Harmonic Mitigation Throughout a Distribution System: A DG Based Solution

N. Pogaku∗ T. C. Green†
Department of Electrical and Electronic Engineering
Imperial College, London. Exhibition Road, SW7 2BT.

Abstract

This paper investigates the use of ancillary services from inverter-interfaced distributed generators (DGs) to achieve harmonic mitigation across a network. The approach is to include the functionality of a resistive active power filter (R-APF) within several DGs. The R-APF provides adjustable damping at harmonic frequencies. In a realistic network, which has feeder sections of different characteristic impedances, it is impractical to damp with precisely that resistance. Instead, feeders are split into harmonic sections based on the standing waves of the highest order harmonic and DG ancillary services are called up for each section. Coordination of services from each DG is arranged through adaptation of the harmonic resistance according to target THD levels. The primary purpose of each DG is the supply of real power and this is respected through a further aspect of the resistance adaptation which reduces the harmonic duty of an individual DG if it approaches the apparent power rating of the inverter. The harmonic VA required of the inverter is dependent on both the chosen harmonic resistance and the harmonic voltage component present at the connection bus. The system is demonstrated through a simulation of an irregular feeder using Simulink and PLECS.

Index Terms Distribution System, Harmonic Propagation, Harmonic Damping, Resistive Active Power Filter (R-APF), Distributed Generation (DG)

∗N. Pogaku is a Ph.D. student in the Department of Electrical and Electronic Engineering, Imperial College, London, UK (email: nagaraju.pogaku@imperial.ac.uk)
†T. C. Green is a reader in the Department of Electrical and Electronic Engineering, Imperial College, London, UK (email: t.green@imperial.ac.uk)
1 Introduction

Harmonic currents originating from non-linear loads cause harmonic voltage drops that can propagate throughout a power distribution system. The propagation of these voltages through feeders depends on the harmonic impedances of the system and if significant capacitance is present then resonant amplification of certain harmonic orders is a particular concern for system operators. There are two obvious ways to tackle this: (i) reduce the harmonic excitation from the loads and (ii) modify the network impedances to move, remove or damp resonance. Although newly purchased individual pieces of equipment should meet standards such as IEC61000, the aggregate load on a distribution feeder can still draw harmonic currents of sufficient magnitude to cause excitation of harmonic resonance. The problem is most likely to occur where line impedances are high, such as in low voltage distribution, and where shunt capacitors have been added for reactive power compensation. The combination of shunt capacitors and line inductance provides resonant amplification of the harmonic voltages across a certain frequency range [1]. The changing nature of loads affects how prominent this problem becomes. Traditional resistive loads (notably heating and lighting loads) are not significant harmonic sources and also provide damping (dissipation) at harmonic frequencies. Modern electronic loads are very different: simple line-frequency rectifiers emit harmonic currents and some loads act as constant current or constant power loads and so do not provide damping at harmonic frequencies. However, loads that employ single-phase unity power-factor pre-regulators often use the voltage waveform shape as the reference for the current waveform and are thus resistive loads to harmonics within their control bandwidth.

Traditional active power filters (APFs) have been proposed [1], [2], [3] for location adjacent to major non-linear loads to provide equal but opposite harmonic currents and thus prevent the flow of harmonic currents into the power network. There are two problems with this approach. First, there is a case for consumer-funded APFs for specific high power non-linear loads but this approach does not suit a collection of many small non-linear loads that individually meet the regulations but which emit correlated low-order harmonic currents. Second, most forms of APF compensate the local load current and do not assist
with existing network voltage distortion [4]. Moreover, those APFs based on instantaneous reactive power theory contribute additional harmonic distortion if the network voltage is distorted or unbalanced.

The utility network operator faces a difficult challenge if the aggregate harmonic distortion interacts with resonances in the network impedances and violates harmonic voltage limits. Conventional means of mitigating harmonics can be difficult to apply because the pattern of harmonic current sources, the configuration of the network and the connection of power factor correction capacitors are all time varying. The challenge is to provide active devices such as power line conditioning APFs with control algorithms that provide harmonic mitigation across a utility network. This requires that the control actions of multiple APFs support each other and that there are no unstable interactions. Ideally this should be done without the need for a communication infrastructure and in particular without the need for central coordination and synchronisation.

In the literature several techniques have been proposed to mitigate the harmonics throughout distribution networks. Some of the reported methods use optimisation algorithms to compute the compensation reference for each APF in a network [5], [6], [7], [8], [9]. This rests on having available harmonic impedance data for each branch and measurements of harmonic voltage at each bus neither of which are easy to achieve in real time. An important theme of work has been the resistive termination at the end of a radial feeder [10], [11], [12], [13]. An acknowledged problem is that if the feeder is electrically long and the harmonic order is high (the harmonic wavelength short) then harmonic standing waves can not be attenuated along the full length of the feeder by resistive termination at the end of the feeder only. If the feeder is a regular LC ladder of a consistent characteristic impedance, the termination with a single resistance would be effective. From the economic point of view, installing dedicated APFs at intervals along a feeder is not attractive to a utility network operator. Nonetheless, resistive APFs which divide a long radial feeder into small electrical sections (at harmonic frequencies) and effectively damp the harmonics in those sections would be attractive from the technical point of view.

In this paper we propose an approach based on multiple (harmonic) resistive elements throughout a network where those elements are additional services provided by the inverters
of distributed generators (DGs). The expected uptake of DG provides an opportunity for several novel ancillary services because DG inverters can be expected to have a sophisticated local controllers, instrumentation and, for much of the time, some unused apparent power rating while the real power export is below the rated value. They will also be present in several locations across the network and often close to load centres that require services. The suggestion is that a distribution network operator (DNO) faced with an out-of-limits harmonic voltage on a feeder could contract DGs to provide harmonic damping services in preference to fitting its own active or passive harmonic mitigation devices.

There are several challenges to address. First, a methodology needs to be developed to establish which DGs are well positioned to supply harmonic mitigation services for a feeder section and should be contracted to do so. Second, a means is required of adaptively varying the harmonic damping to suit the locally observed conditions while maintaining stable operation within a network containing several such devices. Third, a means is required to allow a DG to reduce its harmonic mitigation contribution at times when all (or almost all) of its apparent power rating is required for real power export. Fourth, it may be necessary for DGs to vary their mitigation contribution depending how much mitigation the DNO contracts to buy.

2 Harmonic Mitigation using Voltage Detection Based Resistive Active Power Filters

Damping of a harmonic resonance or attenuation of harmonic standing wave requires a dissipative, that is resistive, element. This resistive element should only respond to harmonic excitation and should not consume power at the fundamental frequency of the distribution network. Akagi [10] has proposed suitable device composed of a shunt APF that is based on voltage detection. The voltage measured at the point of connection is filtered to remove the fundamental component (and perhaps other components). The APF is then controlled to draw a current proportional to the instantaneous voltage.

\[ I_{APF} = K_v \cdot V^h \] (1)
where, $V^h$ is the harmonic component of the voltage at the point of connection and the gain $K_v$ is the conductance offered the harmonic content in the voltage.

The resistive APF (R-APF) can be viewed as both providing a local low impedance path for harmonic currents emitted by loads and providing damping to harmonic voltages propagated from remote distorting loads. The R-APF proposed by Akagi was aimed at utility applications to provide quality power to the consumers by eliminating the propagation of background harmonics in networks with significant power factor correction capacitors. It was concluded in [10] that voltage detection in time domain was the most suitable approach in terms of stability for shunt active filters in power distribution systems.

2.1 Standing Waves in LC Ladder Networks

Wada et al [12] explained a curious phenomenon that may occur in an electrically long distribution feeder having many PFC capacitors. The installation of R-APFs on a feeder was observed to decrease the harmonic voltages at some buses (particularly close to the point of installation) but may increase harmonics at other buses. Wada termed this the *wack-a-mole* effect. The system that was studied was a long transmission line with distributed PFC capacitors at load buses as shown in Fig.1 and it was assumed that the PFC capacitors were of equal value and placed at regular intervals, i.e., $L_{1-2} = L_{2-3} = \cdots L_{9-10} = L$ and $C_2 = C_3 = \cdots C_{10} = C$.

There are two types of harmonic sources to consider in such a network

1. Harmonic voltage source existing at bus1 as shown in Fig.2(a), which represents a propagated distortion from the external system.

2. Harmonic current source existing part way along the feeder as shown in Fig.2(b) which represents a local non-linear load.

The analysis presented approximated the ladder network with a distributed parameter transmission line (similar to a co-axial cable) with negligible series resistance. With the R-APF placed at the far end of the feeder (bus 10 in this example) with a resistance equal to $1/K_v$ at harmonic frequencies, the harmonic voltage standing-wave at any point $x$ on the feeder in the presence of $V^h_S$ at the source end of the feeder (bus 1) is given by,
\[ V_h(x) = \frac{\cosh \gamma (l - x) + K_v Z_c \sinh \gamma (l - x)}{\cosh \gamma l + K_v Z_c \sinh \gamma l} \frac{Z_c I_S^h}{V_S^h} \] (2)

When a harmonic current source \( I_S^h \) is placed at distance \( l_S \) a standing-wave voltage is established that is defined by (3) for the section \( 0 \leq x \leq l_S \)

\[ V_h(x) = \frac{(\cosh \gamma (l - l_S) + K_v Z_c \sinh \gamma (l - l_S)) \sinh \gamma x}{\cosh \gamma l + K_v Z_c \sinh \gamma l} Z_c I_S^h \] (3)

and by (4) for \( l_S \leq x \leq l \)

\[ V_h(x) = \frac{(\cosh \gamma (l - x) + K_v Z_c \sinh \gamma (l - x)) \sinh \gamma l_S}{\cosh \gamma l + K_v Z_c \sinh \gamma l} Z_c I_S^h \] (4)

where, \( Z_c \) is the characteristic impedance of the line (5), \( \lambda \) is the wave length (6) and \( \gamma \) is the propagation constant (7).

\[ Z_C = \sqrt{L/C} \] (5)

\[ \lambda = \frac{1}{h f_1 \sqrt{LC}} \] (6)

\[ \gamma = j w \sqrt{LC} \] (7)

Not only has the series resistance been neglected but so too has shunt impedance of the loads at each bus. This can be considered as a worst case in which the loads are not consuming but the PFC capacitors are left in place. In fact, the network is more realistic than it might seem. Even with a line fully loaded with resistive loads, the shunt resistance is still large compared with typical values for \( Z_C \). It is also important to note that the wavelength of the fundamental is short if either the series inductance or shunt capacitance is large, such as might be the case in low voltage feeders, and that the wavelengths at harmonic frequencies are proportionately shorter.
2.2 Investigation of Example Feeders and Terminations

As a first step, a regular ladder network of 10 buses was investigated based on a 400 V system (whereas Wada used a 6.6 kV example). The chosen system parameters are given in Table 1. Fig.3 shows the harmonic voltage along the feeder when a 5th harmonic voltage source equal to 2.5% of the fundamental magnitude is present at bus 1. The voltage is normalised to the excitation voltage (in other words, the voltage magnification factor has been plotted). Four termination cases are shown. With no termination at bus 10 (open-circuit termination), a standing wave is established that means that the harmonic voltage rises along the feeder to reach approximately 7 times the excitation voltage by bus 10. The short-circuit termination reduces the harmonic voltage at bus 10 to zero and pulls down the harmonic voltage at all of the intermediate buses. Termination with the characteristic impedance gives an almost level compensation but with a slight attenuation due to the non-negligible line resistance. The fourth case is a termination with an adaptive R-APF as suggested by Akagi. The R-APF adjust its gain (conductance) to achieve attenuation of the harmonic voltage to a given value. In this case the target was set equal to the magnitude at the other end of the feeder and flat compensation is achieved.

The results in Fig.3 are a simple case and suggest that short-circuit termination is a good approach. However, the open-circuit case shows that this is a relatively short bus such that even at 5th harmonic it is shorter than a quarter wavelength. Even the move to considering the 7th harmonic, Fig.4 shows a more complex picture. In this case the short-circuit termination causes harmonic amplification at buses close to the source. Termination with the characteristic impedance is better but in practice this is not easy to achieve because the impedance might not be known or might not be an appropriate measure for an irregular network. Fig.4 also shows the result from termination with an adaptive R-APF. Three different target distortion levels were studied. In this case, a target set just below the source level (2% compared to 2.5%) gave the best result but this is not always the case. Although Wada’s ‘wack – a – mole’ is present for some termination resistances because the feeder is longer than 1/4 \( \lambda_7 \), termination with a resistance close to the characteristic impedance result in a flat propagation profile over distances greater than 1/4 \( \lambda_h \)
Fig. 5 shows an example of a harmonic current source at a load bus (bus 5) and with R-APF termination at bus 10. In this case, short-circuit termination is particularly bad (unlike in the case of Fig. 3), open-circuit performs well and termination with the characteristic impedance gives level compensation. The adaptive termination does not perform well in this case because although the target voltage distortion (2.5%) is achieved at the termination bus, it does not result in termination with a resistance close to the characteristic impedance.

A real distribution system presents some problems not apparent in the simple case of the regular ladder network. First, loads change frequently, shunt capacitors are switched to compensate the loads and network switching changes the feeder topology. The characteristic impedances is therefore not a fixed value and will not be known a priori.

2.3 Discussion

The voltage detection based R-APF discussed in this section was aimed at providing damping for the propagation of harmonics in distribution systems. Some important observations are listed below:

1. The analysis given in this section assumes a simplified distribution system. The distribution system is assumed to be radial and PFC capacitors are evenly distributed along the feeder so that it resembles a long transmission line. Practical distribution systems are branched and meshed and the placement of PFC capacitors will be irregular. The analysis discussed in this section may not be valid for the general case.

2. In [12] it was concluded that if the feeder length exceeds $1/4 \lambda_h$ for a given harmonic order $h$, installing R-APF at the end bus of the feeder cannot provide the adequate damping for the propagation of the harmonics in the feeder. The action of R-APF may lead to a - mole effect.

3. The automatic gain adjusted R-APF is a good solution for dynamically varying harmonic propagation. However, installing only a single R-APF at the end of the feeder does not ensure the mitigation of voltage harmonics throughout the feeder in all cases. It is possible for flat compensation to occur beyond $1/4 \lambda_h$ but this requires a distortion target that happens to cause the R-APF to adapt to a resistance approximately
equal to the characteristic impedance.

3  Methodology for Multiple Harmonic Dampers

The conclusion in section 2 was that a single R-APF installed at the end of the radial feeder may not be effective for a wide range of harmonics and system parameters unless a means of adapting to the characteristic impedance. Even so it can not be guaranteed to work if the network is irregular and standing waves form in complex patterns. The proposal is to provide harmonic mitigation in local feeder sections by identifying electrical distances similar to the harmonic quarter wavelengths $1/4 \lambda_h$ of the regular network. From this one can identify the buses at which R-APFs should be deployed to provide damping.

The method proceeds in the following steps.

1. A worse case network model is formed with all PFC capacitors in place and loads impedances assumed infinite.

2. The order, $h^*$ of the highest order harmonic of concern is identified.

3. A harmonic voltage of order $h^*$ is connected at bus1 of the feeder and the propagation of this voltage along the feeder is observed.

4. The locations of the maximum and minimum harmonic voltage (the nodes and anti-nodes of the standing wave) are identified as electrical distances similar in form to $1/4 \lambda_{h^*}$.

5. Adjustable gain R-APFs are connected at or close to each of these locations.

The method will ensure that harmonic voltages of order up to $h^*$ are damped to below the source voltage throughout the feeder. This solution is independent of the location and type (voltage or current) of harmonic source. As noted in section 2, the wave length of any frequency component has an inverse relationship to the capacitance. Hence, this solution remains valid even if any of the capacitors in the feeder are disconnected. Using the no-load condition for the initial assessment of the feeder is appropriate because the harmonic propagation is worst under this condition[10]. Even with full rated loading, the resistance
presented to the harmonics will be significantly greater than the characteristic impedance of the feeder. Hence, the damping provided by the loads for harmonic propagation is insignificant compared to the damping provided by R-APFs.

The test system was modified according to Table. 2 to give a more realistic irregular pattern of feeder sections and bus capacitances. The system parameters chosen here are obtained by slight modification of the system considered in [13]. Line inductances are in the range of 2.5% to 5% and PFC capacitors are in the range of 3.5% to 10.5% compensation. For the test case, the $17^{th}$ harmonic is taken as the highest order harmonic of concern. A $17^{th}$ harmonic voltage source of 2.5% was connected at bus 1 and the propagation of this voltage found to be that shown by the curve marked w.o.r. in Fig.6(a). Voltage maxima were found at buses 3 & 7 and voltage minima at buses 5 & 9 which indicates that the feeder is long in terms of the $17^{th}$ harmonic. The four buses are chosen as sites for resistive damping via an APF. Each of these R-APFs is controlled to keep the individual harmonic voltage level below 2.5% at the point of connection. This configuration was investigated through a time-domain simulation performed using MATLAB-SIMULINK and PLECS [18]. In the presence of these R-APFs, the $17^{th}$ harmonic voltage throughout the feeder kept below the source harmonic voltage as also shown in Fig.6(a). The dashed line indicates the target level, 2.5%, for the individual harmonic voltage of the R-APFs. To test the ability of this system to maintain good harmonic mitigation under varying system parameters, the capacitor at bus 6 was disconnected with the same R-APFs operating. It can be seen from Fig.6(a) that, the $17^{th}$ harmonic voltage throughout the system still maintained below the source harmonic voltage.

Having allocated the R-APF locations on the basis of the propagation of the $17^{th}$ harmonic, it is important to verify that harmonic mitigation of lower order harmonics (with longer $\lambda_h$ values) is indeed ensured. For this test, $13^{th}$, $11^{th}$ and $7^{th}$ harmonic current sources of 14 A each were connected at bus 6, bus 4 and bus 4, respectively and the results are shown in Fig.6(b)-(d). In all these conditions, the harmonic voltages at all the buses are less than the harmonic voltage at the source buses, validating the proposed method. In this particular case, harmonic voltages throughout the system are below the 2.5% target set for the R-APFs for all the harmonic orders below $h^*$. One interesting observation is that, in all
of these cases, the bus where the harmonic source exists is the bus with the peak residual
distortion after compensation by the R-APFs (as indicated with an arrow in Fig.6).

Fig. 7 examines the case of multiple distorting loads present at the same time. This
tests whether the adaptation of the APF resistance to a single value acts to properly damp
more than one source. A 7th harmonic current source was connected at bus 2 and a 13th
harmonic source at bus 6. It can be seen that the 4 R-APFs placed on the basis of the
17th harmonic standing wave provide effective damping throughout the feeder for multiple
sources at lower orders.

Proper choice of the harmonic order to use when choosing locations of the R-APFs
is crucial. If the order is lower than the order of a significant harmonic present in the
real system then the R-APFs will be placed further apart than is necessary to ensure
damping of this higher harmonic. Fig.8 illustrates such a case. The same irregular network
(Table. 2) was considered with the assumption that the 13th harmonic was the highest order
present. Voltage excitation at bus 1 revealed three nodes/anti-nodes and three R-APFs were
connected, one at each of the buses 4, 7 and 9. A 7th harmonic current source of 14 A was
placed at bus 3. Fig.8(a) shows that without the R-APFs a standing wave with a peak
voltage amplitude of 8% is established whereas with the R-APFs operating the voltage
distortion remains below the target value of 2.5% throughout the feeder except at bus 3
itself. Because the excitation is at a lower frequency than the design case the results are
good. Fig.8(b) illustrates what happens when this is not the case. A 23rd order harmonic
current source of 3.5A was connected at bus 2. The voltage standing wave with the R-APFs
active is little different from that with them inactive. In particular, the harmonic voltage at
buses 3 and 5 remains higher than that at bus 2 (where the harmonic source is connected)
indicating that the resonant amplification has not been sufficiently damped.

4 Harmonic Mitigation Services from Distributed Generators

The emphasis on new and renewable forms of energy and the consent difficulties in reinforcing
network infrastructure suggests that more distributed generation will be connected to
networks in the coming years. Several of the forms of DG, such as micro-turbines and fuel cells, are connected to the distribution network via an interfacing inverter. The inverter is a very versatile unit and while its main function will be to control the export of real power it may also be called upon to provide reactive power support as an ancillary service to the DNO or as a condition of connection. This can be taken further by exploiting the control capabilities of the inverter to provide other ancillary services. It is possible that power quality services could be traded in a network containing controllable loads [14]. In terms of the control for R-APF operation, it may or may not be necessary to supplement the DG instrumentation with bus voltage sensors. There is growing interest in designing interface inverters to provide services such as the elimination of harmonic distortion, unbalances and voltage dips [15],[16],[17].

4.1 Integration of DG and R-APF

A schematic showing the addition of an R-APF function to a DG is shown in Fig.9. A standard power controller sets a reference value for the fundamental frequency current. An additional control block, the Harmonic Reference Calculation block, filters the bus voltage and multiplies this by an adaptable gain to form a harmonic current reference. The adaption method follows that described in [13]. The reference $THD^*$ is multiplied with the fundamental voltage $V_1$ to produce a reference harmonic voltage $V_{h*}$. Measured harmonic voltage $V_h$ is then compared with reference $V_{h*}$ in the GainAdj. block, and the gain $K_v$ is adjusted till $V_h$ falls below $V_{h*}$. The sum of the two current references is passed to a standard current control loop. In this diagram it is assumed that the power source provides a stiff DC voltage.

The operation of the R-APF as part of a DG was tested by simulation in the 400V irregular network of Table 2 with 70% loading and a DG connected at the far end of the feeder (bus 10). All the reference calculations were implemented in the $d$-$q$ reference frame. A 7th, a 11th and a 13th order harmonic components of each 2.5% are connected at bus 1 and a 5th harmonic current source of 5.75 A is connected at bus 8. Fig.10 shows what happens when the R-APF action is switched on. This occurs at $t = 0.5 \, s$. The gain of the R-APF function, $K_v$ rises and the harmonic voltage at bus 10 (top axes) falls to below the
3% limit. At $t = 2 \, \text{s}$, the DG is given a fundamental frequency real power reference of 15kW. The instantaneous real power output of the inverter, which is the sum of fundamental and harmonic terms (bottom axes), is seen to be the sum of a slowly varying offset and an oscillatory term.

### 4.2 Adaptive gain adjustment under peak fundamental power demands

The assumption thus far had been that the DG has sufficient apparent power rating to be able to export fundamental real power and import harmonic real power. Under conditions of peak power export the instantaneous phase current may exceed the ratings if the damping current is also large. Since the main objective of a DG is the real power export, the harmonic damping will need to be curtailed in some circumstances to ensure the apparent power rating is not exceeded. The harmonic power required for damping the voltage harmonics varies from time to time and depends on both the amplitude of harmonic voltage at the point of connection and the gain $K_v$. Under the peak fundamental power demand, the harmonic power supplied by the inverter has to be decreased by reducing $K_v$. This can be achieved with an additional current limiting mechanism in the inverter reference calculation which monitors and controls the harmonic power.

A simple current limiter scheme is illustrated in Fig.11. A current limiting outer loop (shown with thick lines in Fig.11) is added to the harmonic current reference block. The current limit $I_{dq}^{max}$ is obtained from the inverter rating. The inverter fundamental current reference $I_{dq}^*$ is subtracted from the total inverter current limit $I_{dq}^{max}$ to produce a harmonic current limit $I_{dq}^{h \, max}$. This reference is then compared with the actual harmonic current $I_{dq}^h$. If $I_{dq}^h$ is less than $I_{dq}^{h \, max}$ then gain adjustment is performed as normal based on the detected harmonic voltage. If $I_{dq}^h$ exceeds $I_{dq}^{h \, max}$, the prevailing value of the gain $K_v$ is decreased until $I_{dq}^h$ falls below its limit and the total inverter current falls below $I_{dq}^{max}$. During this ramping down of $K_v$, the instantaneous current limit applied to the total current reference will be active. Fig. 11 makes clear that the first preference in the allocation of apparent power is given to the fundamental power export. It may not be possible, therefore, to achieve sufficient harmonic damping when peak fundamental power is requirements.

A simulation was performed in a similar way to that in Section 4.1 but this time the
real power export was increased in two steps until the inverter approached its current limit of 29 A\textsubscript{rms}. The current limit is equivalent to 50 A in the power-conserving $d$-$q$ frame used here. Simulation results are shown in Fig.12. Initially the inverter is inactive and the bus voltage is seen to contain over 25 V of harmonic distortion (bottom plot). The inverter is activated at $t = 0.6$ s with a fundamental real power reference of 18 kW (top plot). The real power quickly responds to this and the current limit for harmonic mitigation falls (second plot). Gradually the gain of the R-APF rises $K_v$ (third plot) and the distortion $V_{dq}^h$ falls to its target of 3.0% and the current remains within its limit. It can be observed that when the fundamental power increases from 0 to 18 kW the measured harmonic voltage oscillated slightly because of the high pass filter response characteristics in the harmonic identifier. At $t = 2.05$ s the fundamental real power reference is stepped up to 19.5 kW; the harmonic current limit $I_{dq}^{h, max}$ falls to below the prevailing harmonic current $I_{dq}^h$ and the R-APF gain is reduced in response. The R-APF action is still present but is not longer able to keep the distortion of the bus voltage at its target. The following simple calculation reveals the action taken by adaptable gain controller.

The relation between the inverter apparent power and the various components of the power is given by

$$S = \sqrt{P_1^2 + Q_1^2 + D^h}$$  \hspace{1cm} (8)

where $P_1$ is the fundamental real power, $Q_1$ is the fundamental reactive power (which is set to zero in this case) and $D^h$ is the distortion power. For normal operating conditions of the DG and R-APF function, the dominant term in the distortion power will be the product of fundamental voltage with the harmonic current drawn by the filter. This is the ratings price that must be paid to allow the (dissipative) harmonic damping power, the product of harmonic voltage and harmonic current, to be drawn by the R-APF. The damping power itself is relatively small but, of course, crucial. The remaining term, the product of the fundamental current of the DG and harmonic voltage of the network is small and would be present even if the R-APF function were not present. In Fig. 12 the inverter fundamental power reference was set to 18 kW and full compensation (to the 3% target) was achieved.
with a conductance \((K_v)\) of 1.25 S. From this the approximate distortion power can be calculated as

\[
D^h \simeq \sqrt{3} \times 400 \times \left(\frac{400}{\sqrt{3}} \times 0.03 \times 1.25\right) = 6000 \text{ VAR}
\]

and the apparent power output is

\[
S \simeq \sqrt{18000^2 + 6000^2} = 18974 \text{ VA}
\]

which is well below the 20 kVA limit. Now, to achieve full compensation at a fundamental power reference of 19.5 kW the inverter output has to be

\[
S \simeq \sqrt{19500^2 + 6000^2} = 20400 \text{ VA}
\]

which exceeds the inverter limit. Hence, the current limiter was activated to decrease the distortion power by decreasing the conductance to 0.54 S which led to an increased distortion component in the bus voltage (4.45%). Under this condition, the output distortion power is

\[
D^h = \sqrt{3} \times 400 \times \left(\frac{400}{\sqrt{3}} \times 0.0445 \times 0.54\right) = 3845 \text{ VAR}
\]

and the apparent power output is

\[
S = \sqrt{19500^2 + 3845^2} = 19875 \text{ VA}
\]

which is approximately equal to the inverter rating. This confirms that the adaptable gain controller can achieve a good sharing of fundamental and harmonic powers with a simple current limiting mechanism.

### 4.3 Contribution of multiple DG inverters to harmonic mitigation

The argument has been that there will be several DG inverters in a network and thus if one of them reduces its harmonic mitigation action because of the sort of current limiting described in Section 4.2 the harmonic action of others could be increased. The test to be
described here is whether the adaptation mechanism of the R-APF element can achieve this. Fig. 13 shows the results of a simulation in which two DGs each with an R-APF function are connected at bus 10. Initially DG1 was supplying 19 kW and DG2 was supplying 5 kW. Both DG inverters had spare apparent power capacity and so they shared the harmonic current duty equally and were able to keep the harmonic voltage below the 3.0% target. This is equivalent to connecting two equal (harmonic) resistances in parallel. At $t = 1.12$ s the fundamental power command to DG1 was increased to 19.8 KW and the harmonic current limit within DG1 was decreased to accommodate the increased fundamental current. To realise this, the gain $K_{e1}$ was decreased until the harmonic current met its limit. The consequence of this is that the harmonic content of the bus voltage rises a little and the second inverter responds to this by increasing its R-APF gain, $K_{e2}$ to bring the distortion back to its target value.

4.4 Discussion

The simulation results shown here have demonstrated how a network-wide harmonic mitigation function can be incorporated into normal DG operation using an adaptive implementation of a resistive active power filter. The features that might make this a desirable solution include:

1. The main advantage of using DGs for active harmonic mitigation is that the network operator can avoid using dedicated devices for this function and therefore the solution is cost effective.

2. An adjustable gain R-APF responds only if there is any voltage harmonic component above the set point. If a dedicated R-APF is used for this function it will have to sit idle when harmonic damping is not required. A DG unit used as an R-APF is more suitable for such applications.

3. The proposed method for limiting the harmonic current under conditions of peak fundamental power ensures that the delivery of fundamental power is not restricted by the additional functions. The loss of harmonic mitigation contribution from one DG can be smoothly compensated for by an adjacent DG through its adaptation
algorithm and without the need for central coordination.

5 Conclusions

A scheme for adding resistive APF functionality to distributed generators has been described and used as the basis for a proposal to use multiple R-APF devices to provide harmonic damping throughout a distribution feeder. In conclusion:

1. Damping the propagation of harmonics with a resistive APF is an attractive solution for radial feeders, but a single R-APF at the end of the feeder may not be effective for a wide range of harmonics, especially for electrically long feeders and high harmonic orders.

2. The proposed method is based on multiple R-APFs which divide the feeder into small electrical sections at harmonic frequencies and provide an effective solution for a wide range of harmonics even when the shunt and series impedances of the feeder change as loads or the system configuration change.

3. The proposed method for limiting the harmonic current under peak fundamental power demands ensures that the fundamental power duty is given the first preference. Peak fundamental power demand may reduce the harmonic damping of an individual DG but this may not be a significant problem if more than one DG has been arranged to provide this additional harmonic damping service.

4. The proposed method provides an effective way of sharing the harmonic duty among the adjacent DGs.
References


[18] www.plexim.com
Table 1: System parameters of Fig.1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line Voltage</td>
<td>400 V</td>
</tr>
<tr>
<td>Line Frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Line Inductance (L)</td>
<td>0.2 mH</td>
</tr>
<tr>
<td>Line Resistance</td>
<td>0.05 Ω</td>
</tr>
<tr>
<td>Bus Capacitance</td>
<td>50 μF</td>
</tr>
</tbody>
</table>

Table 2: Parameters of the distribution system analysed in proposed method

<table>
<thead>
<tr>
<th>Line Voltage, Base kVA</th>
<th>400 V, 70kVA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line Frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Line Inductance (mH)</td>
<td>L1-2 0.22, L2-3 0.3, L3-4 0.2, L4-5 0.22, L5-6 0.22, L6-7 0.3, L7-8 0.4, L8-9 0.2, L9-10 0.2</td>
</tr>
<tr>
<td>Capacitance (μF)</td>
<td>C2 50, C3 100, C4 100, C5 150, C6 50, C6' 100, C7 50, C8 150, C9 150, C10 150</td>
</tr>
</tbody>
</table>
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Figure 12: Harmonic current limiting loop response under peak fundamental power demands: (a) fundamental real power reference $P_{ref}^h$. (b) harmonic current $I_{dq}^h$ and its limit $I_{dq}^{h \text{ max}}$. (c) Conductance (gain) $K_v$. (d) harmonic voltage $V_{dq}^h$ and its target $V_{dq}^{\text{target}}$. 
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