise for a given amount of limiting was investigated by means of a simulation. The results of this simulation show that in the system under consideration the dynamic range required on the multiplex highway of a 15 channel system is only twice that required for a single channel.

1. Introduction

This paper describes a simulation of a multiplex system which is intended for use as a switch in the concentrator and expander stages of a digital local telephone exchange. The parameters of the simulation can be changed for other systems using digital orthogonal functions as carriers and results obtained for these other systems. These results will be of particular interest when the maximum allowable signal level on the multiplex highway is an important system constraint.

The system shown in Figure 1a contains the essential elements of a multiplex system using digital orthogonal functions as carriers. The input signals to be multiplexed are sampled and held constant for some time (T) and each multiplied by one of a set of functions which are mutually orthogonal over T. The outputs of the multipliers are then added together in an amplifier. The output of the amplifier is the multiplexed highway signal which is transmitted to demultiplexers where it is again multiplied by the orthogonal function and integrated over T. The output of the integrators is transferred to the output sample hold gates.

This process can be carried out in either the analogue or digital domains and conversion between the two can take place at some point in the system illustrated in Fig. 1b and Fig. 1c. The multiplex highway itself is an attractive position for the converters as only one of these is then required for each multiplex system. This fact has been exploited for many years by the use of time division multiplex (t.d.m.) systems for exactly this purpose. However results to be published show that systems using digital orthogonal functions based on Hadamard matrices have superior noise and cross-talk performance when compared with t.d.m. systems but they have the disadvantage of requiring higher signal levels on the multiplex highway and hence more quantisation levels in the encoding process.

Consider a system with m channels each with a maximum voltage level \( V_i \). When using t.d.m. the maximum level on the multiplex highway at any time is \( V_i \) since only one channel can be non zero at any time. However when using sets of digital orthogonal functions based on Hadamard matrices
all the channels can be non-zero simultaneously hence the maximum level on
the multiplex highway can be mVj. If the step size in the converters is
to remain constant, which must be the case or the quantisation noise—
performance will be degraded, then an increased number of bit (n) is
needed in the converter this is given by

\[ n = \log_2 m \]

Failure to provide this number of bits will result in a limiting of the
highway signal which affects the orthogonality between the channels,
resulting in noise in the inactive channels and limiting of those causing
the overload. However it is difficult to determine a well defined peak
value for speech signals and a simultaneous peak voltage on all the channels
in a system is very infrequent, therefore the noise caused by this mechanism
is difficult to quantify. Hence to gain an insight into the actual
maximum level required on the multiplex highway and hence in the number of
bits in the converters the multiplex system and the speech amplitude
input to each channel was simulated. By running the simulation with
different maximum levels on the multiplex highway the variation of the
noise with respect to this level was found.

2. The Overload Mechanism

A simple way of demonstrating the effect of overload is to consider a
simplified multiplex system for example the fourth order Walsh function
case. The Hadamard matrix of order four on which the Walsh functions are
based is given by

\[ \tilde{W}_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \]

The four rows in the above matrix will correspond to the set of digital
orthogonal functions in the multiplex system. Let the inputs to the four
channels at a given time (T) be the following vector

\[ \vec{D} = \begin{bmatrix} A \\ B \\ C \\ 0 \end{bmatrix} \]

where A > B > C > zero (0). Then the multiplex highway vector will be

\[ \bar{R} = \tilde{W}_4 \times D = \begin{bmatrix} A + B + C \\ A + B - C \\ A - B - C \\ A - B + C \end{bmatrix} \]

The multiplication of this vector by the appropriate functions at the
demultiplexer and integrating will produce ideally the original data
scaled by the matrix normalisation factor, as follows

\[ I_1 = A + B + C + A + B - C + A - B - C + A - B + C = 4A \]
\[ I_2 = A + B + C + A + B - C - A + B + C - A + B - C = 4B \]
\[ I_3 = A + B + C - A - B + C - A + B - C + A - B + C = 4C \]
\[ I_4 = A + B + C - A - B + C + A - B - C - A + B - C = 0 \]
However when the amplitude of the highway signal is restricted not to exceed \( A + B \) for example, then the highway vector becomes altered:

\[
\mathbf{H} \text{ (limited)} = \begin{bmatrix}
A + B \\
A + B \\
A - B - C \\
A - B + C 
\end{bmatrix}
\]

and the output of each channel will be given by

\[
\begin{align*}
I_1 &= A + B + A + B + A - B - C + A - B + C = 4A \\
I_2 &= A + B + A + B - A + B + C - A + B - C = 4B \\
I_3 &= A + B - A - B - A + B + C + A - B + C = 2C \\
I_4 &= A + B - A - B + A - B - C - A + B - C = 2C
\end{align*}
\]

It is apparent from the above that the signal in the third channel has been reduced in amplitude which is itself not serious but in addition part of that channels signal power has appeared in the fourth channel. This channel leakage is not in general intelligible cross talk and will be subjectively noticeable only in the intervals in conversations on other channels, hence the noise is most appropriately treated for the exchange system considered by aggregating it with idle channel noise from other sources.

3. Telephone Speech Statistics

The chosen parameters for speech statistics were originally identified by Holbrook and Dixon (Ref. 1) these are

(a) the activity factor (\( \tau \)) which determines the proportion of total time available a channel carries a speech signal.

(b) the channel volume (\( v \)) this is the long term mean power while active

(c) the instantaneous amplitude (\( x \)) of the channel.

The activity factor of the channel is determined by the channel loading and the proportion of the time a seized channel carries speech, this latter fact has been found empirically to be about 0.4. The loading of the channel depends on the size of the groups of channels and the grade of service to be offered to the customer. For a large group of channels this is typically 0.6 which gives an activity factor \( \tau = 0.4 \times 0.6 = 0.24 \) a value close to 0.25 which is often quoted as typical for this parameter. However, for smaller groups of channels such as 15, the number considered for the digital local exchange the loading is approximately 0.4 giving \( \tau = 0.16 \) and this value is used in the simulation.

The second parameter of interest is the speech volume distribution. The volume of a channel, defined as the long term mean power of the channel while it is active, is a constant for a given connection. The volume scales the selections from the instantaneous voltage distribution to allow for the unique characteristics of different speakers. The probability density function of this parameter is given by Holbrook and Dixon (Ref. 1) as:

\[
p(V) dV = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(V - V_0)^2}{2\sigma^2}\right) dV
\]

where \( V \) is the volume, \( V_0 \) the average of the distribution and \( \sigma \) the standard deviation. These parameters are all measured in decibels and so the distribution is log normal rather than normal. This distribution was
measured by Holbrook and Dixon at the microphone and hence the distribution at the local exchange must be changed by the loss introduced between the telephone and the exchange. This loss is caused by:

(a) the attenuation caused by the local distribution network
(b) the variation in microphone sensitivity with changing current feed
(c) the action of the regulator in the telephone which tends to compensate for the above.

The average volume levels in the local distribution network range between -10dBm0 and -18dBm0 and the standard deviation \( \sigma \) of the volume distribution has been measured by Berry (Ref. 2) for local exchanges in the United Kingdom reporting value ranging between 4dB and 8dB for this parameter, with the lower figures being measured for exchanges on short lines (i.e. Director exchanges in cities). The system to be simulated will be most likely to overload when the volume levels encountered are high. This is most likely when an exchange is connected by short lines. Therefore for the purpose of the simulation it was assumed that the average volume level \( V_0 \) was -12dBm0 and that this was accompanied by a value of standard deviation of 4.1dB. These figures are typical for a city exchange with short lines, and are more stringent than the conditions for the majority of exchanges in the network.

Measurements on the instantaneous amplitude distribution of speaker were also made by Holbrook and Dixon (Ref. 1) and later by Rizoni (Ref. 3) and Richards (Ref. 4). Several theoretical distributions have been proposed as models of the instantaneous speech amplitude distribution, these include the gaussian (Ref. 4), the exponential (Ref. 5) and the gamma distribution proposed by Richards. This particular distribution is given by

\[
p(x) = \frac{k}{2\Gamma(\lambda)} (\lambda x)\lambda - 1 \exp(-\lambda x)
\]

where \( k = \sqrt{\lambda(\lambda + 1)} \). \( \lambda \) is the ratio of instantaneous amplitude to root mean square amplitude and \( \lambda \) a parameter to be adjusted to suit the particular statistics modelled. A value of \( \lambda = 0.2 \) is a close fit to the distribution of telephone speech while \( \lambda = 0.5 \) fits the distribution for speech measured using a high quality microphone. For the purpose of the simulation \( \lambda \) was set to 0.2.

These then are the important statistics which determine the range of amplitudes of a telephone speech waveform, and hence those at the input of the multiplex system. They do not completely characterise the signal as they contain no information about the spectral distribution of the signal. However the spectral content of the signal has no effect on the multiplex system since samples of the signal are generated at the sampling frequency and are treated independently of preceding and following samples.

4. The Simulation of Speech

It follows from the above that the generation of the speech signal for each of the multiplex channels can be divided into three parts:

(a) the decision on whether the channel is active
(b) the generation of an amplitude level
(c) the weighting of the voltage level by the volume selected for that channel.
In order to generate these signals a random number generator was required. This was provided by means of a generator using a multiplicative congruence technique. The numbers thus generated have a rectangular distribution between 0 and 1. The activity of a channel was decided by comparing a random number with the activity factor. If the random number is greater than the activity factor then the channel is inactive whereas if it is less than the activity factor it is active. The generation of the instantaneous amplitude levels is a little more involved. It can be shown that a given probability distribution can be obtained by making selections from the inverted cumulative distribution function (c.d.f.) using random numbers with a rectangular distribution between 0 and 1 (Ref. 6). If the c.d.f. can be obtained analytically this is a fairly simple task. However the gamma distribution is not easily integrable and hence numerical techniques must be used to generate the c.d.f. Hence the first part of the simulation performed a numerical integration on the gamma function in order to generate an array corresponding to its c.d.f. This array was then used in conjunction with the random number generator to generate selections from the gamma distribution.

The volumes of the channels could also be obtained in the same way. However a more direct method is possible. The central limit theorem shows that the distribution of the sum of a number of independent random variables tends towards a Gaussian distribution (Ref. 7). Hence if a number of selections are made from a rectangular distribution the sum of the selections can be treated as Gaussian. The values of the Gaussian distribution are given by

\[ x = \sigma \left( \sum_{i=1}^{n} R_i - n/2 \right) \sqrt{\frac{12}{n}} + \mu \]

where \( \sigma \) is the standard deviation, \( n \) is number of selections \( \mu \) is the mean of the distribution and \( R_i \) are the numbers selected from the rectangular distribution. Thus if \( n = 12, \sigma = 4 \) and \( \mu = 0 \) we have,

\[ x = 4 \left( \sum_{i=1}^{12} R_i - 6 \right) \]

This then can be used to generate samples from the log normal volume distribution. By converting these values which are in decibels to linear form, the results can be used to scale the selections from the speech amplitude distribution.

5. The Simulation of the Multiplex System

It was decided that the set of orthogonal functions to be used in the simulation of the multiplex system would be the set of Walsh functions. These functions were chosen because they are considered a typical set of Hadamard functions and in addition their matrix is simple to generate. The generation is carried out by performing the inverse Walsh-Fourier transform (Ref. 8) on a set of impulses in the transform domain. Each of the set of impulses generates one of the Walsh functions which can then be stored in an array.

The multiplex simulation is exactly equivalent to a hardware multiplex system. In each subinterval of \( T \) the signal generated for each channel is multiplied by the appropriate value taken from the array of functions and then the results from all the channels are added together to form part of a line vector which contains results for all of \( T \). In a similar manner the
line vector is processed to form the demultiplexed signals. The r.m.s. value of the demultiplexed signal for each channel is then calculated from the successive outputs of the demultiplexer. The simulation allowed both the input signals to each channel and the multiplex highway signal, to be limited to predetermined values so that the effect of the limiting on the idle channel noise level can be assessed. In this manner the dynamic range required by the multiplex system was determined. The complete simulation program is shown in A7.4.

6. The Real Time Operation of the Program

This can be described by reference to the flowchart of Fig. 2. Before the simulation run is started the array of multiplexing functions is formed and the cumulative distribution function of the speech amplitudes is generated. The system then reads in information referring to which one of the 15 channels is idle, and the limit set on the multiplex highway. The length of the simulation is determined by two parameters

(a) the length of each run with the volume levels fixed and
(b) the number of times the volume levels are changed.

The program then finds the volume levels for the first run and uses these to scale the amplitudes generated for the first run. Each amplitude may be set at zero depending on the activity of that channel. The amplitudes are then passed through the multiplex system squared and accumulated, so that at the end of the simulation the root mean square value of the voltages is found. The program then generates a second set of volumes, corresponding to a new set of speakers, and uses these to scale the amplitudes generated on the next run. In this way a long term average level of the signal in each channel is built up. The result for the idle channel is the idle channel noise power caused by the overload effect. After a simulation with a preselected overload level the level can be varied for the next simulation until the minimum level which meets the specification is found. In this way the dynamic range required by the system can be determined.

7. Results

The results which follow use as a reference the CCITT p.c.m. specification (Ref. 9) as the transmission performance of a digital local exchange will be required to meet this specification. The first result given in Fig. 3 was obtained with the multiplex system disabled. The simulation program was run with the speech generation routine working normally and the outputs of the 15 channels were summed together to produce composite amplitude levels. The distribution of these levels represents the overall amplitude levels to be found on the multiplex highway. The continuous curve represents the distribution without limiting of individual channels, and the second curve shows the effect of limiting each channel before multiplexing at the level specified for p.c.m. systems (+3dBm0 = 5.6Vrms, Ref 9). It shows that the probability of the combined amplitudes of the channels being high is low and that it falls swiftly when the channels are individually limited before multiplexing.

The result of the full simulations are given in Fig. 4. This shows the variation in the level of noise induced into one channel (normalised to $V_{rms}$) when the level of limiting on the multiplex highway is varied. It also shows a maximum allowable level for the idle channel noise; this was set at 0.001$V_{rms}$ ($= -72$dBm0) which is rather more stringent than that
A simulation of a multiplex system using digital orthogonal functions based on Hadamard matrices has been described. This has shown that this system requires twice the dynamic range (one extra bit in the encoders and decoders) of a t.d.m. system to meet the idle channel noise requirements of a digital local exchange. This is much less than that predicted from simple theoretical considerations. The program described could easily be modified to find the dynamic range requirements for other applications.

8. Acknowledgements

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References

FIG. 2 THE FLOW CHART FOR THE SYSTEM SIMULATION
Figure 3: Instantaneous combined amplitudes for 15 channels.

Figure 4: Idle channel noise as a function of the limit on the multiplex highway.
CHAPTER 1


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CHAPTER 2


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2.5 C.C.I.T.T. Draft Recommendation G712.


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CHAPTER 3


Chapter 4


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CHAPTER 5


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CHAPTER 6

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APPENDIX 1


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APPENDIX 7

COMPUTER PROGRAMS

A7.1 PROGRAM MATRIX
A7.2 PROGRAM EDGES
A7.3 PROGRAM FWFT
A7.4 PROGRAM MWXSIM
A7.1 PROGRAM MATRIX

1600 " THIS PROGRAM CALCULATES THE CROSSCORRELATION
1160 " BETWEEN FUNCTIONS IN A MATRIX OF ORDER M
1200 
1300 DIMENSION FM(18,18),GM(18,18)
1400 DIMENSION G(32),F(32),GN(320),FN(320),Y(320)
1500 " THE PROGRAM READS THE SIZE OF MATRIX AND THE INTERVAL
1600 READ/,M,I
1700 " THE PROGRAM NOW READS THE FUNCTIONS
1800 DO 800 KN=1,M
1900 READ/,FM(KN,K),K=1,M)
2000 KM=0
2100 DO 900 KM=1,M
2200 GM(KN,KM)=FM(KN,KM)
2300 900 CONTINUE
2400 800 CONTINUE
2500 " THE CROSSCORRELATION IS NOW CALCULATED
2600 DO 1100 LO=1,M
2700 LM=0
2800 DO 1100 LM=LO,M
2900 KL=0
3000 PRINT/,LO,LM
3100 DO 1100 KL=1,LM
3200 G(KL)=GM(LO,KL)
3300 F(KL)=FM(LM,KL)
3400 1100 CONTINUE
3500 N=M*I
3600 DO 200 J=1,N
3700 GN(J)=G(1+(J-1)/I)
3800 Y(J)=0
3900 FN(J)=F(1+(J-1)/I)
4000 FN(J+N)=FN(J)
4100 GN(J+N)=GN(J)
4200 200 CONTINUE
4300 DO 100 L=1,N
4400 X=0
4500 DO 100 JE=1,N
4600 X=FN(JE)*GN(JE+L-1)
4700 Y(L)=Y(L)+X
4800 100 CONTINUE
4900 " THE CROSSCORRELATION MATRIX IS PRINTED OUT
5000 PRINT/,Y(N-2),Y(N-1),Y(1),Y(2),Y(3)
5100 1100 CONTINUE
5200 STOP
5300 END
PROGRAM EDGES

1000 " THIS PROGRAM FINDS THE OCCURANCE OF EDGE EFFECTS
1100 "
1200 DIMENSION FM(16,16), GM(16,16)
1300 DIMENSION G(32), F(32), GN(320), FN(320), Y(320)
1400 " THE MATRIX ORDER AND SET OF FUNCTIONS ARE READ IN
1500 READ/M
1600 DO 300 KN=1,M
1700 READ/(FM(KN,K), K=1,,M)
1800 KM=0
1900 DO 900 KM=1,M
2000 GM(KN,KM)=FM(KN,KM)
2100 960 CONTINUE
2200 800 CONTINUE
2300 " THE POSITION OF THE EDGES IS FOUND
2400 DO 1100 LO=1,M
2500 LM=0
2600 DO 1100 LM=LO,M
2700 KL=0
2800 PRINT/ALO,LM
2900 DO 1000 KL=1,M
3000 G(KL)=GM(LO,KL)
3100 F(KL)=FM(LM,KL)
3200 1000 CONTINUE
3300 F(M+1)=F(1)
3400 G(M+1)=G(1)
3500 Z=0
3600 T=0
3700 DO 100 J=1,M
3800 X=F(J+1)-F(J)
3900 Y=G(J+1)-G(J)
4000 Z=X*Y
4100 T=T-Z
4200 100 CONTINUE
4300 " THE EDGE MATRIX IS PRINTED OUT
4400 PRINT/ST
4500 1100 CONTINUE
4600 STOP
4700 END
A7.3 PROGRAM FVFT

1000 " THIS PROGRAM TRANSFORMS BETWEEN THE TIME AND
1100 "WALSH—FFEEQUENCY DOMAINS
1200 ""
1300 DIMENSION DB(256)
1400 DIMENSION XR(9,256)
1500 800 CONTINUE
1600 PRINT 500
1700 500 FORMAT("ENTER LOG OF INPUT VECTOR SIZE")
1800 READ/M
1900 IF(M .LT. 2) GO TO 900
2000 N=2**M
2100 " THE INPUT VECTOR IS READ IN DDS AND CONVERTED TO LINEAR
2200 PRINT 600
2300 600 FORMAT("ENTER INPUT VECTOR")
2400 READ/JCDECK),H=1,N)
2500 DO 50 I=IoN
2600 50 XRC1JI)=10.0**C —DBCI)/20.0)
2700 NH=N/2
2800 DO 100 L=1,N
2900 100 XRCLPJJS)=XRCLAJ)+XRCLJJ2)
3000 JD=JD-1
3100 XRCLP,JD)=XRCLJ)—XRCL,J2)
3200 100 CONTINUE
3300 L=M+1
3400 PRINT 700
3500 700 FORMAT("TRANSFORM COEFFS ARE: ")
3600 PRINT 1000 (XR(L,J),J=1,N)
3700 END
A7.4 PROGRAM MUXSIM

10000 " THIS IS THE COMPLETE SIMULATION PROGRAM
10100 "
10200 DIMENSION A(32), B(32), C(32), D(150), E(150), G(150)
10300 DIMENSION X(61), FX(61), AREA(61), T(11), Y(11), VOLLOG(20)
10400 DIMENSION SIGMA(32), RMS(32)
10500 DIMENSION XR(9,256), W(32,32), RIN(32), OUTPUT(32)
10600 REAL LINE(32)
10700 M=4.
10800 "
10900 " GENERATION OF THE SET OF WALSH FUNCTIONS
11000 "
11100 N=2**M
11200 XR(1,1)=1.0
11300 DO 2000 K1=1,N
11400 NH=N/2
11500 DO 100 L=1,14
11600 LP=L+1
11700 LM=L-1
11800 NY=0
11900 NZ=2**LM
12000 NZ1=2*NZ
12100 NiN=N/NZ1
12200 DO 100 I=1,NZN
12300 NX=NY-1-1
12400 NY=NY-I-NZ
12500 JS=(I-1)*NZI
12600 JD=JS-NZ1+1
12700 DO 100 J=NX,NY
12800 JS=JS+1
12900 J2=J+NH
13000 XR(LP,JS)=XR(LP,J)+XR(LP,J2)
13100 JD=JD-1
13200 XR(LP,JD)=XR(LP,J)-XR(LP,J2)
13300 100 CONTINUE
13400 L=M+1
13500 DO 2100 K2=1,N
13600 2100 W(K1,K2)=XR(L,K2)
13700 XR(1,K1+1)=XR(1,K1)
13800 XR(1,K1)=0.0
13900 2000 CONTINUE
14000 "
14100 " CALCULATION OF THE CUMULATIVE DISTRIBUTION FUNCTION
14200 "OF THE GAMMA DISTRIBUTION
14300 "
14400 RL=0.2
14500 RGAMMA=4.590845
14600 RK=(RL*(RL+1))**0.5
14700 FX(1)=0.5
14800 H=0.1
14800 " PROGRAM MUXSIM CONTINUED
14900 "
15000 X(1)=0.1
15100 DO 101 I=2, 61
15200 T(I)=X(I-1)+H
15300 E=H/10.
15400 DO 200 J=2, 61
15500 T(J)=T(J-1)+E
15600 DO 300 K=2, 11
15700 400 Y(K)=RINT(RN**((RK*T(L)**2.0)*RGAMMA))
15800 SUM1=0.
15900 SUM2=0.
16000 DO 500 I =2, 10
16100 500 SUM1=SUM1+3.0*Y(I)
16200 DO 600 I=3, 9
16300 600 SUM2=SUM2+1.0*Y(I)
16400 AREA(J)=E/3.0*(Y(1)+SUM1+SUM2)/Y(11)
16500 FX(I)=FX(I-1)-AREA(J)
16600 200 CONTINUE
16700 DO 700 M=2, 61
16800 FX(M)=FX(M-1)+AREA(M)
16900 700 CONTINUE
17000 " THE PROGRAM READS THE PARAMETERS OF THE SIMULATION
17100 PRINT 601
17200 601 FORMAT("ENTER LIMIT, CHANNEL IDLE")
17300 READ/LIMIT, IDLE
17400 IF(LIMIT.EQ.0.0) GO TO 2900
17500 PRINT 345
17600 345 FORMAT("ENTER RUN LENGTH, SELECTIONS, RANDOM NUMBER")
17700 READ/IIRUN, IX
17800 ROOT=Z*FLOAT(IIRUN)
17900 DO 1000 M=1, Z
18000 DO 800 I1=2, 11
18100 RI=0.
18200 GENERATION OF THE SET OF VOLUMES
18300 DO 900 M=1, 12
18400 CALL RANNUM(IX, RRAN)
18500 800 CONTINUE
18600 VOLOG(M1)=(RI-6.0)*4.1
18700 900 CONTINUE
18800 FX(I1)=16.0**((VOLOG(M1)/20.0)
18900 800 CONTINUE
19000 FX(IDLE)=0.0
19100 " DETERMINATION OF THE ACTIVE CHANNELS AND
19200 " SELECTION OF SPEECH AMPLITUDES
19300 " GENERATION OF THE SET OF VOLUMES
19400 DO 900 M=1, 12
19500 CALL RANNUM(IX, RRAN)
19600 IF(RRAN.LT.0.16) GO TO 1500
19700 RIN(12)=0.0
20400 " PROGRAM MUXSIM CONTINUED
20500 "
20600 GO TO 1200
20700 1500 ISIGN=0
20800 CALL RANNUM(IX,RRAN)
20900 IF(RRAN.LE.0.5) GO TO 1600
21000 RRAN=RRAN-0.5
21100 ISIGN=1
21200 1600 CONTINUE
21300 DO 1300 I3=1,60
21400 IF(RRAN.GT.FX(I3)) GO TO 1300
21500 C(I2)=FLOAT(I3)-0.5
21600 IF(ISIGN.NE.1)C(I2)=0.5-FLOAT(I3)
21700 " WEIGHTING OF AMPLITUDES BY CHANNEL VOLUMES
21800 "AND LIMITING
21900 RIN(I2)=C(I2)*F(I2)
22000 IF(RIN(I2).GT.56.6)RIN(I2)=56.6
22100 IF(RIN(I2).LT.-56.6)RIN(I2)=-56.6
22200 GO TO 1200 -
22300 1300 CONTINUE
22500 1200 CONTINUE
22600 " THE MULTIPLEX SYSTEM
22700 DO 2300 K3=1,N
22800 OUTPUT(K3)=0.0
22900 LINE(K3)=0.0
23000 DO 2200 K4=1,N
23100 LINE(K3)=LINE(K3)+RIN(K4)*W(K4,K3)
23200 IF(LINE(K3).GT.LIMIT)LINE(K3)=LIMIT
23300 IF(LINE(K3).LT.-LIMIT)LINE(K3)=-LIMIT
23400 2300 CONTINUE
23500 DO 2500 K5=1,11
23600 OUTPUT(K5)=OUTPUT(K5)+LINE(K6)*W(K5,K6)/N
23700 SIGMA(K5)=SIGMA(K5)+(OUTPUT(K5)**2)
23800 2400 CONTINUE
23900 1400 CONTINUE
24000 " CALCULATION OF R.M.S. VALUES
24100 RMS(K8)=SCRTCSIGMACK8)/ROOT)
24200 PRINT 10 (RMS(10),K7=1,N)
24300 10 FORMAT(8E9.2)
24400 DO 2500 L9=1,N
24500 SIGMA(L9)=0.0
24600 GO TO 700
24700 STOP
24800 END
24900 " LEHAR RANDOM NUMBER GENERATOR
25000 SUBROUTINE RANNUM(IX,RRAN)
25100 IX=MOD((IX*8195),33554432)
25200 RRAN=FLOAT(IX)/33554432
25300 END