THE INTERACTION BETWEEN SLAB AND COLUMNS

IN FLAT SLAB CONSTRUCTION

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by

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ABSTRACT

This thesis is concerned with the effect of the overall stiffness of the slab, acting in conjunction with the columns, on the restraining moment at the column due to unequally vertically loaded panels; and on the horizontal deflection of the slab due to wind load.

An elastic analysis is used to determine the moment-rotation relation at the joint of the column and slab. The dissipation of the bending moment from the joint to the field of the slab is computed. Further solutions are obtained for vertical loading on the slab. One panel, four panel and infinite slab are considered.

A small scale perspex model is used to verify the elastic solutions and good agreement is obtained.

Tests were carried out on a full scale one panel reinforced concrete flat slab designed with the help of the elastic solution. The behaviour of the slab subjected to wind as well as vertical load was observed to be satisfactory. Correlation between the experimental results and theory is reasonably close.
A yield line analysis for a flat slab subjected to wind and vertical load is presented. The experiment confirmed the predicted failure mechanism and the ultimate strength.

The effect of slab stiffness on the column restraining moment is studied and a simple effective width method is suggested to estimate the restraining moment at the column head. A suitable bending moment distribution is suggested for the design of the slab to resist this restraining moment and wind moment.

The effective width of the slab to be assumed in calculating the horizontal deflection of the structure is also investigated and an approximate expression is proposed.

The results are related to current design practice, which is found to be less than satisfactory, particularly in respect of wind loading.
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CHAPTER I

INTRODUCTION

Even though flat slab construction was introduced only half a century ago, yet because of its great interest to both theoretical as well as practical engineers, much work has been done on finding out the real behaviour of this type of construction. But most of the analytical solutions on flat slab structures published are mainly concerned with internal panels under vertical and symmetrical loading systems with either pinned or clamped columns supports. In these cases the effect of the restraint from the columns does not enter the problem. However, in the case of an external panel, unsymmetrical loading, or unequal span conditions, the column restraint will play an important part in the behaviour of the structure; for it will affect the proportion of the negative and positive moment in the slab. Moreover, in the case of a flat slab structure without shear walls or cross bracing, the moment due to wind load will be transmitted from the column to the slab. The relative stiffness of the slab at the junction with the column will determine the magnitude of the horizontal deflection of the structure and the moments to be transferred. Hence a study into the interaction between slab and columns in flat slab construction is of fundamental importance.

In 1959, Mr. A. L. Handley, the District Surveyor of the City of London, in checking the side sway requirements for a flat slab
building in London, felt that the present Code of Practice was inadequate in this respect. The "Continuous Frames Method" for designing flat slabs recommended by C.P. 114, 1957, Clause 332, allows the whole width of the slab between the centre-lines of the adjacent panels to be assumed to act as a beam. It is apparent, however, that the slab will not provide the same rigidity and strength at the junction with the column as the imaginary beam. World wide enquiries by the District Surveyor failed to elicit any basis for taking account of slab column interaction and the Civil Engineering Research Association therefore sponsored a research project on this topic at Imperial College. The work described in this thesis represents the first stage of the project.

The problem of column-slab interaction has two main aspects - the overall stiffness of the structure, and the local strength of the slab at the column head. From the stiffness point of view, the moment-rotation behaviour of the slab requires to be elucidated. From the strength point of view the concentration of bending moment and the distribution of punching and torsional shear stresses around the column head, as well as the ultimate strength, have to be determined.

The problem contains many major sub-problems and involves many variables. It is not the object of this thesis to solve all the problems specifically or to find the effect of varying all the parameters, but rather, being a pilot research on the subject, to examine the overall interaction between slab and columns and so
provide some insight into the behaviour, so that specific problems
may be brought out for further studies.

With this background of generality in mind it can be stated that
the object of this work was

(1) To determine the stiffness of flat slabs subjected to restrain-
ing bending moments and wind moments from the columns.

(2) To study the dissipation of the wind moments into the slab, and
the distribution of moment due to transverse load.

(3) To study experimentally the performance of a full scale
reinforced concrete flat slab subjected to vertical, wind, and
combined load conditions, and to relate the observed behaviour
to elastic and yield line analyses.

Review of published data

The flat slab, or beamless floor, is a relatively modern type
of building structure, originated by C. A. P. Turner in 1902. Many
structures of this type were built long before a generally accepted
theory of design was developed. In its infancy the design of flat
slabs was based mainly on the practical experience of the designer and
the results of a number of tests on some of the constructed flat slab
buildings\(^1, 2\). In general, these tests gave a much lower total move-
ment in a panel than would have been expected for a beam and slab
construction. This fact was much publicised by those engineers who
favoured flat slab construction as one of the advantages of this
form of structure, and they attributed this behaviour to "two
dimensional action", "plate action", "dome or arch effect", "Poisson's
ratio effect", etc. Nevertheless, the more conservative engineers
still thought that the underdesign of the flat slab below the usual
statical requirements was unacceptable, and that the mathematical
theory of plates and the effect of Poisson's ratio would lead to a
reduction of the actual bending moment was inconceivable and mysterious.

In 1914, J. R. Nichols(3) pointed out that the numerical sum of
the positive and negative bending moments in an internal panel of an
infinite flat slab supported on circular column heads must be equal
to the total static moment. Assuming the shear to be uniformly
distributed around the arc of the intersection of the column head and
the slab, he derived the formula

\[ M_p + M_n = M_o = \frac{wL^3}{8} \left(1 - \frac{4c}{\pi L} + \frac{c^3}{3L^3}\right) \]

where

- \( M_p \) = positive bending moment across the mid section of the panel
- \( M_n \) = negative bending moment across the column line of the panel
- \( M_o \) = total static moment in the panel
- \( w \) = U.D.L. on the panel
- \( L \) = length or width of the square panel
- \( c \) = diameter of the column head.

This treatment of the moments in flat slabs was severely
criticised by C. A. P. Turner and some others as not taking into account
the difference between the action of a plate and a beam.

Lacking a comprehensive treatment for flat slabs, the code of
practice relies on experimental results. Hence, coefficients of $\frac{1}{10}$, 
0.09 and 0.09 $F$ of $WL$ instead of $\frac{1}{8}$ have been adopted in codes of
different periods and the expression $(1 - \frac{4c}{\pi L} + \frac{c^3}{3L^3})$ has been replaced
by a simpler approximate expression $(1 - \frac{2}{3} \frac{c}{L})^2$ (5, 6). The 1956
ACI code (6) gives

$$M_c = 0.09 F WL \left(1 - \frac{2}{3} \frac{c}{L}\right)^2$$

where $F = 1.15 \frac{c}{L}$, but not less than 1
and $W = wL^2$

Though Nichols confirmed the total static moment, nothing had
yet been written about how this moment would be distributed in the
panel. In 1921, H. M. Westergaard established a theoretical analysis
to calculate the distribution of positive and negative moments through-
cut the slab when considered as an isotropic plate, and his solution
was checked by W. A. Slater by tests on reinforced concrete flat slab
buildings (4). With this information, the code of practice formulated
the distribution coefficients for the positive and negative moments at
the column and middle strips.

Since then several mathematical treatments of different aspects
of flat slab behaviour have been published (7, 8, 9, 10).
V. Lewe and A. Nadai both solved the problem of the deflection of an internal panel of an infinite slab supported on points with uniformly distributed load in combined algebraic, trigonometric and hyperbolic series form. The same slab with concentrated central load was also discussed by Lewe. The assumption that the cross-sectional dimension of the columns is small and can be neglected in so far as deflections and moments at the centre of the plate are concerned simplifies the problem very much. Further study shows that the moments at the panel centre remain practically constant for ratios of column size to panel length up to 0.2. Hence, the assumption that the reactions from the columns are concentrated at the panel corners is sufficiently accurate for the central portion of the panel. The case of replacing the point support by circular or rectangular support was investigated by S. Woinowsky-Krieger, K. Frey, as well as V. Lewe; and the effect of a rigid connection with the column (i.e. clamped support) was evaluated by A. Nadai and F. Tolke for circular columns and by S. Woinowsky-Krieger for a square column section, using Maschelishvili's complex variable method. Their results demonstrated that by assuming a clamped condition within the column head the bending moments around the support were increased numerically while the bending moments in the slab were reduced.

As the double periodical function solutions for these problems were too mathematical and complicated for practical engineers, a numerical approach to these problems was adopted by H. Marcus who
solved the biharmonic equation as two sets of harmonic equations by a finite difference method\(^{(9)}\).

Most of the previously mentioned methods deal with internal rectangular flat slabs loaded uniformly. In 1957, John E. Brotchie gave a theory which was applicable to panels of any shape and with any type of loading and any degree of restraint of fixity between slab and column\(^{(11)}\). He considered the slab as floating on a certain imaginary liquid of a density \(\gamma\) which was related to the stiffness of the slab, \(D\), and an arbitrary length, \(S\), by \(S = (\frac{D}{\gamma})^{1/4}\) and that the column reactions and clamping restraining moment as well as the load were applied to this floating slab and the results were superimposed to give the final value. The solution of the differential equation was a Bessel function, and the numerical values of the function were given in tables of influence coefficients.

With the establishment of electronic digital computers in the last decade, the numerical procedure of solving plate problems again becomes very attractive. This method is applicable to most boundary and loading conditions in flat slab problems including external panels, which have not yet been solved mathematically. In 1959, A. Ang, applying Newmark's Plate Analog concept, developed a distribution procedure for the analysis of continuous rectangular flat slabs\(^{(12)}\). The procedure is formed in a computer programme for the ILLIAC computer. The programme calculates
(1) the deflection influence coefficients and the distribution and carry over factors of a panel

(2) the edge deformations and the total deformation of the panel due to the loading; and this calculation is repeated until the boundary conditions along the edges of all panels are satisfied. W. S. Prescott showed that the effect of openings and edge beams with known flexural and torsional properties can also be investigated by means of the finite difference method with the computer (24).

Besides the advance of analytical work, programmes of systematic experiments on the behaviour of flat slab structures have also been carried out recently in the University of Illinois, U.S.A. and elsewhere. G. T. Mayes, D. S. Hatcher, and J. O. Jirse of the University of Illinois each tested a quarter scale model of a nine-panel reinforced concrete flat slab floor with deep and shallow edge beams. Each model was different from the other in some respect. Mayes' was of the flat plate type, i.e. beamless floor without drop panel or column head; Hatcher's had drop panels and column heads; Jirse's was reinforced with welded wire mesh reinforcement. The test results were given in graphs and tables of deflections, steel stresses and strains, crack patterns and bending moments converted from steel strains and column reactions. The bending moments were then compared with those calculated from theory by the numerical method and those according to the A.C.I. code. Their results showed that the total moments in a panel agreed very well with the static
moments as given by the Nichols' formula and the ultimate strength
was reasonably close to that predicted by the yield line theory
(13, 14).

Little analytical work has been published on the problem of
punching shear around column heads. The shearing strength in
reinforced concrete slabs depends on many variables, for example,
concrete strength, percentage of steel, thickness of slab, size of
column, eccentricity, etc., and it seems that analytical analysis is
hardly feasible. However, experimental work by E. Hognestad,
H. Nylander, and some others has thrown some light on this subject.

In 1956, R. Elstner and E. Hognestad (15) reported the results
of experimental work on the shearing strength of square reinforced
concrete slabs with square central stubs. The following formula for
estimating the shearing strength was suggested:

\[ \frac{v}{f'c} = \frac{P_{\text{shear}}}{7/8 \cdot bdf'c} = 0.035 + \frac{130}{f'c} + \frac{0.07}{\phi} \]

This was revised to:

\[ \frac{v}{f'c} = \frac{333}{f'c} + \frac{0.046}{\phi} \]

where
- \( f'c \) = cylinder strength in psi
- \( b \) = circumference of column
- \( d \) = effective depth of slab
- \( \phi = \frac{P_{\text{shear}}}{P_{\text{flexure}}} \)
- \( P_{\text{shear}} = \text{ultimate shear capacity} \)
P_{flexure} = \text{ultimate flexural capacity of the slab computed by the yield-line theory without regard to a shear failure.}

It was reported that an eccentricity up to half the column size has little or no effect on the ultimate shear strength of a slab subjected to a concentrated load and an increase in the percentage of compression reinforcement likewise has little effect. G. D. Base did similar experiments and observed similar results. He also found that the angle of the plane of rupture changed from a small angle to 45° as the tension reinforcement was increased. He concluded that shear reinforcement in the form of stirrups may be more satisfactory than bent up bars (16).

H. Nylander and S. Kinnunen studied the punching shear strength of circular slab models with circular column stubs reinforced with 2-way or ring and radial steel (17). They derived a theory for estimating the punch load by considering the equilibrium of the shear cone acted upon by the external forces and the stresses in the reinforcement and in the concrete. Effects of different amounts of flexural reinforcement in each type of specimen, and different column diameters were studied. Deflections, steel strains and concrete strains on the bottom surface (i.e. the surface with the stubs) of the slabs were measured and the different stages of flexural cracking, shear cracking, and the ultimate load were observed in the experiment. It was noticed that the first shear crack opened up at a load which
ranged from 45 to 75 per cent of the ultimate load. In general the experimental ultimate load was found higher than that given by the suggested theory and this was accounted for as due to the "dowel effect" of the reinforcing bars.

Similar tests were also done by Israel Rosenthal (18). He applied both central and eccentric loads on circular slabs with or without shear reinforcement. He found that eccentric loading tended to reduce ultimate strength and that shear reinforcement was highly effective in preventing punching failure, suggesting that thin slabs with shear reinforcement could be employed.

The idea of a "shear block" enclosed by a periphery at a distance of the effective thickness of the slab around the face of the column was employed by M. P. Van Buren as well as W. K. Hahn to develop a theory for deriving the maximum unit combined shear stress for bending and punching load in a flat slab. The method is applicable to external and corner columns as well as internal columns and can deal with either circular or rectangular column sections. Provision is made in their theories for overhanging edges and the effect of nearby openings in the slab (19, 20).

The foregoing works cover either general solutions of the slab or the detailed analysis around the column head.

In 1938, H. D. Dewell and H. B. Harnill presented the results of a study into the design of flat slabs and their supporting columns as indeterminate structural frames (25). The following remarks were
made about the inadequacy of the design code:

(1) ACI Code empirical coefficients for the total positive moments for interior panels were exact for the case of uniform load on all panels only. For the case of live load on alternate strips of panels the total positive moment coefficients in the panel had an increase of from 10 to 45 per cent over the coefficients of the ACI Code for stiffness ratio \( \frac{K_c}{K_s} \) ranging from 7.68 to 0.84 and live load to dead load ratio \( \frac{LL}{DL} \) ranging from 1.43 to 4.37 for a certain structure.

(2) Bending in the interior columns, induced by unbalanced loading was not properly provided for.

(3) Variations in design from the requirements of the code might be made provided that the design would satisfy a detailed analysis by the "elastic" method.

Frame analysis has been applied to a flat slab bridge\(^{(26)}\) showing that flat slab construction could be employed even for single strip structures like bridges.

New experimental techniques have made it possible to study flat slab behaviour by means of small scale elastic models. C. G. J. Vreedenburgh and H. van Wijngaerden applied the Moire Fringe Method to determine the distribution of bending moments in \( \frac{1}{50} \) scale perspex flat slab models and obtained very good results\(^{(21)}\).
The differential transformer displacement transducer makes it possible to measure curvatures in small models of slabs from which distributions of bending moment can be found. This technique has been used by Leonhardt(22) and in certain investigations at Imperial College with which the writer has been concerned.

Thus, it can be seen that although there is a considerable literature on flat slab construction there is remarkably little data on column-slab interaction, particularly in respect of wind loading.
CHAPTER 2

ELASTIC ANALYSIS

Exact solutions for the plate equation with boundary conditions other than simply supported or clamped are, in general, considered to be rather difficult by the present known mathematic methods. The boundary conditions usually encountered in flat slab construction are seldom of a simple nature. Special methods like those of Galerkin and Muskhelishvilli have been applied to solve certain problems in this field with great success. But to grasp these methods is a major undertaking.

Numerical methods, however, offer a simple and straightforward approach to most problems, though they give approximate solutions only. Nevertheless, with the aid of the modern electronic computers, good approximations, with an accuracy acceptable to practical engineers, can be obtained. The central finite difference method has, therefore, been adopted in this analysis.

Solution by Finite Difference Method

The central finite difference operators for the plate equations are well known and need no derivation here. For the boundary between slab and column, it is assumed that the region within the perimeter of the column, column head, or drop panel, is of a different stiffness compared with that of the slab itself. The finite difference
operators for the points near this perimeter are derived in accordance
with continuity and equilibrium. A square grid is used and Poisson's
ratio \( \mu \) is included in the coefficients. The column size is
assumed to be zero (i.e., a point-support), 1/6th, or 1/12th of the
span width.

Assuming the slab is of homogeneous, elastic and isotropic
material, the thin plate equation for small deflections is

\[
\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{f}{D}
\]  

(2.1)

The bending moments are given by

\[
m_x = -D \left( \frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right) \]  

(2.2)

\[
m_y = -D \left( \frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right) \]  

(2.3)

\[
m_{xy} = -m_{yx} = D \left( 1 - \mu \right) \frac{\partial^2 w}{\partial x \partial y} \]  

(2.4)

and the shearing forces by

\[
V_x = -D \frac{\partial}{\partial x} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \]  

(2.5)

\[
V_y = -D \frac{\partial}{\partial y} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \]  

(2.6)

The reaction equations for the edges, from the Kirchhoff boundary
conditions, are
\[ r_x = -D \frac{\partial}{\partial x} \left( \frac{\partial^2 w}{\partial x^2} + (2 - \mu) \frac{\partial^2 w}{\partial x \partial y} \right) \] (2.7)

\[ r_y = -D \frac{\partial}{\partial y} \left( \frac{\partial^2 w}{\partial y^2} + (2 - \mu) \frac{\partial^2 w}{\partial x \partial y} \right) \] (2.8)

The above differential equations can be represented by central finite difference operators as follows:

\[ \text{The finite difference operators are shown in the diagrams.} \]
\[ m_y = \frac{D}{a^4} \begin{vmatrix} 1 & -1 & -1 & 1 \\ -\mu & 2\mu & -\mu & \text{x} \text{w} \\ \mu & \mu & -\mu & \mu \\ -1 & 1 & -1 & 1 \end{vmatrix} \tag{2.11} \]

\[ m_{xy} = -m_{yx} = \frac{D}{4a^4} \begin{vmatrix} 1 & -1 & -1 & 1 \\ 1-\mu & -1+\mu & -\mu & 1-\mu \\ -1+\mu & 1-\mu & -\mu & 1-\mu \\ 1-\mu & 1-\mu & -\mu & 1-\mu \end{vmatrix} \tag{2.12} \]

\[ v_x = \frac{D}{2a^4} \begin{vmatrix} 1 & -4 & 4 & -1 \\ 1 & 4 & -4 & 1 \\ 1 & -1 & 1 & -1 \end{vmatrix} \tag{2.13} \]
\[ V_y = \frac{D}{2a^3} x W \quad (2.14) \]

\[ \tau_z = \frac{D}{2a^3} x W \quad (2.15) \]

\[ \tau_y = \frac{D}{2a^3} x W \quad (2.16) \]
For a free edge:

From equation (2.1) and boundary conditions $y_x = 0$ and $y_{xx} = 0$, the operator for a free edge becomes

\[ w = \frac{3a^4}{h} \]  \hspace{1cm} (2.17)

At point of an applied moment:

When there is an applied moment at a point in the plate, a discontinuity in moment is introduced. In order to express the internal moment at the point by finite differences, it is necessary to extend the flexural curve on either side of the point and to introduce an imaginary point at one grid length from the point (Figure 2.1).

Two additional equations are required for the determination of
the two fictitious points.

(i) Total moment at point 0 must equal the applied moment \( M \).

(ii) Slopes at 0 on curves 103 and on 1'03 must be equal.

\[ -m_{03}a + m_{01}a = M \] (2.18)

\[ -\frac{D}{a^2} \left( -w_5 + (2 + 2\mu) w_o - w_i - \mu w_2 - \mu w_4 \right) a \]

\[ + \frac{D}{a^2} \left( -w_{1'} + (2 + 2\mu) w_o - w_i - \mu w_2 - \mu w_4 \right) a = M \]

i.e. \( w_5 - w_{1'} - w_i + w_{1'} = \frac{Ma}{D} \) (2.19)

\[ \frac{w_{1'} - w_i}{2a} = \frac{w_3 - w_i}{2a} \] (2.20)

i.e. \( w_5 - w_{1'} + w_i - w_{1'} = 0 \) (2.21)
ACTUAL AND FICTITIOUS DEFLECTION CURVES
AT THE JUNCTION OF COLUMN AND SLAB

FIG. 2·ii
At the junction between the slab and the drop panel or column head, the deflection curves in the two portions are not of the same curvature. By producing each part of the curve into the other field, two fictitious points are introduced (Figure 2.11). But the moment and the slope for both curves at the junction must be the same. This gives the extra equation for expressing any fictitious point in terms of the actual deflections.

$$m_{s3} = m_{s1}$$

$$\frac{D_1}{a^2} \begin{bmatrix} -\mu & -1 \\ -2+\mu & 1 \end{bmatrix} x \frac{w}{a^2} = \frac{D_2}{a^2} \begin{bmatrix} -\mu & -1 \\ -2+\mu & 1 \end{bmatrix} x \frac{w}{a^2}$$

Coefficients in broken line boxes refer to fictitious points.
i.e.

\[
\begin{align*}
-\mu & = (1 - \frac{D_1}{D_2}) \\
\frac{\mu}{(2+2\mu)(1-\frac{D_1}{D_2})} & = 1 \\
-\mu(1 - \frac{D_1}{D_2}) & = 1
\end{align*}
\]

slope of $\overline{BO_1'}$ = slope of $\overline{BO_1}$, 

\[
\frac{w_3 - w_1'}{2a} = \frac{w_1' - w_1}{2a}
\]

Eliminating $w_3'$ from (2.22) + (2.23) and dividing by $(1 + \frac{D_1}{D_2})$

\[
\begin{align*}
-\mu(1 - \frac{D_1}{D_2}) & = 1 \\
-\frac{(1 - \frac{D_1}{D_2})(1 + \frac{D_1}{D_2})}{(2+2\mu)(1-\frac{D_1}{D_2})} & = 1
\end{align*}
\]

\[
\begin{align*}
-\mu & = (1 - \frac{D_1}{D_2}) \\
\frac{\mu}{(2+2\mu)(1-\frac{D_1}{D_2})} & = 1 \\
-\mu(1 - \frac{D_1}{D_2}) & = 1
\end{align*}
\]

\[
\begin{align*}
-\mu & = (1 - \frac{D_1}{D_2}) \\
\frac{\mu}{(2+2\mu)(1-\frac{D_1}{D_2})} & = 1 \\
-\mu(1 - \frac{D_1}{D_2}) & = 1
\end{align*}
\]
Putting \( \frac{D_1}{D_2} = C_0 \), \( 1 - \frac{D_1}{D_2} = C_1 \), \( (I + \frac{D_1}{D_2})^{-1} = C_2 \)

The above expression is written:

\[
\begin{align*}
\begin{array}{c}
\mu c_x c_y \\
-c_y c_z \\
(2 \cdot 2) c_x c_y \\
-2c_z
\end{array}
\end{align*}
\]

\[x w = 0 \quad (2.25)\]

Similarly, eliminating \( w_1' \) we get:

\[
\begin{align*}
\begin{array}{c}
\mu c_x c_y \\
-2c_x c_y \\
(2 \cdot 2) c_x c_z \\
2c_z
\end{array}
\end{align*}
\]

\[x w = 0 \quad (2.26)\]

Equations for any other points near the junction can be derived in a similar manner.
Moments and shearing forces at a point on the junction of column and slab

Figure 2.iv
The finite difference operators for $v^w$ for a point at the junction as shown in Figure 2.iv are derived as follows:

$$a v_{ov} - a v_{es} + \frac{a}{2} v_{on} - \frac{a}{2} v_{os} - \frac{a}{2} v_{es} + \frac{a}{2} v_{es} + \frac{a}{2} v_{es} = 0 \quad (2.27)$$

$$a v_{ov} x = (m_{oo} - m_{ov} + m_{es} - m_{en}) x$$

$$a v_{es} = m_{ow} - m_{wo} + m_{en} - m_{es}$$

$$\frac{a}{2} v_{on} + \frac{a}{2} v_{on} = \frac{1}{2} m_{mo} + \frac{1}{2} m_{on} - \frac{1}{2} m_{mo} - \frac{1}{2} m_{no} + m_{on} - m_{ne}$$

$$\frac{a}{2} v_{os} + \frac{a}{2} v_{os} = \frac{1}{2} m_{so} + \frac{1}{2} m_{os} - \frac{1}{2} m_{so} - \frac{1}{2} m_{os} + m_{os} - m_{es}$$

$$a v_{es} = \frac{D_x}{a^2} \left[ -w_q + (2 + 2 \mu) w_1 - w_0 - \mu w_5 - \mu w_6 \right]$$

$$- \frac{D_x}{a^2} \left[ -w_1 + (2 + 2 \mu) w_0 - w_3 - \mu w_4 - \mu w_2 \right]$$

$$+ \frac{D_x}{a^2} \left[ (1 - \mu) w_1 - (1 - \mu) w_6 + (1 - \mu) w_2 - (1 - \mu) w_0 \right]$$

$$- \frac{D_x}{a^2} \left[ (1 - \mu) w_6 - (1 - \mu) w_1 + (1 - \mu) w_0 - (1 - \mu) w_6 \right]$$

\[ \text{(2.28)} \]
\[ a_{V_{06}} = \frac{sD_a}{sD_i} x \frac{1}{\alpha^2} x W \quad (2.29) \]

\[ -a_{V_{ow}} = \frac{sD_a}{sD_i} x \frac{1}{\alpha^2} x W \quad (2.30) \]

\[ a_{(V_{on1} + V_{on2})} = \frac{sD_a}{sD_i} x \frac{1}{\alpha^2} x W \quad (2.31) \]
Substituting these patterns into Equation 2.27

\[
G = \frac{1}{x \cdot d \cdot y} \cdot \frac{y \cdot d \cdot x}{y \cdot d \cdot x}
\]
Multiplying by 2, dividing by $D_2$, and replacing the coefficients for the fictitious points, equation (2.33) becomes

$$x'w = 2 \ell_e \frac{t_a^d}{p_i} \quad (2.34)$$

In order to simplify the coefficients, the notation given on page 33 is used. Then equation (2.34) is written

$$x'w = 2 \ell_e \frac{t_a^d}{p_i} \quad (2.35)$$
At the corner of the slab-column junction, the operator is

\[ x \ W \ = \ \frac{2 \ C_6 C_8}{D_1} \ \ \ \ (2.36) \]

The operators for any other points can be derived in a similar manner.
U = POISSONS RATIO
C_0 = D_1 / D_2
C_1 = 1 - C_0
C_2 = x \times \text{DIVIDE}(1, 1 + C_0)
C_3 = C_0 C_2
C_4 = C_1 C_2
C_5 = U C_4
C_6 = 2 + 2 U
C_7 = C_6 C_4
C_8 = 4 - U
C_9 = -4 + 2 U
C_{10} = 0

C_{11} = 1 - 5 C_5 C_5
C_{12} = 4 C_3 + 2 C_3 C_5
C_{13} = 8 + 2 C_5 C_7
C_{14} = 4 C_2 - 2 C_2 C_5
C_{15} = 2 C_3
C_{16} = -16 C_3 - 2 C_3 C_7
C_{17} = 2 + 4 U + 3 U U
C_{18} = 20 + 2 C_17 C_4 C_4
C_{19} = -16 C_3 + 2 C_2 C_7

C_{20} = 5 + 8 U + 5 U U
C_{21} = 20 - 8 C_2 C_4 C_4
C_{22} = -8 + 6 C_5 C_7
C_{23} = 4 C_3 C_2
C_{24} = 2 + 10 C_4 - 2 C_7 + 8 C_7 C_3 - 2 C_5 C_5
C_{25} = 0
C_{26} = 2 - C_5
C_{27} = -8 + C_7
C_{28} = 20 + C_4
C_{29} = 2 + 2 C_5
C_{30} = 0

C_{31} = 2 C_8 C_2 - 4 C_5 C_2
C_{32} = C_9 + 3 C_5 C_7
C_{33} = -C_6 + C_5 C_7
C_{34} = 2 C_8 C_3 + 4 C_5 C_3
C_{35} = 2 U C_2
C_{36} = 2 C_0 C_3 C_5
C_{37} = 4 - C_3 C_5
C_{38} = 4 - C_3 C_6
C_{39} = -8 - C_7
C_{40} = 20 + U C_4
C_{41} = 2 + C_5
C_{42} = 20 + 2 C_4

C_{43} = 3 + 4 U + 2 U U
C_{44} = 20 + 2 C_4 C_4 C_4
With the operators given above, a system of simultaneous equations can be written for the whole field of the slab. In some cases, the number of equations is too big to be solved by a single inversion, hence the method of partitioning of matrices is used to break up the matrix into a number of sub-matrices. The calculation is done by means of the Mercury and Atlas computers in the University of London Computer Unit. The computational procedure for all the solutions given here can be outlined in the following block diagram.
SET LENGTH, WIDTH INDEX NOS.

READ IN PARAMETERS AND CALCULATE CONSTANTS

ARRAY COEFFICIENTS FOR THE EQUATIONS FOR TWO ROWS OF GRID POINTS

WRITE IN AND ARRAY THE CORRESPONDING LOADING TERMS

OPERATION ON THE PARTITIONED MATRICES

STORE THE OPERATED SUB-MATRICES

LAST SET OF SUB-MATRICES?

NO

YES

BACK SUBSTITUTION TO GET SOLUTION OF ALL THE Unknowns - w's.

CALCULATE MOMENTS FROM THE DEFLECTIONS

PRINT OUT w, mx, my

STOP
Numerical Results

Infinite Slab with Point Supports

A rectangular array of columns in an infinite slab is considered (Figure 2.1a), and from this a panel of slab centred about one column can be isolated (Figure 2.1b) Due to symmetry and antisymmetry, it is only necessary to consider one quarter of a panel (Figure 2.1c). The grid size for a 1:1 panel (I/W/1) is shown in Figure 2.2. Applied moments are assumed to be distributed over the mesh length a (or a/2 where the moment is applied along an edge). Distributions of deflection and bending moment are shown in Figures 2.3 to 2.4. The grid for a 1.5:1 panel (I/W/3) is shown in Figure 2.7 and the distributions of deflection and bending moment are shown in Figures 2.8 and 2.9 respectively.

The rotation at the point of application of the bending moment in a slab can be compared with the rotation obtained from simple beam theory. The ratio gives the width of the slab which is effective in providing stiffness.

In calculating the rotation at the point of application of the bending moment it is noted that the 3 point central finite difference expression for slope is not very accurate because of the sharp change of slope at that spot. Some forward finite difference expressions using 4 or more points might give better results. These expressions are derived by differentiating the Lagrange interpolation polynomial
for the deflection curve.

\[ f(w)_n = \sum w_k \frac{(x-x_{-1})(x-x_0) \cdots (x-x_{k-1})(x-x_{k+1}) \cdots (x-x_n)}{(x_k-x_{-1}) \cdots (x_k-x_{k-1})(x_k-x_{k+1}) \cdots (x_k-x_n)} \]

where \( k = -1, 0, 1, 2 \ldots n \)

\[ \frac{df(w)}{dx} = \sum w_k \frac{(x_o-x_{-1})(x_o-x_1) \cdots (x_o-x_{k-1})(x_o-x_{k+1}) \cdots (x_o-x_n)}{(x_k-x_{-1})(x_k-x_1) \cdots (x_k-x_{k-1})(x_k-x_{k+1}) \cdots (x_k-x_n)} \frac{l}{x_k-x_o} \]

\[ + w_o \left( \frac{l}{x_o-x_{-1}} + \frac{l}{x_o-x_1} + \frac{l}{x_o-x_2} + \cdots + \frac{l}{x_o-x_n} \right) \]

where \( k = -1, (\pm 0), 1, 2 \ldots n \)
For example, the slope of the deflection curve at 0 using 5 points, is

\[
\frac{df(x)}{dx} = w_0 \frac{(x_0 - x_i)(x_0 - x_2)(x_0 - x_3)}{(x_0 - x_i)(x_0 - x_2)(x_0 - x_3)} \frac{l}{(x_i - x_0)}
\]

\[+ w_0 \frac{l}{x_0 - x_i} + \frac{l}{x_0 - x_1} + \frac{l}{x_0 - x_2} + \frac{l}{x_0 - x_3} \]

\[+ w_1 \frac{(x_0 - x_i)(x_0 - x_2)(x_0 - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} \frac{l}{(x_i - x_0)}
\]

\[+ w_2 \frac{(x_0 - x_i)(x_0 - x_1)(x_0 - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} \frac{l}{(x_i - x_0)}
\]

\[+ w_3 \frac{(x_0 - x_i)(x_0 - x_1)(x_0 - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} \frac{l}{(x_i - x_0)}
\]

For equal intervals, the slope at 0 using 3 points, 4 points and 5 points are given, respectively, by the following expressions:

3 points: \( \theta_0 = \frac{1}{2a} \left( -w_{-1} + w_1 \right) \)

4 points: \( \theta_0 = \frac{1}{6a} \left( -2w_{-1} - 3w_0 + 6w_1 - w_2 \right) \)

5 points: \( \theta_0 = \frac{1}{12a} \left( -3w_{-1} - 10w_0 + 18w_1 - 6w_2 + w_3 \right) \)
Values of the rotation at the point of the application of the bending moment calculated according to the above formulae together with their corresponding effective width to span width ratios for a range of length width ratios are tabulated below.

<table>
<thead>
<tr>
<th>Length ratio $\alpha$</th>
<th>3 point</th>
<th>4 point</th>
<th>5 point</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta$</td>
<td>Eff. width</td>
<td>$\theta$</td>
</tr>
<tr>
<td>1:2</td>
<td>.770</td>
<td>.108</td>
<td>.640</td>
</tr>
<tr>
<td>3:4</td>
<td>.771</td>
<td>.162</td>
<td>.641</td>
</tr>
<tr>
<td>1:1</td>
<td>.774</td>
<td>.215</td>
<td>.645</td>
</tr>
<tr>
<td>4:3</td>
<td>.832</td>
<td>.267</td>
<td>.703</td>
</tr>
<tr>
<td>2:1</td>
<td>.878</td>
<td>.380</td>
<td>.751</td>
</tr>
</tbody>
</table>

Table 2.1

One Panel Slab with Point Supports

Solutions have been obtained for a single panel slab with a symmetrical system of moments applied at the corners, as shown in Figure 2.10. Distributions of deflection and bending moment are shown in Figures 2.11 to 2.13.

Distributions of deflection and bending moment for a square slab having an antisymmetrical system of bending moments applied at each corner (i.e. corresponding to wind loading) are shown in Figures 2.18 to 2.20. For these solutions the mesh size shown in
Figure 2.10 was used.

**Effect of Grid Size:** In order to examine the accuracy of the finite difference solutions a solution was obtained for a single panel square slab with a central point load (Figure 2.14). The distribution of deflection and bending moment are shown in Figures 2.15 to 2.17. It is seen that the difference between the solutions with a grid size of $L/8$ and $L/12$ is small. Hence the grid size $L/12$ was adopted for subsequent solutions.

**Effect of Poisson's Ratio:** The effect of Poisson's ratio was studied for a single panel square slab having a symmetrical system of corner moments. Distributions of deflection for Poisson's ratio values 0, .1, .35, and .5 are shown in Figures 2.21 and 2.24.

A comparison of deflections and bending moment for two values of Poisson's ratio, 0 and .35, is also given for a square slab with a uniformly distributed load. (Figures 2.25 and 2.26).

**Four Panel Slab with Point Supports**

The one panel slab involves only corner columns whereas a four panel slab supported on nine columns involves the combined effects of an internal column, edge columns, and corner columns. In order to analyse the four panel slab under wind loading conditions, it is necessary to consider the separate effect of bending moments applied at each column; these effects can then be superimposed. The panel arrangement and mesh size for a four-panel square slab are shown in Figure 2.27 and a selection of the loading systems to
be considered is shown in Figure 2.28. Distributions of deflection and bending moment for these loading conditions are shown in Figures 2.29 to 2.38.

In order to investigate the local behaviour of the slab around the column, it is necessary to assume a finite area in the slab which is occupied by the column, or column head. In this section a square slab with a column or column head occupying a square of 1/6th or 1/12th of the span width is studied. These column sizes are chosen because 1/6th of the span width is about the average size of the column head in flat slabs with column heads and 1/12th of the span width is about the average size of a column in the flat plate type of construction. The column size is sub-divided into two grid lengths in the finite difference mesh. The rigidity in the column area may be assumed to be infinite. But in order to make the computer programme more general so that the variation in the moment of inertia of the column region or drop panel may be studied, it is taken as a variable. By studying the effect of the ratio of the rigidity of the slab to that of the column region on the solutions it is found that there is practically no variation in most of the solutions with the rigidity of the column region beyond 100 times the rigidity of the slab. Hence, for convenience, the ratio \( \frac{D_{\text{slab}}}{D_{\text{column region}}} = \frac{1}{1000} \quad (C_o = .001) \) is adopted for representing the infinite rigidity of the column head region in the solutions.
The applied bending moment $M$ is assumed to be equivalent to a couple $M = P \times 2a$. $P$ is assumed to spread over one or two grid widths with the forces applied at the grid points as shown in Figure 2.4.

**FIG. 2.4**

**Infinite Slab with Finite Column Area**

The grid size for the slab with a column head of $1/6$th of the span width ($L/W/7$) is shown in Figure 2.39. The deflection curves and distribution of bending moments are shown in Figures 2.40 and 2.41.
For the slab with a column size of 1/12th of the span width (I/W/9) shown in Figure 2.42, the deflection and bending moments are shown in Figures 2.43 and 2.44.

One Panel Slab with Finite Column Area

A one panel square slab with a rigid column region of 1/6th of the span width is shown in Figure 2.45. The deflection and bending moment distribution for this slab subjected to wind moments (O/W/2) are given in Figures 2.46 and 2.47.

The grid for a one panel square slab with a rigid column region of 1/12th of the span width is shown in Figure 2.48. The distribution of deflection and bending moment for this slab under wind moment (O/W/3) are given in Figures 2.49 and 2.50; the distributions under restraining moment (O/R/5) are shown in Figures 2.51 and 2.52; and the distributions under a uniformly distributed load (O/D/3) are given in Figures 2.53 and 2.54.
<table>
<thead>
<tr>
<th>Solution Number</th>
<th>Slab</th>
<th>Loading</th>
<th>$\alpha$</th>
<th>$a$</th>
<th>$C_0$</th>
<th>Col. Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>I/W/1</td>
<td>Infinite slab</td>
<td>Wind moment at internal column</td>
<td>1</td>
<td>L/12</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>I/W/2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>4/3</td>
<td>L/12</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>I/W/3</td>
<td>&quot;</td>
<td>&quot;</td>
<td>1.4</td>
<td>L/12</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>I/W/4</td>
<td>&quot;</td>
<td>&quot;</td>
<td>2</td>
<td>L/8</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>I/W/5</td>
<td>&quot;</td>
<td>&quot;</td>
<td>.5</td>
<td>L/16</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>I/W/6</td>
<td>&quot;</td>
<td>&quot;</td>
<td>3/4</td>
<td>L/12</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>I/W/7</td>
<td>&quot;</td>
<td>&quot;</td>
<td>1</td>
<td>L/12</td>
<td>0</td>
<td>.001 L/6</td>
</tr>
<tr>
<td>I/W/8</td>
<td>&quot;</td>
<td>&quot;</td>
<td>1</td>
<td>L/12</td>
<td>0</td>
<td>.01 L/6</td>
</tr>
<tr>
<td>I/W/9</td>
<td>&quot;</td>
<td>&quot;</td>
<td>1</td>
<td>L/24</td>
<td>0</td>
<td>.001 L/12</td>
</tr>
<tr>
<td>O/R/1</td>
<td>One panel slab</td>
<td>Restraining moment at four columns</td>
<td>1</td>
<td>L/12</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>O/R/2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>1</td>
<td>L/12</td>
<td>.1</td>
<td></td>
</tr>
<tr>
<td>O/R/3</td>
<td>&quot;</td>
<td>&quot;</td>
<td>1</td>
<td>L/12</td>
<td>.35</td>
<td></td>
</tr>
<tr>
<td>O/R/4</td>
<td>&quot;</td>
<td>&quot;</td>
<td>1</td>
<td>L/12</td>
<td>.5</td>
<td></td>
</tr>
<tr>
<td>O/R/5</td>
<td>&quot;</td>
<td>&quot;</td>
<td>1</td>
<td>L/24</td>
<td>0</td>
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<td>&quot;</td>
<td>&quot;</td>
<td>1</td>
<td>L/12</td>
<td>0</td>
<td>.001 L/6</td>
</tr>
<tr>
<td>O/W/1</td>
<td>One panel slab</td>
<td>Wind moment at four columns</td>
<td>1</td>
<td>L/12</td>
<td>.35</td>
<td></td>
</tr>
<tr>
<td>O/W/2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>1</td>
<td>L/12</td>
<td>0</td>
<td>.001 L/6</td>
</tr>
<tr>
<td>O/W/3</td>
<td>&quot;</td>
<td>&quot;</td>
<td>1</td>
<td>L/24</td>
<td>0</td>
<td>.001 L/12</td>
</tr>
<tr>
<td>O/P/1</td>
<td>One panel slab</td>
<td>Point load at centre</td>
<td>1</td>
<td>L/4</td>
<td>.35</td>
<td></td>
</tr>
<tr>
<td>O/P/2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>1</td>
<td>L/8</td>
<td>.35</td>
<td></td>
</tr>
<tr>
<td>O/P/3</td>
<td>&quot;</td>
<td>&quot;</td>
<td>1</td>
<td>L/12</td>
<td>.35</td>
<td></td>
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*cont'd....*
<table>
<thead>
<tr>
<th>Solution Number</th>
<th>Slab</th>
<th>Loading</th>
<th>$\alpha$</th>
<th>$a$</th>
<th>$C$</th>
<th>Col. Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>O/D/1</td>
<td>One panel slab</td>
<td>Distributed load</td>
<td>1</td>
<td>$L/12$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>O/D/2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>1</td>
<td>$L/12$</td>
<td>.5</td>
<td></td>
</tr>
<tr>
<td>O/D/3</td>
<td>&quot;</td>
<td>&quot;</td>
<td>1</td>
<td>$L/24$</td>
<td>0</td>
<td>.001 $L/12$</td>
</tr>
<tr>
<td>F/W/1</td>
<td>Four panel slab</td>
<td>Wind moment at centre column</td>
<td>1</td>
<td>$L/8$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>F/W/2</td>
<td>&quot;</td>
<td>Wind moment along the edge at centre edge columns</td>
<td>1</td>
<td>$L/8$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>F/W/3</td>
<td>&quot;</td>
<td>Wind moment across the edge at centre edge columns</td>
<td>1</td>
<td>$L/8$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>F/W/4</td>
<td>&quot;</td>
<td>Wind moment at four corner columns</td>
<td>1</td>
<td>$L/8$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>F/D/1</td>
<td>Four panel slab</td>
<td>Up and down distributed load on two strips</td>
<td>1</td>
<td>$L/8$</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

**List of Numerical Solutions**

- **Type of slab**
  - I - Infinite number of panels, Internal column
  - O - One panel slab
  - F - Four panel slab

- **Loading**
  - W - Wind moment
  - R - Restraining moment

- **Conditions**
  - P - Point load
  - D - Distributed load
Discussion

Figure 2.55 reproduces the deflection curves along the column line for the infinite slab with the column assumed to be a point and for the wind moment \( M \) applied across one grid width or two grid widths; with the column size equal to 1/12th of the span width and the rigidity of the column region equal to 100 or 1000 times the slab and the moment \( M \) applied across one or two grid widths; and with a column head size of 1/6th of the span width and the rigidity of the column region equal to 100 or 1000 times that of the slab and the moment \( M \) applied across one or two grid widths.

From this figure, it is observed that with the rigidity of the column region taken into consideration and assumed to have a value 100, 1000 or more times the rigidity of the slab itself, the results of the numerical solutions are practically the same whether the moment is distributed across one or two grid widths. On the other hand, if the rigidity of the column region is not taken into consideration, the way that the moment \( M \) is assumed to apply will affect the solution considerably. Furthermore, the effect of varying the rigidity of the column region from 1 to 100 times the rigidity of the slab produces quite a big difference in the solutions; but from 100 times to 1000 times the difference is negligible.

It is noticed that the deflections at the fictitious point show marked differences in magnitude for various assumptions made at the
column region, though the actual deflection curves inside the slab may be quite close to one another. That is the reason why the slopes for the point-support cases calculated from the 3 point formula are not very accurate, but a formula using more forward points may give better results. However, with the column region assumed 1000 times more rigid (i.e. equivalent to infinite rigidity) the slope is then exactly defined.

The rotation of a simple beam with a rigid portion at the support stretching to the same extent as in the slab is calculated by the same numerical method and is employed to compare with the rotation of the slab. This rotation of the simple beam which was obtained by means of the numerical method was checked with the rotation calculated by the moment-area method and the accuracy of the numerical method was proved to be within 0 to +2 per cent. Results of the comparison are given in the following table.

The figures in the column numbered below denote:

1. Column size
2. Ratio of the rigidity of the slab to that of the column region $C_o$
3. Number of grid widths $M$ is assumed to apply
4. Rotation of the slab, in $(M/D)$
5. Rotation of the beam, in $(ML/EI)$
6. Ratio of the effective width to span width.

<table>
<thead>
<tr>
<th>Column Size</th>
<th>Ratio of slab to column region $C_o$</th>
<th>Number of grid widths $M$</th>
<th>Rotation of slab $(M/D)$</th>
<th>Rotation of beam $(ML/EI)$</th>
<th>Ratio of effective width to span width</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2.ii

From these results, it may be concluded that for an internal square panel in an infinite slab subjected to wind moments from the columns, the effective width of the slab to be assumed as beam in the elastic frame analysis can be taken as 45 per cent of the span width in the case of flat plate construction where the column size is about 1/12th of the span width; and as 55 per cent in the case of a flat slab with column head where the column head size is about 1/6th of the span width.

For different shapes of panels where the span length width ratio is not 1:1, it can be deduced, from Table 2.i, that the effective width increases as the length width ratio increases and decreases as the length width ratio decreases.

Figure 2.56 reproduces the deflection curves along the column
line of the one panel square slab with point-supports and with columns of sizes of 1/6th, 1/8th and 1/12th of the span width and the rigidity of the column region equal to 1000 times the slab. The comparison in this case is different from the infinite slab case because a change in the column dimension also changes the overall dimension of the slab. In spite of this, it is possible to make a comparison of the extent of the slab from the column centre line which can be assumed as effective.

Comparative values are tabulated below.

In the table, column

(1) Column size
(2) Total width of slab, in terms of \( L \)
(3) Rotation of the slab, in \( (N/D) \)
(4) Rotation of the beam, in \( (ML/DL) \)
(5) Effective width / total width
(6) Effective extent of the slab from the column centre line, in \( L \)
(7) Corresponding values to (6) in the case of the infinite slab.

+ Note: \( EI \) of slab cross-section = \( D \times \) total width.

There are two \( M \)'s (one from each corner) applied on the span width of the slab.
Table 2.iii

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>p't</td>
<td>1</td>
<td>1.680</td>
<td>.344</td>
<td>.200</td>
<td>.100</td>
<td>.107</td>
</tr>
<tr>
<td>L/12</td>
<td>13/12</td>
<td>.4517</td>
<td>.238</td>
<td>.527</td>
<td>.244</td>
<td>.235</td>
</tr>
<tr>
<td>L/8</td>
<td>9/8</td>
<td>.3360</td>
<td>.1986</td>
<td>.591</td>
<td>.270</td>
<td>-</td>
</tr>
<tr>
<td>L/6</td>
<td>7/6</td>
<td>.2613</td>
<td>.1656</td>
<td>.634</td>
<td>.287</td>
<td>.285</td>
</tr>
</tbody>
</table>

From columns (6) and (7) in the above table it can be seen that the effective extent of the slab from the column centre line for stiffness is approximately the same for a corner column as for an internal column. From Figure 2.56, it can be seen that the rigidity and size of the column region affects the solution in the same way as in the infinite slab case.

So far, the discussion has been concerned with the effective width of the slab regarding its stiffness. As for its strength in resisting the bending moment at the critical section it is noted from the moment distribution curves, that the intensity of moment is very high at the corner of the column but falls off very rapidly from the sides of the column. The moment is about one third of the maximum value at a distance of half the column size from the side of the column. So there is, according to the elastic analysis, a concentration of moment within the column region. To determine this amount of moment inside the width of the column band shown in Figure 2.vi, the intensities
Concentration of bending moment near column

FIG. 2.vi
of moment at the grid points are assumed to be uniform across one grid width and the bending moments given by the three grid points on the column edge are summed up. The amount of this moment and its ratio to the total moment across that section line are given below.

<table>
<thead>
<tr>
<th>Solution Number</th>
<th>( m_1 ) (( \frac{M}{L} ))</th>
<th>( m_0 ) (( \frac{M}{L} ))</th>
<th>( m_1 ) (( \frac{M}{L} ))</th>
<th>Portion of ( M ) inside B</th>
<th>Percentage of the total moment at that section</th>
</tr>
</thead>
<tbody>
<tr>
<td>I/W/7</td>
<td>2.15</td>
<td>1.90</td>
<td>2.15</td>
<td>.516</td>
<td>62</td>
</tr>
<tr>
<td>O/W/2</td>
<td>2.61</td>
<td>2.64</td>
<td>2.90</td>
<td>.570</td>
<td>68.4</td>
</tr>
<tr>
<td>O/R/6</td>
<td>2.17</td>
<td>2.67</td>
<td>3.45</td>
<td>.600</td>
<td>60</td>
</tr>
<tr>
<td>I/W/9</td>
<td>4.34</td>
<td>3.73</td>
<td>4.34</td>
<td>.517</td>
<td>56.4</td>
</tr>
<tr>
<td>O/W/3</td>
<td>4.20</td>
<td>4.85</td>
<td>6.08</td>
<td>.544</td>
<td>59.4</td>
</tr>
<tr>
<td>O/R/3</td>
<td>3.30</td>
<td>4.60</td>
<td>6.46</td>
<td>.530</td>
<td>53</td>
</tr>
</tbody>
</table>

Table 2.iv

From this result, it is seen that with column size equal to 1/6th of the span width about 60 to 68 per cent of the moment across the whole width of the slab at the critical section is concentrated inside the column band B bounded by lines 1/4 of the column size from the column face; and with column size equal to 1/12th of the span width, about 53 to 60 per cent concentrated inside the band. It is also noticed that the concentration is somewhat less in the case of restraining moment than in the case of wind moment.
If the total moment at the section is equated to that of a rectangular distribution with an ordinate equal to the maximum unit moment at that section, the equivalent width of the slab to be taken for checking the bending stresses can be determined. By subtracting the column width from it, the remaining part gives the portion of the slab beyond the column faces which is active. The ratio of this portion of the slab to the size of the column is calculated and is given in the following table.

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I/W/7</td>
<td>2.15</td>
<td>5/6</td>
<td>.388</td>
<td>.111</td>
<td>.665</td>
</tr>
<tr>
<td>O/W/2</td>
<td>2.90</td>
<td>5/6</td>
<td>.287</td>
<td>.120</td>
<td>.720</td>
</tr>
<tr>
<td>O/R/6</td>
<td>3.45</td>
<td>1</td>
<td>.290</td>
<td>.123</td>
<td>.738</td>
</tr>
<tr>
<td>I/W/9</td>
<td>4.34</td>
<td>11/12</td>
<td>.211</td>
<td>.064</td>
<td>.767</td>
</tr>
<tr>
<td>O/W/3</td>
<td>6.08</td>
<td>11/12</td>
<td>.151</td>
<td>.0675</td>
<td>.810</td>
</tr>
<tr>
<td>O/R/5</td>
<td>6.46</td>
<td>1</td>
<td>.155</td>
<td>.0715</td>
<td>.857</td>
</tr>
</tbody>
</table>

Table 2.v

where column

(1) Solution number
(2) Maximum unit moment
(3) Total moment at critical section from each column
(4) Equivalent width of slab for checking bending stresses

(5) Active portion of slab beyond the sides of the column

(6) Ratio of active portion of slab to size of column.

From the figures in columns (5) and (6), it can be seen that the active portion of the slab beyond the sides of the column is apparently in direct proportion to the column size. Its ratio, therefore, gives a good measure of it.

Besides depending on the column size, the active portion of the slab depends also on the location of the column and the loading condition as well. It varies from .66 to .74 of the column size for a column head size of $1/6\ L$ and from .76 to .86 of the column size for a column of size equal to $1/12\ L$.

All these discussions and conclusions are based on the thin plate analysis. It is clear that the third dimension of the slab will certainly affect the results.

The problem of the shear stress distribution around the column has not been tackled in this analysis because it can be visualized by intuition that it does not depend so much on the plane dimensions of the slab but more on the local dimensions and detailing as well as the slab thickness at the column. This study is outside the scope of this thesis.
SMALL SCALE ELASTIC MODEL INVESTIGATION

In order to verify the numerical solutions obtained by the finite difference method, an elastic model was tested. The model was designed to simulate a one panel square slab supported at four corners by point supports. The material chosen was perspex sheet because it has a smooth surface which is necessary for the instruments employed; it is easy to machine and to join together; and it has a low elastic modulus so that large deformation is produced with a small load.

The model was 12 inches square and 3/8 in. thick. Rectangular strips of 2 1/2" x 1" protruded at the corners for applying the bending moments as well as for providing a small area at the corner for the support.

The slab was supported from below, and also from above, by four cantilever steel rods located in the cone seatings in steel discs embedded in the slab. One was rigid enough to prevent sway, while the rest (3 inches long and 1/16 inch diameter) were flexible enough to allow lateral movement.

Two loading conditions were imposed on this model: one was a system of symmetrical moments applied at the corners to simulate restraining moments; and the other was a system of anti-symmetrical moments to represent wind moments. These
moments were actually produced by hanging weights at 2 inches distance from the support on the protruding strips. In the case of wind moments, the weights on one side of the slab were each applied through a pulley to change their direction. The weights were supported on a platform when they were not applied to the model. The platform was fixed to the ram of a hydraulic jack by means of which it could be raised and lowered. In loading the model the platform was lowered and the weights were then suspended on the model.

All these arrangements are shown in Figure 2.57.

The instrument used to measure the change of curvature at a point due to loading and unloading was a curvature meter. It consisted of a displacement transducer of the differential transformer type mounted vertically at the midpoint between two fixed legs on a supporting body, as shown in the photographs on Figure 2.58. This type of displacement transducer gives a possible discrimination of $1 \times 10^{-6}$ in. on the scale of a BPA multi-meter amplifier. This enabled the relative deflection $\Delta$ between the legs and the transducer core rod to be measured with sufficient accuracy on a grid length $'a'$, which was the distance between the core rod and each leg, of half an inch.
From this deflection $\Delta$, the curvature at the point can be calculated from the finite difference expression:

\[
\frac{1}{R_x} = \frac{\partial^2 w}{\partial x^2} = \frac{w_1 - 2w_o + w_1}{a^4} = \frac{2\Delta x}{a^4}
\]

\[
\frac{1}{R_y} = \frac{\partial^2 w}{\partial y^2} = \frac{w_2 - 2w_o + w_2}{a^4} = \frac{2\Delta y}{a^4}
\]

And the bending moments per unit width, from

\[
m_x = -p \left( \frac{\partial^2 w}{\partial x^2} + \rho \frac{\partial^2 w}{\partial y^2} \right)
\]

\[
m_y = -p \left( \frac{\partial^2 w}{\partial y^2} + \rho \frac{\partial^2 w}{\partial x^2} \right)
\]
The accuracy of these finite difference formulae depends on the deflected shape of the plate and the grid length 'a', and cannot be stated in general terms. For example, the error for a parabolic curve is zero; the error at the peak of a sine curve given by
\[ w = \sin \frac{\pi x}{L} \]
depends on 'a' in the following way.

\[
\begin{array}{c|cccc}
 a & \frac{L}{4} & \frac{L}{8} & \frac{L}{16} & \frac{L}{32} \\
\hline
\text{Percentage error in} \frac{\partial^2 w}{\partial x^2} & 5.04 & 1.28 & 0.32 & 0.09
\end{array}
\]

The dimension for the model has been chosen such that 'a' was \( L/12 \) and \( L/24 \) in the wind moment and restraining moment cases respectively. Therefore the error due to the above approximate expressions was likely to be very small.

The deflections were measured by mechanical dial guages reading to \( 1 \times 10^{-4} \) of an inch. They were supported in a row by a beam which could be shifted along two parallel girders on the supporting frame.

All the measurements obtained from the experiment were converted to values of dimensionless groups so that they could be compared with the numerical values from the finite difference solution. For example, deflections were expressed by values of \( \frac{wD}{M} \) and moments per unit width, by \( \frac{ML}{M} \).
Deflections for the slab subjected to symmetrical bending moments applied at the corners are given in Figures 2.59 to 2.62. The bending moment $m_x$ across certain sections in the slab are shown in Figures 2.63 to 2.65.

The deflections and distributions of bending moments $m_x$ and $m_y$ for the wind moment case are shown in Figures 2.67 to 2.69, Figures 2.70 and 2.71, and Figure 2.72 respectively.

From the Figures, it can be seen that there is good agreement between the elastic model results and the numerical solutions with a finite difference grid size $a = L/12$. However, there are a few points to be noted. The measured deflections in the centre portion of the slab and on the edges normal to the direction of the applied moments are slightly less than the theoretical values. The measured bending moments $m_x$ coincided practically with the theoretical results, but for bending moments $m_y$ there is some scatter of the experimental results. In assessing the magnitude of the discrepancies between the experiments and finite difference solutions it should be noted that moments $m_y$ are plotted to a larger scale than $m_x$. Furthermore, the curvature in the $x$-direction is much larger than that in the $y$-direction, so the absolute error in $\frac{\partial^2 \omega}{\partial x^2}$ (and hence in $\mu \frac{\partial^2 \omega}{\partial x^2}$) is relatively large. Thus the percentage error in $m_y$ is much greater than the percentage error in $m_x$. 
From this experiment, it is seen that the elastic behaviour of the one panel slab as observed here was as predicted by the numerical solution. Hence it could be concluded that the numerical method used in the previous section gives reliable solutions of good accuracy.
CHAPTER 3

EXPERIMENTS ON A FULL SCALE REINFORCED CONCRETE SLAB

DESIGN OF THE TEST SLAB

Introduction

This chapter describes an experiment on a full scale reinforced concrete flat slab structure subjected to distributed loading and to restraining and wind moments from the columns. As is well known, reinforced concrete is not a homogeneous, isotropic, elastic material, and until now there is no theory which can deal exactly with this material. Most analyses are based on the assumptions of the theory of elasticity, as in the previous chapters of this thesis, or on the ultimate load yield line theory. The purpose of this experiment was to compare the actual behaviour with that predicted by elastic and yield line analysis.

It was decided that to begin with a flat slab of the simplest type should be employed. Naturally this led to a one panel slab supported at the four corners by columns without column heads or edge beams - a so-called flat plate type structure. In spite of the fact that a one panel flat
slab like this is seldom built in practice, it is in every sense a flat slab and it offers the most favourable case for purposes of comparison because it can be analysed fairly accurately in its entirety by the numerical method.

The design of a one panel flat slab is not covered by the empirical method in the C.P.114, and has to be based on a rational elastic analysis. The finite difference and the slope-deflection methods were used in the design. Due consideration was given to anything in C.P.114 which might be applicable to this case. No effort was made to emphasize any particular parameter in the design. The aim was to achieve a well-balanced design which was representative of normal practice.
Design Considerations

Overall Dimensions

The laboratory floor has anchorage holes at 2 feet centres. It was therefore convenient to have 12 foot column centres. With columns 1 ft square, the overall dimensions of the slab were 13 ft x 13 ft.

An actual building may have a floor to floor height of 11 to 12 ft. Assuming inflection points occur at the mid-height of the columns, there will be a hinge-ended column of 6 ft long below and above the slab for each floor. Combining the effect of their stiffness in a single lower column results in a column 3 ft long. This is also a suitable height for the arrangement of the supports and loading systems. Due to the arrangement of the ball seating, the actual height from the centre of the supporting ball to the mid-plane of the slab was 3 ft 1\(\frac{1}{2}\) in.

Design Loads

The vertical uniformly distributed load to be assumed in the design included the dead load, superimposed load, and the live load on a typical structure. With a span of 12 ft the thickness of the slab was estimated to be 6 in., giving a self weight of 75 lb/sq. ft. approximately. A superimposed load of 25 lb/sq. ft. and a live load of 100 lb/sq. ft. is
quite common for this type of construction, and a total uniformly distributed load of 200 lb/sq. ft. was assumed as the design vertical load.

The horizontal load to be assumed for a floor in a tall building depends on the wind pressure, the distance of the floor under consideration to the roof, and the transverse span. It was the intention in this experiment to emphasise the effect of wind load within realistic limits. Its magnitude was decided after the next consideration.

Proportion of the Negative and Positive Design Moments in the Test Slab

The minimum total thickness of the slab specified in C.P.114 is 5 in. or L/32, which is 4.5 in. in this case.

It was assumed that:

The total thickness of slab \( t = 6'' \)

Column size \( c = 12'' \) square

The height of the column from the mid-plane of the slab to the centre of the ball at the column end \( h = 37.5'' \)

The elastic modulus of concrete \( E = 4 \times 10^6 \) p.s.i.

The gross section area of concrete was taken for calculating the stiffness of the slab \( D \) and the stiffness of the column \( EI \).
From the finite difference solutions:

Due to the vertical uniformly distributed load \( q \),

unrestrained rotation of the slab corner,
\[
\Theta = 1.2043 \frac{M}{D}
\]

Due to the restraining moments \( M \) applied at the corners
in both directions, rotation of the slab at the corner,
\[
\Theta = \frac{M_h}{3EI}
\]

provided that the ball seating support at the end of
the column is rigidly located in position and that there
is no horizontal movement of the end of the column.

Substituting these quantities into the Slope-Deflection Equation,
\[
1.2043 \frac{M}{D} + \frac{M_h}{3EI} = 0.04452 \frac{qL^3}{D}
\]
or
\[
M (1.2043 + 0.130) = 0.04452 qL^3
\]

\[
M = 0.0334 \frac{qL^3}{\frac{1}{30}} = 0.0283 \frac{qL^3}{D}
\]

It was anticipated that there would be some horizontal
movement of the ball seating on the dynamometer under load
and hence the column would have a certain amount of free
rotation. This would relax the restraining negative moment
redistributing it to the positive section. It was necessary
in the design to make some allowance for this effect, and a
15 per cent reduction and redistribution of the negative
moment was made, leaving the negative restraining moment
\[
M = 85 \text{ per cent} \times 0.0334 \frac{qL^3}{D} = 0.0283 \frac{qL^3}{D} \times \frac{1}{36} qL^3
\]
Hence, the design restraining moment was assumed to be
\( \frac{1}{36} qL^3 \) for each column in each direction (measurements of
column base movement made during the test indicated a moment
reduction varying between 10 and 14 per cent).

With the design uniformly distributed load \( q = 200 \) lbs
per square foot on the slab and restraining moments of
\( M = \frac{1}{36} qL^3 = 115,200 \) lb-ins applied at the corners, the
distribution of the negative and positive moments at the
critical sections, according to the finite difference solutions,
are as shown in Figures 3.1 to 3.2.

From these figures, the total negative moment across the
critical section is -152,200 lb-ins. The total positive
moment across the mid-span is 288,000 lb-ins.

Hence the sum of the negative and positive moments is
\( M_{\text{total}} = 440,200 \) lb-ins.

From the expression given in C.P.114, (for slabs having at
least three bays), the sum of the negative and positive
moments should be

\[
M_o = \frac{1}{10} qL^3 \left( 1 - \frac{2}{3} \frac{c}{L} \right)^2
= \frac{1}{10} \times 200 \times 144 \times 144 \times \left( 1 - \frac{2}{3} \frac{1}{12} \right)^2
= \frac{1}{10} \times 370,000 \text{ lb-ins.}
\]

which is less than the calculated total design moment.
In this case, the static moment coefficient $1/8$ is more appropriate. That gives

$$M_{\text{static}} = \frac{1}{8} qL^3 \left(1 - \frac{2}{3} \frac{c}{L}\right)^2$$

$$= 463,000 \text{ lb-ins.}$$

which is very close to the calculated total design moment.

So the total design moment was taken to be

$$M_{\text{total}} = 440,200 \text{ lb-ins}$$

and the negative and positive proportions were

$$\frac{-152,200}{440,200} = -34.6 \text{ per cent}$$

and

$$\frac{288,000}{440,200} = 65.4 \text{ per cent}$$

respectively.

The wind moment was estimated to be of such amount that it would not reverse the resultant moment at the windward side when combined load was applied. For nearly zero moment at the critical section due to combined load the wind moment should be about 80 per cent of the restraining moment. This gave the magnitude of the wind moment

$$M_w = 80 \text{ per cent} \times \frac{1}{36} qL^3$$

$$= \frac{1}{45} qL^3 \text{ per column.}$$

Therefore the horizontal force per column line is

$$H = \frac{2 M_w}{h} = \frac{2}{37.5} \times \frac{1}{45} \times 200 \times 144 \times 144$$

$$= 4930 \text{ lb.}$$
The total wind load on the floor is 9860 lb. Assuming a wind pressure of 15 psf on the side of the building, this load represents the wind load at a distance from the roof of

\[
\frac{9860}{15 \times 0.2} = 55 \text{ ft. which is about five stories down from the roof. The distribution of the moment at the critical sections due to this wind load are shown in Figures 3.3 and 3.4.}
\]

**Distribution and Detailing of the Slab Reinforcement**

The distribution of reinforcement in the slab was mainly divided into two groups: the middle strip and the column strip, as specified in the Code of Practice. Using 3/8 in. diameter deformed bars with a permissible stress \( t = 30,000 \) psi, the permissible moment of resistance at different spacings and at effective depths corresponding to the two layer of bars, from the expression \( N_t = A t \frac{7}{8} d \), are

<table>
<thead>
<tr>
<th>Spacing:</th>
<th>9&quot;</th>
<th>6&quot;</th>
<th>3&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d = 5.3&quot; ):</td>
<td>20,500</td>
<td>30,600</td>
<td>61,200</td>
</tr>
<tr>
<td>( d = 4.9&quot; ):</td>
<td>19,000</td>
<td>23,400</td>
<td>56,800</td>
</tr>
</tbody>
</table>

In order to suit the combinations of the distribution of moments as shown in Figures 3.1 to 3.4 approximately, the following distributions of reinforcement were adopted.
For sections in the transverse direction of wind

At the mid-span section for positive moments

9" spacing in the Middle strip and
6" spacing in the column strip

At the critical section for negative moment

6" spacing in the column strip only.

It was not necessary to provide any negative reinforcement in the middle strip, as can be seen from the moment distribution diagram. Furthermore, the moment peak at the corner of the column was being neglected. It was considered to be a very localized effect and would be redistributed as local cracking started to occur.

For sections in the direction of the wind

At the mid-span section for positive moment, the distribution of moment would be the same as for vertical load only. The same reinforcement spacing as in the mid-span section in the transverse direction of wind was adopted.

At the critical section for negative moment

3" spacing in the nearer half of the column strip
6" spacing in the further half of the column strip

no negative reinforcement was provided in the middle strip.

Again, the moment peak at the corner of the column was neglected. The same reinforcement spacings were adopted for the windward side as well.
The distributions of the designed moment of resistance in the slab are also shown in Figures 3.1, 3.2, and 3.5 against the distribution of moments by calculation.

The negative steel was well anchored by a hook at the end. The cutting length was 3' 8" from the column centre line which was 3.67/12 = 0.306L. The ACI 318-56 Code specifies a length of 0.30 L. From the bending moment surface, there was at least a length of 8" beyond the negative moment envelop around the column for the 3/8 in. diameter bars.

The positive steel consisted of straight bars extending to two inches from the edge of the slab. Anchorage presents no problem in this case.

Although not required by the calculated bending moments, three long bars at one foot centres were placed in the top of the slab in the column strip and were spliced to the negative bars over a splice length of 15 inches. They would not interfere with the negative steel at the critical section. The detailing of the slab reinforcement is shown in Figure 3.6.

Design of the columns

The columns were designed for the direct load and the bending moments in both directions by means of the column analogy method. A one foot square section reinforced by four one inch diameter deformed bars was adopted. The main problem
here was in the detailing of the reinforcement for anchorage. At the lower end there was a three foot length of column for bond and a hook could be provided at the end for anchorage. But at the upper end, anchorage was more difficult to achieve. If the bars were bent into the slab they would interfere with the slab negative reinforcement and make the analysis of results complicated. Another way was to weld the slab reinforcement to the column reinforcement at the top. But this was not a very common practice. To solve this problem it was decided to provide a column stub 18 inches long above the slab so that the column steel could go straight through and could be anchored properly by a hook. Besides, this stub also represented the root of the upper column, and its presence was desirable for a more realistic cracking pattern on top of the slab.

Analysis of the Shearing Stress at the Junction of the Slab and Column.

The shearing stress in a flat slab is not only caused by the gravity load alone, but the transfer of bending moment from the column to the slab also produces torsional shearing stress. The ACI - ASCE Committee 326 recommended the following formula for analysing the combined shearing stress:

\[ v_u = \frac{V}{b_o d} + \frac{KM}{f'_c} \left( \frac{c}{2} \right) \leq 4.0 \sqrt{\frac{f'_c}{2}} \]
where

\[ M = \text{the total joint moment on the pseudocritical peripheral section about its centroid} \]

\[ J_c = \text{the polar moment of inertia of the pseudocritical peripheral section about its centroid} \]

\[ c = \text{the side of the pseudocritical section perpendicular to the axis of torsion} \]

\[ b_o = \text{effective perimeter of peripheral section} \]

\[ K = \text{a reduction factor on the total moment to obtain the moment transferred by torsional shear stress, found to be 0.2 on the basis of the limited test data available, but may approach zero or take values greater than 0.2 under other conditions.} \]

The formula is for an internal column with bending moment applied in one direction along a symmetrical axis. Assuming that the same approach and the same reduction factor \( K \) can be applied to the case of corner column subjected to bi-axial bending, the following formula is obtained:

\[ V_{u, \text{bod}} = \frac{V}{S_x d} + \frac{2 M_x}{S_x d} + \frac{2 M_y}{S_y d} - \frac{f_c}{c} \]

where \( S_x, S_y = \text{modulus for the periphery of the stress block in the } x, y \text{ direction with regard to the point under consideration.} \]
The dimensions of the stress block and the worst loading case at design load in the test slab are as shown in Figure 3.1.

\[ P = 8485 \text{ Lb.} \]
\[ M_x = 115,000 \text{ Lb.-in.} \]
\[ M_y = 207,200 \text{ Lb.-in.} \]

**Effective depth** \( d = 5.3'' \)

\[ b_o = AB + AC = 29.3'', \quad \frac{AC}{AB} = 1 \]

For point \( A \),
\[ S_y = \frac{AB^3}{6} \left( 1 + 4 \frac{AC}{AB} \right) = 180 \text{ in.}^2 \]

For point \( B \),
\[ S_y = \frac{AB^3}{6} \left( 1 + 4 \frac{AC}{AB} \right) \left( \frac{AB}{AB + 2AC} \right) \]
\[ = 60 \text{ in.}^4 \]

For point \( A \) or \( B \),
\[ S_x = \frac{AC^2}{6} \left( 1 + 4 \frac{AB}{AC} \right) \]
\[ = 180 \text{ in.}^2 \]
Assuming a load factor = 2

At A, \( V_u = 2 \left( \frac{8485}{5.3 \times 29.3} + \frac{23000}{5.3 \times 180} + \frac{41440}{5.3 \times 60} \right) \)

= 244 psi.

At B, \( V_u = 2 \left( \frac{8485}{5.3 \times 29.3} + \frac{23000}{5.3 \times 180} - \frac{41440}{5.3 \times 60} \right) \)

= -103 psi.

Assuming \( f'_c = 4000 \) psi

\[ 4 \sqrt{f'_c} = 4 \sqrt{4000} = 250 \]

\[ \therefore V_u \leq 4 \sqrt{f'_c} \]

Therefore the design was adopted.
Reinforcement steel

The reinforcement used in the slab consisted of 3/8 in. diameter deformed bars which were manufactured by the George Cohen 600 Group Limited under the trade name 'Norhite 60' reinforcing bars. These bars are produced with a hot rolling process and have a high tensile strength, a defined yield point, and a high ductility. The guaranteed minimum yield stress is 60,000 psi and the average percentage elongation is 22. The deformations consist of both transverse and longitudinal ribs giving a good bond strength and crack control property.

The mechanical properties and the stress-strain relationship of these bars were determined by testing six samples in an Amsler Testing Machine. Typical stress-strain curves are shown in Figures 3.7 and 3.8.

The results are given in the following table.

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Yield stress psi</th>
<th>Ultimate Strength psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>73,100</td>
<td>114,500</td>
</tr>
<tr>
<td>2</td>
<td>73,500</td>
<td>112,700</td>
</tr>
<tr>
<td>3</td>
<td>74,000</td>
<td>110,000</td>
</tr>
<tr>
<td>4</td>
<td>73,500</td>
<td>114,000</td>
</tr>
<tr>
<td>5</td>
<td>73,800</td>
<td>---</td>
</tr>
<tr>
<td>6</td>
<td>73,600</td>
<td>---</td>
</tr>
<tr>
<td>Average</td>
<td>73,600</td>
<td>112,800</td>
</tr>
</tbody>
</table>

Table 3.1
Average Modulus of Elasticity = $29.2 \times 10^6$ psi

The reinforcement used for the column was of 1 inch diameter deformed bars of the same type as the slab reinforcement. One specimen of this size of bar was tested and it showed similar properties to the 3/8 in. diameter bars. The ties for the column reinforcement were made from 1/8 in. diameter plain round bars taken from stock.

Concrete

The concrete used for casting the slab was supplied by Ready Mixed Concrete Limited. The mix was specified to give a cube strength of 5,000 psi at 28 days. Ordinary Portland Cement of the Blue Circle Brand, manufactured by the Cement Marketing Company, was used. The coarse aggregate of a maximum size of 3/4 in. was from Egham and was of irregular or round shape. The 3/8 in. gravel was from Felixstowe and was of irregular or round shape. The sand was also from Felixstowe and was of natural shape. The aggregates were basically quartzite in quality. The moisture content in the aggregates varied from 2 to 4 o/o. The water/cement ratio in the mix was 0.55 and the slump was 2 inches.
For a cubic yard of concrete the batch weights were:

<table>
<thead>
<tr>
<th>Material</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement</td>
<td>540 lb</td>
</tr>
<tr>
<td>Sand</td>
<td>1280 lb</td>
</tr>
<tr>
<td>3/8 Aggregate</td>
<td>450 lb</td>
</tr>
<tr>
<td>3/4 Aggregate</td>
<td>1550 lb</td>
</tr>
<tr>
<td>Water</td>
<td>175 lb</td>
</tr>
</tbody>
</table>

Four and a half cubic yards of concrete were required for casting the slab, the strip beams, and the control specimens. It was delivered in two batches.

The control specimens made for determining the properties of the concrete were four 6 in x 12 in. cylinders for finding the stress-strain relationship and compressive strength, four 6 in x 9 in cylinders for splitting strength, four 6 in x 5 in x 38 in rectangular beams for modulus of rupture and ten 6 in. cubes for compressive cube strength. These control specimens were tested in a Denison Machine and in an Amsler Machine according to the methods specified by the British Standard, 1881.

The properties of the concrete are summarized in the following tables.
<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Batch</th>
<th>Date of Testing</th>
<th>Age Days</th>
<th>Cube Strength psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>24-6-64</td>
<td>7</td>
<td>3240</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>15-7-64</td>
<td>28</td>
<td>4730</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>17-11-64</td>
<td>154</td>
<td>7500</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>&quot;</td>
<td>&quot;</td>
<td>7250</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>7620</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>5070</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>21-12-64</td>
<td>188</td>
<td>6900</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>&quot;</td>
<td>&quot;</td>
<td>6850</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>6400</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>&quot;</td>
<td>&quot;</td>
<td>5310</td>
</tr>
</tbody>
</table>

Average of No. 3-10 6610

Table 3.ii Cube strength of Concrete

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Batch</th>
<th>Modulus of Splitting Rupture psi</th>
<th>Modulus of Cylinder Strength psi</th>
<th>Modulus of Modulus of Elasticity psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>520</td>
<td>475</td>
<td>5150</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>546</td>
<td>457</td>
<td>3560</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>520</td>
<td>444</td>
<td>3780</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>496</td>
<td>449</td>
<td>3520</td>
</tr>
</tbody>
</table>

Average

<table>
<thead>
<tr>
<th>Date of Testing Age</th>
<th>Modulus of Splitting Rupture psi</th>
<th>Modulus of Cylinder Strength psi</th>
<th>Modulus of Modulus of Elasticity psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>16-11-64 154</td>
<td>520</td>
<td>456</td>
<td>4000</td>
</tr>
</tbody>
</table>

Table 3.iii Properties of Concrete

The stress-strain curves are shown in Figure 3.9
Fabrication

The base of the column was a 1 ft. square, 1 in. thick steel plate with four $\frac{1}{2}$ inch diameter shear studs welded on the top and with eight 3/8 inch screw holes at the bottom for connection to a ball seating. The forms for the columns were supported 1 ft. above the floor by blocks to make room for the dynamometers. They were properly plumbed and centered and were clamped to the scaffolding frame which was used to support the slab and column framework. The framework for the slab consisted of 1 inch thick plywood panels. They were supported by 2 x 4 inches wooden joists placed at about 12 to 18 inches spacing and clamped on the scaffolding. The formwork was accurately squared and levelled. No camber was introduced in the slab.

Positions of the bottom steel and the loading holes were marked on the formwork. Gaps between the joints were sealed with plasticine. A coat of lacquer was applied to the formwork which was then greased. The column reinforcement cages were dropped into the forms and were held in position by spacers. The positive slab reinforcing bars were placed according to the marks made on the formwork and were tied together at the intersections to form a mat. The mat was supported by 1/2 inch thick
mortar spacer books. Finally, the mats of the negative reinforcement were put in place. Those negative bars which intersected the column bars were inserted and tied to the mats. Steel legs were welded on the mats of negative reinforcement to hold them rigid and at the proper depth.

The leads from the strain-gauges on the reinforcement were passed through holes drilled in the formwork or were held up from above. To provide holes in the slab for the loading rods to pass through, these steel tubes of 5/16 in. diameter and 6 in. long were fixed on the formwork by threaded studs. They were held in position from above by angles tied to the scaffolding.

The required amount of concrete for casting was delivered in two batches, with an interval of one hour. The order of casting was as follows: the columns up to the level of the slab, the portions of the slab around the columns, the central portion of the slab, the upper column stubs. Thus, by the time the upper column stubs were cast the concrete around the columns was strong enough to prevent the concrete in the slab
around the columns flowing due to the pressure from the concrete in the upper column stubs. As the casting of the slab proceeded, the control specimens and the strip beams were also cast. About an equal number of specimens were made from each batch of concrete.

The concrete was vibrated thoroughly with three poker vibrators. A length of steel angle was used for screeding the surface of the slab, using the sides of the slab formwork as screed rails. Due to the presence of the tubes for the loading holes and some of the strain gauge wires coming out of the slab from the top, it was not possible to run the screed over the whole slab. Local screeding was done with trowels and floats. The whole procedure of casting took about two and a half hours.

The curing procedure started about two hours after casting. The whole slab was covered with wet hessian. It was then wrapped with a sheet of polythene. The control specimens and the strip beams were cured in the same way in the laboratory.
The forms for the columns and the sides of the slab formwork were stripped on the following day. The hessian was kept wet for seven days. After that the polythene and hessian were removed and the slab and the other specimens were left in the laboratory without other treatment. The bottom panels of the slab formwork were removed at seven days, but the slab was propped at the centre and the edges by the posts of the scaffolding to prevent further deflection due to creep.

The thickness of the slab was measured by means of a depth micrometer through the loading holes. Since the top of the slab was not very even around the holes, a rectangular bar with two packing legs was put across the hole as a datum. Several readings were taken with the legs placed at 6 inches to 1 foot apart and the mean value was taken as the thickness of the slab at that spot. The variation of thickness in the slab is given in Figure 3.10.
Loading

The vertical uniformly distributed load on the slab was represented by loads applied on thirty six 6 x 6 in. loading pads spaced at 2 ft. centres on top of the slab. The pads were made of 1/2 in. thick rubber packing. Load was transmitted to each of the pads by pulling a high tensile rod of 0.198 in. diameter which passed through the slab and the centre hole of the pad and was rigidly clamped on top of the pad and, at the lower end, to a cross beam, by prestress wire grips of the Gifford-Udall type. The cross beam was fixed at the centre to the ram of a tension jack so that the load from each jack was spread to two loading rods. The eighteen jacks were fixed to three 4 x 2 in. channels which were bolted to the strong floor of the laboratory. Details of the loading arrangement are shown in Figures 3.12 and 3.13.

The horizontal load was applied along the N-S column centre lines at the level of the mid-plane of the slab. Two pillars were bolted to the laboratory floor at the north side of the slab to support two horizontal jacks. 4 x 4 in. steel
plates with bell seatings were fixed to the columns and the pillars. There was a 5/8th in. steel ball at each end of the jack. The jacks were also secured from an overhanging cantilever by means of springs. The arrangement is shown in Figure 3.13.

Pressure for the jacks was supplied by two Amsler loading cabinets, one for each loading system. These cabinets are equipped with lead maintaining devices; hence the loads applied to the slab were kept constant during each test.

Eight of the tension jacks for the vertical load were calibrated for their load-pressure characteristics. The mean value of the eight pressure readings at each load was used to plot the load-pressure relation for the eighteen jacks used for loading the slab. The horizontal jacks were also calibrated.

Strain Measurements

Strains in the reinforcement were measured by electric resistance strain gauges fixed before casting. The gauge used for this purpose was Polyester Gauge PLS-10 manufactured by Tokyo Sokki Kenkyujo Company Limited. It has a gauge length
of 10 mm and a gauge width of 1.5 mm. The overall size of
the gauge is 27 x 4 mm, which is suitable for applying to bars
of 3/8 in. diameter.

To stick strain gauges on deformed bars of small diameter
it was necessary to remove the transverse ribs where the gauge
was to be fixed. This was done by filing. The surface was
then smoothed with emery cloth and cleaned with acetone. A
length of about 2 in. was prepared in this manner for the type
of gauge used. Two strain gauges were stuck at each station
in diametrically opposite positions with the P-2 adhesive
provided by the manufacturer.

Wire leads were soldered to the gauges and were firmly
anchored on the bar with a nylon tie. A coating of P-2 adhesive
was applied around the gauges for moisture protection. In
addition, a thick coat of Silicone Rubber was spread around
that portion of the bar to protect the gauges from mechanical
damage while casting. To test the water-proofing effect of
these castings two samples of deformed bars with strain gauges
fixed in this manner were immersed in water for three days and
the bars were tested in the Amsler Testing Machine. The stress-strain curves obtained showed that the strain gauges were working normally. To test the mechanical protecting effect under actual concrete casting conditions, a beam having reinforcing bars with strain gauges fixed was cast in the laboratory. The beam was tested under pure bending with two equal point loads. The strain gauges were found to work properly. From this evidence, it seemed that this technique of applying strain gauges to reinforcement was satisfactory. After the slab was cast, however, it was found that some of the gauges inside the slab were damaged or affected by moisture. It was appreciated that in casting a slab of this size with so many strain gauges, it was not possible to take as much care in the vicinity of every gauge as when casting a single beam, and still keep up the necessary speed of casting. The effect of moisture or some other factors was that some of the strain gauge readings under no load or a constant load measured by the Automatic Strain Recorder were increasing with time or with the number of times the voltage was applied to the gauges. The reasons for this
behaviour of the strain gauges and the technique of applying strain gauges on reinforcement in concrete structures are still under investigation. In this experiment, three sets of strain readings were taken at no load and at each load increment and after unloading. The readings were plotted against time and a straight line was drawn through all the no load readings for a given strain gauge. A line was drawn through the points for the strain readings under load and parallel to the no load line. The distance between the two parallel lines gave the incremental strain for that load increment. The strain readings estimated in this manner had reasonable values despite the fact that the no load readings were not constant.

Strains on the concrete surface were measured with type PL-60 polyester gauges which had a gauge length of 60 mm. A strain gauge of such length was necessary to give a reliable strain reading on concrete with 3/4 in. aggregate. In applying the strain gauge, the concrete surface was levelled off with a carborundum stone. The dust was brushed off and
the area was cleaned with acetone. A thin layer of P-S adhesive was painted on the surface and left to harden. Then the surface of the P-S coating was smoothed and slightly roughened with emery cloth and cleaned. The strain gauge was stuck on the coating with P-2 adhesive. A one pound weight was placed on the gauge through a pad and foam rubber for about 10 minutes.

Strain gauge positions on the test slab are shown in Figures 3.14 to 3.16. Since the loading was symmetrical about the N-S centre line, it was necessary to instrument half the slab only. In this case the west side half was instrumented fully and only a few check gauges were placed on the other half. The gauges were mainly located at the critical section lines of the slab to give the strains and the bending moments at those sections. The centre line of the slab was the critical section for positive moment, and a line through the column faces was the critical section for negative moment. The strains on the negative critical sections were measured in the column strip only. Strain gauges were also placed on the faces of the columns underneath the slab.
Unlike the embedded strain gauges on the reinforcement the strain gauges applied on the surface of the concrete gave very steady readings.

All the strain gauges on the test slab were connected to a Solartron D.C. Automatic High Speed Digital Strain Recorder. This device recorded strain gauge readings at a rate of ten gauges per second. It had a range of from -15996 to +15996 microstrain and had a resolution of 2 microstrains.

Moment-Strain Relationship of Reinforced Concrete Sections

Due to the limited tensile strength of concrete and progressive cracking of the section, the bending moment in a reinforced concrete slab cannot be calculated directly from the stress in the reinforcement. In order to determine the bending moments in the test slab from the measured reinforcement strains, tests were carried out on a series of rectangular beams with reinforcement placed at different spacings and at different effective depths, duplicating different strips of the test slab, to establish experimentally
the relationship between bending moment and steel strain.

Five rectangular beams each 8 ft. long and 6 in. thick overall were cast with the same concrete as used in casting the slab, and on the same day. The reinforcement used in these beams was also Norhite '60' high tensile deformed bars of 3/8 in. diameter. The other measurements of these beams were:

<table>
<thead>
<tr>
<th>Beam No.</th>
<th>Width b in.</th>
<th>Effective Depth d in.</th>
<th>Bar No.</th>
<th>Spacing in.</th>
<th>No. of Bars in the Beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>4.9</td>
<td>9</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>5.3</td>
<td>9</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>5.3</td>
<td>3</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>5.3</td>
<td>6</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>4.9</td>
<td>6</td>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

Table 3.iv

Two polyester PLS-10 guages were fixed on each of the reinforcing bars in these beams in the manner described above.

These strain guages were read with a manually operated Peckel A. C. bridge. All the gauges in the beams gave stable readings, in spite of the insulation resistance of the guages in one of the beams being only about 0.4 megohm.

The beam was supported on two steel rollers on a span of 6 ft. 10 in. Constant bending moment was applied to the
middle portion of the beam by two equal loads at 2 ft. apart. These equal loads were transmitted from a jack through a cross beam. A load cell was fixed between the ram and the cross beam to measure the applied load. The arrangement is shown in Figure 3.17.

The beams were tested at an age of about nine months after casting. The slab was tested at an age of six months. The age factors for the compressive strength of concrete are 1.20 for six months and 1.22 for nine months, a difference of about 2 per cent. This slight increase of the concrete strength was considered to be negligible.

The loading procedure for testing these beams was intended to be similar to that used for testing the slab. In the first stage, the beam was loaded in very small increments of reinforcement strain (20 microstrains) to about 100 microstrain for two cycles.
Then it was loaded slowly up to the cracking load. This gave approximately a straight line moment-strain relationship for the gross section. In the second stage, the section had become an initially cracked section and the cracks continued to develop. Load was applied to produce strain increments of 10 με up to 200 με for 2 cycles. Then the maximum strain reached was brought up to 300 με and loaded for two cycles. This cycle loading was continued with strain increments of 100 με until the maximum strain reached 1000 με. In the last stage the section was well cracked and behaved more according to simple theory.

The beam was loaded cyclically as before but with increments of 200 με until the yielding strain of the reinforcement was reached and the ultimate moment of the section was determined.

The bending moment applied to the beam was calculated from the load, which was measured by the load cell, and the lever arm. The bending moment from the self weight of the beam and the loading arrangements was added to the applied moment to give the total moment, which was then converted and expressed in lb-in per foot width. The strain corresponding
to the self weight of the beam was interpolated linearly from
the first increment reading of each cycle. The bending moment
was plotted against the net reinforcement strain. Residual
strain was not taken into account in these curves since only
the incremental strains were measured in the test of the slab.
The moment-strain curves obtained from these tests were, in
general, of the shape shown in Figure 3.18. The envelope of
the curves consists of a straight line up to a point corresponding
to the theoretical concrete cracking point and a curve which
approaches the straight line given by the simple no tension
theory; the envelope finally joins a horizontal line which is
the nominal ultimate moment. Because of imperfection in the
concrete section, cracking may start at any earlier stage than
the theoretical cracking point and the initial part of the
moment-strain curve may follow one of the dotted lines. If
the beam is unloaded after reaching a certain stage such as
A, reloading of the beam will approximately follow the line OA
and a further increment from A to B will follow the envelope
curve AB. The initial portion of the line OA generally has a slope greater than the remainder of OA. This may be due to the effect of the 'locked in' strains set up by the last unloading.

During the test, beam No. 1 was accidentally cracked because of a broken valve on the electrical pump. Therefore the result for this beam was not complete and had to be deduced from other tests. Moreover, since the actual average thickness of the slab was 6.3 in. instead of 6 in. as designed, the effective depths of the five beams did not exactly represent the slab, and a simple correction was introduced to compensate for the difference. The assumption was that the bending moment for the same reinforcement strain was proportional to the effective depth.

The moment-strain relations are shown in Figures 3.19 to 3.25.
Deflection Measurements

Vertical deflections on the slab were measured with two precision levels. Wooden scales were fixed vertically on the slab as targets. They were spaced at 2 ft. centres as shown in Figure 3.26.

The precision levels used were type No. N3 made by Wild. These levels can read to 0.001 of an inch and have a range of 1/2 in. If the deflection was larger than 1/2 in., the nearest 1/2 in. or 1/4 in. divisions on the wooden scale was sighted and added to the fine readings from the level scale.

Horizontal deflections were measured on the column faces at the south side of the slab with mechanical dial gauges. Their positions are shown in Figure 3.27.
Reaction Dynamometer

In order to check the bending moment between the column and the slab externally, the reactions at the ends of the columns were measured by dynamometers which could give the three components of force. Several designs for this dynamometer were prepared and considered but it was finally decided that the tripod type was the most suitable. This dynamometer consisted of three rods arranged as a tripod standing on a rigid base. From the axial stresses on the legs of the tripod the components of the force applied at the apex of the tripod with regard to any system of axis can be calculated.

The design load for the dynamometers in this experiment was assumed to be about three times the maximum design column load. The actual figures taken were:

<table>
<thead>
<tr>
<th>Type</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical force</td>
<td>( P_z )</td>
<td>11.5 tons</td>
</tr>
<tr>
<td>Horizontal forces</td>
<td>( P_x )</td>
<td>8.0 tons</td>
</tr>
<tr>
<td></td>
<td>( P_y )</td>
<td>4.8 tons</td>
</tr>
</tbody>
</table>
The sides of the base triangle of the tripod were made the same length as the legs of the tripod. So the dynamometer was in the form of an equilateral tetrahedron. The maximum force in each leg under the assumed maximum loading conditions was 14 tons.

After considering the overall dimensions, sectional area, buckling strength, and the technical difficulties in fabrication and in fixing, the dimensions and the details of the dynamometer were designed and are shown in Figure 3.28. The length of the sides of the tetrahedron came out to be 9.5265 in.

The material used for the legs of the tripod was cold drawn seamless tube of 1 in. outer diameter and No. 6 gauge wall thickness. A tensile test on a specimen of this material showed that its ultimate strength was 44 tons/sq.in., and its yield stress was 32.4 tons/sq. in. The base plate and the ball seating of the dynamometer were made of 1 1/2 in. thick mild steel plates.
The fabrication of the dynamometer was considered to be rather difficult because most of the machining was at an angle and the measurements must refer to the centre of the ball at the apex of the tripod. However, much care was taken to ensure that the dimensions of the parts machined conformed to the design dimensions. Since it was not practical to provide hinge connections at the joints, they were welded. The effect of the fixity at the joints on the axial stresses of the legs subjected to vertical load was found to be 0.5 per cent, which was assumed to be negligible.

Four strain gauges were fixed at the central portion of each leg, two longitudinally and two transversely, in diametrically opposite positions. These four gauges formed the 4 arms of a full bridge circuit to measure axial strain.

To calibrate the dynamometer, it was necessary that a force acting in any direction and at any angle to the vertical axis should be applied at the apex of the tripod. This was accomplished by providing two systems of adjustment in the
calibration arrangement. One was that the jack for the applied force could be fixed at any angle in a vertical plane; and the other was that the tripod could be rotated to any direction on a horizontal plane about its vertical axis. A combination of these two systems of movement gave a compression force which could be in any direction in space within a cone of 45° degrees to the vertical.

A beam supported on two pillars defined the vertical plane of loading. A plate with a universal ball reaction block was clamped on the underside of the beam. This plate could be moved to any position in the beam so that the line joining the ball in the universal ball reaction block on the plate and the ball on the apex of the dynamometer could be fixed at the vertical angle required. A jack with an extension length and a proving ring was fitted between these two balls to apply the load.

A 1 in. thick plate with angular graduations was fixed in a horizontal plane on the laboratory floor under the beam.
The dynamometer was clamped on this plate by a bolt through its centre and was set in the direction required for the calibration. By resolving the force from the jack into components in the directions of the reference axes \((x, y, z)\), the forces \(P_x\), \(P_y\) and \(P_z\), applied to the dynamometer were known. The strain gauge readings on the three legs \(\varepsilon_1\), \(\varepsilon_2\), and \(\varepsilon_3\) were then recorded.

The relationship between the three components of the applied force and the strains can be expressed as follows:

\[
P_x = K_{1x} \varepsilon_1 + K_{2x} \varepsilon_2 + K_{3x} \varepsilon_3
\]
\[
P_y = K_{1y} \varepsilon_1 + K_{2y} \varepsilon_2 + K_{3y} \varepsilon_3
\]
\[
P_z = K_{1z} \varepsilon_1 + K_{2z} \varepsilon_2 + K_{3z} \varepsilon_3
\]

By applying the force in three different directions three different systems of \(P_x\), \(P_y\), and \(P_z\), and three sets of \(\varepsilon_1\), \(\varepsilon_2\), and \(\varepsilon_3\) were obtained. From these the three sets of constants \(K_{1x}\), \(K_{2x}\), \(K_{3x}\); \(K_{1y}\), \(K_{2y}\), \(K_{3y}\); and \(K_{1z}\), \(K_{2z}\), \(K_{3z}\), could be determined.
As an example the result for one of the calibrations is given below.

<table>
<thead>
<tr>
<th>ε₁</th>
<th>ε₂</th>
<th>ε₃</th>
<th>Px</th>
<th>Py</th>
<th>Pz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>-860</td>
<td>-846</td>
<td>-848</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2nd</td>
<td>1072</td>
<td>-1430</td>
<td>-1448</td>
<td>7850</td>
<td>0</td>
</tr>
<tr>
<td>3rd</td>
<td>-604</td>
<td>852</td>
<td>-2062</td>
<td>0</td>
<td>7850</td>
</tr>
</tbody>
</table>

\[ L = \varepsilon \]

<table>
<thead>
<tr>
<th></th>
<th>K_{1x}</th>
<th>0</th>
<th>K_{1x} = 3.097</th>
</tr>
</thead>
<tbody>
<tr>
<td>ε</td>
<td>K_{2x} = 7850 giving K_{2x} = -1.585</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>K_{3x} = 0</td>
<td>K_{3x} = -1.562</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>K_{1y}</th>
<th>0</th>
<th>K_{1y} = -0.011</th>
</tr>
</thead>
<tbody>
<tr>
<td>ε</td>
<td>K_{2y} = 0 giving K_{2y} = 2.709</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>K_{3y} = 7850</td>
<td>K_{3y} = -2.685</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>K_{1z}</th>
<th>11200</th>
<th>K_{1z} = -4.376</th>
</tr>
</thead>
<tbody>
<tr>
<td>ε</td>
<td>K_{2z} = 7960 giving K_{2z} = -4.390</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>K_{3z} = 7980</td>
<td>K_{3z} = -4.402</td>
<td></td>
</tr>
</tbody>
</table>
If the legs of the tripod are entirely identical and their positions are exact, the \( K \)s for the components should have the following relationship.

\[
K_{2x} = K_{3x} = -\frac{1}{2} K_{1x}
\]

\[
K_{1y} = 0 \text{ and } K_{2y} = -K_{3y}
\]

\[
K_{1z} = K_{2z} = K_{3z}
\]

It is seen that the values of the \( K \)s from the calibration show very good agreement to the above relationship.

In order to reduce experimental error, an extra set of strain readings was obtained for a fourth position of the applied force to get new sets of \( K \) values, and the above relationship was also used to get the mean values of the constants which are given below.

<table>
<thead>
<tr>
<th>Dynamometer</th>
<th>( K_{2x} )</th>
<th>( K_{3x} )</th>
<th>( K_{1y} )</th>
<th>( K_{1z} = K_{2z} = K_{3z} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Col. SE</td>
<td>-1.560</td>
<td>2.701</td>
<td>-4.411</td>
<td></td>
</tr>
<tr>
<td>Col. SW</td>
<td>-1.557</td>
<td>2.697</td>
<td>-4.404</td>
<td></td>
</tr>
<tr>
<td>Col. NW</td>
<td>-1.562</td>
<td>2.705</td>
<td>-4.418</td>
<td></td>
</tr>
<tr>
<td>Col. NE</td>
<td>-1.563</td>
<td>2.708</td>
<td>-4.421</td>
<td></td>
</tr>
</tbody>
</table>
DESCRIPTION OF THE TESTS

Testing Procedure

A test on the slab was conducted by applying vertical, wind, or combined load to the slab in increments to a certain maximum load level, measuring steel and concrete strains, vertical and horizontal deflections, and the strains in the reaction dynamometers.

Before any load was applied, there was already an initial vertical load on the slab consisting of the self weight of concrete and the weight of the vertical loading arrangements. This totalled 75 psf which was 3/8 of the design vertical load, 200 psf. The readings at this initial load level were the datum for all the subsequent readings for increments of applied load. The actual readings at this initial load level were deduced by linear readings; and the actual readings for all the subsequent load levels were taken to be the sum of this deduced reading for the initial load plus the reading due to the applied load.

For vertical load the first applied load increment on the slab was always 25 psf which together with the initial load, would make up 100 psf which was the design dead load on the slab and was half of the total design vertical load. Any
further increment of vertical load was at multiples of 50 psf making $\frac{3}{4}$, 1, or $1 \frac{1}{4}$ times the design vertical load. When combined load was applied, a horizontal load of $\frac{3}{8}$ of the design wind load was applied first without applying any vertical load so that the initial vertical load of the slab was matched by the same amount of wind load before reading started. Then both the vertical and the wind loads were increased in the same proportion.

The applied load was monitored on the dial of the Amsler load cabinet according to the pressure calculated from the load-pressure characteristic curve of the jacks. In the case of combined load, two loading cabinets were being operated at the same time by two operators; while one operator called out the fraction of load increment on his system, the other would bring his system to the same fraction before any further fraction of increment was added, so that both systems would reach the required load level at about the same time.

In each test, except for the test to failure, readings were taken at no applied load, at the applied load level, and then at no applied load again. Thus each test consisted of a series of loading and unloading cycles. This loading procedure was adopted because the time interval elapsed for taking a set of reading was quite considerable and it was
necessary to check the initial readings after each loading. But in the test to failure, after the slab was severely overloaded to \(1 \frac{1}{2}\) times the combined design load, load increments were continued until the ultimate load was reached.

At each no load and loaded stage, usually a complete set of readings for all the instruments was taken. But in some cases some of the vertical deflections were omitted. The average time required for taking a complete set of readings of all the instruments was about 20 minutes. As soon as the applied load was on or off and readings could begin a set of strain readings was printed. A second set of strain readings was printed at the middle of the interval and then a third set of readings was printed before a change of loading conditions. The strains on the reaction dynamometers were recorded twice during the interval with a manually operated Peckel A.C. bridge and a full bridge switch box, while the vertical and horizontal deflections were being read once.

After each loading above the design load, the slab was inspected for the development of cracks and their size and extent were recorded. Photographs were taken to record their positions.

The load maintaining device in the loading cabinet was used for most of the tests to keep the applied load constant.
during the test, except in the test to failure. In this test it was necessary to control the displacement rather than the load.

Outline of the Tests

Five main tests were carried out on the slab in different conditions.

Test No. 1  Vertical Load Test

This test was a preliminary test mainly to study the deflection of the slab subjected to vertical load alone when the slab was uncracked. The slab was loaded to a total load of 100 psf, 150 psf, and 200 psf, which were \( \frac{1}{2} \), \( \frac{3}{4} \), and 1 times the design vertical load respectively. A complete set of vertical deflections were recorded. The test was then carried on to the overload range to reach a load of 290 psf (1.45 times the design vertical load) at which load the centre part of the slab started cracking.

Test No. 2  Vertical Load Test

This test was a repetition of the last test except that the condition of the slab was different because part of the slab was then initially cracked. Again, total loads of \( \frac{1}{2} \), \( \frac{3}{4} \), 1, \( \frac{1}{4} \), and 1.45 times the design vertical load were
imposed on the slab. Full sets of strain readings and reactions were recorded. The vertical deflections at the centre and at the mid points of the edges of the slab were read so that they could be compared with the corresponding deflections in the previous test.

Test No. 3. Wind Load Test

In this test, the behaviour of the slab subjected to applied wind load only was studied. There was some vertical load on the slab which was the weight of the slab and the loading arrangements during the test, but this would not greatly affect the reading due to the applied wind load. The horizontal load was applied to the slab at four load levels: 1/4, 1/2, 3/4, and 1 times the design wind load, which was 4930 lb per column line (9860 lb total). Strain readings and horizontal deflections and some of the vertical deflections were recorded.

Test No. 4 Combined Load Test - Design Load and 1 \( \frac{1}{4} \) times Design Load

This was the condition for which the slab was designed. Vertical and wind loads were applied to the structure simultaneously in the ratio of their design values (200 psf and 4930 lb per column line respectively). A complete set
of readings for all the measurements were taken for 1/2 and 1 times the design combined load. Then the slab was overloaded to $1 \frac{1}{4}$ times the design load.

Test No. 5 Combined Load Test to Failure

In this test, after repeating the full design load and $1 \frac{1}{4}$ times the design load conditions, the slab was loaded to $1 \frac{1}{2}$, $1 \frac{3}{4}$, and 2 times the design load. Strains, horizontal deflections, and central and mid-edge vertical deflections of the slab were recorded. Unfortunately the reaction dynamometers were out of order at this stage, and reactions could not be measured. Photographs of the crack patterns were taken.

After that the slab was slowly loaded to failure. After reaching $2 \frac{1}{4}$ times the design load, the horizontal deflection of the slab was so large that the vertical loading rods had an appreciable horizontal arrangement which reduced the net horizontal load on the slab. The test was interrupted and was continued the next day. With a correction added to the applied horizontal load, the structure was brought to failure when the flexural strength of the slab was exhausted and a shear failure occurred at column SW at a combined load just over $2 \frac{1}{4}$ times the design value.
<table>
<thead>
<tr>
<th>Test No.</th>
<th>Date</th>
<th>Vertical Load</th>
<th>Wind Load</th>
<th>Load Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>psf multiple of</td>
<td>multiple of</td>
<td>lb. per</td>
</tr>
<tr>
<td></td>
<td></td>
<td>V.L.</td>
<td>W.L.</td>
<td>column line</td>
</tr>
<tr>
<td>1</td>
<td>26-11-1964</td>
<td>100</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>200</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>290</td>
<td>1.45</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3-12-1964</td>
<td>100</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>200</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>290</td>
<td>1.45</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>4-12-1964</td>
<td></td>
<td>1/2</td>
<td>2465</td>
</tr>
<tr>
<td>4</td>
<td>7-12-1964</td>
<td>100</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>200</td>
<td>1</td>
<td>1/4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>250</td>
<td>4</td>
<td>1/4</td>
</tr>
<tr>
<td>5</td>
<td>8-12-1964</td>
<td>200</td>
<td>11/4</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>250</td>
<td>1</td>
<td>1/4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>300</td>
<td>11/2</td>
<td>13/4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>350</td>
<td>13/4</td>
<td>13/4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>400</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>454</td>
<td>2.27</td>
<td>2.27</td>
</tr>
</tbody>
</table>

(Failure)

The values for the vertical load include the initial load of the slab.

Only major load levels reached are given in the table.

V.L. Design Vertical Load 200 psf
W.L. Design Wind Load 4930 lb per column line
C.L. Design combined load V.L. + W.L.
Behaviour of the Test Slab

Reactions

The column reactions measured during the tests are given in Figures 3.29 to 3.34.

In order to comprehend the behaviour of the slab it is necessary to know how the column reactions were distributed in response to the applied load. Their magnitude and their correspondence to calculated values are also discussed.

In Test No. 1, the average measured vertical reaction ($R_z$) at the design load is 7169 lb. The nominal figure for the applied load should be 7200 lb - a difference of -0.43 per cent from the measured value. The maximum reaction is at column NW and its value is 7263 lb.; the minimum is at column SW and its value is 7020 lb. They deviate from the mean value by about 1.31 per cent and -2.07 per cent respectively. It can be seen that the vertical load is very evenly distributed among the four columns.

The horizontal reactions in the N-S direction ($R_x$) at the design load varied from 2718 lb to 2849 lb. The mean value of the four horizontal reactions ($R_x$) is 2767 lb. The deviation from the mean value is therefore from -1.82 per cent to 3.0 per cent. The horizontal reactions in the E-W direction ($R_y$) have
values from 3041 to 2729 lb, and the mean value at the design load is 2849 lb. The deviation from the mean value is from 6.7 per cent to -4.2 per cent. Hence the horizontal reactions in this direction are less uniform. Also from the mean values of $R_x$ and $R_y$, it can be seen that the horizontal reactions in the E-W direction ($R_y$) are slightly bigger than those in the N-S direction($R_x$). However, they are both less than the calculated horizontal force according to the negative design moment, 3060 lb. by 9.5 per cent and 6.7 per cent respectively. (Design negative restraining moment is $-\frac{1}{36} qL^3$. Observed restraining moment from average measured horizontal reactions is $-\frac{1}{40} qL^3$.)

In the over load test in Test No. 1, after the central span of the slab had started cracking, the horizontal reactions in the two directions changes at a slightly different rate, as can be seen from the reaction measurements at 1.45 times the vertical design load. The reaction in the N-S direction ($R_x$) is a little less than the linear extrapolation from the previous values whereas the reaction in the E-W direction ($R_y$) has exceeded the linear extrapolation by about 6 per cent.

In Test No. 2, the slab was initially cracked; yet the reactions measured are in general very much the same as in
Test No. 1. The vertical reaction \( R_z \) at the four columns varies from 7007 to 7250 lb. The deviation from the average value, 7143 lb. is from -1.9 per cent to 1.5 per cent, showing again a fairly even distribution of the vertical reaction after cracking.

The horizontal reactions in the N-S direction \( R_x \) vary from 2642 to 2851 lb. Their average value is 2762 lb. The individual figures vary from this by -3.1 per cent to 4.6 per cent. The maximum and minimum values of the horizontal reactions in the E-W direction \( R_y \) are 2954 and 2810 lb. Their corresponding deviations from the average value of the four reactions are 3.1 per cent and -1.95 per cent. Again, it is seen that the reactions in the E-W direction \( R_y \) are larger than those in the N-S direction \( R_x \) by about 5 per cent. The average values of the \( R_x \) and \( R_y \), 2726 and 2866 lb. are 11 per cent and 6.3 per cent smaller than the design value 3060 lb. The reason may be that the stiffness of the slab is relatively less than expected at the corners because of cracking around the columns.

In Test No. 3, the horizontal reactions in the N-S direction \( R_x \) vary from 2473 to 2427 lb. Their average value is 2452 lb. at the design load level. Therefore the percentage difference from the mean value is from 0.9 per cent to -1.0 per cent. The calculated wind force in each column is 2465 lb.
Hence each of the measured horizontal reaction agrees very well with the applied value; it is reasonable that this should be so, since these reactions are statically determinate.

The horizontal reaction \( R_y \) in the other direction E-W is very small. The average value at the design wind load is 144 lb. This is about 5.8 per cent of the value of the reaction in the direction of wind. The calculated reaction \( R_y \) is 238 lb. acting inwardly at the south and outwardly at the north side columns. It is seen that the measured reactions are reversed in direction; the explanation for this discrepancy has not yet been found. But since the measured absolute values of these reactions are so small, the main results of the tests will not be affected significantly if their effect is omitted in the following discussion.

The average vertical reaction at design wind load is 1324 lb. upward or downward. This is about 3 per cent higher than the expected value of 1285 lb. from statics.

In Test No. 4, combined loading was imposed on the structure. It can be seen from the reaction measurements that they are not given exactly by superposition of the two separate loading cases. The superposed values for the design load level from the experimental values of tests Nos. 2 and 3, (Figures 9.31 and 3.32) are:
Comparing these values with the values of the reactions at the
south and north side columns in Test No. 4, (Figure 3.33) it
can be seen that at the design load, the horizontal reactions
in the N-S direction \( (R_x) \) are 145 and 77 lb. less than the
superposed values in the south side columns; and are 112 and
303 lb. more in the north side columns. This shows that the
horizontal load was being resisted more by the north side columns
than by the south side columns under combined loading. This
tendency of the north side columns (windward side) to take more
horizontal load than the south side columns under combined load
can be explained by the fact that cracking at the south side
column-slab junction was increased by the wind load, whereas
the cracking was decreased at the north side. Thus the local
stiffnesses will be different and the north side will take a
little more load.

The reaction in the E-W direction \( (R_y) \) from this test
are very close to the superposed values for the north side
columns but are about 4 per cent less than the superposed values
for the south side columns. This slight decrease in the reactions
(\(R_y\)) in the south side columns may again be due to the above mentioned reasons.

The reactions at the design combined load are

\[
\begin{array}{ccc}
R_z & R_x & R_y \\
\hline
\text{Columns SE and SW} & 8485 & 5525 & \mp 3060 \\
\text{Columns NW and NE} & 5915 & -595 & \mp 3060 \\
\end{array}
\]

(\(\mp\) Transverse effect of the wind load is omitted)

It is seen that the measured vertical reactions are close to the design figures. The reactions in the N-S direction (\(R_x\)) have discrepancies of 522 and 531 lb. in columns SE and SW and of 440 and 484 lb. in columns NW and NE respectively. This is because the experimental restraining moment from the column is less than the design restraining moment, as has been observed and described in Test No. 1 and Test No. 2.

The measured horizontal reaction in the E-W direction (\(R_y\)) at columns SE and SW are 453 lb. and 519 lb. less than the reactions due to design restraining moment and those of column NW and NE are respectively 57 lb. more and 41 lb. less than the reactions due to design restraining moment. This may be due to the combined effect of the reduction of the experimental restraining moment and the transverse horizontal reactions due to wind load.
In Test No. 5, only the reactions at the first two loading levels could be recorded. They are reproduced in Figure 3.34. These two loading cases in this test were repetitions of the 1 and 1\frac{1}{4} times design load cases in Test No. 4. In general they show a behaviour which is very similar to that described in the previous section.

**Deflections**

The behaviour of the test structure as seen from its vertical and horizontal deflections is described here. The deflected surface of the slab is compared with that from the finite difference solutions. The load-deflection relationship at certain critical points such as the centre point of the panel and the mid-points in the column lines are plotted and correlated with the calculated deflections.

The finite difference solution for the vertical deflection of the slab under a uniformly distributed load (q) and with negative restraining moment equal to the average experimental value of \(-\frac{1}{40} qL^3\) at the columns (Design value is \(-\frac{1}{36} qL^3\)), gives deflections

- at the centre: 
  \[ w_c = 0.01234 \frac{qL^4}{D} \]  
  \( (3.1) \)

- at the mid-point of the column line: 
  \[ w_c = 0.00667 \frac{qL^4}{D} \]  
  \( (3.2) \)
Two limiting values can be assumed for the stiffness of the slab, $D$.

(1) Gross concrete section:

With $E_{\text{concrete}} = 3.7 \times 10^6$ psi

Poisson's Ratio $\mu = 0$

Mean total thickness of slab = 6.3 in.

$$D = 925 \times 10^6 \text{ lb-in}^2/\text{ft.}$$

(2) Transformed section, completely cracked:

With modular ratio $m = 8$, the stiffness $D$ for different effective depths and bar spacing are given below:

<table>
<thead>
<tr>
<th>Effective Depth (in.)</th>
<th>Spacing of Bars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>9&quot;</td>
</tr>
<tr>
<td>5.6&quot;</td>
<td>106</td>
</tr>
<tr>
<td>5.2&quot;</td>
<td>92</td>
</tr>
<tr>
<td>5.3&quot;</td>
<td>-</td>
</tr>
<tr>
<td>4.9&quot;</td>
<td>-</td>
</tr>
</tbody>
</table>

(unit: $10^6 \text{ lb-in}^2/\text{ft.}$)

The average value of $D$ taken for the slab as a whole

In $N$-S direction $127 \times 10^6 \text{ lb-in}^2/\text{ft.}$

In $E$-$W$ direction $110 \times 10^6 \text{ lb-in}^2/\text{ft.}$
For the horizontal deflections, since the elastic analysis in the previous chapter suggested that 3.5 ft. of the slab width could be assumed as effective in providing stiffness in resisting wind moment deflection, this is the same as the width of the column strips in this particular case. Taking the variation of section inside the column into account, the horizontal deflection of the test slab is given by

$$\Delta_h = 0.129 \frac{MLh}{2EI_{(slab)}} + \frac{Mh^2}{3EI_{(col.)}}$$

The EI values of the column strip and of the column for the two extreme cases are

(1) Gross concrete section

Total thickness of the slab near column-slab junction 6 in.

$$EI_{(slab)} = 2800 \times 10^6 \text{ lb-in}^2/\text{ft.}$$

$$EI_{(column)} = 6400 \times 10^6 \text{ lb-in}^2/\text{ft.}$$

(2) Transformed section, completely cracked

$$EI_{(slab)} = 2 \times 235 + 1.5 \times 132$$

$$= 670 \times 10^6 \text{ lb-in}^2$$

$$EI_{(column)} = 3000 \times 10^6 \text{ lb-in}^2$$
Test No. 1

The vertical deflections at the design vertical load for half of the slab are shown in Figures 3.35 and 3.36. The deflection curves from the finite difference solution according to the experimental restraining moment \( \frac{1}{40} qL^3 \) from the average value of the measured reactions, and taking the gross concrete section stiffness of the slab are plotted for comparison. It is found that the experimental deflection curves are very close to the finite difference solution. Thus it is seen that the slab behaved very much like an isotropic plate before the slab starts cracking.

The load-deflection curve for the centre point of the panel in this test is shown in Figure 3.37, and those for the mid-points of the column lines are shown in Figures 3.38 to 3.41. It is observed that the load-deflection curves for these points in the slab are linear up to the design load level and these linear portions are very close to the lines given by expressions (3.1) and (3.2) with a gross concrete section stiffness. The deflections at these points at the design load are as follows:

<table>
<thead>
<tr>
<th>Point</th>
<th>Experiment</th>
<th>Eq. (3.1), D(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centre panel</td>
<td>12</td>
<td>0.0920&quot;</td>
</tr>
<tr>
<td>Mid-edge</td>
<td>1</td>
<td>0.0585</td>
</tr>
<tr>
<td>&quot;</td>
<td>24</td>
<td>0.0575</td>
</tr>
<tr>
<td>&quot;</td>
<td>15</td>
<td>0.0460</td>
</tr>
<tr>
<td>&quot;</td>
<td>27</td>
<td>0.0455</td>
</tr>
</tbody>
</table>
When the load has exceeded the design load it is noted that the load-deflection curves are no longer linear. At 1.45 times the design load, when the slab cracked along the N-S centre line, the deflections are:

<table>
<thead>
<tr>
<th>Point</th>
<th>Experiment</th>
<th>Eq. (3.1) and (3.2) D(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centre panel</td>
<td>12</td>
<td>0.223&quot; (1.166&quot; (1.012</td>
</tr>
<tr>
<td>Mid-edge</td>
<td>1</td>
<td>0.1375 0.631</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td>0.1275 0.631</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.094 0.546</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>0.0935 0.546</td>
</tr>
</tbody>
</table>

It is noticed that these experimental deflections are only about 1/5 of the deflections calculated from the expressions (3.1) and (3.2) using the completely cracked transformed section stiffness. This is because there is still a great part of the concrete sustaining tension. After unloading the residual deflections at the initial load of 3/8 times the design load were noted to be 0.060" at the centre and 0.02 to 0.036 in. at the midpoints of the column lines.

There was practically no horizontal deflection of the slab under this loading condition.
Test No. 2

The load-deflection curve for the centre panel and the mid-edge points of the slab are shown in Figures 3.37 to 3.41. It can be seen that these curves are linear nearly up to the maximum load reached in this test. But the slope (deflection/load) of these curves are much bigger than the corresponding curves in Test No. 1. This is because the slab has cracked and the stiffness of the slab is reduced. From the slopes of these curves it is found that the stiffness is approximately equal to two thirds of that of the uncracked section. However, the maximum deflections reached in this test are found to be practically the same as those at the same load (1.45 design load) in Test No. 1. Their magnitudes at the design load and at 1.45 times the design load are as follows:

<table>
<thead>
<tr>
<th>Point</th>
<th>Design Load</th>
<th>1.45 V.L.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centre panel</td>
<td>12</td>
<td>0.135&quot;</td>
</tr>
<tr>
<td>Mid-edge</td>
<td>1</td>
<td>0.078</td>
</tr>
<tr>
<td>&quot;</td>
<td>24</td>
<td>0.076</td>
</tr>
<tr>
<td>&quot;</td>
<td>15</td>
<td>0.061</td>
</tr>
<tr>
<td>&quot;</td>
<td>27</td>
<td>0.055</td>
</tr>
</tbody>
</table>

It is noticed that the deflections within the design load in this test are 1.1 to 1.5 times bigger than those in Test No. 1. Again, the horizontal deflection of the slab is zero in this test.
Test No. 3

In this test, only the horizontal load was applied. The horizontal deflection of the slab as measured at points K and K' (Figure 3.27) are shown in Figures 3.42 and 3.43. It can be seen that these load-deflection curves are almost straight lines for the whole range within the design wind load.

Based on the elastic analysis, only the column strip of the slab is effective in providing the stiffness for resisting the side sway, the horizontal deflections as calculated from expression (3.3) by assuming (1) gross concrete sections for the slab and for the column, and (2) transformed cracked section, are

- Gross concrete section \(0.0237\) in.
- Transformed cracked section \(0.0983\) in.

The experimental horizontal deflection at the design wind load is \(0.040\) in. The experimental value is bigger than that for the gross concrete section. After unloading from the design wind load, the residual horizontal deflection was only \(0.003\) in.

The vertical deflections of the slab in this test are shown in Figure 3.45. Most of the vertical deflections in the middle strip are so small that they could not be read by the precision level. The maximum vertical deflection measured
is 0.0105 in. at point 7 on the N-S column line. The deflection curves according to the finite difference solution and assuming the gross concrete section are also shown in Figure 3.45. The experimental deflections are found to be close to the calculated curves.

Test No. 4

The vertical deflections at the design combined load for half of the slab are shown in Figures 3.46 and 3.47. By comparing them with Figures 3.35 and 3.36 it is seen that the vertical deflections in this test are $1 \frac{1}{2}$ to 2 times bigger than those in Test No. 1. This is because the stiffness of the slab is decreased due to cracking. The experimental vertical deflections at the centre and at the mid-edge of the panel are as given below.

<table>
<thead>
<tr>
<th>Point</th>
<th>1 Design Load</th>
<th>$\frac{1}{4}$ Design Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centre panel</td>
<td>12</td>
<td>0.146&quot;</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.203&quot;</td>
</tr>
<tr>
<td>Mid-edge</td>
<td>1</td>
<td>0.089</td>
</tr>
<tr>
<td>&quot;</td>
<td>24</td>
<td>0.118</td>
</tr>
<tr>
<td>&quot;</td>
<td>15</td>
<td>0.088</td>
</tr>
<tr>
<td>&quot;</td>
<td>27</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0925</td>
</tr>
</tbody>
</table>
slopes (deflection/load) are slightly bigger than those in Test No. 2.

The horizontal deflections of the slab in this test are shown in Figures 3.42 and 3.43. It is noticed that these load-deflection curves tend to deviate from linearity after about 3/4 of the design combined load and the horizontal deflection increased considerably from the straight line at a slight overload. This may be due to the fact that the cracking at the south side column-slab junctions opened up under this loading condition. The magnitude of the horizontal deflection at design load and $1\frac{1}{4}$ overload are given below.

<table>
<thead>
<tr>
<th>Point</th>
<th>1 Design Load</th>
<th>$1\frac{1}{4}$ Design Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>At Col. SW</td>
<td>K 0.043&quot;</td>
<td>0.067&quot;</td>
</tr>
<tr>
<td>At Col. SE</td>
<td>K' 0.044&quot;</td>
<td>0.068&quot;</td>
</tr>
</tbody>
</table>

Comparing these results to the last test, it is plain that the horizontal deflection at the design load in this test is about 10 per cent.bigger than that in Test No. 3 when the slab was subjected to wind load only. This slight increase in the horizontal deflection under combined load may be due to the reduction of the local stiffness of the slab because of more extensive cracking rather than to the "Py effect". The load-deflection lines obtained from equation (3.3) for the two cases of section stiffnesses are also shown. It is observed
that the maximum deflection at $1 \frac{1}{4}$ times over load is about mid-way between the two limiting values.

Test No. 5

The load-deflection curves for the vertical deflections at the centre and at the mid-edge points up to the failure load are shown in Figures 3.48 to 3.52. It is noted from these curves that they are linear for the initial portion up to about $1 \frac{1}{2}$ times the design combined load and the rate of deflection with regard to the load in this portion is about twice that of the deflection line for the gross concrete section but is still much less than that of the line for the completely cracked section. This shows that the tension in the concrete has been playing quite a significant part in the section within this range of load throughout all the tests. Beyond that load level, some more parts of the slab cracked and the overall stiffness of the slab dropped considerably. The deflection of the slab increased very rapidly when it was loaded to 2 times the design load. At $2 \frac{1}{4}$ times the design load, which is slightly less than the ultimate load, the vertical deflections at the centre and at the mid-edge points, 1, 15, and 27, except point 24, are all very near to the load-deflection line for the completely cracked section. The reason that the mid-edge point
24 is not deflecting as much as the others is that, according to yield line analysis, the portion of the slab near the north side would not yield; whereas yield lines would pass through or pass near the other three mid-edge points. It can be said that the behaviour of the slab conformed fairly well with the elastic analysis up to about $1 \frac{1}{2}$ times the design load, but as soon as any part in the structure begins to yield the elastic analysis no longer applies and plastic analysis is necessary in order to describe the behaviour. Nevertheless, the experimental results show that the overall behaviour of the slab regarding its vertical deflection lies within the limits of the two extreme conditions of the section by applying the elastic analysis up to twice the design load.

The horizontal deflections of the slab at points K and K' are shown in Figures 353 and 3.54. From these load-deflection curves, it is seen that the rate of deflection with respect to the load up to design load level is bigger than that in Test No. 4 by about 15 per cent. At $1 \frac{1}{4}$ times the design load the horizontal deflection is the same as the maximum deflection reached in Test No. 4, 0.067 in. At $1 \frac{1}{2}$ times the design load, the deflection is 0.100 in. which is twice that at the design load. This is because of the reduction in the local
stiffness due to further development of the cracks at the column-slab junction. Another increment of load to \(1 \frac{3}{4}\) times design load caused the horizontal deflection to increase suddenly to 0.240 in., which is greater than the calculated deflection 0.172 in. from equation (3.3) with cracked section. The reason is that a shear crack has appeared at the column-slab junction of the south side columns and, according to the strain readings, two of the bars at Column SW have passed the yielding point. From then on the local strength and stiffness of the junction in its plastic state governs the behaviour of the horizontal deflection of the slab rather than the overall elastic stiffness. The horizontal deflection at \(2 \frac{1}{4}\) design load reached 0.92 in. just before the slab failed. This is about \(3 \frac{1}{2}\) times the deflection estimated by equation (3.3).
Cracking

In Test No. 1, cracks in the centre of span did not occur until a load of 1.45 times the design vertical load was reached. The estimated cracking load is 1.2 times the design load. This is possible because in a slab, any crack trying to open up in a highly stressed spot would be confined by the nearby less stressed region. Fine cracks may have occurred at some places before this load was reached even though they were not detected.

After Test No. 1, the load was removed and the slab was examined for cracks. On the bottom of the slab, it was found that a fairly obvious crack running straight in the N-S direction went right across the middle of the E-W span, which is the weaker span. On the sides of the slab it was observed that this crack was about .003 in. wide and 3.2 in. deep, i.e. about half of the total thickness of the slab. Near the middle of the east side and west side edges, there were two fine cracks running in the E-W direction at the bottom surface of the slab. They were about 1 ft. long from the edges. On the top surface of the slab, it was found that a fine crack occurred at the joint of each column face and the slab.

The cracks on the top surface and at the bottom surface of the slab observed after Test No. 1 are shown in Figures 3.55 and 3.56 respectively.
From Test No. 2 to the end of Test No. 4, there were no new cracks detected on the slab. The crack pattern remained the same as that observed at the end of Test No. 1.

In Test No. 5, many new cracks were formed after the load exceeded $1 \frac{1}{2}$ times the design combined load. The slab was then examined for cracks at $1 \frac{3}{4}$ times the design load. At the bottom surface it was found that several inclined cracks making a small angle to the E-W direction appeared in the middle strip. The crack pattern on the east side half of the slab was approximately symmetrical to that on the west side half. There were two more cracks running in the N-W direction, one at each side of the previous crack and about 9 in. from it. On the top surface of the slab it was found that there were cracks passing the corner of the upper column stub and inclined at about 45 degrees cutting across the corners of the slab at the south side. On the edge of the slab at the south side, a shear crack appeared at about 5 in. from the side of the column on the top and ended at about 1 ft. from the side of the column at the bottom of the slab. This was a torsion shear crack which was to cause the edge of the slab to shear off from the sides of the columns in an upward direction.
As the load increased the cracks opened to a greater extent and more new cracks formed, following the same general pattern; i.e. at the bottom surface, cracks ran in the N-S direction near the N-S centre line, and inclined cracks ran from the centre of the slab towards the east edge and towards the west edge but inclining towards the north. On the top, cracks developed around the south side columns. Cracks running in the E-W direction were also formed between the band of N-S cracks and between the inclined cracks. The position of the cracks in the perpendicular directions reflected the position and spacing of the reinforced bars. It seems that the bars weaken the concrete section in tension and cracks form more easily along the underside of the bars.

The cracking pattern at the ultimate load is shown in Figures 3.57 to 3.59.
Strains and Stresses

The behaviour of the slab as seen from the strain measurements is discussed in this section. The strains are used instead of the stresses because they are direct records from the experiment and they will be used for obtaining the bending moment distribution. The steel stresses can be easily converted from the strains by multiplying the latter by the Young's Modulus, which is $292 \times 10^6$ psi in this case, and before yielding they will distribute in the same manner as the strains. Therefore, the strain distribution is used for the discussion. But the steel stresses for a few sections are also given to show the order of magnitude. Locations of the strain gauges are shown in Figures 3.14 to 3.16.

Test No. 1

The strains in this preliminary test are not reported here because the loading conditions in Test No. 2 are similar.

Test No. 2

The strains for three loading levels, $1/2$, 1, and 1.45 of the design vertical load are shown in Figures 3.60 to 3.73. Generally speaking, the strain distribution curves at different load levels at each section show a similar and fairly proportional distribution.
The strain distribution in the negative bending moment column sections, A, B, D, and E, (Figures 3.60 to 3.63) show that the steel bars within the column width have a much bigger strain than those outside the column width. This may be due to the cracking of the section as well as the concentration of the restraining moment within the column width. It is observed that at the 1/4 span line, the strain curve drops almost to zero. This confirms that the middle strip is not effective in resisting the negative moment in this one panel slab. The maximum strains in sections A and B at the design load are 544με and 533με respectively, in the central part of the column width. The strains in the bars near the edges of the columns A1 and B1, and at the corners of the columns, A3 and B3, are not as high. But in sections D and E in the other direction, the maximum strains occurred at the bars nearest to the edge - 630 με at D1 and 760με at E1. Bars outside the column width recorded very low strains. Even though the steel percentages in sections D and E are only half that of the other sections, the strain readings in sections D and E are not very much bigger than those in A and B. This may be because the concrete in tension was taking much of the load at the design load level.
The concrete strains at these sections (Figures 3.67 to 3.69) show a distribution similar to the steel strains. They also show a very high peak strain within the column width and a very low strain at 1/4 span. The maximum concrete strain reached at the design load was 190 \( \mu \varepsilon \) in section B'.

The strains on the centre lines of the slab are shown in Figures 3.64 and 3.65. It is seen that the strains in section F are about twice as big as those in section C. This is because section F was cracked while section C was only slightly cracked near the edges. The maximum strain in the positive steel at design load is 384 \( \mu \varepsilon \) at F2. The concrete strains are quite evenly distributed in section C' but are rather irregular in section F' (Figures 3.70 and 3.71). This may be due to the difference in the depth of the crack at various positions.

The concrete strains on the column faces are as shown in Figures 3.72 and 3.73. It is observed that they do not follow plane strain distribution, as the strains near the corner are much bigger. The maximum strain in the columns is -440 \( \mu \varepsilon \) at design load and -738 \( \mu \varepsilon \) at 1.45 times design load.

The steel stresses for sections A, D, and F are converted from the strain readings and are shown in Figures 3.74 to 3.76. The maximum top steel stress is 11,200 psi. The allowable steel
stress, 30,000 psi was nowhere exceeded in the slab at design load. At 1.45 times design load the maximum steel stresses at the negative and positive sections are 34,900 and 19,400 psi respectively.

Test No. 3

When the slab was subjected to wind load alone, only sections A and B recorded appreciable strains. They are shown in Figures 3.77 and 3.78. The strains on the central sections and on the column sections in the other direction were mostly nearly zero, and are not reproduced.

The steel strain in section A due to wind moment is distributed in a manner very similar to the distribution due to the restraining moment induced by vertical load. Maximum strains occurred in the bars in the central portion of the columns and had a value of $370 \mu \varepsilon$ at the design wind load. Maximum concrete strain in the same section (Figure 3.79) is $-175 \mu \varepsilon$ at A. The strain decreases very rapidly outside the column width. The strain distribution at section B is similar to that at section A. From this and the previous test, it can be observed that when a bending moment is transferred from a column to the slab, whether the moment is produced by vertical or wind load, only a very narrow strip of the slab is effective. The main portion is the width within the column
faces; then the same width of slab next to the column face is also taking a considerable amount of it. The remaining part of the column strip may make some contribution, but by no means will the middle strip be counted on to resist the bending moment. The strain distributions on the column faces in this test are shown in Figures 3.81 and 3.82. The maximum compressive strain occurred at the corner of Column SW and the value is only $-184\mu\varepsilon$.

The steel stress distribution for section A is shown in Figure 3.83. The maximum steel stress occurred at A2 and is 10,900 psi at full wind load.

Test No. 4

The strain distributions in the slab at $1/2$, $1$, and $1\frac{1}{4}$ of the design combined load are shown in Figures 3.84 to 3.95.

At section A, which is the most critical negative moment section for the combined load case, the maximum strain in the steel at design load is $976\mu\varepsilon$. Again, as in the separate loading cases, the bars in the central portion of the column are strained most. The other bars at the sides of the column, A1 and A3, acquire a strain of 432 and $566\mu\varepsilon$ respectively. At the bar immediately outside the column, A4, the strain drops rapidly to $120\mu\varepsilon$. From there towards the $1/4$ span line the strain distribution gradually falls to about $20\mu\varepsilon$ at A7.
The variation of the strain in section B, where the moment is approximately zero under this loading condition, is rather small and irregular.

The variations of the strains at the negative sections, D and E, in the other direction are almost the same as for vertical load only. Also it is the same with the positive moment sections. Slight differences in the magnitudes may be due to further opening or closing of the cracks due to the combined load. The concrete strains at the corresponding sections show the same behaviour (Figures 3.90 to 3.93).

The steel stress distributions at sections A and D are shown in Figures 3.96 and 3.97. It is seen that the maximum steel stress is 27,600 psi at A2 at the design combined load. This stress is less than the allowable stress. In the design it was recognised that the peak stress might be greater than the allowable stress, but this did not occur in the experiment. This can be explained by the fact that concrete is taking part of the tension and that the column corner is not as rigid as assumed, and hence the concentration of stress at the corner of the column is relieved. Furthermore, in the design it was assumed that the peak stress would spread out to the nearby bars, and this was not realized at the design load level.
Test No. 5

The variations of the strains in this test at load levels of $1, 1 \frac{1}{2}, 1 \frac{3}{4}$, and 2 times the design combined load are shown in Figures 3.98 to 3.110.

It is observed that most of the strain distributions at all sections show a rapid increase of strain with regard to the load and a marked difference in the shape of the distribution curves between the load levels $1 \frac{1}{2}$ and $1 \frac{3}{4}$ times the design load. This is because the state of the structure had changed a great deal at this load increment as described in Pages 120 to 132. As a result of rapid development of cracking, first yielding has occurred in bars A2 and A3 in section A, and in bar D2 in section D at $1 \frac{3}{4}$ times design load. Bars next to the columns were being pulled to a higher strain. The strain distribution curve at section C at this load level has a different shape from those at the previous load levels and is more irregular. The strain readings at section C lie between 1100 and 740 $\mu$E and those at section F lie between 860 and 1370 $\mu$E at $1 \frac{3}{4}$ times design load.

At twice the design load, more bars at the negative sections yielded. It is noticed that all the top bars within the column region, except those in section B, had reached the
yield strain (2500\(\mu\varepsilon\)). At this load level, no bottom steel had yet reached yield point. The nearest one was F4 which had a value of 1960\(\mu\varepsilon\). The maximum strain in section C was at C3, and its value was 1650\(\mu\varepsilon\).

The concrete strains in sections A' and D' are very irregular because of the shear crack. The concrete strains at the central sections are still much below limiting strain. The maximum value at section C' is -680\(\mu\varepsilon\) at C'1, and in section F', it is -1050\(\mu\varepsilon\) at F'3. There was no sign of any crushing failure in the slab at this load.

Finally, the slab was brought to failure at 2.27 times the design load. The top bars immediately next to the column also yielded but not those further away from the columns. This shows that only those bars within the truncated pyramid of the column formed by the shear crack surfaces will share the redistribution of the peak stress after the maximum stressed bars have yielded. Those bars outside the truncated pyramid of the column remained at low stress.

At the central section C, all of the bars reached yield point, but in section F only the bars in the southern half of the slab yielded. The E-W bars in the northern half of the slab might yield near the edges, but are not at the centre, as can be seen from the crack lines(Figure 3.57)
Crushing of the concrete occurred in the slab around the south side column-slab junctions after the flexured strength had been exhausted.

The steel-stress distribution for sections A, D, and F are shown in Figures 3.111 to 3.113. The extent of the yielded bars in the sections are marked by double lines on the Figures.

**Comparison of Bending Moments**

To obtain the distribution of bending moments in the critical sections of the slab from the strain readings, the result of the strip beam tests described in the previous section are used. First of all, the maximum strain reached previously in a strain gauge is noted. On the appropriate moment-strain relationship diagram, the moment-strain curve for that strain is determined. From this curve, the bending moment corresponding to the strain reading in the test under consideration is interpolated.

For bars without strain gauges, their strains are estimated from the neighbouring strain readings by interpolation.

After getting the bending moment per unit width of slab for each bar in the section, it is plotted according to the spacing to give the bending moment distribution across the section. For comparison purposes, the section is divided into strips and the average bending moment in each strip is evaluated.
and drawn on the same diagram. The positive sections are divided by the 1/4 span line into two strips, the middle strip and the column strip. For the negative sections, the column strip is subdivided into the inner half column strip, which is adjacent to the column, and the outer half column strip, which is next to the middle strip. Their line of division is the 1/8 span line.

Test No. 2

The bending moment distribution at full design vertical load for the negative and positive sections are shown in Figures 3.116 to 3.121.

It is seen that the bending moment distributions in section A and B, and in sections D and E, are quite similar, but there is some difference between the two groups. There is a peak value in the sections A and B which is about 2/3 above the mean value in the inner column strip. In sections D and E, however, there is no distinct peak value but the maximum bending moment is spread quite evenly over the column width. This may be due to the difference in the bar spacing which affected the local distribution of the bending moment.

The mean moments in the inner half column strip in these negative sections varied from 27.0 to 28.5 \( \times 10^3 \) lb-in/ft. They are approximately equal to each other. The value in the
design bending moment distribution for these sections (Figure 3.1) is \(32.27 \times 10^3\) lb-in/ft., so the average experimental value in this strip is 13.5 per cent less than the design value. In the outer column strip, the mean values varied from \(4.6\) to \(7.3 \times 10^3\) lb-in/ft. Their average is about 12.5 per cent less than the corresponding value of the design bending moment distribution. However, if the actual observed restraining bending moment in the experiment, \(-\frac{1}{40} qL^3\), obtained from the average value of the measured reactions is used for calculating the bending moment distributions in the slab, (Figures 3.114 and 3.115), the mean bending moments in the inner half column strip and in the outer half column strip will be \(28.77\) and \(5.45 \times 10^3\) lb-in/ft. respectively. Then it can be seen that the experimental results in these negative sections are very close to the calculated values with the observed restraining moment.

The total negative moment at these sections ranges from \(63.9\) to \(65.1 \times 10^3\) lb-in. Compared to the design value, 76,090 lb-in, they are 16 per cent to 14.5 per cent too low. Compared to the total negative moment in the section calculated according to the observed negative restraining moment in the experiment, \(65.7 \times 10^3\) lb-in., it is seen that they have very good agreement.
From these distributions of the bending moment, it can be observed that most of the restraining moment from the column passes through the inner half column strip. According to average experimental figures, the amount is about 86 per cent of the total moment. And approximately 55 per cent of the total moment is within the column width. It seems, therefore, that about 45 per cent of the moment is transmitted from the column to the slab by torsion of the slab at the side of the column in this case.

In the positive sections, the bending moment distribution in each strip is fairly uniform (Figures 3.120 and 3.121). The mean values in the column strip and in the middle strip are 27,300 and 17,200 lb-in/ft. for section F and 25,600 and 20,700 lb-in/ft. for section C. It is noticed that in section F the column strip has a much higher moment than the middle strip.

The design moments for these strips are 24,060 and 19,920 lb-in/ft. (Figure 3.2). The experimental results are bigger than these except in the middle strip of section F, which is slightly less. The calculated values using the experimental restraining moment are 26,010 and 21,490 lb-in/ft. The experimental results for section C and for the column strip
of section F are reasonably close to the above figures, but the middle strip of section F is 20. per cent too low.

The total positive moments across half the span are 151,700 and 146,900 lb-in. in section C and in section F respectively. The design value is 144,000 lb-in. The measured positive moments are slightly bigger. The calculated value with the observed restraining moment is 155,520 lb-in. This shows that the positive moment gives fairly good agreement.

The total moment in the panel, i.e. the sum of the positive and negative moments, is 432,500 lb-in. in the N-S direction, and is 423,200 lb-in. in the E-W direction. They are only slightly less than the design total moment of 440,200 lb-in or the calculated total moment with the observed restraining moment, 442,400 lb-in.
### Test No. 2

<table>
<thead>
<tr>
<th>Bending Moment in Negative Critical sections</th>
<th>Inner half</th>
<th>Outer half</th>
<th>Total -ve moment in half panel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design, with R.M. $= \frac{1}{36} qL^3$</td>
<td>30,980</td>
<td>8,150</td>
<td>76,090</td>
</tr>
<tr>
<td>Measured,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In section A</td>
<td>27,800</td>
<td>6,500</td>
<td>65,150</td>
</tr>
<tr>
<td>B</td>
<td>28,500</td>
<td>4,600</td>
<td>63,900</td>
</tr>
<tr>
<td>In section D</td>
<td>28,200</td>
<td>6,000</td>
<td>64,400</td>
</tr>
<tr>
<td>E</td>
<td>27,000</td>
<td>7,300</td>
<td>64,950</td>
</tr>
<tr>
<td>Calculated, with observed R.M. $= \frac{1}{40} qL^3$</td>
<td>28,760</td>
<td>5,450</td>
<td>65,700</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bending Moment in Positive Sections</th>
<th>Col. strip</th>
<th>Middle strip</th>
<th>Total +ve moment in half panel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design, with R.M. $= \frac{1}{36} qL^3$</td>
<td>24,060</td>
<td>19,920</td>
<td>144,000</td>
</tr>
<tr>
<td>Measured,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In section C</td>
<td>25,600</td>
<td>20,700</td>
<td>151,700</td>
</tr>
<tr>
<td>In section F</td>
<td>27,200</td>
<td>17,200</td>
<td>146,900</td>
</tr>
<tr>
<td>Calculated, with observed R.M. $= \frac{1}{40} qL^3$</td>
<td>26,010</td>
<td>21,500</td>
<td>155,520</td>
</tr>
</tbody>
</table>
### Total moment vs. Total Moment in Panel (lb-in)

<table>
<thead>
<tr>
<th>Design, with $\frac{1}{36} qL^3$</th>
<th>Calculated, with observed $\frac{1}{480} qL^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R.M. = $\frac{1}{36} qL^3$</td>
<td>R.M. = $\frac{1}{480} qL^3$</td>
</tr>
<tr>
<td>440,200</td>
<td>442,400</td>
</tr>
</tbody>
</table>

**Comparison of bending moments**  
Test No. 2
Test No. 3

Due to wind load alone, only the result of section A is converted for bending moment. The distribution is shown in Figure 3.122.

It is seen that the mean moment in the inner column strip is 27,400 lb-in/ft. and is 11.5 per cent less than the design value. The outer column strip however, has a mean moment of 9,700 lb-in/ft. which is 16 per cent higher than the corresponding design value.

The total wind moment per column calculated from the measured reactions is 92,000 lb-in/which is practically the same as the design value, 92,500 lb-in.

With this moment transferred from the column, the moment at section A should be 84,400 lb-in. The total bending moment at section A measured from the experiment is 79,300 lb-in; the discrepancy is 6 per cent.

The proportion of the bending moment in the inner column strip in this case is 69.3 per cent. The moment within the column width is 40 per cent of the total moment in the section showing that about 60 per cent of the moment is transferred by torsion through the slab at the side of the column in this case.
<table>
<thead>
<tr>
<th>Bending Moment</th>
<th>Inner half strip (lb-in/ft)</th>
<th>Outer half strip (lb-in/ft)</th>
<th>Middle strip (lb-in/ft)</th>
<th>Total Moment half panel (lb-in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design</td>
<td>30,980</td>
<td>8,140</td>
<td>3,480</td>
<td>84,480</td>
</tr>
<tr>
<td>Measured in</td>
<td>27,400</td>
<td>9,700</td>
<td>3,300</td>
<td>79,300</td>
</tr>
</tbody>
</table>

Comparison of bending moments  
Test No. 3
Test No. 4

The moment distribution at the negative and positive critical sections at full design combined load are shown in Figures 3.123 to 3.127.

In section A, the mean value of the moment intensity measured in the inner column strip is 50,600 lb-in/ft. This is 20 per cent less than the design value of 63,250 lb-in/ft. If it is compared with the calculated value with the observed restraining moment, 59,750 lb-in/ft, it is about 15 per cent too low.

In the outer column strip the mean value of the experimental moment distribution is 15,000 lb-in/ft. The design figure is 15,130 lb-in/ft. which is close to the measured moment. The calculated mean moment with the observed restraining moment in this strip is 13,600 lb-in/ft; hence the measured moment is 10 per cent too high.

The total moment across half the panel in this section is 136,700 lb-in. from the measured strain readings. The design figure is 160,570 lb-in. Hence the experimental result is 15 per cent less than the design figure. By comparing it with the calculated total moment with the observed restraining moment, 150,180 lb-in, the measured total moment in this section
The percentage of bending moment in the inner column strip as compared with the total measured moment in section A is 74 per cent. The amount of bending moment inside the column width is about 47 per cent of the total moment in this section. Hence about 53 per cent of the moment in this section is transferred by torsion.

The bending moment distributions in the negative sections D and E, and in the positive sections C and F are similar to the corresponding ones in Test No. 2 when the slab was subjected to vertical load only. The total moments in the negative sections D and E are less than the design value. The calculated distribution according to the average experimental restraining moment, \(-\frac{1}{40} qL^3\), as shown in Figure 3.114 is used for comparing with sections D and E, and that in Figure 3.115 is used for comparing with sections C and F. (Transverse effect of the wind is assumed to be zero). It can be seen that they compare closely with each other.
### Bending Moment in Negative Critical sections

<table>
<thead>
<tr>
<th>Design, with R.M. = ( \frac{1}{36} ) qL³</th>
<th>Inner half col. strip (lb-in/ft)</th>
<th>Outer half col. strip (lb-in/ft)</th>
<th>Total -ve moment in half panel (lb-in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured, in section D</td>
<td>30,980</td>
<td>8,150</td>
<td>76,090</td>
</tr>
<tr>
<td>Measured, in section E</td>
<td>28,760</td>
<td>5,450</td>
<td>65,700</td>
</tr>
<tr>
<td>Calculated, with observed R.M. = ( \frac{1}{40} ) qL³</td>
<td>28,760</td>
<td>5,450</td>
<td>65,700</td>
</tr>
</tbody>
</table>

### Bending Moment in Positive Critical sections

<table>
<thead>
<tr>
<th>Design, with R.M. = ( \frac{1}{36} ) qL³</th>
<th>Col. strip (lb-in/ft)</th>
<th>Middle strip (lb-in/ft)</th>
<th>Total +ve moment in half panel (lb-in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured, in section C</td>
<td>24,060</td>
<td>19,920</td>
<td>144,000</td>
</tr>
<tr>
<td>Measured, in section F</td>
<td>26,010</td>
<td>21,500</td>
<td>155,520</td>
</tr>
<tr>
<td>Calculated, with observed R.M. = ( \frac{1}{40} ) qL³</td>
<td>26,010</td>
<td>21,500</td>
<td>155,520</td>
</tr>
</tbody>
</table>

(Transverse effect due to wind load is omitted)

Comparison of Bending Moments  Test No. 4
### Bending Moment Design

<table>
<thead>
<tr>
<th>Design, with ( R.M. = \frac{1}{36} qL^3 )</th>
<th>Inner half</th>
<th>Outer half</th>
<th>Middle</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>col. strip (lb·in/ft)</td>
<td>63,250</td>
<td>15,130</td>
<td>3,480</td>
<td>160,570</td>
</tr>
<tr>
<td>Measured, in section A</td>
<td>50,600</td>
<td>15,000</td>
<td>4,330</td>
<td>136,700</td>
</tr>
<tr>
<td>Calculated, with observed ( R.M. = \frac{1}{40} qL^3 )</td>
<td>59,750</td>
<td>13,600</td>
<td>3,480</td>
<td>150,180</td>
</tr>
</tbody>
</table>

**Comparison of Bending Moments**  
Test No. 4
Yield Line Analysis for Flat Slabs Subjected To Wind Load

When a flat slab is subjected to wind as well as vertical load, yield line theory can be applied to the design of the slab for a given combination of load, or to the evaluation of the collapse load for a given slab. Depending on the ratio of the vertical load to wind load, the failure mechanisms as shown in Figures 3.128 and 3.129 are two of the most probable ones, provided that the failure is to occur in the slab and not in the columns. The one shown in Figure 3.128 is the ordinary type of sway mechanism with a positive yield line in the span of the slab and a negative yield line at the windward side of the columns. The analysis of this mechanism is simple because it is the same as for plane frames. The second type of sway mechanism as shown in Figure 3.129 is more likely to occur in external panels if the edge of the slab is not strengthened. This mechanism is formed by a series of positive and negative zigzag yield lines in the central span and passing the corners on the windward side of the columns respectively, and another series of parallel positive and negative yield lines along the direction of wind. This makes the failure mechanism look like a series of separated parallel and inclined slabs with the columns, linked together by folded plates. The analysis of this sway mechanism involves two parameters, x and y, as defined in Figure 3.131.
The method of analysis is as follows:

Referring to Figures 3.130 and 3.131

\[ BC = \frac{L - x}{y + b} (b - \lambda) - \gamma \]

\[ EC = x + BC = x + \frac{L - x}{y + b} (b - \lambda) - \gamma \]

\[ HJ = (\lambda + \frac{y + b}{L - x} \gamma) \]

\[ HI = \gamma + \frac{L - x}{y + b} \lambda \]

\[ HI' = \frac{HJ}{y + HJ} L = \frac{\lambda + \frac{y + b}{L - x} \gamma}{y + (\lambda + \frac{y + b}{L - x} \gamma)} \]

\[ GI' = L - HI' \]

Assume deflection at \( E = \delta \)

\[ \delta_A = \frac{y}{y + b} \delta \]

\[ \delta_B = \frac{BC}{EC} (1 + \frac{x}{L - x}) \delta \]

\[ \delta_C = \frac{BC}{L - x} \delta = (\frac{b - \lambda}{y + b} - \frac{\gamma}{L - x}) \delta \]

\[ \delta_1 = (\frac{\gamma}{L - x} + \frac{\lambda}{y + b}) \delta \]

Horizontal deflection \( \Delta = \frac{h}{L - x} \delta \)
The rotations are

$$
\begin{align*}
\Theta_1 &= \frac{1}{L - x} \delta \\
\Theta_2 &= \frac{\delta - \delta_c}{EC} = \frac{1 - \left( \frac{b - \lambda}{y + b} - \frac{x}{L - x}\right)}{x + \frac{L - x}{y + b} \left( \frac{b - \lambda}{y + b} - \gamma \right)} \\
\Theta_3 &= \frac{\delta_B}{BJ} = \frac{\delta_B}{b - HJ}
\end{align*}
$$

Let $q = udl$ on the panel

$p = \text{wind force on the panel}

= k \cdot q (L \times 2b)$

Then, by virtual work, work done by external forces

$$
E = P_\Delta + q \times \frac{1}{3} \left( L \times 2b \delta - 2 \times \frac{1}{2} H I' H J (\delta + \delta_1) + 2 \times \frac{1}{2} b x GI' x \delta_A + \frac{1}{2} \times 2 (b - H J) (x + BC) \delta_B \right)
$$

The energy dissipated by the yield moments

$$
D = 2 \left( \frac{M_{AE}}{n} + \frac{M'}{IC/n} \right) (\Theta_1 + \Theta_2) + \left( \frac{M_{AE}}{t} + \frac{M'}{IC/t} + \frac{M'}{AI/t} + \frac{M_{EC}}{t} \right) \Theta_3
$$

From these equations, $q$ can be determined.
In analysing the ultimate flexural strength of the test slab by means of yield line theory first of all the failure mechanism which is most likely to occur is determined by studying several types of mechanisms approximately. Once a possible yield line pattern is decided, then a detailed analysis is made to determine the exact position of the yield lines and the minimum ultimate load for this pattern.

The ultimate moment of resistance for a section in the slab is calculated from

$$M_{ult} = \frac{1}{2} b d^2 f' c q (1 - 0.59 q)$$

Yield stresses are taken as $$f_y = 73,600 \text{ psi}$$ and $$f'_c = 4,000 \text{ psi}$$

With effective depths varying from 4.9 in. to 5.6 in. and spacings of the bars from 3 in. to 9 in. the ultimate moment per bar varies from 38,000 lb-in. to 44,000 lb-in.

The mechanisms, Type 1 and Type 2, as shown in Figure 3.132 are failure mechanisms without sidesway. Their ultimate loads are found to be 555 psf and 583 psf respectively.

The mechanism Type 4 (Figure 3.133) is a special case of the pattern which has been described before. Since all the edges are external edges in this test slab some of the yield lines or portions of them in the pattern become fictitious.
Hence it seemed to be the most likely collapse mechanism for the test slab. Following the method of analysis given above, the ultimate load for different values of $x$ and $y$ (Figure 3.131) are

<table>
<thead>
<tr>
<th>$x$ (ft)</th>
<th>$y$ (ft)</th>
<th>ult. load (psf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>12</td>
<td>446</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
<td>445</td>
</tr>
<tr>
<td>6</td>
<td>16</td>
<td>445</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td>450</td>
</tr>
</tbody>
</table>

So the ultimate load for the test slab is determined to be 445 psf, which is equal to 2.225 times the design combined load. The failure mode will be mechanism Type 4, with $x = 6$ ft. and $y = 14$ ft.

The experiment confirmed that the failure mode for the test slab is mechanism Type 4. However, it was noticed that near the edges, all the yield lines tended to cross the edge perpendicularly. And because of the torsion on the line $IJ$, the slab sheared off upwardly from the column at the side. Furthermore, the position of the yield line $AE$ is not very well defined since there are several wide cracks situated at that region. The best line chosen gives $x = 5.5$ ft. and $y = 14$ ft.
The ultimate load reached in the experiment is 454 psf, which is 2.27 times the design combined load. The result is 2 percent greater than the calculated value.

So it is seen that the ultimate behaviour of the slab agreed very well with the yield line analysis. The reasons for this agreement are as follows:

(1) The test slab is a single panel. There are no edge beams or adjacent panels and the columns are pinned ended, so that membrane action will not be significant.

(2) Most of the bars yield more or less simultaneously just before the ultimate load is reached, except a few bars near the columns; therefore the effect of work hardening is small.
From the elastic analysis of a one panel square slab and an internal panel of an infinite slab with column size equal to 1/12 of the span width, it is seen that the effective width of the slab for calculating restraining moment is 0.83 and 0.82 of the span width respectively. Using the actual dimensions of the test slab, it is found that this reduction in the effective width of the slab causes only a small change in the restraining moment between the column and the slab. If the moment for zero rotation at the column-slab joint is $M$, then the moment in the column for the case of the one panel slab will be $0.876 M$ by taking the whole width of slab and will be $0.90 M$ by taking the effective width; and for the case of internal panels, the corresponding figures are $0.78 M$ and $0.81 M$ respectively. The difference is only 3 per cent. But the value of the Fixed End Moment (F.E.M.) needs some consideration. Some moment is transmitted to the adjacent panels between the columns, and the slab deflects and rotates between columns, so the clamping moment is actually less than the F.E.M. of a beam. In calculating the moment between column and slab, the
clamping column moment should be used instead of the F.E.M.
The analysis shows that the clamping moment is 89 per cent of
the F.E.M. in the one panel slab, and is 79 per cent of the
F.E.M. in an internal panel of the above mentioned dimensions.
Therefore the restraining moment will be \( \frac{1}{30} qL^3 \) on a column
of the one panel slab (or \( \frac{1}{15} qL^3 \) for the panel) and \( \frac{1}{19} qL^3 \)
for an internal column. The ACI Code clause 1004 (b) 2 requires
a design column moment of \( \frac{1}{30} qL^3 \) and \( \frac{1}{40} qL^3 \) for an external
and internal column respectively. These values are about half
of the corresponding values given above. The figures obtained
from the alternative frame analysis method as given in the Code
are, however, about 10 per cent and 20 per cent bigger than
the values given above according to the elastic plate analysis.
The average measured column restraining moment on the test
slab is \( \frac{1}{40} qL^3 \). This is less than the calculated value
0.0333 qL^3 by 0.0083 qL^3. The horizontal movement of the column
ends accounted for 0.0046 qL^3 and the rest of the discrepancy
0.0037 qL^3 may be due to the redistribution of the maximum
negative moment because of cracking near the column face section.

The result shows that the restraining moment given by
the Code frame analysis method is too big while that given
by the empirical method is too small.
It is suggested that for flat plate structures of similar dimensions a modified frame analysis may be used, taking the effective width to be 80 per cent of the span width of the slab, and taking the clamped column moment to be 90 per cent of the F.E.M. in the case of external panels and 80 per cent of the F.E.M. in the case of internal panels.

The overall stiffness of the slab in resisting horizontal deflection of the structure subjected to wind load depends on the length-width ratio of the panel and the column dimensions. From the results of the elastic solutions of the few cases computed, the following expressions for the effective width of a uniform thickness slab are suggested.

**Internal Panel:**
\[
(0.280 \frac{L - \gamma}{L'} + \left( \frac{2\lambda}{L'} + \frac{\gamma}{L'} \right) + 0.086) L'
\]

**Edge half panel:**
\[
(0.140 \frac{L - \gamma}{L'} + \left( \frac{2\lambda}{L'} + \frac{\gamma}{2L'} \right) + 0.043) L'
\]

where

- \(L\) = length of the panel, between column centres
- \(L'\) = width of the panel, between column centres
- \(2\lambda\) = width of the column, in \(L'\) direction
- \(2\gamma\) = size of the column, in \(L\) direction

(For \(L/L' = 2/3\) to \(3/2\), and \(\gamma/\lambda \leq 1\))

The effective width calculated should not be more than \(L'\) (or \(1/2 L' + \lambda\) for an edge half-panel).
According to the above formula, an internal square panel with a square column of size equal to 1/12 of the span width, the effective width is 0.468 of the panel width. The actual figure from finite difference solution is 0.469 of the panel width.

A simple method of calculating the horizontal deflection is by assuming the column and the effective width of the slab as a frame. The lower limit of the horizontal deflection is obtained by taking the gross concrete sections. However, from the test on the reinforced concrete slab, it is observed that the section near the junction of the column and slab is probably cracked at working load conditions. Hence the upper limit of the horizontal deflection at design load is obtained by assuming the cracked transformed section of the members. The experimental result shows that the actual horizontal deflection at design load is about half way between these two limiting values. So it is proposed that for safe control of the horizontal deflection the value obtained by assuming the properties of the cracked section and the effective width of the slab as given above should not be more than the allowable deflection.

From the elastic analysis and from the result of the experiment on the test slab, it can be observed that the bending moment distribution on the section of the slab at the
face of the column is concentrated near the column. Depending
on the type of loading and on the size of the column, the moment
within the column width can be 40 per cent to 50 per cent of the
total moment at that section line. The portion of moment within
twice the column width is 70 per cent to 85 per cent.

For an edge column it is therefore suggested that to resist
efficiently the moment at the section due to transfer of bending
moment from the column to slab, 45 per cent of the total moment
at that section should be considered to be distributed within
the column width and 80 per cent within twice the column width.

For an internal column it is suggested that 40 per cent
of the total moment at that section should be considered to be
distributed within the column width and 80 per cent within
three times the column width.

The rest, 20 per cent, is assumed to distribute at the rest
of the column strip. The middle strip should be assumed as not
effective in resisting the moment transferred from the column.

The measured and calculated bending moment distribution for
the positive sections in the column strip and middle strip are
close to the 60 per cent and 40 per cent distributions given
in the Codes of Practice.

The portion of the bending moment transferred through torsion
can be from 45 per cent to 60 per cent in the case of a
corner column, as observed in the test slab. It is considered
that means of preventing cracks due to torsion and punching shear at the junction will improve the serviceability and ultimate load of the slab and further research work should be done on this local problem in conjunction with more work on one- and multi-panel flat slabs.

The performance of the test slab showed that the design of a reinforced concrete flat slab according to elastic plate analysis to resist wind and vertical load is satisfactory in strength and in stiffness. At design combined load the measured maximum vertical deflection is 1/900 of the span and the horizontal deflection is 1/750 of the height. The stresses in the reinforcement and in the concrete are less than the allowable stresses. The ultimate strength of the slab is 2.27 times the design load and is very close to that predicted by yield line analysis.
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**NOTATION**

- **L**
  - length of panel normal to direction of applied moment

- **aL**
  - length of panel in direction of applied moment

- **a**
  - grid size

- **w**
  - deflection

- **x, y**
  - rectangular reference co-ordinates

- **t**
  - thickness of slab

- **D = \frac{Et^3}{12(1-\nu^2)}**
  - flexural rigidity of a plate per unit width

- **D_o = \frac{Et^3}{12}**

- **E**
  - modulus of elasticity

- **\nu**
  - Poisson's ratio

- **M**
  - total bending moment

- **m**
  - intensity of bending moment per unit width

- **q**
  - intensity of uniformly distributed load

- **v**
  - shearing force per unit width

- **r**
  - reaction force per unit width

- **\theta**
  - rotation

- **\Delta**
  - deflection

- \( \frac{1}{R_x}, \frac{1}{R_y} \)
  - curvature
I  
moment of inertia

C  
column size

h  
column height

Mw  
wind moment

H  
horizontal force

Px, Py, Pz  
force

ε, ε1, ε2, ε3  
strain

Klx, Kly, Klz, etc.  
calibration constant for dynamometer

Rx, Ry, Rz  
reaction on column

The following notation has also been used for:

L  
length of panel or span of beam

L'  
width of panel

b  
half width of panel or breadth of beam

λ  
half column width

χ  
half column size normal to λ

δ  
deflection

k  
constant

E,D  
energy stored or dissipated

A  
area

t  
tensile stress

n,t  
reference direction suffix
\( M_{\text{ae/n}} \) etc. \( \Rightarrow \) total positive yield moment on line \( \text{ae} \) in n-direction

\( M'_{\text{al/t}} \) etc. \( \Rightarrow \) total negative yield moment on line \( \text{ai} \) in t-direction

Other notation is either defined where it occurs or takes its generally accepted meaning.
INFINITE SLAB SUPPORTED
ON COLUMNS WITH SIDE LOADING

FIG 21

- Width of Panel
- Length of Panel
- Size of FD Grid
- Intensity of Moment

\[ \frac{M}{Q} \]
SLAB I/W/1

ASPECT RATIO \( \alpha = 1 \)

FINITE DIFFERENCE GRID SIZE \( a = L/12 \)

POISSON'S RATIO \( \mu = 0 \)

GRID FOR SLAB I/W/1

SYMMETRY

ANTISYMMETRY

ANTISYMMETRY

ANTISYMMETRY

\( y \)

\( x \)

FD GRID LINES
DEFLECTIONS FOR SLAB 1/W/1

\[ \theta = \frac{7744 \frac{M}{D_o}}{b = 2.15 L} \]
BENDING MOMENT $m_y$ ALONG LINES $y_0 - y_6$ IN SLAB I/W/1
BENDING MOMENT $m_y$ ACROSS LINES X0 - X3 IN SLAB I/W/1
BENDING MOMENT $m_x$ ALONG LINES X0 - X6 IN SLAB I/W/1
SLAB I/W/3

AN ELEMENT OF AN INFINITE FLAT SLAB WITH

ASPECT RATIO \( \alpha = 1.5 \)

FINITE DIFFERENCE GRID SIZE \( \Delta = L/12 \)

POISSON'S RATIO \( \nu = 0 \)

FINITE DIFFERENCE GRID FOR SLAB I/W/3
DEFLECTIONS IN SLAB I/W/3

\[ \theta = 0.860 \frac{M_Y}{D_o} \]

\[ b = 291.2 \]
BENDING MOMENT $M_y$ ACROSS LINES $X_0 - X_9$ IN SLAB I/W/3

$\frac{M_y L}{M_y}$

FIG 2.9
SLAB O/R/3

SQUARE SLAB SUPPORTED AT FOUR CORNERS WITH MOMENTS APPLIED SYMMETRICALLY AT THE CORNERS

FINITE DIFFERENCE GRID SIZE $a = \frac{L}{12}$

POISSON'S RATIO $\mu = 0.35$

GRID LINES FOR SLAB O/R/3
$\epsilon = 2.678 \frac{M}{D}$

$b = 1856 \text{ L}$

DEFLECTIONS OF SLAB O/R/3

FIG 211
BENDING MOMENT $M_x$ ACROSS LINES Y0 - Y6 IN SLAB 0/R/3

FIG 212
BENDING MOMENT $M_y$ ALONG LINES $y_0 - y_6$ IN SLAB 0/R/3

FIG 213
SLAB 0/P/2

SQUARE SLAB SUPPORTED AT FOUR CORNERS WITH CONCENTRATED CENTRAL LOAD

FINITE DIFFERENCE GRID SIZE $\alpha = L/8$

POISSON'S RATIO $\nu = 0.35$

FINITE DIFFERENCE GRID

FIG 2.14
EFFECT OF GRID SIZE ON SOLUTION FOR
CONCENTRATED LOAD AT CENTRE OF SLAB O/P/2

DEFLECTIONS ON THE EDGE

DEFLECTIONS ON LINE $X_0$

FIG 215
$a = \frac{L}{12}$

$\alpha = \frac{L}{4}$

$m_y$ ACROSS LINE AT $\frac{1}{4}$-SPAN

$\frac{m}{p}$

$0$

$0.1$

$0.2$

$0.3$

$a = \frac{L}{4}$

$a = \frac{L}{8}$

$a = \frac{L}{12}$

$m_y$ ACROSS LINE $x_0$

FIG 2.16
$m_x \text{ ALONG THE EDGE}$

$\frac{m}{P}$

$0$

$-0.1$

$-0.2$

$-0.3$

$a = \frac{L}{8}$

$a = \frac{L}{4}$

$a = \frac{L}{12}$

$m_x \text{ ALONG LINE } X_0$

FIG 217
DEFLECTION CURVES FOR SLAB O/W/1

\[ \theta = 1.793 \frac{M}{D_0} \]

\[ b = 0.93 \frac{L}{M} \]
$m_x$ ACROSS LINES $Y_0 - Y_6$

IN SLAB 0/W/1
my ALONG LINES $Y_o - Y_b$
IN SLAB .0/W/1
EFFECT OF POISSON'S RATIO $\mu$ ON SLAB O/R

DEFLECTION ON LINE $X_0$

DEFLECTION ON LINE $X_6$

FIG 221
DEFLECTION ON LINE $y_b$

DEFLECTION ON LINE $y_0$

FIG 222
m_x ACROSS LINE Y_4

m_x ACROSS LINE Y_0

FIG 2.23
EFFECT OF POISSON’S RATIO $\mu$ ON SLAB O/D

DEFLECTION ON LINE $X_6$

DEFLECTION ON LINE $X_0$

FIG 2.25
\[ \frac{m}{qL^4} \times 10^3 \]

\[ m_x \text{ ALONG LINE } x_6 \]

\[ \mu = 0 \]

\[ \mu = 0.35 \]

\[ m_y \text{ ACROSS LINE } x_0 \]

\[ \mu = 0 \]

\[ \mu = 0.35 \]

FIG 2.26
FOUR PANEL SLAB F

$\mu = 0$
LOADING CONDITIONS FOR SLAB F

FIG 2.28
DEFLECTIONS FOR SLAB F/W/1

\[ \frac{w D_0}{M L} \]

\[ \theta = 0.600 \frac{M}{D} \]

\[ b = 0.556 \text{ L} \]

FIG 2.29
$m_x$ ACROSS LINES $Y_0 - Y_8$
IN SLAB F/W/1

FIG 230
\[ \theta = 1.360 \frac{M}{D} \]
\[ b = 0.245 \ L \]

DEFLECTIONS FOR SLAB F/W/2

FIG 231
$m_x$ ACROSS LINES $Y_0 - Y_8$
IN SLAB F/W/2

FIG 2 32
DEFLECTIONS FOR SLAB F/W/3

θ = 640 \frac{MD}{b = 521 L}

FIG 2.33
$m_x$ ACROSS LINES $Y_0 - Y_B$

IN SLAB F/W/3

FIG 234
\[ \frac{\omega D_o}{ML} \]

DEFLECTIONS FOR SLAB F/W/4
m_x ACROSS LINES Y_0 - Y_8
IN SLAB F/W/4
DEFLECTIONS FOR SLAB F/D/1
my across lines \( X_6 \) & \( X_4 \)

in slab F/D/1

FIG 2 38
DEFLECTION CURVES FOR SLAB I/W/7
\[ m_y \text{ ACROSS LINES } X_0 - X_6 \]

IN SLAB I/W/7

FIG 2.41
DEFLECTION CURVES FOR SLAB 1/W/9
$m_y$ ACROSS $x$ - LINES
IN SLAB 1/W/9
DEFLECTION CURVES FOR SLAB 0/W/2
$m_x$ ACROSS LINES $Y_0 - Y_7$

IN SLAB 0/W/2

FIG 247
GRID FOR SLABS
0/W/3
0/R/5
0/D/3
DEFLECTION CURVES FOR SLAB 0/W/3

\[ \frac{\omega D_0}{ML} = 0.5 \]

\[ \theta = 45.168 \frac{M}{D_0} \]
DEFLECTION CURVES FOR SLAB 0/R/5
m_x ACROSS Y-LINES
IN SLAB O/R/5
DEFLECTION CURVES FOR SLAB 0/D/3
$m_x$ ACROSS Y-LINES IN SLAB O/D/3
COMPARISON OF DEFLECTION CURVES ON LINE $X_0$

IN INFINITE SLAB

FIG 2.55
<table>
<thead>
<tr>
<th>POINT</th>
<th>1</th>
<th>ONE</th>
</tr>
</thead>
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<tr>
<td>L/12</td>
<td>0.01</td>
<td>TWO</td>
</tr>
<tr>
<td>L/8</td>
<td>0.01</td>
<td>TWO</td>
</tr>
<tr>
<td>L/6</td>
<td>0.01</td>
<td>TWO</td>
</tr>
</tbody>
</table>

Comparison of deflection curves on line $x_0$

In one panel slab

FIG 2.56
PULLEY

\[ \frac{3}{8} \text{ in} \]

\[ \frac{2}{16} \text{ in rod} \]

\[ \frac{3}{12} \text{ in} \]

\[ \frac{1}{2} \text{ in} \]

\[ 2" \]

\[ 12" \]

\[ 2\frac{1}{2}" \]

\[ 1" \]

\[ 12" \]

\[ \mu = 0.35 \]

\[ E = 0.45 \times 10^6 \text{ psi} \]

\[ t = 0.368 \text{ in} \]

\[ D_{35} = 2135 \text{ lb-in} \]

PERSPEX MODEL OF SLAB 0

FIG 2.57
TRANSDUCER CURVATURE METER

FIG. 2.58
SLAB 0/R/3

--- FINITE DIFFERENCE SOLUTION

○ EXPERIMENTAL RESULTS

DEFLECTIONS ON EDGE X₆

FIG 259
DEFLECTION ON LINE $X_3$

DEFLECTION ON LINE $X_0$
DEFLECTION ON LINE Y₄

DEFLECTION ON LINE Y₆

FIG 2 61
DEFLECTION ON LINE $Y_0$

DEFLECTION ON LINE $Y_2$
\frac{mL}{M}

m_x \text{ ACROSS LINE } Y_4

FIG 2.64
$\frac{mL}{M}$ across line $y_5$
FIG 2.66

my ALONG LINE Y₀

my ALONG LINE Y₃

my ALONG LINE Y₅
SLAB O/W/1

DEFLECTIONS ON LINE X₆

FIG 267
DEFLECTIONS ON LINE $X_3$

DEFLECTIONS ON LINE $X_0$

FIG 2.68
DEFLECTIONS ON LINE $Y_2$

DEFLECTIONS ON LINE $Y_4$

DEFLECTIONS ON LINE $Y_6$

FIG 269
\( m_x \) ACROSS LINE \( Y_2 \)

\( m_x \) ACROSS LINE \( Y_4 \)

FIG 2.70
my ALONG LINE $y_2$

my ALONG LINE $y_4$

my ALONG LINE $y_5$

FIG 2.72
Theoretical moment distribution

--- Distribution of the moment of resistance of the reinforcement in section D

DISTRIBUTION OF DESIGN BENDING MOMENT DUE TO VERTICAL LOAD AT NEGATIVE CRITICAL SECTIONS

Fig. 3.1
Theoretical moment distribution

Distribution of the moment of resistance of the reinforcement in Section C

Distribution of the moment of resistance of the reinforcement in Section F

DISTRIBUTION OF DESIGN BENDING MOMENT DUE TO VERTICAL LOAD AT POSITIVE CRITICAL SECTIONS

Fig. 3.2
DISTRIBUTION OF DESIGN BENDING MOMENT DUE TO WIND LOAD
AT SECTION A

Fig. 3.3
DISTRIBUTION OF TRANSVERSE BENDING MOMENTS DUE TO WIND LOAD

Fig. 3.4
Theoretical moment distribution

Moment distribution assumed in design

Distribution of the moment of resistance of the reinforcement in Section A

DISTRIBUTION OF BENDING MOMENT DUE TO COMBINED LOAD

AT SECTION A
ARRANGEMENT OF REINFORCEMENT IN TEST SLAB

Top Steel

Bottom Steel
TYPICAL STRESS-STRAIN CURVE
FOR SLAB REINFORCEMENT

Fig. 3.7
STRESS-STRAIN CURVE FOR SLAB REINFORCEMENT

Fig. 3.8
STRESS—STRAIN CURVE FOR CONCRETE

Fig. 3.9
Fig. 3.10

VARIATION OF THE SLAB THICKNESS
FIG. 3.12

DETAIL OF VERTICAL LOADING ARRANGEMENT

HALF SCALE
FIG. 3.13

LOADING ARRANGEMENT

SCALE 1" = 1 FT
LOCATION OF THE REINFORCEMENT STRAIN GAUGES

A, B, D, and E - on top steel bars
C and F - on bottom steel bars

Fig. 3.14
A', D', B', and E' - on bottom surface
C' and F' - on top surface

LOCATION OF THE CONCRETE STRAIN GAUGES

Fig. 3.15
LOCATION OF STRAIN GAUGES ON COLUMN FACES

COLUMNS NW

COLUMNS SW

Fig. 3.16
TEST ASSEMBLY FOR STRIP BEAM

FIG. 3.17
1.17.11

APPROXIMATE STRAIGHT LINES

THEORETICAL CRACKING POINT

POSSIBLE EXPERIMENTAL INITIAL PATHS

ULTIMATE MOMENT

BENDING MOMENT

0

STEEL STRAIN
EFFECTIVE DEPTH \( d = 5.6 \text{ in.} \)
BAR SPACING \( = 9 \text{ in.} \)
EFFECTIVE DEPTH  d = 5.6 in.
BAR SPACING  = 6 in.

MOMENT  (10^3 LB-IN/FT)

STEEL STRAIN

FIG. 3.20
EFFECTIVE DEPTH \( d = 5.2 \text{ in.} \)
BAR SPACING \( = 6 \text{ in.} \)
EFFECTIVE DEPTH $d = 5.3 \text{ in.}$
BAR SPACING $= 9 \text{ in.}$
EFFECTIVE DEPTH \( d = 5.3 \text{ in.} \)
BAR SPACING \( = 6 \text{ in.} \)
EFFECTIVE DEPTH \( d = 5.3 \text{ in.} \)
BAR SPACING \( = 3 \text{ in.} \)
EFFECTIVE DEPTH $d = 4.9$ in.
BAR SPACING $= 6$ in.
LOCATION OF VERTICAL DEFLECTION MEASUREMENTS

Fig. 3.26
LOCATION OF HORIZONTAL DEFLECTION DIAL GAUGES

Fig. 3.27
Note:
This design is based upon an equilateral tetrahedron with sides 3.6 cm in length and all machining calculations should be based on this.

MATERIAL:
Mild Steel
Sign Convention

Positive Directions for Column Reactions

Fig. 3.29
<table>
<thead>
<tr>
<th>Load Multiple of V.L.</th>
<th>1/2</th>
<th>1</th>
<th>1.45</th>
</tr>
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<tr>
<td><strong>Rz</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Col. SE</td>
<td>3581.5</td>
<td>7185.0</td>
<td>10616.5</td>
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<td>7020.5</td>
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<td>7116.1</td>
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<td>4434.1</td>
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Unit: Pound

Column Reactions, Test No. 1 - Vertical load only

Fig. 3.30
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<th>1/2</th>
<th>3/4</th>
<th>1</th>
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<th>1.45</th>
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Unit: pound

Column Reactions, Test No. 2 - Vertical load only

Fig. 3.31
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<tr>
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<td>651</td>
<td>978</td>
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<td>Rx</td>
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Unit: pound

Column Reactions, Test No. 3 - Horizontal load only

Fig. 3.32
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<th>3/4</th>
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<th>3/4</th>
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Unit: pound

Column Reactions, Test No. 4 - Combined load

Fig. 3.33
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Unit: pound

Column Reactions, Test No. 5 - Combined load

Fig. 3.34
Deflections on 24-1 and 25-2 in Test No. 1
Deflections on 26-3 and 23-7 on Test No. 1
Load-Deflections calculated with Eq. (3.1)

- Uncracked section
- Cracked section, N-S
- Cracked section, E-W

Deflection at centre of panel, Point 12.
Load-Deflections
Calculated with Eq. (3.2)
- Uncracked section
- Cracked section
E-W

Deflection at Edge Mid-Point 1
Deflection at Edge Mid-Point 24
Deflection at Edge Mid-Point 15

Load-Deflections
Calculated from Eq. (3.2)

- - Uncracked section
--- Cracked section, N-S
Deflection at Edge Mid-Point 27

VERTICAL LOAD (200 psf)

TEST NO.1  TEST NO.2  TEST NO.4

0 0 0
0.1 0.1 0.1
0.2 0.2 0.2
0.3 0.3 0.3

IN.

Deflection at Edge Mid-Point 27
Load-Deflections
Calculated from Eq. (3.3)

- Uncracked section
- - Cracked section, N-S

TEST NO. 3
Horizontal Deflection at Point K

TEST NO. 4
Horizontal Deflection at Point K'

FIG. 3.43

Horizontal Load (9860 LB) vs. Horizontal Displacement

TEST NO. 3

TEST NO. 4

Horizontal Displacement (in.)
Horizontal Deflection at Column Ends J and J'

Fig. 3.44

TEST NO. 3

TEST NO. 4

Horizontal Deflation (9860 LB)

(IN.)

0.005 0.010

0 0.015
Deflections on 26-3 and 23-7 in Test No. 3
Deflections on 24-1 and 25-2 in Test No. 4

Experimental Deflections

Calculated Deflections
with observed restraining moment $-\frac{1}{40} qL^3$
and 0.663 uncracked section stiffness
Deflections on 26-3 and 23-7 in Test No. 4.
Deflection at Centre of Panel, Point 12 in Test No. 5

Load-Deflections
Calculated with Eq. (3.1)

Uncracked section
Cracked section, N-S
Cracked section, E-W
Load-Deflections
Calculated from Eq. (3.2)

- - - Uncracked section

- - - Cracked section, E-W

Deflection at Edge Mid-Point 1 in Test No. 5
Deflection at Edge Mid-Point 24 in Test No. 5
Deflection at Edge Mid-Point 15 in Test No. 5

Load-Deflection
Calculated from Eq. (3.2)

Uncracked section
Cracked section, N-S

Deflection at Edge Mid-Point 15 in Test No. 5
Deflection at Edge Mid-Point 27 in Test No. 5
Load-Deflections
Calculated from Eq. (3.3)

Uncracked section
Cracked section

Horizontal Deflection At Point K in Test No. 5
Horizontal Deflection at Point K' in Test No. 5
Cracks on Bottom Surface After Test No. 1

Fig. 3.55
Cracks on top surface After Test No. 1

Fig. 3.5b
CRACK PATTERN ON BOTTOM SURFACE AT ULTIMATE LOAD

FIG. 3.57
CRACK PATTERN ON TOP SURFACE AT ULTIMATE LOAD
CRACK PATTERN ON THE SIDES AT ULTIMATE LOAD

FIG. 3.59
Steel Strain in Section A in Test No. 2

Fig. 3.60
Steel Strain in Section B in Test No. 2

Fig. 3.61
Steel Strain in Section D in Test No. 2

Fig. 3.62
Steel Strain in Section E in Test No. 2

Fig. 3.63
Steel Strain in Section C in Test No. 2

Fig. 3.64
Steel Strain in Section F in Test No. 2

Fig. 3.65
FIG. 3.66

THIS FIGURE NUMBER HAS BEEN INCLUDED BY ERROR
Concrete Strain in Section A' in Test No. 2

Fig. 3.67
Concrete Strain in Section B' in Test No. 2

Fig. 3.68
Concrete Strain in Section D' in Test No. 2

Fig. 3.69
Concrete Strain in Section C' in Test No. 2

Fig. 3.70
Concrete Strain in Section F' in Test No. 2

Fig. 3.71
Concrete Strain on Column SW in Test No. 2

Fig. 3.72
Concrete Strain on Column NW in Test No. 2

Fig. 3.73
Steel Stress in Section A in Test No. 2

Fig. 3.74
Steel Stress in Section D in Test No. 2

Fig. 3.75
Steel Stress in Section F in Test No. 2

Fig. 3.76
Steel Strain in Section A in Test No. 3

Fig. 3.77
Steel Strain in Section B in Test No. 3

Fig. 3.78
Concrete Strain in Section A' in Test No. 3

Fig. 3.79
Concrete Strain in Section B' in Test No. 3

Fig. 3.80
Concrete Strain on Column SW in Test No. 3

Fig. 3.81
Concrete Strain on Column NW in Test No. 3

Fig. 3.82
Steel Stress in Section A in Test No. 3

Fig. 3.83
Steel Strain in Section A in Test No. 4

Fig. 3.84
Steel Strain at Section B in Test No. 4

Concrete Strain in Section B' in Test No. 4
Steel Strain in Section D in Test No. 4

Fig. 3.86
Steel Strain in Section E in Test No. 4

Fig. 3.87
Steel Strain in Section C in Test No. 4

Fig. 3.88
Steel Strain in Section F in Test No. 4

Fig. 3.89
Concrete Strain in Section A* in Test No. 4

Fig. 3.90
Concrete Strain in Section D' in Test No. 4

Fig. 3.91
Concrete Strain in Section C' in Test No. 4

\[ \text{Concrete Strain in Section C'} \text{ in Test No. 4} \]
Concrete Strain in Section F' in Test No. 4

Fig. 3.93
Concrete Strain on Column SW in Test No. 4

Fig. 3.94
Concrete Strain on Column NW in Test No. 4

Fig. 3.95
Steel Stress in Section A in Test No. 4

Fig. 3.96
Steel Stress in Section D in Test No. 4

Fig. 3.97
Steel Strain in Section A in Test No. 5

FIG. 3 98
Steel Strain in Section B in Test No. 5

Fig. 3.99
Steel Strain in Section E in Test No. 5

FIG. 3.101
Steel Strain in Section C in Test No. 5

Fig. 3.102
Steel Strain in Section F in Test No. 5

Fig. 3.103
Concrete strain in Section A' in Test No. 5

Fig. 3.104
Concrete Strain in Section D' in Test No. 5

Fig. 3.105
Concrete Strain in Section E in Test No. 5

Fig. 3.106
Concrete Strain in Section C' in Test No. 5

Fig. 3.107
Concrete Strain in Section F' in Test No. 5

Fig. 3.108
Concrete Strain in Column SW in Test No. 5

FIG. 3.109
Concrete Strain on Column NW in Test No. 5

Fig. 3.110
Steel Stress in Section A in Test No. 5

Fig. 3.111
Steel Stress in Section D in Test No. 5

Fig. 3.112
Steel Stress in Section F in Test No. 5

Fig. 3.113
Distribution of Moment at Column Negative Sections
Due to UDL with observed Restraining Moment
\[ M_R = -\frac{1}{4\alpha} qL^3 \]

Fig. 3.114
Calculated Moment distribution
Average moment distribution in strips

Distribution of Moment at Central Positive Sections
Due to UDL with Restraining Moment - \( \frac{1}{40} qL^3 \)

Fig. 3.115
Distribution of Measured Moment in Section A in Test No. 2

Fig. 3.116
Distribution of Moment in Section B in Test No. 2

Fig. 3.117
Distribution of Moment in Section D in Test No. 2

Fig. 3.118
Distribution of Moment in Section E in Test No. 2

Fig. 3.119
Distribution of Moment in Section C in Test No. 2

Fig. 3.120
Distribution of Moment in Section 3 in Test No. 2

Fig. 3.121
Distribution of Moment in Section A in Test No. 3

Fig. 3.122
Distribution of Moment in Section A in Test No. 4
Distribution of Moment in Section D in Test No. 4

Fig. 3.124
Distribution of Moment in Section E in Test No. 4

Fig. 3.125
Distribution of Moment in Section C in Test No. 4

Fig. 3.126
Distribution of Moment in Section F in Test No. 4

Fig. 3.127
Sway Failure Mechanism in Flat Slab

Fig. 3.128
Sway and Valley Failure Mechanism in Flat Slab

Fig. 3.129
SECTION A-A

SECTION B-B

SECTION C-C

Elevation Sections In Figure 3.129

Fig. 3.130
Half Panel of Slab in Figure 3.129

Fig. 3.131
Failure Mechanism of Test Slab

Type 1

Type 2

Fig. 3.132
Failure Mechanism of Test Slab

Fig. 3.133
BOTTOM SURFACE AFTER FAILURE

FIG. 3.134