Quantum Friction – Fact or Fiction?

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Abstract. Two parallel dielectric plates separated by vacuum interact through zero point charge fluctuations and experience friction when the plates are in relative motion and the vacuum sheared. Even at the absolute zero of temperature residual quantum fluctuations remain due to the zero point energy giving rise to ‘quantum friction’. In a recent paper the reality of these fluctuations is questioned and the existence of quantum friction called into question. Here we refute this assertion.

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1. Introduction

In a recent paper Philbin and Leonhardt [1] (henceforth referred to as PL) calculated the frictional forces due to electromagnetic fluctuations between two perfectly flat parallel dielectric surfaces separated by vacuum. In what was claimed to be an exact calculation their conclusion was that, at zero temperature where the only fluctuations are quantum in nature, friction is precisely zero. This result contradicts a substantial body of earlier work and in this paper I argue for the correctness of the earlier results and point to errors in the reasoning of PL.

It is well known from classical electromagnetism [2-5] that a distribution of electrical charges outside a dielectric surface induces image charges of the opposite sign which in turn exert an attractive force on the external charge distribution. It is also well known that if the charges move parallel to the surface, the images lag behind tending to pull the charges back. This frictional work done by the distribution is dissipated in the electrical resistance of the dielectric.

Now consider two perfectly flat parallel dielectric surfaces separated by vacuum. On each surface the zero point energy of the electrons will give rise to charge fluctuations and these induce attractive images charges in the other dielectric. This mechanism is responsible for the Van der Waals forces between two neutral dielectrics. They are the most long range of the forces between neutral surfaces and are entirely quantum in nature and their reality is well established. Building on the Van der Waals argument, several authors have argued [6-14] that when two surfaces are set in relative parallel motion, the Van der Waals forces acquire a frictional component as the image charges lag behind the
original fluctuations. Other authors consider the case of a neutral atom moving parallel to a dielectric surface [15,16] and also find finite friction at absolute zero. This is the frictional force denied by PL.

The point at issue here is the reality of zero point charge fluctuations. To the extent that they are as ‘real’ as a collection of classical charges moving over a surface then quantum friction undoubtedly exists. PL argue that quantum charge fluctuations are special in some way and cannot be used to arrive at a correct description of quantum friction.

The strategy I follow is to propose a simple model dielectric, consistent with the laws of electromagnetism, which can be solved analytically in a full quantum treatment that makes no assumptions about zero point fluctuations. Since PL claim their result is exact and applies to all such models, if only one model contradicts their conclusions then evidently their result is in error.

2. A simple model

A commonly adopted model electrical permittivity is given by,

\[ \varepsilon = 1 - \frac{\omega_p^2}{\left(\omega^2 + i\gamma\right)} \]  

where \( \omega_p \) is the plasma frequency. It describes the response of a free electron gas such as is found in metal. We shall assume that \( \gamma \), representing loss processes, is vanishingly small, and causality demands that it is always positive.

![Figure 1](image-url)

**Figure 1.** Surface plasmon dispersion with wave vector. The excitations are asymptotic to \( \omega_p/\sqrt{2} \) at large wave vectors, and to the light line at small values of \( k \).

The model supports charge density oscillations localised at the surface known as surface plasmons, with a frequency asymptotic to \( \omega_p/\sqrt{2} \) and defined by \( k \) their 2D wave vector in the surface plane.

\[ q_k(r) = A_{c,k} \cos(k \cdot r - \omega t) + A_{s,k} \sin(k \cdot r - \omega t) \]  

(2)
We show their dispersion in figure 1. The surface plasmons are responsible for all interactions with external electromagnetic fields. We shall assume that the material has no magnetic activity so that the magnetic permeability $\mu = 1$.

Quantum mechanically surface plasmons are described by the following Hamiltonian,

$$ H_0 = \sum_{g=s,c} \left( \frac{1}{2} + a_g^+ a_g \right) \hbar \omega_{sp}(k) $$

(3)

where $a_g^+ a_g$ are creation and annihilation operators for the surface plasmons of sin symmetry when $g = s$ and of cos symmetry when $g = c$.

![Figure 2](image)

**Figure 2.** Two perfectly flat parallel dielectric surfaces separated by a distance $d$ in relative motion with velocity $v$. The surface plasmons interact across the gap.

Next introduce a second dielectric surface parallel to the first separated by a distance $d$ as illustrated in figure 2. The new Hamiltonian describes the second surface and its time dependent interaction with the first surface:

$$ H = \sum_{g} \left[ + \left( \frac{1}{2} + a_g^+ a_g \right) \hbar \omega_{sp}(k) \right] + H_{\text{int}} $$

(4)

where,

$$ H_{\text{int}} = \sum_k \left[ -\frac{1}{2} (a_{c_k}^* - a_{c_k}) (b_{c_k}^* - b_{c_k}) \cos(k \cdot vt) + e^{-\frac{|k|}{\hbar} \omega_{sp}(k)} \right] $$

(5)

The interaction term decays exponentially with surface separation, and the time dependent factor is due to the relative motion of the two interacting surface plasmons:
rather like riding over cobbles tones. In the limit of $v \to 0$ the surface plasmons hybridise and form a bond across the gap to give the attractive Van der Waals force. As a consequence of flat surfaces, momentum is conserved and only surface plasmons having equal and opposite wave vectors interact; all other interactions average to zero.

Each oscillator in the quantum system is described by states

$$|n; bgk\rangle \exp\left(-i \left[n + \frac{1}{2}\right] \hbar \omega_{sp} t / \hbar\right)$$

(6)

where $n$ is the number occupancy of the state, $a$ or $b$ refers to the surface on which the oscillator is located, $g$ identifies the nature of the state (sin or cos) and $k$ is the wave vector. We assume that the temperature is zero Kelvin so that the system is initially in its ground state,

$$|\Psi_0\rangle = \prod_{g=s,c, k} |0; agk\rangle \exp\left(-i \frac{1}{2} \hbar \omega_{sp} t / \hbar\right) \prod_{g=s,c, k} |0; bgk\rangle \exp\left(-i \frac{1}{2} \hbar \omega_{sp} t / \hbar\right)$$

(7)

However interaction between the surfaces will create excitations of equal and opposite wave vectors on opposite surfaces, absorbing mechanical energy from the driving mechanism and hence creating friction. These excitations are highly correlated and are the plasmonic equivalent of optical qbits. The total friction is the sum over the energy of all qbits created. Note, that since we start the system in the ground state, each wave vector can make only a zero or positive contribution to energy dissipation. Therefore if any wave vector makes a positive contribution there will be an overall finite frictional effect. We shall choose to work in the regime where,

$$k \gg k_0 = \omega/c_0$$

$$k^{-1} \ll d \ll (\omega/c_0)^{-1}$$

(8)

thus ensuring that relativistic effects are negligible, and that the interaction is weak enough to be accurately treated in first order perturbation theory.

Next consider first order corrections to the ground state,

$$|\Psi_1\rangle = |\Psi_0\rangle + \prod_{g=s,c, k} |0; agk\rangle e^{-i\frac{1}{2} \hbar \omega_{sp} t / \hbar} \prod_{g=s,c, k'} |0; bgk'\rangle e^{-i\frac{1}{2} \hbar \omega_{sp} t / \hbar}$$

$$\times \left[ + A(t; cek) |1; ac k\rangle |1; bck\rangle + A(t; skk) |1; ask\rangle |1; bsk\rangle \right] e^{-i3h\omega_{sp} t / \hbar}$$

(9)

where the second term represents the four difference types of cross excitation possible at wave vector $k$. Substituting in the time dependent Schrödinger equation and applying first order perturbation theory in the usual way [17], we deduce, in the limit of large $t$, 

...
\[
\lim_{t \to \infty} |A(t; cc k)|^2 = \lim_{t \to \infty} |A(t; ss k)|^2 = \lim_{t \to \infty} |A(t; sc k)|^2 = \lim_{t \to \infty} |A(t; cs k)|^2
\]
\[
= \frac{\pi t |\omega_{sp}|^2}{8v} \delta \left( k_x + \frac{|\omega_{sp}|}{v} \right) + \delta \left( k_x - \frac{|\omega_{sp}|}{v} \right) e^{-2|k|d}
\]

In other words the number of excitations grows linearly with time corresponding to a constant amount of frictional work being done. From the energy of the excitations we calculate after integrating over all possible excitations,

\[
F_x = \frac{\hbar \omega_{sp}^3}{2\pi v^2} \int_{-\infty}^{+\infty} \exp \left( -2d \sqrt{\left( \frac{2\omega_{sp}}{v} \right)^2 + k_y^2} \right) dk_y
\]

an expression that is manifestly non zero and therefore proves our case for non zero friction.

### 3. The poor man’s friction formula

In an earlier paper [10] I calculated that the frictional force due to electrical zero point fluctuations is given by,

\[
F_x = \frac{\hbar}{4\pi} \int_0^\infty k_x dk_x \int_{-\infty}^{+\infty} e^{-2kd} dk_y
\]

\[
\times \int_0^{k_{x,v}} \frac{\Im R_2(\omega) \Im R_1(\omega - k_x v)}{\left| 1 - R_1(\omega - k_x v) R_2(\omega) e^{-2kd} \right|^2} d\omega
\]

where I have corrected an erroneous factor of two in the original. The reflection coefficients to p-polarised radiation are \( R_1(\omega), R_2(\omega - k_x v) \) at frequency \( \omega \), and at the Doppler shifted frequency \( \omega - k_x v \). The wave vector of the radiation in the plane of the surface is \( (k_x, k_y) \). For magnetically active media there is a second contribution with s-polarised reflection coefficients used in placed of p-polarised ones. The formula has been the basis of many subsequent calculations and has been extended to finite temperatures by Persson and Volokitin [13].

This formula follows from classical electromagnetism together with the assumption that zero point fluctuations of charge have the same status as classical charges. Although not a rigorous quantum mechanical treatment (hence the designation of ‘poor man’s friction’) the derivation was considerably simpler than those in previous works and also more general.
We now compare the values given by (12) with those calculated in (11) which of course made no assumptions about quantum fluctuations. First we express the reflection coefficient for p polarisation in terms of the dielectric function,

\[
\lim_{k \to \infty} R(k, \omega) = \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 1}
\]  

(13)

and from (1),

\[
\Im R(\omega) = \Im \left( \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 1} \right) = \pi \omega_{sp} \left[ \delta(\omega - \omega_{sp}) - \delta(\omega + \omega_{sp}) \right]
\]  

(14)

The denominator in (12) represents multiple scattering corrections, but since we have assumed weak interaction between the surfaces we shall neglect this correction and calculate,

\[
F_x = \frac{\hbar \omega_{sp}^3}{2 \pi v^2} \int_{-\infty}^{+\infty} \exp \left( -2d \sqrt{\left( \frac{2\omega_{sp}}{v} \right)^2 + k_y^2} \right) dk_y
\]  

(15)

in agreement with the quantum mechanical treatment reported in (11).

4. Reconciliation

At first sight it is surprising that PL conclude that there is zero friction because the formalism they use is almost identical to the one I used to derive the ‘poor man’s friction’ result presented above. This approach treats multiple scattering exactly within classical electromagnetism, but introduces quantum fluctuations in an ad hoc fashion rather than in a full quantum, but more complex, formalism as given in section 2.

To understand how different conclusions come about from almost identical formalisms let us consider how I arrived at (12). In my earlier paper, like PL, I start with an integration over frequency along the entire positive real axis,

\[
F_x = \frac{\hbar L^2}{\pi (2\pi)^2} \int_0^{+\infty} k_x dk_x \int_{-\infty}^{+\infty} \exp(-2kd) dk_y
\]  

(16)

\[
\times 2 \int_0^{+\infty} \left[ \Im R_{1pp}(\omega + k_x v) - \Im R_{1pp}(\omega - k_x v) \right] \Im R_{2pp}(\omega) d\omega
\]

Up to this point I am in agreement with PL. In fact apart from differences in notation our formulae are almost identical.
Figure 3. Left: cuts in the complex plane for the three reflection coefficients appearing in equation (16). Note that the Doppler shift moves the cuts by $\pm k_x v$. Right: the position of the cuts assumed by PL.

The reflections coefficients, $R_{pp}(\omega)$, have analytic structure in the complex frequency plane characterised by a cut that runs below the positive real axis and above the negative real axis. However because of the Doppler shift in the moving frame the cuts are shifted by $\pm k_x v$ as shown in figure 3. This is where the two derivations part company because PL assume that all cuts fall at $\Re\omega = 0$ as shown to the right of figure 3 and as also shown in figure 2 of their paper.

Figure 4. Rotation of the contour for frequency integration should include a contribution from the Doppler cut. This contribution does not appear in PL.

Consequently when in the next step PL rotate the contour of integration by $90^\circ$ they miss the Doppler induced green cut sticking out into the positive frequency axis. See figure 4. This cut is responsible for the quantum friction.

In conclusion: a straightforward quantum mechanical calculation for a simple model surface shows that friction is finite even at zero temperature and is in quantitative agreement with most previous approaches to the problem, but in contradiction to the conclusions of PL. The cause of the discrepancy is identified as a failure by PL correctly to account for modification of analytic structure found in the complex frequency plane when two surfaces are in relative motion.
References