BVCM: a comprehensive and flexible toolkit for whole system biomass value chain analysis and optimisation – mathematical formulation

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Abstract

This paper presents the novel MILP formulation of the Biomass Value Chain Model (BVCM), a comprehensive and flexible optimisation toolkit that models a large number of bioenergy system pathways. The model accounts for the economic and environmental impacts associated with the end-to-end elements of a pathway: crop production, conversion technologies, transport, storage, local purchase, import (from abroad), sale and disposal of resources, as well as CO_2 sequestration by CCS technologies and forestry. It supports decision-making around optimal use of land, biomass resources and technologies with respect to different objectives, scenarios and constraints. Objectives include minimising cost, maximising profit, minimising GHG emissions, maximising energy/exergy production or any combination of these. These objectives are combined with a number of scenarios (such as including different CO_2 prices, different technology and climate scenarios, import scenarios, waste cost scenarios), different credits (e.g. by-product and end-product, CCS and forestry carbon sequestration) and a number of constraints such as minimum levels of energy production and maximum environmental impacts.

The toolkit includes an extensive database of different biomass technologies including pre-treatment, densification, liquid and gaseous fuel production, heat and power generation (separately or combined, biodedicated or co-fired), waste-to-energy conversion and carbon capture and sequestration. A large number of resources are considered including a variety of bio-resources (e.g. energy crops such as Miscanthus and SRC willow, arable crops such as winter wheat, sugar beet and oilseed rape and short and long rotation forestry), intermediates, products, by-products and wastes.

The BVCM is a spatio-temporal model: currently it is configured for the UK using 157 square cells of length 50 km and the planning horizon is from the 2010s to the 2050s, with seasonal variations considered. The framework is data-driven so the model can be easily extended: for example adding new resources, technologies, transport modes etc. or changing the time horizon and the location to another country is only a matter of changing the data. Results of example UK case studies are presented to demonstrate the functionality of the model.

Keywords: biomass value chains; bioenergy; waste conversion; resource-technology network; modelling; optimisation

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1. Introduction

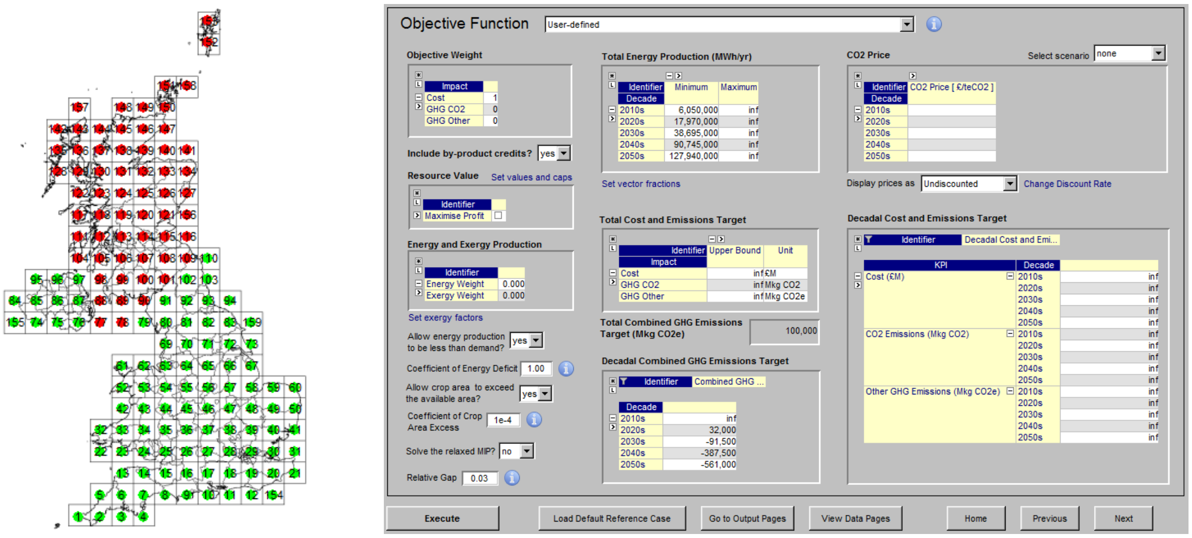
Most energy system studies in the UK indicate a prominent role for bioenergy in the coming decades, especially if the UK is to meet its emission targets [1, 2, 3, 4]. Bioenergy is a complex and controversial subject [5]. When deployed properly, it has the potential to help secure energy supply, mitigate climate change and create development opportunities particularly in the rural areas. However, when implemented poorly, it could negatively impact the climate and nature conservation as well as heighten land-use conflicts (e.g. food cultivation vs. bioenergy production). It is therefore important to understand fully the end-to-end elements that comprise the bioenergy system: from crops and land use, conversion of biomass to useful energy vectors, the manner in which it is integrated into the energy system (e.g. into transport fuel or into generation of heat or electricity), its interaction with systems outside the bioenergy boundary, and also the environmental and social impacts.

The Biomass Value Chain Model (BVCM), which was commissioned and funded by the Energy Technologies Institute (ETI), is a comprehensive and flexible toolkit that models a large number of bioenergy pathways, currently configured for the UK and over a time horizon of 50 years, from the 2010s to the 2050s. It supports decision-making around optimal use of biomass resources and bioenergy technologies with respect to different objectives such as minimum cost, maximum profit, minimum GHG emissions, maximum energy/exergy production or a combination of these objectives. The model accounts for the economic and environmental impacts associated with the end-to-end elements of a pathway: crop production, conversion technologies, transport, storage, local purchase, import (from abroad), sale and disposal of resources, as well as CO_2 sequestration by CCS technologies and forestry. Being a spatio-temporal model, the BVCM considers the dynamics and spatial dependence of system properties such as resource availability and demand, determines where and when to invest in conversion technologies (accounting for technology retirements) and how to operate them, allocates crops to the available land and determines the logistical interconnections.

The BVCM toolkit comprises the following:

- mixed-integer linear programming (MILP) model implemented in the AIMMS modelling platform [6] and solved using the CPLEX MIP solver [7];
- databases, provided as a series of Excel workbooks, that are used to store all of the data concerning technologies, resources, yield potentials, waste potentials etc. along with a data extraction tool;
- graphical user interface (GUI), also implemented in AIMMS, for configuring and performing optimisations and visualising the results; and
- tools implemented in Excel for further analysis of the results.

This paper, which focuses on the first and third components, describes the MILP mathematical formulation of the BVCM model. The other components provide flexibility to the tool, for example the databases and the data extraction tool enable the model to be data-driven: the model is easily extensible and applicable over different spatial and temporal scales, e.g. changing the scale of the model or extending it to include more technologies and resources is simply a matter changing the input data. The GUI, on the other hand, is very useful for scenario shaping as it enables the definition of a large number of "what-if" scenarios. For example, the user can: choose from the different settings for climate scenario, biomass yields, costs, efficiencies, imports, wastes etc.; specify the size of land available for biomass growth and whether or not to apply different constraints on land; restrict technologies and resources to certain locations; perform regional analysis; and perform many more functions. Figure 1(a) shows part of the "Cell Selection" page in the GUI, which can be used to define the region of interest to be considered in a scenario and Figure 1(b) gives a screenshot of the "Objective Function" page where the objective of the optimisation and the targets on energy production and emissions can be defined. Visualisation is also a very important aspect of the tool and the GUI has been designed to display the results of the optimisation in the most intuitive way: maps are used whereever possible to display the location and size of biomass plantations, technologies, resource storage, import of resources, waste utilisation and transport of resources. The toolkit also includes a stochastic analysis module wherein uncertainties in key parameters (e.g. biomass yields and costs, technology costs and efficiencies) can be specified as distributions rather than fixed numbers and a set of solutions is generated by sampling from these distributions. This allows the identification of more robust solutions, i.e. solution features such as resources and technologies that appear in a large number of different scenarios.



Objective Function: User-defined

Objective Weight

Impact	
Cost	1
GHG CO2	0
GHG Other	0

Include by-product credits? yes

Resource Value Set values and caps

Identifier	
Maximise Profit	☐

Energy and Exergy Production

Identifier	
Energy Weight	0.000
Exergy Weight	0.000

Set exergy factors

Allow energy production to be less than demand? yes

Coefficient of Energy Deficit 1.00

Allow crop area to exceed the available area? yes

Coefficient of Crop Area Excess 1e-4

Solve the relaxed MIP? no

Relative Gap 0.03

Total Energy Production (MWh/yr)

Identifier / Decade	Minimum	Maximum
2010s	6,050,000	inf
2020s	17,970,000	inf
2030s	38,695,000	inf
2040s	90,745,000	inf
2050s	127,940,000	inf

Set vector fractions

Total Cost and Emissions Target

Identifier / Impact	Upper Bound	Unit
Cost	inf	£M
GHG CO2	inf	Mkg CO2
GHG Other	inf	Mkg CO2e

Total Combined GHG Emissions Target (Mkg CO2e) 100,000

Decadal Combined GHG Emissions Target

Identifier: Combined GHG ...

Decade	
2010s	inf
2020s	32,000
2030s	-91,500
2040s	-387,500
2050s	-561,000

CO2 Price Select scenario none

Identifier / Decade	CO2 Price [£/teCO2]
2010s	
2020s	
2030s	
2040s	
2050s	

Display prices as Undiscounted Change Discount Rate

Decadal Cost and Emissions Target

Identifier: Decadal Cost and Emi...

KPI	Decade	
Cost (£M)	2010s	inf
	2020s	inf
	2030s	inf
	2040s	inf
	2050s	inf
CO2 Emissions (Mkg CO2)	2010s	inf
	2020s	inf
	2030s	inf
	2040s	inf
	2050s	inf
Other GHG Emissions (Mkg CO2e)	2010s	inf
	2020s	inf
	2030s	inf
	2040s	inf
	2050s	inf

(a) (b)

Figure 1: Examples of using the GUI to define the optimisation problem: (a) the "Cell Selection" map, where green circles indicate cells that are included in the problem and red circles are excluded; (b) the "Objective Function" page, which is used to define the objective of the optimisation, the targets on energy production and GHG emissions and also allow the user to configure some solver settings.

The main contribution of this paper is the novel and, to date, most comprehensive MILP model of biomass value chains. There is no model that can address all of the issues relevant to bioenergy, but we believe that the BVCM considers the largest number of issues (simultaneously) that need to be addressed in biomass value chains, such as land allocation (accounting for the spatial and temporal variation in biomass yields and available land areas), transport and storage of resources, imports, staged investment and retirement of technologies, co-product and end-product values, carbon capture and sequestration through technologies (with transport of CO_2 to sequestration sites) or by planting forestry, CO_2 price scenarios and uncertainty (although in the interest of space, the stochastic-analysis procedure is not presented in this paper). The BVCM is a multi-vector model that considers more pathways from biomass to energy than any other model: it decides what types of biomass to grow, where and when, and what forms of energy (e.g. heat, electricity, liquid and gaseous fuels) to produce in order to achieve a given objective subject to constraints on demands, emissions and so on. It considers a large number of technologies, such as pre-processing, power generation, heat production, combined heat and power, biomass to gaseous and liquid fuels and CCS. It also considers a variety of primary feedstocks including food and energy crops, forestry and waste resources (e.g. municipal solid waste), the availability of which are considered on a spatial and decadal basis.

The rest of the paper is structured as follows: Section 2 surveys the previous literature on biomass supply chain modelling and highlights the need for a comprehensive and flexible model that is applicable for a wide range of scenarios. The problem statement is given in Section 3 and the model structure is described in Section 4. The most important part of this paper is the MILP model formulation, which is discussed in detail in Section 5. Example case studies are described in Section 6. Finally, some concluding remarks are made in Section 7. A brief overview of the AIMMS implementation, cost discounting and unit conversion factors used in the model are also provided in the Supplementary Material.

2. Literature review

There is a growing body of literature on modelling of biomass supply chains. The need to satisfy the ever increasing demand for energy (in the form of electricity, heat, gas or transport fuel) while also reducing GHG emissions has driven a considerable amount of research into low-carbon and carbon neutral technologies. Among these are a number of technologies based on the conversion of biomass, such as gasification, anaerobic digestion, biomass-fired boilers and power plants, biomass to ethanol (and other transport fuels). It is natural, then, to ask which combination of technologies and types of biomass, and their locations, give rise to the most efficient (e.g. cost effective) provision of energy. A good overview of the issues and challenges faced in biomass supply chains is given by Mafakheri and Nasiri [8]. Mitchell [9] provides a concise list of the earlier biosystems models and describes the development of many of them.

De Meyer et al. [10] analysed the biomass supply chain publications between 1997 and 2012 and categorised them according to the optimisation approach taken, the level of the main decision variables (i.e. strategic or tactical) and the objective function. They found that the majority of publications are focused on an economic objective and out of these 49 publications, only one used a multi-criteria decision analysis framework, 41 used a mathematical programing framework and the remaining seven used heuristics. Of the 41 publications using mathematical programming, 32 used mixed-integer linear programming (MILP) and the remainder used non-linear programming (NLP), linear programming (LP) or integer programming (IP). The reason for these results is that a number of descrete decisions need to be made in biomass supply chains, such as whether or not a transport link exists between two locations and the number of plants to be installed at each location: these are binary and integer decisions, respectively. These descrete decisions combined with the continuous decisions, such as how much biomass to harvest in any location and the rate of operation of each of the technologies, gives rise to a mixed-integer problem (MIP). If all constraints and objective function are linear (MILP), this is a significantly easier problem to solve than an MINLP, where at least one of the constraints or objective function are non-linear. As the BVCM is an MILP formulation, and the majority of other work in this area is also based on mixed-integer linear programming, this literature review will be confined to similar models.

Yue et al. [11] categorised biomass supply chain publications according to type of problem being solved (e.g. supply chain design, planning and operation, multi-objective optimisation etc.) and the application area (e.g. first, second, third generation biofuel technologies, algae to biofuels and biomass to heat and/or power). According to their classification, the BVCM covers all application areas apart from algae to biofuels (which can be added in a future version) and falls under several of their problem definitions: supply chain design and technology selection (where to locate biomass production and technologies, what transport routes are taken), planning and operation (when seasonality is considered), decentralised production (the model will decide whether centralised or decentralised production is most effective), multi-objective optimisation (the objective function is a user-defined weighted sum of several impacts – cost, GHG emissions, energy production etc.) and uncertainties.

Almost all existing models use a multi-echelon structure, many of which break the supply chain into three echelons for biomass harvesting and storage, preprocessing and storage, and energy conversion (cf. Figure 1 in Mafakheri and Nasiri [8]), with transport between each echelon. Note that these are not the normal echelons associated with manufacturing supply chains: these include warehouses, distribution centres and sometimes demand centres and the products are typically unchanged when transported across the echelons; whereas in the bio-energy context, material is converted at each echelon (raw biomass to densified, densified biomass to energy etc.). Čuček et al. [12] followed this approach and also included an echelon for demand locations. Zhang et al. [13] also used three similar echelons, with harvested biomass in the form of raw forest residue and road-side chippings, both of which can be transported independently to the preprocessing sites or directy to integrated sites, which can process raw biomass or preprocessed biomass. In Tittmann et al. [14], the pre-processing echelon is ignored, yielding a simpler model but one that is unable to explore the trade-off between densifying the biomass at source and saving on transport costs, transporting biomass as-received and saving on investments into densification technologies or even converting the biomass on site. Lin et al. [15] focused only on bio-ethanol production using a similar echelon structure and model formulation as Zhang et al. but with only one form of raw biomass. As with Zhang et al., the available biomass is a given input and no account of land use is made. This "strategic model" is complemented by the "tactical model" of Shastri et al. [16], that optimises farm equipment selection, transportation vehicles and biomass harvesting and delivery schedules, given pre-selected farms, processing facilities and biomass flows. More recently, Lin et al. [17] integrated the two models.

Čuček et al. [18] extended their earlier model [12] to a 4-layer multi-echelon model with 12 one-month periods, seasonality, purchase of raw materials and intermediate storage (with losses). Several types of biomass are considered and their availability is determined by the allocation of areas in which to grow them multiplied by yields (independent of location). Areas for the production of food are also considered and it is assumed that the demands for food and biofuels always exceed their production. A number of technologies are included that can convert the biomass to several biofuels and by-products. They present hypothetical studies using regions containing 16 cells (4x4) and 36 cells (6x6).

You et al. [19] focused only on ethanol production from a generic biomass feedstock with fixed availability. Efficiencies for the ethanol and by-product are calculated using Aspen Plus simulations and then used in the 4-layer multi-echelon multi-objective MILP, which considers cost, GHG emissions and social benefits (job creation), to generate Pareto curves. It considers 12 one-month periods and storage of biomass including storage losses. They restrict each bio-refinery site to one technology.

Zhang and Hu [20], Shabani and Sowlati [21] and Santibañez-Aguilar et al. [22] also developed multi-echelon models with 12 one-month intervals. Zhang and Hu presented a 3-echelon model to produce bio-gasoline from corn stover via fast pyrolysis with upgrading to drop-in biofuels; Shabani and Sowlati considered forestry residues to produce electricity; and Santibañez-Aguilar et al. developed a multi-objective model, considering cost, emissions and social impacts, focussing on biofuels from a variety of biomass feedstocks.

Osmani and Zhang [23] also considered a time horizon of one year but with only four seasons. Their model is specific for bio-ethanol from crop residues and woody biomass with uncertainty in the supply, demand and prices. Duarte et al. [24] considered a longer time horizon of 5 one-year periods, from 2013 to 2017. Their model is specific to the conversion of coffee cut stems to bio-ethanol, which is blended with gasoline. van Dyken et al. [25] extended the eTransport model [26] to include biomass and optimised only the operation of the system over 12 one-week periods. The eTransport model, however, can consider both investment and operation but for computational efficiency it decomposes the problem into two parts: an investment model that determines the infrastructure and set of technologies using dynamic programming and an operational model that then minimises the cost of meeting the predefined energy demands using MILP.

There are many examples of steady-state multi-echelon MILP biomass supply chain models. These include: Akgul et al. [27] for bio-ethanol from corn; Balaman and Selim [28] specific for bio-electricity from corn silage and animal manure via anaerobic digestion and biogas CHP; Čuček et al. [12] described above; Elia et al. [29] for generic liquid fuel from crop residues, switchgrass, forest residues, coal and natural gas considering only a generic conversion process; Elia et al. [30] for bio-fuels (gasoline, diesel and jet fuel) from forestry residue; Marvin et al. [31] for production of ethanol from agricultural residues; Zamboni et al. [32, 33] for bio-ethanol from corn with cost minimisation in part 1 and with part 2 extending the objective function to include GHG emissions; and Frombo et al. [34] for production of heat from forestry residues, with the excess heat production being converted to electricity and sold to the grid.

By splitting the supply chain into echelons, the spatial representation of the chain can be simplified: a number of dedicated harvesting, preprocessing and conversion sites are defined. The alternative is to use a full spatial representation of the study area without explicit echelons, typically employing a grid of square cells, where harvesting, preprocessing and conversion can take place in any cell. The advantage of the former approach is that the size of the problem can be easily controlled by limiting the number of sites in each of the echelons; the advantage of the full spatial representation is that, because all elements of the supply chain can be located anywhere, there is less chance of overlooking a potential location for any of the elements. Some multi-echelon models, however, also use a full spatial representation. An example of which is the hydrogen supply chain model developed by Almansoori and Shah [35] for Great Britain, which is divided into 34 square regions of approximately 108 km in length. Echelons for primary manufacturing of hydrogen (from biomass, natural gas, coal and electricity), storage sites and fuelling stations for fuel cell vehicles are included in the model. Dunnett et al. [36] adapted this model to investigate the trade-offs between centralised and decentralised preprocessing of the biomass for a biomass-to-ethanol supply chain. Zamboni et al. [32] also applied the methodology of Almansoori and Shah to model bio-ethanol supply chains in northern Italy, with various modifications of this model being made up to the work of Akgul et al. [27]. Giarola et al. [37] also focused on biomass supply chains in northern Italy, this time considering both first and second generation bio-refineries using a multi-objective optimisation with a more detailed finance model.

There are also models that consider temporal variation but do not account for the spatial distribution of system properties. For example, Dunnett et al. [38] considered the operation of a biomass-to-heat supply chain by utilising the STN (State-Task Network) representation introduced by Kondili et al. [39], to model the relationship between the different materials in the supply chain and the processes that transform them, and extending their scheduling formulation [39, 40] to be applicable to bioenergy supply chains. Zamboni et al. [41] also developed a multi-period (10 one-year periods) non-spatial model specific to ethanol production from wheat. The model determines the production of wheat based on the nitrogen dosage level (from a set of discrete levels) and the end use options for DDGS (animal feed and CHP fuel).

The models mentioned above (except [38]) are based on multi-echelon representation which can be inflexible if the pathways are fixed, as they usually are, i.e. there are no pathways in which an end-product can be used as an input to a technology. For example, bio-SNG can be an end-product from a gasification technology and at the same time an input to another technology such as a bio-SNG boiler (i.e. bio-SNG can be an intermediate). One exception is the work of Kim et al. [42], who recognised that some intermediates can also be sold. They resolved this issue by including an additional echelon: the "conversion1" plants produce intermediates that can either be sold or sent to "conversion2" plants that will convert them to gasoline or bio-diesel. Although this resolves the issue to some extent, it is not a complete solution because further echelons would need to be added if more than one processing step is required to convert an intermediate product to a final product, which would be quite cumbersome, and the most general situation of being able to convert one resource to another *and back again* would not be possible (such a situtation may arise, for example, when considering both liquid and gaseous bio-hydrogen, which should be able to interconvert from one form to another in order for the model to decide the form in which to transport and store the hydrogen at different stages of the chain). Another exception is the model of Čuček et al. [18], already described above, which is also much more flexible: there are four echelons, L1 to L4, and the pathways are allowed to recycle within layers L2 (collection and pre-processing) and L3 (bio-refineries) as well as allowing material to be transported back from L3 to L2.

An alternative approach, which can represent the different pathways in a more flexible manner, is to use a similar representation to the STN of Kondili et al. [39], in which the states are replaced by resources and the tasks by technologies, hence becoming a Resource-Technology Network (RTN). A model based on this approach was developed by N. J. Samsatli as part of the BP Urban Energy Systems project at Imperial College [43]. The aggregate formulation (peak and average periods) has been used in a number of publications [44, 45, 46]. One advantage of RTN formulations over multi-echelon formulations is that in the former, resources can be stored at any point in the pathway, whereas in the latter, storage constraints have to be written explicitly for each echelon. More details about the RTN are given in Section 4.

Overall, the models described above are somewhat limited in a number of ways. Most are restricted to a particular purpose: many are concerned with biofuels [18, 19, 20, 22, 23, 24, 27, 29, 30, 31, 32, 33, 36, 37, 41] (most of these considered only bio-ethanol), few (separately) considered electricity [21, 28], heat [34, 38] or hydrogen [35], and none considered multiple end-vectors. Apart from [18, 38, 43], all of the above models have fixed pathways due to their multi-echelon nature. Often the biomass is restricted to one or two varieties or is just treated as a lumped resource; usually the land area is not considered and the biomass availability is a fixed parameter; in the few cases where biomass yields are considered and land areas allocated, the effects of climate and soil conditions on yield are generally not explicitly considered (e.g. Čuček et al. [18]). On the technology side, a limited number of technologies are considered and while some approaches consider staged investment, few consider capacity retirements (which will be important for longer planning periods, such as that in the BVCM, which includes data upto and including the 2050s). A number of models only allow one technology per location, which may not be optimal.

There is a need for a comprehensive and flexible model that can be used to optimise biomass value chains by simultaneouslty considering all end vectors (e.g. heat, electricity, liquid and gaseous fuels) using a variety of feedstock such as energy crops, conventional crops and forestry resources on a national scale with a sufficiently detailed spatial representation in order to determine the optimal use of land, transport of resources, location of technologies, emissions etc. It is also important to consider the future energy mix *throughout* the transition towards a lower-carbon economy and to determine what role bioenergy can play: i.e. determining how to progress from the current energy system to the future one is just as important as determining what that future energy system should be. Hence a pathway model is required that can account for climate predictions and their impact on biomass yields (also on a spatial level). A model that can determine the most effective biomass value chains to achieve specified targets (e.g. a low cost and low carbon bioenergy system) is an essential element in assessing the role of biomass in mitigating climate change. The BVCM was developed with the aim of addressing all of these issues and is described in the following sections.

3. Problem statement

Since the BVCM is principally a bioenergy pathway model, it must be able to determine what crops to grow (and where to grow them) in each decade and what technologies to use to convert the crops to end-use energy vectors given any set of targets for bioenergy production, which may be overall whole-system energy targets, targets for each energy vector and even targets at the regional level. The pathway element of the model refers to both the ability to determine the production pathway from crops to bioenergy, i.e. which technologies are used in any particular decade, and also the pathway taken over time from the initial state to the final energy system, which is determined by investing in technologies and changing land use each decade. The energy systems and the pathways between them are determined in order to minimise a combination of whole-system cost and environmental impact (collectively referred to as impacts).

A more specific, but not exhaustive, definition of the problem is:

- Given:
    - A spatial representation of the region to be considered (e.g. the whole of the UK) that consists of a number of cells characterised by:
        - The total area of each cell
        - The coordinates of the centroid of each cell (which are used to calculate the distance between each pair of cells)
        - The area of each type of land cover within each cell that is available for growing energy crops
        - An existing set of transport infrastructures
    - A set of biomass feedstocks characterised by:
        - A yield potential (odt[1]/ha/yr) for each cell, in each decade for a number of climate and technological scenarios
        - The fraction of the annual yield occurring in each season
        - Impacts (cost, GHG emissions etc.) for planting, growing and harvesting
        - Information on storage: capital and operating impacts, maximum number of seasons over which a resource can be stored, fraction of stored amount that is lost
    - A set of technologies capable of converting the biomass feedstocks to final energy vectors, via any number of intermediates, with the properties (for each decade, with the exception of existing capacity):
        - Minimum and maximum capacity of a single plant, along with availability in hours per year

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[1] Oven dry tonne

        - Whether a technology is available for investment in a particular decade (to account for technologies that are not yet available/sufficiently developed or technologies that will be phased out in the future)
        - Efficiency of each technology (defined in terms of conversion factors from an input set of resources to an output set of resource)
        - Capital and operating impacts (fixed and variable)
        - Operating and economic lifetimes. The former is the physical lifespan of the technology and the latter is the number of years over which the investment costs are annualised
        - Maximum number of plants that can be built each year for the region of interest (build rate)
        - Existing capacity for each cell (the initial state of the energy system) and the year the existing capacity retires

- Determine:
    - The land area in each cell, for each decade, allocated to the production of each bioenergy feedstock (crop)
    - The amount of each resource being stored in each cell, in each season and each decade
    - The rate of transport of each resource between all cells, in each season and in each decade
    - The rate of import of resources to each cell, in each season and in each decade
    - The number, capacity and location of each technology investment in each decade
    - The rate of operation of each technology in each cell and in each season and each decade

- Subject to:
    - Minimum energy production constraints
        - For each end-use vector in each decade
        - For total energy
    - Satisfaction of local demands for resources
        - E.g. demand for heat in each cell in each decade must be met

- In order to:
    - Minimise/maximise an objective function that may include any or all of the following
        - Cost
        - Profit (defined as revenue minus cost)
        - GHG emissions
        - Total energy production
        - Total exergy production

4. Model structure

In the BVCM, energy pathways are represented by Resource-Technology Networks (RTNs), comprising resources, technologies, technology modes and their interconnections. Resources are any material or energy resource, e.g. primary biomass feedstocks, intermediates, end-vectors, by-products and wastes. Technologies represent a physical facility or device capable of performing a specific set of technology modes. A technology mode represents the processes occurring in a technology that convert a given set of input resources to a different set of output resources. Each technology can perform a number of different technology modes, for example a power plant may have a number of modes representing the different biomass feeds that the plant can use to generate power or a CHP plant can produce different different ratios of heat to power and these can be represented by different technology modes. All of these resources, technologies and modes, with their interconnections, represent all of the possible energy pathways in the system. Figure 2 shows an example of an RTN diagram with resources represented by circles, technology modes represented by rectangles with solid lines and technologies represented by rectangles with dashed lines. The diagram demonstrates the two integral characteristics of an RTN: the capability of technologies to perform multiple modes and the various alternative network pathways (i.e. resource-technology-mode configurations) that generate the same output state. These characteristics enable the flexibility of the BVCM to consider a wide range of different feedstocks and technologies with multiple operating modes to generate different energy vectors such as heat, electricity, transport fuels, hydrogen and bio-methane.

The RTN is based on the State Task Network (STN) introduced by Kondili et al. [39] in order to represent batch process recipes in the context of scheduling of multi-purpose batch chemical plants.

In the example in Figure 2, only a few resources, technologies and modes are shown but in the actual model there are many resources and technologies. In the current BVCM version, there are 93 resources and 69 distinct technologies, most of which are available at three different scales (small, medium and large) with multiple modes – the number of combinations of techology, size and mode is well in excess of 1200. The combinatorial nature of the links between resources, technologies and modes results in a very large number of possible bioenergy chains. Figure 3 shows all of the pathways for SRC willow; similar diagrams exist for each of the feedstocks but are not shown in the interest of space.

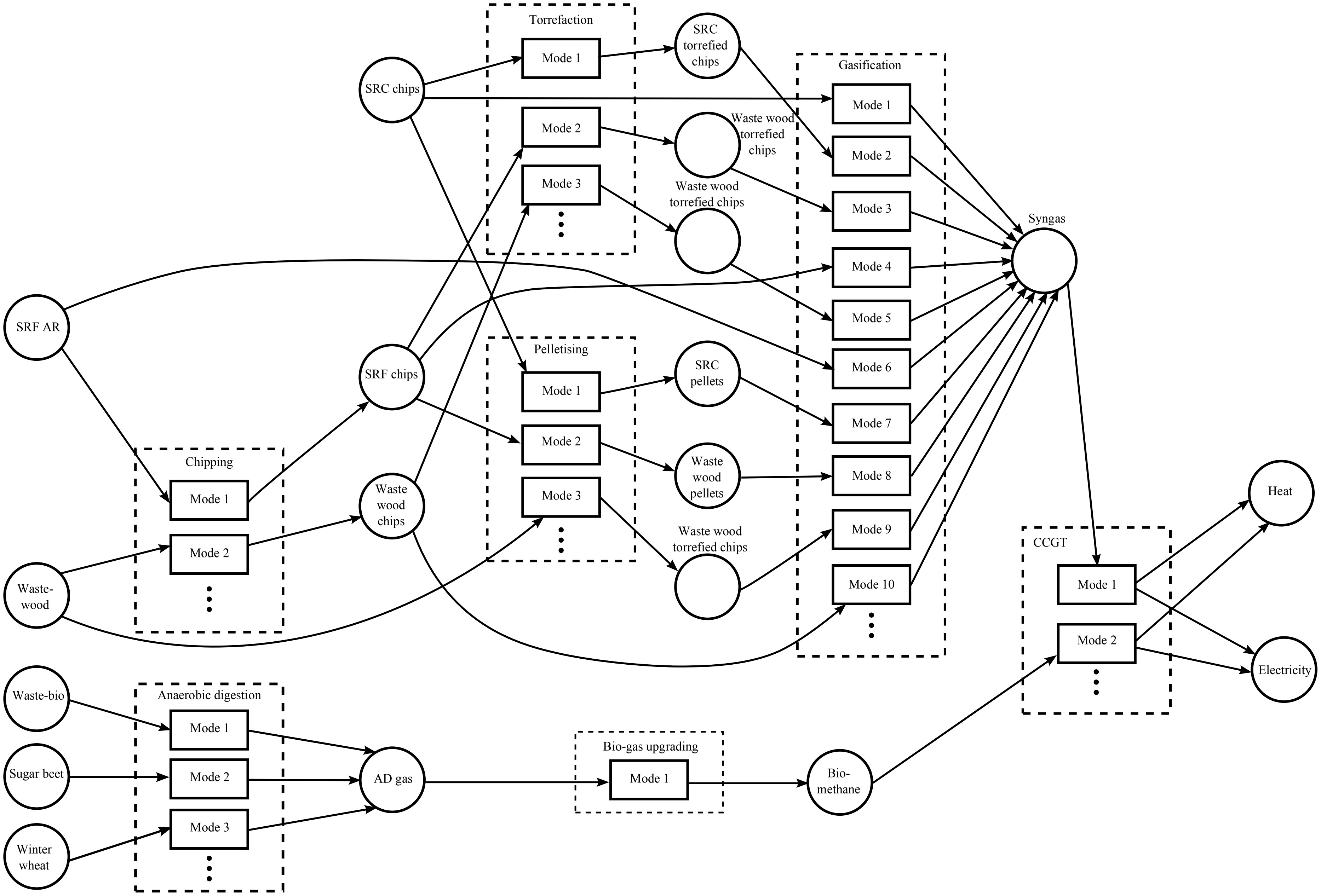


Figure 2: A simple example of an RTN structure in the BVCM with circles representing resources, rectangles with dashed lines representing technologies and rectangles with solid lines representing technology modes.

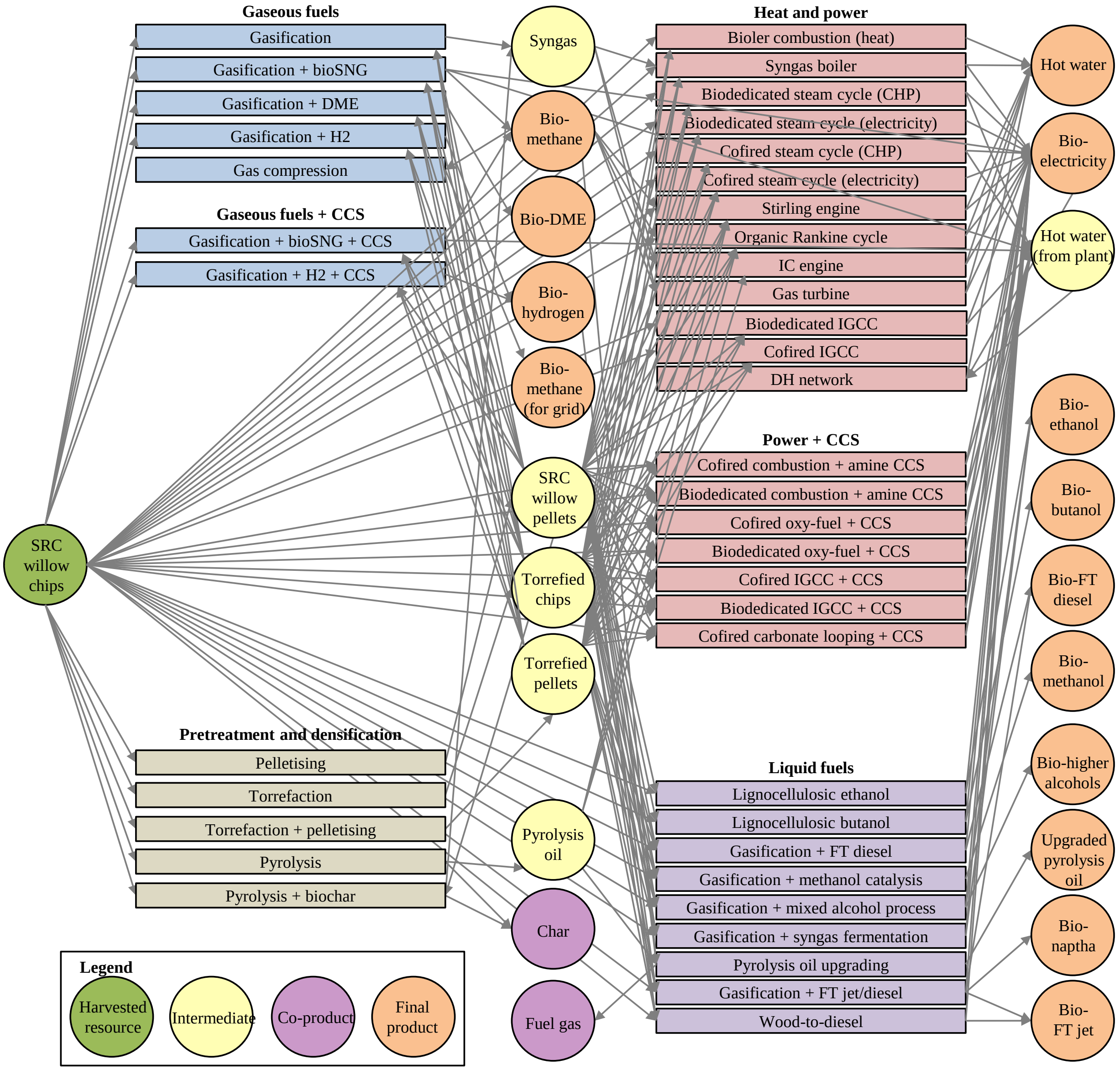


Figure 3: Resource-Technology Network showing all of the possible pathways for SRC willow (similar diagrams exist for each of the primary feedstocks)

The subsequent subsections describe the elements in the BVCM: temporal and spatial representation, resources, technologies, infrastructure for transport of resources and the objective function.

4.1. Time

The BVCM considers the strategic development of the biomass value chain from the 2010s to the 2050s (although this can easily be extended beyond). Time is represented on two levels: decadal and seasonal. Each decade d is an element of the set $\mathbb{D}$ of decades. Investment decisions, land-use changes, technology improvements and yield enhancements take place on a decadal basis. For example the annual yields of any crop may be different from one decade to the next but are assumed to be the same in each year within that decade.

The seasonal level accounts for the variation of biomass production throughout the year. Each season t is an element of the set $\mathbb{T}$ of seasons, which may include only one season (i.e. the whole year – therefore, in this simple case, seasonality is not considered), two seasons (winter/spring and summer/autumn) or all four seasons. When more than one season is considered, storage is modelled to account for the intermittent supply of crops.

4.2. Space

In general, the region of interest can be divided into a number of cells, which may be of any shape and size. Each cell represents a geographical location and may have a dynamic demand for various resources. A cell may host different technologies for converting and storing resources. It may also contain infrastructure connections with other cells for transport of resources and external connections for import and export of resources. Examples of information that may vary with location include demand, resource availability, land cover and built environment. Hence, data for these properties must be given for each cell, c, in the set of cells $\mathbb{C}$.

The BVCM is currently configured for the UK by dividing it into 157 square cells of length 50 km. This spatial resolution is sufficiently high to account for regional variations in biomass yield, costs and GHG emissions and to allow an appropriately detailed representation of transport networks (e.g. the trade-off between converting biomass to energy in-situ versus densifying the biomass and transporting it to a more centralised conversion plant) without being so high that the model becomes intractable.

4.2.1. Land area allocation

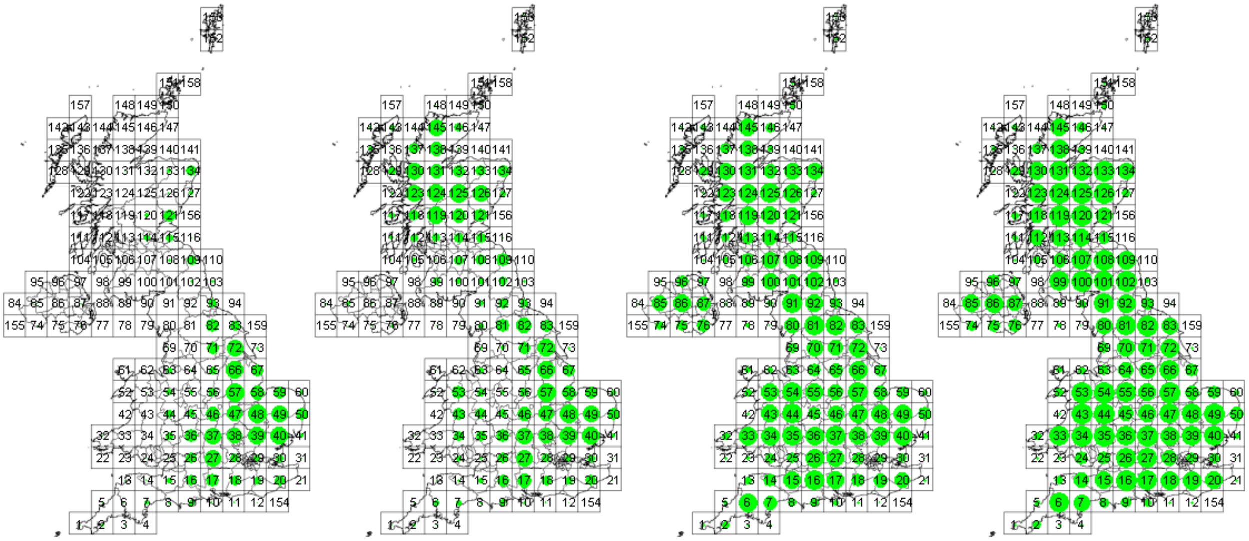
Through a number of constraints, the model provides flexibility in defining different scenarios for land available for bioenergy production (the remainder being available for other land uses, e.g. food production).

The BVCM categorises land use into four "levels": each land level is represented in the model by the index k, which is an element of the set $\mathbb{K}$ of land levels. Table 1 defines the BVCM categories in terms of the land classifications in the CORINE Land Cover (CLC) 2006 map [47]. The area under each land category was obtained by summing the corresponding categories in the CLC map. Figure 4 shows the average available area in each land level plotted on a 50 km × 50 km grid map. Against these categories, a "level of aggression" in the potential allocation of different existing land to bioenergy feedstock production can be defined in two ways:

1. The specification of the overall level (from 1 to 4 and on a cumulative basis, i.e. Level 2 includes Level 1, Level 3 includes Levels 1 and 2 etc.)
2. The specification of the fraction of the land in each level that is available for bioenergy.

Table 1: BVCM land categories and the corresponding classification in CORINE Land Cover 2006 map

BVCM	CORINE Land Cover Map
Level 1: easy and established technology	2.1 Arable land 2.4 Heterogeneous agricultural areas
Level 2: pioneering plant establisment	BVCM Level 1 plus: 3.2 Shrub and/or herbaceous vegetation association 3.3 Open spaces with little or no vegetation
Level 3: challenging from a techno-economic and ecological aspect	BVCM Level 2 plus: 2.2 Permanent crops 2.3 Pastures
Level 4: last resort	BVCM Level 3 plus: 3.1 Forests 1.4 Artificial non-agricultural vegetated areas



Level 1 Level 2 Level 3 Level 4

Figure 4: Average available area in each land level (defined in Table 1), shown on a 50 km × 50 km grid map, on a cumulative basis (i.e. Level 2 includes Level 1, Level 3 includes Levels 1 and 2 etc.). The total areas available are: Level 1 ≈ 7.60 Mha; Level 2 ≈ 13.15 Mha; Level 3 ≈ 19.99 Mha; Level 4 ≈ 22.28 Mha.

4.2.2. Yield potentials

In the BVCM, the yield potentials of each biomass crop $r \in \mathbb{R}^B$, were calculated at a 1 km×1 km level based on the "low" and "medium" scenario from the UK Climate Projections 2009 (UKCP09) [48] with three technology improvement pathways ("best" , "business as usual" and "worst"), depending on a series of factors such as on-farm improvements and how the gap between theoretical yields and on farm attainable yields evolves, to generate a total of six yield scenarios for each biomass resource. Each yield scenario is an element s of the set $\mathbb{S}$ of yield scenarios.

These were then aggregated to the 50 km level while excluding yields in 1 km level cells that fall in any of the following (user-selected) categories (based on the classification given in Lovett et al. [49]):

- None
- Basic 3w: excludes land areas with elevation greater than 250 m, slope greater than 15% and topsoil organic carbon greater than 30%

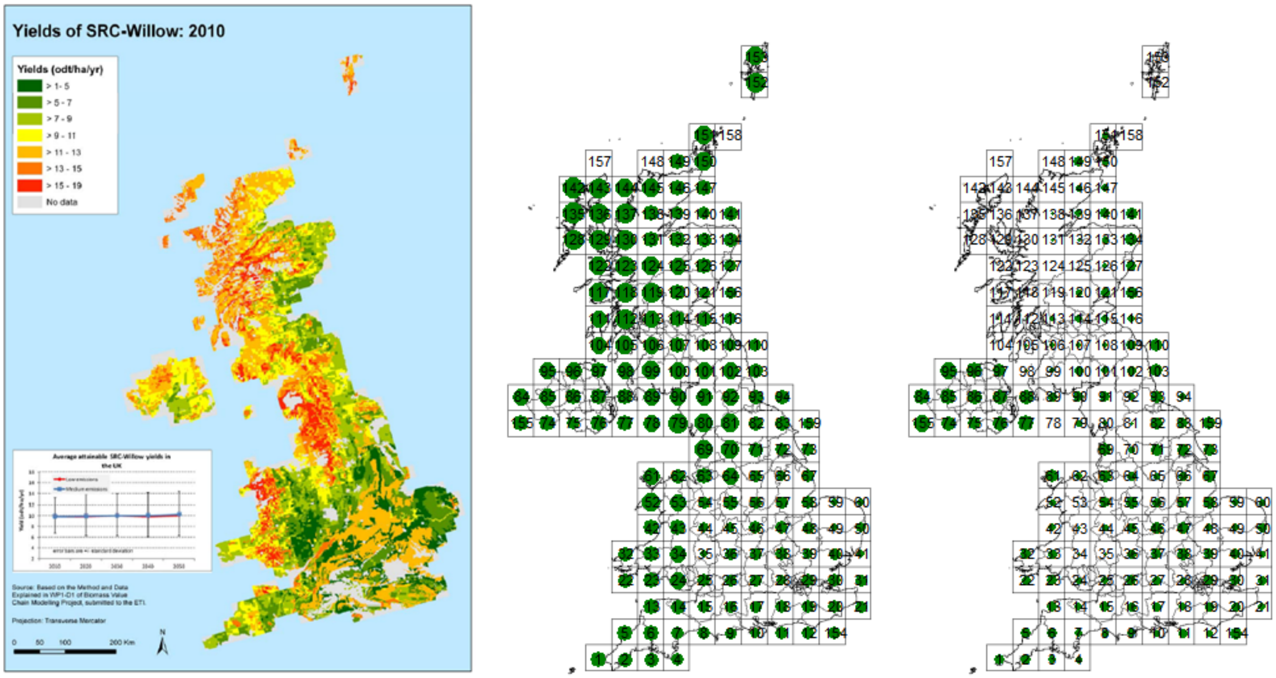
- UKERC 7w: Basic 3w plus 7 additional constraint masks to exclude urban areas/roads/rivers, parks, scheduled monuments/world heritage sites, designated areas, cultural heritage areas and natural and semi-natural habitats

- UKERC 7: UKERC 7w and also excluding existing woodland

- UKERC 9w: UKERC 7w constraint plus areas with high naturalness score (>75% or >65% inside national parks/areas of outstanding natural beauty)

- UKERC 9: UKERC 9w and also excluding existing woodland.

Example yield maps at the 1-km level (unfiltered) and 50-km level are shown in Figure 5.



(a) (b) (c)

Figure 5: Yield potential of SRC willow: (a) 1-km level unfiltered (data provided by the University of Southampton and GIS mapping by EIFER); (b) 50-km level unfiltered; and (c) 50-km level filtered using UKERC 9 constraints.

4.3. Resources

Resources refer to any distinct material or energy stream considered in the value chain: biomass feedstocks, intermediates, end-products, co-products and wastes. A resource can be consumed or produced by a technology, transported from one cell to another, imported from abroad to specific locations (e.g. "ports") and stored when seasonality is considered. Some resources, e.g. electricity and gas, can be purchased from a non-biomass related infrastructure (i.e. the "grid"), e.g. to cover times when biomass production is not sufficient to meet demands; similarly, some resources can be sold to generate revenue. Each resource is characterised by a set of properties (e.g. lower heating value, density and composition). Although for biomass feedstocks these properties may depend on the location and decade in which they are grown, the properties of all reasources are assumed to be independent of location and time.

In the model, each resource is an element r of the set $\mathbb{R}$ of resources. Each resource can be a member of a number of different subsets, which include the following:

$\mathbb{R}^B$ is the set of biomass feedstocks, e.g. Miscanthus, which are resources that can be grown on particular areas of land

$\mathbb{R}^{Ro}$ is the subset of biomass feedstocks that rotate with winter wheat

$\mathbb{R}^F$ is the set of forestry resources, which is a subset of $\mathbb{R}^B$, with specific rules for planting and harvesting to account for the longer time required to produce biomass

$\mathbb{R}^W$ is the set of waste resources that can be utilised in waste-to-energy technologies, such as municipal solid waste (MSW)

$\mathbb{R}^{GD}$ is the set of resources whose demands can be assumed to be independent of location because there exists an infrastructure to transport them easily and cheaply, e.g. electricity and natural gas

$\mathbb{R}^{LD}$ is the set of resources with location-dependent demands, e.g. heat

$\mathbb{R}^Q$ is the set of resources that can be transported from one cell to another

$\mathbb{R}^I$ is the set of resources that can be imported

$\mathbb{R}^S$ is the set of resources that can be stored

$\mathbb{R}^{TF}$ is the set of transport fuels

The user is able to specify the elements included in each of the subsets above, in order to define a specific scenario for optimisation.

The BVCM distinguishes between "green" and "brown" resources by explicitly defining in the Technology Database a mode of a technology (discussed in Section 4.4) that consumes or produces these resources. "Green" resources are end products from a bio-technology; they may have demands and their production contributes towards the bioenergy production target that the user can set. "Brown" resources, on the other other hand, are produced by conventional (e.g. fossil) technologies; in the BVCM, they do not have demands that count towards the bioenergy target. These "brown" resources are present so that the model can choose to use e.g. grid electricity or natural gas (which may be needed to operate the technologies) instead of building technologies to produce them from biomass.

In addition, the resources are classified into a number of families with similar properties. These are used to apply specific constraints to groups of resources that belong to the same family and also to perform sensitivity analyses at the family level. It also allows the resources to be grouped conveniently in the GUI. The resource families are:

- Arable crops, e.g. winter wheat, oilseed rape, sugar beet
- Energy crops, e.g. Miscanthus, short rotation coppice (SRC) willow
- Forestry, e.g. short rotation forestry (SRF), long rotation forestry (LRF)
- Wastes, e.g. waste-wood, waste-bio (includes food wastes)

- Intermediates, e.g. waste wood chips, pyrolysis oil, syngas, AD gas
- Co-products, e.g. DDGS, digestate, glycerine, sugar beet sugar and pulp
- Final vectors, e.g. bio-electricity, bio-heat, bio-methane, bio-ethanol, bio-hydrogen
- Miscellaneous, e.g. chemicals, such as hexane, urea and sulphuric acid, that are used as inputs to some technologies

4.4. Technologies

A technology represents any type of plant that can convert one or more input resources to one or more output resources, e.g. a power plant or a gasification plant. Most bioenergy technologies can process multiple feedstocks or produce multiple outputs: each distinct set of input resources that can be processed or output resources that can be produced by the same physical plant represents a *mode* of that technology. Some examples of technologies with multiple modes are:

- the pelletising technology, which can process SRC willow chips into SRC willow pellets, winter wheat straw into winter wheat pellets, SRF into SRF pellets and so on (as can be seen in Figure 2);
- the boiler combustion technology, which can convert a number of feedstocks, such as biomass (as received, chips or pellets) and waste wood into heat;
- the sugar bio-refinery technology, which can convert sugar beet into a number of end-products and by-products: bio-ethanol, bio-electricity, sugar beet sugar and sugar beet pulp.

The technologies are grouped into 12 families in order to allow a batch of similar technologies to be conveniently included or excluded in a scenario and also to be able to apply constraints and perform sensitivity analysis on a family level rather than on an individual level. The technology families are:

- Densification, e.g. chipping, pelletising, oil extraction
- Thermal pre-treatment, e.g. torrefaction, pyrolysis, mechanical biological treatment (MBT)
- Anaerobic digestion, e.g. anaerobic digestion, biogas upgrading
- Gasification , e.g. gasification (generic), gasification (bioSNG), gasification (H_2)
- 1G biofuels, e.g. 1G bio-ethanol, 1G bio-diesel, 1G bio-butanol
- 2G biofuels, e.g. lignocellulosic bio-ethanol, lignocellulosic bio-butanol, gasification (FT diesel), lignocellulosic bio-refinery (Inbicon)
- Heating, e.g. boiler combustion, syngas boiler, district heating (DH) network
- CHP onsite, e.g. Stirling engine, organic Rankine cycle, internal combustion engine
- CHP for district heating, e.g. gas turbine, steam cycle, IGCC
- Power, e.g. CCGT, plasma gasification, incineration, pyroliquid bio-refinery (Ensyn)
- Power + CCS, e.g. oxyfuel, chemical looping, combustion + amine
- Gaseous + CCS, e.g. gasification (bioSNG) + CCS, gasification (H_2) + CCS

4.4.1. Technology efficiency

A technology can operate in multiple modes using different inputs and outputs. The input or output upon which the maximum capacity of the technology is based is referred to as the main input or the main output. The maximum capacity for the technology is independent of the mode but always refers to the main input of each mode or the main output of each mode. For example, the maximum capacity of a CCGT plant would be based on the main output, bio-electricity, while the inputs for each mode might be syngas, natural gas or bio-methane.

The efficiency of each mode of a technology is represented by specifying a coefficient for each resource associated with that technology mode. When a technology runs at a particular rate, the rate of production or consumption of a resource is the conversion factor multiplied by the rate of operation of the technology. The conversion factors for each technology are normalised based on how the maximum capacity of the technology is specified. For example, if the capacity is specified per unit of main input, then the conversion factors are scaled so that the conversion factor for the main input is -1. This means that if a technology is running at a particular rate, then the rate of production of the main input is -1 multiplied by the rate of operation of the technology (i.e. it is consumed). Similarly, the rate is multiplied by the conversion factors of other input and output resources to obtain their rates of production. Conversely, if the capacity is specified per unit of main output, then the conversion factors are scaled to make the conversion factor of the main output equal to 1. Therefore, the rate of production of main output is the rate of operation of the technology multiplied by 1, and the rates of production of other inputs and outputs are the rate of operation of the technology multiplied by the respective conversion factors.

For co-fired technologies, the conversion factors represent the total output of a resource from the technology, e.g. the rate of electricity production from a co-fired plant when it is fed with all of its inputs - e.g. coal and biomass. The co-firing fraction represents the part of that production rate that is due to the biomass and therefore the actual rate of output produced from biomass is the cofiring fraction multiplied by the conversion factor of the main output multiplied by the production rate of the technology.

4.5. Resource transport

Transport modes are represented by the index $l \in \mathbb{L}$, of which four are considered in the BVCM: $\mathbb{L} \equiv \{\text{road, rail, inland waterways, close coastal shipping}\}$. In the BVCM, transport between cells is limited to adjacent cells (von Neumann neighbourhood [50]). Transport over longer distances is achieved by making several neighbour-to-neighbour transfers along the route between the source and destination cell. The road and rail networks were modelled using OpenStreetMap [51] while the distribution of inland waterways (canals and navigable rivers) was modelled using WaterWaysWorld [52]. The feasible transport connections, $\Gamma_{cc'l}$, were determined from these maps, an example of which is shown in Figure 6 for barge transport. The meshing of the road network with the BVCM cellular representation gives an average tortuosity per cell, which was then used to convert straight line distances to expected travel distances. The road network tortuosities and railway length are illustrated in Figures 7(a) and (b).

With respect to coastal shipping, only a single type of ship carrier is considered as ship emissions do not change much with scale. Unlike the inland transport modes, ship transport is not restricted to adjacent cells and instead transport from one port to any other port is allowed. The existing UK major ports were identified (see Figure 7(d)) and represented in the BVCM by a set $\mathbb{C}^{\text{ship}} \subset \mathbb{C}$.

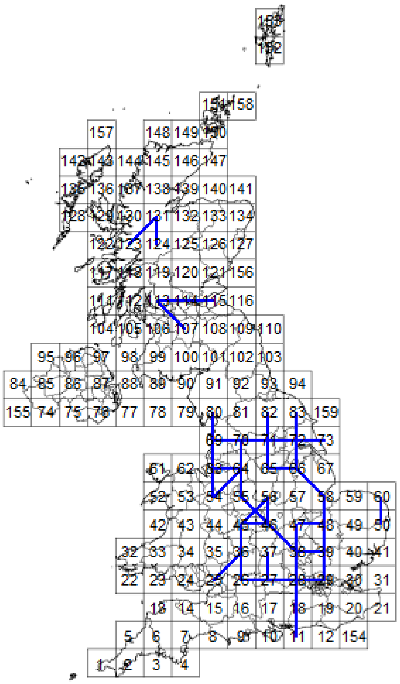
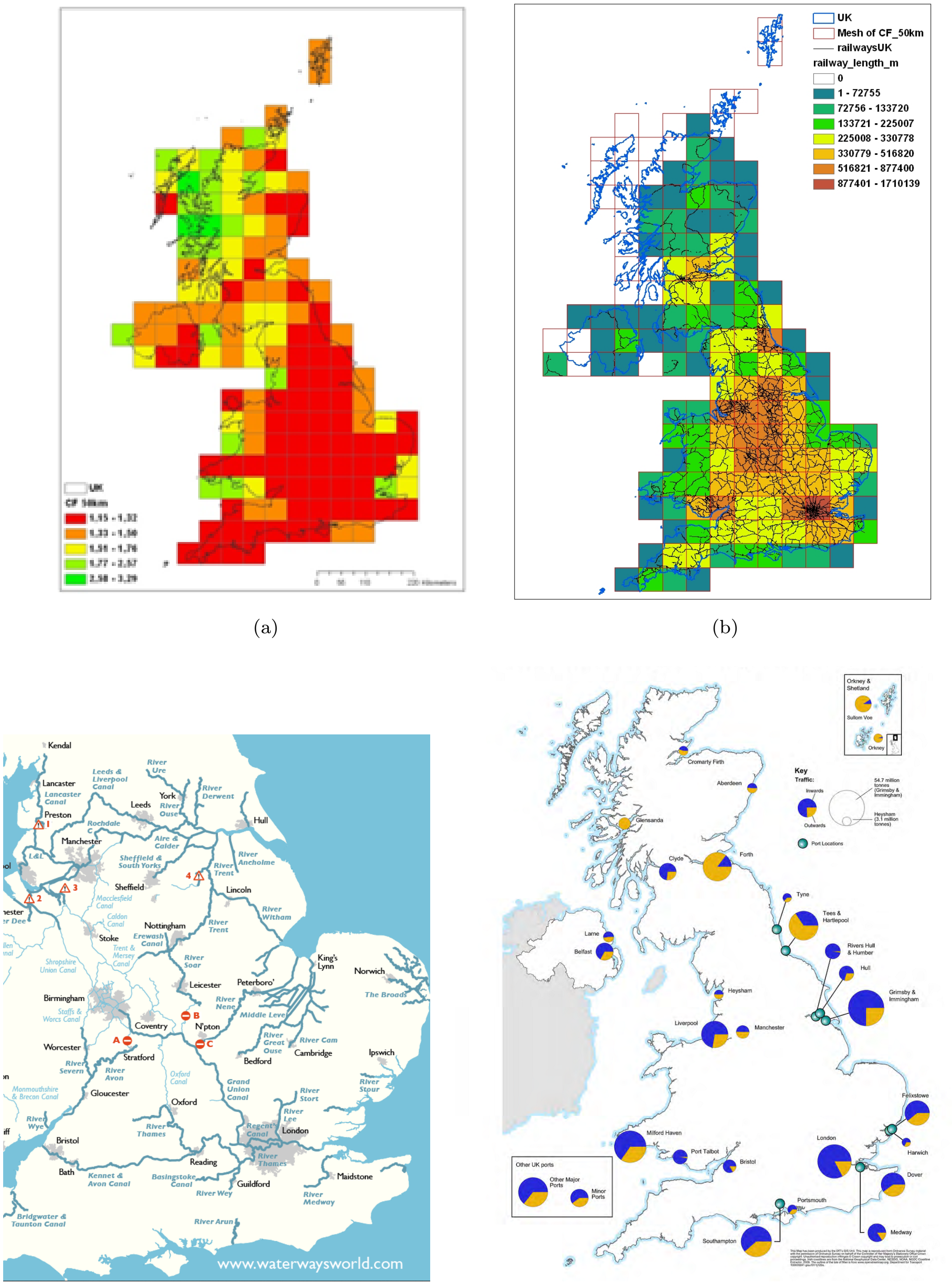


Figure 6: Representation of feasible inland waterways transport connections in the BVCM. (The user may double click on any link to disable/enable its use in an optimisation run.)

4.6. Objective function

All of the activities associated with the provision of energy through the biomass value chain give rise to a number of financial and environmental impacts. For example, planting, growing and harvesting of energy crops incur a cost and the use of machinery also results in CO_2 emissions; building and operating technologies for converting resources also obviously incur capital and operating costs, along with other environmental impacts. Whether the impacts are cost, GHG emissions, air quality indicators or anything else, they all arise in similar ways from the activities of the biomass value chain: they are a function of one or more decision variables in the problem, e.g. the amount of capacity of a technology installed, the rate of operation of a technology, the rate of transport of a resource and so on. Each type of impact, i, is an element of the set of impacts $\mathbb{I}$. Currently, there are three impacts in the BVCM: cost, CO_2 emissions and other GHG emissions; and it is straight forward to include addional impacts such as life-cycle assessment indicators, air quality indicators and so on. Parameters then define how much each impact is increased (or decreased) by each activity in the value chain and the value of each impact, i, is calculated for each group of related activities: capital impact, operating impact, transport impact etc.

The objective function is therefore the weighted sum over all impacts of the total value chain impact (capital + operating + transport + ... impacts). The values of the weights are user defined and therefore allow a variety of objective function scenarios to be considered: minimise cost, minimise GHG emissions or any combination of the impacts. The weights can also be calculated automatically using CO_2 prices to convert the environmental impacts into monetary impacts (cost). The objective function also includes other indicators of the value chain performance: total energy production and total exergy production, in terms of the user-defined end-use vectors, with appropriate user-specified weights. This then allows maximisation of total energy production as an objective function (actually, minimising the negative of the total energy production). Full details of the objective function formulation are given in Section 5.12.



(a) (b) (c) (d)

Figure 7: Some properties of the UK transport networks: (a) road tortuosity (provided by EIFER); (b) railway length (provided by EIFER); (c) inland waterways; (d) traffic through UK ports [53].

5. Mathematical formulation

The starting point for the model is an energy balance (here, referred to as a resource balance, since resources represent both energy carriers and other materials). Since there can potentially be a large number of resources, writing a detailed resource balance, in each cell for every time period, for every resource would result in a very large problem. This is because the transport of each resource would need to be tracked between each pair of cells. However, there are some resources for which the transport infrastructure already exists and the difficulty and cost of transport is so low that a detailed account of their transport need not be made. Resources such as electricity and natural gas are examples of resources that can be transported easily and cheaply. Therefore, for computational efficiency, resources are divided into two subsets: "global demand resources", $\mathbb{R}^{GD}$, and "balance resources", $\mathbb{R}^{\text{bal}}$. The former set represent resources such as electricity and natural gas, which can be transported easily and therefore it is only necessary to consider their "global" demand (i.e. the total demand for the area of interest); the latter represents all resources that require a more detailed account of transport and therefore should be included in the detailed resource balance. The set of balance resources, $\mathbb{R}^{\text{bal}}$, is therefore defined as:

$$\mathbb{R}^{\text{bal}} = \mathbb{R} - \mathbb{R}^{GD} \tag{1}$$

Two resource balances are provided for the local demand resources. If the resource cannot be stored *or* if only one season is being considered (i.e. there is no seasonality), then the following resource balance is written:

$$\begin{aligned}
&\left(\frac{B_{rcd}|_{r\in\mathbb{R}^B} + B_{cd}^{\text{straw}}|_{r=\text{Straw}}}{1-\mu_r}\right)\left(f_{rt}^{\prime B}\frac{\nu^{dpY}}{\nu_t^{dps}}\right) + WF_{cdt}x_{rcd}|_{r\in\mathbb{R}^W} + WAC_{cdt}|_{r=\text{WasteAll}} \\
&\qquad + \sum_{j\in\mathbb{J}}\sum_{m=1}^{M_j}\mathscr{P}_{jmcdt}\alpha_{jmdr}\gamma_{jmdr} + \sum_{c'|\Gamma_{c'cl}=1}\sum_{l\in\mathbb{L}}Q_{rc'cldt}|_{r\in\mathbb{R}^Q} - \sum_{c'|\Gamma_{cc'l}=1}\sum_{l\in\mathbb{L}}Q_{rcc'ldt}|_{r\in\mathbb{R}^Q} \\
&-D_{rcdt} + RP_{rcdt} - RS_{rcdt}^{L} + RI_{rcdt}|_{r\in\mathbb{R}^I\wedge c\in\mathbb{C}^{\text{ship}}} = RD_{rcdt} \qquad \forall r\in\mathbb{R}^{\text{bal}} - \mathbb{R}^S,\ c\in\mathbb{C},\ d\in\mathbb{D},\ t\in\mathbb{T}
\end{aligned} \tag{2}$$

For resources that can be stored, the following resource balances are written when seasonality is considered (either 2 or 4 seasons are modelled):

$$\begin{aligned}
&\left(\frac{B_{rcd}|_{r\in\mathbb{R}^B} + B_{cd}^{\text{straw}}|_{r=\text{Straw}}}{1-\mu_r}\right)\left(f_{rt}^{\prime B}\frac{\nu^{dpY}}{\nu_t^{dps}}\right) + WF_{cdt}x_{rcd}|_{r\in\mathbb{R}^W} + WAC_{cdt}|_{r=\text{WasteAll}} \\
&\qquad + \sum_{j\in\mathbb{J}}\sum_{m=1}^{M_j}\mathscr{P}_{jmcdt}\alpha_{jmdr}\gamma_{jmdr} + \sum_{c'|\Gamma_{c'cl}=1}\sum_{l\in\mathbb{L}}Q_{rc'cldt}|_{r\in\mathbb{R}^Q} - \sum_{c'|\Gamma_{cc'l}=1}\sum_{l\in\mathbb{L}}Q_{rcc'ldt}|_{r\in\mathbb{R}^Q} \\
&-D_{rcdt} + RP_{rcdt} - RS_{rcdt}^{L} + RI_{rcdt}|_{r\in\mathbb{R}^I\wedge c\in\mathbb{C}^{\text{ship}}} = \left(\frac{I_{rcdt} - I_{rcdN^{\mathbb{T}}} + S_{rcdt}^{loss}}{\mathscr{R}_t^I}\right) + RD_{rcdt} \\
&\qquad\qquad \forall r\in\mathbb{R}^S,\ c\in\mathbb{C},\ d\in\mathbb{D},\ t=1
\end{aligned} \tag{3}$$

$$\begin{gathered}
\left(\frac{B_{rcd}|_{r\in\mathbb{R}^B} + B^{\text{straw}}_{cd}|_{r=\text{Straw}}}{1-\mu_r}\right)\left(f'^{B}_{rt}\frac{\nu^{dpY}}{\nu^{dps}_t}\right) + WF_{cdt}x_{rcd}|_{r\in\mathbb{R}^W} + WAC_{cdt}|_{r=\text{WasteAll}} \\
+\sum_{j\in\mathbb{J}}\sum_{m=1}^{M_j}\mathscr{P}_{jmcdt}\alpha_{jmdr}\gamma_{jmdr} + \sum_{c'|\Gamma_{c'cl}=1}\sum_{l\in\mathbb{L}} Q_{rc'cldt}|_{r\in\mathbb{R}^Q} - \sum_{c'|\Gamma_{cc'l}=1}\sum_{l\in\mathbb{L}} Q_{rcc'ldt}|_{r\in\mathbb{R}^Q} \\
-D_{rcdt} + RP_{rcdt} - RS^L_{rcdt} + RI_{rcdt}|_{r\in\mathbb{R}^I\wedge c\in\mathbb{C}^{\text{ship}}} = \left(\frac{I_{rcdt} - I_{rcd,t-1} + S^{loss}_{rcdt}}{\mathscr{R}^I_t}\right) + RD_{rcdt} \\
\forall r\in\mathbb{R}^S,\ c\in\mathbb{C},\ d\in\mathbb{D},\ t=2,\ldots,N^{\mathbb{T}} \qquad (4)
\end{gathered}$$

The terms in each of these constraints are summarised below and described in more detail in the subsequent subsections. The dimensions of each term are units of resource per rate basis and they apply to each resource r, in cell c, during season t of decade d.

- $\left[\left(B_{rcd}|_{r\in\mathbb{R}^B} + B^{\text{straw}}_{cd}|_{r=\text{Straw}}\right)/\left(1-\mu_r\right)\right]\left(f'^{B}_{rt}\nu^{dpY}/\nu^{dps}_t\right)$ is the rate of biomass production (grown and harvested) – the "bar" notation indicates that the term only appears when the balance is written for a specific resource: B_{rcd} only appears for biomass resources ($r\in\mathbb{R}^B$) and B^{straw}_{cd} only appears for straw ($r=\text{Straw}$)
- $WF_{cdt}x_{rcd}|_{r\in\mathbb{R}^W}$ is the net rate of production of waste resources through the separation of "Waste-All"
- $WAC_{cdt}|_{r=\text{WasteAll}}$ is the rate of consumption of "Waste-All"
- $\sum_{j\in\mathbb{J}}\sum_{m=1}^{M_j}\mathscr{P}_{jmcdt}\alpha_{jmdr}\gamma_{jmdr}$ is the net rate of resource production due to the operation of technologies
- $\sum_{c'|\Gamma_{c'cl}=1}\sum_{l\in\mathbb{L}} Q_{rc'cldt}|_{r\in\mathbb{R}^Q}$ is the rate of transport of resource into the cell c from other cells c'
- $\sum_{c'|\Gamma_{cc'l}=1}\sum_{l\in\mathbb{L}} Q_{rcc'ldt}|_{r\in\mathbb{R}^Q}$ is the rate of transport out of the cell c to other cells c'
- D_{rcdt} is the demand for the resource
- RP_{rcdt} is the rate of resource purchased (e.g. utilisation of grid electricity)
- RS^L_{rcdt} is the rate of sale of the resource
- $RI_{rcdt}|_{r\in\mathbb{R}^I\wedge c\in\mathbb{C}^{\text{ship}}}$ is the rate of import of the resource from abroad
- $\left(I_{rcdt} - I_{rcd,t-1} + S^{loss}_{rcdt}\right)/\mathscr{R}^I_t$ is the rate of change of the inventory level
- RD_{rcdt} is the rate of resource disposal

Since detailed transport of global demand resources is not explicitly considered, for computational efficiency, the resource balance for global demand resources consists only of constraints ensuring that the total net production exceeds the demand (see constraints 33 to 34 in Section 5.4).

5.1. *Biomass production*

Crop production is modelled using a yield potential, Y_{rscd}, which is the annual production (in oven-dry tonnes per hectare) that can be achieved in each cell, c, in each decade, d, for a given yield scenario, $s \in \mathbb{S}$. The actual production of a crop, B_{rcd}, is the planted area, A_{rcd}, multiplied by the yield potential:

$$B_{rcd} \leq A_{rcd} Y_{rscd} \Upsilon^{Y}_{rd} \qquad \forall r \in \left(\mathbb{R}^{B} - \mathbb{R}^{F}\right),\, s = s^{\star},\, c \in \mathbb{C},\, d \in \mathbb{D} \tag{5}$$

where $s^{\star} \in \mathbb{S}$ is the user-selected yield scenario. The factor Υ^{Y}_{rd} is a yield uplift, which allows the user to specify a number of "what if" scenarios based around the yield potential without needing to modify the raw yield data, e.g.: what would happen if yields increased by 10% per decade? Constraint 5 does not apply to the forestry resources, $r \in \mathbb{R}^{F}$, as these are modelled differently, as described in Section 5.1.5.

5.1.1. *Straw production*

There are three resources associated with winter wheat: WW total, WW grain and WW straw. The first resource is used to model the use of the whole crop in energy production, i.e. use of the grain and the straw together. The second and third resources represent the case where the grain is separated from the straw and both can be used separately to produce energy. Yields for WW whole crop and WW grain are provided, along with the amount of straw that is useable after the grain has been harvested, represented as a fraction of the grain yield, ψ_c. The amount of straw produced, B^{straw}_{cd}, in association with the production of WW grain is therefore:

$$B^{\text{straw}}_{cd} \leq \psi_c B_{\text{WW grain},cd} \qquad \forall c \in \mathbb{C}, d \in \mathbb{D} \tag{6}$$

5.1.2. *Crop rotation*

To maintain soil fertility, certain crops need to be grown in rotation with winter wheat. These are defined by the set $\mathbb{R}^{Ro} \subset \mathbb{R}^{B} - \mathbb{R}^{F}$ and by the number of years that winter wheat must be planted for each year that the crop is grown, y^{Ro}_{r}. Here it is assumed that for each rotated pair of crops the ratio of areas planted each year is equal to the ratio of number of years that each crop is planted in the rotation. Therefore crop rotations can be modelled using the following constraint on the areas planted.

$$\sum_{r \in \mathbb{R}^{Ro}} y^{Ro}_{r} A_{rcd} = A_{\text{WW total},cd} + A_{\text{WW grain},cd} \qquad \forall c \in \mathbb{C},\, d \in \mathbb{D} \tag{7}$$

5.1.3. *Establishment*

Certain crops require a period of establisment before their full yield potential is realised. For these crops, the first decade of planting results in a fraction, f^{BE}_{r}, of the full yield potential:

$$B_{rcd} \leq \left[\left(A_{rcd} - A_{rc,d-1}\right) f^{BE}_{r} + A_{rc,d-1}\right] Y_{rscd} \Upsilon^{Y}_{rd} \qquad \forall r \in \left(\mathbb{R}^{B} - \mathbb{R}^{F}\right),\, s = s^{\star},\, c \in \mathbb{C},\, d \in \mathbb{D} \tag{8}$$

For crops that do not required an establishment period, f^{BE}_{r} is set to 1 and constraint 8 reduces to constraint 5.

5.1.4. Land area allocation

The land area in each cell that can be allocated for biomass production should not exceed the maximum available area, which depends on the user-specified “selected overall area level”, $k^\star$ (the different area levels are discussed in Section 4.2.1):

$$\sum_{r \in \mathbb{R}^B} A_{rcd} \leq \sum_{k' | k' \leq k} f^A_{k'd} \left(A^{cum}_{ck'} - A^{cum}_{c,k'-1} \right) \qquad \forall c \in \mathbb{C},\, k = k^\star\, d \in \mathbb{D} \tag{9}$$

where A^{cum}_{ck} is the sum of the areas from levels 1 to k inclusive (cumulative), obtained from the CLC 2006 map, and therefore $A^{cum}_{ck} - A^{cum}_{c,k-1}$ is the area for land category k. The area available for growing crops in each cell for each land category, k, is this area in land category k, $A^{cum}_{ck} - A^{cum}_{c,k-1}$, multiplied by the user-specified fraction of the area in this category that can be used for biomass production, $f^A_{k'd} \in [0, 1]$. The purpose of this fraction is to account for the area required to grow other resources, such as food and timber, so that biomass production for energy cannot displace these existing requirements. However, it is not difficult to account for food etc. directly in the model by adding new resources to the set $\mathbb{R}^B$ (some food crops, such as winter wheat, are already present), providing their yield potentials and demands, making them an element of the set of “local demand resources” and setting appropriate values for their maximum selling rate. If all other crops have been included, $f^A_{k'd}$ can be set to 1 and energy crops will compete fairly with other land uses for area in the model.

The total area allocated for biomass production in each decade can also be constrained over all cells by specifying the total available area in each decade, A^{tot}_d:

$$\sum_{r \in \mathbb{R}^B} \sum_{c \in \mathbb{C}} A_{rcd} \leq A^{tot}_d \qquad \forall d \in \mathbb{D} \tag{10}$$

A similar constraint on land area can be written for each crop. A pre-defined parameter, A^{max}_{rdu}, is used to give an increasing maximum available area for each crop in each decade (i.e. ramp-up rates) in order to restrict the pace of land-use change to reasonable values.

$$\sum_{c} A_{rcd} \leq A^{max}_{rdu} \qquad \forall r \in \mathbb{R}^B,\, d \in \mathbb{D},\, u = u^\star \tag{11}$$

where $u^\star \in \mathbb{U}$ is the user-selected ramp-up scenario, from the different ramp-up rate scenarios, $\mathbb{U} \equiv \{\text{none, low, medium, high}\}$, provided for UK energy crops (i.e. Miscanthus and SRC willow). These scenarios are based on the results of E4Tech’s energy crops economics and uptake project commissioned by the ETI in 2013 (R. Taylor, personal communication, 28 November 2013). They are defined as follows:

- None: no limit is imposed on the area allocated for each crop
- Low: “conservative” scenario where the growth of the industry is linear based on current deployment trends (870 ha/yr) and no future acceleration
- Medium: “realistic” scenario where the planting rate is growing at 30% per year
- High: “stretch” scenario where the planting rate is expanding at 50% per year

Constraint 12 restricts the growth of a particular crop to certain land categories,

$$\sum_{r\in\mathbb{R}^B|\lambda_r=k} A_{rcd} \leq \sum_{k'\leq k} f^A_{k'd}\left(A^{cum}_{ck'} - A^{cum}_{c,k'-1}\right) \qquad \forall k \leq k^\star,\ c\in\mathbb{C},\ d\in\mathbb{D} \tag{12}$$

where λ_r is the area level upto which biomass resource, $r \in \mathbb{R}^B$, can be grown. For example, food crops are restricted to area level 1, so $\lambda_r = 1$; energy crops can be planted in any level, so $\lambda_r = 4$.

Finally, the planting of a crop, $r \in \mathbb{R}^B$, can be restricted to certain locations, c, using constraint 13.

$$A_{rcd} \leq A^{\text{cell}}_c AP_{rc} \qquad \forall r\in\mathbb{R}^B,\ c\in\mathbb{C},\ d\in\mathbb{D} \tag{13}$$

Here, A^{cell}_c is the total area of each cell ($= 2.5\times10^5$ ha for a 50 km x 50 km square cell) and AP_{rc} is a binary parameter that is equal to 1 if a crop, $r \in \mathbb{R}^B$, is allowed to grow in cell c; 0 otherwise.

5.1.5. Forestry

Since forestry resources, $r \in \mathbb{R}^F$, are not annual crops, their yields cannot be represented on an annual basis. Hence, the data and results for these resources are organised based on the planting and reporting decades. So if "P" and "H" represent the planting and harvesting decades, respectively, then the forestry sets, $\phi \in \mathbb{F}$, can be visualised in Table 2. Also, although the main yield of forestry resources occurs 20 years after planting, there is also a small amount of wood produced ("thinnings" and "stub removal") in the decades either side: this is represented by a "t" and an "s", respectively, in Table 2.

Table 2: Forestry sets representation in the BVCM

Planting period	2010s	2020s	2030s	2040s	2050s
2010s (set 1)	P	t	H	s	
2020s (set 2)		P	t	H	s
2030s (set 3)			P	t	H

Forestry production is the sum over all forestry sets, $\phi \in \mathbb{F}$, of the product of the forestry yield potential, $Y^F_{r\phi scd}$, and the forestry planting area, $A^F_{r\phi cd}$:

$$B_{rcd} \leq \sum_{\phi} A^F_{r\phi cd} Y^F_{r\phi scd} \Upsilon^Y_{rd} \qquad \forall r\in\mathbb{R}^F,\ s=s^\star,\ c\in\mathbb{C},\ d\in\mathbb{D} \tag{14}$$

The forestry resources include short rotation forestry (SRF), long rotation forestry (LRF) and LRF for CO_2 sequestration. The first two are grown for energy production: nearly all of the trees are harvested and used as inputs to technologies; hence, the CO_2 sequestration rate (tCO_2/ha/yr) for these forestry resources is low (since all of the CO_2 fixed by the trees is assumed to be released when the wood is converted to energy). The last one, on the other hand, is grown for CO_2 sequestration purposes (i.e. afforestation): none of the trees are harvested, hence the yields are zero but the CO_2 sequestration rate is high and this offsets the CO_2 emissions of the rest of the system.

The land allocated to afforestation (i.e. LRF for CO_2 capture), for a particular set $\phi \in \mathbb{F}$, is assumed to be constant from the planting decade to the end of the time horizon (i.e. the land committed for a forestry set ϕ cannot be converted to any other use – it cannot be increased because planting more area in a later decade would involve a different planting set ϕ). The land for afforestation may increase over the decades if there are different forestry sets planted in different decades. On the other hand, the land allocated for forestry grown for energy production (i.e. SRF and LRF) is committed for that purpose within the start decade, $d^S_{r\phi}$, and end decade, $d^E_{r\phi}$ associated with a particular forestry set $\phi \in \mathbb{F}$ but can be available for any other use before the start decade, $d^S_{r\phi}$, and after the end decade, $d^E_{r\phi}$.

$$A^F_{r\phi cd} = \begin{cases} A^F_{r\phi c,d-1} & \forall r = \mathrm{LRF\,for\,CO_2\,capture} \in \mathbb{R}^F,\ \phi \in \mathbb{F},\ c \in \mathbb{C},\ d > \phi \\ A^F_{r\phi c,d-1} & \forall r \neq \mathrm{LRF\,for\,CO_2\,capture} \in \mathbb{R}^F,\ \phi \in \mathbb{F},\ c \in \mathbb{C},\ d^S_{r\phi} < d \leq d^E_{r\phi} \\ 0 & \forall r \neq \mathrm{LRF\,for\,CO_2\,capture} \in \mathbb{R}^F,\ \phi \in \mathbb{F},\ c \in \mathbb{C},\ d < d^S_{r\phi} \vee d > d^E_{r\phi} \end{cases} \tag{15}$$

The total land area allocated to each forestry resource, $r \in \mathbb{R}^F$, in each cell c, during decade d, is the sum of the planting areas over all forestry sets, ϕ.

$$A_{rcd} = \sum_{\phi} A^F_{r\phi cd} \qquad \forall r \in \mathbb{R}^F,\ c \in \mathbb{C},\ d \in \mathbb{D} \tag{16}$$

CO_2 is sequestered whenever a set of LRF is planted in a given cell, the rate of which is given by the CO_2 accumulation rate, $F_{r\phi scdi}$, expressed in tCO$_2$/ha/year. The carbon accumulation rate data were estimated based on Sitka spruce, a tree species in the UK considered to have the greatest potential per hectare for long-term carbon storage. Also, Sitka spruce makes up around one quarter of the total woodland area in Great Britain [54], the largest share for any species found in the UK, thus it is a good choice as a representative species. The BVCM covers only 5 decades, so the analysis only considers the first 50 years of tree growth which have been divided into 5 10-year periods to suit the temporal resolution in the BVCM. The rate, $F_{r\phi scdi}$, includes the carbon accumulation in the tree only and does not include soil carbon. The total CO_2 sequestered by forestry, I^{FS}_{di}, in MkgCO$_2$ during decade d is given by constraint 17. The parameter ν^{YpD} converts the right hand side of constraint 17 from an annual to a decadal basis while the factor, 1000, in the denominator converts tonne to Mkg.

$$I^{FS}_{di} = \sum_{r \in \mathbb{R}^F} \sum_{\phi \in \mathbb{F}} \sum_{c \in \mathbb{C}} A^F_{r\phi cd} F_{r\phi scdi} \nu^{YpD} / 1000 \qquad \forall d \in \mathbb{D},\ i = \mathrm{GHG\_CO_2} \tag{17}$$

5.1.6. Biomass production impact

Associated with the production of biomass is a number of impacts: cost, GHG emissions etc. arising through the various activities related to the production of each biomass resource: e.g. pre-planting, establishment, harvesting, use of fertilisers and pesticides, seeds, fuel, machinery and labour. As with biomass yields, these impacts have been estimated at the 1 km×1 km level for each yield scenario, $s \in \mathbb{S}$, and aggregated to the 50 km×50 km level. These impacts are represented by the parameter BPI_{rscdi}, which is the value of impact $i \in \mathbb{I}$ of producing one unit of resource $r \in \mathbb{R}^B$ in cell $c \in \mathbb{C}$ during decade $d \in \mathbb{D}$. In the BVCM, the units are £/odt for monetary impacts and kgCO$_2$e/odt for GHG emissions.

In addition, the model can include an opportunity cost (gross margin), which represents the premium to the farmer for planting speciality crops (e.g. SRC willow and Miscanthus) rather than traditional crops (e.g. wheat). This is represented by the parameter GM_{rscd} and expressed in £/odt.

The overall unit impact, BPI^{GM}_{rscdi}, can be calculated from both the unit impact, BPI_{rscdi}, and the gross margin, GM_{rscd}, as defined by equation 18:

$$BPI^{GM}_{rscdi} = BPI_{rscdi} + \max\left(0, GM_{\text{WW grain},scd}\frac{Y_{\text{WW grain},scd}}{Y_{rscd}} - GM_{rscd}\right)$$
$$\forall r \in \mathbb{R}^B, s \in \mathbb{S}, c \in \mathbb{C}, d \in \mathbb{D}, i \in \mathbb{I} \qquad (18)$$

For some resources, some of the impacts depend only on the area planted or only on the mass of crop produced (independent of location). For example, for forestry the impacts depend only on the area planted and for sugar beet there is an additional step, e.g. processing of sugar, which depends only on the mass of sugar that is processed. Therefore, extra parameters are needed for these additional impacts: the additional impact per hectare only (e.g. due to machinery) is represented by BPI^{ha}_{rdi}; for forestry, because the impact may depend on the decade at which it is planted/harvested (i.e. the set $\phi \in \mathbb{F}$), the parameter is $BPI^{F}_{r\phi di}$; and the impact per unit mass only is denoted by BPI^{t}_{rdi}.

The total biomass production impact during decade d is therefore:

$$I^{BP}_{di} = \left(\sum_{r\in(\mathbb{R}^B-\mathbb{R}^F)} \sum_{c}\left[\left(BPI^{GM}_{rscdi} + BPI^{t}_{rdi}\right)\Upsilon^{BPI}_{rdi} A_{rcd} Y_{rscd} \Upsilon^{Y}_{rd} + BPI^{ha}_{rdi}\Upsilon^{BPI}_{rdi} A_{rcd}\right]\right.$$
$$\left. + \sum_{r\in\mathbb{R}^F}\sum_{\phi}\sum_{c} BPI^{F}_{r\phi di}\Upsilon^{BPI}_{rdi} A^{F}_{r\phi cd}\right)\varsigma_i ADF_{di} DDF_{di} \qquad \forall s = s^{\star},\, d \in \mathbb{D},\, i \in \mathbb{I} \qquad (19)$$

where Υ^{BPI}_{rdi} is a crop cost uplift/downlift factor, which allows different scenarios with different crop costs to be performed without needing to change the raw crop cost data. The factor ς_i converts the units of the decadal impacts (e.g. I^{BP}_{di}) to £M or MkgCO$_2$e and ADF_{di} and DDF_{di} are factors used to discount costs back to 2010. Cost discounting is discussed in the Supplementary Material.

5.2. Wastes

In the BVCM, the waste resources that can be converted to useful energy are defined by the set $\mathbb{R}^W$, which currently includes:

- "Waste-Bio": kitchen and green waste
- "Waste-Wood": wood and furniture
- "Waste-Plastics": plastic film and dense plastics
- "Waste-Paper & textiles": paper, card and textiles
- "Waste-Other": any waste that does not fall into the above categories
- "Waste-All": mixture of the above 5 waste resources

Intermediate waste resources, which are processed from the waste resources, $r \in \mathbb{R}^W$, are also considered. For example, "Waste-RDF" is produced from "Waste-All" by the Mechanical Biological Treatment (MBT) technology, "Waste-Wood-Pellets" is produced from "Waste-Wood" by the pelletisation technology etc.

The BVCM data on wastes were based on the data generated from the ETI "Energy from Waste" (EfW) project [55]. Waste potentials in the UK from the EfW project were reprocessed and aggregated for to the 50 km×50 km resolution in the BVCM, with the potential of "Waste-All" being the sum of the potentials of the constituent wastes. As the waste potentials from the EfW project were given for the years 2007 and 2050 only, the decadal estimates were obtained by assuming that waste potentials evolve linearly from 2007 to 2050. Also, the data for wastes do not account for seasonality, thus it was assumed that the generation of wastes is constant throughout the year.

It was assumed that transport of "Waste-All" is not allowed across admistrative borders. Therefore, "Waste-All" cannot be transported between cells in the BVCM. Also, the composition of "Waste-All" (calculated using the waste potentials) varies with location but it was assumed that the efficiencies of technologies that use "Waste-All" as an input (e.g. incineration, MBT and plasma gasification) are independent of the composition of "Waste-All".

The rate of consumption of "Waste-All" cannot be greater than the total waste potential:

$$WAC_{cdt} \leq \sum_{r \in \mathbb{R}^{CW}} WP_{rcd}/\mathscr{R}^{Y} \qquad \forall c \in \mathbb{C},\, d \in \mathbb{D},\, t \in \mathbb{T} \tag{20}$$

where WAC_{cdt} is the "Waste-All" consumption rate in units of resource per rate basis, WP_{rcd} is the waste potential of a constituent waste resource, $r \in \mathbb{R}^{CW}$, in units of resource per year and $\mathscr{R}^{Y}$ is a unit conversion factor defined so that both sides of constraint 20 are expressed in units of resource per rate basis (see the Supplementary Material for its definition).

For waste resources, $r \in \mathbb{R}^{W}$, the resource balance given in equations 2 simplifies to:

$$RD_{rcdt} = WF_{cdt}x_{rcd} + WAC_{cdt}|_{r=\text{WasteAll}} + \sum_{j \in \mathbb{J}} \sum_{m=1}^{M_j} \mathscr{P}_{jmcdt}\alpha_{jmdr} \qquad \forall r \in \mathbb{R}^{W},\, c \in \mathbb{C},\, d \in \mathbb{D},\, t \in \mathbb{T} \tag{21}$$

where RD_{rcdt} is the rate of resource disposal (see Section 5.10), WF_{cdt} is the rate at which "Waste-All" is separated into its constituents, x_{rcd} is the mass fraction of constituent wastes in "Waste-All" (the mass fraction for "Waste-All" is set to -1) and the last term represents the net production of resources due to the activity of technologies (here, only the technologies that consume wastes will contribute).

When equation 21 is written for $r =$ "Waste-All", the first and third terms on the right-hand side will be negative if "Waste-All" is separated or if a technology consumes "Waste-All", respectively. These negative terms must be balanced by the second term, the "Waste-All" consumption rate, which cannot exceed the total waste potential (equation 20). When the equation is written for the constituent wastes, $r \in \mathbb{R}^{CW}$, the first term on the right hand side will be positive if any "Waste-All" is separated, the second term is always zero and the third term will be negative if any technology consumes the component waste. For those components that are produced by the separation of "Waste-All" but not consumed by technologies, the resource disposal, RD_{rcdt}, must be positive to balance their generation.

5.2.1. Waste utilisation impact

There are costs and emissions associated with the utilisation of wastes. The costs, which could be positive or negative, correspond to the gate fees charged at different waste treatment, recovery and disposal facilities [56].

In the BVCM, there are different pre-defined cost scenarios for wastes, $w \in \mathbb{W} \equiv \{\text{none, low, medium, high}\}$; each scenario defines the cost of "Waste-All" and of each constituent waste, $r \in \mathbb{R}^{CW}$. The unit waste utilisation impact, WUI_{ridw}, represents the (monetary or emissions) impact of utilising one unit of waste resource, $r \in \mathbb{R}^{W}$, during decade d for a waste cost scenario w; its units in the BVCM are £/t or kgCO$_2$e/t.

It was assumed that wastes are available in mixed form, i.e. "Waste-All" with a low (typically negative) cost, or already separated at a much higher cost. However, to avoid double counting the availablity of wastes, the model was formulated as though all wastes were mixed as "Waste-All" and to utilise any individual waste, the "Waste-All" must be separated. Data obtained for the costs of "Waste-All" and the separate constituents, were used to calculate the cost of separating the "Waste-All" such that the cost of the "Waste-All" plus the separation cost is equal to the total cost of the indivudual constituents. If only one constituent is used, the remaining ones are disposed of and incur a disposal cost that is the negative of the waste cost, thus the system only pays for the wastes that are actually utilised in technologies.

The impact of separating one unit of "Waste-All" into its components, $WASI_{cdiw}$, is the difference between the mass-fraction weighted sum of the unit impacts of the constituent wastes and the unit impact of "Waste-All".

$$WASI_{cdiw} = \sum_{r \in \mathbb{R}^{CW}} x_{rcd} WUI_{ridw} - WUI_{WasteAll,idw} \qquad \forall c \in \mathbb{C},\, d \in \mathbb{D},\, i \in \mathbb{I},\, w = w^{\star} \tag{22}$$

The decadal impact associated with the utilisation of wastes, I^{WU}_{di}, is the unit impact of "Waste-All" multiplied by the rate of consumption of "Waste-All" plus the unit separation impact multiplied by the fractionation rate of "Waste-All", expressed in £M or MkgCO$_2$e per decade:

$$\begin{aligned} I^{WU}_{di} = \varsigma_i ADF_{di} DDF_{di} \Bigg[& \sum_{c \in \mathbb{C}} \sum_{t \in \mathbb{T}} \Big(WUI_{\text{WasteAll},idw} WAC_{cdt} + WASI_{cdiw} WF_{cdt} \Big) \mathscr{R}^{I}_{t} \\ & + \sum_{r \in \mathbb{R}^{W}} \sum_{c \in \mathbb{C}} \sum_{t \in \mathbb{T}} RDI_{rid} RD_{rcdt} \mathscr{R}^{I}_{t} \Bigg] \qquad \forall d \in \mathbb{D},\, i \in \mathbb{I} \end{aligned} \tag{23}$$

where $\mathscr{R}^{I}_{t}$, defined in the Supplementary Material, is a factor that converts the impacts from per rate basis (the user can select which time unit to use: hour, day or year) to a per season basis; the summation over all seasons $t \in \mathbb{T}$ then converts the impacts to an annual basis.

The negative values for some of the waste costs may result in wastes being utilised in a technology with the resulting (intermediate) resource then not being used in the rest of the chain (i.e. abandoned or discarded for free). For example, in order to benefit from the negative cost of waste wood, the chipping technology could convert it to wood chips which are not used elsewhere. To avoid this situation, it was necessary to account for resource disposal explicitly, the modelling of which is discussed in Section 5.10.

5.3. Resource conversion

In the resource balance given by constraints 2 to 4, the fourth term on the left hand side, $\sum_{j \in \mathbb{J}} \sum_{m=1}^{M_j} \mathscr{P}_{jmcdt} \alpha_{jmdr} \gamma_{jmdr}$, represents the net production rate of resource $r \in \mathbb{R}$ due to the operation of all technologies in cell c during decade d in season t. The variable $\mathscr{P}_{jmcdt}$ is the rate of operation of technology j when operating in mode m. The parameter α_{jmdr} is the rate of consumption ($\alpha_{jmdr} < 0$) or production ($\alpha_{jmdr} > 0$) of resource

r per unit rate of operation of the technology. For co-fired technologies (i.e. technologies, such as power stations, that use biomass in combination with conventional fossil fuel), the fraction of the main outputs of each technology mode that is derived only from biomass is given by the parameter γ_{jmdr}. For example, for a coal power plant that is cofired with 10% biomass, $\gamma_{jmdr} = 0.1$ for $r =$ electricity and $\gamma_{jmdr} = 1$ for all other resources. Also, $\gamma_{jmdr} = 1$ for all technologies that are not cofired. This allows the BVCM to account only for energy produced by biomass, which was its original purpose (and therefore the constraints on energy, discussed in section 5.4, relate only to energy produced directly from biomass).

The maximum production rate of a technology is the installed capacity, CJ_{jcd}, multiplied by the availablity, $\mathscr{A}_{jd}$, which is the fraction of a year during which the technology can be operated. The sum of the production rates of all modes of a technology must not exceed this maximum production rate:

$$\sum_{m=1}^{M_j} \mathscr{P}_{jmcdt} \leq CJ_{jcd}\mathscr{A}_{jd} \qquad \forall j \in \mathbb{J},\, c \in \mathbb{C},\, d \in \mathbb{D},\, t \in \mathbb{T} \tag{24}$$

The capacity installed in each cell c may change each decade due to investments and retirements. These are tracked through the following capacity balance constraint:

$$CJ_{jcd} = CJ_{jc,\,d-1} + CJ^0_{jc} + CJI_{jcd} - CJR_{jcd} \qquad \forall j \in \mathbb{J},\, c \in \mathbb{C},\, d \in \mathbb{D} \tag{25}$$

CJ_{jcd} is the capacity of technology j in cell c at the beginning of decade d, the parameter CJ^0_{jc} is the existing capacity of technology j in cell c (i.e. the capacity of any technologies that are already present in the cell), CJI_{jcd} is the additional capacity made available at the beginning of decade d due to investments in technology j in cell c and CJR_{jcd} is the amount of capacity of technology j retired in cell c at the beginning of decade d.

Integer variables, NJI_{jcd}, are used to determine the number of technologies invested in and the actual capacity increase is constrained by the minimum and maximum capacities of a single technology:

$$CJI_{jcd} \geq C^{min}_{jd} NJI_{jcd} \qquad \forall j \in \mathbb{J},\, c \in \mathbb{C},\, d \in \mathbb{D} \tag{26}$$

$$CJI_{jcd} \leq C^{max}_{jd} NJI_{jcd} \qquad \forall j \in \mathbb{J},\, c \in \mathbb{C},\, d \in \mathbb{D} \tag{27}$$

Capacity retirements in each decade are due to retirement of existiting technologies (the parameter CJR^{min}_{jcd}) and retirement of technology investments made by the optimisation:

$$CJR_{jcd} = CJR^{min}_{jcd} + \sum_{d' \in \mathbb{D}} CJI_{jcd'} RF_{jdd'} \qquad \forall j \in \mathbb{J},\, c \in \mathbb{C},\, d \in \mathbb{D} \tag{28}$$

The parameter $RF_{jdd'}$ defines the fraction of capacity retired in decade d of a technology, j, that was built in decade d'. Table 3 gives an example definition of $RF_{jdd'}$ for a technology with a lifetime of 14 years.

Table 3: Retirement fractions for a technology with a lifetime of 14 years.

Investment decade, d'	Retirement decade, d 2010s	2020s	2030s	2040s	2050s
2010s	0	0.6	0.4	0	0
2020s	0	0	0.6	0.4	0
2030s	0	0	0	0.6	0.4
2040s	0	0	0	0	0.6
2050s	0	0	0	0	0

Two further constraints can be applied to limit the rate at which technologies can be built. First, individual technologies can only be built at a rate of BR_{jd} plants per year and only if that technology is available in that decade ($a_{jd} = 1$):

$$\sum_{c\in\mathbb{C}} NJI_{jcd} \leq \nu^{YpD} a_{jd} BR_{jd} \qquad \forall j \in \mathbb{J},\, d \in \mathbb{D} \tag{29}$$

Second, there is an upper limit on the total capacity of all technologies belonging to family, $j_F \in \mathbb{J}_F$, that can be built in each decade:

$$\sum_{j\in\mathbb{J}|\theta_{j_F j}=1 \,\wedge\, a_{jd}=1} \sum_{c\in\mathbb{C}} CJI_{jcd}\alpha_{jmdr^{MO}_{jm}} \mathscr{U}^{\text{MWh}}_{r^{MO}_{jm}} \mathscr{R}^h \leq BR^F_{j_F d}\nu^{YpD} \qquad \forall j_F \in \mathbb{J}_F,\, d \in \mathbb{D} \tag{30}$$

where $BR^F_{j_F d}$ is the family build-rate for technology family j_F in decade d, given in MW of main output per year. CJI_{jcd} is the invested capacity of a technology j, which may be on an input or output basis, which is multiplied by $\alpha_{jmdr^{MO}_{jm}}$ to give the capacity on an output basis. The factor $\mathscr{U}^{\text{MWh}}_{r^{MO}_{jm}}$ converts the output capacity from units of resource to MWh and $\mathscr{R}^h$ converts from per rate basis to per hour (defined in the Supplementary Material), so that the units of the left hand side of constraint 30 are in MW. The outer summation is over all technologies, j, that belong to the family j_F ($\theta_{j_F j} = 1$ if technology j belongs to family j_F, $\theta_{j_F j} = 0$ otherwise) and that are available for investment in decade d ($a_{jd} = 1$).

5.3.1. Technology capital impact

The decadal technology capital impact, I^{TC}_{di}, includes the cost and emissions associated with building new technologies:

$$I^{TC}_{di} = \varsigma_i DDF_{di} \sum_{j\in\mathbb{J}} \sum_{c\in\mathbb{C}} TCI_{jdi} CJI_{jcd} TDF_{jdi} \sum_{j_F\in\mathbb{J}_F} \Upsilon^{TCI}_{j_F d}\theta_{j_F j} \tag{31}$$

where TCI_{jdi} is the unit impact associated with the construction of technology j in decade d, expressed in terms of £ or kgCO$_2$e per unit of capacity. CJI_{jcd} represents the capacity invested for technology j at the beginning of decade d. TDF_{jdi} and DDF_{di} are factors that discount costs back to 2010 and are discussed in the Supplementary Material. The factor ς_i converts the units of the impacts to M£ or MkgCO$_2$e per decade. The user-specified parameter $\Upsilon^{TCI}_{j_F d}$ is a capital cost uplift factor, which enables the user to refine the cost of technologies when performing sensitivity studies without needing to modify the raw data. This parameter is applied at the technology family level $j_F \in \mathbb{J}_F$ since it is expected that the changes will occur at this level; $\Upsilon^{TCI}_{j_F d} > 1$ denotes an increase in cost. The binary parameter $\theta_{j_F j}$ indicates the family association of a technology (i.e. $\theta_{j_F j} = 1$ if technology j is a member of family j_F, $\theta_{j_F j} = 0$ otherwise, so the sum selects the family to which the technology belongs and applies that uplift factor).

5.3.2. Technology operation impact

The decadal impact associated with the operation of technologies, I^{TO}_{di}, comprise production- and capacity-related elements (i.e. variable and fixed costs, respectively) and is given by equation 32:

$$I^{TO}_{di} = \varsigma_i ADF_{di} DDF_{di} \left(\sum_{j \in \mathbb{J}} \sum_{c \in \mathbb{C}} TOI^{f}_{jdi} CJ_{jcd} + \sum_{j \in \mathbb{J}} \sum_{m=1}^{M_j} \sum_{c \in \mathbb{C}} \sum_{t \in \mathbb{T}} TOI^{v}_{jdi} \mathscr{P}_{jmcdt} \nu^{dps}_{t} / \nu^{dpY} \right) \tag{32}$$

where TOI^{f}_{jdi} represents the fixed annual operational and maintenance impact for technology j and TOI^{v}_{jdi} denotes the variable annual production impact for technology j (excluding cost and emissions due to the consumption of input resources – these are taken into account through the production of the inputs by technologies further up the value chain or through resource purchase); both parameters are expressed in £ or kgCO$_2$ per unit of capacity per year. The fixed cost component is multiplied by the total capacity of technology j in cell c during decade d while the variable cost component is multiplied by the total processing rate of technology j operating in all modes m in cell c in decade d during season t. The term ν^{dps}_{t}/ν^{dpY} converts the per rate basis impacts to per season, which are then summed over all seasons t to obtain the annual impact.

5.4. Demand and energy production

The annual average rate of production P^{G}_{rd}, in units of resource per rate basis, of a global demand resource $r \in \mathbb{R}^{GD}$ in decade d is given by equation 33. The factor, $\mathscr{R}^{P}_{t}$, defined in the Supplementary Material, converts the units of the summand to units of resource per season, which then becomes units of resource once summed over all seasons and is finally converted back to units of resource per rate basis.

$$P^{G}_{rd} = \sum_{j \in \mathbb{J}} \sum_{m=1}^{M_j} \sum_{c \in \mathbb{C}} \sum_{t \in \mathbb{T}} \mathscr{P}_{jmcdt} \alpha_{jmdr} \gamma_{jmdr} \mathscr{R}^{P}_{t} \qquad \forall r \in \mathbb{R}^{GD},\, d \in \mathbb{D} \tag{33}$$

The annual average production rate of a resource $r \in \mathbb{R}^{GD}$ in decade d must be greater than or equal to the fraction of the annual average demand that must be fulfilled by biomass as given by constraint 34. Here, $\mathscr{D}^{G}_{rd}$ is a parameter that represents the UK annual average demand for resource $r \in \mathbb{R}^{GD}$ in decade d, which must be specified in units of resource per rate basis; β_{rd} represents the minimum fraction of the demand for resource $r \in \mathbb{R}^{GD}$ that must be satisfied by biomass.

$$P^{G}_{rd} \geq \beta_{rd} \mathscr{D}^{G}_{rd} \qquad \forall r \in \mathbb{R}^{GD},\, d \in \mathbb{D} \tag{34}$$

The annual average rate of production P^{L}_{rd} of a local demand resource $r \in \mathbb{R}^{LD}$ in decade d, expressed as units of resource per rate basis, is defined by equation 35. Here, the variable D_{rcdt}, which appears in the resource balance given by constraints 2 to 4, determines how much of the demand for a resource $r \in \mathbb{R}^{LD}$ in cell c in decade d during season t is met.

$$P^{L}_{rd} = \sum_{c \in \mathbb{C}} \sum_{t \in \mathbb{T}} D_{rcdt} \mathscr{R}^{P}_{t} \qquad \forall r \in \mathbb{R}^{LD},\, d \in \mathbb{D} \tag{35}$$

Similar to constraint 34, the annual average production rate of a resource $r \in \mathbb{R}^{LD}$ in decade d must be greater than or equal to the fraction of the annual average demand that must be satisfied by biomass, as given by constraint 36. The parameter $\mathscr{D}^{L}_{rcd}$ represents the average annual demand for resource $r \in \mathbb{R}^{LD}$ in cell c in decade d, in units of resource per basis. The average annual demand in each cell is summed to obtain the UK average annual demand. The user-specified parameter β_{rd} represents the minimum fraction of the demand for resource $r \in \mathbb{R}^{LD}$ that must be fulfilled by biomass.

$$P^{L}_{rd} \geq \beta_{rd} \sum_{c \in \mathbb{C}} \mathscr{D}^{L}_{rcd} \qquad \forall r \in \mathbb{R}^{LD},\, d \in \mathbb{D} \tag{36}$$

The total energy production is the sum of the global production and the local production, which is given by equation 37, expressed in MWh per year.

$$P^{E,tot}_{d} = \sum_{r \in \mathbb{R}^{GD} | \chi_r \neq 0} P^{G}_{rd} \mathscr{U}^{\text{MWh}}_{r} \mathscr{R}^{Y} + \sum_{r \in \mathbb{R}^{LD} | \chi_r \neq 0} \sum_{c \in \mathbb{C}} \sum_{t \in \mathbb{T}} D_{rcdt} \mathscr{U}^{\text{MWh}}_{r} \mathscr{R}^{P}_{t} \mathscr{R}^{Y} \tag{37}$$

The total energy production in decade d must be greater than or equal to the user-defined energy production target as given by constraint 38. Here, $P^{E,\,min}_{d}$ represents the minimum total energy production (MWh per year) during decade d.

$$P^{E,tot}_{d} \geq P^{E,\,min}_{d} \qquad \forall d \in \mathbb{D} \tag{38}$$

In some cases, for example when maximising profit, energy can be overproduced. To avoid unrealistically high energy production, an upper bound can be specified using the parameter $P^{E,\,max}_{d}$ (MWh per year) and the constraint below:

$$P^{E,tot}_{d} \leq P^{E,\,max}_{d} \qquad \forall d \in \mathbb{D} \tag{39}$$

The total exergy production in decade d is defined similarly to the total energy production (equation 37) but with an exergy factor included in the summations:

$$P^{X,tot}_{d} = \sum_{r \in \mathbb{R}^{GD}} \chi_r P^{G}_{rd} \mathscr{U}^{\text{MWh}}_{r} \mathscr{R}^{Y} + \sum_{r \in \mathbb{R}^{LD}} \sum_{c \in \mathbb{C}} \sum_{t \in \mathbb{T}} \chi_r D_{rcdt} \mathscr{U}^{\text{MWh}}_{r} \mathscr{R}^{P}_{t} \mathscr{R}^{Y} \tag{40}$$

The parameter χ_r represents the exergy factor of resource r, which is the fraction of energy that is available to do useful work. For example, for electricity $\chi_r = 1$ and for hot water $\chi_r \approx 0.268$ (assuming a typical temperature of 90°C for the district heating network [57]).

The user may also specify the minimum rate of production of each energy vector by specifying the minimum fraction of the total energy production. For transport fuels, the constraint is:

$$P^{TF}_{d} = \sum_{r \in \mathbb{R}^{TF}} P^{G}_{rd} \mathscr{U}^{\text{MWh}}_{r} \mathscr{R}^{Y} \geq p^{TF,\,min}_{d} P^{E,tot}_{d} \qquad \forall d \in \mathbb{D} \tag{41}$$

where P^{TF}_{d} is the total production of transport biofuels (MWh per year), which include all $r \in \mathbb{R}^{TF}$, and $p^{TF,\,min}_{d}$ is the minimum fraction of the total energy production. Similar constraints, variables and parameters exist for bio-heat, bio-methane, bio-electricity and bio-hydrogen (constraints 42 to 45).

$$P_d^{Heat} = P^L_{\text{Hot water, d}}\mathscr{R}^Y \geq p_d^{Heat,\,min} P_d^{E,tot} \qquad \forall d \in \mathbb{D} \tag{42}$$

$$P_d^{Gas} = P^G_{\text{Bio}-\text{methane, d}}\mathscr{R}^Y \geq p_d^{Gas,\,min} P_d^{E,tot} \qquad \forall d \in \mathbb{D} \tag{43}$$

$$P_d^{Elec} = P^G_{\text{Bio}-\text{electricity, d}}\mathscr{R}^Y \geq p_d^{Elec,\,min} P_d^{E,tot} \qquad \forall d \in \mathbb{D} \tag{44}$$

$$P_d^{H_2} = P^G_{\text{Bio}-\text{hydrogen, d}}\mathscr{R}^Y \geq p_d^{H_2,\,min} P_d^{E,tot} \qquad \forall d \in \mathbb{D} \tag{45}$$

5.5. Resource transport

The rate of transport of a resource $r \in \mathbb{R}^Q$ from cell c to cell c' via transport mode $l \in \mathbb{L}$ during decade d in season t is defined by the positive variable $Q_{rcc'ldt}$. In the resource balance given by constraints 2 to 4, the fifth and sixth terms, $\sum_{c'|\Gamma_{c'cl}=1}\sum_{l\in\mathbb{L}} Q_{rc'cldt}|_{r\in\mathbb{R}^Q} - \sum_{c'|\Gamma_{cc'l}=1}\sum_{l\in\mathbb{L}} Q_{rcc'ldt}|_{r\in\mathbb{R}^Q}$, represent the *net* rate of transport of resource $r \in \mathbb{R}^Q$ into cell c from other cells c'. The parameter $\Gamma_{cc'l} = 1$ represents the feasible transport connections between cell c and cell c' for a transport mode l; $\Gamma_{cc'l} = 0$ if transport is not allowed.

The flow rate of resources is constrained by a user-specified parameter Q_{rl}^{max}, which denotes the maximum rate of transport of resource $r \in \mathbb{R}^Q$ via a transport mode l:

$$Q_{rcc'ldt} \leq Q_{rl}^{max}\Gamma_{cc'l} \qquad \forall r \in \mathbb{R}^Q;\ c,\, c' \in \mathbb{C};\ l \in \mathbb{L};\ d \in \mathbb{D};\ t \in \mathbb{T} \tag{46}$$

5.5.1. Resource transport impact

The decadal impact associated with the transport of resources in decade d is given by equation 47:

$$I_{di}^{Q} = \varsigma_i ADF_{di} DDF_{di} \sum_{r\in\mathbb{R}^Q}\ \sum_{c,\, c'\in\mathbb{C}}\ \sum_{l\in\mathbb{L}}\ \sum_{t\in\mathbb{T}} TrOI_{rldi} Q_{rcc'ldt} ADD_{cc'l} \mathscr{U}_r^t \mathscr{R}_t^I \qquad \forall d \in \mathbb{D},\, i \in \mathbb{I} \tag{47}$$

where $TrOI_{rldi}$ is the unit impact of transporting resource $r \in \mathbb{R}^Q$ by transport mode $l \in \mathbb{L}$ in decade d, expressed in terms £ or kgCO$_2$e per tonne per km. The cost component comprises a fixed cost for loading and unloading; charter cost including hire, labour and overheads; and a fuel cost. GHG emissions are based on the Biograce efficiencies [58] multiplied by the carbon intensity of the fuel.

In equation 47, $ADD_{cc'l}$ represents the actual logistic distance between cells c and c' travelled by transport mode l expressed in km. These were obtained by converting straight line distances to expected travelled distances using the tortousity factors. Since the resource flow rate, $Q_{rcc'ldt}$, is expressed in terms of units of resource per rate basis, unit conversion factors are needed: $\mathscr{U}_r^t$ to convert the units of resource to tonne and $\mathscr{R}_t^I$ to convert the per rate basis to per seasonal basis, which are then summed over all seasons t to get the per annual basis.

5.6. Resource import

In the resource balance defined by constraints 2 to 4, the last term on the left hand side represents the rate of import, RI_{rcdt}, of resource $r \in \mathbb{R}^I$ through existing UK major ports, $c \in \mathbb{C}^{\text{ship}} \subset \mathbb{C}$.

Although in general any resources can be imported, the BVCM currently considers the import of biomass resources only. This allows the model to analyse the role of biomass import as part of the future UK energy mix, which is important considering the current reliance of the UK on biomass imports. Four import scenarios are pre-defined: $\sigma \in \mathscr{I} \equiv \{\text{None, Low, Medium, High}\}$, which allow the user explore these issuses. The impacts of importing a resource, $r \in \mathbb{R}^I$, and its availability depend on the selected import scenario, $\sigma^\star$:

- None: no import of resources
- Low: low availability, high price
- Medium: medium availability, medium price
- High: High availability, low price

These were derived from global supply-cost curves for a number of generic groups of biomass, $g \in \mathscr{G} \equiv \{\text{Energy crops, Forestry and sawmill residues, Small roundwood, Agricultural residues}\}$. The parameter RG_{rg} denotes the membership of each resource r in the import group g: $RG_{rg} = 1$ if resource r is a member of group g, $RG_{rg} = 0$ otherwise. The maximum rate of import of each group g for import scenario σ in decade d is represented by the parameter $RI^{max}_{gd\sigma}$. This sets the upper bound for the total import of all resources r in the import group g for the selected import scenario $\sigma^\star$:

$$\sum_{r\in\mathbb{R}^I}\sum_{c\in\mathbb{C}^{\text{ship}}}\sum_{t\in\mathbb{T}} RI_{rcdt} RG_{rg} \frac{\nu_t^{dps}}{\nu^{dpY}} \mathscr{U}_r^{\text{MWh}} \leq \frac{RI^{max}_{gd\sigma}}{\mathscr{R}^Y} \qquad \forall \sigma = \sigma^\star \in \mathscr{I},\ g \in \mathscr{G},\ d \in \mathbb{D} \tag{48}$$

where the variable RI_{rcdt} is the rate of import of resource $r \in \mathbb{R}^I$ in cell $c \in \mathbb{C}^{\text{ship}}$ in season t during decade d, expressed in terms of units of resource per rate basis. The other symbols in constraint 48 are unit conversion factors: in the BVCM the $RI^{max}_{gd\sigma}$ data are given in MWh/year so these are converted to MWh per rate basis using the factor $\mathscr{R}^Y$. Similarly, the left hand side of constraint 48 is converted to MWh per rate basis using the factor $\mathscr{U}_r^{\text{MWh}}$.

In any given year, each port can only receive and send a certain total amount of resource. These maximum inward and outward capacities, given in tonnes per year, are specified by the Department for Transport [53] and are represented in the BVCM by the parameters PC_c^{in} and PC_c^{out}, respectively. Constraint 49 models the maximum inward capacity by summing all inward transport of resources in $r \in \mathbb{R}^Q$ by ship and all imported resources in $r \in \mathbb{R}^I$. Constraint 50 limits the outward capacity in a similar manner.

$$\sum_{r\in\mathbb{R}^Q}\sum_{c'\in\mathbb{C}^{\text{ship}}}\sum_{t\in\mathbb{T}} Q_{rc'c,\text{ship},\text{dt}} \frac{\nu_t^{dps}}{\nu^{dpY}} \mathscr{U}_r^t + \sum_{r\in\mathbb{R}^I}\sum_{t\in\mathbb{T}} RI_{rcdt} \frac{\nu_t^{dps}}{\nu^{dpY}} \mathscr{U}_r^t \leq \frac{PC_c^{in}}{\mathscr{R}^Y} \qquad \forall c \in \mathbb{C}^{\text{ship}},\ d \in \mathbb{D} \tag{49}$$

$$\sum_{r\in\mathbb{R}^Q}\sum_{c'\in\mathbb{C}^{\text{ship}}}\sum_{t\in\mathbb{T}} Q_{rcc',\text{ship},dt} \frac{\nu_t^{dps}}{\nu^{dpY}} \mathscr{U}_r^t \leq \frac{PC_c^{out}}{\mathscr{R}^Y} \qquad \forall c \in \mathbb{C}^{\text{ship}},\ d \in \mathbb{D} \tag{50}$$

5.6.1. Resource import impact

The decadal impact due to import of resources, I_{di}^{RI}, for a selected import scenario, σ^*, is given by equation 51.

$$I_{di}^{RI} = \varsigma_i ADF_{di} DDF_{di} \sum_{r \in \mathbb{R}^I} \sum_{c \in \mathbb{C}^{\text{ship}}} \sum_{t \in \mathbb{T}} RII_{rid\sigma} \frac{RI_{rcdt}}{1-\mu_r} \mathscr{R}_t^I \qquad \forall \sigma = \sigma^\star \in \mathscr{I},\, d \in \mathbb{D},\, i \in \mathbb{I} \tag{51}$$

whre $RII_{rid\sigma}$ represents the unit impact associated with the import of resource $r \in \mathbb{R}^I$, during decade d for an import scenario $\sigma \in \mathscr{I}$, expressed in £ or kgCO$_2$e per units of resource on an oven-dry basis. The cost and emission will vary depending on the actual country of origin of the feedstock. However, in the BVCM the origin of the import was not taken into account, instead the data for $RII_{rid\sigma}$ were based on typical exporting countries. The price paid for biomass landed at a UK port typically consists of biomass production cost (raw unprocessed biomass) in the country of origin, processing cost, transport cost (usually by road/rail and sea) and profit margins with respect to the international supply chains. The GHG emissions for imported resources, on the other hand, include carbon dioxide, methane and nitrous oxide emissions (calculated in kgCO$_2$ equivalent) due to cultivation, harvesting, drying, processing and transport of resources [59].

In equation 51 the unit impact is multiplied by the variable RI_{rcdt} representing the rate of import of resource $r \in \mathbb{R}^I$ in cell $c \in \mathbb{C}^{\text{ship}}$ in decade d during season t. The term in the denominator, $(1-\mu_r)$, converts the rate of resource import from a wet basis to a dry basis.

5.7. Resource purchase

Some resources, e.g. chemicals and fossil energy resources such as natural gas and electricity, can be purchased from a "grid" that exists in all cells. Note that bio-resources are usually not included in this because their "purchase" is already considered as biomass production or resource import, as discussed in Sections 5.1 and 5.6.

In the resource balance given by constraints 2 to 4, the variable RP_{rcdt} represents the rate at which resource r is purchased from the grid in cell c in decade d during season t. The maximum rate of purchase, RP_{rd}^{max} (units of resource per year), imposes a cap on the rate of resource r that can be bought from the grid in decade d as given by constraint 52. Again, the terms ν_t^{dps}/ν^{dpY} and $\mathscr{R}^Y$ are unit conversion factors used to express both sides of the constraint in terms of units of resource per rate basis.

$$\sum_{c \in \mathbb{C}} \sum_{t \in \mathbb{T}} RP_{rcdt} \nu_t^{dps} / \nu^{dpY} \leq RP_{rd}^{max} / \mathscr{R}^Y \qquad \forall r \in \mathbb{R},\, d \in \mathbb{D} \tag{52}$$

5.7.1. Resource purchase impact

The decadal impact due to purchase of resources, I_{di}^{RP}, is given by equation 53:

$$I_{di}^{RP} = \varsigma_i ADF_{di} DDF_{di} \sum_{r \in \mathbb{R}^{LD}} \sum_{c \in \mathbb{C}} \sum_{t \in \mathbb{T}} RPI_{rid} RP_{rcdt} \mathscr{R}_t^I \qquad \forall d \in \mathbb{D},\, i \in \mathbb{I} \tag{53}$$

where RPI_{rid} represents the unit impact associated with the purchase of resource $r \in \mathbb{R}^{LD}$ during decade d, expressed in £ or kgCO$_2$e per units of resource. The unit purchase costs and emissions were collected from the literature and existing models: for example the unit purchase impacts of key energy resources such as electricity, natural gas, diesel and hydrogen from 2010 to 2050 were obtained from ETI–ESME, while those for chemicals such as hexane, sulphuric acid, caustic soda were taken from current market trading data and the future prices were derived by scaling up with gasoline prices from ETI–ESME.

5.8. Resource sale

Of the net production of a resource, some satisfies demands, some is sold and the rest may be disposed of (at a cost). Both the amount that is sold and the amount that satisfies demands contribute towards the revenue.

In the resource balance given by constraints 2 to 4, the variable RS^L_{rcdt} represents the rate of sale of resource $r \in \mathbb{R}^{LD}$ in cell c in decade d during season t. For resource $r \in \mathbb{R}^{GD}$, the rate of sale in decade d is represented by the variable RS^G_{rd}. To avoid the system being driven towards overproduction of certain resources with high values and to account for the limited market for these resources, a user-specified cap, RS^{max}_{rd}, expressed in terms of units of resource per year, is used to limit the rate of sale of resource $r \in \mathbb{R}$ in decade d as given by constraint 54. Here, the factors ν^{dps}_t/ν^{dpY} and $\mathscr{R}^Y$, which were already explained above, are used to express each term in the constraint as units of resource per rate basis.

$$RS^G_{rd}|_{r\in\mathbb{R}^{GD}} + \sum_{c\in\mathbb{C}}\sum_{t\in\mathbb{T}} RS^L_{rcdt}|_{r\in\mathbb{R}^{LD}}\nu^{dps}_t/\nu^{dpY} \le RS^{max}_{rd}/\mathscr{R}^Y \qquad \forall r\in\mathbb{R},\, d\in\mathbb{D} \tag{54}$$

For resource $r \in \mathbb{R}^{GD}$, the rate of sale, RS^G_{rd}, is the total production rate of that resource in excess of the demand, as given by constraint 55. Again, the upper limit on RS^G_{rd} is given by RS^{max}_{rd} as defined by constraint 54.

$$P^G_{rd} - \beta_{rd}\mathscr{D}^G_{rd} \ge RS^G_{rd} \qquad \forall r\in\mathbb{R}^{GD},\, d\in\mathbb{D} \tag{55}$$

For hot water, the production in excess of demand cannot be sold, i.e. $RS^{max}_{\text{Hot water, d}} = 0$. Based on constraint 36, the total production of hot water may exceed its demand. However, this excess production is not given any value, i.e. based on constraint 54, $RS^L_{\text{Hot water},cdt} = 0$.

5.8.1. Resource value

The decadal revenue is given by equation 56, where RV_{rdi} represents the unit value metric assigned to resource $r \in \mathbb{R}$ which is accrued to the system whenever that resource fulfils demand or whenever excess production of that resource is sold. As indicated by index i in the mathematical representation, this unit resource value has three components: cost, CO_2 emissions and non-CO_2 emissions. In other words, in addition to receiving monetary value for the sale of resources, GHG emissions credits may also be obtained if green/bio- resources displace supply of a resource by conventional methods. The data for RV_{rdi} were collected from existing models and the literature. For example, the prices for most of the bioenergy end vector produced (such as bio-electricity, bio-methane, bio-diesel and bio-hydrogen) from 2010 to 2050 were obtained from ETI ESME. The prices of co-products (such as glycerine, rapeseed meal, DDGS and winter wheat straw) were determined from current market trading data assuming that future prices stay constant. Animal slurries and digestate, which offset each other as input/output to anaerobic digestion, were assumed to have zero cost, as was char – the value of which was assumed to be for CO_2 sequestration only. Similarly, for GHG emissions credits, most of the bioenergy end vectors were assumed to displace their corresponding fossil vector in ETI–ESME (for example, bio-alcohols and bio-naptha displace fossil gasoline; bio-methane and fuel gas displace fossil natural gas) resulting in avoided fossil emissions. Some of the fossil carbon intensity data were determined from UK's carbon calculator for biofuels (based on Biograce standard values) [58]. The emission credits for most of the co-products were also obtained from the Department for Transport's carbon calculator [59], with credits arising due to displaced animal feed, substituted fertiliser or avoided crop growth altogether.

In equation 56, the rate at which resource $r \in \mathbb{R}$ meets demand is represented by the terms D_{rcdt} and $\beta_{rd}\mathscr{D}^G_{rd}$, while the rate of sale of resource $r \in \mathbb{R}$ (produced in excess of the demand) is represented by RS^L_{rcdt} and RS^G_{rd}. The user-specified binary parameter, $\vartheta_r = 1$ if the credit for resource r is to be included in the revenue, $\vartheta_r = 0$ otherwise.

$$I^R_{di} = \varsigma_i ADF_{di} DDF_{di} \sum_{r\in\mathbb{R}} \sum_{t\in\mathbb{T}} \vartheta_r RV_{rdi} \left[\sum_{c\in\mathbb{C}} \left(RS^L_{rcdt} + D_{rcdt}\right)|_{r\in\mathbb{R}^{LD}} + \left(RS^G_{rd} + \beta_{rd}\mathscr{D}^G_{rd}\right)|_{r\in\mathbb{R}^{GD}} \right] \mathscr{R}^I_t \tag{56}$$

5.9. Resource storage

The right hand side of the resource balance represents the net excess production of a resource. This must either be stored for use in a later season or discarded (at a cost). The storage term comprises the inventory of resource at the end of season t, I_{rcdt}, and the amount of resource that is lost during storage (e.g. spoilage), S^{loss}_{rcdt}. The storage loss is assumed to be proportional to the amount of resource in storage:

$$S^{loss}_{rcdt} = \frac{4\varrho_r}{N^{\mathbb{T}} N^S_r} I_{rcdt} \tag{57}$$

The factor multiplying the storage inventory is determined using the number of seasons that the resource can be stored, N^S_r, and the fraction of resource that will be lost if it is stored for the full duration, ϱ_r.

The change in inventory over season t is the amount at the end of the season minus the amount at the beginning (which is equal to the amount at the end of the previous season) minus how much is lost. To obtain a net rate of increase in inventory, this change is divided by a factor $\mathscr{R}^I_t$, which depends on the number of seasons being modelled and the time unit being used.

Since each decade is assumed to be composed of 10 identical years, the inventory must not increase year on year. Therefore, in the resource balance for the first season, $t = 1$, the change in inventory is calculated using the inventory at the end of the final season: $I_{rcdN^{\mathbb{T}}}$. This results in a cyclic inventory profile over the year.

Storage capacity is modelled using a capacity balance:

$$S_{rcd} = S_{rcd-1} + \Delta S_{rcd} \qquad \forall r \in \mathbb{R}^S,\ c \in \mathbb{C},\ d \in \mathbb{D} \tag{58}$$

where S_{rcd} is the storage capacity for resource r in cell c at the beginning of decade d; S^{max}_{rc0} is the initial storage capacity; and ΔS_{rcd} is the amount of new capacity installed at the beginning of decade d.

The storage inventory is therefore bounded from above by the storage capacity:

$$I_{rcdt} \leq S_{rcd} \qquad \forall r \in \mathbb{R}^S,\ c \in \mathbb{C},\ d \in \mathbb{D},\ t \in \mathbb{T} \tag{59}$$

5.9.1. *Storage impact*

Storage of resources and investment in storage capacity affect the economic and environmental performance of the system through the following impacts.

The decadal storage operational impact I^{SO}_{di} is given by equation 60, where SOI_{rdi} is the unit impact of storing resource $r \in \mathbb{R}^S$ in decade d. For example, for biomass feedstocks, SOI_{rdi} (£ or kgCO$_2$e per units of resource per season) includes the unit cost and emission due to moving, handling and settling the feedstocks in the storage location.

$$I^{SO}_{di} = \varsigma_i ADF_{di} DDF_{di} \sum_{r \in \mathbb{R}^S} \sum_{c \in \mathbb{C}} \sum_{t \in \mathbb{T}} \frac{4}{N^{\mathbb{T}}} SOI_{rdi} I_{rcdt} \qquad \forall d \in \mathbb{D},\, i \in \mathbb{I} \tag{60}$$

The decadal storage capital impact I^{SC}_{di} is given by equation 61, where SCI_{rdi} (£ or kgCO$_2$e per units of resource) is the unit storage capital impact.

$$I^{SC}_{di} = \varsigma_i ADF_{di} DDF_{di} \sum_{r \in \mathbb{R}^S} \sum_{c \in \mathbb{C}} SCI_{rdi} \Delta S_{rcd} \qquad \forall d \in \mathbb{D},\, i \in \mathbb{I} \tag{61}$$

5.10. *Resource disposal*

In order to account properly for the production of resources that have little or no value (that are by-products from the production of some other valuable resource), a disposal impact (penalty) can be associated with each resource. Also, including disposal cost in the model prevents negative-cost feedstocks (e.g. waste wood – the utilisation of which generates revenue) being used in a very cheap technology (e.g. chipper or combustion boiler) and then the product not being used anywhere (this is a situation that can arise with an alternative formulation of the resource balance, where the resource disposal term is not included and the constraint is an inequality so that any excess production of a resource can be discarded for free).

In the resource balance given in constraints 2 to 4, the variable RD_{rcdt} represents the rate at which resource $r \in \mathbb{R}$ is being disposed of in cell c in decade d during season t. The decadal impact of disposing resources, I^{RD}_{di}, is given by equation 62, where RDI_{rid} is the unit resource disposal impact, expressed as £ or kgCO$_2$e per units of resource.

$$I^{RD}_{di} = \varsigma_i ADF_{di} DDF_{di} \sum_{r \notin \mathbb{R}^W} \sum_{c \in \mathbb{C}} \sum_{t \in \mathbb{T}} RDI_{rid} RD_{rcdt} \mathscr{R}^I_t \qquad \forall d \in \mathbb{D},\, i \in \mathbb{I} \tag{62}$$

For waste resources, it is worth noting that not all of the constituent wastes, $r \in \mathbb{R}^{CW}$, will be used, thus they must be discarded. In this case, their "disposal" contributes to the disposal cost by an amount exactly the negative of their contribution to the waste utilisation cost due to the separation technology. Therefore, any waste that is unused does not contribute at all to the objective function, i.e. the model only "pays" for the wastes that it consumes. For better clarity, waste disposal costs are included in the waste utilisation impact (equation 23) rather than in the resource disposal impact (hence the restriction $r \notin \mathbb{R}^W$ in the summation in equation 62).

5.11. *CCS technologies and CO$_2$ transport*

Carbon capture and storage is modelled by allowing certain modes of technologies to capture CO$_2$ at a rate proportional to the operation of the technology (kgCO$_2$ per MWh of output), represented by the parameter

$\varpi_{jmdi}^{\text{CCS}}$ (which is equal to zero if the mode of the technology does not capture CO_2). The captured CO_2 must then be transported via pipeline to a limited number of sequestration sites, $c \in \mathbb{C}^{\text{seq}}$, where the amount sequestered gives rise to CCS credits, which are deducted from the total CO_2 emissions of the system. However, there are additional impacts associated with the transport of the captured CO_2. Both the CCS credits and transport impacts contribute to the decadal CCS impacts, I_{di}^{CCS}, defined by equation 65.

The rate of CO_2 capture in any cell is given by equation 63, where the first three factors in the summation give the rate of production of the main output of mode m of technology j (as described in Section 4.4). This is then converted to MWh per rate basis using the factor $\mathscr{U}_r^{\text{MWh}}$ and then to kgCO_2 captured per rate basis by multiplying with $\varpi_{jmdi}^{\text{CCS}}$; $\mathscr{R}_t^I$ converts to total CO_2 captured per season and the factor $\varsigma_{\text{GHG\_CO2}}$ converts to MkgCO_2.

$$V_{cdt}^{\text{captured}} = \sum_{j \in \mathbb{J}} \sum_{m=1}^{M_j} \mathscr{P}_{jmcdt} \alpha_{jmdr_{jm}^{MO}} \gamma_{jmdr_{jm}^{MO}} \mathscr{U}_{r_{jm}^{MO}}^{\text{MWh}} \varpi_{jmdi}^{\text{CCS}} \mathscr{R}_t^I \varsigma_{\text{GHG\_CO2}} \qquad \forall c \in \mathbb{C},\, d \in \mathbb{D},\, t \in \mathbb{T} \tag{63}$$

The CO_2 balance is given by equation 64. The first term on the left hand side is the amount of CO_2 captured, as defined above, the second is the rate of transport of CO_2 (MkgCO_2/season) into cell c. The first term on the right hand side is the rate of transport of CO_2 out of the cell and the final term is the rate of CO_2 sequestration (which is restricted to the sequestration sites, $c \in \mathbb{C}^{\text{seq}}$). For convenience, instead of defining another transport mode for pipeline, the connection rules for CO_2 are the same as the "truck" mode since all cells are connected to their adjacent neighbour.

$$V_{cdt}^{\text{captured}} + \sum_{c' | \Gamma_{c'c,\text{truck}}=1} Q_{c'cdt}^{CO_2} = \sum_{c' | \Gamma_{cc',\text{truck}}=1} Q_{cc'dt}^{CO_2} + V_{cdt}^{\text{sequestered}}|_{c \in \mathbb{C}^{\text{seq}}} \qquad \forall c \in \mathbb{C},\, d \in \mathbb{D},\, t \in \mathbb{T} \tag{64}$$

5.11.1. CCS Impacts

The decadal impacts due to operation of CCS technologies and transport of CO_2 are given by equation 65. The cost component includes the cost of CO_2 transport, where CTI_d is the cost of transporting 1 Mkg of CO_2 from one cell to another (approximately 80 km, taking tortuosity into account). The CO_2 impact is the negative of the total CO_2 sequestered in each decade, which is the sum over all seasons of the amount sequestered in each season multiplied by the number of years in a decade, ν^{YpD}.

$$I_{di}^{\text{CCS}} = \begin{cases} \varsigma_i ADF_{di} DDF_{di} CTI_d \sum_{c,\, c' \in \mathbb{C}} \sum_{t \in \mathbb{T}} Q_{cc'dt}^{CO_2} & \forall d \in \mathbb{D},\, i = \text{Cost} \\ -\nu^{YpD} \sum_{c \in \mathbb{C}^{\text{seq}}} \sum_{t \in \mathbb{T}} V_{cdt}^{\text{sequestered}} & \forall d \in \mathbb{D},\, i = \text{GHG\_CO2} \\ 0 & \forall d \in \mathbb{D},\, i = \text{GHG\_Other} \end{cases} \tag{65}$$

5.12. Objective function

The objective function, Z, to be minimised, is a weighted sum of the individual components of the total impact (cost, CO_2 emissions and other GHG emissions) and the energy and exergy production, defined by equation 66.

$$Z = \sum_{d \in \mathbb{D}} \sum_{i \in \mathbb{I}} \zeta_{\kappa^\star di} I_{di}^{tot} - \omega^E \sum_{d \in \mathbb{D}} P_d^{E,tot} - \omega^X \sum_{d \in \mathbb{D}} P_d^{X,tot} \tag{66}$$

where $\kappa^\star$ is the selected CO_2 price scenario; I_{di}^{tot}, defined by equation 67, is the total impact for decade d; $P_d^{E,tot}$ is the total energy production in decade d, given by equation 37; and $P_d^{X,tot}$ is the total exergy production in decade d, given by equation 40.

$$I_{di}^{tot} = I_{di}^{BP} + I_{di}^{TC} + I_{di}^{TO} + I_{di}^{Q} + I_{di}^{RP} + I_{di}^{RI} + I_{di}^{SC} + I_{di}^{SO} + I_{di}^{\mathrm{CCS}} - I_{di}^{FS} + I_{di}^{WU} + I_{di}^{RD} - I_{di}^{R} \qquad \forall d \in \mathbb{D},\, i \in \mathbb{I} \tag{67}$$

The terms in the definition of the decadal total impact are:

- I_{di}^{BP}: decadal biomass production impact, defined by equation 19
- I_{di}^{TC}: decadal technology capital impact, defined by equation 31
- I_{di}^{TO}: decadal technology operating impact, defined by equation 32
- I_{di}^{Q}: decadal transport operating impact, defined by equation 47
- I_{di}^{RP}: decadal resource purchase impact, defined by equation 53
- I_{di}^{RI}: decadal resource import impact, defined by equation 51
- I_{di}^{SC}: decadal storage capital impact, defined by equation 61
- I_{di}^{SO}: decadal storage operating impact, defined by equation 60
- I_{di}^{CCS}: decadal CCS impact, defined by equation 65
- I_{di}^{FS}: decadal forestry CO_2 sequestration impact, defined by equation 17
- I_{di}^{WU}: decadal waste utilisation impact, defined by equation 23
- I_{di}^{RD}: decadal resource disposal impact, defined by equation 62
- I_{di}^{R}: decadal revenue, defined by equation 56

The weights are specified by the user in order to define a number of different objectives, for example: minimise cost, maximise profit, minimise emissions, maximise energy/exergy production. GHG emissions can be treated as individual components of the objective function (so effectively a multi-objective optimisation) or as costs by multiplying the GHG emissions by a CO_2 price, $q_{\kappa d}$. In the BVCM a number of CO_2 price scenarios are defined: $\kappa \in \mathscr{K} \equiv \{\text{None, Low, Medium, High}\}$. The objective weights (for the total impacts) in equation 66 depend on the selected CO_2 price scenario and are given by:

$$\zeta_{\kappa di} = \begin{cases} \omega_i^I & \forall \kappa = \text{None},\ d \in \mathbb{D},\ i \in \mathbb{I} \\ 1 & \forall \kappa \neq \text{None},\ d \in \mathbb{D},\ i = \text{Cost} \\ q_{\kappa d} DDF_{di} & \forall \kappa \neq \text{None},\ d \in \mathbb{D},\ i \neq \text{Cost} \end{cases} \tag{68}$$

When the CO_2 price scenario $\kappa =$ None is selected, the individual weights, ω_i^I, for the impacts must be provided and $\zeta_{\kappa di} = \omega_i^I$. Depending on the values of these, the objective can be to minimise cost, minimise emissions or any linear combination of these. To minimise cost, ω^I_{Cost} is set to 1 and other weights are set to zero. To maximise profit, $\omega^I_{\text{Cost}}$is set to 1, $\omega^I_{i\neq\text{Cost}}$ is set to zero and ϑ_r is set to 1 for resources r whose values are to be included in the revenue, e.g. end vectors and by-products.

When the CO_2 price scenario $\kappa \neq$ None, the total impact weights, $\zeta_{\kappa di}$, are set to 1 for the cost component while those for the GHG components are set equal to the CO_2 price, discounted back to 2010 using the factor DDF_{di}.

6. Example case studies

Several studies using energy system models, e.g. UK MARKAL/TIMES [4] and ETI–ESME [1] have indicated that bioenergy can be an important part of the energy mix that will enable the UK to meet its energy and climate change objectives, such as the 2020 renewables targets [60] and the 2050 carbon reduction targets [61]. Although these models provide useful perspectives in terms of the contribution of bioenergy to the UK energy system and to the decarbonisation targets, they are aggregated in nature and do not have the granularity needed to perform an analysis that will show where, how and when bioenergy technologies can be deployed for the greatest overall benefit to the future UK energy system.

This section describes example case studies using the BVCM to determine the most effective pathway that meets an average level of energy demand and desirable GHG savings required from the UK bioenergy sector. We note that three scenarios are not sufficient to gain any firm insights from the model. However, the Energy Technologies Institute are currently exploring a large number of scenarios using the BVCM and will publish, in the near future, a comprehensive insights paper on the role that biomass will play in achieving the UK's energy and emissions targets in 2050. Therefore, in the interest of space and because the focus of this paper is the novel MILP model, the objective of this section is only to demonstrate the functionalities of the BVCM rather than to provide insights into any specific scenarios.

6.1. Inputs

The target values for energy production and emission savings required from the UK bioenergy sector are given in Table 4. It was assumed that up to 10% of UK land can be used for biomass production, with only 2% of the land in Level 1 available for bioenergy while 15% of the land in Levels 2 to 4 can be used. The aim is to determine the optimal allocation of crops to available land, choice of technologies, where and when they are deployed, transport and import of resources and what form of energy to produce.

Table 4: Decadal targets for bioenergy production and GHG emission savings used in the examples, and representative values for CO_2 prices used in the second example.

Decade	Average bioenergy demand (TWh/yr)	GHG emission targets ($MtCO_2e$)	CO_2 price (£/tCO_2e)
2010s	6.05	no target	23.20
2020s	17.97	32	48.60
2030s	38.70	-91.5	141.00
2040s	90.74	-387.5	473.25
2050s	127.94	-561	733.30

The biomass yield potentials for the "medium" climate scenario and "business as usual" yield scenario are selected and filtered using the UKERC 9 land constraint. The cultivation of food crops such as winter wheat, sugar beet and oil seed rape is restricted to Level 1 land while energy crops like Miscanthus and SRC willow can be grown in any of the levels. Also, the establishment yields for Miscanthus and SRC willow are specified such that only upto 55% and 78%, respectively, of the yield potential is realised in the first decade of planting (R. Taylor, personal communication, 16 September 2013).

Resources can be transported by road, rail, inland waterways and close costal shipping and can be imported into major UK ports (with impacts and maximum rates defined by the "medium" scenario, as described in Section 5.6). For waste potentials, the impacts are defined by the "medium" scenario. For CCS technologies, initial sequestration sites have been selected as Peterhead and Humberside (cells 134 and 72, respectively), as suggested by the ETI's UK Storage Appraisal Project [62]. Finally, a discount rate of 3.5% and a finance rate of 8% were used.

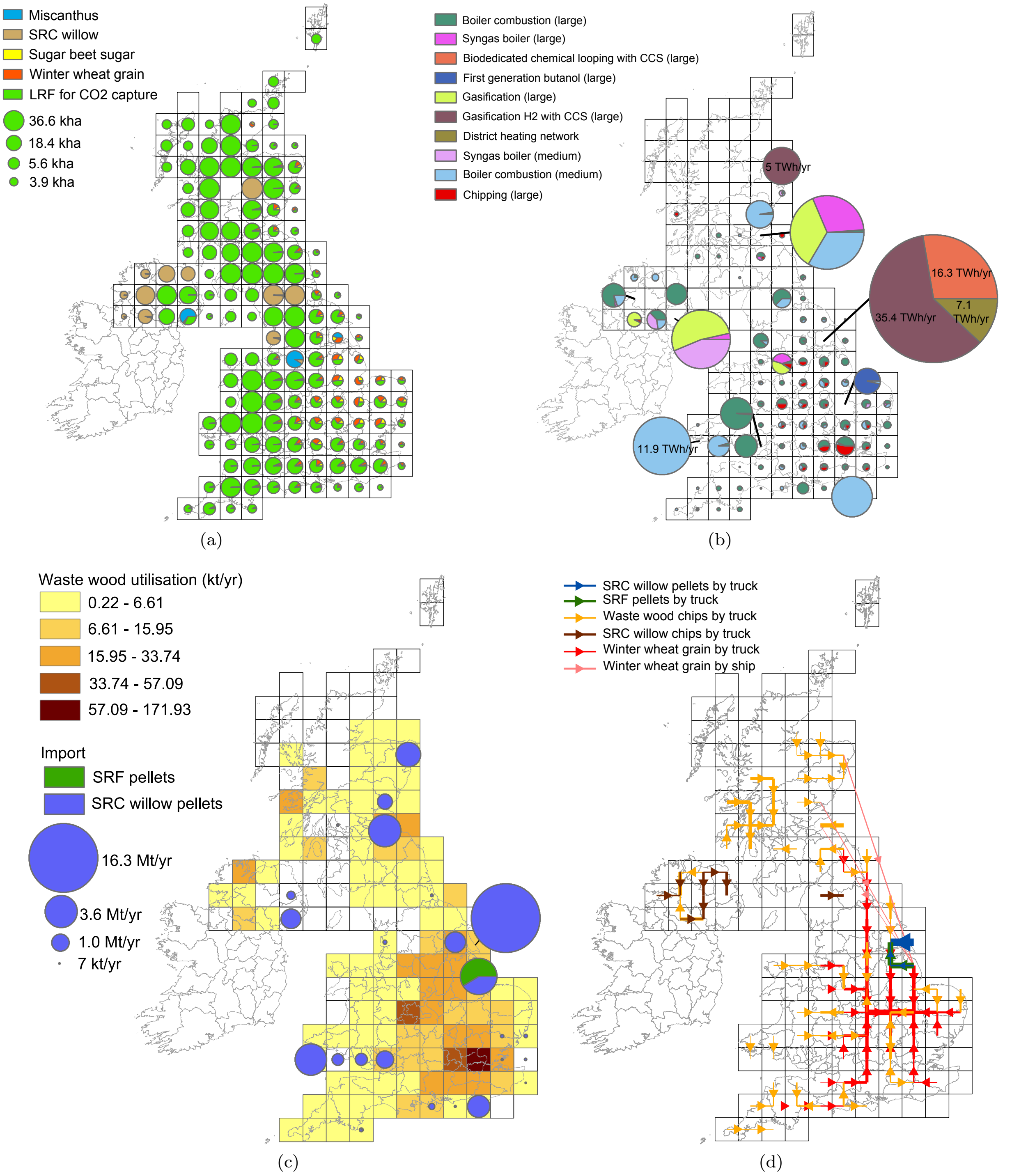
The first set of results is from an example case where the objective was to minimise cost, accounting for co-product credits. The second set of results illustrates an example case where the profit was maximised considering a medium CO_2 price scenario and revenue can be obtained from the sale of final products and co-products. The third scenario is similar to the first but considers minimisation of GHG emissions in order to demonstrate the CCS technologies and CO_2 transport to sequestration sites.

6.2. Results for an example cost-minimisation case

The results of the example cost-minimisation case are summarised in the maps shown in Figure 8, which show the optimal allocation of land, location of technologies, utilisation of wastes, imports and resource transport for the 2050s (maps such as these can be generated for all decades) and in the graphs shown in Figure 9, which show the breakdown of emissions and costs over the 5-decade time horizon along with the energy production and feedstock mixes for each decade.

Figure 8(a) shows the allocation of land for the 2050s. The pie chart in each cell indicates the relative areas of land allocated to planting of biomass; the size of the pie chart indicates the total area of land allocated in each cell. The majority of the allocated land in the 2050s is dedicated to growing "LRF for CO2 capture" (green), which contributes towards meeting the GHG emissions targets. Energy crops are grown in some areas, notably SRC willow (brown) in the north and west of Northern Ireland, part of Scotland and in north-west of England, near the border with Scotland; Miscanthus (blue) is grown in south-east Northern Ireland and in north-west England; winter wheat (red) and sugar beet (yellow) are grown in rotation throughout the eastern and central parts of England.

Figure 8(b) shows the location of technologies in the 2050s. A large proportion of the energy demands is met through production of heat (i.e. hot water) (see Figure 9(c)) and so it can be seen that in the 2050s some of the larger technologies are "Boiler combustion medium" (light blue), "Boiler combustion large" (dark green), and syngas boilers (magenta and voilet). The first two technologies are using various biomass feedstocks; the syngas boilers are fed with syngas that is produced from the "Gasification Large" technologies (lime). CCS technologies are located at the two key CCS sequestration sites, Peterhead and Humberside. Of the winter wheat and sugar beet that are being grown in England, the winter wheat is transported to the 1st generation butanol plant in Cambridgeshire indicated by the dark blue circle (see also the red and pink arrows in Figure 8(d)) and the sugar beet is converted to sugar and sold to obtain co-product credits (it is assumed that co-products sold would displace production of these resources by conventional means and thus reduce GHG emissions).



(a)

(b)

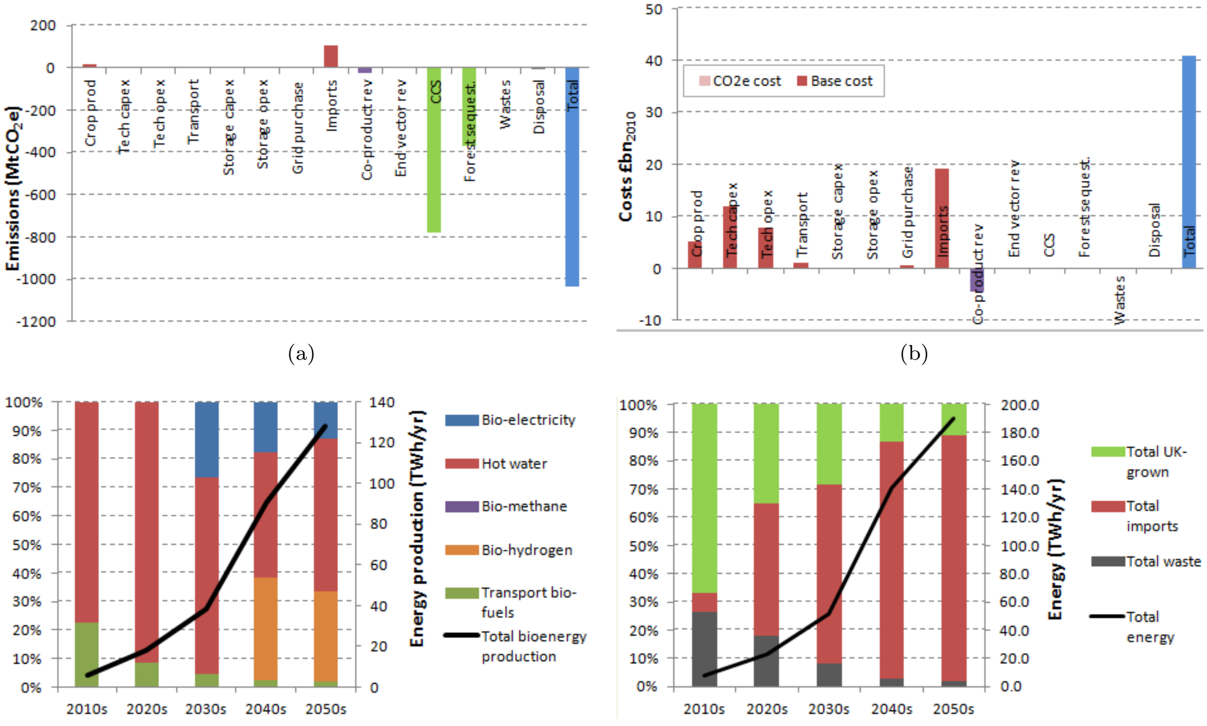
(c)

(d)

Figure 8: Results of the example cost-minimisation case showing the spatial distribution of resources and technologies in the 2050s: (a) land allocation; (b) technologies; (c) import and waste utilisation and (d) transport of resources.

Figure 8(c) shows the imports of biomass (circles) and utilisation of wastes (shading). The majority of imports are of SRC willow pellets (most of which arrives close to the Humberside CCS sequestration site and is transported there – as indicated by the blue arrow in Figure 8(d)). Some SRF pellets are also imported and transported to Humberside. Most of the waste utilisation is in central and eastern England, though by 2050 the waste potential is relatively small (see Figure 9(d)).

Finally, the transport of resources is shown in Figure 8(d). The SRC willow (brown arrows) grown in Northern Ireland is transported to technologies located in the south-east of Northern Ireland; the two cells close to the Scottish border both supply SRC willow to boilers located in the eastern cell of the two. Waste wood chips (yellow arrows) are distributed throughout the UK. The SRC willow chips and SRF pellets being imported are transported to the Humberside CCS sequestration site. As mentioned before, the red and pink arrows indicate transport of winter wheat by truck and close coastal shipping, respectively; it is all directed to and consumed in the 1st generation butanol plant.



(a) (b)

(c) (d)

Figure 9: Results of the example cost-minimisation case: (a) bioenergy system GHG emissions (cumulative 2010s - 2050s); (b) bioenergy system costs (cumulative 2010s - 2050s); (c) energy production mix; and (d) feedstock mix.

Figure 9(a) shows the breakdown of GHG emissions over the whole 5-decade time horizon, with the total GHG emissions indicated by the bar on the far right. The largest GHG emissions are from imports but these are more than compensated by the emissions credits from CCS technologies and forestry sequestration. CCS provides roughly twice the GHG emissions credits that forestry sequestration does.

Figure 9(b) shows the breakdown of costs (at 2010 levels) over the whole 5-decade time horizon, with the total cost on the far right. The largest component of the costs is from imports, with technology capital and operating costs second and third. About 90% of the cost of producing the biomass is offset by the co-product credits, coming from DDGS (Dried Distillers Grains with Solubles) and sugar beet sugar.

Figures 9(c) and 9(d) show the energy production and feedstock mixes, respectively. In the first two decades, most of the energy is provided as heat, with some transport biofuels. Transport fuels are almost phased out and replaced by electricity and hydrogen; the proportion of hot water is also reduced. The total energy provided is indicated by the black line; it meets the targets in Table 4. Initially, the energy is provided by UK-grown crops and wastes. The waste potentials are projected to fall off significantly by the 2040s, so their utilisation also falls (in most scenarios, waste wood is fully utilised). In this example, the cost-optimal solution is to import a higher proportion of biomass in later decades and to reduce the proportion of energy from UK-grown biomass to just over 10% of the the total energy production in the 2050s. However, due to the increasing energy demands (cf. Table 4), the absolute amount of energy produced from UK-grown biomass from the 2010s to the 2050s increases from around 7 TWh/yr to around 21 TWh/yr. The total energy content of the biomass grown and imported is shown by the black line in Figure 9(d); this is greater than the energy targets due to the efficiency of the conversion technologies.

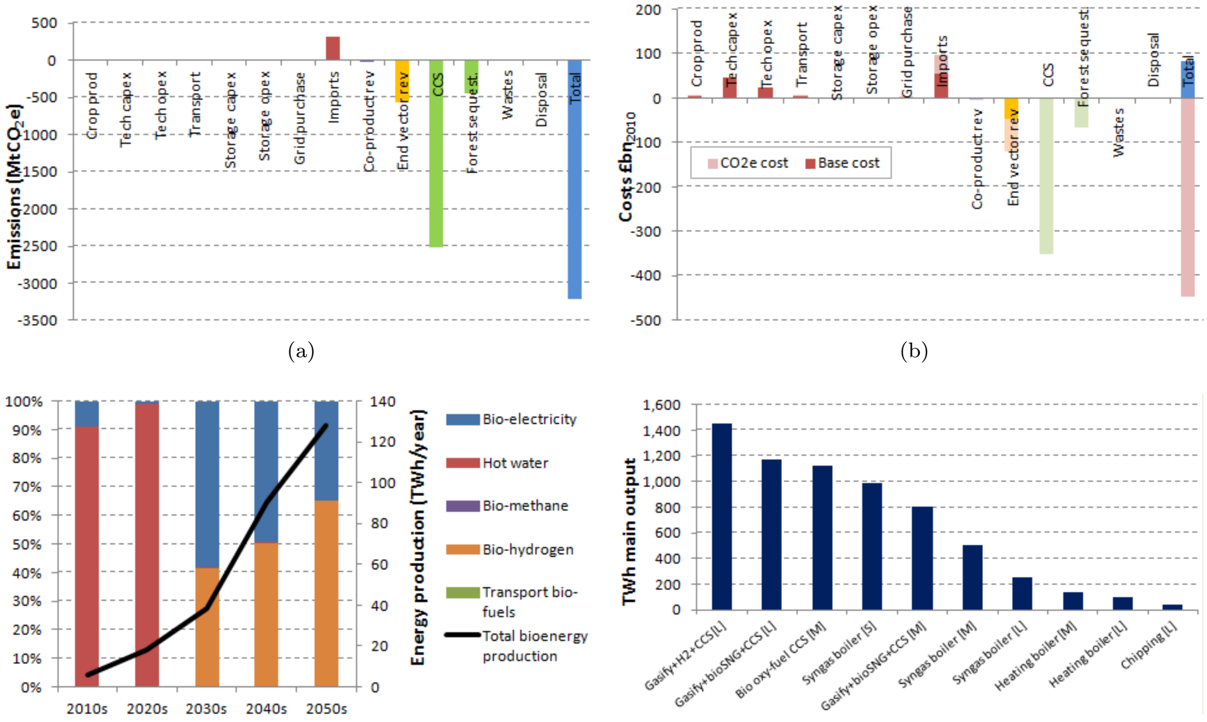
6.3. Results for an example profit-maximisation case

In the profit-maximisation case, all of the data were the same apart from the energy production targets given in Table 4, which became the upper bound on energy production to avoid overproduction and the medium CO_2 price scenario was applied. The revenue from the sale of end-products and co-products was included in the objective function this time.

The results are summarised briefly in Figure 10, which shows the breakdown of the GHG emissions (negative values are actually GHG emissions credits), breakdown of costs, bioenergy mix and top 10 technology utilisation. In addition to CCS and forestry CO_2 sequestration credits, the GHG emissions credits from the sale of end-products also contribute to meeting the GHG emissions savings targets given in Table 4. The system's GHG emissions are more negative than that of the cost-minimisation case due to the medium CO_2 price scenario driving the profit. The forestry CO_2 sequestration has increased slightly but CCS emissions credits have more than trippled.

Because a CO_2 price scenario is being applied, both the base costs and emissions costs can be seen in Figure 10(b). The base costs, represented by the dark bars, indicate the actual cost of the techologies, raw materials, transport etc.; the CO_2 emissions costs, represented by the light bars, indicate the cost of emitting CO_2 (emissions multiplied by the CO_2 price). In this scenario, significant profit is being made from the credits gained by capturing and sequestering the CO_2 resulting from the gasification with CCS and power generation with CCS technologies (see Figure 10(d), where gasification technologies are more dominant and bio-oxyfuel power generation with CCS replaces chemical looping).

In the early decades the bioenergy mix is mostly in the form of heat from the combustion of bio-methane in syngas boilers but in the later decades bio-electricity and bio-hydrogen are more dominant. Compared with the minimum-cost scenario, no transport fuels are produced, more hot water is produced in the first two decades but it is phased out completely thereafter; bio-electricity and bio-hydrogen are produced in more equal proportions.



(a) (b)

(c) (d)

Figure 10: Results for an example profit-maximisation case: (a) bioenergy system GHG emissions (cumulative 2010s - 2050s); (b) bioenergy system costs, including CO$_2$ costs/revenues (cumulative 2010s - 2050s); (c) bioenergy mix; (d) top 10 technology utilisation (cumulative 2010s to 2050s).

6.4. Results for an example GHG-minimisation case

In the GHG-minimisation case, the input data were identical to the cost-minimisation case but since the GHG emissions were being minimised, an upper bound on total cost was imposed to prevent solutions with unrealistically high system costs. Figure 11 illustrates the network that transports CO$_2$ from the CCS technologies to the sequestration sites (in this case, Humberside and Peterhead, indicated by the green shaded squares). The Peterhead sequestration site is fed by CO$_2$ captured in three main locations: one in the same cell and two further south, in Scotland. Two smaller sources of CO$_2$ also feed into Peterhead. In England, the Humberside sequestration site is fed by CO$_2$ captured in a number of locations: six main sources are piped to Humberside and a medium source of CO$_2$ originates at the Humberside cell itself. Several smaller sources spread around England and Wales also feed into the Humberside site, along with a small source in south-west Scotland. Although not shown on the map, the main CCS sites correspond to major ports where different biomass feedstocks are imported.

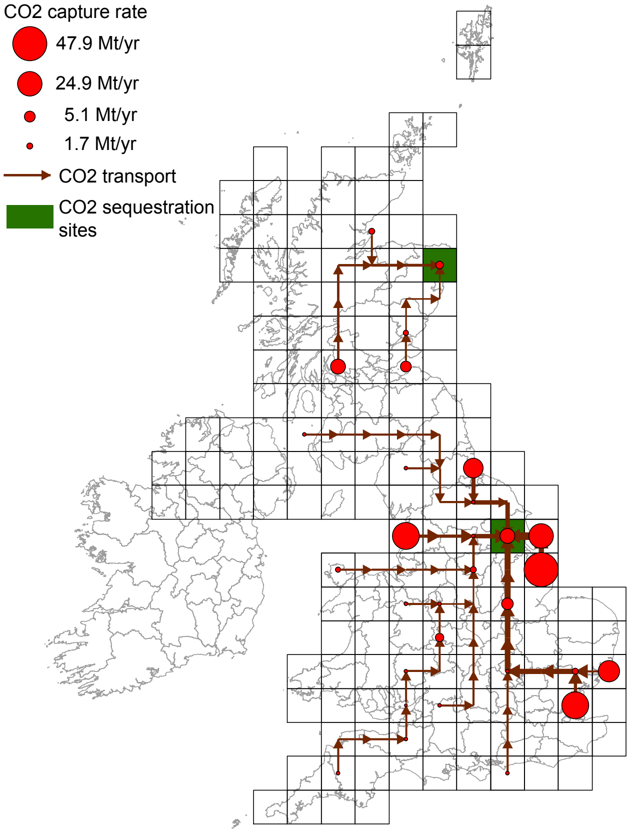


Figure 11: CO_2 captured by the CCS technologies and its transport to the sequestration sites in the 2050s for the example GHG-minimisation case.

7. Conclusions

Biomass is expected to play a significant role in the future energy mix if the UK is to meet its GHG emissions targets while satisfying an increasing demand for energy. As there are many alternative pathways from biomass to energy, a comprehensive and flexible model is required to choose the most effective route accounting for all end-to-end elements in the pathways: land use, biomass production (including arable crops, energy crops and forestry); import, conversion, transport, storage, purchase, sale and disposal of resources; CCS technologies; utilisation of waste resources (e.g. municipal and industrial solid waste). The most effective route depends on the objective function chosen and the constraints imposed on the system. Therefore, the model must support a large number of different scenarios. The model also needs to allow energy production in a variety of forms: not just electricity, heat or bio-fuel alone but all of them simultaneously so that the biomass value chain does not neglect any end-product energy vector and is not biased towards any particular one.

In this paper, the novel mathematical formulation of the Biomass Value Chain Model (BVCM) was presented. To date, to the authors' knowledge, BVCM is the most comprehensive and flexible model for whole system optimisation of biomass value chains. It is a spatio-temporal model that includes a large number of resources and technologies and determines the optimal allocation of land to crop production in each decade and what technologies to use to convert the crops to end-use energy vectors given any set of targets for bioenergy production, which may be overall whole-system energy targets, targets for each energy vector and even targets at the regional level. A variety of objective functions can be formulated in the BVCM: minimise cost, minimise GHG emissions, maximise overall system profit, maximise energy production and maximise exergy production. These can be combined to form a custom objective function, e.g. combining cost and GHG emissions with different weights in order to generate a Pareto curve. Constraints can also be customised by the user either by selecting from pre-defined scenarios or by defining new constraints. For example, the user can select which final energy vectors are included in a scenario or can allow the model to choose the combination that results in the optimal performance of the chain.

To account for the large uncertainty in some of the data, a stochastic functionality is also included in the BVCM, which allows the key inputs (e.g. yields, technology efficiencies, costs) to be sampled from a random distribution and to produce distributions in the outputs. More robust solutions can be obtained by selecting technologies that always appear in the mix. The BVCM also has a very user friendly interface allowing users to configure a scenario easily, run the optimisation and analyse and visualise results.

Currently, the default data set is for the UK, represented by 157 square cells of length 50 km, over the period of 50 years, in decades from the 2010s to the 2050s. Land use is modelled using four levels according to the the CORINE Land Cover 2006 map. There are 93 different resources comprising bio-resources, wastes, intermediates, final products and co-products. The bio-resources included are: winter wheat, sugar beet, oilseed rape, SRC willow, Miscanthus, short and long rotation forestry. Yields and impacts for these crops are provided for six different scenarios: all of the combinations of two UKCP09 climate scenarios (low and medium) and three technology improvement scenarios (worst case, business as usual and best case). Each of these scenarios are further filtered according to the UKERC land constraints. The user can refine these further by specifying how much of the available area can be used for growing crops and up to which land-use level they can be grown. For the waste resources, waste potentials for the UK are provided and there are three cost scenarios. There are 69 distinct technologies, at different scales with multiple modes (more than 1200 combinations in total), including: pre-treatment and densification; gaseous fuel production; liquid fuel production; heat, power and combined heat and power generation; waste-to-energy; and carbon capture technologies. Resources can be transported by road, rail, inland waterways and close-costal shipping and can be imported into major UK ports with three import scenarios relating to the impacts and availability of the resources. Also, three different CO_2 price scenarios are provided for use when the objective function is set to consider the monetary value of GHG emissions.

The BVCM is data-driven, so it can easily be extended to include other resources, technologies, etc. by adding to the database. It can be applied to other countries simply by providing a different data set for the available land areas, yield potentials (and impacts), waste potentials and so on. All of these require no reformulation of the model.

Three example scenarios using different objective functions, given energy production and emissions savings targets, were presented to demonstrate the capabilities of the model. In each case, a number of different forms of energy were produced (heat, electricity, hydrogen and transport fuels) using a variety of feedstocks and technologies. This illustrates the need to include all end energy vectors in the model; something that very few existing models do.

Future enhancements to the model include: consideration of emissions due to land-use change by integrating the results of the ELUM project [63] (currently underway); the inclusion of third-generation bioenergy technologies, such as aquatic biomass; and inclusion of social impacts.

Overall, the BVCM is a very useful toolkit for roadmapping the future biomass value chain pathways and it can provide valuable insights on how to implement bioenergy systems without negative sustainability related impacts. Indeed, the Energy Technologies Institute are currently using the BVCM to perform a large number of studies in order to gain such insights for the UK, which will be published in the near future. The purpose of this paper was to present the model formulation in full so that the insights can be understood in the context of the capabilities and limitations of the model.

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References

[1] C. Heaton, Modelling low-carbon energy system designs with the ETI ESME model. http://www.eti.co.uk/wp-content/uploads/2014/04/ESME_Modelling_Paper.pdf, [Online; accessed 8-August-2014] (2014).

[2] G. Day, Modelling the UK energy system: practical insights for technology development and policy making. http://eti.co.uk/downloads/related_documents/2012_12_10_GD_Modelling_the_UK_energy_system_FINAL.pdf, [Online; accessed 5-August-2014].

[3] H. M. Government, The carbon plan: Delivering our low carbon future. https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/47613/3702-the-carbon-plan-delivering-our-low-carbon-future.pdf, [Online; accessed 6-August-2014] (December 2011).

[4] P. Usher, N. Strachan, Examining decarbonisation pathways in the 2020s on the way to meeting the 2050 emissions target. http://discovery.ucl.ac.uk/1298585/1/CCC%20MARKAL%20Final%20Report%20-%20UCL%20Nov10.pdf, [Online; accessed 6-August-2014] (2011).

[5] Department for Transport, Department of Energy and Climate Change and Department for Environment Food and Rural Affairs, UK bioenergy strategy. https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/48337/5142-bioenergy-strategy-.pdf, [Online; accessed 6-August-2014] (2012).

[6] J. Bisschop, Copyright Paragon Decision Technology B. V, AIMMS, Optimization Modeling. Paragon Decision Technology (1999).

[7] IBM ILOG, CPLEX 12.5 (2013).

[8] F. Mafakheri, F. Nasiri, Modeling of biomass-to-energy supply chain operations: Applications, challenges and research directions, Energy Policy 67 (2014) 116–126.

[9] C. P. Mitchell, Development of decision support systems for bioenergy applications, Biomass and Bioenergy 18 (2000) 265–278.

[10] A. De Meyer, D. Cattrysse, J. Rasinmäki, J. Van Orshoven, Methods to optimise the design and management of biomass-for-bioenergy supply chains: A review, Renewable and Sustainable Energy Reviews 31 (2014) 657–670.

[11] D. Yue, F. You, S. W. Snyder, Biomass-to-bioenergy and biofuel supply chain optimization: Overview, key issues and challenges, Computers and Chemical Engineering 66 (2014) 36–56.

[12] L. Čuček, H. L. Lam, J. J. Klemeš, P. S. Varbanov, Z. Kravanja, Synthesis of regional networks for the supply of energy and bioproducts, Clean Techn Environ Policy 12 (2010) 635–645.

[13] Y. Zhang, G. Hu, R. C. Brown, Integrated supply chain design for commodity chemicals production via woody biomass fast pyrolysis and upgrading, Bioresource Technology 157 (2014) 28–36.

[14] P. W. Tittmann, N. C. Parker, Q. J. Hart, B. M. Jenkins, A spatially explicit techno-economic model of bioenergy and biofuels production in california, Journal of Transport Geography 18 (6) (2010) 715 – 728.

[15] T. Lin, L. F. Rodríguez, Y. N. Shastri, A. C. Hansen, K. Ting, GIS-enabled biomass-ethanol supply chain optimization: model development and Miscanthus application, Biofuels, Bioproducts and Biorefining 7 (3) (2013) 314–333.

[16] Y. N. Shastri, A. C. Hansen, L. F. Rodríguez, K. C. Ting, Optimization of Miscanthus harvesting and handling as an energy crop: Biofeed model application, Biological Engineering 3 (2) (2010) 37–69.

[17] T. Lin, L. F. Rodríguez, Y. N. Shastri, A. C. Hansen, K. C. Ting, Integrated strategic and tactical biomass biofuel supply chain optimization, Bioresource Technology 156 (2014) 256 – 266.

[18] L. Čuček, M. Martin, I. Grossmann, Z. Kravanja, Multi-period synthesis of optimally integrated biomass and bioenergy supply network, Computers and Chemical Engineering 66 (2014) 57–70.

[19] F. You, Optimal design of sustainable cellulosic biofuel supply chains: multiobjective optimisation coupled with life cycle assessment and input-output analysis, AIChE 58 (2012) 1157–1180.

[20] L. Zhang, G. Hu, Supply chain design and operational planning models for biomass to drop-in fuel production, Biomass and Bioenergy 58 (2013) 238–250.

[21] N. Shabani, T. Sowlati, A mixed integer non-linear programming model for tactical value chain optimization of a wood biomass power plant, Applied Energy 104 (2012) 353–361.

[22] J. Santibañez-Aguilar, J. González-Campos, P.-O. J.M., M. Serna-González, M. El-Halwagi, Optimal planning and site selection for distributed multiproduct biorefineries involving economic, environmental and social objectives, Journal of Cleaner Production 65 (0) (2014) 270 – 294.

[23] A. Osmani, J. Zhang, Economic and environmental optimization of a large scale sustainable dual feedstock lignocellulosic-based bioethanol supply chain in a stochastic environment, Applied Energy 114 (2014) 572–587.

[24] A. Duarte, W. Sarache, Y. Costa, A facility-location model for biofuel plants: Applications in the Colombian context, Energy 72 (2014) 476–483.

[25] S. van Dyken, B. H. Bakken, H. I. Skjelbred, Linear mixed-integer models for biomass supply chains with transport, storage and processing, Energy 35 (2010) 1338–1350.

[26] B. H. Bakken, H. I. Skjelbred, O. Wolfgang, eTransport: Investment planning in energy supply systems with multiple energy carriers, Energy 32 (2007) 1676–1689.

[27] O. Akgul, A. Zamboni, F. Bezzo, N. Shah, L. G. Papageorgiou, Optimization-based approaches for bioethanol supply chains, Industrial and Engineering Chemistry Research 50 (2011) 4927–4938.

[28] S. Balaman, H. Selim, A network design model for biomass to energy supply chains with anaerobic digestion systems, Applied Energy 130 (2014) 289–304.

[29] J. A. Elia, R. C. Baliban, X. Xiao, C. A. Floudas, Optimal energy supply network determination and life cycle analysis for hybrid coal, biomass, and natural gas to liquid (CBGTL) plants using carbon-based hydrogen production, Computers & Chemical Engineering 35 (8) (2011) 1399 – 1430.

[30] J. Elia, R. Baliban, C. Floudas, B. Gurau, M. Weingarten, Hardwood biomass to gasoline, diesel, and jet fuel: 2. supply chain optimisation framework for a network of thermochemical refineries, Energy & Fuels 27 (2013) 4325–4352.

[31] W. Marvin, L. Schmidt, S. Benjaafar, D. Tiffany, P. Daoutidis, Economic optimization of a lignocellulosic biomass-to-ethanol supply chain, Chemical Engineering Science 67 (2012) 68–79.

[32] A. Zamboni, N. Shah, F. Bezzo, Spatially explicit static model for the strategic design of future bioethanol production systems. 1. cost minimization, Energy Fuels 23 (2009) 5121–5133.

[33] A. Zamboni, F. Bezzo, Shah, Spatially-explicit static model for the strategic design of future bioethanol production systems. 2. multi-objective environmental optimization, Energy & Fuels 23 (2009) 5134–5143.

[34] F. Frombo, R. Minciardi, M. Robba, R. Sacile, A decision support system for planning biomass-based energy production, Energy 34 (2009) 362–369.

[35] A. Almansoori, N. Shah, Design and operation of a future hydrogen supply chain: Multi-period model, International Journal of Hydrogen Energy 34 (19) (2009) 7883 – 7897.

[36] A. J. Dunnett, C. S. Adjiman, N. Shah, A spatially explicit whole-system model of the lignocellulosic bioethanol supply chain: an assessment of decentralised processing potential, Biotechnology for Biofuels 1.

[37] S. Giarola, A. Zamboni, F. Bezzo, Spatially explicit multi-objective optimisation for design and planning of hybrid first and second generation biorefineries, Computers & Chemical Engineering 35 (9) (2011) 1782 – 1797, energy Systems Engineering.

[38] A. Dunnett, C. Adjiman, N. Shah, Biomass to heat supply chains. applications of process optimization, Trans IChemE, Part B, Process Safety and Environmental Protection 85 (2007) 419–429.

[39] E. Kondili, C. C. Pantelides, R. W. H. Sargent, A general algorithm for short-term scheduling of batch-operations .1. MILP formulation, Computers & Chemical Engineering 17 (1993) 211–227.

[40] C. C. Pantelides, Unified frameworks for optimal process planning and scheduling, in: D. W. T. Rippin, J. Hale (Eds.), Proceedings of the Second Conference on Foundations of Computer-Aided Operations, CACHE Publications, 1994, pp. 253–274.

[41] A. Zamboni, R. J. Murphy, J. Woods, F. Bezzo, N. Shah, Biofuels carbon footprints: Whole-systems optimisation for GHG emissions reduction, Bioresource Technology 102 (16) (2011) 7457 – 7465.

[42] J. Kim, M. J. Realff, J. H. Lee, C. Whittaker, L. Furtner, Design of biomass processing network for biofuel production using an MILP model, Biomass and Bioenergy 35 (2) (2011) 853 – 871.

[43] N. Samsatli, M. Jennings, Optimization and systems integration, in: J. Keirstead, N. Shah (Eds.), Urban Energy Systems: An Integrated Approach, Routledge, 2011, Ch. 9, pp. 157–184.

[44] C. Weber, J. Keirstead, N. Samsatli, N. Shah, D. Fisk, Trade-offs between layout of cities and design of district energy systems, in: 23rd International Conference on Efficiency, Cost, Optimization, Simulation and Environmental Impact of Energy Systems, Lausanne, 2010.

[45] J. Keirstead, C. Calderon, Capturing spatial effects, technology interactions, and uncertainty in urban energy and carbon models: Retrofitting newcastle as a case-study, Energy Policy 46 (2012) 253 – 267.

[46] J. Keirstead, N. Samsatli, A. M. Pantaleo, N. Shah, Evaluating biomass energy strategies for a UK eco-town with an MILP optimization model, Biomass and Bioenergy 39 (2012) 306 – 316, biorefinery.

[47] CORINE Land Cover Map for 2006 (CLC 2006). http://www.eionet.europa.eu/, [Online; accessed 11-August-2014].

[48] Met Office, UK Climate Projections. http://ukclimateprojections.metoffice.gov.uk/, [Online; accessed 6-August-2014].

[49] A. Lovett, G. Sünnenberg, T. Dockerty, The availability of land for perennial energy crops in great britain, GCB Bioenergy 6 (2) (2014) 99–107.

[50] E. W. Weisstein, von neumann neighborhood. from mathworld–a wolfram web resource. http://mathworld.wolfram.com/vonneumannneighborhood.html, [Online; accessed 6-August-2014].

[51] OpenStreetMap, http://www.openstreetmap.org, [Online; accessed 9-May-2014].

[52] WaterWaysWorld, Widebeam map. http://www.waterwaysworld.com/images/widebeam_map.png, [Online; accessed 9-May-2014].

[53] UK ports and traffic (PORT01). https://www.gov.uk/government/statistical-data-sets/port01-uk-ports-and-traffic#table-port0102, [Online; accessed 9-May-2014].

[54] J. Moore, Wood properties and uses of sitka spruce in britain, Tech. rep., Forestry Commission (2011).

[55] Energy Technologies Institute, Energy from Waste Project. http://www.eti.co.uk/project/energy-from-waste/, [Online; accessed 6-August-2014].

[56] WRAP, Gate Fees Report 2013. http://www.wrap.org.uk/sites/files/wrap/Gate_Fees_Report_2013_h%20(2).pdf, [Online; accessed 9-May-2014].

[57] I. Dincer, M. A. Rosen, Exergy: Energy, Environment , and Sustainable Development, Elsevier, Oxford, UK, 2007.

[58] Biograce. http://www.biograce.net/, [Online; accessed 6-August-2014].

[59] The UK solid and gaseous biomass carbon calculator. https://www.ofgem.gov.uk/publications-and-updates/uk-solid-and-gaseous-biomass-carbon-calculator, [Online; accessed 6-August-2014].

[60] UK renewable energy roadmap. https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/48128/2167-uk-renewable-energy-roadmap.pdf, [Online; accessed 11-August-2014] (2011).

[61] Climate Change Act 2008. http://www.legislation.gov.uk/ukpga/2008/27/contents, [Online; accessed 11-August-2014].

[62] G. Day, Optimising the location of CCS in the UK, Online (2014).
URL `http://www.eti.co.uk/wp-content/uploads/2014/03/Optimising_the_location_of_CCS_in_the_UK1.pdf`

[63] Ecosystem Land Use Modelling & Soil C Flux Trial (ELUM). http://www.ceh.ac.uk/sci_programmes/elum-project.html, [Online; accessed 11-August-2014].

Nomenclature

Indices and sets

$c, c' \in \mathbb{C}$	Spatial cells
$c \in \mathbb{C}^{\text{seq}} \subset \mathbb{C}$	CCS sequestration cells
$c \in \mathbb{C}^{\text{ship}} \subset \mathbb{C}$	Ports for coastal shipping
$d \in \mathbb{D}$	Decades
$g \in \mathscr{G}$	Resources group for import
$i \in \mathbb{I}$	Impacts (or key performance indicators): currently, $\mathbb{I} \equiv \{\text{Cost, GHG\_CO}_2\text{, GHG\_Other}\}$
$j \in \mathbb{J}$	Technologies
$j_F \in \mathbb{J}_F$	Technology families
$k \in \mathbb{K}$	Land classifications based on the CORINE Land Cover Map
$l \in \mathbb{L}$	Transport modes
$m \in \mathbb{M}_j$	Modes of technology j, $\mathbb{M}_j \equiv \{1, \ldots, M_j\}$
$r \in \mathbb{R}$	Resources
$r \in \mathbb{R}^B \subset \mathbb{R}$	Crops
$r \in \mathbb{R}^{CW} \subset \mathbb{R}^W$	Components of "Waste-All"
$r \in \mathbb{R}^{GD} \subset \mathbb{R}$	Global demand resources

$r \in \mathbb{R}^F \subset \mathbb{R}^B$	Forestry resources
$r \in \mathbb{R}^I \subset \mathbb{R}$	Resources that can be imported from abroad
$r \in \mathbb{R}^{LD} \subset \mathbb{R}$	Local demand resources
$r \in \mathbb{R}^Q \subset \mathbb{R}$	Resources that can be transported
$r \in \mathbb{R}^{Ro} \subset \mathbb{R}^B$	Crops that rotate with winter wheat
$r \in \mathbb{R}^S \subset \mathbb{R}$	Resources that can be stored
$r \in \mathbb{R}^{TF} \subset \mathbb{R}$	Transport fuels
$r \in \mathbb{R}^W \subset \mathbb{R}$	Waste resources
$s \in \mathbb{S}$	Yield scenarios
$t \in \mathbb{T}$	Seasons
$u \in \mathbb{U}$	Land area ramp-up rate scenarios
$w \in \mathbb{W}$	Waste cost scenarios
$y \in \mathbb{Y}$	Years
$\kappa \in \mathscr{K}$	CO_2 price scenarios
$\phi \in \mathbb{F}$	Forestry sets
$\sigma \in \mathscr{I}$	Import scenarios

Parameters

A_c^{cell}	Total area of each cell [ha]
A_{ck}^{cum}	Available area of cell c at level k (cumulative) [ha]
A_{rdu}^{max}	Maximum land area allocated for crop $r \in \mathbb{R}^B$ in decade d [ha]
A_d^{tot}	Total available area for all crops in decade d [ha]
$ADD_{cc'l}$	Actual logistic distance between cells c and c' for transport mode l [km]
ADF_{di}	Factor that discounts annual payments within a decade d to the beginning of that decade
AP_{rc}	Binary parameter: 1 if crop $r \in \mathbb{R}^B$ is allowed in cell c, 0 otherwise
a_{jd}	Binary parameter: 1 if technology j is avaialable for use in decade d, 0 otherwise
BPI_{rscdi}	Unit impact of producing crop $r \in \mathbb{R}^B$, excluding opportunity costs, for a yield scenario s in cell c in decade d [£/odt or kgCO_2e /odt]
$BPI_{r\phi di}^F$	Unit impact of producing forestry resource $r \in \mathbb{R}^F$ in set ϕ in decade d [£/ha/yr or kgCO_2e /ha/yr]
BPI_{rscdi}^{GM}	Unit impact of producing crop $r \in \mathbb{R}^B$, including opportunity costs, for a yield scenario s in cell c in decade d [£/odt or kgCO_2e/odt]
BPI_{rdi}^{ha}	Additional unit impact per hectare of producing crop $r \in \mathbb{R}^B$ in decade d [£/ha/yr or kgCO_2e /ha/yr]
BPI_{rdi}^t	Additional unit impact per tonne, independent of location, of producing crop $r \in \mathbb{R}^B$ in decade d [£/t or kgCO2e /t]
BR_{jd}	Maximum number of technology j at maximum capacity per year in decade d
$BR_{j_F d}^F$	Maximum capacity at which technologies belonging to family j_F can be built in decade d [MW main output/yr]
CJ_{jc}^0	Existing capacity of technology j in cell c [unit of capacity]
C_{jd}^{min}	Minimum production capacity of technology j in decade d [unit of capacity]
C_{jd}^{max}	Maximum production capacity of technology j in decade d [unit of capacity]
CJR_{jcd}^{min}	Minimum capacity of technology j retired in cell c in decade d [unit of capacity]

CTI_d	Unit $\mathrm{CO_2}$ transport cost [£/1 $\mathrm{MkgCO_2}$/80 km]
DDF_{di}	Factor that discounts payments back to 2010
$d^E_{r\phi}$	Harvesting decade of forestry set ϕ
$d^S_{r\phi}$	Planting decade of forestry set ϕ
EL_{jd}	Economic life of technology j purchased at the beginning of decade d (years)
$F_{r\phi scdi}$	Rate of $\mathrm{CO_2}$ accumulation of forestry resource $r \in \mathbb{R}^F$ in set ϕ for a yield scenario s in decade d [$\mathrm{tCO_2}$/ha/yr]
f'^B_{rt}	Fraction of annual yield of crop $r \in \mathbb{R}^B$ produced in season t
f^A_{kd}	Fraction of land area of level k available for biomass production in decade d
f^{BE}_r	Establishment yield fraction of crop $r \in \mathbb{R}^B$
GM_{rscd}	Gross margin for producing crop $r \in \mathbb{R}^B$ for a yield scenario s in cell c in decade d [£ /odt]
$k^\star$	Selected land area level
$\mathrm{LHV_r}$	Lower heating value of resource r [GJ/t]
N^S_r	Number of seasons that resource r can be stored
$N^{\mathbb{T}}$	Number of seasons (in a year)
M_j	Number of modes of technology j
PC^{in}_c	Maximum inward capacity of a port $c \in \mathbb{C}^{\mathrm{ship}}$ [t/yr]
PC^{out}_c	Maximum outward capacity of a port $c \in \mathbb{C}^{\mathrm{ship}}$ [t/yr]
$p^{Gas,\,min}_d$	Fraction of the total energy production as bio-methane
$P^{E,\,min}_d$	Minimum total energy production [MWh/yr]
$P^{E,\,max}_d$	Maximum total energy production [MWh/yr]
$p^{Elec,\,min}_d$	Fraction of the total energy production as bio-electricity
$p^{Heat,\,min}_d$	Fraction of the total energy production as bio-heat
$p^{H_2,\,min}_d$	Fraction of the total energy production as bio-hydrogen
$p^{TF,\,min}_d$	Fraction of the total energy production as transport biofuels
Q^{max}_{rl}	Maximum transport rate of resource r by transport mode l [unit of resource/rate basis]
$q_{\kappa d}$	$\mathrm{CO_2}$ price for a scenario κ in decade d [£/kg]
RDI_{rid}	Unit disposal impact [£/unit of resource or $\mathrm{kgCO_2e}$/unit of resource]
$RF_{jdd'}$	Fraction of capacity of technology j retired in decade d after capacity was first installed in decade d'
RG_{rg}	Binary parameter: 1 if resource r is a member of import group g, 0 otherwise
$RI^{max}_{gd\sigma}$	Maximum rate of import of resource group g in decade d for an import scenario σ [MWh/yr]
$RII_{rid\sigma}$	Unit impact of importing resource $r \in \mathbb{R}^I$ in decade d for an import scenario σ [£/odt or $\mathrm{kgCO_2e}$/odt]
RP^{max}_{rd}	Maximum rate of purchase of resource r from the “grid” in decade d [unit of resource/yr]
RPI_{rid}	Unit impact of importing resource r in decade d [£/unit of resource or $\mathrm{kgCO_2e}$/unit of resource]
RS^{max}_{rd}	Maximum rate of sale of resource r in decade d [unit of resource/yr]
RV_{rdi}	Unit value of resource r in decade d [£/unit of resource or $\mathrm{kgCO_2e}$/unit of resource]
SCI_{rdi}	Unit storage capital impact [£/unit of resource or $\mathrm{kgCO_2e}$/unit of resource]
SOI_{rdi}	Unit storage operational impact [£/unit of resource/season or $\mathrm{kgCO_2e}$/unit of resource/season]
$s^\star$	Selected yield scenario
TCI_{jdi}	Unit technology capital impact [£/unit of capacity or $\mathrm{kgCO_2e}$/unit of capacity]
TDF_{jdi}	Techology Discount Factor: discounts the capital cost of technology j to the beginning of purchased decade d

TOI^{f}_{jdi}	Fixed unit operating impact for technology j in decade d [£/unit of capacity/yr or kgCO$_2$e/unit of capacity/yr]
TOI^{v}_{jdi}	Variable unit operating impact for technology j in decade d [£/unit of capacity/yr or kgCO$_2$e/unit of capacity/yr]
$TrOI_{rldi}$	Unit impact of transport of resource $r \in \mathbb{R}^{Q}$ by mode l in decade d [£/t/km or kgCO$_2$e/t/km]
$u^{\star}$	Selected land area ramp-up rate scenario
$WASI_{cdiw}$	Unit impact of separating “Waste-All” into its components, [£/odt or kgCO$_2$e/odt]
WP_{rcd}	Waste potential of waste resource $r \in \mathbb{R}^{CW}$ in cell c in decade d [unit of resource/yr]
WUI_{ridw}	Unit waste utilisation impact of resource $r \in \mathbb{R}^{W}$ in decade d for a cost scenario w [£/t or kgCO$_2$e/t]
x_{rcd}	Mass fraction of component $r \in \mathbb{R}^{CW}$ of “Waste-All” in cell c in decade d
Y_{rscd}	Maximum yield of resource $r \in \mathbb{R}^{B}$ for a yield scenario s in cell c in decade d [odt/ha/yr]
$Y^{F}_{r\phi scd}$	Maximum yield of resource $r \in \mathbb{R}^{F}$ in forestry set ϕ for a yield scenario s in cell c in decade d [odt/ha/yr]
y^{Ro}_{r}	Number of winter wheat years in a single year of rotated crop $r \in \mathbb{R}^{B}$
β_{rd}	Minimum fraction of the demand for resource r to be satisfied by biomass in decade d
$\Gamma_{cc'l}$	Feasible transport connections between cells c and c' for a transport mode l
Υ^{BPI}_{rdi}	Uplift/downlift factor for the cost of producing crop $r \in \mathbb{R}^{B}$ in decade d
$\Upsilon^{TCI}_{j_F d}$	Uplift/downlift factor for the capital cost of technology family j_F in decade d
Υ^{Y}_{rd}	Uplift/downlif factor for the yield potential of resource $r \in \mathbb{R}^{B}$ in decade d
α_{jmdr}	Conversion factor for resource r in technology j when operating in mode m in decade d
γ_{jmdr}	Cofiring fraction for resource r in technology j when operating in mode m in decade d (1 for all technologies except for cofired technologies)
$\zeta_{\kappa di}$	Objective function weight including CO$_2$ price for a scenario κ in decade d
$\theta_{j_F j}$	Binary parameter: 1 if technology j belongs to family j_F, 0 otherwise
ϑ_{r}	Binary parameter: 1 if the value of resource r is to be included in the revenue, 0 otherwise
ι	Discount rate
λ_{r}	Land area level upto which crops $r \in \mathbb{R}^{B}$ can be planted
μ_{r}	Mass fraction of water in resource r
$\varpi^{\mathrm{CCS}}_{jmdi}$	Unit CCS CO$_2$ credits [kgCO$_2$/MWh output]
ρ_{r}	Density of resource r [kg/m^3]
ϱ_{r}	Total fraction of resource r lost after storing for the full number of seasons
ς_{i}	Scaling factor ($1{\times}10^{-6}$)
τ	Finance rate
ν^{dps}_{t}	Number of days in season t
ν^{dpY}	Number of days in a year
ν^{YpD}	Number of years in a decade
ν^{hpd}	Number of hours in a day
ν^{hpY}	Number of hours in a year
$\sigma^{\star}$	Selected import scenario
χ_{r}	Exergy per unit energy for resource r
ψ_{c}	Ratio of straw to grain mass in cell c
ω^{E}	Weight of energy production in the objective function
ω^{I}_{i}	User-specifed objective function weight for impact i

ω^X	Weight of exergy production in the objective function
$\mathscr{A}_{jd}$	Fraction of a year at which technology j is available for operation in decade d
$\mathscr{D}^G_{rd}$	Annual average demand for resource $r \in \mathbb{R}^{GD}$ during decade d [unit of resource per rate basis]
$\mathscr{D}^L_{rcd}$	Annual average demand for resource $r \in \mathbb{R}^{LD}$ in cell c during decade d [unit of resource per rate basis]
$\mathscr{R}$	Rate (or time unit) basis [hour, day or year]
$\mathscr{R}^h$	Time unit conversion factor [rate basis per hour]
$\mathscr{R}^Y$	Time unit conversion factor [rate basis per year]
$\mathscr{R}^I_t$	Time unit conversion factor [rate basis per season]
$\mathscr{R}^P_t$	Time unit conversion factor [year per season]
$\mathscr{U}_r$	Unit of resource r [t, m^3 or MWh]
$\mathscr{U}^{\text{MWh}}_r$	Factor that converts the units of resource r to MWh
$\mathscr{U}^t_r$	Factor that converts the units of resource r to tonne

Positive variables

A_{rcd}	Area allocated to crop $r \in \mathbb{R}^B$ in cell c and decade d [ha]
$A^F_{r\phi cd}$	Area allocated to forestry resource $r \in \mathbb{R}^F$ in set ϕ in cell c and decade d [ha]
B_{rcd}	Rate of growth and haversting of resource $r \in \mathbb{R}^B$ in cell c and decade d [odt/rate basis]
B^{straw}_{cd}	Rate of production of winter wheat straw in cell c in decade d [odt/rate basis]
CJ_{jcd}	Total capacity of technology j in cell c and decade d [unit of capacity]
CJI_{jcd}	Capacity investment in technology j in cell c and decade d [unit of capacity]
CJR_{jcd}	Capacity retirement of technology j in cell c and decade d [unit of capacity]
D_{rcdt}	Demand for resource $r \in \mathbb{R}^{LD}$ that is satisfied in cell c in decade d during season t [unit of resource/rate basis]
I_{rcdt}	Amount of resource $r \in \mathbb{R}^S$ in storage in cell c in decade d during season t [unit of resource]
I^{BP}_{di}	Impact of biomass production in decade d [£M/decade or MkgCO$_2$e /decade]
I^{CCS}_{di}	Impact of CCS in decade d [£M/decade or MkgCO$_2$e /decade]
I^{FS}_{di}	Impact of forestry CO$_2$ sequestration in decade d [MkgCO$_2$e/decade]
I^Q_{di}	Impact of transport operation in decade d [£M/decade or MkgCO$_2$e /decade]
I^R_{di}	Revenue from the sale of resources in decade d [£M/decade or MkgCO$_2$e /decade]
I^{RD}_{di}	Impact of disposing of resources in decade d [£M/decade or MkgCO$_2$e /decade]
I^{RI}_{di}	Impact of importing resources in decade d [£M/decade or MkgCO$_2$e /decade]
I^{RP}_{di}	Impact of purchasing resources in decade d [£M/decade or MkgCO$_2$e /decade]
I^{SC}_{di}	Impact of investing in storage capacity in decade d [£M/decade or MkgCO$_2$e /decade]
I^{SO}_{di}	Impact of operating the storage in decade d [£M/decade or MkgCO$_2$e /decade]
I^{TC}_{di}	Impact of investing in new technologies in decade d [£M/decade or MkgCO$_2$e /decade]
I^{TO}_{di}	Impact of operating and maintaining the technologies in decade d [£M/decade or MkgCO$_2$e /decade]
I^{tot}_{di}	Total impact in decade d [£M/decade or MkgCO$_2$e /decade]
I^{WU}_{di}	Impact of utilising waste resources in decade d [£M/decade or MkgCO$_2$e /decade]
P^{Gas}_d	Annual bio-methane production in decade d [MWh/yr]
$P^{E,tot}_d$	Annual total energy production in decade d [MWh/yr]
P^{Elec}_d	Annual bio-electricity production in decade d [MWh/yr]

P_d^{Heat}	Annual bio-heat production in decade d [MWh/yr]
$P_d^{H_2}$	Annual bio-hydrogen production in decade d [MWh/yr]
P_d^{TF}	Annual transport biofuel production in decade d [MWh/yr]
P_{rd}^{L}	Annual average rate of production of local demand resource $r \in \mathbb{R}^{LD}$ [unit of resource/rate basis]
$P_d^{X,tot}$	Annual total exergy production in decade d [MWh/yr]
$Q_{rcc'ldt}$	Rate of transport of resource $r \in \mathbb{R}^{Q}$ between cells c and c' using mode l in decade d during season t [unit of resource/rate basis]
$Q_{cc'dt}^{\mathrm{CO_2}}$	Rate of $\mathrm{CO_2}$ transport from capture cell c to sequestration cell c' during decade d in season t [$\mathrm{MkgCO_2}$/season]
RD_{rcdt}	Rate of disposal of resource r in cell c in decade d during season t [unit of resource/rate basis]
RI_{rcdt}	Rate of import of resource r from port $c \in \mathbb{C}^{\mathrm{ship}}$ in decade d during season t [unit of resource/rate basis]
RP_{rcdt}	Rate of purchase of resource $r \in \mathbb{R}^{LD}$ from the "grid" in cell c in decade d during season t [unit of resource/rate basis]
RS_{rd}^{G}	Rate of sale of resource $r \in \mathbb{R}^{GD}$ in cell c in decade d during season t [unit of resource/rate basis]
RS_{rcdt}^{L}	Rate of sale of resource $r \in \mathbb{R}^{LD}$ in cell c in decade d during season t [unit of resource/rate basis]
S_{rcd}	Storage capacity for resource r in cell c at the beginning of decade d [unit of resource]
S_{rcdt}^{loss}	Amount of resource $r \in \mathbb{R}^{S}$ lost in storage in cell c in decade d during season t [unit of resource]
$V_{cdt}^{\mathrm{captured}}$	Rate of $\mathrm{CO_2}$ capture in cell c during decade d in season t [$\mathrm{MkgCO_2}$/season]
$V_{cdt}^{\mathrm{sequestered}}$	Rate of $\mathrm{CO_2}$ sequestration in cell $c \in \mathbb{C}^{\mathrm{seq}}$ during decade d in season t [$\mathrm{MkgCO_2}$/season]
WAC_{cdt}	The total rate of utilisation (i.e. in technologies) and separation of "Waste-All" [unit of resource/rate basis]
WF_{cdt}	Rate at which "Waste-All" is separated into its components in cell c in decade d during season t [unit of resource/rate basis]
ΔS_{rcd}	Amount of storage capacity for resource r in cell c added in decade d [unit of resource]
$\mathscr{P}_{jmcdt}$	Rate of operation of technology j in mode m in cell c in decade d during season t [unit of main input or output/rate basis]

Free variables

I_{di}^{WU}	Impact of utilisation of wastes in decade d [£M/decade or $\mathrm{MkgCO_2e}$ /decade]
P_{rd}^{G}	Annual average rate of production of global demand resource $r \in \mathbb{R}^{GD}$ in decade d [unit of resource/rate basis]
Z	Objective function

Integer variables

NJI_{jcd}	Number of technologies j invested in cell c in decade d