Nucleon Isobar Production in High Energy

$k^*$-p and p-p Interactions

by

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ABSTRACT

This thesis is a study of nucleon isobars produced in four-body final states from the collisions of 10 GeV/c negative kaons and 16 GeV/c protons in hydrogen.

The early chapters briefly summarise the experimental details and the techniques used to extract the events of interest. Cross-sections are also presented.

The general features of the reactions are described in some detail and these are related to isobar production. An attempt is made to extract the partial cross-sections for the four-body final state induced by 16 GeV/c protons.

The last chapters are chiefly concerned with the three-body isobars. Several t-channel models are considered in an effort to describe the data, before discussing the resonant properties of the low mass p π⁺ π⁻ system.

Finally an investigation is carried out as to whether it is s-channel or t-channel helicity that is conserved in the diffractive production of this low mass pπ⁺ π⁻ system.


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PREFACE

The author joined the High Energy Physics Group at Imperial College in the October of 1966 and mainly spent the following six months attending the necessary post-graduate lecture courses.

For a brief period the author assisted with a low energy $\kappa$-deuterium experiment, then in progress in the group, before becoming involved with the 16 GeV/c proton-proton experiment, the results of which form part of this work.

The author later also joined the 10 GeV/c $\kappa^-$-proton experiment and was then responsible, with his colleagues, for those samples of film allocated to Imperial College in both experiments.
1.1 The 16 GeV/c Proton-Proton Experiment

This experiment is a collaboration between Imperial College and Cambridge University. The bubble chamber photographs that form the basis of the work were taken during September 1966 at the CERN Proton Synchrotron (CPS) in the CERN 2m. Hydrogen Bubble Chamber (CHBC). Originally a total of 100,000 pictures was requested, but owing to a chamber fault that developed during the course of the run, only 50,000 were actually taken. These were divided equally between the two groups.

The experiment was designed for the study of one reaction only, namely,

\[ pp \rightarrow pp \pi^+ \pi^- \]  

(1)

To avoid a large wastage of measuring and computing effort, an attempt was made fairly early in the data processing cycle to extract those events likely to have occurred via reaction (1). This will be described in somewhat more detail in a later chapter.

1.2 The 10 GeV/c k^-p experiment

For this experiment, Imperial College is collaborating with four other European groups. The full names and addresses of the institutes are listed under reference (1).
The first pictures were taken early in 1965, again at the CPS but in the 1.5m British National Hydrogen Bubble Chamber (BNHBC). The incident momentum of 10.12 GeV/c was at that time the highest obtained for negative kaons. Since then, three further exposures have taken place, all in the CERN 2m. chamber. For convenience, these runs are known by numbers allocated at CERN and will be referred to as experiments 75, 10, 12 and 13 in ascending chronological order. To date, the collaboration has a total of 708,000 pictures, of which the Imperial College share is 116,000. For comparison, the CERN allocation of film is 343,000 photographs.

At the present time, the status of the various experiments is as follows. Experiments 75 and 10 are completely finished. Experiment 12 has been almost completed by all the groups, except Imperial College. Our sample of 27,000 pictures from this experiment was divided into two sections. The first section, which was the author's responsibility, was measured on manual machines. This part of the experiment is completely finished and forms part of the total number of events available to the collaboration.

The second section of Experiment 12 was measured with the Imperial College H.P.D. (2),(3) At that time, the automatic measuring system was in a state of development, so although the results have been disappointing, this was not altogether unexpected. A large amount of remeasuring remained to be done, but this is now almost complete and it is hoped that the whole experiment will be finished very shortly.

Experiment 13 is still being processed by all the groups. At Imperial College, we have measured all the events on the H.P.D. In this case, 57,000 photographs are involved. The first measurement
pass has been completed and preliminary results look very promising.

The channels that are to be studied in this work are the four-body final states,

$$k^- p \rightarrow k^- p \pi^+ \pi^- \quad (2)$$

$$\rightarrow \bar{k}^0 p \pi^- \pi^0 \quad (3)$$

$$\rightarrow \bar{k}^0 n \pi^+ \pi^- \quad (4)$$

Channel (2) bears a direct comparison with channel (1), in the proton-proton experiment. In reaction (3), the incident $k^-$ leads to a $\bar{k}^0$ in the final state, while in reaction (4) both incident $k^-$ and $p$ undergo charge exchange. Unfortunately, the channel with only proton charge exchange,

$$k^- p \rightarrow k^- n \pi^+ \pi^0$$

involves two unseen neutral particles in the final state and hence cannot be extracted from the data with any reasonable degree of purity.

1.3 Nucleon Isobars

The first nucleon resonance to be established was the $\Delta^{++}$ (1236), by Fermi and co-workers \(^{(4)}\) in 1952. Since then their number has grown at an almost alarming rate. The current issue of the "Review of Particle Properties" \(^{(5)}\) lists no fewer than 13 well established isotopic spin $\frac{1}{2}$ resonances and 9 with isotopic spin $3/2$. There are in addition several further candidates \(^{(6)}\) awaiting confirmatory evidence.
The properties of these isobars (mass, width, spin, parity, elasticity) are invariably determined from low energy pion-proton interactions. If the centre-of-mass energy of the collision is somewhere near the resonance mass, then the incoming particles have the possibility to form this excited state, which subsequently decays into the observed reaction products. These are usually called "formation experiments."

For two-body final states a technique called phase-shift analysis has been evolved to extract the resonance parameters from the experimental data. The scattering amplitude is written as a sum over partial wave amplitudes, where each of these corresponds to a definite orbital angular momentum, \( \ell \), total spin, \( J \), and isotopic-spin, \( I \). The idea is then to determine these amplitudes at each energy and to connect them with a unique solution. By examining these solutions as a function of energy, the existence of resonances can be detected.

Groups at CERN \(^7\), Berkely \(^8\), Glasgow \(^9\) and Saclay \(^10\) have performed such an analysis using experimental data from laboratories throughout Europe and the U.S.A. The solutions are more or less in agreement.\(^{11}\)

For three body final states a more phenomenological approach must be adopted. The partial wave analysis is performed assuming that the reaction proceeds exclusively through intermediate quasi-two-body states.

In the channels of which this thesis is a study, nucleon isobars can be expected to consist of only one out of two or three particles in the intermediate state. In this respect, the experiments may be referred to as "production experiments."
Production experiments do not form an ideal tool for the study of isobars. At these higher energies, the accuracy of measurement is limited, so that closely spaced resonant states will not be resolved. In addition there is often a large background from competing processes, which is difficult or impossible to separate.

Nevertheless, nucleon isobars do manifest themselves quite strongly in the channels that are being considered, so that a study of their properties and underlying production mechanisms must be worthwhile. It is also very interesting to compare the results with the phase-shift analyses.

1.4 Theories

The theoretical effort in understanding strong interaction physics has basically developed along two lines.

The first method may be loosely termed "statics" and involves classifying the observed particles and resonance states according to certain group symmetry schemes. One attempts to assign particles to certain multiplets of a group and from the underlying group symmetry one hopes to "explain" the properties of the multiplet members. One spectacular success of the SU(3) symmetry scheme was the prediction of the strangeness, $S = -3$, particle, the $\Omega^-$, as the tenth and last member of the decuplet containing the $\Delta^{++}$ (1236). This particle was eventually seen in a bubble chamber experiment. As a matter of interest, seven definite examples of $\Omega^-$ production have been seen in the 10 GeV/c experiment.

The dynamical approach is at present on a much less sure theoretical foundation. There is essentially no theory of the dynamics
of the strong interaction and any progress has inevitably been of a phenomenological nature. Theories then become "models" and one hopes that a given model will fit a given reaction over a wide range of energies, with the same values of any unknown parameters. Progress may be termed "spectacular" if a model is successful for many reactions at different energies.

In this thesis various models have been applied to the data with varying degrees of success. Most of these models are very crude and can be expected to fit only the gross features of the data. However, even with this limited degree of success, one feels that one is beginning to understand something of the production processes responsible for the various final states.
CHAPTER 2

INSTRUMENTATION

2.1 The Beams

The first 10 GeV/c $k^-$ exposure utilized the "02" beam at the CPS. The 16 GeV/c proton-proton and later $k^-$ experiments were run using the "U3" beam line (this has been subsequently modified to "U4"). The construction and operation of these beam lines has been extensively described elsewhere, so only a brief description of the "U3" beam will be given here.

During picture taking for the proton experiment the beam was operated in an unseparated mode. The protons in the CPS were accelerated to 16 GeV/c and then deflected onto an internal target. The shape of the elastic differential cross-section at high energies is such that the majority of secondaries are produced in a narrow forward cone. The elastically scattered protons continued to circle the CPS in deflected orbits and were subsequently ejected into the beam line using a fast kicker magnet.

For the $k^-$ exposure, the radio-frequency (RF) separators were used. The CPS was operated at about 20 GeV/c and the protons deflected onto an external target. Using the fast ejection system, all 20 bunches circulating in the CPS could be extracted in about 2 $\mu$sec. The external target had the advantage that the secondaries produced at 0° could be utilized.

The components of the "U3" beam are shown schematically in Fig. 2.1. The distance between target and bubble chamber is about 165 m.
The angular acceptance of the beam is defined by collimators C1 and C2 in the horizontal (± 7.5 m rad) and vertical (± 5.0 m rad) planes respectively. The four quadrupoles Q1 to Q4 serve to focus the beam horizontally at the momentum slit C4 and to provide a vertical magnification such that the opening angle at the first RF cavity is about 1 mrad. The two bending magnets M1, M2 together with C4 provide a momentum resolution of better than 0.3%. After quadrupole Q5 and bending magnets M3 and M4 the beam is reasonably dispersion free and parallel in the vertical plane. The quadrupoles Q6 and Q7 image the target in both planes to the centre of RF1.

Each of the RF cavities exerts a sinusoidal force in the vertical plane. Evidently particles of different mass will take different times to reach RF2. Hence the relative phase of the fields in the two cavities can be adjusted so that unwanted particles hit the beam stopper. The quadrupoles Q8 to Q11 image C3 onto the beam stopper and have overall magnification -1. Thus in an ideal situation unwanted particles should be given equal and opposite deflections in the two cavities.

Collimator C5 immediately behind the beam stopper redefines the vertical acceptance, and serves to somewhat reduce the background.

The next stage of the beam provides a further momentum analysis. Quadrupoles Q12 and Q13 give horizontal and vertical foci in collimators C6 and C7 respectively. C6 is imaged by the quadrupole doublet Q14, Q15 in the 1 mm horizontal collimator C8, in front of the bubble chamber. The momentum analysis is provided by bending magnets M5 and M6 and collimator C8. The final momentum resolution should be better than 0.5%.
Quadrupoles $Q_{14}$, $Q_{15}$ also shape the beam in the vertical plane for entry into the bubble chamber. Vertical bending magnets $VM_7$ and $VM_8$ are used to obtain the correct level and entry angle.

For the proton experiment, the pion and muon contamination can be expected to be negligible because of the extraction technique.\(^{(15)}\) At 10 GeV/c, the problem is more serious however. In Experiment 75 the pion contamination was as high as 10% of all particles reaching the bubble chamber, but this was reduced to 5% for subsequent exposures. The muon contamination is of the order of 20%, but this is much less important since these are not strongly interacting particles.

The parameters of the "U3" beam line for Experiment 12 are summarised in Table 2.1.

2.2 The Bubble Chambers

The pictures for Experiment 75 were taken in the 1.5 m British National Hydrogen Bubble Chamber, but all the other experiments used the CERN 2m Hydrogen Bubble Chamber.

A bubble chamber essentially consists of a large volume of liquid; hydrogen in our case. If this liquid can be brought into a superheated state, then bubbles will form along the path of any charged particles passing through the chamber. A high magnetic field is usually applied throughout the chamber volume so that the particle tracks will in general be helices. Photographs of the tracks can then be taken. Two different views of a given track are necessary to enable the reconstruction in space to be performed, but three cameras are usually used so that the spatial reconstruction is over-determined.
For a general discussion of bubble-chambers and the principles involved, refs. (19) and (20) should be consulted.

Both the 1.5 m. BNHBC (21) and CERN 2 m. HBC (22) have been extensively described elsewhere, so it is proposed here to give only a brief description of the 2m. chamber, used to obtain most of the experimental data for this thesis.

The sides of the liquid hydrogen container are glass, the top and bottom metal. Small crosses are etched on the glass windows (the majority are on the inner faces) and these are used as reference points for the later reconstruction of the particle tracks, as well as survey marks for the determination of the chamber constants.

The chamber is illuminated by three flash tubes on one side, and photographed on the other. The illumination is of the dark field type, i.e. the light actually scattered by the bubbles is detected. The CERN 2m. HBC was originally designed with a four camera system, but recently the number has been reduced to three.

The liquid hydrogen is brought into a super-heated state by a lowering of the pressure. The chamber is allowed to expand upwards, against a piston and the frequency of this expansion can be varied to suit the experimental requirements. (The 1.5 m. BNHBC used a gas expansion technique). The complete cycle time of a bubble chamber is typically 1 sec. The photographs are taken about 1 msec. after the beam entry to allow the bubbles to grow to the required size, about 150 \textmu. The average number of bubbles per centimetre for beam tracks is 10 – 15.

Table 2.2 lists some of the important parameters of the 2m. chamber as set up for Experiment 12.
2.3 **Film Quality**

During the course of a run it is important that the film quality should be frequently checked. This then allows adjustments to be made in the beam and bubble chamber operating conditions.

To provide a rough check a test strip is cut from the end of each roll of film (about 1500 frames) and this is developed in the experimental area. The bubble density of beam tracks can then be determined with the aid of a microscope. In addition to this, one complete view of each roll is processed as quickly as possible to provide more detailed information on unwanted particle contamination.

During picture taking for the 16 GeV/c proton experiment a fault developed in the injection system necessitating frequent adjustments of collimator C₃ (see section 2.1). As a result about half the film is of poor quality with a rather high number of beam tracks per frame. The first half of the photographs averaged 10 - 15 protons per picture, but for the second half the number was 15 - 20. The operating conditions tended to be very variable throughout, so that many frames had an impossibly high number of beam tracks and had to be discarded.

For the k⁻ experiments the contamination of unwanted particles was monitored throughout the runs. Frequent scans were made for interactions involving the visible decay of a neutral strange particle and using known cross-sections the pion contamination could be estimated. The numbers of such V⁰ events actually seen will evidently depend on the size of the chamber. From the dimensions of the region scanned for interactions in Experiment 12, the number of kaon interactions which should give rise to a visible V⁰ was calculated to be 22%, and the number of pion interactions to be 7%.
Suppose we see \( N \) interactions of which \( n \) have an associated \( \nu^0 \).

Then,

\[
\begin{align*}
    n &= 0.22 \, a \, N + 0.07 \, (1 - a) \, N \\
    \text{(2.1)}
\end{align*}
\]

where \( a \) is the ratio of the number of kaons to the total number of pion and kaon tracks. Anti-proton contamination is assumed to be small.

Furthermore, knowing the total \( k^- \) and \( \pi^- \) cross-sections at 10 GeV/c and the number of interactions, the muon contamination can be estimated.

Suppose for \( N_0 \) beam tracks there are \( N \) interactions of all types. Then the cross-section is given by,

\[
\sigma = \frac{N}{A \rho N_0 l} \tag{2.2}
\]

where \( A \) is Avagadro's number, \( \rho \) is the density of liquid hydrogen and \( l \) is the length of the chamber scanned for interactions.

We can also write,

\[
\sigma = a \sigma_k + (1 - a) \sigma_\pi \tag{2.3}
\]

where \( \sigma_k, \sigma_\pi \) are the total \( k^- \) and \( \pi^- \) cross-sections.

Thus \( N_0 \) can be found and compared with the observed number of beam tracks to arrive at the muon contamination.

For experiment 12, the average number of \( k^- \) tracks per picture was about 10 and the pion contamination of the order of 5%. The muon contamination was calculated to be 20%.

* See section 4.5.
TABLE 2.1

"U3" BEAM PARAMETERS FOR EXP. 12

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Primary beam momentum</td>
<td>20.6 GeV/c</td>
</tr>
<tr>
<td>Target (copper)</td>
<td>2(v) x 1(h) x 150 mm³</td>
</tr>
<tr>
<td>Production angle</td>
<td>0°</td>
</tr>
<tr>
<td>Maximum collection angles at target:</td>
<td></td>
</tr>
<tr>
<td>Horizontal</td>
<td>± 7.5 m rad</td>
</tr>
<tr>
<td>Vertical</td>
<td>± 5.0 m rad</td>
</tr>
<tr>
<td>Solid angle</td>
<td>150 μsr</td>
</tr>
<tr>
<td>Relative momentum bite</td>
<td>± 0.25 %</td>
</tr>
<tr>
<td>Length of beam</td>
<td>165.5 m</td>
</tr>
<tr>
<td>Cavity separation</td>
<td>50.0 m</td>
</tr>
<tr>
<td>RF frequency</td>
<td>2,855 MHz</td>
</tr>
</tbody>
</table>
### TABLE 2.2

**CERN 2m. HYDROGEN BUBBLE CHAMBER**

**Windows:**
- **Dimensions**: 217 x 77 x 17 cm³
- **Weight**: 660 kg
- **Absolute refractive index**: 1.5168

**Optical System:**
- **Light sources**: 3
- **Condenser systems**: 3
- **Cameras**: 4

**Illuminated Region**
- **Front plane**: 52 x 162 cm²
- **Back plane**: 60 x 192 cm²
- **Depth**: 50 cm
- **Useful volume**: 465 l

**Film (unperforated)**: 50 mm
- **Scattering angle**: 7°
- **Central plane demagnification**: 1 : 13.3
- **Working temperature**: 27°K
- **Magnetic field**: 17.343 k gauss
FIG. 2.1  LAYOUT OF U3 BEAM

Collimators:

Horizontal

Vertical

Magnets:

Horizontal bending magnet  M

Vertical bending magnet  VM

Quadrupole  Q

Target  T

Radio frequency cavity  RF

Beam stopper  BS

Hydrogen bubble chamber  HBC
CHAPTER 3

DATA EXTRACTION

3.1 Scanning

In the 10 GeV/c experiment each roll of film was scanned twice for interactions of all topologies. The "topology" of an event is defined as a three digit number, \(2mn\), where \(2\) is the number of outgoing tracks, \(m\) the number of charged decays and \(n\) the number of neutral decays. The only events that were not recorded were "100's" corresponding to the \(k^-\) decay,

\[k^- \rightarrow \pi^- \pi^0 \pi^0\]

and "000's", which are characteristic of events of the type,

\[k^- p \rightarrow k^0 n + m\pi^0, \quad m \geq 0\]

and \[k^- p \rightarrow \Lambda + m\pi^0, \quad m \geq 1\]

where the \(k^0\) or \(\Lambda\) does not decay inside the visible chamber volume.

A fiducial region was defined, about 90 cms. long in space, and only events with production vertex within this region were recorded. This limitation was imposed to ensure that all secondary tracks were of sufficient length to allow the momentum to be measured to better than 10%. For events involving the visible decay of a strange particle, the fiducial region was extended to a length of about 120 cms, and these events were only recorded as such if the secondary vertex lay within this larger region.
Events with two or more visible strange particle decays, "rare" events, were recorded if the production vertex was within the extended fiducial volume. The cross-section for this reaction class is very small and this relaxation of the criteria was done simply to obtain the greatest number of events possible.

When one of the two scanners failed to find an event or the two scans disagreed as to its interpretation, then this event was looked at by a physicist. This partial third scan was called a "check-scan". At the end of the check-scan for each roll, a complete set of scan cards existed, one for each event, on which was recorded all the information relevant to that event. This information was later punched and subsequently transferred to magnetic tape in the form of a "master-list". Such a master-list allowed the progress of each event through the experiment to be followed at each stage and, in principle at any rate, should have prevented biases because of "lost" events.

The detailed scanning criteria in operation for all the 10 GeV/c k^- experiments can be found in the work of D. P. Dallman, ref. (17).

At 16 GeV/c the scanning instructions were somewhat different. Since the object of the experiment was to study only the reaction,

\[ pp \rightarrow pp \pi^+ \pi^- \quad (1) \]

only four prong events with no associated visible strange particle decays were recorded. A full scan of all topologies on about 7,000 frames showed that these four prong events form about 40% of all pp interactions at 16 GeV/c. However from published data (23) it was known in advance
that reaction (1) accounts for only 20%\(^{(24)}\) of the total four prong sample at this energy.

Hence to avoid a huge wastage of measuring and computing time, a preliminary selection of those events likely to belong to reaction class (1) was attempted at the scanning stage.

Events were roughly measured using a "D-MAC" digitizing system\(^{(25)}\) fitted to an ordinary scanning projector. The events were measured on one view only and the vertex, followed by two points on each track were digitized. Beam tracks were not measured.

These three points on each track were sufficient to define a circle in the x-y plane,

\[
(x - a)^2 + (y - b)^2 = p^2
\]  

(3.1)

and hence the radius of curvature, \(p\), could be estimated.

To 'calibrate' the machine a large number of beam tracks were first digitized and since their momentum is well known (see Sections 3.3) the conversion factor, in GeV/cm., could be estimated.

If the event really belongs to reaction class (1) the projected sum of the 3-momenta in the x-y plane must satisfy

\[
S = \sum_{i=1}^{4} P_{xy} \geq P_B
\]  

(3.2)

where \(P_B\) is the projected beam momentum. Since the beam protons entered the chamber with very small dip angle, \(P_B\) may be taken as the nominal beam momentum of 16 GeV/c.

Events with one or more unseen neutral particles can be expected to have \(S\) less than \(P_B\). We show in Fig. 3.1 the distribution in \(S\) for
a sample of 966 events and it can be seen that there is a peak around 16 GeV/c. It was eventually decided to impose a cut-off at 10 GeV/c and events having a value of S less than this were not subsequently measured. We shall see in Section 4.4 that this value for the cut-off was a very reasonable choice.

The rough digitizations were punched on paper tape and a facility was also incorporated to record useful "comments". For example, the fact that an event had a track with greater than minimum ionization or a track that stopped in the chamber could be noted. Those events that had a short straight track (visible sagitta less than 2 mm.) were passed straight to the measuring stage without any pre-digitization.

There were 27 examples of three prong events seen in the scanning. These may come about for several possible reasons. There may be a 5-ray produced on one of the tracks very close to the vertex, one of the secondary particles may undergo an interaction or there may be a very slow proton with insufficient energy to be visible. *

Events with a missing negative particle were not processed any further. However, those events with a missing positive track were passed through the system, a missing proton being simulated by a zero range track, with suitable errors. It will be seen in Section 4.3 that these events are very important and that we unfortunately have a small bias against them.

* Various other possibilities exist also. The event may be a genuine k+ decay or there may be a small amount of helium contamination in the chamber and we are seeing a proton nucleus interaction.
The scanning and rough measuring was performed at the same time and the majority of the film was only scanned once. However, in order to calculate the cross-section (see Section 4.5) a sample of film (ten half-rolls), selected for its better quality, was scanned twice and then check scanned, as in the 10 GeV/c \( k^- \) experiment. The number of beam tracks on each frame was also recorded.

The information from the "D-MAC" machines was analysed by computer program (26) to find the candidates for measurement. A measurement list was produced and this formed the basic master list for the experiment. It was found that just over 50% of all four prongs were rejected by the program, thus representing a considerable saving in measuring time.

3.2 Measuring

The events of that part of the kaon experiment (12) which was the author's responsibility and those of the proton experiment were all measured on the two National Measuring Machines (27) belonging to the Imperial College group.

Each machine essentially consists of a moving stage bearing Moiré fringe digitizers. The displacement of the stage (parallel to the film length) records the x-coordinate and the displacement of the carriage bearing the lens records the y-coordinate. A projection system displays a magnified image of the film onto a screen.

To measure an event the operator begins by setting the pointer on the first fiducial mark. The counters are then set to fixed values and all subsequent digitizations are with reference to this point.
Three other fiducial points are then measured followed by the event itself. The co-ordinates of the vertex and any other identifiable points (such as secondary vertices or the end point of a track that stops in the chamber) are digitized and about ten points on each track. The aim is to space these points along the entire length of the track so obtaining the maximum sagitta and minimum error in the curvature. This procedure is then repeated for the other two views.

The least count that can be recorded on these machines is $2\mu$ in film or $27\mu$ in space for the CERN 2m. chamber.

The digitizations from the machines are punched on paper tape and then processed by the program BIND. This converts the data into a form suitable for the reconstruction program as well as making certain logical checks on the format of the information (whether the correct number of fiducials have been measured on each view etc.). The BIND output is on magnetic tape.

For part of the time during the measuring of the film for both experiments, the machines were on-line to the group's PDP-6 computer. These logical checks could then be made in real time. A rough circle fit was also performed for each track and the operator could be asked to repeat any that were unsatisfactory. When the machines were on-line the digitizations were written directly to magnetic tape.

For the 16 GeV/c proton experiment only those three or four prong events selected by the program D-MAC were actually measured. In Experiment 12 all events except those of topology 410, 610 etc. were measured.
3.3 Geometric Reconstruction and Kinematic Fitting

The geometric reconstruction and kinematic fitting of the events were performed using the CERN programs THRESH (29,30) and GRIND (29,31) respectively. It is not proposed to give here a description of the detailed workings of these very large and complicated programs. As well as the original reports (30,31) and TC Manuals (29) there exist good summaries in the works listed under Ref. (17). In this section we simply give a very brief outline of what each program actually does.

The input to THRESH is essentially a series of co-ordinate pairs for each view of a given event, corresponding to the measured positions of the fiducials and labelled points and digitizations along the tracks. Using the known fiducial and camera positions and certain other optical constants, THRESH reconstructs the positions of labelled points in space and finds the best values of the variables $1/p$, $\lambda$ and $\phi$ for each track. $p$, $\lambda$ and $\phi$ are the curvature, dip and azimuthal angle of a track at its mid-point. The variable $1/p$ rather than $p$ is used because the former is normally distributed. The points are constrained to be on a helix and the best values of the parameters are found by the method of least squares.

THRESH makes certain approximations for the reconstruction. The tracks are supposed to describe pure helices which implies that the magnetic field is constant throughout the chamber volume. The maximum deviation of the field from its central value is in fact about 2%, but no more than 1% in the middle region of the chamber.

Changes in the momentum of a track due to energy loss through multiple Coulomb scattering are neglected. Because of this
deficiency very low momentum tracks had to be measured only along part of their length in order that a helix could be reconstructed.

The errors that THRESH assigns to the track parameters are based on the root-mean-square deviation (residual) of the measured points from the best helix fit. These are known as "internal" errors. They will not be the true errors because of the factors mentioned above. For very fast tracks the Coulomb scattering is comparatively unimportant compared with the measuring error. We show in Fig. 3.2 the distribution in the residuals of about 500 beam tracks for Experiment 12. The median of the distribution is at 6.5μ.

For the second half of Experiment 12 and for Experiment 13 a mass dependent version of THRESH is being used. Corrections for a non-uniform field and energy losses due to multiple Coulomb scattering are now taken care of inside THRESH itself. While the older version of the program was being run, these corrections were applied in GRIND.

If for any reason THRESH fails to satisfactorily reconstruct an event, an error flag is set. These failing events then have to be remeasured. Ideally one would like to have an overall pass rate of greater than 90% and this is usually achieved after three measurement cycles.

The output from THRESH is written onto magnetic tape in a suitable format for input to GRIND.

The purpose of GRIND is to identify a given event uniquely and to find the true values of the momentum and energy of each track, together with the corresponding errors.

GRIND tries various hypotheses for the event, where a hypothesis is an assignment of definite particle masses to each track.
The conservation of energy and momentum lead to four constraint equations which can be written,

\[ f_k (x_i, m_i) = 0, \ k = 1,4 \]  \hspace{1cm} (3.3)

where the \( x_i \) are the true values of the variables \( x_i^o \), reconstructed in THRESH, and \( m_i \) are the particle masses.

In practice there exists a second class of variables, termed "unmeasured". THRESH may not have been able to find a reasonable value for one of the parameters of a track (for example the track may be very short so a determination of the momentum is not possible) or the hypothesis may correspond to a reaction where there is an unseen neutral particle. We shall call the true values of these unmeasured variables \( y_i \) and the starting values, \( y_i^o \), are obtained by solving the constraint equations,

\[ f_k (x_i^o, m_i; y_i^o, m_i') = 0, \ k = 1,4 \]  \hspace{1cm} (3.4)

using the masses \( m_i, m_i' \) assigned by the particular hypothesis.

A chi-squared function is written of the type,

\[ x^2 = \sum_{ij} (x_i - x_i^o) G_{ij} (x_j - x_j^o) \]

\[ + \sum_k (y_k - y_k^o) G_{kk} (y_k - y_k^o) \]  \hspace{1cm} (3.5)

where \( G_{ij}^{-1} \) is the error matrix for well measured variables and \( G_{kk}^{-1} \) is that for unmeasured variables. The latter matrix is assumed diagonal, since correlations between badly measured or unknown variables would
not have much meaning.

The problem then is to minimize this chi-squared function, subject to the constraint equations,

\[ f_k (x, m; y, m') = 0 \quad k = 1,4 \]  \hspace{1cm} (3.6)

The solution is not trivial since the constraint equations are non-linear and an iterative procedure has to be adopted. When the solution has been found or GRIND decides that no solution is possible, the next hypothesis is tried, until the list is exhausted.

The number of degrees of freedom of a fit is given by,

\[ n_d = 4 - N \]  \hspace{1cm} (3.7)

where \( N \) is the number of unknown variables. If \( N \) is less than four a fit is possible and, if successful, is referred to as an \( n_d \) - constraint fit (\( n_d \)-C). When \( N \) equals four, then it is often possible to find a solution for each of the competing hypothesis. Clearly there is no way of estimating the relative "goodness" of these fits so these events are put into the pool of "reject events" and play no further part in the analysis.

When an event has two or more missing neutral particles, in principle GRIND should not find a solution. These events are termed "no-fits". (We shall be discussing the overlap between "fit" and "no-fit" events in the next chapter). The neutral particle system is termed \( \pi^0 \) and its energy and momentum, and hence its mass, is obtained by solving the constraint equations (3.4), for the given hypothesis.
The number and type of hypotheses which are to be attempted in GRIND are supplied as input data to the program. Evidently we only consider those that conserve charge, strangeness and baryon number. In addition certain other event types with small cross-sections compared to competing hypotheses are neglected.

For example, in Experiment 12 the reactions,*

\[ k^-p \rightarrow k^-p k^+k^- \]  
(2a)

and  
\[ k^-p \rightarrow k^-pp \bar{p} \]  
(2b)

are neglected compared with

\[ k^-p \rightarrow k^-p \pi^+\pi^- \]  
(2)

At 16 GeV/c the reactions

\[ pp \rightarrow pp k^+ k^- \]  
(1a)

and  
\[ pp \rightarrow pp p \bar{p} \]  
(1b)

can also be expected to have small cross-sections \(^{(34)}\) compared with

\[ pp \rightarrow pp \pi^+\pi^- \]  
(1)

In the 16 GeV/c proton experiment only the 4C hypotheses corresponding to reaction (1) were attempted. The 1C hypotheses were not tried in order to save computing and measuring time. This will be justified more fully in Chapter 4.

* These \(k^-p\) and \(pp\) reactions were in fact extracted later using a technique due to Ehrlich et al. \(^{(32)}\). The cross-sections \(^{(33)}\) for channels (2a), (2b) and (2) at 10 GeV/c were found to be 30, 3 and 880 \(\mu b\) respectively.
GRIND is usually supplied with some information about the beam parameters for a given experiment. From a single measurement of a beam track, the error in the momentum may be greater than 10% and it is clear that the true momentum is known to much greater accuracy.

The values of the beam momentum, $P_0$, dip, $\lambda_0$, and azimuthal angle, $\phi_0$, at a point $(x_D, y_D, z_D)$ are specified in the input data to GRIND. The values of the dip and azimuthal angles at the interaction vertex $(x, y, z)$ can then be obtained from,

$$\lambda = \lambda_0 + C_\lambda (a - a_D)$$

$$\phi = \phi_0 + C_\phi (y - y_D) + C_L L_D$$

where $L_D$ is the track length from the plane $x = x_D$ to the apex of the event. The beam momentum can also be corrected for energy loss due to multiple Coulomb scattering.

The values of the beam parameters $P_0$, $\lambda_0$ and $\phi_0$ and of the constants $C_\lambda$, $C_\phi$, $C_L$ are found by measuring a large number of beam tracks and passing them through THRESH. The average values can then be found (17) as a function of $x$, $y$, $z$.

The final beam momentum for the $k^-$ experiment was $10.03 \pm 0.05$ GeV/c and for the proton experiment $16.080 \pm 0.075$ GeV/c.

In both experiments the value of the beam momentum was imposed, though a check was first made to see if the measured value agreed, within errors, with the title value. For Experiment 12 weighted averages of the beam dip and azimuthal angle were used. In general these angles are quite well measured so the weighting tends to favour
the measured values. In the 16 GeV/c experiment the measured values of
the angles were used throughout because of the poor beam definition
and difficulties with chamber constants[17].

It was mentioned earlier in this section that the helix fit
errors given by THRESH are not the true errors. GRIND in fact uses
"external" errors defined by,

$$\Delta \rho = \frac{8 \cdot f_0}{(L \cos \lambda)^2}$$

$$\Delta \lambda = \frac{9 \cdot f_0}{L \cos \lambda}$$

$$\Delta \phi = \frac{4.16 f_0}{L \cos \lambda}$$

(3.9)

where $L$ is the length of the track. The constant $f_0$ is used throughout
GRIND to "scale" the errors and is the theoretical setting error of the
measuring machines. In both experiments a value for $f_0$ of 75µ in space
was used. (This value is somewhat larger than the true setting error
of about 54µ, found by repeated settings on a given fiducial point,
but should allow for general uncertainties in the chamber constants,
etc.)

For each track a comparison is made between internal and
external errors. If they do not agree within specified limits an
error flag is set and the track is considered badly measured. Such
events then have to be remeasured.
3.4 Event Interpretation

The errors involved in the measurement of high energy bubble chamber events are such that GRIND usually cannot arrive at a unique conclusion. The problem then is to decide the correct interpretation of an event from the list of competing hypotheses.

Because the list of criteria which has been developed for the k^- experiments to assist in this "choicing" is rather extensive, a program called AUTOGRIND(35) has been used to simplify the task. AUTOGRIND reads the GRIND output tape and prints out the fits successfully attempted in GRIND. Details of failing hypotheses are also given. The program then chooses those fits and/or no-fits which satisfy the given criteria.

The event is also visually examined on the scan-table to check that the observed bubble-density of a given track is consistent with that predicted by the fit. It is well known that the ionization of a particle track is inversely proportional to the momentum and directly proportional to the mass. The actual form of the relationship is,

\[ I = I_o \left( 1 + \frac{m^2}{p^2} \right) \]  

(3.10)

where \( I_o \) is the minimum ionization (i.e. that achieved by a particle travelling at the velocity of light), \( m \) the assumed rest mass and \( p \) the momentum for the given track.

Thus for a track of given momentum one can hope to distinguish between the various mass assignments. In fact the upper limit for deciding \( \pi/k \) ambiguities is about 0.8 GeV/c and that for \( \pi/p \) ambiguities about 1.5 GeV/c.
Any hypothesis that does not have all tracks consistent with the observed bubble density is immediately rejected.

The list of criteria operative for Experiment 12 is fully discussed in the work of D. P. Dallman (17), and it is not proposed to go into too much detail here. However some pertinent points will be presented.

Only a maximum of three competing hypotheses is allowed and events are assigned a weight equal to the reciprocal of the number of fits selected. To bring this about certain assumptions have to be made.

Fits are preferred to no-fits and 4C fits to 1C fits. The justification for preferring 4C to 1C fits cannot be based on statistical grounds alone if both hypotheses have a similar chi-squared probability. The ambiguity usually only arises in the case of a missing π⁰, since this particle has a relatively small mass. However, if an event fits both hypotheses the π⁰ must necessarily have very low momentum. We show in Fig. 3.3 the laboratory momentum distributions for the π⁻ and π⁰ from channel (3).

Any contamination of 4C fits in this reaction would show up as a peak just above zero in the π⁰ distribution. Conversely any loss of events into the 4C channel would result in a depletion of the π⁰ distribution. There is certainly no accumulation of events at zero laboratory momentum, but it is more difficult to prove that fits have not been lost. While there is no reason to expect the π⁻ and π⁰ distributions to be the same, they are sufficiently similar for us to believe that few events have been wrongly assigned to the 4C channels.

With the same sort of argument one can justify the acceptance of 4C fits over no-fits and here the separation should be even better.
However, the no-fit contamination in the 1C channels may be quite high. This is especially true for missing neutron events.

We show in Fig. 3.4 the missing mass squared distributions for a sample of four prong events from the kaon experiment. These plots give some idea of the resolution obtained in these channels. Figs. 3.4 (a), (b) and (c) correspond to those events fitting the 1C hypothesis with missing $\pi^0$, $k^0$, and $n$ respectively. The 4C sample has been removed and all events have been checked for ionization consistency. No other selections have been applied. The missing mass squared limits used in the experiment are shown as dashed lines.

A lower chi-squared probability limit of 1% for 4C fits and 5% for 1C fits was imposed. In addition, where there were two or more competing hypotheses, a fit was rejected if its chi-squared probability was less than one third of the value for the best fit. If more than three hypotheses still remained, the three with the highest probabilities were chosen. Assuming the external errors have been correctly estimated, the probability distribution for a "clean" sample of events should be flat. The distribution for "mis-fit" events will be peaked at low values. Hence these selections should preferentially remove the mis-fit events.

In the case where more than three no-fits were acceptable, a special NOFIT AMBIG hypotheses was selected where all the tracks were treated as pions, unless positively identified otherwise by ionization.

In the 16 GeV/c proton experiment the problem was much simpler. As mentioned in Section 3.3, only the 4C hypotheses were attempted in GRIND. It then only remained to ensure that the predicted bubble-density was consistent with that observed. A 1% chi-squared probability
cut-off was also imposed.

In this experiment a slightly different procedure was adopted in deciding ambiguities in that the fit with the highest probability was always chosen. In fact no more than 5% of the events were ambiguous so no serious bias should be introduced with this method.

3.5 POST-KINEMATIC PROGRAMS

For each successful fit attempted in GRIND, AUTOGRIND punches a SLICE card containing enough information to identify the event and the hypothesis on the GRIND output tape. Those cards corresponding to the accepted fits are separated and used as input to the program SLICE (29), together with the GRIND output tape. SLICE calculates any quantities of interest and writes them out in a specified format on a data summary tape (DST). This DST then forms the basis of any further analysis of the experiment.

The analysis was actually performed using the program SUMX (29) which can generate the distributions of any specified quantities in the form of histograms or scatter plots. Cuts on the data may also be imposed in the program.
FIG. 3.1

- 16.0 GeV/c
- 965 events
Experiment 12: Beam Track Residuals

Median 6.5 μ

479 Beam Tracks

FIG. 3.2
\[ K^{-}p \rightarrow \bar{K}^{0}p\pi^{-}\pi^{0} \]

(a)

(b)

FIG. 3.3
FIG. 3.4

(a) $K^- p \rightarrow K^- p \pi^+ \pi^- \pi^0$

(b) $K^- p \rightarrow p \pi^+ \pi^- \pi^- K^0$

(c) $K^- p \rightarrow \eta K^- \pi^+ \pi^+ \pi^-$
4.1 Scanning Efficiencies

From the results of the check-scan described in Section 3.1 it is possible to estimate the number of events that have been missed in the first two scans and hence to determine the total number of interactions in a given fiducial volume.

Suppose the true number of events is $N$ and $N_1$, $N_2$ are the numbers actually detected in the first and second scans respectively. Assuming that all events have an equal probability of being seen, we can write the detection efficiencies as,

$$e_1 = \frac{N_1}{N} \quad (4.1)$$

$$e_2 = \frac{N_2}{N}$$

The number of events seen in both scans is given by,

$$N_{12} = N e_1 e_2 \quad (4.2)$$

and hence,

$$N = \frac{N_1 N_2}{N_{12}} \quad (4.3)$$

Thus the probability that an event is found at all is given by,

$$e = \frac{N_1 + N_2 - N_{12}}{N}$$

or

$$e = \frac{N_{12}}{N_1 N_2} (N_1 + N_2 - N_{12}) \quad (4.4)$$

This scanning efficiency varies with the event topology.
since some event types are more difficult to detect than others. Hence each topology is usually treated separately. In addition the assumption of equal detection probability for all events is not strictly correct, so a small correlation may exist between the two scans. For example, a two prong event with one very short track on a frame where there are a large number of beam tracks will be very difficult to see. This effect is minimised to some extent by insisting that the two independent scans are done on two different stereo-views.

The numbers of events in the various topologies found in the two experiments are presented in Table 4.1. The figures are for the Imperial College sample of film in both cases.

4.2 Topology Cross-Sections

This section refers only to the $k^{-}$ experiments. The procedure for obtaining cross-sections at 16 GeV/c was somewhat different.

Since the muon contamination is not well known the total $k^{-}$ path length cannot be found. Hence to obtain cross-sections it is necessary to normalize to the results from counter experiments. The total cross-section $^{(23)}$ used is $22.6 \pm 0.2$ mb for 10.1 GeV/c $k^{-}p$ interactions.

The corrected number of events of each topology is first calculated using the known scanning efficiencies. However two special cases must be considered.

Events of topology 000 were not scanned for since previous experience $^{(17)}$ had shown that the detection efficiency for these
events was unacceptably low. Nevertheless, the number of such events can be estimated as follows: events of this type must involve either an unseen $k^0$ or $\Lambda$ (including $\Lambda$ from a $\Xi^0$ decay) to conserve strangeness. Using the charged/neutral branching ratios the relative numbers can be calculated from the fits to events of topology 001.

Secondly for elastic scattering there is a serious loss of events where the recoil proton has a very short range. Moreover, the detection efficiency is a function of the azimuthal angle, $\theta$, of the proton about the beam direction; since tracks moving either towards or away from the cameras will have small projected path lengths. The number of events fitting the elastic hypothesis was examined as a function of both $|t|$, where $t$ is the four momentum transfer from proton to proton, and $\theta$. For a given $|t|$ interval, the distribution in $\theta$ should be uniform assuming an unpolarized target. Hence, the number of lost events can be estimated.

However below $|t| = 0.06$ GeV$^2$ there is a large loss of events at all azimuthal angles. This loss can be corrected for by extrapolating the experimental distribution in $|t|$ to $|t| = 0$. The method is more fully described in a paper$^{(37)}$ published by the collaboration.

Having found the true number of events of each topology the corresponding cross-section can be calculated. The cross-sections together with the corrected numbers of events are listed in Table 4.2. Since the analysis of Experiment 12 has not been completely finished by all the groups the cross-sections are from Experiments 75 and 10 data only. Unless otherwise stated the event totals include Experiment 12.
In principle to calculate the cross-section for a particular reaction one has simply to find the ratio of the number of events in this reaction to the total number of the same topology. No corrections are necessary for "unmeasurable" events, assuming that these are not biased towards particular reactions. Of course further corrections may be necessary because of deficiencies in the selection criteria (see Section 4.4) or uncertainties in the interpretation of events.

This method is particularly advantageous for the $k^-$ experiment where new data is being constantly added.

4.3 **Systematic Errors**

Clearly before one can give individual channel cross-sections it must be established that there are no systematic biases that could lead to either an over- or under-estimation of the number of fits. Such biases would also be reflected in the final fitted values of the variables for each track and result in serious distortions in the distributions of the kinematic variables.

The values assigned to the external errors in GRIND are very important since if these are too small, events will be lost and if too large, too many fits will result.

Assuming a clean sample of events of a given channel has been isolated, the chi-squared probability distribution should be flat. These distributions are shown in Fig. 4.1 for the reactions (1) to (4), where the distributions corresponding to the 4C channels (1) and (2) have been "corrected" for the 1% probability cut-off. No corrections have been applied to the 1C distributions. The large peak present in all four channels at high probabilities implies that
the external errors used were too large. At lower probabilities the distributions for the 1C channels are relatively uniform.

The low probability peak present in both 4C fit channels is more serious. It either implies a loss of such events or that the channels are contaminated by other reactions.

A detailed investigation of this problem for the 16 GeV/c experiment (see K. Pongpoonsuksri, Ref. 17) showed that it could be traced to a large extent to poor optical constants for the CERN 2m chamber. (The distribution corresponding to Fig. 4.1(b) for Experiment 75 alone, which was run in the 1.5m BNHBC, shows no low probability peak). A spurious curvature in the x-z plane was found to exist, sufficient to induce a systematic error in the z-component of momentum of about 30 MeV. A bias of this order can lead to a loss of the highly constrained events from channels (1) and (2). The effect was particularly noticeable for the class of events having one very fast secondary track and these tended to populate the low probability peak.

In both experiments a careful examination of such quantities as the missing-man squared and missing energy was undertaken and any events which were likely to belong to reactions (1) and (2) were sent for remeasurement. While not entirely satisfactory it is to be hoped that this procedure avoided too large a loss of 4C fits.

The distributions in the stretch functions of the track variables provide a good test for systematic biases. If \( x_m \) is the measured value of a quantity and \( x_f \) the final fitted value, the stretch function is defined as,
The beam stretch functions for events fitting the 4C channels provide a particularly sensitive test. These are shown in Fig. 4.2 for a sample of events from channel (1). The distribution for $1/p$, $\lambda$ and $\phi$ are plotted where $p$ is the beam momentum, $\lambda$ the dip and $\phi$ the azimuthal angle. If there are no systematic errors the distributions should have zero mean. If the random errors have been correctly estimated they should be Gaussian functions with unit standard deviation. The theoretical curves are also shown in Fig. 4.2, normalized to the number of events.

The experimental distributions are generally too broad, reflecting the fact that too large external errors were used. Also the beam dip stretch function is shifted somewhat to negative values. However in general the results are acceptable.

In the proton experiment the symmetry of initial and final states in the overall centre of mass system enables us to perform further tests for the detection of systematic errors.

The centre-of-mass production angular distributions are shown in Fig. 5.9 for the $p$, $\pi^+$ and $\pi^-$, and these should be symmetric about zero. We define the quantity,

$$R = \frac{F - B}{F + B}$$

for each particle, where $F$ is the number of events in which the particle goes forward and $B$ the number in which it goes backward. The values of $R$ are listed in Table 4.3 and it will be seen that we
have an excess of events with a backward going \( \pi^+ \). To investigate this problem further we define two invariant proton-proton four-momentum transfers, one from the target to the slower of the two protons in the laboratory frame, the other from the beam to the faster proton. These are shown in Fig. 4.3 and a loss of events with very slow protons is evident. These peripheral events are of the type where the other particles tend to go forward in the laboratory and could account for the poor \( \pi^+ \) forward/backward ratio.

Another possible source of error is the misidentification of final state protons and \( \pi^+ \). For slower particles this should not arise since the track bubble density will allow us to make a distinction. A scatter-plot (not shown) of the individual proton production angles in the centre-of-mass reveals that there are 18 events in which both protons go backwards and none where both go forward. This excess almost certainly arises from a confusion of the fast proton and \( \pi^+ \) tracks.

One encouraging feature is that we do not appear to be suffering a loss of events with one very fast proton. Such a loss would lead to an excess of forward pions.

In conclusion it should be pointed out that the apparent loss of events could in reality be due to a contamination of mis-fit events from the tail of the probability distributions. However all our investigations\(^{(17)}\) have led us to believe that this is not the case. The most likely explanation is that we have a scanning bias against events with a very short proton track. In any event the overall contamination is reasonably low.
4.4 Biases in the Reactions Studied

We have seen the effect of the small systematic bias in the reconstruction of events is not sufficient to seriously affect the 4C channels (1) and (2). The bias is also presumably present for the one constraint channels (3) and (4) but in this case is more difficult to estimate. We shall however assume that it is of roughly the same order.

Since we prefer 4C fits over 1C fits the only ambiguities present in reactions (1) and (2) are internal. Because of the peripheral nature of the events, these are almost invariably of the type where there are two fast forward going particles in the laboratory frame, with roughly the same momentum. For the \( k^- \) experiment the ambiguity will be between the \( k^- \) and \( \pi^- \) and for the proton experiment between \( p \) and \( \pi^+ \). In the \( k^- \) reaction these events will mainly contribute to the low mass \( k^- \pi^+ \pi^- \) region and hence should not bias our studies of the isobars. In channel (1) we can at least obtain some idea of the effect of any possible bias by comparing fast and slow proton distributions.

As explained earlier the procedure for dealing with the ambiguities was different in the two experiments. In the kaon experiment (12) events were assigned a weight equal to the reciprocal of the number of acceptable fits. In the proton experiment, the fit with the highest probability was taken. Since in the latter experiment there was only a total of 69 ambiguous events and we can suppose at least half of these must have been selected correctly any bias is small.

The 1C channels (3) and (4) may be expected to have a
greater contamination. Fig. 4.4 shows the missing mass squared distributions for events having acceptable fits to the channels.

\[ k^- p \rightarrow p k^0 \pi^- + \text{neutrals} \]  
\[ k^- p \rightarrow k^0 \pi^+ \pi^- + \text{neutrals} \]

These hypotheses can in principle be ambiguous with each other but the number of such ambiguous fits is relatively small. Where a 4C fit to the reaction,

\[ k^- p \rightarrow p k^0 \pi^- \]

exists, this is taken to be the correct identification of the event.

The main source of bias will come from a loss of events into the no-fit category or an inclusion of no-fits in the fit channels. In Fig. 4.4 the shaded events correspond to those assigned to the no-fit channels by the criteria, the unshaded events to those in the fit channels (3) and (4). The contamination in these channels can be estimated from the distributions.

Suppose we assume that the distributions in the missing mass squared should be symmetric and that the lower half of such a distribution corresponds to a relatively pure sample of events. Then the number of wrongly included fits can be found by reflecting the lower half into the upper half and calculating the difference. The number of lost events can be found by fitting a Gaussian curve to the peaks and extrapolating beyond the upper missing mass squared cut-off. Using these methods it is estimated that about 3% of genuine fits have been excluded from channel (3) and 10% from channel (4). The contamination of "bad" events is calculated to be of the order of 5%
and 15% respectively for the two channels.

One further bias that may exist for the proton experiment is a possible loss of events through genuine 4C candidates being rejected by the D-MAC program. This was checked in two ways.

Firstly for a sample of film consisting of five half rolls, all events were measured irrespective of whether they were rejected or not. It was found that only one event from the reject class fitted the 4C hypothesis out of a total of about 200.

Secondly we show in Fig. 4.5 the distribution of the projected sum of the momenta as measured on the D-MAC machine for a sample of events belonging to channel (1) (the sample does not include the five half rolls mentioned above). This plot fully justifies the choice of 10 GeV/c as the cut-off point.

From both of these tests we estimate that the maximum number of lost events cannot exceed 1% of the total sample.

4.5 Cross-Section for \( pp \rightarrow pp \pi^+ \pi^- \)

Because of the very small beam contamination in the proton experiment we have attempted to determine the cross-section for channel (1) absolutely.

Suppose \( N \) protons are incident in a given time on a target of cross-sectional area \( a \), with \( n \) scattering centres per unit volume. Then the number of protons that interact in a distance \( dx \) is given by,

\[
dN = -N n a \frac{\sigma_p}{a} dx \tag{4.7}
\]

\[
= -N n \sigma_p dx \tag{4.8}
\]
where $\sigma_T$ is the total cross-section presented by each scattering 
centre.

Integrating equation (4.8) we find for the number of protons 
remaining after distance $x$,

$$N = N_0 e^{-n \sigma_T x} \quad (4.9)$$

where $N_0$ is the initial number.

Suppose we are interested in a particular class of reactions 
whose cross-section is $\sigma_\alpha$. Then the number of such reactions between 
$x$ and $x + dx$ is,

$$dN_\alpha = N_0 e^{-n \sigma_T x} \frac{\sigma_\alpha}{n} \sigma_\alpha \, dx \quad (4.10)$$

and hence the total number in a fiducial volume of length $L$ is,

$$N_\alpha = N_0 \frac{\sigma_\alpha}{\sigma_T} \int_0^L e^{-n \sigma_T x} \, dx$$

or

$$N_\alpha = N_0 \sigma_\alpha \int_0^L (1 - e^{-n \sigma_T x}) \, dx \quad (4.11)$$

Now $n$, the number of scattering centres per unit volume is given by 
$A p$, where $A$ is Avagadro's number and $p$ is the density of the target 
material, in our case liquid hydrogen. Thus $n$ is of order 
$0.4 \times 10^{23} \text{cm}^{-3}$. The total proton-proton cross-section at 16 GeV/c is 
about 40 mb, i.e. $4 \times 10^{-26} \text{cm}^2$. The fiducial volume is about 100cm 
long so $n \sigma_T L$ is of order 0.16. Hence to a good approximation we 
may write,

$$N_\alpha = N_0 \frac{\sigma_\alpha}{\sigma_T} L (1 - \frac{1}{2} n \sigma_T L) \quad (4.12)$$

Hence,
\[
\sigma_a = \frac{N_a}{N_0} \frac{\sigma}{n L} (1 + \frac{1}{n} \sigma T L)
\]  

(4.13)

In the expression for the cross-section for a particular reaction, the total cross-section enters as a small correction. To this extent the calculation is not an absolute one.

We have calculated the cross-section for reaction (1) using the detailed results from the ten half-rolls for which beam tracks were recounted on every frame. The numbers of events scanned and measured together with the number of 4C fits are given in Table 4.4. For comparison the statistics for the total Imperial College sample of film is given in Table 4.5.

The number of 4C fits, \( N \), has to be corrected for scanning losses, unmeasurable events and the effects of the 1\% probability cut-off. The value obtained for the cross-section was,

\[ \sigma = 1.68 \pm 0.07 \text{ mb} \]

We show in Fig. 4.6 a plot of the cross-section for reaction (1) versus the laboratory momentum. The data points at the other momentum are taken from the papers listed under Ref. (38). It can be seen that our cross-section point is in very good agreement with the results of other authors.

4.6 \( k^- \) Channel Cross-Sections

We have seen that to obtain the channel cross-section for the \( k^- \) induced reactions one simply needs to obtain the ratio of the "good" events in the sample to the total events in the particular topology. However, when the event involves a neutral \( k^0 \), there will be corrections necessary for the cases where the decay is not seen.
For channels (3) and (4) these events should be included in the no-fit reactions,

\[ k^- p \rightarrow p n^0 \]
and \[ k^- p \rightarrow n^+ n^0 \]

The lost events are estimated by weighting each observed event with the inverse of the probability that the \( k^0 \) decays within the illuminated region. In addition a correction is also necessary for cases where the decay occurs too close to the apex of the event to be detected. The minimum observable length (projected into the film plane) used was 3mm. The weight \( W \) used is given by,

\[
\frac{1}{W} = \exp \left( -\frac{L_{\text{min}}}{L \cos \lambda} \right) - \exp \left( -\frac{L_{\text{max}}}{L} \right) \]  

where \( L_{\text{min}} \) is the minimum observable length, \( L_{\text{max}} \) is the distance along the line of flight to the edge of the illuminated region and \( \lambda \) is the dip angle. \( L \) is the length corresponding to a \( k^0 \) with mean life-time \( \tau \) and is given by,

\[
L = \frac{\tau p}{m} \]  

where \( m \) is the mass and \( p \) the momentum.

This weighting procedure is intended to reduce biases in the sample against very slow or fast \( k^0 \)'s. A further correction is necessary for a cross-section calculation, to account for the neutral and three particle decay modes. The latter mode has a much longer mean life and will not be in general observable inside the chamber.

The full determination of the channel cross-section has to date only been performed for Experiments 75 and 10. These are summarised in Table 4.6 together with the number of events in each
channel. For completeness the proton experiment is included.

4.7 Mass Resolution

The mass resolution obtained for the effective mass of a group of particles is evidently of great importance in assessing whether fluctuations above background in the mass distribution have any significance. It will also determine to what extent we can expect to resolve closely spaced resonant states.

As a means of illustrating the point we show in Fig. 4.7 the distribution of the effective mass of the $\pi^-$ with the slow proton ($p_s$) in the laboratory and with the fast proton ($p_f$). The relatively narrow bin width of 25 MeV has been used for these plots. The mass of $p_s\pi^-$ shows a very great deal of structure with three clear peaks above background at masses of about 1225, 1475 and 1660 MeV. This structure is not at all evident in the $p_f\pi^-$ mass distribution, and it would be very difficult to estimate the widths of these resonances from this plot. This difference is of course understandable because slower tracks have small measurement errors. When these distributions are plotted in the larger bin width of 50 MeV (see Fig. 5.17(a)) they appear to be compatible, though of course less information is available compared with Fig. 4.7(a).

The situation for the three body effective mass, $p\pi^+\pi^-$, in channel (1) is much better. When the fast proton is involved we are essentially examining the missing mass to the well measured slow proton. In this case both distributions are fully compatible.

One can estimate the mass resolution for a given effective mass combination in several ways.
The obvious method is to use the fitted errors of the track parameters as given by GRIND. The square of the effective mass of two particles is given by,

$$M_{12}^2 = (E_1 + E_2)^2 - (p_1 + p_2)^2$$

$$= m_1^2 + m_2^2 + 2E_1E_2 - 2p_1p_2 \cos \theta_{12}$$  \hspace{1cm} (4.16)

where \(E_i, p_i, m_i\) are the energy, momentum and mass of the \(i\)th track and \(\theta_{12}\) is the angle between them.

If the dip and azimuthal angle of track \(i\) are \(\lambda_i, \varphi_i\) respectively, we have

$$\cos \theta_{12} = \cos \lambda_1 \cos \lambda_2 \cos (\varphi - \varphi_2) + \sin \lambda_1 \sin \lambda_2$$  \hspace{1cm} (4.17)

Using the normal laws of error propagation \(\delta M_{12}\) can be calculated.

For the proton experiment we have estimated the two body effective mass resolution at a mass of 1.5 GeV to be less than 10 MeV for the \(p_s \pi\) combination and of order 20 MeV for the \(p_f \pi\) combination. Because of the lower primary momentum for the 10 GeV/c experiment the results are somewhat better. The resolution for a \(K^- \pi\) mass of 1.4 GeV in the 4C channel (2) is about 10 MeV. The 1C fits are somewhat worse and here the result is of order 17 MeV.

A second approach is to fit a well established resonance (preferably of narrow width) with a Breit-Wigner shape and to estimate the resolution from,

$$\Gamma_f^2 = \Gamma_0^2 + \delta M^2$$  \hspace{1cm} (4.18)

where \(\Gamma_f\) is the fitted width and \(\Gamma_0\) the natural width. (This formula is of course only strictly correct for the sum of two Gaussian
distributions, but it should not be too bad an approximation).

The $K^0 \pi^-$ effective mass distribution from channels (3) and (4) has been fitted with the sum of two Breit-Wigners. The fitted $K^*(890)$ width was 58 MeV compared with the generally accepted value of 50 MeV, giving a resolution of order 30 MeV in these two channels.
TABLE 4.1

(a) **Experiment 12**

<table>
<thead>
<tr>
<th>Topology</th>
<th>Scan 1 (%)</th>
<th>Scan 2 (%)</th>
<th>Overall (%)</th>
<th>Events Found</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>85.3</td>
<td>81.7</td>
<td>97.3</td>
<td>4626</td>
</tr>
<tr>
<td>400</td>
<td>94.8</td>
<td>92.5</td>
<td>99.6</td>
<td>4331</td>
</tr>
<tr>
<td>600</td>
<td>95.8</td>
<td>93.8</td>
<td>99.7</td>
<td>1352</td>
</tr>
<tr>
<td>001</td>
<td>42.6</td>
<td>65.2</td>
<td>80.0</td>
<td>124</td>
</tr>
<tr>
<td>201</td>
<td>91.9</td>
<td>86.3</td>
<td>98.9</td>
<td>1324</td>
</tr>
<tr>
<td>401</td>
<td>94.6</td>
<td>92.6</td>
<td>99.6</td>
<td>1129</td>
</tr>
<tr>
<td>601</td>
<td>96.0</td>
<td>92.6</td>
<td>99.7</td>
<td>293</td>
</tr>
<tr>
<td>210</td>
<td>91.1</td>
<td>84.8</td>
<td>98.6</td>
<td>143</td>
</tr>
<tr>
<td>410</td>
<td>95.9</td>
<td>90.7</td>
<td>99.6</td>
<td>401</td>
</tr>
<tr>
<td>610</td>
<td>93.4</td>
<td>89.4</td>
<td>99.3</td>
<td>151</td>
</tr>
<tr>
<td>≥ 8 prong</td>
<td>95.2</td>
<td>93.6</td>
<td>99.7</td>
<td>181</td>
</tr>
<tr>
<td>Rare</td>
<td>89.8</td>
<td>84.1</td>
<td>98.4</td>
<td>380</td>
</tr>
<tr>
<td>300</td>
<td>67.8</td>
<td>68.5</td>
<td>89.9</td>
<td>118</td>
</tr>
</tbody>
</table>

(b) **16 GeV/pp**

<table>
<thead>
<tr>
<th>Topology</th>
<th>Scan 1 (%)</th>
<th>Scan 2 (%)</th>
<th>Overall (%)</th>
<th>Events Found</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>81.1</td>
<td>83.8</td>
<td>97.0</td>
<td>5024</td>
</tr>
</tbody>
</table>

* Figures only for the 10 half rolls which were scanned twice
** Includes 300
TABLE 4.2

TOPOLOGICAL CROSS-SECTIONS AT 10 GeV/c

<table>
<thead>
<tr>
<th>Topology</th>
<th>Events</th>
<th>Cross-Section (µb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>57000</td>
<td>8206 ± 110</td>
</tr>
<tr>
<td>400</td>
<td>43505</td>
<td>6160 ± 86</td>
</tr>
<tr>
<td>600</td>
<td>10533</td>
<td>1962 ± 38</td>
</tr>
<tr>
<td>001</td>
<td>1685</td>
<td>277 ± 10</td>
</tr>
<tr>
<td>201</td>
<td>13592</td>
<td>1768 ± 21</td>
</tr>
<tr>
<td>401</td>
<td>10618</td>
<td>1477 ± 18</td>
</tr>
<tr>
<td>601</td>
<td>1558</td>
<td>322 ± 15</td>
</tr>
<tr>
<td>210*</td>
<td>857</td>
<td>287 ± 12</td>
</tr>
<tr>
<td>410*</td>
<td>1841</td>
<td>619 ± 16</td>
</tr>
<tr>
<td>610*</td>
<td>443</td>
<td>323 ± 15</td>
</tr>
<tr>
<td>≥ 8 prongs*</td>
<td>393</td>
<td>287 ± 15</td>
</tr>
<tr>
<td>Rare*</td>
<td>1301</td>
<td>437 ± 40</td>
</tr>
</tbody>
</table>

* Does not include Experiment 12 data.
### TABLE 4.3

<table>
<thead>
<tr>
<th>Particle</th>
<th>$\frac{F - B}{F + B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$\frac{36}{5634}$</td>
</tr>
<tr>
<td>$\pi^+$</td>
<td>$\frac{117}{2817}$</td>
</tr>
<tr>
<td>$\pi^-$</td>
<td>$\frac{15}{2817}$</td>
</tr>
</tbody>
</table>

### TABLE 4.4

STATISTICS FOR TEN HALF-ROLLS USED IN CROSS-SECTION CALCULATION

<table>
<thead>
<tr>
<th>Frames</th>
<th>Events</th>
<th>No. Meas.</th>
<th>Unmeas.</th>
<th>No. 4C Fits</th>
</tr>
</thead>
<tbody>
<tr>
<td>7500</td>
<td>5024</td>
<td>2687</td>
<td>302</td>
<td>413</td>
</tr>
</tbody>
</table>

### TABLE 4.5

STATISTICS FOR TOTAL IMPERIAL COLLEGE DATA

<table>
<thead>
<tr>
<th>Frames</th>
<th>Events</th>
<th>No. Meas.</th>
<th>Unmeas.</th>
<th>No. 4C Fits</th>
</tr>
</thead>
<tbody>
<tr>
<td>27000</td>
<td>16733</td>
<td>9013</td>
<td>973</td>
<td>1460</td>
</tr>
</tbody>
</table>
### TABLE 4.6

<table>
<thead>
<tr>
<th>Final State</th>
<th>No. of Unique Fits</th>
<th>No. of Ambig. Fits</th>
<th>Total Fits</th>
<th>Total Events</th>
<th>Total Decay Wt.</th>
<th>Average Decay Wt.</th>
<th>Cross-Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>pdp π⁺n⁻</td>
<td>2817</td>
<td>-</td>
<td>2817</td>
<td>2817</td>
<td>-</td>
<td>-</td>
<td>1.69 ± 0.07</td>
</tr>
<tr>
<td>pk⁻n⁺π⁻</td>
<td>5246</td>
<td>1645</td>
<td>6891</td>
<td>6068</td>
<td>-</td>
<td>-</td>
<td>0.883 ± 0.022*</td>
</tr>
<tr>
<td>pK⁻π⁻π⁺</td>
<td>1514</td>
<td>12</td>
<td>1526</td>
<td>1520</td>
<td>1658.9</td>
<td>1.091</td>
<td>0.55 ± 0.03*</td>
</tr>
<tr>
<td>nK⁻π⁺π⁻</td>
<td>785</td>
<td>16</td>
<td>801</td>
<td>793</td>
<td>842.8</td>
<td>1.063</td>
<td>0.31 ± 0.02*</td>
</tr>
</tbody>
</table>

* Cross-section figures taken from Experiments 75 and 10 only.*
FIG. 4.1

(a) $pp \rightarrow pp \pi^+ \pi^-$

(b) $K^- p \rightarrow K^- p \pi^+ \pi^-$
$K^- p \rightarrow K^0 p \pi^- \pi^0$

$K^- p \rightarrow K^0 n \pi^+ \pi^-$

**FIG. 4.1**
BEAM STRETCH FUNCTIONS

\[ pp \rightarrow pp \pi^+ \pi^- \]

(a) $\frac{1}{P}$

(b) $\lambda$

(c) $\phi$

FIG. 4.2
FIG. 4.3

(a) \[ t \text{ (Target, slow proton)} \]

(b) \[ t \text{ (Beam, fast proton)} \]
Experiment 12. Missing Mass

Squared Distributions

(a) $K^-p \rightarrow p K^0 \pi^- + \text{neutrals}$

(b) $K^-p \rightarrow K^0 \pi^+ \pi^- + \text{neutrals}$

FIG. 4.4
$pp \rightarrow pp \pi^+ \pi^-$

Sum of projected measured momenta

264 events + 30 overflows

FIG. 4.5
FIG. 4.6

- Ref. 38
- This experiment

$\text{Cross-section (m.b.)}$

$\text{P}_{\text{LAB}}$ (GeV/c)
FIG. 4.7

NO. OF EVENTS / 0.025 GeV

NO. OF EVENTS / 0.025 GeV

(b)

M(p, n') GeV

M(p, π') GeV
CHAPTER 5

GENERAL FEATURES OF THE REACTIONS

5.1 Introduction

The channels which we are considering exhibit the typical features of all high energy hadronic interactions. The distributions in the kinematic variables violently disagree with the predictions of a pure statistical model (phase-space) and in particular the invariant mass plots indicate copious production of both two and three body resonances. The $p p n^+$ final state is reached almost exclusively through intermediate states involving at least one nucleon isobar, whereas the $k N n^+$ channels are produced via intermediate states involving either meson resonances, both strange and non-strange, or nucleon isobars. The partial cross-sections for strange baryon resonance production in these channels are extremely small.

5.2 Centre-of-mass variables

In figs. 5.1 to 5.8 are shown the centre-of-mass longitudinal and transverse* momentum distributions of the final state particles, referred to the direction of the incident proton or kaon.

The pure statistical model predicts that any spatial component of the momentum of a given particle should be normally distributed with zero mean and a standard deviation that increases as the mass of the

* The transverse momenta are of course invariant under the Lorentz Transformation from laboratory to centre-of-mass systems.
particle. Furthermore, the spatial components are predicted to be uncorrelated so that the distribution in the transverse momentum of any particle is given by,

\[ N(p_t) \, dp_t = \frac{1}{2\sigma_t^2} \, p_t \, e^{-p_t^2/2\sigma_t^2} \, dp_t \]  \hspace{1cm} (5.1)

where \( \sigma_t \) is the standard deviation of the distribution of a single component.

The mean of such a distribution is given by,

\[ < p_t > = \sqrt{\frac{\pi}{2}} \, \sigma_t \]  \hspace{1cm} (5.2)

so that the mean transverse momentum also increases with increasing particle mass.

The longitudinal momentum \( p^\ast \) distributions show the final state protons in channel (1) to be produced strongly forwards and backwards. The kaon in channels (2), (3) and (4) is produced strongly forwards and the nucleon backwards. Clearly phase-space is quite inadequate to describe the data and some more sophisticated matrix element is needed that must certainly favour events of a peripheral nature.

The pions from all channels in general tend to be produced with smaller values of longitudinal momentum and the distributions peak somewhere in the neighbourhood of zero. For channel (1) of course, these distributions should be exactly symmetric about zero because of the symmetry in the initial state.
The means and standard deviations for the pion $p^*_L$ distributions from channel (1) are listed in Table 5.1 together with the mean for the proton distribution. Because of the final state symmetry we have in fact evaluated $<|p^*_L|>$ for the protons. A Gaussian curve with the parameters listed in Table 5.1 does not particularly well describe either of the pion distributions, which are rather too sharply peaked for a good fit to be obtained. This is particularly true of the $\pi^-$ distribution. The values of chi-squared obtained for the fits are given in Table 5.1.

The mean longitudinal momenta for the various final state particles in the kaon induced reactions are listed in Table 5.2.

The mean transverse momenta for all the particles from the four reactions are listed in Table 5.3. It can be seen that the mean proton transverse momentum is roughly the same for channels (1) to (3) and of order 370 MeV/c. For channel (4), the mean neutron transverse momentum is considerably higher. The pions generally seem to have a lower mean than either the nucleon or kaon within a given channel, whereas the value for the kaon is of the same order as the nucleon. It is of interest that the mean transverse momenta in channel (4) are consistently higher than those in other channels, this being the only channel where diffractive type mechanisms are not allowed.

One further anomaly is that the mean transverse momentum for the positive pion in reactions (1) and (2) is considerably less than that for the negative pion. This illustrates the strong influence resonance production has on these distributions. If one recalculates the means for channel (1) only considering those events where both $p^-\pi^+$ masses are greater than 1.4 GeV (to remove the strong $\Delta^{++} (1236)$ signal)
one obtains $496 \pm 18$ MeV/c for the $\pi^+$ and $336 \pm 12$ MeV/c for the $\pi^-$. In this situation it is the $\pi^+$ distribution which is peaked to larger values.

Broadly speaking our results are of the same order as those found by other authors (39) at different energies.

It has been known (40) for some time that this quantity $\langle p_t \rangle$ is very roughly $350$ MeV/c for all secondaries produced in hadron collisions, independent of the primary energy. It has become common practice to assume that the shape of the distribution is also the same for all secondaries and given by (5.1).

An unbiased maximum likelihood estimate of $\sigma_t$ is,

$$\sigma_t = \left( \frac{1}{2N} \sum_{i=1}^{N} p_{t_i}^2 \right)^{\frac{1}{2}}$$  \hspace{1cm} (5.3)

with error,

$$\delta \sigma_t = \frac{\sigma_t}{\sqrt{4N}}$$  \hspace{1cm} (5.4)

where $N$ is the number of events in the sample. We have calculated curves for all the particles with values of $\sigma_t$ given by (5.3) and the results are summarised in Table 5.3.

Clearly the fits with one or two exceptions are extremely poor. The nucleon distribution is always badly described and this applies also to the mesons in channels (1) and (2). The situation for the kaons and pions is somewhat better in channels (3) and (4), quite good fits being obtained for the $K^0$ and $\pi^+$ in the latter channel.

As an example of the type of fit obtained the curves for the particles in channel (2) are shown in Fig. 5.4.
With the advent of larger statistics, recent workers have attempted to measure more precisely the transverse momentum spectra for all secondaries produced in hadron collisions. Some authors, for example Cocconi et al. (41) and Orear (42), have claimed that the spectra are well represented by an expression of the form,

\[ N(p_t) \, dp_t = \frac{1}{p_0} \frac{p_t}{p_0} e^{-\frac{p_t}{p_0}} \, dp_t \quad (5.5) \]

However, Friedlander (43), using a very large compilation of data has shown that this form is not adequate for all values of \( p_t \). He also pointed out that \( <p_t> \) may increase with increasing primary energy. In a systematic survey of the transverse momentum spectra of pions, kaons and baryons produced in 12.5 GeV/c proton-proton interactions, Ratner et al. (44) confirmed that equation (5.5) does not provide an adequate parametrization of the data, but rather the distributions obey a law such as (5.1).

From (5.5) an unbiased maximum likelihood estimate of \( p_0 \) is given by,

\[ p_0 = \frac{1}{2N} \sum_{i=1}^{N} p_{t_i} \quad (5.6) \]

with error,

\[ \delta p_0 = \frac{p_0}{\sqrt{2N}} \quad (5.7) \]

We have evaluated these expressions for the events in all four channels and the results are displayed in Table 5.3 together with the values of chi-squared for the fits. Although the fits cannot be
described as particularly good, they are clearly very much better for channels (1) and (2) than those provided by the Boltzman type distribution, (5.1). The baryon spectra are better parametized in all four channels. Some typical curves are shown for channel (3) in Fig. 5.6.

In conclusion it should be mentioned that the results of the authors quoted have been for distributions summed over all final multiplicities. We on the other hand have isolated four body final states in which resonance production is particularly strong.

The centre-of-mass production angular distributions show the striking forward or backward peaking of the final state baryons and kaons in perhaps a rather more convenient form. These are displayed in Figs. 5.9 to 5.12 for channels (1) to (4) respectively. The pion distributions are also shown and in general these exhibit both a forward and a backward peak. We shall be analysing these distributions in somewhat more detail in Chapter 6.

5.3 Effective Mass Distributions

The effective mass distributions for some two and three body combinations of interest are displayed in Figs. 5.13 to 5.21. Those combinations not shown have an essentially smooth behaviour with no apparent resonance structure.

Figure 5.13 shows the two pion effective mass in the four channels. A substantial p signal is present for the three kaon channels, but this is not so evident in channel (1). However, when we make the selection,

\[ M(pn^+) > 1.4 \text{ GeV} \]

for both pn⁺ combinations, in an attempt to remove events having isobars
present, the $\rho^0$ stands out more strongly. This is shown in Fig. 5.14 and a significant $f^0$ signal can also be seen.

The $K\pi$ mass spectra for the three kaon induced reactions are shown in Fig. 5.15. Very strong $k^*(890)$ production is apparent in all three channels with an appreciable amount of $k^*(1420)$. In channel (3), the $k^*$ resonances can be observed in two charged states.

The $p\pi^+$ mass spectra for reactions (1) and (2), shown in Fig. 5.16, exhibit copious production of the $\Delta^{++}(1236)$ isobar. There is no evidence to suggest that any other $I = 3/2$ isobar is being produced.

When we come to look at the mass spectra for other $N\pi$ combinations the situation is not so clear. The $p\pi^-$ effective mass distributions are plotted in Figs. 5.17 (a), (b) and (c) for channels (1) to (3) respectively. In each case there is some indication of structure in the low mass $p\pi^-$ system. We have already seen in Section 4.7, that for channel (1), looking only at the $p\pi^-$ combination with the slow proton in the laboratory shows up three very clear peaks at around 1225, 1450 and 1650 MeV. For reaction (2) we have attempted to improve the resolution by requiring that the absolute value of the four-momentum transfer, $t$, from target proton to $p\pi^-$ system should be less than 0.6 GeV$^2$. The resulting distribution is shown in Fig. 5.18 and the three peak structure is now much more evident. In channel (3) the $p\pi^-$ mass distribution has an accumulation of events at low masses but there is little evidence for any structure. In this case, the application of any $|t|$ selection does not improve the resolution.

In reaction (4) there is some evidence for enhancements in the $n\pi^+$ system, but the statistics are rather limited. The distribution is
shown in Fig. 5.17 (e). The \( n\pi^- \) mass spectrum, shown in Fig. 5.17 (f), seems to have an essentially smooth behaviour.

There is a reasonably strong \( \Delta^+(1236) \) signal in channel (3) and the corresponding \( p\pi^- \) mass distribution is shown in Fig. 5.17 (d). In this channel we have the possibility of a quasi-two-body intermediate state, namely

\[
k^- p \rightarrow \Delta^+(1236) k^-(890)
\]

We show in fig. 5.19 (a) the \( p\pi^- \) mass spectrum with the requirement that the \( k^0\pi^- \) mass be in the \( k^*(890) \) band defined by,

\[
0.82 < M(k^0\pi^-) < 0.96 \text{ GeV}
\]

The \( \Delta^+ \) peak is still very much in evidence. Similarly the distribution in the \( k^0\pi^- \) mass is displayed in Fig. 5.19 (b) where the \( p\pi^- \) mass is constrained to lie in the \( \Delta^+(1236) \) region defined by,

\[
1.15 < M(p\pi^-) < 1.35 \text{ GeV}
\]

A strong \( k^-(890) \) signal can be seen with some evidence for \( k^-(1420) \).

We now turn to the three body mass combinations. The \( k\pi\pi \) mass spectra for the three kaon induced reactions are shown in Fig. 5.20. The well known \( Q \) and \( L \) meson are very much in evidence in channels (2) and (3), but no structure can be seen in the distribution for channel (4).

The \( N\pi\pi \) mass distributions for the four channels are plotted in Fig. 5.21. In channels (1) and (2) there is an accumulation of events
at the low mass ends of the spectra, with evidence for possible isobars at around 1450 and 1700 MeV. The $p\pi^-\pi^0$ and $n\pi^+\pi^-$ distributions shown in Fig. 5.21 (c) and (d) show no evidence for any low mass structure.

5.4 The Isobars

In this section we attempt to identify the nucleon enhancements seen in our data with the isobars reported in the phase-shift analyses (5).

There is little doubt that the large enhancement seen in the $p\pi^+$ mass distributions (Fig. 5.16) at around 1225 MeV in channels (1) and (2) is the $I = 3/2 \Delta^{++} (1236)$ resonance. However, the identification of the enhancements in other charged modes of the $N\pi$ system is not so obvious.

We have seen that the low mass $p\pi^-$ spectra in channels (1) and (2) (see Fig. 5.17) show definite evidence for a three peaked structure (Such a structure cannot be ruled out for the $p\pi^-$ and $n\pi^+$ distributions in channels (3) and (4), but here the statistics are more limited). The first enhancement in these distributions is at around 1225 MeV and can presumably be associated with the neutral mode of the $\Delta(1236)$. However, at least part of the enhancement may be a reflection of the structure at around 1400 MeV in the $p\pi^+\pi^-$ mass distributions. The low available phase-space for the decay of this object would force the $p\pi^-$ mass to lie in the $\Delta^0$ region. (The same comments apply equally to the $\Delta^{++} (1236)$, but here the cross-section is much larger).

The other two peaks in the distributions are at around 1470 and 1670 MeV. Since there is no evidence for these enhancements in the $I = 3/2 p\pi^+$ system, they may be assumed to have $I = \frac{1}{2}$. A number of
possible resonances could be contributing in this region. The lower mass object could be associated with the $P_{11}$ (1470) or $D_{13}$ (1520) or perhaps a mixture of these two. We shall see in Chapter 10 that the $p^n\pi^-$ enhancement at 1700 MeV could also be contributing to the structure.

There are three well established resonances in the region of 1670 MeV: the $D_{15}$ (1670), $F_{15}$ (1688) and $S_{11}$ (1700), and any combination of these may be present.

It should be stressed at this stage that although these enhancements may be referred to in the course of this work as single objects, we cannot demonstrate that this is in fact the case. Rather there may be many resonances contributing with possible interference effects.

The $p^n\pi^0$ mass distribution of channel (3), shown in Fig. 5.17(d), shows a significant $\Delta^+$ signal, but no evidence of any $I = \frac{1}{2}$ enhancement. We have seen that there is no low mass $p^n\pi^0$ structure in this channel, so this effect cannot be simply a reflection.

It will be seen in the next chapter that the low mass enhancements in the $p^n\pi^-$ systems of channels (1) and (2) probably have $I = \frac{3}{2}$. The first such enhancement at around 1450 MeV may again be due to some combination of the $P_{11}$ (1470) and $D_{13}$ (1520), both of which have significant three body decay modes (5). We shall see in Chapters 10 and 11 that there is good evidence for believing that the low mass $p^n\pi^-$ system is produced in accordance with the Gribov-Morrison parity rule,

$$\Delta P = (-') \Delta J$$ (5.8)
where $\Delta J, \Delta P$ are the change in spin and parity relative to the incident proton. If this is the case the only candidate for the 1700 MeV enhancements is the $F_{15} (1688), J^P = 5/2^+$. There is the possibility of the $P_{11} (1780)$ with $J^P = \frac{3}{2}^+$, but the mass is rather high.

For the high statistics channels (1) and (2) we have attempted to parametrize the mass spectra in terms of relativistic Breit-Wigner functions with hand drawn backgrounds.

In the case of a two body isobar, the Breit-Wigner function was written,

$$BW(M) = \frac{M \Gamma(M)}{(M^2 - M_o^2)^2 + M_o^2 \Gamma^2(M)}$$

(5.9)

where $M_o$ is the resonance mass and $M$ the mass of the $p\pi$ combination.

For the $\Delta(1236)$ resonance, an energy dependent width of the form suggested by Jackson (46) was used:

$$\Gamma(M) = \left(\frac{q}{q_o}\right)^3 \frac{\rho(M)}{\rho(M_o)} \Gamma_o$$

(5.10)

where $\Gamma_o$ is the natural width, $q$ is the momentum of either of the decay products from a $p\pi$ system of mass $M$ and $q_o$ the momentum from a system of mass $M_o$. $\rho$ is a slowly varying function of the form

$$\rho(x) = \frac{(x + m_p)^2 - m_n^2}{x^2}$$

(5.11)

where $m_p, m_n$ are the proton and pion masses.
For the other pπ resonances, an energy independent width was assumed.

To fit the three-body distributions an energy dependent width was used of the form,

$$\Gamma(M) = \frac{R_3(M)}{R_3(M_0)} \cdot \int_0^1$$

(5.12)

where $R_3(x)$ is the available phase-space for a system of mass $x$ decaying into $p\pi^+\pi^-$.  

The effective mass distributions were parametrized in terms of an expression,

$$f(M) = \sum_{k=1}^{n} b_k \frac{E(M)A(M)}{\int E(M)A(M)dM} + (1 - \sum_{k=1}^{n} b_k) \frac{A(M)}{\int A(M)dM}$$

(5.13)

where $A(M)$ is the hand-drawn background function and $b_k$ the fraction of resonance $k$. The sum runs over the number of resonances used in the fit. The expression (5.13) is normalized to unity and so is in the form of a probability density. Hence we may write the log-likelihood function,

$$\ln \mathcal{L} = \sum_{i=1}^{N} \ln f(M_i)$$

(5.14)

where $N$ is the number of events. This function is then maximised by varying the masses and widths and percentages $b_k$.

The fitted masses and widths of the various enhancements are given in Table 5.4, together with an estimate of the cross-section for their production.

In fitting the pπ mass spectra, only the combination with the slow proton was used in channel (1) and in channel (2) a selection
It will be noted that the production cross-sections quoted for channel (1) have a sum which is greater than the total reaction cross-section, but of course this method takes no account of double resonance production or of cases where a three body isobar has a quasi-two-body decay mode.

In general the fitted widths and cross-sections, and to a lesser extent the masses, depend very strongly on the form of the background used, and this is reflected in the relatively large errors quoted. A more sophisticated method for finding the reaction cross-sections in channel (1) will be described in Chapter 7 and it will be seen there that the values obtained with this simple minded procedure are not too unreasonable.

5.5 Four-Momentum Transfer Distributions

We now turn to a discussion of the distributions in the invariant four-momentum transfer, \( t \), from the initial state beam or target to various combinations of final state particles.

A difficulty arises in channel (1) because of the identical particles in the initial and final state. If we are interested in the distribution of \( t \) from proton to proton for example, then four values of this quantity can be calculated for each event. We showed earlier in the chapter that the final state baryons are produced very strongly forwards and backwards, and since we believe that hadron collisions at this sort of energy are dominated by peripheral mechanisms, to a good approximation we can associate the fast proton in the laboratory
with the beam proton and the slow proton with the target. Because the
final state protons are so well separated, as evidenced by the \( p_\perp(p) \)
(Fig. 5.1(a)) and \( \cos \theta_p \) (Fig. 5.9(a)) distributions, this method leads
to almost no ambiguities. Throughout the rest of this work, unless
stated otherwise, it is to be understood that this association of the
protons has been made.

The distributions in \( t \) for various combinations of final
state particles in the four channels are shown in Figs. 5.22 to 5.25.
In general all these distributions exhibit a strong peaking at low
values of \( |t| \). This peaking is most pronounced within a given channel
for the distribution in the four momentum transfer from baryon to baryon.
Those from kaon to kaon are less sharp.

Another striking feature is that the distributions in \( t \) from
initial proton to the \( np \) combinations of channel (4) are much flatter
than the corresponding distributions for the \( pn \) combinations in channels
(1) and (2).

We have tried to put these ideas on a more quantitative basis
by fitting the differential distributions in \( t' \) to a form,

\[
\frac{d\sigma}{dt'} = A e^{-at'}
\]  

(5.15)

where,

\[
t' = |t - t_{\text{min}}|
\]  

(5.16)

and \( |t_{\text{min}}| \) is the minimum value of \( |t| \) allowed for a given event and is
obviously a function of both the mass of the combination being considered
and the recoiling mass. The minimum value of \( t' \) is now of course zero,
so that the distributions for various mass combinations can be compared
on a more equal basis.

The values of $a$ for both two and three body baryon systems have been determined as a function of the mass of the system. The results are plotted in Figs. 5.26 to 5.29. The distributions were fitted in the range,

$$0.0 \leq t' < 0.4 \text{ GeV}^2$$

and acceptable chi-squared probabilities were obtained for all the data points shown.

The first striking feature to be noted is that in all channels and for all particle combinations there is a general decrease in the slope $a$, as the mass of the combination increases.

Eisenberg and Lyons (55) have recently investigated this phenomenon and showed that it is largely kinematic in origin. The Regge-pole theory as such does not reproduce the decrease in slope with mass, for a given reaction, unless impossibly large values for the slopes of the Regge trajectories are assumed.

Consider for example the reaction,

$$pp \rightarrow \Delta^{++} p\pi^-$$

and suppose this is dominated by the two diagrams shown in Fig. 5.30. To avoid any problems with duality (47), (48) we suppose that the $p\pi^-$ vertex in diagram (a) does not include any quasi-diffraction scattering, which will be counted in (b). Now looking at the distribution in $t'$ to the $p\pi^-$ combination is only physically meaningful so long as diagram (a) dominates. This will mainly occur at low $p\pi^-$ mass. As the mass increases more and more of diagram (b) may contribute so that the slope of the $t'$ distribution for $p\pi^-$ decreases (since evidently the $p\pi^-$
combination will cease to be produced "peripherally").

Another interesting point is that the very fact of choosing $t'$ as a variable, rather than $t$, may cause this variation of slope with mass. If the matrix element for a process can be written,

$$|A|^2 \sim e^{\mu t}$$  \hspace{1cm} (5.17)

where $\mu$ is a constant, then the slope of the distribution in $t'$ will be $\mu$, independent of the mass of the pion system.

However, if the matrix element should really be represented by an expression,

$$|A|^2 \sim \frac{1}{(t - m_\pi^2)^2}$$  \hspace{1cm} (5.18)

where $m_\pi$ is the mass of the pion, the experimental value of the slope of the distribution in $t'$ will decrease with increasing pion mass (since the distance from the boundary of the physical region to the pion pole increases with increasing pion mass).

It was pointed out in a paper published by the collaboration that, within the errors, these distributions of slope versus mass show no structure corresponding to that in the mass distributions. This paper mainly considered meson systems where the dominant resonances tend to be rather better separated, but Figs. 5.26 to 5.29 show that this in general is also the case for baryon systems.

The largest values of the slope $a$ are observed for the low mass $p^+\pi^-$ systems produced in channels (1) and (2). The slopes for
the low mass $p\pi^-\pi^0$ and $n\pi^+\pi^-$ systems in channels (3) and (4), where no structure is observed, are much lower.

The $\Delta^{++}(1236)$ resonance is produced in the first two channels with slope of order $8$ GeV$^{-2}$ and the low mass $p\pi^-$ system with slope of order $7$ GeV$^{-2}$. The two body baryon mass combinations in channels (3) and (4) tend to be produced with somewhat smaller slopes.
### TABLE 5.1

**pp→ppπ⁺π⁻: Longitudinal Momentum Distributions**

<table>
<thead>
<tr>
<th>Particle</th>
<th>Mean $p_\parallel$ (MeV/c)</th>
<th>St. Dev (MeV/c)</th>
<th>$\chi^2$/ND</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>$1874 \pm 8$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\pi^+$</td>
<td>$1.2 \pm 12$</td>
<td>$616 \pm 9$</td>
<td>90/31</td>
</tr>
<tr>
<td>$\pi^-$</td>
<td>$8 \pm 13$</td>
<td>$678 \pm 9$</td>
<td>107/33</td>
</tr>
</tbody>
</table>

### TABLE 5.2

**Mean Longitudinal Momentum**

<table>
<thead>
<tr>
<th>Channel</th>
<th>Particle</th>
<th>Mean (MeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2)</td>
<td>$p$</td>
<td>$-1,456 \pm 7$</td>
</tr>
<tr>
<td></td>
<td>$\kappa^-$</td>
<td>$1,009 \pm 8$</td>
</tr>
<tr>
<td>$\pi^+$</td>
<td></td>
<td>$200 \pm 7$</td>
</tr>
<tr>
<td>$\pi^-$</td>
<td></td>
<td>$248 \pm 9$</td>
</tr>
<tr>
<td>(3)</td>
<td>$p$</td>
<td>$-1,606 \pm 10$</td>
</tr>
<tr>
<td></td>
<td>$\kappa^0$</td>
<td>$852 \pm 13$</td>
</tr>
<tr>
<td>$\pi^+$</td>
<td></td>
<td>$439 \pm 13$</td>
</tr>
<tr>
<td>$\pi^-$</td>
<td></td>
<td>$315 \pm 15$</td>
</tr>
<tr>
<td>(4)</td>
<td>$\pi^-$</td>
<td>$1,238 \pm 22$</td>
</tr>
<tr>
<td></td>
<td>$\kappa^0$</td>
<td>$.929 \pm 20$</td>
</tr>
<tr>
<td>$\pi^+$</td>
<td></td>
<td>$.87 \pm 20$</td>
</tr>
<tr>
<td>$\pi^-$</td>
<td></td>
<td>$.395 \pm 23$</td>
</tr>
</tbody>
</table>
### Table 5.3

Parameters of Transverse Momentum Distributions

<table>
<thead>
<tr>
<th>Channel</th>
<th>Particle</th>
<th>Mean $P_t$ (MeV/c)</th>
<th>$\sigma_t$ (MeV/c)</th>
<th>$\chi^2$/ND</th>
<th>$P_0$ (MeV/c)</th>
<th>$\chi^2$/ND</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$p$</td>
<td>$378 \pm 3$</td>
<td>$318 \pm 2$</td>
<td>$441/24$</td>
<td>$189 \pm 2$</td>
<td>$148/24$</td>
</tr>
<tr>
<td></td>
<td>$\pi^+$</td>
<td>$277 \pm 4$</td>
<td>$241 \pm 2$</td>
<td>$613/19$</td>
<td>$139 \pm 2$</td>
<td>$145/19$</td>
</tr>
<tr>
<td></td>
<td>$\pi^-$</td>
<td>$353 \pm 5$</td>
<td>$295 \pm 3$</td>
<td>$161/21$</td>
<td>$177 \pm 3$</td>
<td>$83/21$</td>
</tr>
<tr>
<td></td>
<td>$p$</td>
<td>$360 \pm 3$</td>
<td>$299 \pm 2$</td>
<td>$380/23$</td>
<td>$186 \pm 2$</td>
<td>$95/23$</td>
</tr>
<tr>
<td></td>
<td>$K^-$</td>
<td>$381 \pm 3$</td>
<td>$319 \pm 2$</td>
<td>$370/24$</td>
<td>$197 \pm 3$</td>
<td>$77/24$</td>
</tr>
<tr>
<td></td>
<td>$\pi^+$</td>
<td>$282 \pm 2$</td>
<td>$237 \pm 2$</td>
<td>$430/22$</td>
<td>$141 \pm 2$</td>
<td>$60/22$</td>
</tr>
<tr>
<td></td>
<td>$\pi^-$</td>
<td>$351 \pm 3$</td>
<td>$292 \pm 2$</td>
<td>$342/22$</td>
<td>$179 \pm 3$</td>
<td>$137/22$</td>
</tr>
<tr>
<td>(2)</td>
<td>$p$</td>
<td>$364 \pm 6$</td>
<td>$302 \pm 4$</td>
<td>$103/20$</td>
<td>$182 \pm 3$</td>
<td>$97/20$</td>
</tr>
<tr>
<td></td>
<td>$K^-\pi^+$</td>
<td>$368 \pm 6$</td>
<td>$305 \pm 4$</td>
<td>$86/190$</td>
<td>$184 \pm 3$</td>
<td>$67/19$</td>
</tr>
<tr>
<td></td>
<td>$\pi^-$</td>
<td>$318 \pm 5$</td>
<td>$261 \pm 3$</td>
<td>$70/16$</td>
<td>$159 \pm 3$</td>
<td>$125/16$</td>
</tr>
<tr>
<td></td>
<td>$K^0\pi^0$</td>
<td>$317 \pm 5$</td>
<td>$264 \pm 3$</td>
<td>$103/17$</td>
<td>$159 \pm 3$</td>
<td>$97/17$</td>
</tr>
<tr>
<td>(3)</td>
<td>$n$</td>
<td>$447 \pm 10$</td>
<td>$377 \pm 6$</td>
<td>$209/21$</td>
<td>$225 \pm 6$</td>
<td>$85/21$</td>
</tr>
<tr>
<td></td>
<td>$K^0n$</td>
<td>$505 \pm 9$</td>
<td>$402 \pm 7$</td>
<td>$30/18$</td>
<td>$256 \pm 7$</td>
<td>$151/18$</td>
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<td></td>
<td>$\pi^+$</td>
<td>$430 \pm 9$</td>
<td>$352 \pm 6$</td>
<td>$46/18$</td>
<td>$216 \pm 5$</td>
<td>$93/18$</td>
</tr>
<tr>
<td></td>
<td>$\pi^-$</td>
<td>$382 \pm 8$</td>
<td>$313 \pm 5$</td>
<td>$142/18$</td>
<td>$191 \pm 5$</td>
<td>$112/18$</td>
</tr>
</tbody>
</table>

(a) For fit to formula (5.3)

(b) For fit to formula (5.5)
### TABLE 5.4

(a) $pp \rightarrow pp\pi^+\pi^-$ at 16 GeV/c

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Decay products</th>
<th>Mass (MeV)</th>
<th>Width (MeV)</th>
<th>Cross-Section (µb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta^{++}(1236)$</td>
<td>$p\pi^+$</td>
<td>1231 ± 4</td>
<td>100 ± 15</td>
<td>770 ± 20</td>
</tr>
<tr>
<td>$\Delta^0 (1236)$</td>
<td>$p\pi^-$</td>
<td>1220 ± 12</td>
<td>98 ± 25</td>
<td>200 ± 60</td>
</tr>
<tr>
<td>$N^*!^0(1470)$</td>
<td>$p\pi^-$</td>
<td>1482 ± 10</td>
<td>78 ± 15</td>
<td>170 ± 20</td>
</tr>
<tr>
<td>$N^*!^0(1688)$</td>
<td>$p\pi^-$</td>
<td>1670 ± 10</td>
<td>58 ± 10</td>
<td>90 ± 15</td>
</tr>
<tr>
<td>$N^*+(1470)$</td>
<td>$p\pi^+\pi^-$</td>
<td>1405 ± 10</td>
<td>150 ± 25</td>
<td>450 ± 75</td>
</tr>
<tr>
<td>$N^*+(1700)$</td>
<td>$p\pi^+\pi^-$</td>
<td>1706 ± 10</td>
<td>160 ± 30</td>
<td>190 ± 60</td>
</tr>
</tbody>
</table>

(b) $k^-p \rightarrow pk\pi^+\pi^-$ at 10 GeV/c

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Decay products</th>
<th>Mass (MeV)</th>
<th>Width (MeV)</th>
<th>Cross-section (µb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta^{++}(1236)$</td>
<td>$p\pi^+$</td>
<td>1232 ± 5</td>
<td>130 ± 20</td>
<td>177 ± 18</td>
</tr>
<tr>
<td>$\Delta^0 (1236)$</td>
<td>$p\pi^-$</td>
<td>1250 ± 15</td>
<td>150 ± 65</td>
<td>40 ± 10</td>
</tr>
<tr>
<td>$N^*!^0(1470)$</td>
<td>$p\pi^-$</td>
<td>1500 ± 10</td>
<td>65 ± 20</td>
<td>20 ± 15</td>
</tr>
<tr>
<td>$N^*!^0(1688)$</td>
<td>$p\pi^-$</td>
<td>1685 ± 10</td>
<td>100 ± 35</td>
<td>35 ± 5</td>
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<tr>
<td>$N^*+(1470)$</td>
<td>$p\pi^+\pi^-$</td>
<td>1425 ± 10</td>
<td>140 ± 20</td>
<td>60 ± 8</td>
</tr>
<tr>
<td>$N^*+(1700)$</td>
<td>$p\pi^+\pi^-$</td>
<td>1705 ± 10</td>
<td>100 ± 15</td>
<td>33 ± 10</td>
</tr>
</tbody>
</table>
$pp \rightarrow pp \pi^+ \pi^-$

5634 combs.

**FIG. 5.1(a)**
\[ p\bar{p} \rightarrow p\bar{p} \pi^+ \pi^- \]

2817 events

(b)

(c)

FIG. 5.1
FIG. 5.2

(a) 5634 combs.

(b) 2817 events

(c) 2817 events

$pp \rightarrow pp \pi^+ \pi^-$

NO. OF EVENTS / 0.05 GeV/c

$p_t (P) \text{ GeV/c}$

$p_t (\pi^+) \text{ GeV/c}$

$p_t (\pi^-) \text{ GeV/c}$
$K^- p \rightarrow p K^- \pi^+ \pi^-$

6068 events

**FIG. 5.3**

(a) $p_t^*(p)$ GeV/c

(b) $p_t^* (K^-)$ GeV/c
$K^- p \rightarrow pK^+ \pi^+ \pi^-$

6068 events

FIG. 5.3

(d)

$N_{\text{O.EV}} / 0.1 \text{GeV/c}$

$p^*_T (\pi^+) \quad \text{GeV/c}$

$N_{\text{O.EV}} / 0.1 \text{GeV/c}$

$p^*_T (\pi^-) \quad \text{GeV/c}$
$K^- p \rightarrow p K^- \pi^+ \pi^-$  6068 events

FIG. 54

(a) $p_t(p)$ GeV/c

(b) $p_t(K^-)$ GeV/c

(c) $p_t(\pi^-)$ GeV/c

(d) $p_t(\pi^-)$ GeV/c
\begin{align*}
K^-p &\rightarrow pK^0\pi^-\pi^0 & \text{1653 events}
\end{align*}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig5.5a}
\caption{(a)}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig5.5b}
\caption{(b)}
\end{figure}
$K^- p \rightarrow p \bar{K}^0 \pi^- \pi^0$  1653 events

**FIG. 5.5**
$K^- p \rightarrow p K^0 \pi^- \pi^0$

1653 events

**FIG. 5.6**
\[ \text{K}^- \text{p} \rightarrow \text{nK}^\circ \pi^+ \pi^- \quad 884 \text{events} \]

**Fig. 5.7**

(a)

(b)
$K^- p \rightarrow nK^0 \pi^+ \pi^-$  

884 events

NO. OF EVENTS / 0.10 GeV/c

FIG. 5.7

(d)

NO. OF EVENTS / 0.10 GeV/c

$p_T^* (\pi^+) 1.2$ GeV/c

$p_T^* (\pi^-) 1.2$ GeV/c
$K^- p \rightarrow nK^\circ \pi^+ \pi^-$

884 events

(a) 

(b) 

(c) 

(d) 

FIG. 5.8
$pp \rightarrow pp \pi^+ \pi^-$

**FIG. 5.9**

(a) $p$

(b) $\pi^+$

(c) $\pi^-$
FIG. 5.10

$K^-p \rightarrow pK^- \pi^+ \pi^-$

(a) $p$

(b) $K^-$

(c) $\pi^+$

(d) $\pi^-$

NO. OF EVENTS / 0.1

$\cos \theta^*$

$\cos \theta^*$
\( K^- p \rightarrow n \bar{K}^0 \pi^+ \pi^- \)

**Fig. 5.12**

(a) \( n \)

(b) \( \bar{K}^0 \)

(c) \( \pi^+ \)

(d) \( \pi^- \)
**FIG. 5.13**

- **pp \( \rightarrow \) pp \( \pi^+ \pi^- \)**
  - (a) Histogram showing the distribution of invariant mass for \( \pi^+ \pi^- \) in pp collisions.

- **K\(^-\)p \( \rightarrow \) pK\(^-\)\( \pi^+ \pi^- \)**
  - (b) Histogram showing the distribution of invariant mass for \( \pi^+ \pi^- \) in K\(^-\)p collisions.

The histograms display the number of events per 0.05 GeV bin for the invariant mass \( M(\pi^+\pi^-) \) in both cases.
\[ K^- p \rightarrow p K^0 \pi^- \pi^- \]

**FIG. 5.13**

\[ K^- p \rightarrow n K^0 \pi^+ \pi^- \]
$pp \rightarrow pp \pi^+ \pi^-$

803 events

Both $M(p\pi^+) > 1.4$ GeV

**FIG. 5.14**
(a) $K^- p \rightarrow p K^0 \pi^- \pi^-$

(b) $K^- p \rightarrow p K^0 \pi^- \pi^0$

**FIG. 5.15**
FIG. 5.15

K⁻ p → pK⁺π⁻π⁰

(d)

K⁻ p → nK⁺π⁺π⁻

(c)
$pp \rightarrow pp \pi^+\pi^-$

(a)

$K^- p \rightarrow p K^- \pi^+\pi^-$

(b)

FIG. 5.16
**FIG. 5.17**

(a) $pp \rightarrow pp\pi^+\pi^-$

(b) $K^-p \rightarrow pK^-\pi^+\pi^-$
$K^- p \rightarrow p K^0 \pi^- \pi^0$

![Graph (c)](image)

![Graph (d)](image)

FIG. 5.17
\begin{align*}
K^- p &\rightarrow n\bar{K}^0 \pi^+ \pi^- \\
\text{FIG. 5.17}
\end{align*}
FIG. 5.18
$|t| < 0.6 \text{ GeV}^2$

3134 events

(a) $K^*^- (890)$ Selected. 661 events

(b) $\Delta^+ (1236)$ Selected. 287 events
FIG. 5.20

$K^- p \rightarrow p K^- \pi^+ \pi^-$

(a)
$K^- p \rightarrow nK^0 \pi^+ \pi^-$

**FIG. 5.20**

$M(K^0\pi^+\pi^-)$ GeV

$K^- p \rightarrow pK^0 \pi^- \pi^0$

$M(K^0\pi^-\pi^0)$ GeV
$pp \rightarrow pp \pi^+ \pi^-$

**FIG. 5.21 (a)**
FIG. 5.21

\[ K^- p \rightarrow p K^- \pi^+ \pi^- \]
$K^- p \rightarrow p \bar{K}^0 \pi^+ \pi^-$

$K^- p \rightarrow n \bar{K}^0 \pi^+ \pi^-$

**FIG. 5.21**
\[ pp \rightarrow pp \pi^+ \pi^- \]

**FIG. 5.22**
$K^- p \rightarrow p K^- \pi^+ \pi^-$

**FIG. 5.23**
$K^- p \rightarrow p K^- \pi^+ \pi^-$

**FIG. 5.23**

The diagram shows the distribution of events as a function of $-t$ for the reaction $K^- p \rightarrow p K^- \pi^+ \pi^-$. The $x$-axis represents $-t$ in units of GeV$^2$, and the $y$-axis represents the number of events per 0.05 GeV$^2$. The graph is divided into two parts, labeled (c) and (d), with (c) showing a broader distribution and (d) a more concentrated one at lower $-t$ values.
FIG. 5.24

\[ K^- p \rightarrow p K^0 \pi^- \pi^0 \]

-126 -

NO. OF EVENTS / 0.05 GeV^2

-0.5

0

1

2

3

4

5

60

80

100

20

40

60

80

100

NO. OF EVENTS / 0.05 GeV^2

FIG. 5.24
\( K^- p \rightarrow p K^0 \pi^- \pi^0 \)

**Figure 3.6**

- **Graph (c):**
  - Title: No of Events / 0.05 GeV^2
  - X-axis: \(-t_{p \rightarrow p \pi^0}\) GeV^2
  - Y-axis: No of Events / 0.05 GeV^2

- **Graph (d):**
  - Title: No of Events / 0.05 GeV^2
  - X-axis: \(-t_{p \rightarrow p \pi^-}\) GeV^2
$K^- p \rightarrow n\bar{K}^0 \pi^+ \pi^-$

**FIG. 5.25**

(a) 

(b)
$K^- p \rightarrow n \bar{K}^0 \pi^+ \pi^-$

FIG. 5.25
$pp \rightarrow pp \pi^+ \pi^-$

**FIG. 5.26**
Figure 5.26

The graph shows the distribution of $pp \rightarrow pp \pi^+ \pi^-$ events as a function of $M(p\pi^+)$. The data points are plotted on a two-dimensional graph with $a \text{ GeV}^{-2}$ on the y-axis and $M(p\pi^+) \text{ GeV}$ on the x-axis. The graph includes a label (c) indicating a specific condition or parameter.
$K^- p \rightarrow p K^- \pi^+ \pi^-$

(a)

(b)

FIG. 5.27
$K^- p \rightarrow p K^- \pi^+ \pi^-$

FIG. 5.27
$K^- p \rightarrow p K^0 \pi^- \pi^0$

**FIG. 5.28**
$K^- p \rightarrow n \bar{K}^0 \pi^+ \pi^-$

**FIG. 5.29**

(a) $M (n \pi^+ \pi^-) \text{GeV}$

(b) $M (n \pi^-) \text{GeV}$

(c) $M (n \pi^+) \text{GeV}$
$pp \rightarrow \Delta^{++} p \pi^-$

**FIG. 5.30**

![Diagram showing the reaction $pp \rightarrow \Delta^{++} p \pi^-$](image-url)
CHAPTER 6

ISOBAR PRODUCTION MECHANISMS

6.1 Centre-of-Mass Production Angle Correlations

We showed in Chapter 5 that the protons in channel (1) and kaons and nucleons in channels (2), (3) and (4) are produced very strongly forwards and backwards. This effect is not so evident for the pions, but a definite forward backward peaking does exist.

We show in Figs. 6.1 to 6.4 scatter plots of the pion centre-of-mass production angles for the four channels. It will be seen that there is a pronounced tendency to populate the corners of the plots and hence the events rather clearly separate themselves into two categories:

(A) Both pions are produced in the same hemisphere.
(B) The pions are produced in opposite hemispheres.

For channel (1) there can be no further sub-division of these categories because of the symmetry in initial and final states. However, for the kaon induced reactions one can subdivide the categories according to whether the pions are beam-like or target-like as follows:

(A1) Both pions produced backwards.
(A2) Both pions produced forwards.
(B1) The $\pi^-$ is produced forwards and other pion backwards.
(B2) The $\pi^-$ is produced backwards and other pion forwards.

The two protons in channel (1) and kaon and nucleon in channels (2) to (4) are almost invariably produced in opposite hemispheres. In fact we showed for channel (1) in Section 4.3 that those few events where this is not the case have almost certainly been mis-identified. For the
kaon experiment, there are a small number of events where the nucleon and kaon are produced in the same hemisphere, but these will be ignored.

The numbers of events in the various categories are listed in Table 6.1 for the four channels.

It is useful to interpret this division by hemisphere as indicative of the form of the production mechanism operating. Thus for channel (1) categories (A) and (B) would correspond to the two graphs shown schematically in Figs. 6.5 (a) and (b), respectively. Note that the two pn vertices in diagram (B) are not intended to include quasi-diffraction scattering. This mechanism would be incorporated in (A) by duality (47), (48).

It is interesting to observe that in channels (1) and (2) category (B) requires $I=1$ exchange while $I=0$ exchange can contribute to category (A). In channel (3), the diagram corresponding to category (A2) can have $I=0$ exchange contributing but that corresponding to (A1) cannot. This latter grouping contains few events and this is reflected in the $p\pi^-\pi^0$ mass distribution (shown in Fig. 5.21(c)) which shows no evidence for any low mass structure. Conversely we can say that where $I=0$ exchange is allowed in category (A), the corresponding three body mass distributions show an accumulation of events at low mass.

Thus we believe that for channels (1) to (3), category (A) is dominated by $I=0$ exchange where this is possible and hence the low mass $p\pi\pi$ and $K\pi\pi$ systems have $I=\frac{1}{2}$.

One can also see that these low mass systems must have $I=\frac{1}{2}$ without considering possible 'exchange mechanisms'.

To be definite let us consider the $(N\pi\pi)$ system produced in the kaon channels and suppose that this is produced with $I=3/2$. Now the
incoming $k^-p$ system has $I=0$ or $I=1$, but only the latter state can be responsible for producing an $I=3/2$ $(Nnn)$ combination in a reaction of the type,

$$k^-p \rightarrow k^- (Nnn)$$

The conservation of $I$-spin implies a well defined branching-ratio, namely

$$\frac{k^- p \rightarrow k^- (Nnn)^+}{k^- p \rightarrow k^- (Nnn)^0} = 1$$

The $p^+\pi^-$ mass distribution, shown in Fig. 5.21(b) for channel (2), has a large accumulation of events at the low mass end of the spectrum. If the isotopic spin of this low mass system is $3/2$, there ought to be significant enhancements in the low mass $n^+\pi^-$ and $p^-\pi^0$ distributions. These are shown in Figs. 5.21(c) and (d) and no appreciable accumulation of events is evident. It is of course conceivable that the internal $I$-spin couplings in the $(Nnn)^0$ system favour a large branching mode into $n^0\pi^0$, but this is not very likely. Unfortunately, the $n^0\pi^0$ system would be contained in the no-fit channel,

$$k^- p \rightarrow k^- Z^0$$

and it is impossible to extract the mass distribution. However, a plot of the mass of $Z^0$ (not shown) provides no evidence of any accumulation of events at low masses.

We thus conclude that the low mass $p^+\pi^-$ systems produced in channels (1) and (2) and $K^-\pi$ systems in channels (2) and (3) have $I=\frac{3}{2}$.

We have stated that $I=1$ exchange is required for category (B) in channels (1) and (2) and we will be presenting evidence in Section 6.3 that in fact pion exchange is responsible. However the $K(890)$ resonance is believed to be mainly produce via $\omega$ exchange\(^{(49)}\). Thus where $I=0$ ex-
change is allowed in category (B), \(\omega\) exchange could well dominate. Thus \(\omega\) exchange might be the main contribution to category (B1) in channels (3) and (4).

This subdivision by hemispheres is perhaps not quite so meaningful for channel (4) where both categories (A1) and (A2) contain sizeable fractions of events even though \(I=0\) exchange is not allowed for the corresponding diagram. However, as mentioned earlier neither the \(K^0\pi^+\pi^-\) or \(n\pi^+\pi^-\) distributions show any evidence for any low mass structure. However, it is worthy of note that category (B2) is depopulated, where this grouping would require the existence of \(I=2\) exchange.

In the case of channel (1), other groups have performed a similar classification at different energies. We define the ratio,

\[
\sigma = \frac{\text{No. of events in category A}}{\text{Total no. of events}}
\]  
(6.1)

and Table 6.2 contains the values of \(\sigma\) published by other authors (50), (51) as well as the value found in this experiment: It can be seen that the value of \(\sigma\) seems to increase with increasing energy.

6.2 POMERON EXCHANGE

We have seen that an \(I=0\) exchange mechanism is responsible for the large accumulation of events at low \(p\pi^+\pi^-\) and \(K\pi\pi\) masses. We shall now see that the reactions in fact proceed by Pomeron exchange or differentiation dissociation.

Morrison (52) has observed that for sufficiently high energy (beam momentum greater than about 4 GeV/c) the cross-section for most reactions can be fitted by an expression of the form,

\[
\sigma = A \frac{n}{p_{LAB}}
\]  
(6.2)

where \(p_{LAB}\) is the incoming laboratory momentum. For quasi-two-body (54)
(or two-body) reactions $n$ is supposed to be indicative of the type of exchange operating.

Pomeron exchange reactions have an essentially energy independent cross-section, so $n$ is expected to be small in this case. For meson exchange $n$ is of order $1.5 - 2.0$ and strange-meson and baryon exchange reactions have somewhat higher $n$ values.

This constancy of the cross-section for Pomeron exchange reactions is reflected in the increasing importance with energy of category (A) in channel (1), since the total channel cross-section is still falling off fairly rapidly in this energy range (see Fig. 4.6).

We show in Fig. 6.6 the variation of the cross-sections for the reactions

$$pp \rightarrow p N^+ (1470)$$  \hspace{1cm} \text{(a)}

$$pp \rightarrow p N^+ (1688)$$  \hspace{1cm} \text{(b)}

as a function of energy. The data points are from Ref. (53). With one exception (Connolly et al.) the experimental cross-sections are taken from counter and spark-chamber experiments. Note that the cross-sections are for all decay modes of the $N^+$, including two-body decays.

Evidently within the experimental errors, the cross-sections for reactions (a) and (b) may be assumed to be constant. We have in fact fitted an expression of the form (6.2) to the plots shown in Fig. 6.6 (a) and (b). The values of $n$ obtained are,

$$n = -0.10 \pm 0.30 \quad \text{for reaction (a)}$$

$$n = 0.02 \pm 0.15 \quad \text{for reaction (b)}.$$  

It is of interest to compare these cross-sections with the results quoted in Table 5.4 for the 16 GeV/c experiment. These results are of course only for the $p\pi^+\pi^-$ decay mode of the $N^+$. However, if
we assume that both enhancements decay exclusively into $\Delta\pi$ (see Chapter 10), then we should observe $5/9$ of the three-body production cross-section in channel (1). Moreover, assuming that the isobars decay equally into the two and three body modes (5), we obtain production cross-sections of $1.62 \pm 0.27$ and $0.68 \pm 0.22$ mb respectively for the two channels (a) and (b). The latter figure looks not too unreasonable, but the cross-section for the $N^*(1470)$ is somewhat larger than that quoted in Ref. (53) in this energy region. Two points can be made however.

Firstly there may be some $N^*(1520)$ present in our data which we are unable to separate. This isobar is normally resolved in the cross-counter experiments (53) and has a production section about one third of the $N^*(1470)$ cross-section at 16 GeV/c. Secondly in the region of any threshold enhancement the question of background is extremely important. In drawing a background for the fits described in Section 5.4 we have been guided by the results of the reaction channel analysis dealt with in a later chapter. Here virtually the whole of the diffractive region has been parameterised in terms of resonances.

However, it should be pointed out that in Fig. 6.6(a) all but one of the high energy data points come from a single experiment (Anderson et al. (53)) so that at least one might expect the background to be subtracted in a consistent fashion. In the light of the duality concept (47), (48) this is all one can probably hope to do in these situations.

Diffraction dissociation reactions are extremely peripheral in nature, consistent with the idea that the pomeron has the quantum numbers of the vacuum. The slope, $\alpha$, of the four momentum transfer distributions of reactions where Pomeron exchange is allowed is generally much higher.
than for reactions where it is forbidden.

From Figs. 5.27(a) and 5.28(a) we see that the slopes for the distributions in $t'$ in the region of the 1470 MeV enhancement in channels (1) and (2) are $11.2 \pm 0.6$ and $10.4 \pm 0.9$ GeV$^{-2}$ respectively. For the 1700 MeV enhancement the values of the slopes are $7.6 \pm 0.6$ and $6.0 \pm 0.9$ GeV$^{-2}$. The very high value of the slope just above threshold is characteristic of Pomeron exchange.

6.3 ONE-PION-EXCHANGE

A very large part of the data in channels (1) and (2) can be understood in terms of the one-pion-exchange model. This will be considered in more detail in Chapter 8. In this section we merely want to show that the production mechanism is in fact consistent with one pion exchange.

In Fig. 6.7 we present Chew-Low plots for the reactions

$$pp \to \Delta^{++}, p\pi^-$$

$$k^-p \to \Delta^{++}, k^-\pi^-$$

where the mass of the $p\pi^+$ has been required to lie in the $\Delta^{++}$ band defined by

$$1.15 < M(p\pi^+) < 1.30 \text{ GeV}.$$ 

It will be seen that there is a concentration of events along the Chew-Low boundary up to quite high values of $p\pi^-$ or $k^-\pi^-$ mass. This constitutes evidence for peripheral production of the $\Delta^{++}$ (1236).

One way to investigate the nature of the exchanged object is to look at the decay angular distributions of events in the $\Delta^{++}$ mass region. If the exchanged particle is scalar, definite restrictions are put on the values of the density matrix elements.
We define a coordinate system as follows. In the overall centre-of-mass, the y axis is defined as normal to the plane containing the incident proton and Δ^++ (production-plane). The Z-axis is defined to be along the incident proton direction in the Δ^++ rest-frame. θ and ϕ are then the polar and azimuthal angles of the decay proton from the Δ^++ in its own centre-of-mass.

The general angular distribution of an object of spin 3/2 decaying into a spin 1/2 baryon and spin 0 meson is given by,

\[ W(\cos \theta, \phi) = \frac{3}{4\pi} \left( \frac{1}{6} (1 + 4p_{33}) + \frac{1}{2} (1 - 4p_{33}) \cos^2 \theta \right) 
- \frac{3}{\sqrt{3}} \Re p_{3-1} \sin^2 \theta \cos 2\phi 
- \frac{3}{\sqrt{3}} \Im p_{31} \sin 2\theta \cos \phi \]

(6.3)

where parity is assumed to be conserved in the decay.

Assuming the exchanged object has spin zero, the maximum spin projection of the Δ^++ along the incident proton direction (the quantization axis) is \(\frac{3}{2}\). Thus we require,

\[ p^+_{3m} = p^-_{m-3} = 0 \]  

(6.4)

for all values of m, and in particular

\[ p_{33} = 0 \]  

(6.5)

which implies,

\[ p_{11} = \frac{1}{2} \]  

(6.6)

Thus for the exchange of a spin zero particle the decay distribution reads

\[ W(\cos \theta, \phi) = \frac{1}{8\pi} (1 + 3 \cos^2 \theta) \]

(6.7)

It is more difficult to predict the form of the decay angular
distribution in the case of vector exchange. However, Stodolsky and Sakurai\(^{(57)}\) have treated the \(\Delta p p\) vertex in analogy to the \(\Delta \gamma p\) vertex assuming a magnetic dipole transition \(M1\) dominates. The predicted values of the density matrix elements are,

\[
\rho_{33} = \frac{3}{8} \\
\text{Re } \rho_{3-1} = \frac{\sqrt{3}}{8} \\
\text{Re } \rho_{31} = 0
\]

We show in Fig. 6.8 the distributions in \(\cos \phi\) and \(\phi\) for the \(\Delta^{++}\) produced in channels (1) and (2). The polar angle distributions exhibit a large asymmetry, with an excess of events at \(\cos \phi = 1\) corresponding to the proton from the \(\Delta^{++}\) preferentially lying along the incident proton direction. Such an asymmetry is of course incompatible with the decay of a pure spin state and implies a large background under the \(\Delta^{++}\) signal. We have attempted to correct for this background by two methods.

Firstly the density matrix elements of the \(p\pi^+\) system were determined not only in the \(\Delta^{++}\) region defined previously but also in a control region defined by

\[1.30 \leq M(p\pi^+) \leq 1.45 \text{ GeV}\]

These regions will be referred to as I and II respectively. Suppose that there are a total of \(N_T\) events in region I and \(n_T\) in region II of which \(N_B\) and \(n_B\) do not have a \(\Delta^{++}\). Then assuming that the "density matrix elements" of the background do not depend on the \(p\pi^+\) mass, the experimental "density matrix elements" in the two regions can be written

\[
\rho = \frac{(N_T - N_B) \rho_B + N_B \rho_B}{N_T}
\]
\[ \rho_{II} = \frac{(n_T - n_B) \rho_R + n_B \rho_B}{n_T} \]  \hspace{1cm} (6.9)\\

where \( \rho_R \) are the density matrix elements of the \( \Delta^{++} \) and \( \rho_B \) those of the background.

The numbers of background events in each of the two regions may be estimated from the results of the fits described in Section 5.4 and hence equations (6.9) solved for \( \rho_R \).

The second approach is to assume that those events with \( \cos \theta \) less than zero constitute a reasonably pure sample of \( \Delta^{++} \) events and to determine the density matrix elements with these events only.

We have in fact tried both methods.

The experimental density matrix elements have been calculated using the maximum likelihood technique. The likelihood function for the \( i \) th. event is written,

\[ L_i = W(\cos \phi_i, \phi) \]  \hspace{1cm} (6.10)\\

where \( W \) is given by equation (6.3) and the logarithm of the grand likelihood function,

\[ \ln L = \sum_{i=1}^{N} \ln L_i \]  \hspace{1cm} (6.11)\\

maximised with respect to the density matrix elements. \( N \) is the number of events in the sample.

The density matrix elements were evaluated as functions of \( \Delta^2 \), the absolute value of the square of the four-momentum transfer from incident proton to \( \Delta^{++} \). For each range of \( \Delta^2 \), the \( p\pi^+ \) mass distribution was fitted with a P-wave relativistic Breit-Wigner function and a hand drawn background. In this way the numbers of \( \Delta^{++} \) events in regions I and II could be estimated.
The results are displayed in Figs. 6.9 and 6.10. The first figure corresponds to using the background subtraction method, the second to using the reduced sample with $\cos \theta$ less than zero. Where an event in channel (1) had both $p\pi^+$ combinations in the $\Delta^{++}$ region, that combination with the smaller value of $\Delta^2$ was used (see Section 9.4 on this point).

6.4 Separation of Events

To study the two types of production mechanisms individually, one would like to isolate samples of events which proceed dominantly through the diagrams illustrated in Fig. 6.5 for channel (1). The pion centre-of-mass production angle classification discussed earlier does this to some extent, but is really quite crude and there is probably substantial contamination in any one category.

We show in Fig. 6.11 the $p\pi^+\pi^-$ mass distribution of channel (1) where only that proton having the greater four-momentum transfer from its associated initial state proton is plotted. It can be seen that the background from 'wrong' combinations has been very much reduced and a comparison with Fig. 5.2(a) shows that the low mass structure is no less significant. A similar selection could be applied to channel (2) to remove the diffractively produced $K^+\pi^+\pi^-$ events.

However, in the case of channel (1) we are plotting every event in Fig. 6.11, so there must be a background from one-pion-exchange type processes. The sample obtained from channel (2) in this way would also be contaminated.

One possible way of removing the background is to impose a selection on the four-momentum transfer from incident proton to the $p\pi^+\pi^-$
system. In Fig. 6.12 we show the $p\pi^+$ mass distribution for channel (2) where,

$$|t(p, p\pi^+)| < 0.6 \text{ GeV}^2$$

Again we have succeeded in significantly reducing the background under the two enhancements.

Alternatively assuming that the dominant decay mode of the low mass $p\pi^+\pi^-$ system is $\Delta^{++}\pi^-$, we could only plot those events where the mass of the $p\pi^+$ combination is in the $\Delta^{++}$ mass region, defined in Section 6.3. The resulting distributions are shown in Fig. 6.13 for channels (1) and (2).

In the case of channel (1) it is easy to show that the background from one-pion-exchange type processes is still significant. We show in Fig. 6.14, for the same sample of events, the $p\pi^-$ mass distribution where the proton used is not the one in the $\Delta^{++}$ mass-region. Considerable low mass isobar structure is evident. This contamination is probably not so serious for channel (2) since the $K^-\pi^-$ cross-section is supposed to be relatively small (see Section 8.6).

We show in Fig. 6.15 the 'other half' of the $p\pi^-$ mass distribution of channel (1), where the proton plotted is the one which, when taken with the $\pi^+$ has a mass in the $\Delta^{++}$ region. Again the low mass isobar structure appears to be present but to what extent this is a reflection of the low mass enhancement in the $p\pi^+\pi^-$ system is uncertain.

For those events not involving the production of a $\Delta^{++}$ it is equally difficult to separate the two types of mechanisms. We show in Figs. 6.16(a) and (b) the distributions in the $p_1\pi^-$ and $p_2\pi^-$ mass, where the proton labelled $p_1$ has the greater four momentum transfer from its associated initial state proton. Only events with neither $p\pi^+$ combination in the $\Delta^{++}$ mass band have been considered. Not surprisingly the
distribution in $p_1\pi^-$ mass shows an accumulation of events at the lower end of the spectrum, since this system is likely to form part of the diffractively produced three particle system. The corresponding plot for $p_2\pi^-$ shows some isobar structure and these events can presumably be associated with diagram 6.5(b).

We now turn to the separation of the two one-pion-exchange graphs obtained by interchanging the $\pi^+$ and $\pi^-$. For those events involving $\Delta^{++}$ production in channel (1) this is relatively simple. There are a total of 1473 events having at least one $p\pi^+$ combination in the $\Delta^{++}$ mass band and only 29 events have both. In the case of channel (2) such a separation is not possible. As we shall see in Section 8.6 a considerable fraction of events have both the $p\pi^+$ mass in the $\Delta^{++}$ region and the $k^-\pi^+$ mass in the $k^*^{0}$ (890) region.

Perhaps we can summarize the situation by saying that the production of events in channels (1) and (2) may be understood in terms of a single one-pion-exchange type diagram. This is shown in Fig. 6.5(b) in the case of channel (1). Diffractive production of the $p\pi^+\pi^-$ or $k^-\pi^+\pi^-$ systems can thus be accounted for by allowing diffractive scattering at one or other of the two vertices. This is the duality picture at any rate.

This being the case, however, one might expect that the 1700 MeV enhancement in the $p\pi^+\pi^-$ system would have a decay mode,

$$N^{*+} \rightarrow N^{*0} (1470) \pi^+$$

corresponding to quasi-diffractive scattering at the $p\pi^+$ or $k^-\pi^+$ vertex.

We show in Fig. 6.17 the $p\pi^+\pi^-$ mass distribution of channels (1) and (2) where the $p\pi^-$ mass is required to lie in the $N^{*0} (1470)$ region defined by,
1.40 \leq M(p\pi^-) \leq 1.55 \text{ GeV}

A clear peak at around 1700 MeV can be seen. We shall be discussing this phenomenon in more detail in Chapter 10.
Table 6.1
DIVISION BY HEMISPHERES

<table>
<thead>
<tr>
<th>Final State</th>
<th>Category</th>
<th>No. of events</th>
</tr>
</thead>
<tbody>
<tr>
<td>ppπ⁺π⁻</td>
<td>A</td>
<td>1750</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>1067</td>
</tr>
<tr>
<td>pk⁻π⁺π⁻</td>
<td>A1</td>
<td>1139</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>2623</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>1254</td>
</tr>
<tr>
<td></td>
<td>B2</td>
<td>1052</td>
</tr>
<tr>
<td>pk°π⁻π⁻</td>
<td>A1</td>
<td>81</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>923</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>236</td>
</tr>
<tr>
<td></td>
<td>B2</td>
<td>414</td>
</tr>
<tr>
<td>nk°π⁺π⁻</td>
<td>A1</td>
<td>156</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>321</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>333</td>
</tr>
<tr>
<td></td>
<td>B2</td>
<td>73</td>
</tr>
</tbody>
</table>
Table 6.2
VALUES OF $r$ FOR CHANNEL (1) AT DIFFERENT ENERGIES

<table>
<thead>
<tr>
<th>Beam-momentum (GeV/C)</th>
<th>Reference</th>
<th>$r$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.92</td>
<td>(50)</td>
<td>49</td>
</tr>
<tr>
<td>16.08</td>
<td>This experiment</td>
<td>$62^{+}_{-2}$</td>
</tr>
<tr>
<td>19.0</td>
<td>(51)</td>
<td>67</td>
</tr>
</tbody>
</table>
FIG. 6.1  \( pp \rightarrow pp \pi^+ \pi^- \)

FIG. 6.2  \( K^- p \rightarrow p K^- \pi^+ \pi^- \)
FIG. 6.3 $K^- p \rightarrow p \overline{K}^0 \pi^- \pi^0$

FIG. 6.4 $K^- p \rightarrow n \overline{K}^0 \pi^+ \pi^-$
A: Both pions in same hemisphere

B: Pions in opposite hemispheres

FIG. 6.5
FIG. 6.6

Data from Ref. (53)

(a) $pp \rightarrow pN_{1470}^{*+}$

(b) $pp \rightarrow pN_{1688}^{*+}$
FIG. 6.7(a)
FIG. 6.7(b)
\[ \Delta^{++} \text{ Decay Angular Distributions} \]

\[ \text{pp} \rightarrow \Delta^{++} \rho \pi^- \]

\[ \textbf{FIG. 6.8} \]
$$\Delta^{++} \text{ DENSITY MATRIX ELEMENTS}$$

$$pp \rightarrow \Delta^{++} p \pi^- \quad K^- p \rightarrow \Delta^{++} K^- \pi^-$$

FIG. 6.9
\( \Delta^{++} \) DENSITY MATRIX ELEMENTS

\[ pp \to \Delta^{++} p \pi^- \quad K^- p \to \Delta^{++} K^- \pi^- \]

\[ \rho_{33} \]

\[ \text{Re} \rho_{3-1} \]

\[ \text{Re} \rho_{31} \]

\( \Delta^2 \text{ GeV}^2 \)

FIG. 6.10
FIG. 6.12 \( K^- p \rightarrow p K^- \pi^+ \pi^- \)
\[ |t(p \rightarrow p \pi^+ \pi^-)| < 0.6 \text{ GeV}^2 \]
3657 events

FIG. 6.11 \( p p \rightarrow p p \pi^+ \pi^- \)
Most peripheral \( p \pi^+ \pi^- \) combination only
2817 events
$K^+ p \rightarrow pK^- \pi^+ \pi^-$

$\Delta^{++} \text{Selected}$

(b) 2177 events

$M(p\pi^+\pi^-)$ GeV

(a) 1502 Combinations

$pp \rightarrow pp \pi^+ \pi^-$

$\Delta^{++} \text{Selected}$

FIG. 6.13
FIG. 6.14 \[ pp \rightarrow pp \pi^+\pi^- \]

MASS \( p_1 \pi^- \) when \( p_2 \pi^+ \) in \( \Delta^{**} \)

1502 combinations

NO. OF EVENTS / 0.05 GeV

FIG. 6.15

MASS \( p_2 \pi^- \) when \( p_2 \pi^+ \) in \( \Delta^{**} \)

1502 combinations

NO. OF EVENTS / 0.05 GeV

M(\( p_1 \pi^- \)) GeV

M(\( p_2 \pi^- \)) GeV
$$pp \rightarrow pp \pi^+ \pi^-$$

(a) MASS ($p_1 \pi^-$) $\Delta^{++}$ Removed where

$$|t(p \rightarrow p_1)| > |t(p \rightarrow p_2)|$$

1344 events

(b) MASS ($p_2 \pi^-$) $\Delta^{++}$ Removed where

$$|t(p \rightarrow p_2)| < |t(p \rightarrow p_1)|$$

1344 events

FIG. 6.16
$pp \rightarrow pp \pi^+\pi^-$

$K^-p \rightarrow pK^-\pi^+\pi^-$

---

**FIG. 6.17**
CHAPTER 7

REACTION CROSS-SECTIONS FOR pp → ppπ⁺π⁻

7.1 Introduction

For several reasons a satisfactory basis for the determination of the various partial channel cross-sections in a four-body final state is extremely difficult to find.

The first approximation is to fit the one-dimensional mass spectra with an incoherent sum of Breit-Wigner functions plus a background term. This method has already been utilized in Chapter 5 to obtain the mass and widths of the observed isobars. However in order to obtain a relatively clean resonance sample it may be necessary to select only part of the data. Consequently any estimate of the amount of a particular resonance present may be subject to a large uncertainty. Also with this method it is difficult to obtain information about double resonance production or cascade decays of resonances. Either one must fit two-dimensional distributions or particular selections must be imposed on the data.

By far the greatest difficulty in obtaining reaction cross-sections is the determination of the background. When numerous resonances are present, these will obviously be reflected in all distributions and hence it must be preferable to fit the whole of the data in order to take account of this. A good example of this point is the low mass π⁺π⁻ enhancement at 1470 MeV in channels (1) and (2). The phase-space available for the decay of this enhancement is very small, so that the π⁺ and π⁻ masses will necessarily lie
in the region of the $\Delta^{++}(1236)$ and $\Delta^{0}(1236)$ isobars.

For these reasons we have chosen to fit the density of phase-space for all events. This inherently contains all correlations between four momenta.

Having said all this, it is as well to point out that we cannot hope to be too ambitious because of the large number of reaction channels present, some of which having extremely small cross-section. It is to be hoped that we can provide a model which describes the general features of the data in broad outline, and takes account of the dominant production mechanisms.

7.2 The Model

We have attempted to parametrize the matrix element for channel (1) in terms of diffraction dissociation and one meson exchange. The forms that have been chosen for these mechanisms are the simplest possible consistent with their main phenomenological features. A similar type of analysis has been performed by P.G. Wohlmert et al.\textsuperscript{(58)}, who obtained excellent fits to their data for $\pi^- p \rightarrow p n^+ \pi^- \pi^-$ at 3.9 GeV/c.

Consider the reaction (1),

$$P_a P_b \rightarrow P_1 P_2 n^+ \pi^-$$

A possible one-pion exchange graph is illustrated in Fig. 7.1(a) and is represented by a term,

$$B = V_I (p_1, \pi^+) \frac{1}{(\Delta^2 - \pi^2)^2} V_{II} (p_2, \pi^-)$$

(7.1)

where $\Delta^2$ is the square of the four-momentum transfer from $p_a$ to $p_1 \pi^-$. 
and $V_I$, $V_{II}$ are vertex factors.

If a resonance occurs at a vertex, then the corresponding $V$ is taken as a relativistic Breit-Wigner function. Where the combination is non-resonant, an $s$-wave interaction is assumed and we write for example,

$$V_{II}(p_2, \pi^-) = \frac{m_{\pi^+}^2}{p_2^2}$$  \hspace{1cm} (7.2)

In the case of the $\Delta^{++}(1236)$ we have modified the Breit-Wigner function by the angular correlation factor predicted for spin zero exchange. Thus we write,

$$V_I(p_1, \pi^+) = \frac{m_{p_1\pi^+}}{q} \frac{\Gamma(m_{p_1\pi^+}) A(\theta)}{(m_{p_1\pi^+}^2 - m_\Delta^2)^2 + m_\Delta^2 \rho^2(m_{p_1\pi^+})}$$  \hspace{1cm} (7.3)

where $m_\Delta$ is the mass of the $\Delta^{++}$ and $\Gamma$ the energy dependent width given by (5.10). $q$ is the momentum of $p_1$ in the $p_1\pi^+$ rest-frame and $A(\theta)$ is the decay distribution given by,

$$A(\theta) = 1 + 3\cos^2 \theta$$  \hspace{1cm} (7.4)

where $\theta$ is the polar angle in the usual Gottfried-Jackson frame (see Section 6.3). Other resonances are assumed to have $s$-wave decay.

The mechanism for diffraction dissociation is illustrated in Fig. 7.1(b), and is parametrised by an expression,

$$B = V(p_1, \pi^+, \pi^-) e^{at(p_b, p_2)}$$  \hspace{1cm} (7.5)

where $t$ is the four momentum transfer from $p_b$ to $p_2$ and $a$ is the slope of the elastic proton proton differential cross-section at 16 GeV/c. The value of $a$ was taken as 8.5 GeV$^{-2}(60)$. The vertex factor $V$ is taken to be a Breit-Wigner function,
or a product of such functions for a sequential decay. Only decays into $\Delta^{++}\pi^-$ have been considered and the angular distribution for the $\Delta^{++}$ (equation (7.4)) inserted on the assumption that, since the events are so peripheral, the direction of the $p\pi^+\pi^-$ resonance follows that of the incident proton. For these three body resonances an energy dependent width of the form (5.12) was used.

It should be noted at this point that we are parametrising the whole of the diffractive amplitude in terms of resonances. That is to say, in any given process no background to the resonance occurs, except that which is a reflection from other processes. This relates to the principle of "strong duality"(61).

In the one-pion-exchange graph of Fig. 7.1(a) both $p\pi$ vertices are to be understood as not including any diffractive scattering. Such a process would be included in Fig. 7(b) by duality (47), (48).

We have so far not considered the fact that there are two identical particles in both the initial and final states of reaction (1). We have attempted to symmetrize the matrix element for a given process by writing it as the sum of four terms corresponding to the interchange of both initial and final state protons.

The mechanism responsible for the production of the $\rho^0$ and $f$ mesons is not obvious. We saw in Section 5.3 that the $\rho^0$ and $f$ signals in the $\pi^+\pi^-$ mass distribution were considerably enhanced when we made the selection

$$M(p\pi^+) > 1.4 \text{ GeV}$$

on both $p\pi^+$ combinations. For events satisfying this requirement, we
have examined a scatter plot of the four-momentum transfers from proton to proton (not shown) and find no significant correlation between them. We have hence parametrised the matrix element for the production of these mesons as,

\[ B = V(\pi^+, \pi^-) \left( e^{+} e^{-} + e^{+} e^{-} \right) \]

(7.6)

where \( V \) is a Breit-Wigner function.

The reactions considered in the analysis are listed in Table 7.1. Reactions (3) to (9) are understood to proceed via one-pion-exchange, (10) to (13) via diffraction dissociation. Reaction (14) leading to the uncorrelated \( pp\pi^+\pi^- \) final state was considered to be pure phase-space, and the corresponding matrix element taken as constant.

7.3 The Fitting

Consider again the process,

\[ pp \rightarrow pp\pi^+\pi^- \]

For a given event, the four momenta of the outgoing particles, which we shall denote by \( x_i \), define a point in a 16-dimensional invariant momentum phase-space (only 8 of these dimensions are of course independent).

Suppose that \( T_j(x_i) \) represents the transition amplitude for the process \( j \); then an unnormalised phase-space density may be written at each point \( x_i \),

\[ \tau = \sum_j a_j^2 |T_j(x_i)|^2 \]

(7.7)
where the sum runs over the number of competing processes and the constants $a_j$ are proportional to the relative strengths of the various reactions.

The probability of an event occurring within $x_i$ and $x_i + dx_i$ is then given by,

$$W(a_j, x_i) dx_i = \frac{\sum_k a_k^2 |T_k(x_i)|^2 dx_i}{\sum_k a_k^2 |T_k(x)|^2 dx}$$

where,

$$|T_j (x_i)|^2 = B_j^{i}$$

(7.8)

(7.9)

The $B_j^{i}$ are given by expressions (7.1), (7.5) and (7.6).

Hence,

$$W(a_j, x_i) dx_i = \frac{\sum_k a_k^2 B_k^{i} dx_i}{\sum_k a_k^2 S_k^{i}}$$

(7.10)

The likelihood for the $i$ th event is written,

$$L_i = \frac{\sum_k a_k^2 B_k^{i}}{\sum_k a_k^2 S_k^{i}}$$

(7.11)

and the grand likelihood function given by,

$$\mathcal{L} = \sum_{i=1}^{N} L_i$$

(7.12)

where $N$ is the total number of events.

One then attempts to maximise $\ln \mathcal{L}$ with respect to the parameters $a_j$. 
The percentage of a reaction $j$ is given by,

$$b_j = \frac{a_j^2 S_j}{\sum_k a_k^2 S_k}$$

(7.13)

It should be noted that with this formalism the constraints,

$$\sum_j b_j = 1$$

$$0 \leq b_j \leq 1$$

(7.14)

are automatically satisfied for all $a_k$.

If $P$ is the number of reactions considered then evidently only $P-1$ of the $a_k$'s are independent, so $a_P$ may be set equal to unity.

The procedure just outlined of course assumes that the competing processes add incoherently. This is probably not too bad an approximation, because the one-pion-exchange and diffractive mechanisms are expected to populate rather different regions of phase space. Thus Fig. 7.1(b) will mainly contribute to low $p^+\pi^-\pi^-$ masses and Fig. 7.1(a) to high masses.

The model also takes no account of possible interference effects between resonances produced by the same mechanism. It is worth stressing once again that each enhancement seen in the data has been parametrised in terms of a Breit-Wigner function. We make no claim to identify these resonances (except the $\Delta(1236)$) with the isobars reported in the phase-shift analyses (5), nor can we demonstrate that only one resonance is contributing to each enhancement.

In the actual fitting the values of the masses and widths
of the resonances were kept fixed. For the p and f mesons the values listed in the tables\textsuperscript{(5)} were used, whereas the masses and widths of the isobars were taken from the results of the fits described in Section 5.4 and listed in Table 5.4. The advantage of fixing these parameters is that the integrals defined in equation (7.9) may be computed before the actual fitting is performed.

The integrals were evaluated using the Monte-Carlo program FOWL\textsuperscript{(62)} suitably modified\textsuperscript{(63)} for high energy interactions. FOWL was also used to obtain the predictions of the model for the distributions in the various kinematic quantities of interest. The curves so obtained were normalized to the data.

7.4 Results of the Analysis

The relative amounts of the fourteen channels considered in the analysis are listed in Table 7.1 together with the corresponding cross-sections. The errors quoted are one standard deviation estimates determined in the fitting program, together with a 1\% systematic error added to each. This systematic error is intended to cover any uncertainty in the forms of the matrix elements used. However our assumption of no background under a resonance in a given process is crucial, since any background addition would drastically alter the cross-sections for $N^{*+}(1470)$ and $N^{*+}(1700)$, for example.

A comparison with the results of the fitting to the mass projections, listed in Table 5.4, for the resonance production cross-sections, shows that the two methods give fair agreement.

It is interesting to compare also the total amount of diffraction dissociation predicted by the model with the number of
events in which both pions are in the same hemisphere. Reactions (10) to (13) account for 55.5 ±5.4% of the total reaction cross-section whereas 62 ±2% of events occur with both pions produced either forwards or backwards. These numbers are obviously in good agreement.

From the results quoted in Table 7.1 we can also estimate the branching ratios of the two π⁺π⁻ enhancements.

Defining,

\[ r = \frac{\text{No. of events with } N^*^{++} \pi^+ \pi^-}{\text{No. of events with } N^*^{++} - \text{all}} \]  \hspace{1cm} (7.15)

we obtain a value of \( r = 51 ±14\% \) for the 1470 MeV enhancement and 16 ±9% for that at 1700 MeV. The former figure may not have too much significance because of the restricted phase-space available for the decay but the latter figure is in good agreement with the more detailed analysis described in Chapter 10. In particular both analyses agree that this enhancement does not have a dominant \( \Delta^{++} \pi^- \) decay mode.

We now consider the predictions of the model for the distributions in the various kinematic variables. These are represented as full curves on the diagrams for channel (1) in Chapter 5.

The broad features of the \( p_L^* \) and \( p_t \) distributions, shown in Figs. 5.1 and 5.2, are reasonably well described. However, the experimental \( p_L^* \) distributions for the pions are rather narrower than those predicted by the model. For the protons, the model predicts rather too much peaking at either end of the distribution. The transverse momentum plots show that the model correctly accounts for the fact that particles tend to be produced with low values of this variable. Not suprisingly, the production angular distributions shown in Fig. 5.9 are well reproduced also.
The experimental invariant mass distributions are in reasonable agreement with the predictions of the model. The two pion distribution is shown in Fig. 5.13(a) and the *π* distributions in Figs. 5.16(a) and 5.17(a). The poorest fit obtained is that to the *π* mass distribution, shown in Fig. 5.21(a). The two low mass enhancements are quite well described, but above 2.0 GeV the predicted curve deviates significantly from the data. This may be because diffractive processes are still important in the 2.0 GeV region and our model takes no account of this.

Perhaps the greatest success of the model is that it correctly reproduces the shape of the four-momentum transfer distributions, shown in Fig. 5.22. It is a well known feature of the one-pion-exchange model that it fails to account for the shape of the t-distribution, in this case t(p, *π*). Normally phenomenological form-factors have to be introduced to obtain reasonable agreements. However, in our model no such form-factors were considered.
TABLE 7.1

REACTION CROSS-SECTIONS FOR \( pp \rightarrow ppn^{+}\bar{\pi}^{-} \)

<table>
<thead>
<tr>
<th>No.</th>
<th>Reaction</th>
<th>Fraction (%)</th>
<th>Cross-Section ((\mu b))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( pp^0 )</td>
<td>3.7 ± 1.7</td>
<td>62 ± 29</td>
</tr>
<tr>
<td>2</td>
<td>( ppf )</td>
<td>0.6 ± 0.4</td>
<td>10 ± 7</td>
</tr>
<tr>
<td>3</td>
<td>( p\pi^-\Delta^{++}(1236) )</td>
<td>14.9 ± 2.1</td>
<td>250 ± 35</td>
</tr>
<tr>
<td>4</td>
<td>( p\pi^+\Delta^{0}(1236) )</td>
<td>3.1 ± 1.4</td>
<td>52 ± 24</td>
</tr>
<tr>
<td>5</td>
<td>( p\pi^+N^*(1470) )</td>
<td>2.4 ± 1.6</td>
<td>40 ± 27</td>
</tr>
<tr>
<td>6</td>
<td>( p\pi^+N^*(1688) )</td>
<td>2.5 ± 1.6</td>
<td>42 ± 27</td>
</tr>
<tr>
<td>7</td>
<td>( \Delta^{++}(1236)\Delta^{0}(1236) )</td>
<td>4.6 ± 1.8</td>
<td>77 ± 30</td>
</tr>
<tr>
<td>8</td>
<td>( \Delta^{++}(1236)N^*(1470) )</td>
<td>6.0 ± 2.0</td>
<td>100 ± 34</td>
</tr>
<tr>
<td>9</td>
<td>( \Delta^{++}(1236)N^*(1688) )</td>
<td>6.7 ± 1.8</td>
<td>113 ± 30</td>
</tr>
<tr>
<td>10</td>
<td>( pN^*(1470) )</td>
<td>16.3 ± 3.2</td>
<td>274 ± 54</td>
</tr>
<tr>
<td>11</td>
<td>( pN^*(1700) )</td>
<td>18.7 ± 3.1</td>
<td>314 ± 52</td>
</tr>
<tr>
<td>12</td>
<td>( pN^*(1470) )</td>
<td>17.0 ± 2.6</td>
<td>286 ± 44</td>
</tr>
<tr>
<td>13</td>
<td>( pN^*(1700) )</td>
<td>3.5 ± 1.5</td>
<td>59 ± 25</td>
</tr>
<tr>
<td>14</td>
<td>( pp\pi^+\pi^- )</td>
<td>~0</td>
<td>~0</td>
</tr>
</tbody>
</table>
(a) One-pion-exchange

(b) Diffraction dissociation

FIG. 7.1
CHAPTER 8

ONE-PION-EXCHANGE AND THE I=3/2 kπ CROSS-SECTION

8.1 INTRODUCTION

We have seen in Chapter 6 that there is considerable evidence that a large fraction of the data in channel (1) can be interpreted in terms of a simple one-pion-exchange (OPE) mechanism. Such a mechanism is illustrated in Fig. 7.1(a).

In this Chapter we shall consider in some detail the implications of the OPE model for the sub-reaction,

\[ pp \rightarrow \Delta^{++} \pi^- \]  

(a)

This sub-set of the data has been chosen for several reasons. Firstly the channel (1) is dominated by \( \Delta^{++} \) production and this fact can be used to considerably simplify the calculations. Since there are only relatively few cases where both \( p\pi^+ \) mass combinations lie in the \( \Delta^{++} \) mass band, there is only one diagram to be calculated for each event in general. If we had chosen to consider the whole of channel (1), there would be two such diagrams corresponding to the exchange of the \( \pi^+ \) and \( \pi^- \), together with possible interference effects.

Secondly, selecting \( \Delta^{++} \) events reduces the number of free parameters to be fitted. The \( p\pi^+ \) vertex may be considered to be dominated by p-wave scattering, so only one off-shell parameter has to be introduced.

Lastly it is intended later in the Chapter to apply the model to channel (2) in an attempt to extract the \( k^-\pi^- \) on-shell scattering cross-section. It is to be hoped that by selecting only those events in which a \( \Delta^{++} \) is produced we shall have isolated a sample that proceeds dominan-
tly through the mechanism illustrated in Fig. 8.1(a) and that pion exchange predominates for sufficiently low four-momentum transfers.

8.2 $^\pi_+$ OFF-SHELL SCATTERING

In order to restrict ourselves to the region of validity of the OPE model, we have selected only those events for which

$$\Delta^2 (p, p\pi^+)< 0.4 \text{ GeV}^2.$$ 

where $\Delta^2$ is now understood to be positive in the physical region. This is in accord with the results of Section 6.3 where we saw that the $\Delta^{++}$ density matrix elements were in good agreement with the values predicted for spin zero exchange over this range of four-momentum transfer.

We have attempted to study the $p\pi^+$ off-shell scattering by dividing the $p\pi^+$ mass spectrum into intervals of approximately equal numbers of events and examining the angular distribution in each interval separately.

We write each distribution as,

$$W(\cos \vartheta) = \sum_{\ell} \left( \frac{A_{\ell}}{A_0} \right) P_\ell (\cos \vartheta)$$

(8.1)

where $\vartheta$ is the polar angle of the decay proton in the rest-frame of the $p\pi^+$ system, defined in Section 6.3 and the $P_\ell$ are the Legendre Polynomials. $\ell$-values up to ten have been considered in the expansion.

The shape parameters are then given by,

$$\frac{A_{\ell}}{A_0} = (2\ell + 1)<P_\ell (\cos \vartheta)>$$

(8.2)

where the statistical uncertainty is,

$$\sum \left( \frac{A_{\ell}}{A_0} \right) = (2\ell + 1) \left\{ \frac{<P_\ell^2>}{<P_\ell^2>} - <P_\ell^2> \right\}^{1/2}$$

(8.3)

and $N$ is the number of events in the $p\pi^+$ mass interval.
The first three moments are shown as a function of $p\pi^+$ mass in Fig. 8.2. The other moments not shown are consistent with zero in the $\Delta^{++}$ mass region defined by,

$$1.15 \leq M(p\pi^+) \leq 1.30 \text{ GeV}$$

This region is indicated by arrows in Fig. 8.2.

We have made a comparison with physical on-mass-shell $\pi^+ p$ elastic scattering in the following way. The experimental angular distributions were computed from the results of the CERN phase-shift analysis (CERN I solution) and the shape parameters evaluated using equation (8.2). This was done at various values of $p\pi^+$ mass and smooth curves drawn through the points so obtained. These curves are shown in Fig. 8.2.

It will be seen that the agreement between on-shell and off-shell data for the first two moments is generally quite poor, especially for the $L = 1$ moment. At higher masses this is to be expected of course, since the number of cases in which we are associating the 'wrong' proton with the $\pi^+$ will be increasing. However, we have confirmed that this is not the reason for the disagreement in the $\Delta^{++}$ region by looking at the $p\pi^+$ moments in channel (2). Here there is no confusion among the final state particles and a similar behaviour (not shown) was found.

A possible 'explanation' has been suggested by Colton and Schlein (64) who pointed out that the effect is probably due to the off-mass-shell behaviour of the cosine of the scattering angle. The point is that if one compares the on- and off-shell scattering at equal values of the four-momentum transfer from proton to proton, then the scattering angle is different in the two cases.

A simpler explanation is that we may be considering the 'wrong diagram'. Consider the OPE diagrams for the kaon channel illustrated.
in Fig. 8.1(a) and (b). An analysis of the $\pi^+p$ scattering would
assume that events were occurring as in diagram (a). However, suppose
in diagram (b) the $\pi^+$ diffractively scattered off the kaon. Then the
$\pi^+\pi^-$ mass might be small and consequently the $p\pi^+$ mass also. The
polar angle distribution would then be peaked towards $\cos \theta = 1$ as
seen in Fig. 6.8(a) for the proton channel. The on-shell data on
the other hand predicts a negative value of $<\cos \theta>$ for small $p\pi^+$ mass.

We shall, however, ignore these 'small discrepancies' and say
that in the region of the $\Delta^{++}$ the experimental $\pi^+p$ scattering is reason-
ably approximated by off-shell scattering.

We have similarly evaluated the moments of the $\pi^-p$ system for
those events satisfying both the $\Delta^{++}$ and $\Delta^2$ selections.

The shape parameters up to $s = 10$ for the $\pi^-p$ scattering are
shown in Fig. 8.3 together with the on-shell data. Here the agreement
is really very good.

We conclude then that diagram 7.1(a) forms the main contribution
to the production of the $\Delta^{++}p\pi^-$ final state within the limits imposed
by the $\Delta^2(p, \pi^+)$ selection.

8.3 THE OPE MODEL

The differential cross-section for the OPE process illustrated
in Fig. 7.1(a) is written \(^{(65)}\),

$$
\frac{d^3\sigma}{dm_p^+ dm_p^- d\Delta^2} = \frac{1}{2\pi^2 Q^2} \frac{m^2}{p\pi^-} \frac{q(q(m\pi^+ \Delta^2)\sigma(m\pi^+ \Delta^2))}{2} \\
\frac{1}{(\Delta^2 + m^2)^2} \frac{m^2}{p\pi^-} \frac{q(q(m\pi^- \Delta^2)\sigma(m\pi^- \Delta^2))}{2} 
$$

\[(8.4)\]
where $Q$ is the incident momentum and $W$ the total energy in the centre-of-mass. The $q$'s are the off-shell momenta in the $p\pi^+$ rest frames and the $\sigma$'s the off-shell cross-sections. It is necessary in addition to divide the right hand side of (8.4) by a factor $(4\pi c)^2$ to obtain the correct dimensions.

The pole approximation consists of evaluating all quantities at the pole $A^2 = -m^2$, except the propagator. Thus on-mass-shell $p\pi\pi\pi$ elastic cross-sections would be used in (8.4). Such a procedure is not expected to reproduce the data (65), (66), even if $A^2$-dependent form factors are introduced.

### 8.4 OFF-SHELL CORRECTIONS

Many quasi-two-body reactions have successfully been fitted by Wolf (67) using the OPE model, over a wide range of energies. The off-mass-shell cross-sections were related to their on-shell values by a method given by Durr and Pilkuhn (68).

This method can be extended to three or four body final-states if each two-body vertex is treated as a sum over partial waves, each specified by orbital angular momentum $l$ and total spin $J$.

We introduce Wolf's (67) form factor,

$$G(A^2) = \frac{c - m^2}{c + A^2}$$

(8.5)

which has a 'weak' $A^2$-dependence and write the differential cross-section as,

$$\frac{d^3\sigma}{dm_+ dm_- dA^2} = \frac{1}{2\pi^3 (2\pi)} \frac{m^2 q(m^+, A^2)\sigma(m^+, A^2)}{p_{\pi^+} p_{\pi^-} p_{\pi^+} p_{\pi^-}} \cdot \frac{G^2(\Delta^2)}{(\Delta^2 + m_{\pi}^2)^2} \frac{m^2 q(m^-, A^2)\sigma(m^-, A^2)}{p_{\pi^+} p_{\pi^-} p_{\pi^-}}$$

(8.6)
The $p\pi^+$ vertex in Fig. 7.1(a) is assumed to be dominated by $\Delta^{++}$ (1236) production and hence $p$-wave scattering. The $p\pi^+$ on- and off-shell cross-sections are then related by:

\[
q_{\text{p} \rightarrow \text{p}, \Delta^{++}}(s) \sigma_{\text{p} \rightarrow \text{p}, \Delta^{++}}(t)
\]

\[
= q_{\text{p} \rightarrow \text{p}, \Delta^{++}}(s) \sigma_{\text{p} \rightarrow \text{p}, \Delta^{++}}(t)
\]

\[
\left[\frac{(m_p^+ + m_\pi^2 + \Delta^2)}{p_{\pi^+}}\right] \cdot \left[\frac{(m_p^+ + m_\pi^2 - m_\pi^2)}{p_{\pi^+}}\right]
\]

\[
1 + R_\Delta^2 \cdot \frac{q^2(m_p^+, \Delta^2)}{p_{\pi^+}}
\]

\[
= \left[\frac{(m_p^+ + m_\pi^2 + \Delta^2)}{p_{\pi^+}}\right] \cdot \left[\frac{(m_p^+ + m_\pi^2 - m_\pi^2)}{p_{\pi^+}}\right]
\]

\[
1 + R_\Delta^2 \cdot \frac{q^2(m_p^+, \Delta^2)}{p_{\pi^+}}
\]

\[
(8.7)
\]

In a non-relativistic treatment of the problem, $R_\Delta$ would be the range of the potential. At relativistic energies $R_\Delta$ is supposed to be given by,

\[
R_\Delta \ll \frac{1}{m_x}
\]

\[
(8.8)
\]

where $m_x$ is the mass of the lowest state which can be exchanged between the proton and virtual pion. In practice, however, $R_\Delta$ is treated as an adjustable parameter to be fitted.

We write for the $p\pi^-$ vertex,

\[
q_{\text{p} \rightarrow \text{p}, \Delta^{++}}(s) \sigma_{\text{p} \rightarrow \text{p}, \Delta^{++}}(t)
\]

\[
= q_{\text{p} \rightarrow \text{p}, \Delta^{++}}(s) \sigma_{\text{p} \rightarrow \text{p}, \Delta^{++}}(t)
\]

\[
\left[\frac{(m_p^- + m_\pi^2 + \Delta^2)}{p_{\pi^-}}\right] \cdot \left[\frac{(m_p^- + m_\pi^2 - m_\pi^2)}{p_{\pi^-}}\right]
\]

\[
1 + R_\Delta^2 \cdot \frac{q^2(m_p^-, \Delta^2)}{p_{\pi^-}}
\]

\[
= \left[\frac{(m_p^- + m_\pi^2 + \Delta^2)}{p_{\pi^-}}\right] \cdot \left[\frac{(m_p^- + m_\pi^2 - m_\pi^2)}{p_{\pi^-}}\right]
\]

\[
1 + R_\Delta^2 \cdot \frac{q^2(m_p^-, \Delta^2)}{p_{\pi^-}}
\]

\[
(8.9)
\]

where

\[
\tilde{\kappa} = \frac{q_{\text{p} \rightarrow \text{p}, \Delta^{++}}(s) \sigma_{\text{p} \rightarrow \text{p}, \Delta^{++}}(t)}{q_{\text{p} \rightarrow \text{p}, \Delta^{++}}(s) \sigma_{\text{p} \rightarrow \text{p}, \Delta^{++}}(t)}
\]

\[
(8.10)
\]
and the $A_{LJ}^{\pi^-p}$ are the $\pi^-p$ partial wave amplitudes. These can be related to the pure $I$-spin amplitudes,

$$A_{LJ}^{\pi^-p} = \frac{1}{3} \left[ 2 \ A_{LJ}^{I=3/2} + A_{LJ}^{I=1} \right]$$

where

$$A_{LJ}^{I} = \frac{2i \delta_{LJ}^{I}}{\eta_{LJ}^{I}} e^{i \delta_{LJ}^{I}} - 1$$

and $\eta$ and $\delta$ are the absorption parameter and phase-shift angle as determined from the phase-shift analyses$^7$.

We can relate the off-mass-shell amplitudes to the physical amplitudes as follows.

Define

$$W = \frac{(m - m_\pi)^2 + \Delta^2}{(m - m_\pi)^2 - m_\pi^2}$$

and

$$Q = \frac{q^2 (m - m_\pi, \Delta^2)}{q^2 (m - m_\pi, m_\pi^2)}$$

Then,

$$|A_{\pi^-p}^{o_\frac{1}{2} \ p\pi} (m - m_\pi, \Delta^2)|^2 = W |A_{\pi^-p}^{\pi^-p} (m - m_\pi, m_\pi^2)|^2$$

$$|A_{\pi^-p}^{o_\frac{1}{2} \ p\pi} (m - m_\pi, \Delta^2)|^2 = Q \left[ 1 + R_{1}^{2} \frac{q^2 (m - m_\pi, m_\pi^2)}{p_\pi^2} \right]$$

$$\cdot |A_{\pi^-p}^{\pi^-p} (m - m_\pi, m_\pi^2)|^2$$

(8.15)
\begin{equation}
|A_{13/2}^{-}(m_{p\pi^{-}}, \Delta^2)|^2 = WQ \left[ \frac{1 + R_{p3/2}^2 q^2(m_{p\pi^{-}}, m_{\pi}^2)}{1 + R_{p3/2}^2 q^2(m_{p\pi^{-}}, \Delta^2)} \right] \\
\cdot |A_{13/2}^{-}(m_{p\pi^{-}}, m_{\pi}^2)|^2
\end{equation}

(8.16)

Similar modifications may be written for the $d_{3/2}$, $d_{5/2}$, $f_{5/2}$, \ldots waves.

### 8.5 COMPARISON WITH EXPERIMENTAL DATA

Below a $p\pi^-$ mass of 1.6 GeV, the only waves which contribute significantly to physical $\pi^-p$ scattering are the $s_{1/2}$, $p_{1}$, $p_{3/2}$ and $d_{3/2}$. However, above this mass the off-shell correction factors given in equations (8.14) to (8.16) are rapidly approaching unity.\(^\text{(65)}\)

In order to determine the parameters, $c$, $R_{\Delta'}$, $R_{p_{1}'}$, $R_{p_{3/2}}$ and $R_{d_{3/2}}$ a maximum likelihood fit to the data was performed using equation (8.6) together with the appropriate off-shell corrections.

Only those events with $p\pi^-$ mass less than 1.6 GeV were considered, where these also satisfied the $\Delta^2$ and $\Delta^{**}$ selections previously defined. Where an event had both $p\pi^+$ combinations in the $\Delta^{**}$ mass band, the combination with the lower value of $\Delta^2$ was taken.

The likelihood function for the $i$ th. event was written,

\begin{equation}
L_i = \frac{\left( \frac{d^3}{dm_{p\pi^+} dm_{\pi^-} d\Delta^2} \right)_i}{\int \frac{d^3}{dm_{p\pi^+} dm_{\pi^-} d\Delta^2} \ \frac{d^3}{dm_{p\pi^+} dm_{\pi^-} d\Delta^2}}
\end{equation}

(8.17)

and the grand likelihood function,
\[ \ln \mathcal{L} = \sum \ln L_i \]  

was then maximised with respect to the unknown parameters.

Unfortunately, the statistical significance of the sample of events is rather poor and it was found that the likelihood was only a weak function of the parameters. Hence it was decided to use the results of a maximum likelihood fit performed by Colton \(^{(65)}\) on a much larger sample of data from a proton-proton experiment at 6.6 GeV/c incident momentum. The values of the parameters used are listed in Table 8.1.

The predicted distributions in the \( p\pi^- \) mass and \( \Delta^2 \) are shown in Figs. 8.4 and 8.5 together with the experimental data. We also show in Fig. 8.6 the \( p\pi^+ \) mass distribution in 10 MeV intervals.

The same limits were of course applied in the calculation as to the data. Above a \( p\pi^- \) mass of 1.6 GeV the on-shell cross-sections were used in (8.6), without any off-shell corrections. The physical \( \pi^+ p \) elastic cross-sections were taken from the data compilation of Ref. (69) and the phase-shifts from the CERN I solution \(^{(7)}\).

It should be noted that the predicted curve for the \( p\pi^- \) mass spectrum exhibits no discontinuity at 1.6 GeV, so our procedure of using on-shell cross-sections above this point seems justified. In general, the agreement between the data and the theoretical curves is remarkably good. In particular, the overall normalization is correctly reproduced. In a maximum likelihood fit to the data, the normalization is not of course a free parameter, since it cancels out in the expression (8.17). This correct prediction of the cross-section must be regarded as a big success for the model.
Gellert et al. (70) have claimed that at 6.6 GeV/c the OPE model as presented in this Chapter successfully accounts for the low mass \(\Delta^+\pi^-\) spectrum as a kinematic reflection of the high mass \(p\pi^-\) system. We shall be considering such 'kinematic effects' in more detail in Chapter 9, so only a brief discussion will be given here.

In order to calculate the \(p\pi^+\pi^-\) mass distribution it is necessary to include information on the angular distribution at the \(p\pi^-\) vertex in Fig. 7.1(a). The matrix element is written,

\[
|M|^2 = 4|\mathcal{A}_{p\pi^+}(m, \Delta^2)|^2 \frac{g_2^2(\Delta^2)}{(\Delta^2 + m_{\pi^-}^2)^2} |\mathcal{A}_{p\pi^-}(m, \Delta^2)| (8.19)
\]

where the \(\mathcal{A}\)'s are the amplitudes for virtual pion-proton scattering.

Following Jacob (72) we can write,

\[
|\mathcal{A}_{\pi^0}(m_{p\pi}, \Delta^2)|^2 = 64\pi^2 m_{\pi}^2 \frac{q(m_{p\pi}, \Delta^2)}{q(m_{p\pi}, m_{\pi}^2)} \frac{d\sigma}{d\Omega}(m_{p\pi}, \Delta^2) (8.20)
\]

where \(d\sigma/d\Omega\) is the \(\pi\pi\) differential elastic cross-section.

These were calculated from the phase-shifts (7) with the appropriate Durr-Pilkuhn (68) off-shell corrections. The \(p\pi^+\) vertex was again assumed to be dominated by \(p\)-wave scattering. Above a mass of 2.1 GeV the \(p\pi^-\) vertex was taken to be diffractive and the differential cross-section written,

\[
\frac{d\sigma}{d\Omega}(m_{p\pi^-}, m_{\pi}^2) = \frac{1}{16\pi^2} q(m_{p\pi^-}, \Delta^2) q(m_{p\pi^-}, m_{\pi}^2) \sigma_{T}^{\pi p}(m_{p\pi^-}, m_{\pi}^2) \sigma_{T}^{\pi p}(m_{p\pi^-}, m_{\pi}^2) (8.21)
\]

where \(\sigma_T\) is the total \(\pi^-p\) cross-section (69) and \(t\) is the four-momentum transfer from proton to proton. The slope, \(a\), of the differential cross-section was taken as 7.7 GeV\(^{-2}\) (73).
We show in Fig. 8.7 the p^+p^- mass distribution with the selections previously defined. The theoretical curve was calculated using the program FOWL(62),(63), and equation (8.19) and was normalized to the cross-section obtained by integrating equation (8.4).

The model predicts a broad low mass enhancement, centred at around 1600 MeV, with width of order 500 MeV. The double peaked structure of the experimental distribution is not reproduced.

It should be noted however, that physical π^-p scattering has no azimuthal angular dependence, whereas the off-shell scattering does. We have expanded the angular distribution of the π^-p scattering,

\[ W(\cos \theta, \phi) = \sum_{\lambda, m} \langle Y_{\lambda}^m \rangle Y_{\lambda}^m (\cos \theta, \phi) \]  

(8.22)

where the angles \( \theta \) and \( \phi \) were defined in Section 6.3. The significant moments with \( m \neq 0 \) are shown in Fig. 8.8.

It has been demonstrated by Gellert et al.\(^{(70)}\) that if this experimental azimuthal angular dependence is explicitly incorporated in the model, the predicted low mass enhancement becomes somewhat narrower. This procedure is not of course entirely satisfactory, but simple OPE type models offer no other prescription.

### 8.6 THE K^-π^- CROSS-SECTION

Recently there have been several attempts\(^{(74)}\) to extract the physical \( I = 3/2 \) \( K\pi \) cross-section from off-mass-shell scattering over a \( K\pi \) mass range from threshold to about 2.2 GeV. However, the values reported vary from roughly 2 mb to 70 mb.

It has been shown by Trippe et al.\(^{(75)}\) that in the reactions,

\[ k^+p \rightarrow (K^+\pi^-) \Delta^{++} \]
the $k^+p \rightarrow (k^0\pi^0) \Delta^{++}$ moment has the same behaviour within the experimental errors. Since these two $k\pi$ systems are different admixtures of $I = \frac{1}{2}$ and $I = 3/2$ states, one is lead to suppose that the $I = 3/2$ amplitude must be relatively small. Thus the $k^-\pi^-$ cross-sections quoted by Cho et al. (74) of about one to two mb. seem most reasonable.

We have already seen that for the reaction,

$$pp \rightarrow \Delta^{++} \pi^-$$

the OPE model with Durr-Pilkuhn (68) off-shell corrections reproduces the $\pi^-\pi^+$ mass distribution both in shape and overall normalization.

Using this model we have attempted to extract the physical $k^-\pi^-$ cross-section from the reaction,

$$k^-p \rightarrow \Delta^{++} k^-\pi^-$$

The method used was a modified Chew-Low extrapolation (76) similar to that successfully applied by Ma et al. (77) to the reaction

$$pp \rightarrow \pi^+n$$

at 6.6 GeV/c, to obtain the physical $\pi^+p$ cross-section in the region of the $\Delta^{++}$. The agreement between their extrapolated values and the measured cross-sections was extremely good.

In order to isolate a reasonably pure sample of events which can be described by the OPE mechanism illustrated in Fig. 8.1(a) the following selections have been applied to the data of channel (2).

$$1.15 \leq M(\pi^+n) \leq 1.30 \text{ GeV}$$

$$\Delta^2(p, \pi^+n) < 0.6 \text{ GeV}^2$$

Unfortunately there is a considerable background problem in this channel which was not present for reaction (1). There we saw that
there were only a few events where both $p\pi^+$ combinations had mass
in the region of the $\Delta^{++}$. In Fig. 8.9 is shown the $k^-\pi^+$ mass dis-
tribution for those events of channel (2) satisfying the above selection
criteria. It can be seen that there is a substantial $k^*^0$ (890) signal
with some evidence for $k^*^0$ (1420). If one accepts that there is also
a broad s-wave $k^-\pi^+$ interaction\(^{(75)}\) under the $k^*^0$ (890) peak, then one
might believe that the 'background' amplitude, as illustrated by Fig.
8.1(b), almost completely dominates the reaction (b).

8.7 THE MODEL

The OPE model gives for the differential cross-section for the
diagram of Fig. 8.1(a),

$$
\frac{d^3\sigma}{dm_{p\pi^+} dm_{k^-\pi^-} d\Delta^2} = \frac{1}{4\pi^3(QW)^2} \left[ \frac{m_{p\pi^+}^2}{m_{p\pi^+}} q(m_{p\pi^+}, \Delta^2) (m_{p\pi^+}, \Delta^2) \right]
\frac{G^2(\Delta^2)}{(\Delta^2 + m_{\pi^-}^2)^2} \left[ \frac{m_{k^-\pi^-}^2}{m_{k^-\pi^-}} q(m_{k^-\pi^-}, \Delta^2) (m_{k^-\pi^-}, \Delta^2) \right]
$$

(8.23)

where $G^2(\Delta^2)$ is the form-factor and the notation is similar to that used
in Section 8.3.

The $\pi^+p$ off-shell cross-sections are related to their on-shell
values by equation (8.7) where the value of $R_\Delta$ used is given in Table 8.1.
The $k^-\pi^-$ scattering is assumed to be s-wave, so no off-shell corrections
are used\(^{(68)}\). The phenomenological form-factor $G^2(\Delta^2)$ is given by (8.5)
where we have used Wolf's\(^{(67)}\) 'universal' value of $2.3$ GeV\(^2\) for the constant
$c$.

The modified pole-extrapolation procedure\(^{(77)}\) consists of extra-
polating the quantity,
to the pion pole, $\Delta^2 = -m_{\pi}^2$ \((\frac{d\sigma}{d\Delta^2})_{\text{OPE}}\) is obtained from (8.23) by setting the $k^-\pi^-$ cross-section equal to unity and integrating over $m_{\pi^+}$ and $m_{k^-\pi^-}$.

\((\frac{d\sigma}{d\Delta^2})_{\text{EXP}}\) is the experimental differential cross-section.

At the pion-pole, $\sigma''$ should be given by,

\[ \sigma'' = \sigma \left( m_{k^-\pi^-}, m_{\pi^+}^2 \right) \quad (8.25) \]

The experimental data is divided into regions of $k^-\pi^-$ mass and $\Delta^2$. For each $k^-\pi^-$ mass interval, the quantity $\sigma''$ is calculated as a function of $\Delta^2$, and the resulting data points may then be fitted to an expression of the form,

\[ \sigma'' = a + b \Delta^2 \quad (8.26) \]

which can be evaluated at the pole, $\Delta^2 = -m_{\pi}^2$, to give the on-shell cross-section.

If the theoretical \((\frac{d\sigma}{d\Delta^2})_{\text{OPE}}\) accurately reproduces the shape of the experimental distribution, then the coefficient $b$ should be close to zero. If on the other hand the theoretical curve is a bad fit to the data, then higher order terms may have to be included in (8.26). A very high statistics experiment would be needed to determine these higher order terms to sufficient accuracy.

The Durr-Pilkuhn formalism\(^{(68)}\) assumes that off-shell effects are not important for $s$-wave $k^-\pi^-$ scattering near threshold. However, Love-lace\(^{(78)}\) has pointed out that from current algebra one can show that such off-shell effects will be present. However, by allowing $\sigma''$ to depend on $\Delta^2$ in the extrapolation, we do not insist that the OPE model be exactly correct.
The method we have applied to our data differs from that outlined above in that we have attempted to correct for the very large "background" from the process illustrated in Fig. 8.1(b).

Suppose that in a given interval of kπ mass and Δ^2 the background process contributes \( \frac{Δσ_B}{ΔΔ^2} \) to the numerator in equation (8.24), where ΔΔ^2 is the width of the Δ^2 interval in question.

Then the function which is extrapolated to the pole is,

\[
\sigma^n = \frac{(dσ_2)}{dΔ^2} \exp \left( \frac{Δσ_B}{ΔΔ^2} \right)
\]

where again the kπ cross-section is set equal to unity in evaluating the denominator.

The calculation of the background contribution was carried out as follows. The reaction (b) was assumed to be dominated by the two OPE graphs illustrated in Fig. 8.1. The matrix element was then written as the incoherent sum,

\[
|M|^2 = |A^{pπ^+}(m_{pπ^+}, Δ^2)|^2 \frac{g^2(Δ^2_1)}{(Δ^2_1 + m^2_n)^2} |A^{kπ^+}(m_{kπ^+}, Δ^2_1)|^2
\]

\[
+ |A^{pπ^+}(m_{pπ^+}, Δ^2)|^2 \frac{g^2(Δ^2_2)}{(Δ^2_2 + m^2_n)^2} |A^{kπ^+}(m_{kπ^+}, Δ^2_2)|^2
\]

(8.28)

where Δ^2_1 is the absolute value of the square of the four-momentum transfer from incident proton to the pπ^+ system and Δ^2_2 from incident proton to pπ^- system. The rest of the notation is as in equation (8.19).

The amplitudes can be expressed in terms of the angular dis-
tributions using equation (8.20). These were calculated for the $p^+\pi^-$ vertices in exactly the same way as in Section 8.5. The $k^-\pi^+$ scattering was assumed to be $s$-wave so,

$$\frac{d\sigma}{d\Omega}_{k^-\pi^+} (m_{k^-\pi^+}, \Delta^2) = \frac{\sigma_{k^-\pi^+}}{4\pi}$$ (8.29)

The $k^-\pi^+$ vertex was assumed to be dominated by $s$-, $p$- and $d$-wave scattering and the differential cross-section written (75),

$$\frac{d\sigma}{d\Omega}_{k^-\pi^+} (m_{k^-\pi^+}, m_{\pi^+}^2) = (\frac{d\sigma}{d\Omega})_s + (\frac{d\sigma}{d\Omega})_p + (\frac{d\sigma}{d\Omega})_d$$ (8.29)

These differential cross-sections were represented by $s$-, $p$- and $d$-wave Breit-Wigner functions constrained to their respective unitarity limits at the resonance masses and multiplied by the appropriate angular distributions predicted for pseudo-scalar exchange. The masses and widths of the $k^*$ (890) and $k^*$ (1420) as given in the tables (5) were used for the $p$- and $d$-wave Breit-Wigner functions respectively. The parameters of the $s$-wave Breit-Wigner were taken from Lovelace's Veneziano model fits (78), where we used 900 and 214 MeV for the mass and width respectively.

Thus we have,

$$\frac{d\sigma}{d\Omega}_s = \left(\frac{4}{9}\right) \frac{4\pi\lambda^2}{(m_s^2 - m_{k^-\pi^+}^2)^2 + (\Gamma_s^0 m_s)^2} \cdot \frac{1}{4\pi}$$ (8.30)

where

$$\Gamma_s^0 = \Gamma_s \cdot \frac{m_s}{m_{k^-\pi^+}}$$ (8.31)

and where $m_s, \Gamma_s^0$ are the mass and width of the $s$-wave Breit-Wigner function. $q_s$ is the decay momentum from the $k^-\pi^+$ system and $q_s$ that from a
system with mass $m_s$.

Similarly we have,

$$
\frac{d\sigma}{d\Omega} = \left(\frac{4}{9}\right) \cdot 12\pi \lambda^2 \frac{(\gamma_{p-m}^2)}{(m_p^2 - m^2) + (\gamma_{p-p})^2} \cdot \frac{3}{4\pi} \cos^2 \theta
$$

(8.32)

where $\theta$ is the angle between incoming and outgoing $k^-$ in the $k^-\pi^+$ rest-frame. The width is given by,

$$
\Gamma_p = \Gamma_{p}^0 \frac{(q_p)^3}{m_{k^-\pi^+}}
$$

(8.33)

Lastly,

$$
\frac{d\sigma}{d\Omega} = \frac{1}{4} \cdot \left(\frac{4}{9}\right) \cdot 20\pi \lambda^2 \frac{(\gamma_{d-m})^2}{(m_d^2 - m^2) + (\gamma_{d-d})^2} \cdot \frac{5}{16\pi} \left(9 \cos^4 \theta - 6 \cos^2 \theta + 1\right)
$$

(8.34)

where

$$
\Gamma_d = \Gamma_{d}^0 \frac{(q_d)^5}{m_{k^-\pi^+}}
$$

(8.35)

We have introduced a factor $\frac{1}{4}$ into (8.34) to take account of the three body decay mode of the $K^* (1420)$ resonance.

The $p$- and $d$-wave differential cross-sections have to be modified by the appropriate off-shell corrections (68),

$$
\frac{d\sigma}{d\Omega} (m_{k^-\pi^+}, \Delta_2) = \left(\frac{q(m_{k^-\pi^+}, \Delta_2)}{q(m_{k^-\pi^+}, \Delta_2)}\right)^2 \left[\frac{1}{1 + R_p^2 q^2(m_{k^-\pi^+}, \Delta_2)}\right]
$$
The values of $R_p$ and $R_d$ were taken from the paper by Trippe et al.\(^{(75)}\) and set at 2.3 and 2.2 GeV\(^{-1}\) respectively.

Using FOWL\(^{(62)}\),\(^{(63)}\) the contributions from the two terms in (8.28) were evaluated individually in the specified regions of $k^{-}n^+$ mass and $\Delta^2$. The contribution from the first term can be found absolutely by integration of (8.23). Hence that from the second term can be determined.

The experimental values of "a" as determined from (8.27) are plotted in Fig. 8.10 as a function of $\Delta^2$, for the four regions of $k^{-}n^-$ mass considered. The small number of events involved is reflected in the relatively large errors on the data points, making any extrapolation to the pion pole subject to a very great uncertainty. However, from Fig. 8.10 it is clear that the $k^{-}n^-$ cross-section is relatively small, and certainly our data is inconsistent with the results of some experiments (Urvater et al., De Baere et al., Ref. 74) where cross-sections in excess of 10 mb. were found.

An attempt was made to fit the values of "a" to an expression linear in $\Delta^2$, as given by (8.26). However, a very poor fit was obtained
in the lowest $k^-\pi^-$ mass region. Thus we have used a quadratic function,
\[ \sigma = a + b\Delta^2 + c\Delta^4 \] (8.38)
except in the highest mass region, where a linear form was considered sufficient.

The values of the $I = 3/2$ $k^-\pi^-$ cross-section obtained by extrapolating to the pion pole are given in Table 8.2.

To obtain the predictions of the model the matrix element given by (8.28) was calculated using FOWL, with the values of the $k^-\pi^-$ cross-section listed in Table 8.2. The theoretical curves for $\Delta^2$ and $M(k^-\pi^-)$ are shown in Fig. 8.11, together with the experimental distributions. In general the agreement is rather good, except that the overall normalization of the model is a little low.
Table 8.1

**DURR-PILKERN OFF-SHELL CORRECTIONS FROM REF. (65)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>$1.16 \pm 0.48$ GeV$^2$</td>
</tr>
<tr>
<td>$R_\Delta$</td>
<td>$3.49 \pm 0.79$ GeV$^{-1}$</td>
</tr>
<tr>
<td>$R_{P^+}$</td>
<td>$0.32 \pm 0.89$ GeV$^{-1}$</td>
</tr>
<tr>
<td>$R_{p3/2}$</td>
<td>$2.31 \pm 0.25$ GeV$^{-1}$</td>
</tr>
<tr>
<td>$R_{d3/2}$</td>
<td>$&gt; 30$ GeV$^{-1}$</td>
</tr>
</tbody>
</table>

Table 8.2

**THE $K^-\pi^-$ CROSS-SECTION**

<table>
<thead>
<tr>
<th>Mass of $K^-\pi^-$ (GeV)</th>
<th>Cross-section (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.63 - 1.00</td>
<td>$5.2 \pm 4.7$</td>
</tr>
<tr>
<td>1.00 - 1.35</td>
<td>$2.4 \pm 5.1$</td>
</tr>
<tr>
<td>1.35 - 1.80</td>
<td>$1.3 \pm 2.3$</td>
</tr>
<tr>
<td>1.80 - 2.60</td>
<td>$1.2 \pm 2.1$</td>
</tr>
</tbody>
</table>
OPE DIAGRAMS

\[ K^- p \rightarrow p K^- \pi^+ \pi^- \]

(a)

(b)

FIG. 8.1
$p p \rightarrow p p \pi^+ \pi^-$

$p\pi^+$ Moments; $\Delta^2 < 0.4$ GeV$^2$

FIG. 8.2
$pp \rightarrow \Delta^{++} p \pi^-$

$A_1/A_0$

$A_2/A_0$

$A_3/A_0$

$A_4/A_0$

$A_5/A_0$

FIG. 8.3
FIG. 8.3 (continued)
pp → Δ⁺⁺ p π⁻

Δ² < 0.4 GeV²

1032 events
\[ pp \rightarrow \Delta^{++} p \pi^- \]

\[ \Delta^2 < 0.4 \text{ GeV}^2 \]

FIG. 8.5

FIG. 8.6

1032 events

Mass of $\Delta^{++}$ GeV
FIG. 8.7

\[ pp \rightarrow \Delta^{++} p \pi^- \]

\[ \Delta^2 < 0.4 \text{ GeV}^2 \]

1032 events
FIG. 8.8

$pp \rightarrow \Delta^{++} p \pi^-$

$\rho \pi^-\text{-Moments } \Delta^2 < 0.4 \text{ GeV}^2$

$\langle \text{Re} Y_1^1 \rangle$

$\langle \text{Re} Y_2^1 \rangle$

$\langle \text{Re} Y_3^1 \rangle$

$\langle \text{Re} Y_4^1 \rangle$

$\langle \text{Re} Y_5^1 \rangle$

$M(\rho \pi^-) \text{ GeV}$
$K^- p \rightarrow pK^- \pi^+ \pi^-$

$\Delta^*\text{ Selected}$

$\Delta^2 < 0.6 \text{ GeV}^2$

1235 events

FIG. 8.9
CHEW-LOW EXTRAPOLATION CURVES

(a) $M(K^-\pi^-) < 1.0 \text{ GeV}$

(b) $1.0 \leq M(K^-\pi^-) < 1.35 \text{ GeV}$

(c) $1.35 \leq M(K^-\pi^-) < 1.8 \text{ GeV}$

(d) $1.80 \leq M(K^-\pi^-) < 2.6 \text{ GeV}$

FIG. 8.10
\(- 209 -

K^- p \rightarrow \Delta^{++} K^- \pi^-

\Delta^2 < 0.6 \text{ GeV}

1235 events

FIG. 8.11
CHAPTER 9

OTHER t-CHANNEL MODELS

9.1 INTRODUCTION

For some time the question of whether the low mass $A\pi\pi$ system (where $A$ is $\pi$, $k$ or $p$), produced in four body final states, is a genuine resonance or 'kinematic effect', or some combination of these, has been a subject of considerable controversy\(^{(80)}\). These enhancements appear to have the general properties of resonances. They have definite masses and widths, which do not change with incident beam momentum, and definite spin parity. (The latter statement is at least true for meson systems. The situation is not so clear for baryons - see Chapter 10 on this point). We saw in Chapter 6 that the production of the low mass $p\pi\pi$ is consistent with a diffractive mechanism and this is also true for the $\pi\pi\pi$ and $k\pi\pi$ systems. The enhancements are produced at very small four-momentum transfers and their production cross-section is almost energy independent. If one likes to picture the production mechanism in terms of the one particle exchange graph shown in Fig. 9.1(a) then the quantum numbers of the exchanged object must be those of the vacuum. ($J^P = 0^+, I^G = 0^+$)

9.2 DECK-TYPE MECHANISMS

In 1961, Drell and Handler\(^{(81)}\) proposed a mechanism to explain the bump in the energy spectrum of inelastically scattered protons emerging from proton-nucleus collisions at given scattering angles \(^{(82)}\).
In 1964, this idea was reformulated by Deck (83) to explain in a quantitative fashion the low mass enhancement at about 1080 MeV (the $A_1$ peak) in the $\pi p$ mass spectrum from the reaction, 

$$\pi^+ p \rightarrow \pi^+ \rho^0 p$$

without requiring a resonance interpretation. The so-called Deck-mechanism for this reaction is illustrated in Fig. 9.1(b) where it is assumed that the mechanism responsible for the $A_1$ bump is of a peripheral nature and dominated by one-pion-exchange. Deck assumed that the virtual $\pi p$ scattering could be approximated by the physical on-shell amplitude. For sufficiently high mass, the $\pi p$ cross-section is characterised by a pronounced diffractive peak at low four momentum transfers. Thus the virtual pion could be expected to be scattered forwards in the direction of the $\rho$ to produce a low mass enhancement.

Using detailed calculations along these lines, Deck was able to reproduce the shape of the $A_1$ peak in a 3.65 GeV/c $\pi^+ p$ experiment (84). However, the cross-section he obtained was only about a quarter of the experimental cross-section. In all fairness, it should be pointed out that the model in its original form was extremely crude. Spins were not considered, the $\rho$ meson width was neglected, as were also absorption effects in the initial and final states and form factors associated with the exchanged pion. There could in addition be another diagram contributing, where the $\rho$ meson is diffractively scattered from the target nucleon. This is depicted in Fig. 9.1(c).

Nevertheless, there are some authors (85) who have claimed some success with this limited model.

It is worthy of note that the calculated cross-section for the $\pi p$ enhancement exhibits a sharp rise at threshold. This suggests that an $s$-wave interaction is dominant, and since the $\rho$ has $J^P = 1^-$, the Deck
background must have $J^P = 1^+$. Thus before duality \(^{(47), (48)}\) if an $A_1$ resonance, probably also with $J^P = 1^+$ was occurring, there could be considerable interference effects between resonance and background.

In an attempt to describe the low mass $k\pi\pi$ enhancement at around 1300 MeV, Ross and Yam \(^{(86)}\) introduced a model involving the coherent sum of three diffractive-type diagrams. They supposed that the incident $k$ dissociated into a $k^*(890)$ and a pion and then each process corresponded to the virtual diffraction scattering of the $\pi$, $k^*$ and $k$ with the target nucleon. The three diagrams are illustrated in Fig. 9.2 for this process,

\[ k^- p \rightarrow k^{*0}(890) \pi^- p \]

In a $k^- p$ experiment at 4.6 and 5.0 GeV/c, Dornan et al. \(^{(87)}\) were able to explain the $k\pi\pi$ enhancement entirely in terms of such a model. However, in other experiments at 5.5 \(^{(88)}\) and 12.6 \(^{(89)}\) GeV/c, the authors claimed that the Ross-Yam model did not adequately describe their data and required a resonance interpretation.

In general, simple diffraction models of this kind have had only a limited success in explaining the low mass $\pi\pi\pi$ and $k\pi\pi$ enhancements. The peak predicted by the models is invariably too broad and moreover the width is a function of the energy.

9.3 THE $p\pi\pi$ SYSTEM AND DECK-TYPE MECHANISMS

We now turn to a discussion of the low mass $p\pi\pi$ system, produced in the reactions,

\[ pp \rightarrow (p\pi^+\pi^-) p \]  \hspace{1cm} (1)
\[ k^- p \rightarrow (p\pi^+\pi^-) k^- \]  \hspace{1cm} (2)

The dominant process for the kinematic enhancement is usually
assumed to be of the type

$$pp \rightarrow \Delta^{++} (1236) \pi^- p \quad (1a)$$

$$k^- p \rightarrow \Delta^{++} (1236) \pi^+ k^- \quad (2a)$$

In contrast to the $A_1$ and $k^*$ (1320) resonances which have spin-parity $J^P = 1^+$ consistent with s-wave production by the Deck-type process, the low mass $p\pi\pi$ enhancement is usually identified with the $P_{11} (1470), J^P = \frac{1}{2}^+$, the so-called Roper resonance $^{(90)}$. A Deck mechanism which is dominantly s-wave would give $J^P = 3/2^-$. There is actually a well established resonance $^{(5)}$ with $J^P = 3/2^-$ at 1520 MeV, but this is d-wave.

The 1470 enhancement was originally observed in high energy proton-proton counter experiments at CERN $^{(91)}$ and Brokhaven $^{(92)}$ together with the then well established isobars $^{(93)}$ at 1510 and 1690 MeV. Certain unusual features concerning the 1470 MeV effect were noted. As the laboratory scattering angle was increased, the bump at 1470 MeV decreased in magnitude very quickly until only a well defined signal at 1510 MeV remained. It was found that the 1510 and 1690 MeV enhancements had a roughly similar dependence on the scattering angle, $\theta$, for larger values of this angle. However, at very small $\theta$, the 1470 MeV signal was dominant. This behaviour is of course perfectly consistent with that expected for a diffractively produced resonance. Nevertheless, Resnick $^{(94)}$ attempted to 'explain' the effect using the quasi-diffraction model of Drell and Hilda $^{(81)}$. He obtained rough agreement with the experimental cross-section and its $\theta$-dependence, but was not able to account for its detailed position and width.
9.4 **THE ROSS-YAM MODEL**

The Ross-Yam diagrams (86) for the two channels of interest are shown in Figs. 9.3. In order to isolate a sample of events most likely to be produced via these mechanisms, certain selections have been imposed on the data.

Those events containing a $\Delta^{++}$ (1236) have been extracted by requiring the mass of the $p\pi^+$ combination to lie in the range,

$$1.15 < M(p\pi^+) < 1.30 \text{ GeV}$$

A selection has also been imposed on the effective mass of the other two particles in each channel. We have in fact required,

- $M(p\pi^-) > 1.40 \text{ GeV}$ for channel (1)
- $M(k\pi^-) > 1.30 \text{ GeV}$ for channel (2)

This is in order to reduce the background from non-diffractive $\Delta^{++}$ events (for example, double isobar production in reaction (1)).

Lastly, in the spirit of all peripheral models, we have required,

$$\Delta^2 < 1.0 \text{ GeV}^2$$

where $\Delta^2$ is the absolute value of the four-momentum transfer from incident proton to the $\Delta^{++}$.

After these cuts have been imposed, there are only 12 events in channel (1) which have both $p\pi^+$ combinations satisfying the $\Delta^{++}$ selection. In order to be definite, we have chosen that combination having the smaller $\Delta^2$.*

* Of course to be strictly correct for channel (1) one should consider four terms for each of the diagrams illustrated in Fig. 8.3. These would correspond to an interchange of the initial and final state protons. However, as shown in Ref. (65), three of the terms can be made very small with suitable cuts on the data. In any event, the model at best is only a very crude one.
We show in Fig. 9.4 the $k^+\pi^-$ and $k^+\pi^-$ mass distributions with the above selections imposed. There is no evidence for diffractive production of the $k^-\pi^-$ system ($Q$ and $L$ mesons) and only a small $k^+\pi^-$ signal is present.

In order to demonstrate that baryon exchange processes can be neglected we make use of the Van Hove hexagons, which represent correlations between the centre-of-mass longitudinal momenta of a three body final state in a rather elegant fashion. Since at higher energies the transverse momenta are relatively small, the hexagons tend to be populated only close to the boundary. All correlations can be expressed in terms of the angle $\theta$, which is defined in Fig. 9.5 for the channel (2a). Because of the initial state symmetry in channel (1a), we restrict $\theta$ to lie in the range $0 < \theta < 180^\circ$. Events having values of $\theta$ outside these limits are rotated through $180^\circ$.

The distributions in the angle $\theta$ for reactions (1a) and (2a) are shown in Figs. 9.6 (a) and (b) respectively. Both distributions exhibit a large peak at $120^\circ$ corresponding to the production of the $\pi^-$ with small longitudinal momentum. Baryon exchange processes would be expected to contribute around $\theta = 60^\circ$. For channel (2a) there is a small secondary peak at $180^\circ$. This can be associated with events having low values of longitudinal momentum of the $k^-$, and hence probably not produced via the mechanisms illustrated in Fig. 9.3.

The mass selections on the $p\pi^-$ and $k^-\pi^-$ systems which we have defined are of course a compromise. To be really sure that these vertices are diffractive, the lower limits should perhaps be increased somewhat.

We thus conclude that to a fairly good approximation, diagram
We follow the notation of Ross and Yam(86) and label the initial and final state particles as in Fig. 9.3. These symbols will also be used to denote the four-momenta of the particles.

The amplitude is written,

\[ T_{\lambda^\alpha \lambda^\beta} = \bar{u}(\beta, \lambda) (M^\mu_a + M^\mu_c) u(b, \lambda^\prime_b) \]  (9.1)

where \( M^\mu_a \), \( M^\mu_c \) are the contributions from diagrams (a) and (c) respectively, and \( \lambda^\beta_b \), \( \lambda^\beta_b^\prime \) are the helicities of the incident proton and final state \( \Delta^{++} \). \( u(b, \lambda^\prime_b) \) is the conventional Dirac spinor for a spin \( \frac{1}{2} \) particle.

Following Rarita and Schwinger(96) the vector spinors, \( u^\mu(\beta, \lambda^\beta_b) \), are formed from Dirac spinors and polarization vectors, \( \epsilon_c \), by means of the usual Clebsch-Gordon coefficients. Thus we have,

\[ u^\mu(\beta, \lambda) = \sum C(1, \frac{1}{2}, 3/2; \lambda - \lambda^{'}, \lambda) \epsilon_c(\lambda^{'}, \lambda^{'}, \beta) \]  (9.2)

Using the Feynman rules for the construction of such diagrams we write,

\[ M^\mu_a = \frac{(q^2_{\Delta \pi \pi})}{4\pi} \frac{b^\mu}{t_{b\beta} - m^2_{\pi}} A^a \]  (9.3)

\[ M^\mu_c = \frac{(q^2_{\Delta \pi \pi})}{4\pi} \frac{\gamma^\mu(\beta + \sigma + m_p)}{S_{\beta\sigma} - m^2_{p}} A^c \]

where

\[ t_{b\beta} = (b - \beta)^2 \]

\[ S_{\beta\sigma} = (\beta + \sigma)^2 \]  (9.4)

and \( \beta = \gamma^\mu \gamma^\mu \) etc.
\[ \frac{g \alpha p n}{4\pi} \] is the \( \Delta^{++} p \pi^- \) coupling constant and \( A_a, A_c \) represent the virtual diffraction amplitudes. These are supposed to be pure imaginary with no spin-flip term and written,

\[
A_a = i \frac{2}{\Delta \sigma} \sum_{\alpha + \sigma = 0} \sigma^T \alpha_{\sigma \alpha} e^{i B t a \alpha}
\]

\[
A_c = i \frac{2}{\Delta \sigma} \sum_{\alpha + \sigma = 0} \sigma^T \alpha_{\sigma \alpha} e^{i B t a \alpha}
\] (9.4)

where \( \sigma^T \) is the appropriate total cross-section and \( B, D \) the slopes of the diffraction peaks. We use suffices of the type \( \alpha + \sigma = 0 \) to denote that a quantity is to be evaluated in the \( \alpha \sigma \) rest-frame.

Summing over final spins and averaging over initial spins, we have for the total matrix element,

\[
|T| = \frac{1}{2} \sum_{\beta, \lambda_b} T_{\beta \lambda_b} \lambda_b^2
\] (9.6)

\[
= \sum_{\lambda_b} \bar{u} (\beta, \lambda_{\beta}) (M_a + M_c) \ u (\beta, \lambda_{\beta})
\]

\[
\cdot \left( \bar{\lambda}_{a, \beta} + \bar{\lambda}_{c, \beta} \right)
\] (9.7)

\[
= \sum_{\lambda_b} \bar{u} (\beta, \lambda_{\beta}) (M_a + M_c) \ u (\beta, \lambda_{\beta})
\]

\[
\cdot \left( \bar{\lambda}_{a, \beta} + \bar{\lambda}_{c, \beta} \right)
\] (9.8)

where

\[ \bar{\lambda} = \bar{\lambda}_o \ M^+ \bar{\lambda}_o \] (9.9)

The expression (9.8) involves quantities such as \( \xi_a^\mu, \xi_a^\sigma \mu \) which are Lorentz invariants. Hence these can be evaluated in the \( \beta (\Delta^{++}) \) rest-frame where the polarization vectors have particularly simple forms:
The evaluation of (9.8) is essentially straightforward but rather tedious.

If we define,

\[
N_a = \left( \frac{g^2 \Delta m}{4\pi} \right)^2 \frac{1}{t_{b\beta}} m^2 \pi^2 \left| \frac{a}{a + \sigma} \right|^2 \sqrt{S_{a\sigma}}
\]

and

\[
N_c = \left( \frac{g^2 \Delta m}{4\pi} \right)^2 \frac{1}{s_{b\sigma}} m^2 \pi^2 \left| \frac{a}{a + \epsilon} \right|^2 \sqrt{S_{ab}}
\]

then the final expression for the matrix element is,

\[
\left| T \right|^2 = \frac{4}{3} N_a^2 \left| b \right|^2 m^2 \left( E_b + m_p \right)
+ \frac{4}{3} N_c^2 \left| a \right|^2 \left[ E_b \left[ (m_\Delta + m_p)^2 - m_p^2 \right]
\right.
+ 2 \left( E_b E_{\sigma} - \vec{p}_b \cdot \vec{p}_\sigma \right) (m_\Delta + m_p)
\left. + 2 E_{\sigma} \left[ m_p (m_\Delta + m_p) + (E_b E_{\sigma} - \vec{p}_b \cdot \vec{p}_\sigma) \right]
\right.
+ m_p \left[ (m_\Delta + m_p)^2 + m_p^2 \right]
\left. + \frac{8}{3} N_{a,b}^2 (\vec{p}_b \cdot \vec{p}_\sigma) \left[ E_b (m_\Delta + m_p) + m_p E_{\sigma} \right]
\right\}

(9.11)
This form is reasonably convenient for computational purposes, so no attempt has been made to express it in terms of invariants. All momenta and energies are understood to be evaluated in the $\beta(\Delta^+)$ rest-frame.

To calculate the predictions of the model, events were generated with the program FOWL$^{(62)}$ and weighted according to (9.12). The same selections were applied to the Monte-Carlo events as to the data.

No attempt was made to extract an overall cross-section from the model and so the $\Delta p\pi$ coupling constant was set equal to unity. Values of 22.5, 35.0 and 38.7 mb. were used for the total $k^-p$, $\pi^-p$ and $pp$ cross-sections respectively. For the total $k^-\pi^-$ cross-section we used the asymptotic value predicted by the factorization of Regge-pole residues$^{(98)}, (99)$

$$\sigma^T(k^-\pi^-) = \frac{\sigma^T(k^-p) \sigma^T(\pi^-p)}{\sigma^T(pp)}$$

Assuming that the asymptotic $k^-p$, $\pi^-p$ and $pp$ cross-sections are 22, 25, and 37 mb. respectively gives a value for the $k^-\pi^-$ cross-section of 15 mb.

The values for the slopes of the diffractive peaks were taken as follows: $k^-p$, 7.7 GeV$^{-2}$; $\pi^-p$, 7.7 GeV$^{-2}$; $pp$, 8.5 GeV$^{-2}$. In the absence of further information we have set the $k^-\pi^-$ slope equal to 6.0 GeV$^{-2}$.

9.5 PREDICTIONS OF THE ROSS-YAN MODEL

The distributions in the various kinematic quantities of
interest produced by FOWL were smoothed by eye and roughly normalized to the experimental data. The predicted curves are shown in Figs. 9.6 to 9.12 and are labelled I.

We in fact investigated the effects of including in the matrix element a form factor such as that given by (8.5). This might be expected to compensate in some way for our neglect of off-shell effects, but in practice no difference was observed for most of the distributions. The curves shown in Figs. 9.6 to 9.12 have actually been calculated including such a form factor.

The distributions in the $p\pi^+\pi^-$ mass are shown in Figs. 9.7(b) and 9.8(b) for the two channels. Clearly the model absolutely fails to reproduce the low mass spectrum. On the other hand the $p\pi^-$ and $k\pi^-$ mass distributions shown in Figs. 9.7(a) and 9.8(a) are reasonably well fitted.

The four momentum transfer distributions are shown in Figs. 9.9 and 9.10. The model does not predict the peaking at low values of four momentum transfer from proton to $\Delta^{++}$ and the predicted distributions in four momentum transfer from proton to proton and $k^-$ to $k^-$ are rather too broad.

Perhaps the most significant failure of the model is its inability to account for the shape of the distributions in the $\pi^-p$ and $k^-\pi^-$ azimuthal angles ($\phi$). These are shown in Figs. 9.11(a) and 9.12(a). The co-ordinate system used is exactly the same as that defined in Section 8.2. For channel (2a) the y-axis is taken as normal to the plane containing the incident proton and outgoing $\Delta^{++}$ and the z-axis is along the incident $k^-$ direction, in the $k^-\pi^-$ rest system. $\phi$ is then the rotation angle of the outgoing $k^-$ about the incident $k^-$ direction, in the $k^-\pi^-$ rest
frame. Zero azimuthal angle is defined with respect to the x-axis. The angle $\phi$ is similarly defined for the outgoing proton in channel (1a).

The data for both channels shows an accumulation of events at zero azimuthal angle, whereas the model predicts a depletion*. This of course is reflected in the distribution of the $\Delta^{++}\pi^-$ mass.

Also shown in Figs. 9.11 and 9.12 are the distributions in the Toller angle $\psi^{(100)}$. This is defined as the angle between the plane containing the incident and outgoing kaons (or protons) and the plane containing the incident proton and outgoing $\Delta^+$, in the $\pi^-$ rest-frame. Thus with reference to Fig. 9.3,

$$\psi = \cos^{-1} \left( \frac{\hat{a} \cdot \hat{a} \wedge \hat{b} \wedge \hat{\pi}}{| \hat{a} \wedge \hat{a}| \cdot | \hat{b} \wedge \hat{\pi}|} \right) \quad (9.14)$$

in the $\sigma$ rest-frame.

The distributions in this angle are adequately reproduced.

Lastly we look at the predictions of the model for the distributions in the Van Hove angle $\phi$, shown in Fig. 9.6. The calculated curves exhibit a peak at around $120^\circ$, but this is somewhat displaced to the right of the experimental data.

9.6 MULTI-REGGE-MODELS

The original multiperipheral model as proposed by Amati et al. (101) was difficult to calculate and yielded few predictions that could be tested experimentally. The possibility of Reggeising the exchanges was*.

* The calculation of a single diagram in an OPE type model will of course yield a uniform azimuthal angular distribution. Such a distribution can only become non-uniform when two or more diagrams are involved.
considered as early as 1963 by Ter-Martirosyan\textsuperscript{(102)} and by Kibble\textsuperscript{(103)}, and since then it has been developed theoretically by many authors.

Chan, Kajantie and Ranft\textsuperscript{(104)} calculated the amplitude for the three body final state process shown in Fig. 9.13. By performing a partial wave analysis in the t-channel and then crossing to the s-channel they showed that for sufficiently large s the amplitude is given by,

\[
A \sim \beta_a(t_{13}) \beta_b(t_{25}) \beta(t_{13}, t_{25}, w)
\]

\[
\int_a (t_{13}) s_{34} \int_b (t_{25}) s_{45}
\]

where \(\alpha, \beta\) and \(\int\) are respectively trajectory parameters, residue functions and signature factors. The notation for the kinematic invariants, \(s\) and \(t\), is explained in Fig. 9.13.

The function \(\beta(t_{13}, t_{25}, w)\) represents the coupling of the particle 4 to two Regge trajectories. The angle \(w\) is defined in the 4-rest-frame and given by (9.14).

The prescription of Chan et al. for more than three particles in the final state becomes extremely complicated. However, using an alternative method of Reggeisation proposed by Toller\textsuperscript{(100)}, Bali, Chew and Pignotti\textsuperscript{(105)} succeeded in writing an amplitude for the general case. We shall be using their amplitude for the three body channels (1a) and (2a) in Section 9.7.

The range of validity of the multi-Regge model in its original form is limited in that it is only strictly applicable to regions of phase-space where all the invariant sub-energies \((s_{ij})\) are large. Most high energy reactions, however, are dominated by strong resonance production and consequently some of the \(s_{ij}\) will be small for many of the events.
Nevertheless in a later paper Chan and co-workers (107) applied the model to the reactions

\[ \pi^+ p \rightarrow \pi^+ \pi^- p \quad \text{at 8 GeV/c} \]
\[ \pi^- p \rightarrow k_1^0 k_1^0 n \quad \text{at 12 GeV/c} \]

with some success.

There have since been several approaches to overcome this stringent condition on the allowed values of the invariant subenergies.

The first method is due to Chan, Loskiewicz and Allison (106) and is popularly called the CLA model. In this model the structure of low mass clusters is assumed to be governed by phase-space and the amplitude interpolates smoothly between the multi-Regge limit and low energy cluster production. The resonance features of a reaction would not of course be reproduced.

With this model Chan, Loskiewicz and Allison fitted with one set of parameters a large number of single particle distributions for a variety of high energy \( np \) and \( kp \) interactions. Since then the CLA model has been applied to their data by several experimental groups. For example, in a joint paper (108) the 10 GeV/c \( k^- p \) collaboration together with an 8 GeV/c \( \pi^+ p \) and 16 GeV/c \( \pi^- p \) collaboration successfully accounted for many of the features of reactions involving multiple pion production. The work of Bassompierre et al. (109) and Caso et al. (110) was equally successful.

However, Bialas et al. (111) have pointed out that although the model may fit single particle distributions very well, it fails to take account of some of the detailed correlations.

Recently Veneziano (112) has constructed an amplitude for two body reactions which exhibits Regge-pole behaviour in the high energy limit and resonance production features at low energies. This amplitude,
although non-unitary, has certain rather desirable properties. It is analytic, crossing-symmetric and obeys the duality principle (47)(48).

The amplitude has been generalized to processes with three (113) or more (114) particles in the final state, but computational difficulties restrict its range of application to three-bodies. However in a paper (115) published by the 10 GeV/c K⁺p collaboration, a dual diffractive model has been developed for the production of the low mass Υππ system in channels (2) and (3). In this model the coupling of the incoming and outgoing protons to the Pomeron is factored out, and the rest of the amplitude written in terms of a Veneziano-type five-point function (113).

However, the model is not immediately applicable to the diffractive production of the πππ system. The spins of the protons cannot be taken into account and in any case the complexity of the baryon trajectories means that no clear prescription can be formulated.

The alternative to these two methods is to use the implications of duality (48), (49). Although the multi-Regge model is only strictly valid for high invariant masses of the sub-systems, it must in some average sense provide a description of the amplitude at low invariant masses. Some of the more interesting details may be lost with this approach, but nevertheless the main phenomenological features should be reproduced.

9.7 THE DOUBLE-REGGE MODEL

The double-Regge model has been widely applied to quasi-three-body final states with a view to describing the low mass πππ, Kππ or πππ enhancements. For example, the following reactions have been
studied with some success:

\[
\begin{align*}
pp &\rightarrow \Delta^{++} p\pi^- & \text{at } 6.6^{(116)}, 19.0^{(117)}, 28.5^{(118)} \text{ GeV/c} \\
\pi^+ p &\rightarrow \Delta^{++} \pi^+ \pi^- & \text{at } 13.1^{(119)} \text{ GeV/c} \\
\pi^- p &\rightarrow \Delta^{++} \pi^- \pi^- & \text{at } 6.0^{(120)} \text{ GeV/c} \\
k^- p &\rightarrow \Delta^{++} k^- \pi^- & \text{at } 12.6^{(121)} \text{ GeV/c} \\
k^+ p &\rightarrow \Delta^{++} k^+ \pi^- & \text{at } 9.0^{(122)} \text{ GeV/c} \\
k^- p &\rightarrow k^{*0} (890) \pi^- p & \text{at } 7.3^{(123)}, 12.6^{(121)} \text{ GeV/c} \\
k^+ p &\rightarrow k^{*0} (890) \pi^+ p & \text{at } 7.3^{(125)}, 12.7^{(126)} \text{ GeV/c} \\
\pi^+ p &\rightarrow \rho^0 \pi^- p & \text{at } 13.1^{(127)} \text{ GeV/c} \\
\pi^- p &\rightarrow \rho^0 \pi^+ p & \text{at } 13.0^{(128)}, 20.0^{(128)} \text{ GeV/c}
\end{align*}
\]

Some authors\(^{(124),(129),(130)}\) have preferred to analyse the whole four body final state when a \(\Delta^{++}\) \((1236)\) resonance is involved, by using the known on-shell scattering at the \(p\pi^+\) vertex. In the case of channel \((1)\), the \(\pi^- p\) on-shell data could also be used\(^{(124),(129)}\).

All authors agree that the model predicts a low mass enhancement in the diffractively produced three particle system and hence by duality\(^{(48),(49)}\) implies the existence of resonances.

Following Berger\(^{(131)}\) we have applied the double-Regge model of Bali, Chew and Pignotti\(^{(105)}\) to the channels \((1a)\) and \((2a)\). Although the \(\Delta^{++}\) was considered as a stable particle, it was hoped that the main features of the model would be reproduced.

Only the diagram illustrated in Fig. 9.13 was calculated, the contributions from baryon exchange graphs being neglected for the same reasons as those discussed in Section 9.4. Also one has to be careful in adding graphs in a double-Regge model. In a region where two or more graphs can contribute strongly, such an addition can involve double
counting. It is better in these regions to evaluate just one dia-
gram and hope that this will represent all processes in an average
sense.

Using the notation of Fig. 9.13, the amplitude for this
graph, summed and averaged over spins is written,

\[\sum |M|^2 = N_o \left(\frac{\cosh \gamma_b}{s_{o_b}}\right) 2\alpha_b(t_{25}) \phi_b(t_{25}) \phi(t_{13}, t_{25}, \omega)\]

\[\cosh \gamma_a = \frac{2\alpha_a(t_{13})}{s_{o_a}} \phi(t_{13})\]  \hspace{1cm} (9.16)

where \(\gamma_a, \gamma_b\) describe the coupling of trajectories \(\alpha_a, \alpha_b\) to the two
vertices and \(\phi\) describes the coupling of the two Regge trajectories to
the physical pion. The angle \(\omega\) is defined in (9.14).

We have also,

\[\cosh \gamma_b = s_{45} - t_{13} - m_2^2 - (m_4^2 - t_{25}^2 - m_4^2 - t_{13}^2 - t_{25}^2)\]  \hspace{1cm} (9.17)

\[\cosh \gamma_a = s_{34} - t_{25} - m_{1/2}^2 - (m_4^2 - t_{13}^2 - t_{25}^2)\]  \hspace{1cm} (9.18)

where we have used the fact that \(m_1 = m_3\), and following Berger(131)
some kinematic factors in the expressions for \(\cosh \gamma\) have been absorbed
into the \(\phi\) functions. It is interesting to note that for sufficiently
high \(s_{34}, s_{45}\), the Bali, Chew, Pignotti amplitude reduces to the one
of Chan, Kajantie and Ranft(104).

For sufficiently high \(s_{34}\), Pomeron exchange should be the main
contribution at the 13 vertex. Then to couple properly at the middle
vertex, \(\alpha_b\) should have \(\Lambda\)-parity \(-1\); i.e. \(\pi, \Lambda_1, \Lambda_2\). The coupling of a
Pomeron with two pions is favoured by a factor ten to one(131).
Thus $\alpha_a$ is chosen to be the Pomeron and $\alpha_b$ the pion trajectory.

We assume that the Pomeron is a fixed pole with $\phi_p = 1$ and write,

$$\phi_{13}(t) = e^{-at_1}$$

(9.19)

The pion trajectory is assumed to be linear and written,

$$\alpha_\pi(t_{25}) = \alpha_\pi'(t_{25} - m_\pi^2)$$

(9.20)

with $\alpha_\pi' = 1$

Since we have no information on how two Regge trajectories couple to a physical pion, we neglect any $\omega$-dependence of $\phi(t_{13}, t_{25}, \omega)$ and then 'absorb' $\phi$ into the parameterization (9.19) and $N_o$, which is supposed to be a slowly varying function. Finally we set $N_o$ equal to a constant.

$s_o$ and $s_{o'}$ are two scale parameters. It is usual to set $s_o$ equal to unity and vary $s_{o'}$. In practice it was found that the effects of such a variation were very small, so $s_{o'}$ was also set equal to unity.

The coupling at the $25$ vertex is given by,

$$\phi_{25}(t) = \frac{(\alpha_\pi')^2 m_\pi^2}{2(1-\cos(\pi \alpha_\pi))}$$

(9.21)

which expression now incorporates the Reggeised pion propagator, the signature factor and residue function, and any kinematic factors.

To ensure that Pomeron exchange is the dominant contribution at the $13$ vertex, we have imposed the same selections on the data as in Section 9.4.

The slope for $\pi^- p$ diffractive scattering was set to $7.7 \text{ GeV}^2$. 
in (9.19) and that for \( k^-\pi^- \) scattering at 6.0 GeV\(^2\).

The calculation of the effects of the matrix element (9.16) was performed with FOWL\(^{(62)},(63)\) and the resulting histograms smoothed by eye.

9.8 **PREDICTIONS OF THE DOUBLE-REGGE MODEL**

The calculated curves are shown in Figs. 9.6 to 9.12 and are labelled II. They have been roughly normalised to the data.

The \( \Delta^{++}\pi^- \) spectra are shown in Figs. 9.7(b) and 9.8(b) and it can be seen that the model predicts a low mass enhancement at around 1600 MeV with width of order 900MeV. Of course it was not expected that the double peaked structure seen in the data could be reproduced. Nevertheless the model is clearly describing the \( \Delta^{++}\pi^- \) system in some average sense and the agreement with experiment is quite impressive considering the approximations we have introduced. The rather poor normalization is due to the accumulation of events at high \( \Delta^{++}\pi^- \) mass. These events in fact cannot be considered as proceeding via the mechanism illustrated in Fig. 9.13 since they have low \( p\pi^- \) or \( k^-\pi^- \) mass.

In this case Pomeron exchange cannot be considered as the dominant contribution to the \( pp \) or \( k^-k^- \) vertex. (The normalization could have been improved somewhat a posteriori, by increasing the lower limits on the \( p\pi^- \) and \( k^-\pi^- \) mass selections. However, we have chosen not to do this).

The \( p\pi^- \) and \( k^-\pi^- \) mass distributions are shown in Figs. 9.7(a) and 9.8(a). It can be seen that the model predicts an accumulation of events at higher masses. Thus it would appear that the agreement between the data and the predictions of the Ross-Yam model was rather
The four momentum transfer distributions shown in Figs. 9.9 and 9.10 are rather well described. The agreement between theory and experiment for channel (1a) is excellent considering the poor normalization. For channel (2a) the distribution in four-momentum transfer from kaon to kaon is not too well reproduced. However, this could probably be taken care of by increasing the value used for $a$, the slope of the $K^{-}\pi^{-}$ diffraction peak.

The azimuthal angles at the $p\pi^{-}$ and $K^{-}\pi^{-}$ vertices are shown in Figs. 9.11(a) and 9.12(a). In view of the fact that the model correctly describes the low mass $\Delta^{++}\pi^{-}$ spectrum, it is not surprising that these distributions are very well described. The model also correctly accounts for the peaking in the Toller angle $\omega$, shown in Figs. 9.11(b) and 9.12(b). It will be remembered that the Ross-Yam model also predicted the peaking in these distributions, so this variable cannot be regarded as a very sensitive test.

Lastly we consider the distributions in the Van Hove angle shown in Fig. 9.6. Again these distributions are not particularly well described, although the agreement is somewhat better in channel (2a).

In the paper(108) on the CLA model published by the 10 GeV/c $K^{-}p$ collaboration, it was demonstrated that a considerable amount of baryon exchange (about 56%) was needed in order to correctly reproduce the distribution for channel (2a). (Here, however, no selections were imposed on the data, other than the $\Delta^{++}$ mass-selection). Hence it may be that a certain amount of baryon exchange is needed in the double-Regge model to account for the Van Hove angular distributions.
FIG. 9.1
FIG. 9.2
FIG. 9.3
\[ K^- p \rightarrow p K^- \pi^+ \pi^- \]  \[ \Delta^{++} \text{ Selected} \]
\[ \Delta^2(p, \Delta^{++}) < 1.0 \text{ GeV}^2 \]
\[ M(K^-\pi^-) > 1.3 \text{ GeV} \]

874 events

**FIG. 9.4**
\[ K^- p \rightarrow \Delta^{++} K^- \pi^- \]

\[ (p p \rightarrow \Delta^{++} p \pi^-) \]
(a) $pp \rightarrow \Delta^{++} p \pi^-$

1151 events

(b) $K^- p \rightarrow \Delta^{++} K^- \pi^-$

874 events

FIG. 9.6
$pp \rightarrow \Delta^{++}p\pi^-$

FIG. 9.7

1151 events
$K^- p \rightarrow \Delta^{++} K^- \pi^-$

874 events

FIG. 9.8

(a) $M(K^-\pi^-)$ GeV

(b) $M(\Delta^{++}\pi^-)$ GeV
pp → Δ⁺⁺pπ⁻  

1151 events

FIG. 9.9
$K^- p \rightarrow \Delta^{++} K^- \pi^-$

874 events

FIG. 9.10
pp → Δ⁺⁺γ −

FIG. 9.11
$K^- p \rightarrow \Delta^{++} K^- \pi^-$  

874 events

**FIG. 9.12**
DOUBLE REGGE DIAGRAM

FIG. 9.13
CHAPTER 10

RESONANCE FEATURES OF THE $p\pi^+\pi^-$ SYSTEM

10.1 INTRODUCTION

In this Chapter we investigate the properties of the two enhancements in the low mass $p\pi^+\pi^-$ system produced in channels (1) and (2). It is not of course claimed that these enhancements correspond to definite resonance states with unique spin and parity. Indeed as we shall see two or more isobars could be contributing in each region. Thus any properties of these enhancements which may be referred to must be regarded as some sort of average over all contributions.

10.2 BRANCHING RATIOS

The identification of the decay modes of these resonances is not clear. Because the 1470 MeV enhancement is just above the $\Delta^{++}(1236)\pi^-$ threshold, it is extremely difficult to separate this mode from the uncorrelated $p\pi^+\pi^-$ decay. There is also some confusion in the 1700 MeV region. Little or no evidence for any $\Delta^{++}\pi^-$ decay has been seen in some experiments (132), while others (133) put the branching fraction at around 70%. Plano (6) has pointed out that groups with beam momentum less than 10 GeV/c are mainly consistent with no $\Delta^{++}\pi^-$ decay, whereas those with beam momentum above 10 GeV/c see a substantial branching ratio.

In Figs. 6.13(a) and (b) are plotted the $p\pi^+\pi^-$ mass distributions of channels (1) and (2) where the mass of the $p\pi^+$ is required to lie in the $\Delta^{++}$ mass region defined by,
$1.15 < M(pn^+) < 1.30$ GeV

These two distributions have been fitted with the sum of two Breit-Wigner functions and a hand drawn background. By comparing the numbers of events above background obtained from these fits with the results of the fits to the unselected spectra (see Section 5.4), we may estimate the branching ratios,

$$r = \frac{N^+_{*} \to \Delta^{++} \pi^-}{N^+_{*} \to \text{all three body modes}} \quad (10.1)$$

For the 1700 MeV peak we find $r = 32 \pm 15\%$ and $41 \pm 13\%$ respectively for the two channels. The branching ratios obtained for the 1400 MeV enhancement are consistent with 100% in both cases. However as we have already pointed out, events in the 1400 MeV region are nearly all kinematically constrained to have the pn$^+$ mass inside the $\Delta^{++}$ mass band. Hence this figure of 100% may not have much significance in implying any intermediate $\Delta^{++} \pi^-$ state.

The analysis has been extended slightly for the 1700 MeV enhancement using the technique of Willmann et al. (133). The available phase-space for the 1700 MeV 'resonance' with the $\Delta^{++}$ mass selection imposed, has been calculated with and without the assumption that the dominant decay is $\Delta^{++} \pi^-$. 

Defining

$$F_{ps} = \int \frac{d\delta}{\Delta} \quad (10.2)$$

and

$$F_{\Delta} = \int \frac{|M|^2 \ d\delta}{\int |M|^2 \ d\delta} \quad (10.3)$$
Then,
\[ \frac{N_A}{N} = (1-r) F_{ps} + r F_A \]  
(10.4)

where \( \int_{\Delta} d\phi \) is understood to mean integration over the region of phase-space defined by the \( \Delta^{++} \) mass-selection and \( |M|^2 \) is a p-wave Breit-Wigner function for the decay \( \Delta^{++} \rightarrow p\pi^+ \). \( N \) and \( N_A \) represent the observed numbers of events above background in the 1700 MeV peaks before and after the \( \Delta^{++} \) mass selection is imposed. These were taken from the results of the analysis described above.

We obtain values for the branching ratio of \( r = 30 \pm 15\% \) in channel (1) and \( r = 25 \pm 20\% \) in channel (2). With this method Willmann et al. \(^{(133)} \) quote a figure of \( 70 \pm 15\% \) for data from a \( \pi^+p \) experiment at 13.1 GeV/c.

We have also attempted to analyse the \( p\pi^+\pi^- \) Dalitz plot using a method proposed by Barnes et al. \(^{(132)} \). The total \( p\pi^+\pi^- \) mass distribution in each channel was divided into intervals of 75 MeV and each bin analysed independently.

The amplitude for the decay of the \( p\pi^+\pi^- \) system is written,
\[ |M|^2 = x \frac{|A_{\Delta}|^2}{\sum_i |A_{\Delta}^i|^2 \int d\phi^i} + \frac{1-x}{\sum_i \int d\phi^i} \]  
(10.5)

where the summations are over the events in a given mass interval and \( x \) is the fraction of events decaying via \( \Delta^{++}\pi^- \). The integrations are over the Dalitz plot for the \( i \)th event.

The matrix element for the subsequent \( \Delta^{++} \) decay is written,
\[ |A_{\Delta}|^2 = |F_{BW}|^2 \frac{1}{4} \left\{ (1+b) + 3(1-b) \cos^2 \phi \right\} \]  
(10.6)

where \( |F_{BW}|^2 \) is a p-wave relativistic Breit-Wigner function and \( \phi \) is the
angle between the outgoing proton and the \( \Delta^{++} \) direction in the \( \Delta^{++} \) rest frame.

The parameter \( b \) is related to the ratio of the square of the helicity amplitudes \( |g_{3/2}|^2 \) and \( |g_2|^2 \). We have,

\[
\frac{b}{2-b} = \frac{|g_{3/2}|^2}{|g_2|^2}
\]

where

\[
|g_{3/2}|^2 + |g_2|^2 = \frac{1}{2}
\]

If the helicity \( \frac{1}{2} \) amplitude dominates the decay, then the expression in brackets in (10.6) reduces to the usual form, \( 1+3 \cos^2 \phi \).

We have neglected any possible decay into \( \Delta^0 \pi^+ \), which is suppressed by a factor 9 to 1, assuming that the low mass \( \pi^+ \pi^- \) system has \( I = \frac{1}{2} \), consistent with a diffractive production mechanism.

The fitting was performed with the maximum likelihood technique, where the likelihood function for the \( k \) th event is written,

\[
L_k = \left( \frac{|M_k|^2}{\int |M|^2 \, d\phi} \right)
\]

The function to be maximised is then \( \sum_k \ln L_k \). The allowed ranges of the parameters \( x \) and \( b \) are,

\[
0 \leq x \leq 1
\]
\[
0 \leq b \leq 2
\]

The numbers of \( \Delta^{++} \) and non-\( \Delta^{++} \) events in each mass bin, estimated from the parameter \( x \), are shown in Figs. 10.1 and 10.2 for channels (1) and (2) respectively. Clear peaks above background can be seen in the region of 1700 MeV for both the \( \Delta^{++} \) and the uncorrelated \( \pi^+ \pi^- \) decays. It is interesting to note however, that in the case of
channel (1) the peaks occur in different mass bins. For events decaying into $\Delta^{++}\pi^-$ the peak lies in the range 1750 - 1825 MeV, while for the background events it is in the 1675 - 1750 MeV mass bin. In fact the fit to the spectrum obtained by requiring the $p\pi^+$ combination to satisfy the $\Delta^{++}$ mass selection yielded a somewhat higher mass and smaller width than the fit to the unselected spectrum. The values actually obtained are,

$$M = 1750 \pm 10 \text{ MeV}, \quad \Gamma = 110 \pm 25 \text{ MeV}.$$ 

These should be compared with the values listed in Table 5.4.

In channel (2) the results of the fits to the two spectra are more compatible. The values obtained for those events satisfying the $\Delta^{++}$ mass selection are,

$$M = 1708 \pm 12 \text{ MeV}, \quad \Gamma = 78 \pm 20 \text{ MeV}.$$ 

Unfortunately the uncertainty in the background shape renders any conclusions highly speculative.

Using hand drawn background estimates in Figs. 10.1 and 10.2 and assuming that the peaks observed in diagrams (a) and (b) represent the same effect, we obtain figures for the branching ratio of the 1700 MeV enhancement of $r = 35 \pm 15\%$ in channel (1) and $r = 33 \pm 20\%$ in channel (2).

We saw in Section 6.4 that there was a possibility that the 1700 MeV enhancement could be understood in terms of a Deck(83) type mechanism where the $\pi^+$ is diffractively scattered off the incident $p$ or $k^-$ and forms a low mass system with an $N^*(1470)$. This is completely analogous to the more usual Deck graph for the production of the 1470 enhancement where a diffractively scattered $\pi^-$ combines with the $\Delta^{++}$. 
If this hypothesis is true it could explain the different branching ratios obtained for the 1700 MeV enhancement at different beam energies as due to varying interference effects between the $\Delta^{++}$ and $N^{*0}_{(1470)}$ decay modes\(^{(6)}\).

However, in the same way that the limited available phase-space precludes a determination of the $\Delta^{++} \pi^-$ branching ratio of the 1470 enhancement, it is extremely difficult to sort out these effects in the 1700 MeV region.

Nevertheless we have attempted a simple minded analysis of $p\pi^+\pi^-$ Dalitz plots in the two channels as a function of the mass of the $p\pi^+\pi^-$ system.

The likelihood for the $k$th event is written as \((10.9)\) where
\[
|M|^2 = \frac{x_1 |A_\Delta|^2}{\sum_i |A_\Delta|^2 \, \, d\theta^1} + \frac{x_2 |A_{1470}|^2}{\sum_i |A_{1470}|^2 \, \, d\theta^1} + \frac{1 - x_1 - x_2}{\sum_i |A_{1470}|^2 \, \, d\theta^1}
\]
and $x_1, x_2$ are the percentages of $\Delta^{++}$ and $N^{*0}_{(1470)}$ events respectively. The matrix elements for the subsequent $\Delta^{++}$ and $N^{*0}_{(1470)}$ decays are written as Breit-Wigner functions with no decay angular distribution factors. As before a $p$-wave relativistic Breit-Wigner is used for $|A_\Delta|^2$, but for $|A_{1470}|^2$ we use an $s$-wave relativistic Breit-Wigner function.

The grand likelihood function $\sum \ln L_1$ was then maximised with respect to $x_1$ and $x_2$.

The results of the fits are displayed in Figs. 10.3 for chan-
nels (1) and (2) respectively. Clearly very little $\Delta^{++}\pi^-$ decay mode is now required in the 1700 MeV region, but a substantial branching fraction into $N^{*0}$ (1470) $\pi^+$ is predicted. However, we do not attach too much significance to these results, since in the 1470 MeV region they differ considerably from the values predicted by the first fit described (Figs 10.1 and 10.2). This only serves to emphasise the difficulties involved in separating decay modes of a resonance just above threshold.

10.3 ANGULAR DISTRIBUTIONS

We have already seen in Section 6.4 that for channel (1) the resonance features of the interaction are shown most clearly by considering only the more peripheral of the two $p^+\pi^-$ combinations. The following analysis will therefore be concerned with that combination. In the case of channel (2) we impose no selections on the data.

The angular distribution of the normal to the $p^+\pi^-$ decay plane, in the $p^+\pi^-$ rest system, has been expanded in a series of spherical harmonics, using the usual Gottfried-Jackson (134) set of axes.

We write,

$$I(\phi, \theta) = \sum_{l,m} A_{l,m} Y^m_l (\phi, \theta)$$

(10.12)

where the $z$-axis is defined as the associated incoming proton direction and the $y$-axis is the normal to the plane containing the other proton or $k^-$ and its associated incoming particle. The angles $\theta$ and $\phi$ are then the polar and azimuthal angles of the normal to the $p^+\pi^-$ plane.
The mass spectrum of each channel was divided into intervals of 100 MeV and the coefficients $A_{lm}$ evaluated in each bin.

Now under reflection in the x-z plane (the 'production plane') the spherical harmonics transform as,

$$\text{Re} \gamma^m_l (\cos \theta, \phi) - \text{Re} \gamma^m_l (\cos \theta, -\phi) = \text{Re} \gamma^m_l (\cos \theta, \phi)$$

Hence if parity is conserved in the production process all the imaginary moments must be zero. We find this to be the case within the experimental errors. We also in fact find that all the real moments with $m \neq 0$ are consistent with zero. This constitutes evidence for spin zero exchange.

We show in Figs. 10.5 and 10.6 the moments for the two channels with $m=0$, as a function of $p^+\pi^-$ mass. Only the moments up to $l=6$ are shown, those of higher order being consistent with zero over the mass range considered.

The $l=2$ moment is negative in all $p^+\pi^-$ mass intervals. Hence even in the 1400 MeV region there is some evidence that spin $J \geq 3/2$ is contributing, but above 1700 MeV this is certainly required. In the case of channel (1) all other moments appear to be small and consistent with zero. However, there is some suggestion that the $l=4$ moment of channel (2) may be becoming significant above 1700 MeV. This would require some spin $J \geq 5/2$.

The Birmingham-Glasgow-Oxford collaboration\(^{(135)}\) have pointed out that more information is to be gained by looking at the decay angular distributions in a co-ordinate system originally devised for single pion production reactions at low energy\(^{(136)}\). In the $p^+\pi^-$
centre-of-mass, the z-axis is taken as normal to the decay plane and the y-axis is along the π− direction in this plane. The incoming proton associated with the p+n system then defines the polar and azimuthal angles.

The mass spectra have again been divided into intervals of 100 MeV and the decay angular distributions for each bin expanded in a series of spherical harmonics. The m = 0 moments are of course the same as in the Gottfried-Jackson(134) system, since the polar angles, θ, are identical. We show in Figs. 10.7 and 10.8 several of the moments with m ≠ 0 for the two channels. Those not shown are consistent with zero over the whole mass range.

Under reflection in the decay plane, the spherical harmonics transform as,

$$Y_{l}^{m} (\cos \theta, \phi) \rightarrow (-)^{\ell+m} Y_{l}^{m} (\cos \theta, \phi)$$

Hence since we believe that the decay of the p+n system is parity conserving, the fact that all moments with (\ell+m) odd are consistent with zero is evidence for no interference between exchanged particles of opposite parity.

Above 1800 MeV the real and imaginary parts of \(Y_{4}^{1}\) depart significantly from zero, thus requiring two waves of opposite parity. However, below this mass the results are not so clear cut. There is good evidence for believing that the imaginary \(Y_{4}^{1}\) moment is non-zero for channel (2), but not for channel (1). Similarly the real \(Y_{3}^{1}\) moments differ for the two channels. In channel (2) they are significant above 1700 MeV, but consistent with zero in the case of channel (1) (not shown).

Moments with \(\ell = 4\) and \(m \neq 0\) are certainly not required below
1800 MeV in either channel.

10.4 SPIN ANALYSIS USING DECAY PLANE NORMAL

We have attempted to determine the spin of the enhancements at around 1400 and 1700 MeV, assuming that these are definite resonance states. Possible interference effects have been neglected in the following analysis. We have also made no attempt to correct for any 'background' under the resonances. This is consistent with the duality picture(47), (48).

The two mass regions of interest have been defined by the following selections:

'1400' region: 1.35 < M (pπ⁺π⁻) < 1.55 GeV.

'1700' region: 1.60 < M (pπ⁺π⁻) < 1.80 GeV.

For each region we have looked at the polar angular distributions of the normal to the decay plane in the pπ⁺π⁻ centre-of-mass. The z-axis is again the associated initial proton direction.

Following Berman and Jacob (137), the decay distribution of an object of spin J can be written,

\[
I(\phi) = 2\pi \sum_{M=0}^{J} \sum_{m=0}^{J} \rho_{mm} \sum_{J=0}^{J+} (\phi) R_{M}^{+} \tag{10.15}
\]

where \(\phi\) is the polar angle and the \(R_{M}^{+}\) are related to the phenomenological decay amplitudes, \(F_{M}^{+}\), summed over final state proton helicities and integrated over the decay Dalitz plot.

\[\rho_{mm}\] is the spin density matrix and the \(Z\) functions are defined by,

\[
Z_{mm}^{JM+} (\phi) = [d_{mM}^{J} (\phi)]^{2} + [d_{m-M}^{J} (\phi)]^{2} \tag{10.16}
\]

where the d's are the usual rotation matrices.
It should be noted that the significance of this analysis is
not affected by any possible Δ⁺⁺π⁻ decay mode.

The folded polar angular distributions in the decay normal
for the 1400 MeV region are shown in Figs. 10.9(a) and 10.10(a) for
channels (1) and (2) respectively. These distributions have been
fitted to the theoretical expressions for spin \( J = \frac{3}{2} \) and \( J = 3/2 \).

From (10.15) we have,

\[ \begin{align*}
J = \frac{3}{2}: & \quad I(\phi) = \text{constant} \\
J = 3/2: & \quad I(\phi) = \frac{1}{4} \left[ p_{11} + G(\frac{3}{2} - p_{11}) \right] (1 + 3 \cos^2 \phi) \\
& \quad + \frac{3}{4} \left[ G p_{11} - (\frac{3}{2} - p_{11}) \right] (1 - \cos^2 \phi)
\end{align*} \]  

where \( G = \frac{R_{3/2}^+}{R_{5/2}^+} \)  

The fitting was performed with the maximum likelihood technique
and the chi-squared for the best fit evaluated.

The results for the 1400 MeV region are summarised in Table
10.1(a) and the predicted curves shown in Figs. 10.9(a) and 10.10(a).

Clearly the uniform distribution required if \( J = \frac{3}{2} \) is perfectly adequate
to describe our data, although the chi-squared probability in the case
of channel (2) is rather low. However, there is no good reason for
preferring spin 3/2 over \( \frac{3}{2} \).

We next consider the 1700 MeV enhancement. Evidently from the
moments analysis of Section 10.3, spin \( J \geq \frac{3}{2} \) is required, although of
course there may be some spin \( J = \frac{3}{2} \) contributing.

The folded polar angular distributions for events in the
1700 MeV regions of the two channels are shown in Figs. 10.9(b) and
10.10(b). Fits have been performed assuming \( J = 3/2, 5/2 \) and \( 7/2 \).
Following Rhode et al. (138), we have imposed the values $\rho_{mm} = 0$ for $m \geq 3/2$ and $J \geq 5/2$. This serves to reduce the number of free parameters in the fits and is justified on two grounds.

Firstly we have shown that the production of the 1700 MeV enhancement is consistent with spin zero exchange and thus $\rho_{11}$ is expected to be relatively large.

Secondly, since the low mass $\pi^+\pi^-$ system is produced extremely peripherally, the $\pi^+\pi^-$ direction lies very close to the incident direction (the quantization axis). Thus the maximum spin projections are $+3/2$ and the $+1/2$ for channels (1) and (2) respectively.

With the conditions imposed, the theoretical expressions for spin 5/2 and 7/2 are,

$$J = 5/2; I(\phi) = (10 \cos^4 \phi - 4 \cos^2 \phi + 2) + G(-15 \cos^4 \phi + 14 \cos^2 \phi + 1) + G'(5 \cos^4 \phi - 10 \cos^2 \phi + 5)$$ (10.20)

where $G' = \frac{R^+_{5/2}}{R^+_{1/2}}$ (10.21)

$$J = 7/2; I(\phi) = (175 \cos^6 \phi - 165 \cos^4 \phi + 45 \cos^2 \phi + 9) + 15G \sin^2 \phi(21 \cos^4 \phi - 6 \cos^2 \phi + 1) + 5G' \sin^4 \phi(35 \cos^2 \phi + 1) + 35G'' \sin^6 \phi$$ (10.22)

where $G' = \frac{R^+_{7/2}}{R^+_{1/2}}$ (10.23)

A summary of the fits is given in Table 10.1(b) and the resulting curves shown in Figs. 10.9(b) and 10.10(b). All three spin assignments provide acceptable fits to the data of both channels.
10.5 **SPIN ANALYSIS OF SEQUENTIAL DECAY DISTRIBUTIONS**

We next turn to a discussion of the sequential decay process, 

\[ N^+ \rightarrow \Delta^{++} \pi^- \rightarrow p \pi^+ \]

Analysis of the angular distribution of such a double decay provides a more powerful means of determining the spin of the \( p \pi^+ \pi^- \) system. These distributions are based upon the four angles, \( \theta, \phi \) and \( \theta', \phi' \). \( \theta \) and \( \phi \) are the polar and azimuthal angles of the \( \Delta^{++} \) momentum in the Gottfried-Jackson frame (as defined in Section 10.3). \( \theta' \) and \( \phi' \) are the polar and azimuthal angles of the decay proton in the \( \Delta^{++} \) rest-frame. These angles are defined by taking for \( z' \)-axis the \( \Delta^{++} \) direction in the \( p \pi^+ \pi^- \) rest system and for the \( y' \)-axis, the normal to the plane containing the \( z \) and \( z' \) axes.

We show in Fig. 10.11 the polar angular distributions of the \( \Delta^{++} \) for the 1400 and 1700 MeV regions in channel (1). Events were chosen according to the usual \( \Delta^{++} \) mass selection, 

\[ 1.15 < M(p\pi^+) < 1.30 \text{ GeV}. \]

A strong asymmetry is present in these distributions, corresponding to the \( \Delta^{++} \) travelling predominantly in the incident proton direction. Such an asymmetry is inconsistent with the decay of a pure \( p \pi^+ \pi^- \) state.

We saw previously that by rejecting what were considered as 'wrong' \( p \pi^+ \pi^- \) combinations on a peripheral basis, the low mass structure was virtually unaffected. If we use the same basis for rejecting 'wrong' events from Fig. 10.11, the resulting distributions become more symmetric. These are shown as dashed histograms in Fig. 10.11 and only those events contributing to these symmetric distributions are used in the following analysis.
In the case of channel (2) no selection other than that on the $p\pi^+$ mass was found to be necessary.

We have again performed a moments analysis for the decay $N^*+ - \Delta^{++}\pi^-$. The moments for channels (1) and (2) are shown as a function of $p\pi^+\pi^-$ mass in Figs. 10.12 and 10.13 respectively. Only the $m = 0$ moments are shown, all others being consistent with zero. It will be seen that there are significant contributions from a greater number of the $Y^0_0$ than was the case in Section 10.3, when we examined the distribution of the decay normal.

Both the $\ell = 2$ and $\ell = 4$ moments are significant above 1700 MeV. The $\ell = 6$ moment is hardly present, but as Willmann et al. have pointed out, if $J^P = 7/2^-$ and only the orbital angular momentum wave with $\ell = 2$ contributes to the decay, then the coefficient of $Y^0_6$ may vanish. The $\ell = 1$ and $\ell = 3$ moments contribute above 1700 MeV, but more significantly in channel (1). This suggests that interference between different angular momentum states may be present, but in the following analysis this will be neglected.

The theoretical expressions for the distributions in $e$ and $e'$ for spin $J$ are

\begin{align}
I(e) &= \sum_{\lambda' > 0} \sum_{m > 0} \rho_{\lambda \lambda'} \sum_{m = -\lambda}^{\lambda} |F_{\lambda}|^2 \\
I(e') &= \sum_{m > 0} \rho_{\frac{3}{2} \frac{3}{2}} \sum_{m = -\frac{3}{2}}^{\frac{3}{2}} |F_{\frac{3}{2}}|^2
\end{align}

(10.24, 10.25)

If $J = \frac{3}{2}$, then there is only one term in the expression for $I(e)$ corresponding to $\lambda = \frac{3}{2}$, $m = \frac{3}{2}$ and only one term in $I(e')$, with $m = -\frac{1}{2}$.
The density matrix for the $\Delta^{++}$ is understood to be integrated over any possible $\phi$ dependence, i.e.

$$\rho_{mm}^{\prime} = \int \rho_{mm}^{\prime} (\phi) \, d(\cos \phi)$$

(10.26)

$F_\Lambda$ is the helicity amplitude for the initial decay, $N^{*+} \rightarrow \Delta^{++}\pi$.

We can rewrite (10.24) as,

$$I(\phi) = \sum_{mm} [R_{mm} \int J^{3/2+} (\phi) + \rho_{mm} \int J^{1/2+} (\phi)]$$

(10.27)

where

$$R = \frac{|F_{J}^{J}|^2}{|F_{J/2}^{J}|^2} = \frac{\rho_{11}^{*}}{3 - \rho_{11}^{*}}$$

(10.28)

The folded polar angular distributions in $\phi$ and $\phi'$ for the 1400 MeV regions are shown in Figs. 10.14 and 10.15 for channels (1) and (2) respectively. These distributions have been fitted jointly with the theoretical expressions for spin $J = \frac{1}{2}$ and $J = \frac{3}{2}$. These are,

$$J = \frac{1}{2}; \quad I(\phi) = \text{constant}$$

(10.29)

and

$$J = \frac{3}{2}; \quad I(\phi) = 1 + 3 \cos^2 \phi$$

(10.30)

$$J = 3/2; \quad I(\phi) = (\frac{1}{2} - \rho_{11}^{*})[3 + 3 \cos^2 \phi] + 3R(1 - \cos^2 \phi) + \rho_{11}^{*}[3(1 - \cos^2 \phi) + R(1 + 3 \cos^2 \phi)]$$

(10.31)

and

$$I(\phi') = (3/4 - \rho_{11}^{*}) + 3(\rho_{11}^{*} - 1/4) \cos^2 \phi'$$

(10.32)

Table 10.2 summarises the results of the joint fits. The chi-squared probability for the fit to both the distributions is given under (c), while the individual chi-squared probabilities are listed under (a) and (b) for the distributions in $\phi$ and $\phi'$ respectively.
The curves predicted by the fits are shown in Figs. 10.14 and 10.15.

For \( J = \frac{1}{2} \), the parent decay, \( N^* \to \Delta^{++} \pi^- \), is required to be uniform in \( \cos \theta \) and the data from both channels is perfectly consistent with this. However, the distribution for the daughter decay should have the form \( 1 + 3 \cos^2 \theta' \), and the fits are clearly very poor. If we assume \( J = \frac{3}{2} \) then the results of the joint fits are acceptable.

These results are in agreement with those of Rhode et al.\(^{(138)}\) who also found that the \( J = \frac{1}{2} \) assignment was not consistent with their data. Evidently if the 1400 MeV enhancement has \( J = \frac{1}{2} \), then \( \Delta^{++} \pi^- \) cannot be the principal decay mode.

We now consider the sequential decay distributions for events in the 1700 MeV region. These are shown in Figs. 10.16 and 10.17 for the two channels.

Joint fits to these distributions have been performed assuming spin \( J = 3/2, 5/2 \) and \( 7/2 \), where again we have imposed the condition \( \rho_{mn} = 0 \) for \( m \geq 5/2 \) and \( J \geq 5/2 \). The theoretical expressions for spin \( J = 5/2 \) and \( 7/2 \) are:

\[
J = 5/2: I(\theta) = (\frac{1}{2} - \rho_{11})\left[ R\left(-15\cos^4 \theta + 14\cos^2 \theta + 1\right) \right. \\
+ \left. \frac{1}{2}(45\cos^4 \theta - 38\cos^2 \theta + 9) \right] \\
+ \rho_{11}\left[ R\left(10\cos^2 \theta - 4\cos \theta + 2\right) \right. \\
+ \left. (-15\cos^4 \theta + 14\cos^2 \theta + 1) \right] \\
(10.33)
\]

\[
J = 7/2: I(\theta) = (\frac{1}{2} - \rho_{11})\left[ 15\sin^2 \theta \cos^4 \theta - 21\cos^2 \theta - 6\cos \theta + 1 \right] \\
+ (567\cos^6 \theta - 805\cos^4 \theta + 261\cos^2 \theta + 9) \\
+ \rho_{11}\left[ R\left(175\cos^6 \theta - 165\cos^4 \theta + 45\cos^2 \theta + 9\right) \right]
\]
The distributions for \( \alpha' \) are the same as (10.32).

The results are presented in Table 10.3 and these show that
spin \( J = 5/2 \) or 7/2 is preferred in both channels, but that spin
\( J = 3/2 \) cannot be ruled out. The curves resulting from the fits
are shown in Figs. 10.16 and 10.17.

10.6 PARITY OF THE ENHANCEMENTS

We have seen that for events in the 1400 MeV regions of both
canals, reasonable fits to the data are provided by the \( J = \frac{3}{2} \)
assignment, assuming an uncorrelated \( p\pi^+\pi^- \) decay mode. One is there-
fore tempted to associate this enhancement with the \( P_{1/2}(1470) \), \( J^P = \frac{3}{2}^+ \), seen in the phase-shift analyses(5). However, if the dominant
decay mode is \( A^{++}\pi^- \) then the data does not support the \( J = \frac{3}{2} \) assign-
ment, but good fits are obtained for \( J = 3/2 \).

Using results from Jacob and Wick(139), we can write a formal
relationship between the helicity amplitudes \( F_\lambda \), and the LS coupling
amplitudes \( B_L \), for the decay of an object of spin \( J \) into a spin-3/2
particle and spin-\( \frac{1}{2} \) particle:

\[
F_\lambda = \sum_L B_L \sqrt{\frac{2L+1}{2J+1}} C(L, 3/2, J; 0, \lambda) \quad (10.35)
\]

Now if we assume that the spin is 3/2 and that the decay
occurs via only the lower of the two possible orbital angular momentum
states for a definite parity, then the values of \( R \) are predicted to be
\( 2/9 \) and \( 1 \) for \( J^P = 3/2^+ \) and \( 3/2^- \) respectively. We obtain values of
\( R \) (see Table 10.2) of \( 1.2 \pm 0.2 \) at 16 GeV/c and \( 1.1 \pm 0.2 \) at 10 GeV/c,
so we prefer the \( J^P = 3/2^- \) assignment.
The assumption that the lower angular momentum state dominates should be quite good because of the restricted phase-space available for the decay. Hence if $J^p = 3/2^-$, $\ell = 0$ should predominate in this region. However, the moments analysis for channel (1) (see Fig. 10.12) shows that the $\ell = 1$ moment may be significant close to threshold.

We can similarly apply this type of analysis to the 1700 MeV region. Assuming that the lower angular momentum wave dominates the decay, we can compare the predicted values of $R$ for the various spin-parity assignments, with the experimental results. A summary is presented in Table 10.4, and it can be seen that the spin-parity series $3/2^-$, $5/2^+$, $7/2^-$ is preferred. This is in agreement with the Gribov-Morrison Parity Rule \cite{45} for diffractively produced $p\pi^+\pi^-$ systems.
### Table 10.1
FITS TO DECAY PLANE NORMAL

(a) 1400 MeV region

<table>
<thead>
<tr>
<th>J</th>
<th>No. free parameters</th>
<th>At 16 GeV/c</th>
<th>At 10 GeV/c</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\rho_{11}$</td>
<td>$x^2$-prob. (%)</td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>51.5</td>
</tr>
<tr>
<td>$3/2$</td>
<td>2</td>
<td>$0.49 \pm 0.06$</td>
<td>70.4</td>
</tr>
</tbody>
</table>

(b) 1700 MeV region

<table>
<thead>
<tr>
<th>J</th>
<th>No. free parameters</th>
<th>At 16 GeV/c</th>
<th>At 10 GeV/c</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\rho_{11}$</td>
<td>$x^2$-prob. (%)</td>
</tr>
<tr>
<td>$3/2$</td>
<td>2</td>
<td>$0.5 \pm 0.06$</td>
<td>16.5</td>
</tr>
<tr>
<td>$5/2$</td>
<td>2</td>
<td>$\frac{1}{2}$</td>
<td>11.0</td>
</tr>
<tr>
<td>$7/2$</td>
<td>3</td>
<td>$\frac{1}{2}$</td>
<td>19.4</td>
</tr>
</tbody>
</table>
Table 10.2

JOINT FITS TO SEQUENTIAL DECAY: 1400 MeV REGION

(a) At 16 GeV/c

<table>
<thead>
<tr>
<th>J</th>
<th>$x^2$-prob. (%) (a)</th>
<th>$x^2$-prob. (%) (b)</th>
<th>$x^2$-prob. (%) (c)</th>
<th>$\rho_{11}$</th>
<th>$\rho_{11}'$</th>
<th>R</th>
<th>No. free parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>32.8</td>
<td>0.3x10^{-6}</td>
<td>0.9x10^{-5}</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\infty$</td>
<td>0</td>
</tr>
<tr>
<td>3/2</td>
<td>24.9</td>
<td>18.7</td>
<td>16.0</td>
<td>0.36$^+$</td>
<td>0.27$^+$</td>
<td>1.2$^+$</td>
<td>2</td>
</tr>
</tbody>
</table>

(b) At 10 GeV/c

<table>
<thead>
<tr>
<th>J</th>
<th>$x^2$-prob. (%) (a)</th>
<th>$x^2$-prob. (%) (b)</th>
<th>$x^2$-prob. (%) (c)</th>
<th>$\rho_{11}$</th>
<th>$\rho_{11}'$</th>
<th>R</th>
<th>No. free parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>73.5</td>
<td>0.5x10^{-7}</td>
<td>0.9x10^{-5}</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\infty$</td>
<td>0</td>
</tr>
<tr>
<td>3/2</td>
<td>71.6</td>
<td>51.8</td>
<td>70.5</td>
<td>0.42$^+$</td>
<td>0.26$^+$</td>
<td>1.1$^+$</td>
<td>2</td>
</tr>
</tbody>
</table>
Table 10.3
JOINT FITS TO SEQUENTIAL DECAY: 1700 MeV REGION

(a) At 16 GeV/c

<table>
<thead>
<tr>
<th>J</th>
<th>$x^2$-prob. (%) (a)</th>
<th>$x^2$-prob. (%) (b)</th>
<th>$x^2$-prob. (%) (c)</th>
<th>$\rho_{11}$</th>
<th>$\rho_{11}'$</th>
<th>$R$</th>
<th>No. free parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/2</td>
<td>16.9</td>
<td>30.8</td>
<td>17.7</td>
<td>0.50</td>
<td>-0.02</td>
<td>1.6</td>
<td>2</td>
</tr>
<tr>
<td>5/2</td>
<td>53.7</td>
<td>80.2</td>
<td>77.4</td>
<td>0.39</td>
<td>-0.02</td>
<td>1.3</td>
<td>2</td>
</tr>
<tr>
<td>7/2</td>
<td>79.2</td>
<td>81.9</td>
<td>91.1</td>
<td>0.36</td>
<td>-0.07</td>
<td>1.2</td>
<td>2</td>
</tr>
</tbody>
</table>

(b) At 10 GeV/c

<table>
<thead>
<tr>
<th>J</th>
<th>$x^2$-prob. (%) (a)</th>
<th>$x^2$-prob. (%) (b)</th>
<th>$x^2$-prob. (%) (c)</th>
<th>$\rho_{11}$</th>
<th>$\rho_{11}'$</th>
<th>$R$</th>
<th>No. free parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/2</td>
<td>16.2</td>
<td>30.3</td>
<td>16.9</td>
<td>0.50</td>
<td>-0.03</td>
<td>1.8</td>
<td>2</td>
</tr>
<tr>
<td>5/2</td>
<td>48.6</td>
<td>78.4</td>
<td>72.8</td>
<td>0.29±</td>
<td>0.10</td>
<td>1.0</td>
<td>2</td>
</tr>
<tr>
<td>7/2</td>
<td>45.8</td>
<td>73.4</td>
<td>67.5</td>
<td>0.40±</td>
<td>0.05</td>
<td>1.2</td>
<td>2</td>
</tr>
</tbody>
</table>
### Table 10.4
PARITY ASSIGNMENTS FOR 1700 MeV REGION:

<table>
<thead>
<tr>
<th>J^P</th>
<th>Dominant l-value</th>
<th>R</th>
<th>Experimental value At 16 GEV/c</th>
<th>Experimental value At 10 GEV/c</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/2^+</td>
<td>1</td>
<td>1/9</td>
<td>1.6±0.3</td>
<td>1.8±0.3</td>
</tr>
<tr>
<td>3/2^-</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5/2^+</td>
<td>1</td>
<td>3/2</td>
<td>1.3±0.3</td>
<td>1.0±0.2</td>
</tr>
<tr>
<td>5/2^-</td>
<td>2</td>
<td>1/6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7/2^+</td>
<td>3</td>
<td>1/5</td>
<td>1.2±0.2</td>
<td>1.2±0.3</td>
</tr>
<tr>
<td>7/2^-</td>
<td>2</td>
<td>9/5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
$pp \rightarrow pp \pi^+ \pi^-$

Fig. 10.1

(a) $\Delta^{++}$ Events

(b) Non-$\Delta^{++}$ Events

Number of Events per 75 MeV

$M(p \pi^+ \pi^-)$ GeV
$\not\!K^- p \rightarrow p K^- \pi^+ \pi^-$

(a) $\Delta^{++}$ EVENTS

(b) NON $\Delta^{++}$ EVENTS

FIG. 10.2
$pp \rightarrow pp \pi^+ \pi^-$

(a) No. of $\Delta^{++}$ events

(b) No. of $N^*(1470)$ events

(c) No. of $p\pi^+\pi^-$ events

FIG. 10.3
$K^- p \rightarrow p K^- \pi^+ \pi^-$

(a) No. of $\Delta^{++}$ events

(b) No. of $N^{*+}(1470)$ events

(c) No. of $p \pi^+ \pi^-$ events

FIG. 10.4
FIG. 10.5

pp → pp π⁺ π⁻
$K^- p \rightarrow p K^- \pi^+ \pi^-$

FIG. 10.6
$pp \rightarrow pp \pi^+ \pi^-$
$K^- p \rightarrow p K^- \pi^+ \pi^-$

**FIG. 10.8**
FIG. 10.9

(a) "1400" Region

I: $J = \frac{1}{2}$

II: $J = \frac{3}{2}$

(b) "1700" Region

I: $J = \frac{3}{2}$

II: $J = \frac{5}{2}$

III: $J = \frac{7}{2}$

Polar Angular Distributions of Normal

(pp→pp $\pi^+ \pi^-$)
FIG. 10.10  $K^{-}p \rightarrow pK^{-}\pi^{+}\pi^{-}$  Polar Angular Distributions of Normal

(a) "1400" Region

I: $J = 1/2$
II: $J = 3/2$

266 events

(b) "1700" Region

I: $J = 3/2$
II: $J = 5/2$
III: $J = 7/2$

460 events

NO. OF EVENTS / 0.1

00 0.5 1.0

$\cos \theta$

00 0.5 1.0

$\cos \theta$
FIG. 10.12

pp \rightarrow pp \pi^+ \pi^-

\Delta^{**} Selected

$M(p \pi^+\pi^-) \text{ GeV}$

$A_{10}$

$A_{20}$

$A_{30}$

$A_{40}$

$A_{50}$

$A_{60}$
$K^- p \rightarrow p K^- \pi^+ \pi^-$

$\Delta^{++}$ Selected

FIG. 10.13
FIG. 10.14  \( pp \rightarrow pp \pi^+ \pi^- \) Sequential Decay Distributions

"1400" Region  276 events

(a)  
(b)  

I: \( J = 1/2 \)  
II: \( J = 3/2 \)
FIG. 10.15  

$K^- p \rightarrow p K^- \pi^+ \pi^-$  Sequential Decay Distributions

"1400" Region  222 events

(a)  

(b)  

I: $J = \frac{1}{2}$  
II: $J = \frac{3}{2}$
FIG. 10.16  \( pp \rightarrow pp \pi^+ \pi^- \) Sequential Decay Distributions

"1700" Region  230 events

(a)

(b)

I: \( J = 3/2 \)
II: \( J = 5/2 \)
III: \( J = 7/2 \)
FIG. 10.17  \( K^- p \rightarrow pK^- \pi^+ \pi^- \)  
Sequential Decay Distributions

"1700" Region  243 events

(a)  
(b)  

I: \( J = 3/2 \)  
II: \( J = 5/2 \)  
III: \( J = 7/2 \)
CHAPTER 11

HELICITY CONSERVATION FOR THE LOW MASS
\( p\pi^+\pi^- \) SYSTEM

11.1 INTRODUCTION

Consider the diffractive reaction

\( a + b \to a' + b \)

where \( a' \) may or may not be the same as \( a \).

It has for a long time been assumed that 'by definition' the Pomeron must couple as a \( J^P = 0^+ \) object. This implies that the component of spin of \( a' \) must be the same as that of \( a \), referred to the \( a \) direction in the \( a' \) rest-frame. This is known as \( t \)-channel helicity conservation.

If on the other hand the spin components of \( a' \) and \( a \) are equal, referred to their separate directions in the rest-frame of \( a' \), then \( s \)-channel helicity is said to be conserved.

In an experiment using a polarized photon beam, Ballam et al. (140) have demonstrated that \( s \)-channel helicity is conserved in \( p^0 \) photoproduction, and there is also some evidence to suggest that this is the case in \( \pi N \) elastic scattering (141). There are in addition some theoretical reasons (142) for believing that \( NN \) and \( \pi N \) elastic scattering conserve \( s \)-channel helicity.

The 10 GeV/c \( K^-p \) collaboration together with groups working on 8 GeV/c \( \pi^+p \) and 16 GeV/c \( \pi^+p \) experiments have recently published a paper (143) investigating the diffractive production of the \( Q \) and \( A_1 \) mesons. The experimental evidence shows that it is \( t \)-channel and not \( s \)-channel helicity
that is conserved in these reactions. In the case of $A_2^-$ production, this conclusion has been confirmed by the Illinois group\textsuperscript{(144)} using a compilation of data from $\pi^-p$ experiments at beam momenta from 5 to 25 GeV/c.

The experimental results obtained to date are consistent with the observation of Bialas et al.\textsuperscript{(145)} that perhaps s-channel helicity is only conserved for diffractive processes in which there is no change of spin.

In this chapter we investigate the diffractive production of the low mass $p\pi^+\pi^-$ system in channels (1) and (2). We shall see that in general helicity cannot be conserved in the s-channel.

11.2 TESTS FOR HELICITY CONSERVATION

In the rest-frame of the $p\pi^+\pi^-$ system the momenta of the three particles are coplanar. A normal to this plane can be defined, which we shall call the $p\pi^+\pi^-$ normal. The vector sum of the momenta of the three particles in the overall centre-of-mass defines another direction, called the $p\pi^+\pi^-$ direction. Lastly we define the direction of the associated incident proton, termed the incident direction.

Tests for helicity conservation\textsuperscript{(145),(146)} concern the distribution of the azimuthal angle of the $p\pi^+\pi^-$ normal about either the $p\pi^+\pi^-$ direction ($\phi_s$) or the incident direction ($\phi_t$), when evaluated in the $p\pi^+\pi^-$ rest-frame. Assuming that the target protons in channels (1) and (2) and beam proton in channel (1) are unpolarized, then if s-channel helicity is conserved the distribution in $\phi_s$ should be uniform. If it is t-channel that is conserved, then the distribution in $\phi_t$ should be uniform.

We may similarly define angles $\phi_{sp}$ and $\phi_{tp}$ for the proton of the $p\pi^+\pi^-$ system, and if required, azimuthal angles for the $\pi^+$ and $\pi^-$. Each
of these should be uniformly distributed if helicity is conserved in the corresponding channel. (The plane containing the $p\pi^+\pi^-$ direction and the incident direction is used to define zero azimuthal angle in all cases. Fig. 11.1 explains the notation more fully. The distributions in $\phi_s$, $\phi_{sp}$ etc. are usually referred to as helicity frame distributions and those in $\phi_t$, $\phi_{tp}$ etc. as Gottfried-Jackson frame distributions).

The distributions in $\phi$ are shown in Figs. 11.2 and 11.3 for the two channels. In the case of channel (1) only the $p\pi^+\pi^-$ combination with the lower effective mass has been used. For channel (2) events have only been considered if,

$$M(p\pi^+\pi^-) < M(k^+\pi^0\pi^-) + m_p - m_k$$

(11.1)

This procedure has been adopted in order to extract $p\pi^+\pi^-$ systems most likely to have been produced diffractively.

Figs. 11.2 and 11.3 show that there is a considerable difference between the distributions in $\phi_{sp}$ and $\phi_{tp}$ for both channels. The former distributions show a marked peak at around $180^\circ$, while the latter are essentially uniform.

It might be objected at first sight that the mass selection defined above could distort the distribution in $\phi_{sp}$. However, suppose we consider channel (2). Then it is evident that the mass of the $k^-\pi^+\pi^-$ system is independent of $\phi_{sp}$ and hence any selection based on $M(k^-\pi^+\pi^-)$ cannot affect the distribution in $\phi_{sp}$. A similar argument can be applied to the channel (1).

The distributions in the azimuthal angle of the normal to the $p\pi^+\pi^-$ plane do not show such a significant deviation from uniformity.
Nevertheless the distributions in the helicity frame are less uniform than those in the Gottfried-Jackson frame for both channels.

We next consider those events having a $p\pi^+$ mass in the $\Delta^{++}$ mass region defined by,

$$1.15 \leq M(p\pi^+) < 1.30 \text{ GeV}.$$  

Azimuthal angles ($\phi_{s\Delta}$ and $\phi_{t\Delta}$) can be defined for the $\Delta^{++}$ in the same way as $\phi_{sp}$ and $\phi_{tp}$. The corresponding distributions for channels (1) and (2) are shown in Figs. 11.4 and 11.5 respectively, and it can be seen that in both cases the distribution in the helicity frame is much less uniform than that in the Gottfried-Jackson frame.

Thus for both channels the experimental evidence implies that $s$-channel helicity cannot be conserved in general for diffractively produced $p\pi^+\pi^-$ systems.

11.3 AZIMUTHAL ANGULAR DISTRIBUTIONS AS A FUNCTION OF $p\pi^+\pi^-$ MASS

We have attempted to explore the azimuthal angular dependence in the two frames as a function of the mass of the $p\pi^+\pi^-$ system. To this end we have divided the low mass $p\pi^+\pi^-$ spectra of the two channels into three intervals.

$$M(p\pi^+\pi^-) < 1.55 \text{ GeV} \quad (A)$$
$$1.55 \leq M(p\pi^+\pi^-) < 1.80 \text{ GeV} \quad (B)$$
$$1.80 \leq M(p\pi^+\pi^-) < 2.05 \text{ GeV} \quad (C)$$

The distributions in $\phi_{sp}$ and $\phi_{tp}$ for each of these intervals are shown in Figs. 11.6 and 11.7 for channels (1) and (2) respectively, where the mass selections defined in Section 11.2 have been imposed.

Since the helicity and Gottfried-Jackson frames coincide when
\[ |t| = |t|_{\text{min}}, \] where \( t \) is the four momentum transfer from the incident proton to the \( p^+n^- \) system, we have imposed a selection, \( |t| > 0.05 \) GeV\(^2\), in each of the three mass intervals. The resulting distributions are shown shaded in Figs. 11.6 and 11.7.

In the case of channel (1) the distributions in the helicity frame are much less uniform than those in the Gottfried-Jackson frame for all three mass intervals. However, in channel (2), this may not be true for region (A).

The interval (A) is of special interest in that if spin \( \frac{1}{2} \) dominates this region, one expects a uniform distribution in the azimuthal angle, independent of the choice of axes. The fact that the distribution for channel (1) is significantly non-uniform in the helicity system suggests that other spins may be contributing.

The \( \Lambda^{++} \) mass selection has been applied to events in each of the three regions and the corresponding distributions in \( \phi_s \Lambda \) and \( \phi_t \Lambda \) are shown in Figs. 11.8 and 11.9. For the two higher mass regions (B) and (C), the Gottfried-Jackson frame seems to be the preferred system for both channels. It is difficult to draw any firm conclusions about region (A). In the case of channel (1) neither frame appears to be the preferred system, while for channel (2) the distributions are consistent with uniformity in both frames.

We have also attempted to find the system in which the angular distributions do not depend on azimuthal angle in a somewhat different way.

Suppose \( \phi_p \) are the polar and azimuthal angles of the proton with respect to a co-ordinate system which is obtained from the helicity frame by a rotation \( \chi \), about the y-axis (see Fig. 11.1) or from the Gott-
fried-Jackson frame by a rotation $X_J$.

The $p^+n^-$ mass spectrum was divided into intervals of 150 MeV and in each bin an expansion written of the form,

$$ W(e_p, \theta_p) = \frac{1}{4\pi} \sum_k \left( \frac{A_k}{A_0} \right) P_k (\cos \theta_p) $$

where,

$$ \left( \frac{A_k}{A_0} \right) = (2l + 1) < P_k (\cos \theta_p) > $$

$l$ values up to a maximum of ten were used in the analysis. The 'preferred' system is found by maximising the log-likelihood function,

$$ \chi^2 = \sum_i \frac{1}{i} W(e_p^i, \theta_p^i) $$

with respect to the rotation angle $\chi$. The sum is over events in the $p^+n^-$ mass bin.

The values obtained for $\chi^H$ and $\chi^J$ are shown in Figs. 11.10 and 11.11 as a function of $p^+n^-$ mass. Clearly for channel (1) the Gottfried-Jackson frame is very close to the preferred system over the entire mass range considered. The results for channel (2) are not so clear, and only at higher masses does the Gottfried-Jackson frame consistently appear to be the better choice.

This analysis has also been performed for the azimuthal angle of the $\Delta^{++}$. The values obtained for $\chi^H$ and $\chi^J$ in channels (1) and (2) are shown in Figs. 11.12 and 11.13 respectively. Above a mass of 1.45 GeV the Gottfried-Jackson frame seems to be the better choice in both channels.
11.4 FURTHER TESTS FOR HELICITY CONSERVATION

Recently, Bialas et al. (145) have devised some tests for the combined assumptions of helicity conservation and Gribov-Morrison parity rule (45), for the reactions,

\[ \text{Ap} \rightarrow \text{A (N)} \]
\[ \text{Ap} \rightarrow \text{A (\Delta)} \]

where A is \( \pi, k \) or p. The N and \( \Delta \) systems are understood to be produced diffractively. If A is a proton, then there is an additional assumption necessary, that the helicity dependence of the pN or p\( \Delta \) vertex factors from the AA vertex.

These tests have been applied to the reactions,

\[ \text{pp} \rightarrow \text{p (A++Tr-)} \]  
\[ \text{kp} \rightarrow \text{k (e + n -)} \]

where events have been selected according to the A++ mass selection defined in Section 11.2.

We define two coordinate systems. Suppose that \( \theta \) and \( \phi \) are the polar and azimuthal angles of the \( \Delta^{++} \) in either the Gottfried-Jackson or helicity systems. Then \( \theta' \) and \( \phi' \) are the polar and azimuthal angles of the decay proton, in the \( \Delta^{++} \) rest-frame, where the \( z' \) axis is along the \( \Delta^{++} \) direction. The \( y' \) axis is taken as normal to the \( z \) and \( z' \) axes.

Bialas et al. derived linear relations on double moments of the type,

\[ \sum_{mM} l_L^L = \langle \gamma^L (e, \phi) \text{Re} \gamma^L (e', \phi') \rangle \]

(11.5)

assuming a maximum spin of the \( \Delta^{++} \) system. These relations are as follows.

\[ J_{\max} = 5/2 \]
\[
\begin{align*}
Z_{00}^{02} + \sqrt{5} Z_{00}^{22} + 3 Z_{00}^{42} &= \frac{1}{\sqrt{5}} \left( Z_{00}^{00} + \sqrt{5} Z_{00}^{20} + 3 Z_{00}^{40} \right) \quad \text{(A)} \\
Z_{00}^{12} + \sqrt{\frac{1}{3}} Z_{00}^{32} &= \frac{1}{\sqrt{\frac{1}{3}}} \left( Z_{00}^{10} + \sqrt{\frac{1}{3}} Z_{00}^{30} \right) \quad \text{(B)} \\
Z_{-11}^{42} &= \sqrt{2} Z_{-22}^{42} \quad \text{(C)} \\
J_{\text{max}} = \frac{3}{2}: \\
Z_{00}^{02} + \sqrt{5} Z_{00}^{22} &= \frac{1}{\sqrt{5}} \left( Z_{00}^{00} + \sqrt{5} Z_{00}^{20} \right) \quad \text{(D)} \\
Z_{00}^{12} &= \frac{1}{\sqrt{5}} Z_{00}^{10} \quad \text{(E)} \\
Z_{-11}^{22} &= Z_{-22}^{22} \quad \text{(F)}
\end{align*}
\]

There is the additional requirement that \(\frac{E_L}{m_M} = 0\) for \(J > 2J_{\text{max}}\)

These formulae are valid for any value of \(|t|\) and mass of the \(\Delta^{++}\pi^-\) system, at fixed incoming energy, with the assumptions previously stated.

We in fact found that the majority of the \(\frac{E_L}{m_M}\) are consistent with zero, for \(p\pi^+\pi^-\) masses less than 2 GeV. Only relations (A) or (D) provide a significant test of the assumptions. It can in addition be shown that the relations (C) or (F) alone provide a unique test of the Gribov-Morrison parity rule. If we assume a rule,

\[
\Delta P = (-)^{J+1}
\]

then relations (A) and (B) or (D) and (E) are unchanged and (C) and (F)
are replaced by,
\[ Z_{42}^{\pm} = -\sqrt{2} Z_{22}^{42} \quad (C') \]
\[ Z_{-11}^{22} = -Z_{-22}^{22} \quad (F') \]

Thus using (A) and (D) we cannot distinguish between the spin-parity series
\[ \frac{1}{2}^+, \, \frac{3}{2}^-, \, \frac{5}{2}^+ \quad \text{and} \quad \frac{1}{2}^-, \, \frac{3}{2}^+, \, \frac{5}{2}^- \],
but only prove that there is no admixture.

We have evaluated the left- and right-hand sides of the relations (A) and (D) in both the helicity and Gottfried-Jackson frames, for each of the mass intervals previously defined. Unfortunately, the limited statistics available do not permit any definite conclusions to be drawn. The results for relation (A) are shown in Fig. 11.14 and it can be seen that within the experimental errors, neither frame is preferred.
Co-ordinate Frames.

FIG. 11.1
$pp \rightarrow pp \pi^+ \pi^-$

Azimuthal Angular Distributions

FIG. 11.2
$K^-.p \rightarrow pK^-\pi^+\pi^-$

Azimuthal Angular Distributions

FIG. 11.3
FIG. 11.4  \[pp \rightarrow pp \pi^+ \pi^-\]

\(\Delta^{++}\) Azimuthal Angular Distributions

(a)  \(\phi_{s\Delta}\)

(b)  \(\phi_{t\Delta}\)
FIG. 11.5 \[ K^-p \rightarrow pK^-\pi^+\pi^- \]

\[ \Delta^{++} \text{ Azimuthal Angular Distributions} \]

(a) \[ \phi_{s\Delta} \]

(b) \[ \phi_{t\Delta} \]
FIG. 11.6

pp → ppπ⁺π⁻

(a) \(\Phi_{sp \ in \ A}\)

(b) \(\Phi_{tp \ in \ A}\)

(c) \(\Phi_{sp \ in \ B}\)

(d) \(\Phi_{tp \ in \ B}\)

(e) \(\Phi_{sp \ in \ C}\)

(f) \(\Phi_{tp \ in \ C}\)
FIG. 11.8

pp → pp π⁺π⁻

(a) $\phi_{s\Delta}$ in A

(b) $\phi_{t\Delta}$ in A

(c) $\phi_{s\Delta}$ in B

(d) $\phi_{t\Delta}$ in B

(e) $\phi_{s\Delta}$ in C

(f) $\phi_{t\Delta}$ in C

NO. OF EVENTS / 36°

0° 180° 360°
FIG. 11.9

\[ K^- p \rightarrow pK^- \pi^+ \pi^- \]
FIG. 11.10

PP \rightarrow \text{pp } \pi^{+} \pi^{-}

(a) \chi_{H}

(b) \chi_{J}

M(p\pi^{+}\pi^{-}) \text{ GeV}
$$K^- p \rightarrow p K^- \pi^+ \pi^-$$

(a) $\chi_H$

(b) $\chi_J$

FIG. 11.11
$pp \rightarrow pp \pi^+ \pi^-$

(a) $\chi^\Delta_H$

(b) $\chi^\Delta_J$

FIG. 11.12
$K^- p \rightarrow p K^- \pi^+ \pi^-$

FIG. 11.13
(a) $p p \rightarrow \Delta^{++} p \pi^-$

Helicity Frame

Gottfried-Jackson Frame

(b) $K^- p \rightarrow \Delta^{++} K^- \pi^-$

$M(\Delta^{++}\pi^-)$ GeV

- LHS of (A) : $\times$ RHS of (A)

FIG. 11.14
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Abbreviations used for journals:

N.C.: Nuovo Cimento
N.P.: Nuclear Physics
P.L.: Physics Letters
P.R.: Physical Review

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