Calling Circles: Network Competition with Non-Uniform Calling Patterns*

Steffen Hoernig† Roman Inderst‡ Tommaso Valletti§

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Abstract

We introduce a flexible model of telecommunications network competition with non-uniform calling patterns, accounting for the fact that customers tend to make most calls to a small set of similar people. Equilibrium call prices are distorted away from marginal cost, and competitive intensity is affected by the concentration of calling patterns. Contrary to previous predictions, jointly profit-maximizing access charges are set above termination cost in order to dampen competition if calling patterns are sufficiently concentrated. We discuss implications for regulating access charges as well as on- and off-net price discrimination.

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†shoernig@novasbe.pt. Nova School of Business and Economics, INOVA, Universidade Nova de Lisboa; and CEPR.

‡inderst@finance.uni-frankfurt.de. University of Frankfurt, Imperial College London and CEPR.

§t.valletti@imperial.ac.uk. Imperial College London, University of Rome II and CEPR.

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1 Introduction

Modern communication networks allow users to easily establish a large number of links, both on the same network and across networks. Still, most users' contacts are often limited to a small fraction of all other users. Of these, frequent contact is only made with an even smaller set of people.[1] These tend to be similar users, such as friends and family, or people with close social links, such as neighbors, students at school or university, or business user groups. This affinity is reinforced as networks' brand positioning often appeals to a particular age group, social stratum or professional class, such as Vodafone's, which is directed more to professionals and wealthier people, or Virgin Mobile's in the UK, which typically attracts young people.[2] The introduction of tariffs specifically targeting the youth segment has been a particularly popular recent trend.[3] Also, when mobile operators sponsor sports events or clubs, subscribers are expected to sort according to their respective allegiance.[4] Sobolewski and Czajkowski (2012) present evidence for Poland that a high concentration of on-net calls is driven indeed by social preferences and interactions, rather than only by low on-net prices.[5]

These observations contradict a standard assumption that is frequently made in the literature on competition between telecommunications networks: the assumption of uniform calling patterns, where each subscriber is assumed to call each other subscriber with equal likelihood (e.g., Armstrong, 1998; Laffont et al., 1998a, 1998b). As calls between networks (off-net calls) involve the payment of access charges (also called termination rates), the assumption of a uniform calling pattern has consequences on how access charges impact on the market outcome. When networks competing in multi-part tariffs can set access charges jointly, Gans and King (2001) find that these would be set below cost, which in turn leads to higher prices for on-net than for off-net calls. Furthermore, the resulting ratios of on-net

[1] Shi et al. (2012) find, for a Chinese cellular network, that people make most of their calls (80%) to a very small proportion (20%) of their contacts.

[2] See http://www.ofcom.org.uk/research/cm/cmr09.

[3] For example, in Greece there has been vigorous competition in this area. Heavy marketing campaigns started in 2005, lasting for six years, which promoted tariffs called "CU" (Vodafone), "What's Up" (Cosmote) and "Free to Go" (Wind).

[4] In Portugal, the Benfica football club even created its own mobile virtual network operator, Benfica Telecom, hosted on the network of its sponsor TMN.

[5] More precisely, the authors show that the overall market share of an operator is not, per se, an important determinant of consumer choice. What really matters is the presence of closely related people, even when the difference between on- and off-net prices in Poland is taken into account.

to off-net calls would lie below each network's market share, which contradicts the stylized fact of heavily on-net-biased calling patterns (cf. Birke and Swann 2006). In the UK, a recent report by Ofcom (2011) shows that, although on- and off-net prices have become similar, the share of on-net mobile calls is still significantly higher than what should be expected under a uniform calling pattern.

In this article we address the above evidence by introducing non-uniform calling patterns in a tractable model of network competition. Customers differ in their preferences for a particular network, and instead of stipulating that each subscriber calls any other subscriber with the same likelihood, we suppose that it is more likely that subscribers with similar preferences are called than those further away in preference space. In doing so, we allow for the whole spectrum ranging from zero to perfect correlation between a consumer's brand preferences and calling pattern. We analyze how such concentrated calling patterns, with the resulting higher fraction of on-net calls, affect the equilibrium outcome.

When calling patterns are uniform, economic theory predicts that under multi-part tariffs call prices should be set equal to (perceived) marginal costs. Instead, with non-uniform calling patterns, we find that networks optimally deviate from such marginal-cost pricing in order to price discriminate: On-net prices are set above, and off-net prices below, perceived marginal costs in order to extract rents related to consumers' concentrated calling behavior. Our simple general call pricing formula relates the deviation from marginal-cost pricing to the difference between the calling pattern of a network's marginal subscriber, who is just indifferent between joining this and a competitor's network, and the average calling pattern of subscribers on the same network.

We identify important implications for networks' choice of profit-maximizing access charges. As mentioned above, under a uniform calling pattern, networks would choose access charges below cost. This induces off-net prices to be below on-net prices, leading to negative tariff-mediated network effects (Laffont et al., 1998b). Consumers then prefer to join a smaller rather than a larger network, which dampens competition. However, when the calling pattern is concentrated, the proportion of on- and off-net calls of the marginal subscriber is less closely tied to market shares. This diminishes the role of tariff-mediated network effects, whereas profits from off-net calls gain in importance. Indeed, if we find that if calling patterns are sufficiently concentrated then networks will choose an access charge above cost.

We contribute to the literature that analyzes how network effects affect the intensity of competition (e.g., Katz and Shapiro, 1985). Applied to communication networks, Gabrielsen and Vagstad (2008) and Calzada and Valletti (2008) introduce the notion of calling clubs to account for the fact that subscribers tend to call only a limited number of people, and analyze how subscribers are deterred from switching between networks. However, they assume that clubs only consist of subscribers with identical preferences, so that two calling clubs are always completely disjunct from each other. In our model clubs overlap, which seems more realistic. Dessein (2003, 2004) allows for heterogeneity between high- and low-volume consumers and studies the effects of net call inflows or outflows of specific consumer groups. In Dessein's framework there is scope for call prices to differ from perceived marginal costs if either the access charge is far from cost or if customer heterogeneity is large. In our model, we find that call prices are always distorted away from marginal cost if the calling pattern is not uniform. As we consider a different type of heterogeneity, our work is complementary to Dessein's.

Our model of calling patterns may be useful also for wider applications in the area of network economics (cf. Farrell and Klemperer, 2007). We combine exogenous call preferences for particular consumers with network effects that are induced endogenously through tariffs. The latter feature distinguishes our article from other recent contributions on network effects, such as Fjeldstad et al. (2010). With this article we share, however, the following ingredients. Consumers are located on the Salop circle and have similar preferences (in our setting: call preferences) as nearby consumers.[6] The difference in preference intensity between the marginal and inframarginal consumers makes it optimal for firms to distort marginal prices, which in our setting relates to both on-net and off-net prices. Again, a crucial feature in our setting is, however, that network effects are endogenously induced through the choice of these prices.

Our analysis also contributes to a large literature, starting with Armstrong (1998) and Laffont et al. (1998a, 1998b), on how networks gain from choosing (unregulated) reciprocal access charges. As we noted above, an important puzzle is the prediction of Gans and King (2001) that with two-part tariffs and discrimination between on- and off-

[6] Baron (2010) employs a model where citizens are spatially distributed along the Salop circle, with the probability of two citizens making "contact" a diminishing function of (geographic or socioeconomic) distance. As in our model, agents are more likely to interact with neighbors than with strangers.

net calls networks would jointly choose access charges below cost, and off-net prices below on-net prices. In our model, with sufficiently concentrated calling patterns, on-net prices will be below off-net prices.[7] Our finding provides a rationale for policy invention aimed at *reducing* access charges. There is only scope for such a policy intervention in the standard model with uniform calling patterns if other changes are made: Lopez and Rey (2012) show that an incumbent can set a high access charge in order to foreclose entry; in Jullien et al. (forthcoming) high access charges arise from price discrimination between heavy and light users[8] and in Armstrong and Wright (2009) from the simultaneous interconnection of mobile and fixed networks.

The rest of this article is organized as follows. Section 2 introduces the model and our notion of concentrated calling patterns. In Section 3 we derive equilibrium call prices. Section 4 determines the market equilibrium. In Section 5 networks choose reciprocal access charges. Section 6 discusses the scope for various policy interventions, such as imposing restrictions on access charges or on price discrimination between on- and off-net calls. Section 7 offers some concluding remarks.

2 A Model of Competition with Non-Uniform Calling Patterns

We consider competition between two interconnected telephone networks, 1 and 2, indexed by $i \neq j \in \{1, 2\}$. Both networks incur a fixed cost f to serve each subscriber. The marginal cost of providing a minute of a telephone call is $c \equiv c_O + c_T$, where c_O and c_T denote the costs borne by the originating and terminating network, respectively. As a result, the total marginal cost of an on-net call initiated and terminated on the same network is c. Networks pay each other a reciprocal access charge a when a call initiated on network i is terminated on a different network j. The access mark-up is equal to

[7] In the existing literature, such a pricing structure is obtained in Jeon et al. (2004) by introducing a utility from receiving calls, but disappears once again when access charges are endogenized (Cambini and Valletti, 2008). See also Hermalin and Katz (2011).

[8] See, however, Hurkens and Jeon (2012), who find below-cost access charges in a model with elastic subscription demand.

$m \equiv a - c_T$. Thus, for an off-net call, the network marginal cost is still c, but the perceived marginal cost for the network that initiates the call is $c + a - c_T = c + m$.

Networks can discriminate between on-net and off-net calls through the use of multi-part tariffs:

$$T_i(q_{ii}, q_{ij}) = F_i + p_{ii}q_{ii} + p_{ij}q_{ij},$$

where F_i is the fixed monthly subscription fee that consumers pay to network i, p_{ii} and q_{ii} are the price and quantity of on-net call minutes, and p_{ij} and q_{ij} are the respective price and quantity for off-net call minutes from network i to network j. The advantage of this simple multi-part tariff structure, apart from being comparable to the existing literature, is that we will be able to go a long way in characterizing the equilibrium even with general calling patterns.[9]

Consumer preferences over networks and call demand. A mass 1 of consumers is located uniformly around a preference space given by a Salop circle of circumference 2. Consumers' locations are indexed as $x \in X \equiv [-1, 1]$, where the points -1 and 1 coincide. The two networks' own attributes are represented by their locations at the two points $x_1 = 0$ and $x_2 = 1 (= -1)$. If a consumer at location x subscribes to network 1, he bears a disutility proportional to the distance between his own location and that of the network: $\tau |x|$ with $\tau > 0$. If the consumer subscribes to network 2, the respective disutility is $\tau (1 - |x|)$.

Consumers receive a fixed utility u_0 from being connected. We assume that u_0 is large enough so that all consumers connect to some network. This convenient assumption may be understood as reflecting the fact that most markets for fixed or mobile telephony are highly saturated. Once a call is placed, its length depends on the call price. Given prices per minute p_{ii} for on-net and p_{ij} for off-net calls, consumers demand calls of length $q_{ii} = q(p_{ii})$

[9] Our concept of calling patterns could allow networks to second-degree price discriminate through the use of nonlinear tariffs. In fact, without additional restrictions, networks can perfectly price discriminate through a menu that specifies different "forced quantities" of on-net and off-net minutes together with the respective fixed transfers, ensuring incentive compatibility through the different likelihoods with which consumers make on-net and off-net calls. Still, pricing results similar to those in Proposition 1 below arise when realistic restrictions are imposed, such as that a menu can only condition on aggregate on- and off-net call minutes instead of separately on the individual number and duration of calls. Therefore, we have opted to follow the literature (cf. Laffont et al., 1998b, or Gans and King, 2001) in assuming competition in multi-part tariffs.

and $q_{ij} = q(p_{ij})$, with demand elasticity $\eta(p) = -pq'(p)/q(p)$. The level of indirect utility associated with this demand function is denoted by $v_{ii} = v(p_{ii})$ and $v_{ij} = v(p_{ij})$, with $dv/dp = -q$ and $i \neq j \in \{1, 2\}$.

Calling patterns. The novel ingredient in our model is that consumers' individual calling patterns are not necessarily uniform. We stipulate that their calling patterns are symmetric around their own location. This implies that, for a given pair of consumers, the same number of calls is exchanged in either direction. The calling pattern of each consumer is then represented by a density $h : [0, 1] \to \mathbb{R}$ where $h(z)$ indicates the probability density that the customer at a given location calls someone at distance z (in preference space). Denote by H the corresponding distribution function. For technical reasons which will become apparent below we also assume that h is left-continuous at $z = 1$, i.e., that $\lim_{z \nearrow 1} h(z) = h(1)$.

For the uniform calling pattern we have $h^u(z) = 1$ and $H^u(z) = z$. We will compare calling patterns using the following definition:

Definition 1 *The calling pattern h is* more concentrated *than the calling pattern $\tilde{h}$ if $\tilde{h}$ first-order stochastically dominates h, i.e., if and only if*

$$H(z) \geq \tilde{H}(z)$$

holds for all $z \in [0, 1]$.

Hence, a more concentrated calling patterns shifts probability mass closer to the customer's own location, whereas a more dispersed calling pattern achieves the opposite. Define

$$\mu_h = \frac{\int_0^1 z dH(z)}{\int_0^1 z dH^u(z)} = 2 \int_0^1 z dH(z),$$

which measures the relative dispersion of calling pattern h, as compared to the uniform calling pattern h^u. Let the relative weight of calls to the farthest consumer (located at the opposite side of the circumference), again as compared to the uniform calling pattern, be

$$\hat{\mu}_h = \frac{h(1)}{h^u(1)} = h(1).$$

The following result follows from the definition of a concentrated calling pattern.

Lemma 1 *If calling pattern h is more concentrated than calling pattern $\tilde{h}$, we have*

$$\mu_h \leq \mu_{\tilde{h}} \quad \textit{and} \quad \hat{\mu}_h \leq \hat{\mu}_{\tilde{h}}.$$

Proof. See Appendix.

For simplicity, we will call any calling pattern "concentrated" if it is more concentrated than the uniform calling pattern. Thus any concentrated calling pattern h has $0 \leq \mu \leq 1$ and $0 \leq \hat{\mu} \leq 1$ (if there is no ambiguity we drop the index), and more concentration decreases either one or both of these values.

As a next step we derive the share of on-net calls of a client of network 1 at location x. We focus on situations where consumers behave symmetrically on the two symmetric arcs adjoining the networks. We assume that all consumers expect that the subscribers of network 1 are located symmetrically about its location at zero, that is, on $[-\hat{x}, \hat{x}]$, for some $\hat{x} \in [0, 1]$, and that all other consumers are subscribers of network 2. If the consumer at location $\hat{x}$ is indifferent between the offers of networks 1 and 2, so is the consumer at location $-\hat{x}$. Network 1's market share is thus $\hat{x}$, and that of network 2 is $1 - \hat{x}$.[10] If $\hat{x} \leq 1/2$, the share of on-net calls by consumer $x \in [-\hat{x}, \hat{x}]$ of network 1 is given by

$$G(\hat{x}|x) = \frac{1}{2}\left[H(\hat{x} - x) + H(\hat{x} + x)\right], \tag{1}$$

which becomes $G(\hat{x}|x) = \hat{x}$ for the uniform calling pattern. If $\hat{x} > 1/2$ and $|x| \leq 1 - \hat{x}$, the expression for $G(\hat{x}|x)$ is still identical to (1); if $1 - \hat{x} \leq |x| \leq \hat{x}$, we obtain

$$G(\hat{x}|x) = 1 - \frac{1}{2}\left[H(2 - \hat{x} - |x|) - H(\hat{x} - |x|)\right], \tag{2}$$

as now also consumers with very distant preferences from x are reached through on-net

[10] Below we provide conditions for when, in equilibrium, there will be a unique indifferent customer $\hat{x}$ such that all $x \in [-\hat{x}, \hat{x}]$ subscribe to network 1 and all others subscribe to network 2.

calls.[11] For given $\hat{x}$, the aggregate number of on-net calls on network 1 becomes

$$L_{11}(\hat{x}) = \int_{-\hat{x}}^{\hat{x}} G(\hat{x}|x)\frac{1}{2}dx.$$

For $\hat{x} \leq 1/2$, using expression (1), this can be written as

$$\begin{aligned} L_{11}(\hat{x}) &= \frac{1}{2}\int_0^{\hat{x}} \left[H(\hat{x}-x)+H(\hat{x}+x)\right]dx \\ &= \frac{1}{2}\int_0^{2\hat{x}} H(z)dz. \end{aligned}$$

For $\hat{x} > 1/2$ and $|x| > 1-\hat{x}$, using now expression (2), we obtain

$$\begin{aligned} L_{11}(\hat{x}) &= \frac{1}{2}\int_0^{1-\hat{x}} \left[H(\hat{x}-x)+H(\hat{x}+x)\right]dx + \int_{1-\hat{x}}^{\hat{x}} \left(1-\frac{1}{2}\left[H(2-\hat{x}-x)-H(\hat{x}-x)\right]\right)dx \\ &= \frac{1}{2}\int_0^{2-2\hat{x}} H(z)dz + 2\hat{x} - 1. \end{aligned}$$

Summing up, we have

$$L_{11}(\hat{x}) = \begin{cases} \frac{1}{2}\int_0^{2\hat{x}} H(z)dz & \text{if } \; 0 \leq \hat{x} \leq \frac{1}{2} \\ \frac{1}{2}\int_0^{2-2\hat{x}} H(z)dz + 2\hat{x} - 1 & \text{if } \; \frac{1}{2} \leq \hat{x} \leq 1 \end{cases},$$

which is continuous and continuously differentiable for all $\hat{x} \in [0,1]$, with

$$L'_{11}(\hat{x}) = \begin{cases} H(2\hat{x}) & \text{if } \; 0 \leq \hat{x} \leq \frac{1}{2} \\ 2 - H(2-2\hat{x}) & \text{if } \; \frac{1}{2} \leq \hat{x} \leq 1 \end{cases}.$$

[11] In a previous version of the article consumers were distributed on the Hotelling line, following much of the literature. Although this avoids a case distinction in terms of x, any functional specification of calling patterns must accommodate consumers located close to the boundaries. All results then go unchanged.

In particular, $L_{11}(\hat{x})$ is continuous and continuously differentiable at $\hat{x} = 1/2$ because we obtain, using either branch, that

$$L_{11}(1/2) = \frac{1}{2}\int_0^1 H(z)dz = \frac{1}{2} - \frac{\mu}{4},$$

with the left- and right-derivatives

$$L'_{11}(1/2)^- = H(1) = 1, \; L'_{11}(1/2)^+ = 2 - H(1) = 1.$$

Intuitively, $L_{11}(1/2) = \frac{1}{2} - \frac{\mu}{4}$ means that the number of on-net calls in a symmetric equilibrium increases with a less dispersed calling pattern, whereas the change in on-net calls due to a shift in the indifferent consumers is independent of the calling pattern.

The aggregate number of off-net calls originating on network 1 is $L_{12}(\hat{x}) = \hat{x} - L_{11}(\hat{x})$, with $L'_{12}(1/2) = 0$. In a similar manner, for network 2 we define on-net calls $L_{22}(\hat{x})$ and off-net calls $L_{21}(\hat{x})$, with $L_{22}(\hat{x}) + L_{21}(\hat{x}) = 1 - \hat{x}$.

Calling pattern example. In what follows, we will obtain explicit expressions for our equilibrium characterization by using the following family of calling patterns. With probability $(1 - \lambda)$ a consumer places a call randomly and thus adopts the uniform calling pattern h^u, whereas with probability λ each customer calls his calling club. The latter is represented by an interval $[0, \varepsilon]$, where $0 < \varepsilon < 1$, and a uniform calling pattern constrained to this interval: $h^\varepsilon(z) = 1/\varepsilon$ for $z \in [0, \varepsilon]$ and $h^\varepsilon(z) = 0$ otherwise. The considered calling pattern is now a convex combination of these two patterns:

$$h^\lambda(z) = (1 - \lambda)h^u(z) + \lambda h^\varepsilon(z).$$

Thus λ describes in a direct manner the correlation between a subscriber's brand preferences and his calling pattern.

Observe now that the two parameters μ_h and $\hat{\mu}_h$ are linear functionals in h, which has the useful implication that their value at any mixture of calling patterns is immediately obtained as a mixture of the corresponding values. As is easily checked, given the simple nature of h^u and h^ε, we obtain $\hat{\mu} = 1 - \lambda$ and $\mu = 1 - \lambda + \lambda\varepsilon$.

Recall that μ measures the dispersion of calls, where $\mu = 1$ for the uniform calling pattern. In this example, it is intuitive that μ is strictly lower when it is more likely that any customer makes a call to his calling club (higher λ) or when the club is more concentrated (lower ε). Furthermore, the expression for $\hat{\mu}$ is particularly revealing. With a uniform calling pattern we have $\hat{\mu} = 1$. This clearly corresponds to $\lambda = 0$ or the absence of calling clubs. At the other extreme, when a customer *only* makes calls to his calling club and no random calls, then he does not call the consumer that is farthest from him, so that $\hat{\mu} = 0$ when $\lambda = 1$. Finally, for $\varepsilon < 2\hat{x}$ (which holds in the symmetric equilibrium that we characterize) we have for the aggregate number of on-net calls that

$$L_{11}(\hat{x}) = (1 - \lambda)\,\hat{x}^2 + \lambda\left(\hat{x} - \frac{\varepsilon}{4}\right). \tag{3}$$

The latter implies, for example, that if we only had club calls ($\lambda = 1$) then, as club size decreases to zero in the limit, only on-net calls are made, $L_{11}(\hat{x}) = \hat{x}$ and $L_{12}(\hat{x}) = 0$.

Utility. Given $\hat{x}$, for any consumer $x \in [0, 1]$ the net utility from subscribing to network 1 is given by

$$V_1(x, \hat{x}) = u_0 + v_1(x, \hat{x}) - F_1 - \tau x,$$

where

$$v_1(x, \hat{x}) = G(\hat{x}|x)v(p_{11}) + [1 - G(\hat{x}|x)]\,v(p_{12}).$$

Recall that on network 1 a consumer of type x makes a share of on-net calls $G(\hat{x}|x)$ and a complementary share of off-net calls $1 - G(\hat{x}|x)$. If the consumer subscribes, instead, to network 2, his utility is

$$V_2(x, \hat{x}) = u_0 + v_2(x, \hat{x}) - F_2 - \tau(1 - x),$$

with

$$v_2(x, \hat{x}) = [1 - G(\hat{x}|x)]\,v(p_{22}) + G(\hat{x}|x)v(p_{21}).$$

For consumers in $[-1, 0]$ similar expressions follow after indexing adjustments.

Market game. At $t = 1$, for any given reciprocal access charge, networks compete for consumers by simultaneously making contract offers T_i. At $t = 2$, consumers subscribe and place calls. At this stage, all payoffs are realized. In Section 5, we will also consider an initial stage $t = 0$ where networks jointly choose a profit-maximizing reciprocal access charge.

3 Using On- and Off-Net Tariffs for Price Discrimination

Given the contract T_1, each subscriber at location $x \in [-\hat{x}, \hat{x}]$ of network 1 yields profits equal to the sum of the fixed part F_1, the call profits

$$\pi_1(x, \hat{x}) = G(\hat{x}|x)(p_{11} - c)q(p_{11}) + [1 - G(\hat{x}|x)]\,(p_{12} - c - m)q(p_{12}),$$

and the termination profits

$$R_{12}(x, \hat{x}) = [1 - G(\hat{x}|x)]\, mq(p_{21}),$$

where the symmetry of calling patterns implies that the number of incoming off-net calls is equal to the number of outgoing off-net calls $1 - G(\hat{x}|x)$. Thus the number of calls between networks is balanced, $L_{12}(\hat{x}) = L_{21}(\hat{x})$ for any given $\hat{x}$, whereas the actual traffic in terms of call minutes differs according to networks' call prices.

The profits that network 1 obtains from a given subscriber at location x are then

$$\Pi_1(x, \hat{x}) = \pi_1(x, \hat{x}) + F_1 + R_{12}(x, \hat{x}) - f,$$

and total profits of network 1 are

$$\begin{aligned}\overline{\Pi}_1(\hat{x}) &= \int_0^{\hat{x}} \Pi_1(x,\hat{x})dx \\ &= \hat{x}\left(F_1 - f\right) + L_{11}(\hat{x})(p_{11}-c)q(p_{11}) \\ &\quad + L_{12}(\hat{x})\left[(p_{12}-c-m)q(p_{12}) + mq(p_{21})\right].\end{aligned} \tag{4}$$

Similar expressions are obtained for network 2.

Optimal prices. Take the networks' market shares $\hat{x}$ and $1-\hat{x}$, as given. We consider how networks optimally choose on- and off-net prices so as to maximize profits, holding these market shares constant.

More specifically, we consider the following program. We take as given the gross utility level that the marginal subscriber at $\hat{x}$ must obtain:

$$V_1(\hat{x},\hat{x}) \geq \overline{V}.$$

Note that in equilibrium the utility level $\overline{V}$ will be determined by the offer of the competing network, $\overline{V} = V_2(\hat{x},\hat{x})$. For given $\hat{x}$ and $\overline{V}$, we solve for the choices p_{11} and p_{12} that maximize $\bar{\Pi}_1$. We first relax this program by only considering the participation constraint of the marginal consumer $x = \hat{x}$ (and equivalently of the consumer at $-\hat{x}$) but not those of consumers $-\hat{x} < x < \hat{x}$, and then state a sufficient condition for when (both on- and off-equilibrium) the solution to the relaxed program is indeed a solution to the original one.

Define now

$$\hat{L}_{11}(\hat{x}) = \hat{x}G(\hat{x}|\hat{x})$$

as the total number of on-net calls on network 1 that would arise if all subscribers of network 1 had the same calling pattern as the marginal subscriber $\hat{x}$, and let $\hat{L}_{12}(\hat{x}) = \hat{x} - \hat{L}_{11}(\hat{x})$ be the corresponding off-net calls. For network 2 we let $\hat{L}_{22}(\hat{x}) = (1-\hat{x})(1-G(\hat{x}|\hat{x}))$ and $\hat{L}_{21}(\hat{x}) = (1-\hat{x})G(\hat{x}|\hat{x})$.

Proposition 1 *Take the relaxed program of the two networks, where for each network* *only the participation constraints of the marginal subscribers bind. Then, network i's price for on-net calls satisfies*

$$\frac{p_{ii} - c}{p_{ii}} = \frac{1}{\eta(p_{ii})} \left(1 - \frac{\hat{L}_{ii}(\hat{x})}{L_{ii}(\hat{x})}\right), \tag{5}$$

whereas that for off-net calls satisfies

$$\frac{p_{ij} - c - m}{p_{ij}} = \frac{1}{\eta(p_{ij})} \left(1 - \frac{\hat{L}_{ij}(\hat{x})}{L_{ij}(\hat{x})}\right). \tag{6}$$

When τ is sufficiently large, then for any market share $\hat{x}$ these expressions characterize the optimal prices for the two networks in the unrelaxed program.

Proof. See Appendix.

When the calling pattern of the average infra-marginal subscriber is the same as that of the marginal subscriber $\hat{x}$, as is the case with the uniform calling pattern h^u, then $\hat{L}_{ii}(\hat{x}) = L_{ii}(\hat{x})$ and $\hat{L}_{ij}(\hat{x}) = L_{ij}(\hat{x})$. Proposition 1 then yields the standard perceived marginal-cost pricing result $p_{ii} = c$ and $p_{ij} = c + m$. Yet, when the marginal subscriber makes more off-net calls and fewer on-net calls than the average subscriber, then $p_{ii} > c$ (the on-net price is distorted upwards) and $p_{ij} < c + m$ (the off-net price is distorted downwards). Intuitively, raising the on-net price above marginal cost and lowering the off-net price below marginal cost allows the network to extract more of the information rent of infra-marginal subscribers, who will not switch networks when they have to cede slightly more of their surplus. The pricing formula in Proposition 1 trades off the increase in profits that is made from infra-marginal subscribers with the compensation that must be given to the marginal subscriber in terms of an adjusted fixed fee.

To further foster the intuition, take the case with symmetric market shares, $\hat{x} = 1/2$, and also suppose for the moment that access is priced at cost, $m = 0$. In this case, suppose now that, contrary to the derived optimal tariffs, a network would charge uniform prices. Note that the marginal subscriber now cares equally about on-net and off-net call charges, and so does not mind if the on-net charge rises so long as there is a corresponding reduction in the off-net charge. However, the infra-marginal users of the respective network make fewer off-net calls, so that the network's profits are increased if the on-net call charge is raised and the off-net charge reduced.

Symmetric market shares. As the environment is symmetric, for simplicity we investigate symmetric equilibria. Thus the market equilibrium will also be symmetric with $\hat{x} = 1/2$. From the definition of more concentrated calling patterns, the respective numbers of on-net calls, $L_{11}(1/2) = L_{22}(1/2) = 1/2 - \mu/4$, are strictly higher when calling patterns are more concentrated. Likewise, the respective numbers of off-net calls, $L_{12}(1/2) = L_{21}(1/2) = \mu/4$, are strictly lower. As, given symmetry, the marginal customer at $\hat{x} = 1/2$ always makes half of his calls on-net and the other half off-net, we have, regardless of how concentrated calling patterns are, $\hat{L}_{11}(1/2) = (H(0) + H(1))/4 = 1/4$ and $\hat{L}_{12}(1/2) = 1/4$. From these observations we obtain immediately that as calling patterns become more concentrated relative to h^u prices in Proposition 1 become more distorted: The multiplier $1 - \frac{\hat{L}_{ii}(1/2)}{L_{ii}(1/2)} > 0$ in expression (5) increases, which pushes p_{ii} above c, and the multiplier $1 - \frac{\hat{L}_{ij}(1/2)}{L_{ij}(1/2)} < 0$ in expression (6) decreases, which pushes p_{ij} below $c + m$.

Substituting the above expressions, we obtain the following equilibrium call prices:

Corollary 1 *With symmetric market shares, on-net and off-net prices are*

$$\begin{aligned} \frac{p_{ii} - c}{p_{ii}} &= \frac{1}{\eta(p_{ii})} \frac{1-\mu}{2-\mu}, \\ \frac{p_{ij} - c - m}{p_{ij}} &= -\frac{1}{\eta(p_{ij})} \frac{1-\mu}{\mu}, \end{aligned} \tag{7}$$

We see clearly that the equilibrium distortions of call prices under two-part tariffs depend exclusively on the dispersion of calls, i.e., on a simple global measure of calling patterns.

4 Market Equilibrium

For our following analysis, we first derive two useful results linking networks' optimizing behavior to calling patterns. More precisely, we first consider how the average number of on-net calls per subscriber changes as the marginal subscriber is shifted away from the symmetric market share $\hat{x} = 1/2$:

$$\frac{d}{d\hat{x}}\left(\frac{L_{11}(\hat{x})}{\hat{x}}\right)\bigg|_{\hat{x}=1/2} = 2 - 4L_{11}(1/2) = \mu.$$

Thus under a concentrated calling pattern the average number of on-net calls changes less than under the uniform calling pattern. Similarly, the change in the number of on-net calls by the marginal consumer at $\hat{x}$ is given by

$$\frac{dG(\hat{x}|\hat{x})}{d\hat{x}}\bigg|_{\hat{x}=1/2} = \hat{\mu}, \tag{8}$$

i.e., the relative weight of calls to the farthest consumer.

For our following analysis both the global parameter μ and the local parameter $\hat{\mu}$ will play key roles. We will see that they embody all the information about calling patterns that is necessary to determine the market equilibrium and the jointly profit-maximizing choice of access charges. We already know that both parameters decrease as the calling pattern becomes more concentrated, but now the point of view is a different one: A shift of the marginal customer around $\hat{x} = 1/2$ has a smaller impact on the average number of on-net calls per subscriber because they are less closely tied to the identity of the marginal costumer; and also the number of on-net calls of the marginal costumer is less closely tied to market share. Somewhat loosely speaking, the decrease in both μ and $\hat{\mu}$ results from the fact that, as calling patterns become more concentrated, market shares are less relevant for how much customers make use of on-net instead of off-net calls.

The marginal cost of expanding market share. Throughout the subsequent analysis we assume that a unique and symmetric equilibrium in pure strategies exists.[12] As the marginal consumer must be indifferent between the offers of the two networks, i.e.,

[12] A proof of existence and uniqueness of equilibrium follows the same steps as Laffont et al. (1998b). Transport cost τ must be high enough in order to guarantee unique equilibrium market shares and concave profits. Full details are available in the web appendix.

$V_1(\hat{x}, \hat{x}) = V_2(\hat{x}, \hat{x})$, we have

$$F_1 = F_2 + v_1(\hat{x}, \hat{x}) - v_2(\hat{x}, \hat{x}) + \tau(1 - 2\hat{x}). \tag{9}$$

From this we obtain

$$\frac{dF_1}{d\hat{x}} = \frac{dG(\hat{x}|\hat{x})}{d\hat{x}} (v_{11} + v_{22} - v_{12} - v_{21}) - 2\tau,$$

which at a symmetric equilibrium candidate and after substituting from (8) becomes

$$\left.\frac{dF_i}{d\hat{x}}\right|_{\hat{x}=1/2} = -2 \left[\tau - \hat{\mu} \left(v_{ii} - v_{ij}\right)\right]. \tag{10}$$

Expression (10) captures how expensive it is for a network to shift the marginal subscriber and capture market share. In the standard Hotelling model without network effects, the respective marginal cost would be just 2τ. This remains true when on- and off-net call prices are identical, as then $v_{ii} = v_{ij}$. On the other hand, when off-net calls are more expensive than on-net calls, $v_{ii} > v_{ij}$ holds and tariff-mediated network effects are created. It then becomes *less* expensive for a network to expand its market share. For $p_{ii} > p_{ij}$ and thus $v_{ii} < v_{ij}$ the opposite holds. Importantly, the marginal cost of expanding a network's market share (10) is affected by the value of $\hat{\mu}$ if there are tariff-mediated network effects. Recall that $\hat{\mu} = 1$ holds with uniform calling patterns, whereas it is lower with concentrated calling patterns. Thus tariff-mediated network effects, as captured in expression (10), become gradually less important as calling patterns become more concentrated. This observation will be important below when we analyze how access charges are optimally chosen such as to dampen competition.

Equilibrium profits. We will now derive networks' fixed fees and profits in a symmetric equilibrium. Given the tariff of network 2 and the optimal structure of call prices discussed above, network 1 maximizes its profits by adjusting its fixed fee, or equivalently, its market share.

In a symmetric equilibrium, we have $p_{ii} = p_{jj}$ and $p_{ij} = p_{ji}$. It is then convenient to denote the per-call profits from on-net calls by

$$r_{ii} \equiv (p_{ii} - c)q(p_{ii}).$$

Also consider

$$(p_{ij} - c - m)q(p_{ij}) + mq(p_{ji}), \tag{11}$$

which represents network i's profits from an exchange of one pair of off-net calls with network j. In a symmetric equilibrium we have that $p_{ij} = p_{ji}$, so that quantities are equal: $q(p_{ij}) = q(p_{ji})$ when $\hat{x} = 1/2$. Then, expression (11) simplifies to

$$r_{ij} \equiv (p_{ij} - c)q(p_{ij}).$$

In the proof of Proposition 2 we substitute $dF_1/d\hat{x}$ from (10) and, using symmetry, solve the first-order condition for profit-maximization to obtain

$$F^* = f + \tau - r_{ii} - \hat{\mu}\left(v_{ii} - v_{ij}\right). \tag{12}$$

The equilibrium fixed fee increases in the per-customer fixed cost and in the transport cost, as usual. In fact, in a standard Hotelling model without network effects, we would have $F^* = f + \tau$. There are now two differences: On-net and off-net prices may be different, so that $v_{ii} \neq v_{ij}$, and on-net revenues are not zero, given that prices are not set equal to cost when calling patterns are non-uniform. Substituting F^* back into expression (4) for profits leads to the following outcome.

Proposition 2 *In a symmetric equilibrium, profits for each network are equal to*

$$\bar{\Pi}^* = \frac{1}{2}\left[\tau - \hat{\mu}(v_{ii} - v_{ij}) + \frac{\mu}{2}\left(r_{ij} - r_{ii}\right)\right]. \tag{13}$$

Proof. See Appendix.

In the standard Hotelling model, or when on- and off-net prices are the same, profits are equal to $\tau/2$. Thus, the first term in expression (13) captures as usual how profits depend on the substitutability of networks' services. The second term in expression (13) describes the effect of the tariff-mediated network effects on the marginal subscriber. If the term $\hat{\mu}(v_{ii} - v_{ij})$ is positive, as a result of off-net prices above on-net prices, then these network effects are positive and it is easier to capture market share. When on-net prices are above off-net prices, on the other hand, then these effects are negative, and it is more costly to capture market share. Importantly, when it is easier to capture market share, competition is more intense, and equilibrium profits are lower. As we can see, the relevance of this term depends on the relevance of the calling club of the *marginal* consumer, as described by marginal call weight $\hat{\mu}$.

We come now to the third term in expression (13), which indicates how profits due to infra-marginal subscribers change with the difference between on- and off-net prices. Starting from symmetric market shares, when a network deviates and captures more market share, it increases the number of on-net calls at the expense of decreasing the number of both outgoing and incoming off-net calls. As $r_{ij} - r_{ii}$ captures the true difference in profits between off-net and on-net calls, i.e., evaluated at the true marginal cost, the third term in (13) captures how a small increase in market share impacts profits by turning off-net into on-net calls. As this effect works through all subscribers on a given network, its importance depends on the *average* behavior of subscribers, as described by the dispersion of calls μ.

The second and third terms in expression (13) capture the tariff-induced *costs* and *benefits* from acquiring customers. Both costs and benefits decrease for more concentrated calling patterns. Therefore the impact of a more concentrated calling pattern on network profits is ambiguous, at least for given access charges.

Waterbed effect. Before we ask in the next section how networks would optimally set the access price so as to maximize equilibrium profits, we shed more light on our results by considering the so-called waterbed effect. As we discuss in more detail below, in many jurisdictions around the world, access charges are subject to some form of wholesale regulation. Although access charges directly affect off-net call prices and networks' termination revenues, policy-makers also have a practical interest in understanding how their intervention may influence the structure of *other* prices, which is frequently referred to as a waterbed or seesaw effect. This rebalancing of the pricing structure can be studied empirically using tariff data (see, e.g., Genakos and Valletti, 2011).

In our model of competition in multi-part tariffs, the access price affects off-net prices directly and fixed fees (12) indirectly through v_{ij}. Suppose now that the elasticity of demand for call minutes is constant, $\eta(p) \equiv \eta > 0$. From Corollary 1 we find the equilibrium off-net price

$$p_{ij} = (c+m)\,\frac{\mu\eta}{\mu\,(\eta-1)+1},$$

which does not depend on $\hat{\mu}$ but increases in m and μ (the denominator is positive because $\mu \leq 1$). For the fixed fee we obtain

$$\frac{dF^*}{dm} = -\hat{\mu} q_{ij}\frac{dp_{ij}}{dm} = -\hat{\mu}\frac{p_{ij}q_{ij}}{c+m} < 0.$$

Thus a more concentrated calling pattern dampens the waterbed effect through a lower $\hat{\mu}$, because additional marginal subscribers will make relatively more off-net calls than inframarginal subscribers. These marginal customers become less attractive with a higher access charge, so that the compensation in the fixed fee offered is lower.

Apart from the direct effect through $\hat{\mu}$ there is also an indirect effect of a more concentrated calling pattern, namely through a lower dispersion μ which decreases p_{ij}. Then revenues $p_{ij}q_{ij}$ decrease (increase) if demand is $\eta > 1$ ($\eta < 1$). Thus we can conclude that under a more concentrated calling pattern the waterbed effect on fixed fees will be unambiguously smaller if call demand is elastic, whereas the two effects go in opposite directions if it is inelastic.

We can demonstrate this potential trade-off more clearly for the family of calling patterns h^{λ}. We obtain

$$\frac{dF^*}{dm} = -(1-\lambda)\,(c+m)^{-\eta}\left(1-\frac{1}{\eta}+\frac{1}{\eta}\frac{1}{1-\lambda\,(1-\varepsilon)}\right)^{\eta-1}.$$

In the limit when consumers make *only* calls to their club ($\lambda \to 1$), the fixed fee is *independent* of the access charge, such that there is *no* waterbed effect.[13] On the other

[13] Of course, the access charge has an effect on off-net prices also in this case.

hand, for $\lambda < 1$ a smaller size ε of the calling club reduces (increases) the absolute value of the waterbed effect if $\eta > 1$ ($\eta < 1$).

5 Dampening Competition through Access Charges

In this section we will determine the jointly profit-maximizing reciprocal access charge a. This is the access charge that networks would want to negotiate if they were free to choose between themselves.

Joint profits in a symmetric equilibrium are equal to $2\bar{\Pi}^*$. There are now two opposing effects to consider, corresponding to the second and third terms in expression (13). We referred to these terms as the costs and benefits of capturing additional market share. Take first the costs, i.e., the second term in expression (13). Decreasing the access charge pushes off-net prices down, leading to a decrease in the utility difference $v_{ii} - v_{ij}$. This makes joining the larger network less attractive for customers and thus dampens competition. From this perspective, networks should choose a low a.

However, lower off-net prices decrease the profits $r_{ij} = (p_{ij} - c)q_{ij}$ from making and receiving off-net calls, at least as long as the off-net price induced by a lies below the monopoly call price[14] $p_M = \arg\max_p[(p - c)q(p)]$. Recall that a does not affect the price of on-net calls, p_{ii}. Hence, when a and the off-net price p_{ij} increase, the difference between off-net and on-net profits, $r_{ij} - r_{ii}$, increases as well. From expression (13) this reduces the benefits of acquiring consumers and increases profits $\bar{\Pi}^*$, thus from this perspective networks should choose a high a.

The first effect is stronger when $\hat{\mu}$ is large, which is the case for a less concentrated calling pattern. Then, a shift of the marginal customer has a larger effect on his share of on-net calls. The second effect also increases with a less concentrated calling pattern, i.e., with a larger μ, because then more off-net calls will be transformed into on-net calls as the marginal customer changes. In the proof of the Proposition 3, we show that the off-net price that the networks wish to jointly implement through their choice of the access charge

[14] For simplicity, we stipulate here that this problem has a unique finite solution p_M. This corresponds to $\eta > 1$ in the constant elasticity case, with $p_M = c\eta/(\eta - 1)$.

is indeed strictly decreasing in $\hat{\mu}$ and strictly increasing in μ:

$$\frac{p_{ij} - c}{p_{ij}} = \frac{1}{\eta(p_{ij})} \left(1 - \frac{2\hat{\mu}}{\mu}\right). \tag{14}$$

When calling patterns are uniform, we obtain immediately $p_{ij} < c$ and $a < c_T$, as in Gans and King (2001). In order to obtain the optimal access charge under non-uniform calling patterns we substitute the equilibrium price from Proposition 1 into (14).

Proposition 3 *The jointly profit-maximizing access charge is*

$$a^* = c_T + c\frac{1 - 2\hat{\mu}}{\mu\left[\eta(p_{ij}) - 1\right] + 2\hat{\mu}}. \tag{15}$$

It exceeds termination cost if and only if the calling pattern is sufficiently concentrated,

$$\hat{\mu} < \frac{1}{2}. \tag{16}$$

Proof. See Appendix.

Whether the profit-maximizing access charge is above or below termination cost is thus closely tied to the probability of calling the farthest subscriber, as summarized by the parameter $\hat{\mu}$. Interestingly, it does not depend on other properties of the calling pattern, such as its dispersion μ (the exact level of the profit-maximizing access charge depends on μ, though).

When calling patterns are sufficiently concentrated, the result that Gans and King (2001) obtained with a uniform calling pattern is overturned, as then the access charge is chosen *above* cost in order to dampen competition. In other words, the balance between the costs and benefits of expanding market share changes for more concentrated calling patterns, with the benefits of higher off-net prices outweighing their cost.

Expression (15) for a^* is only implicit, given that $\eta(p_{ij})$ on the right-hand side depends on the access charge, too. This is however no longer the case with isoelastic call demand, $\eta(p_{ij}) = \eta$ - a specification that we have already made use of when discussing the waterbed effect. Then we can substitute η into (15) to immediately obtain the optimal access charge.

Proposition 3 provides a clear threshold indicating when the profit-maximizing access charge is above or below termination cost c_T, namely whether $\hat{\mu} < \frac{1}{2}$ or $\hat{\mu} > \frac{1}{2}$. Due to the confounding effect of dispersion μ, the optimal access charge need not change monotonically as we make calling patterns more concentrated. Still, this is the case in our example. There, where we use the calling pattern h^λ, we have

$$a^* = c_T + c\frac{2\lambda - 1}{(\eta + 1)(1 - \lambda) + (\eta - 1)\lambda\varepsilon}. \tag{17}$$

The access charge margin is strictly negative if and only if consumers make a random call less than half of the time ($\hat{\mu} = 1 - \lambda < 1/2$). Furthermore, a^* increases in λ, so that the access charge strictly increases as it becomes more likely that consumers call their respective calling clubs. A smaller club size ε leads to an a^* further away from c_T if $\eta > 1$, or closer to c_T if $\eta < 1$. Thus in our example the dispersion of calls affects the access charge differently depending on whether call demand is elastic or inelastic.

On-net versus off-net prices. Having determined the jointly profit-maximizing access charge, we can finally return to the question of when on-net prices will be lower than off-net prices if networks adopt this access charge. Although price discrimination reduces off-net prices below the respective cost, the access charge effect pushes in the opposite direction. The following result shows that the outcome is determined by the relative strength of these two countervailing effects.

Proposition 4 *The on-net price is lower than the off-net price, at the profit-maximizing access charge, if and only if*

$$\hat{\mu} < \frac{1}{2 - \mu}\frac{\mu}{2}. \tag{18}$$

Proof. See Appendix.

For our example calling pattern h^λ, we can transform (18) into a condition that clarifies the relation between the probability of club calls λ and the size of the club ε. To this end, we first state the corresponding equilibrium prices:

$$\frac{p_{ii} - c}{p_{ii}} = \frac{1}{\eta}\frac{\lambda(1 - \varepsilon)}{\lambda(1 - \varepsilon) + 1}, \quad \text{and} \quad \frac{p_{ij} - c - m}{p_{ij}} = \frac{1}{\eta}\frac{\lambda(1 - \varepsilon)}{\lambda(1 - \varepsilon) - 1}. \tag{19}$$

It is clear that more club calls λ and a smaller club size ε move prices away from perceived marginal cost. Substituting for the optimal choice a^* from (17), the on-net price is below the off-net price if and only if

$$\lambda > \lambda^* = \frac{1 - 3\varepsilon + \sqrt{9 - 14\varepsilon + 9\varepsilon^2}}{4(1-\varepsilon)}. \tag{20}$$

A necessary condition for (20) to hold is $\lambda \geq 1/2$, i.e., calling clubs must be sufficiently relevant for off-net calls to be more expensive than on-net calls, and the access charge a^* must not be below cost. This condition is stricter when the calling club size ε is small, because less dispersion of calls made to calling clubs implies higher on-net and lower off-net prices, which counters the effect of the rising access charge.

Recall from the Introduction that there is evidence that calls are more concentrated on-net than what would be predicted by market shares alone. In our model there are two forces at work, namely consumers' calling patterns and the relation between on- and off-net prices. Whereas the former indicate the number of calls, the latter determine their length. For all $\lambda > \lambda^*$ both the calling pattern h^λ and the price difference work towards increasing the imbalance in terms of call minutes. For instance, suppose that calling clubs comprise 30% of all consumers, so that $\varepsilon = 0.3$. Then, we obtain $\lambda^* = 88\%$: To ensure that on-net prices are below off-net prices more than 88% of all calls must be club calls.

The share of on-net calls in all calls, $L_{11}(\hat{x})/\hat{x}$ in (3) at $\hat{x} = 1/2$, then becomes 81%. A related measure is the imbalance ratio

$$\frac{L_{ii}(1/2)/\hat{x}}{L_{ij}(1/2)/(1-\hat{x})} = \frac{2-\mu}{\mu} = \frac{1+\lambda(1-\varepsilon)}{1-\lambda(1-\varepsilon)}.$$

Whereas for the uniform calling pattern it is equal to 1, for the above numbers it is equal to 4.2 at $\lambda = \lambda^*$.[15] Furthermore, it increases with more likely club calls and a smaller club size. If measured in minutes instead of calls, at $a = a^*$ and $\lambda \geq \lambda^*$ the imbalance ratio

[15] For Italy in 2011, e.g., the subscriber and on-net call shares compared as follows: TIM 36% vs. 70%, Vodafone 34% vs. 78%, Wind 20% vs. 70%, and H3G 10% vs. 33%. These data imply imbalance ratios of, respectively, 4.2 (TIM), 6.9 (Vodafone), 9.2 (Wind), and 4.4 (H3G).

will be even larger, at

$$\frac{L_{ii}(1/2)/\hat{x}}{L_{ij}(1/2)/(1-\hat{x})}\frac{q(p_{ii})}{q(p_{ij})}=\frac{1+\lambda\left(1-\varepsilon\right)}{1-\lambda\left(1-\varepsilon\right)}\left(\frac{\eta+\frac{1-\lambda-\varepsilon\lambda}{1-\lambda+\varepsilon\lambda}}{\eta-\frac{\lambda(1-\varepsilon)}{1+\lambda-\varepsilon\lambda}}\right)^{-\eta}.$$

6 Regulation

Equilibrium consumer surplus is

$$CS=u_0+2L_{ii}(1/2)v_{ii}+2L_{ij}(1/2)v_{ij}-F^*-4\int_0^{1/2}\tau x\frac{1}{2}dx.$$

Substituting for F^* from (12), we obtain

$$CS=u_0-f-\frac{5}{4}\tau+\left(r_{ii}+v_{ii}\right)+\left(\frac{\mu}{2}-\hat{\mu}\right)\left(v_{ij}-v_{ii}\right). \quad (21)$$

In a standard Hotelling model consumer surplus is equal to total welfare minus τ, because the latter is then equal to networks' total profits. Different on- and off-net prices lead to the following differences: First, $r_{ii}+v_{ii}$ is not equal to first-best surplus because the on-net price is not at marginal cost; and, second, there is the additional term $\left(\frac{\mu}{2}-\hat{\mu}\right)\left(v_{ij}-v_{ii}\right)$ capturing network effects. We discuss these terms in more detail below. Total social welfare is given by the sum of consumer surplus and networks's profits:

$$W=2\bar{\Pi}^*+CS.$$

In what follows, we consider various policy interventions aimed at maximizing either total welfare W or consumer surplus CS.

Regulating Access Charges

We ask next how the networks' profit-maximizing access charge compares to the access charge that a social planner would optimally set.

Setting access charges to maximize welfare. Suppose that a policy maker's objective is to maximize total welfare W by setting the access a. As is intuitive and shown more formally in the proof of the subsequent proposition, the socially optimal level of a is such that off-net prices are at network cost. Substituting $p_{ij} = c$ into the off-net price (7) yields the access charge

$$a^W = c_T + \frac{c}{\eta(c)} \frac{1-\mu}{\mu}. \tag{22}$$

Proposition 5 *Welfare is maximized at $a = a^W$, which exceeds termination cost c_T for all concentrated calling patterns. The profit-maximizing access charge a^* is above the socially optimal level if and only if*

$$\hat{\mu} < \mu/2.$$

Proof. See Appendix.

From condition (18), which is stricter than $\hat{\mu} < \mu/2$ if $\mu < 1$, we can see that whenever networks would opt for an access charge below the socially optimal value then the resulting on-net price would be above the off-net price. Thus for only slightly concentrated calling patterns the qualitative findings of Gans and King (2001) continue to hold.

On the other hand, we have $a^W > c_T$ for all concentrated calling patterns. Intuitively, it is efficient to set the access charge above cost because networks set off-net prices below perceived costs as a discrimination device. Proposition 5 also shows that the market outcome results in an access charge above the efficient access charge unless the farthest consumers are called very little.

For the calling pattern h^λ, we obtain

$$a^W = c_T + \frac{c}{\eta} \frac{\lambda(1-\varepsilon)}{1-\lambda(1-\varepsilon)},$$

so that the welfare-maximizing access charge a^W is strictly higher when calling clubs become more relevant (higher λ) or when they become less dispersed (lower ε). Furthermore, we have $a^* > a^W$ whenever

$$\lambda > \frac{1}{1+\varepsilon}. \tag{23}$$

Hence, for our example we would predict that a welfare-maximizing regulator would want to set access charges strictly *below* the unconstrained market equilibrium level whenever localized calling patterns are sufficiently important. This case does not arise for uniform calling patterns ($\lambda = 0$).

Setting access charges to maximize consumer surplus. Suppose now that the social planner maximizes consumer surplus. His optimal choice is then immediately obtained by looking at the last term in expression (21): When $\hat{\mu} < \mu/2$ the social planer would want to push it down all the way to zero (bill-and-keep) in order to maximize v_{ij}. Instead, when $\hat{\mu} > \mu/2$, the social planner would want to push up the access charge as far as possible, essentially fully choking off the demand for off-net calls. Hence, the social planner's program to maximize consumer surplus gives rise to a bang-bang solution.

This extreme outcome is clearly due to the stylized nature of our model, as it abstracts, for instance, from elastic participation in the market. Still, it illustrates the previously discussed interaction between calling pattern concentration and how the intensity of competition is affected by tariff-induced network effects. In particular, despite shutting down off-net communications and the associated surplus when $\hat{\mu} > \mu/2$, the network effect intensifies competition for the market via lower fixed fees, to the benefit of consumers.

Proposition 6 *When $\hat{\mu} < (>)\ \mu/2$ holds, consumer surplus CS is strictly decreasing (increasing) in the access charge a.*

Incidentally, when $\hat{\mu} < \mu/2$ holds the access charge that prevails in the unregulated equilibrium will be too high *both* from the perspective of maximizing welfare and from the perspective of maximizing consumer surplus. This is another instance where concentrated calling patterns matter, as this possibility would never arise under a uniform distribution.

Imposing Uniform Pricing

In our model networks price discriminate between on-net and off-net calls. A common theme in the IO literature is to analyze how networks' ability to price discriminate affects welfare and consumer surplus. In what follows we explore the prohibition of price discrimination given its practical relevance.

The degree of on-net/off-net price discrimination has in fact been a subject of concern for a number of competition authorities and regulators in Europe and elsewhere. For example, in April 2008, Germany's Federal Competition Authority (Bundeskartellamt) initiated proceedings against the two largest incumbent mobile telephone network operators, T-Mobile and Vodafone, "on the suspicion of the abuse of a joint dominant position on grounds of price differentiation between calls within their own networks (on-net) and calls to other networks (off-net)."[16] Whereas the German Competition Authority discontinued its investigations by the end of 2009, similar complaints against price differentiation between on-net and off-net calls have been made in other countries such as Austria, Belgium (related to the German case), Greece, Italy, or Turkey. In 2011, the New Zealand Commerce Commission also expressed the concern that tariff-mediated network effects could be used strategically to stifle market competition and secure market power.

In our setting, an immediate consequence of uniform pricing is that networks' joint profits are always equal to τ, irrespective of how the access charge is set. This is a standard result in Hotelling models, though formally it also follows immediately from expression (13) for $\bar{\Pi}^*$, after substituting $v_{ii} = v_{ij}$ and $r_{ij} = r_{ii}$ when $p_{ii} = p_{ij} \equiv p^u$. For each network the optimal symmetric uniform price maximizes average perceived efficiency

$$\frac{1}{2}\left[v(p^u) + q(p^u)(p^u - c)\right] - mq(p^u)L_{ij}(1/2),$$

which results in $p^u = c + m\mu/2$. That is, with a more concentrated calling pattern the equilibrium uniform price will be closer to cost. Both total welfare and consumer surplus are maximized when $m = 0$ so that the optimal access charge is equal to cost: $a = c_T$, with resulting uniform price $p^u = c$. We now take these observations as a benchmark against which we compare price discrimination.

When through access charges are set equal to cost, it is immediate from the preceding observations that uniform pricing achieves the highest feasible welfare. From Proposition

[16] See Bundeskartellamt (2010), Examination of Possible Abuse of a Dominant Position by T-Mobile and Vodafone by Charging Lower On-net Tariffs for Mobile Voice Telephony Services, online at: http://cms.bundeskartellamt.de/wEnglisch/download/pdf/Fallberichte/B07-170-07-engl.pdf.

1 we know that with price discrimination the optimal on-net price is always set inefficiently high, independently of the access charge. Thus the first best cannot be achieved under price discrimination with a non-uniform calling pattern, and the joint imposition of uniform pricing and cost-based access maximizes welfare.

It is more interesting to compare consumer surplus in the two cases. Denote the net surplus achieved per on-net call by

$$w_{ii} = v(p_{ii}) + q(p_{ii})(p_{ii} - c),$$

In addition, let $w^{\max} = v(c)$ be the maximum feasible surplus under uniform pricing, obtained when $m = 0$.

Whereas price discrimination clearly reduces welfare, its impact on consumer surplus is more subtle. As we repeatedly observed, through creating tariff-induced network effects, a difference between the price of on-net calls and that of off-net calls also affects the degree of competition. Using expression (21), we see that consumer surplus is strictly higher than the maximal consumer surplus under uniform pricing if

$$\left(\frac{\mu}{2} - \hat{\mu}\right)(v_{ij} - v_{ii}) > w^{\max} - w_{ii}. \tag{24}$$

With uniform calling patterns, both the LHS and the RHS of the above expression are equal to zero when access is regulated at cost. Thus price discrimination and uniform pricing lead to the same result both for welfare and for consumer surplus. This is no longer true for non-uniform calling patterns. At $m = 0$ we have from $p_{ij} < p_{ii}$ for $\mu < 1$ that $v_{ij} > v_{ii}$, i.e., a *necessary* condition for discrimination to improve consumer surplus is that $\hat{\mu} < \mu/2$. Interestingly, this was also the condition for the equilibrium access charge a^* to exceed the welfare maximizing access charge a^W. Although generally a higher concentration of calling patterns has an ambiguous effect on this condition, in our example $\hat{\mu} < \mu/2$ holds if and only if calling patterns are sufficiently concentrated.

Proposition 7 *Suppose that the access charge is set equal to cost: $a = c_T$. Then a ban on price discrimination between on-net and off-net calls increases total welfare. However, it can decrease consumer surplus when condition (24) holds, for which it is a necessary* *requirement that $\hat{\mu} < \mu/2$.*

7 Concluding Remarks

We introduce a flexible model of network competition with non-uniform calling patterns. The model allows us to analyze the implications of concentrated calling patterns on equilibrium outcomes as well as profit-maximizing reciprocal access charges.

We show how equilibrium tariffs depend on calling patterns. Concentrated calling patterns can help to explain the call imbalance ratios that are observed in practice. Our main focus, however, is to analyze whether and when networks would choose reciprocal access charges above cost when left to their own devices. With uniform calling patterns, it is known that networks would choose access charges below cost. We show that this result is reversed if calling patterns are sufficiently concentrated: Profit-maximizing access charges are set above cost because sustaining high off-net prices becomes relatively more important than suppressing network effects. Our results on above-cost access charges also imply that, contrary to other results in the literature, at the profit-maximizing reciprocal access charge on-net prices can be below off-net prices. We analyze how these different results are obtained from the interaction of two effects: Competition is dampened either when it becomes relatively more expensive to capture the marginal customer or when having the marginal customer is less profitable. We show how the strength of either effect changes when calling patterns become more concentrated.

Furthermore, information obtained on the concentration of calling patterns should guide optimal regulation. We explore this issue with respect to both access charge regulation and a prohibition of discriminating between on-net and off-net calls. In particular, we derive conditions for when the welfare maximizing access charge is strictly lower than the one prevailing in an unregulated market, and when an obligation of uniform pricing can increase welfare and/or consumer surplus.

As in much of the literature on network competition, we restrict consideration to a model with only two networks. This has the additional benefit of making our findings comparable to extant results. Also, we are able to offer a simple definition of our concept of more concentrated calling patterns. A consumer is identified by his brand preference over networks in familiar Salop fashion and by his preference for calling few people. These two preferences are at least somewhat correlated, so that consumers prefer to call those with similar brand preferences. This modeling was motivated by evidence on social relations and networks' brand positioning that targets specific consumer groups.

Our modelling ideas readily extend to more general settings involving, in particular, more than two competing networks. The Salop setting allows for localized competition between multiple networks, maintaining the above assumptions about calling patterns. We leave a generalization in this direction to future work.

Appendix: Omitted Proofs

Proof of Lemma 1. First, we obtain

$$2-2\int_0^1 H(t)dt = 2-2\left[tH(t)\right]_0^1 + 2\int_0^1 tdH(t) = \mu_h.$$

Then $H(z) \geq \tilde{H}(z)$ for all $z \in [0,1]$ implies $\int_0^1 H(t)dt \geq \int_0^1 \tilde{H}(t)dt$ and thus $\mu_h \leq \mu_{\tilde{h}}$.

Second, by applying l'Hôpital's rule and using left-continuity of h at $z=1$ we have

$$\lim_{z\nearrow 1} \frac{1-H(z)}{1-z} = \lim_{z\nearrow 1} h(z) = \hat{\mu}_h.$$

As weak inequalities are preserved by taking limits and division by the positive term $(1-z)$, it follows that $H(z) \geq \tilde{H}(z)$ for z close to 1 implies $\hat{\mu}_h \leq \hat{\mu}_{\tilde{h}}$. **Q.E.D.**

Proof of Proposition 1. Given constant market shares, network 1's marginal customer is determined by the condition $V_1(\hat{x},\hat{x}) = \overline{V}$, which can be restated as $F_1 = v_1(\hat{x},\hat{x}) + u_0 - \tau\hat{x} - \overline{V}$. Substituting $\hat{L}_{11}(\hat{x}) = \hat{x}G(\hat{x}|\hat{x})$ and $\hat{L}_{12}(\hat{x}) = \hat{x}(1-G(\hat{x}|\hat{x}))$ into network 1's profits leads to

$$\begin{aligned}\overline{\Pi}_1(\hat{x}) &= \hat{L}_{11}(\hat{x})v(p_{11}) + L_{11}(\hat{x})(p_{11}-c)q(p_{11}) \\ &\quad +\hat{L}_{12}(\hat{x})v(p_{12}) + L_{12}(\hat{x})(p_{12}-c-m)q(p_{12}) + const.,\end{aligned}$$

where the last term on the right-hand side does not depend on p_{11} or p_{12}. We obtain from the maximization of the relevant terms with respect to p_{11} the first-order condition

$$(p_{11} - c)q'(p_{11}) + \left(1 - \frac{\hat{L}_{11}(\hat{x})}{L_{11}(\hat{x})}\right) q(p_{11}) = 0,$$

which solves for the expression (5) presented in the proposition. The result for the off-net price is derived similarly.

Finally, we state a sufficient condition that allows us to ignore the participation constraint of all subscribers with location $-\hat{x} < x < \hat{x}$, i.e., when indeed, as presumed in the relaxed program, $V_1(x, \hat{x}) \geq V_2(x, \hat{x})$ for all $x \leq \hat{x}$. We have

$$V_1(x, \hat{x}) - V_2(x, \hat{x}) = [v_1(x, \hat{x}) - v_2(x, \hat{x})] + \tau(1 - 2x) - [F_1 - F_2],$$

where

$$\begin{aligned} v_1(x, \hat{x}) - v_2(x, \hat{x}) &= \{G(\hat{x}|x)v(p_{11}) + [1 - G(\hat{x}|x)]\, v(p_{12})\} \\ &\quad - \{[1 - G(\hat{x}|x)]\, v(p_{22}) + G(\hat{x}|x)v(p_{21})\}. \end{aligned}$$

A sufficient condition for $V_1(x, \hat{x}) \geq V_2(x, \hat{x})$ holding for all $-\hat{x} < x < \hat{x}$ is that

$$\frac{\partial}{\partial x}[V_1(x, \hat{x}) - V_2(x, \hat{x})] \leq 0,$$

which is equivalent to

$$\frac{\partial G(\hat{x}|x)}{\partial x}[v(p_{11}) - v(p_{12}) + v(p_{22}) - v(p_{21})] \leq 2\tau. \tag{25}$$

Although condition (25) is not stated in terms of the primitives alone, as the terms $v(\cdot)$ depend on the respective prices, note that τ does not enter call prices. Hence, holding all else constant, we can always choose the degree of horizontal differentiation τ (and jointly the fixed utility from participation u_0) large enough, ensuring that all consumers participate and that (25) holds everywhere (cf. also Laffont et al., 1998b). **Q.E.D.**

Proof of Proposition 2. As calls are balanced with $L_{12}(\hat{x}) = L_{21}(\hat{x})$, and framing the first-order condition in terms of the market share rather than the fixed fee, we have

$$\frac{d\overline{\Pi}_1(\hat{x})}{d\hat{x}} = F_1 - f + \hat{x}\frac{dF_1}{d\hat{x}} + L'_{11}(\hat{x})r_{11} + L'_{12}(\hat{x})r_{12} = 0.$$

Note next that $L'_{11}(\hat{x}) + L'_{12}(\hat{x}) = 1$, which yields the first-order condition at a symmetric equilibrium candidate

$$F^* - f + r_{ii} + \frac{1}{2}\left.\frac{dF_1}{d\hat{x}}\right|_{\hat{x}=1/2} + L'_{ij}(1/2)(r_{ij} - r_{ii}) = 0.$$

Using $L'_{ij}(1/2) = 0$ and substituting finally for $dF_1/d\hat{x}$ from (10) yields expression (12).

To obtain equilibrium profits, in a symmetric equilibrium, and after substituting for F^* from (12), profits (4) become

$$\begin{aligned}\bar{\Pi}^* &= \frac{1}{2}(F^* - f + r_{ii}) + L_{ij}(1/2)(r_{ij} - r_{ii}) \\ &= \frac{1}{2}[\tau - \hat{\mu}(v_{ii} - v_{ij})] + \frac{\mu}{4}(r_{ij} - r_{ii}).\end{aligned}$$

Q.E.D.

Proof of Proposition 3. When differentiating profits in (13) w.r.t. m, note first that at a symmetric equilibrium $\hat{x} = 1/2$ does not change. Moreover, from Proposition 1 only off-net but not on-net prices change with m. Note further that $dp_{ij}/dm > 0$ and that profits do not directly depend on m. Therefore we can equivalently maximize $\bar{\Pi}^*$ over p_{ij}.

We obtain[17]

$$
\begin{aligned}
2\frac{d\bar{\Pi}^*}{dp_{ij}} &= -\hat{\mu} q_{ij} + \frac{\mu}{2}\left[q_{ij} + (p_{ij} - c)\, q'_{ij}\right] \\
&= \left[\frac{\mu}{2}\left(1 - \frac{p_{ij} - c}{p_{ij}}\eta(p_{ij})\right) - \hat{\mu}\right] q_{ij} = 0,
\end{aligned}
$$

or

$$
\frac{p_{ij} - c}{p_{ij}} = \frac{1}{\eta(p_{ij})}\left(1 - \frac{2\hat{\mu}}{\mu}\right). \tag{26}
$$

This condition can be solved for p_{ij} as

$$
p_{ij} = c\frac{\mu\eta(p_{ij})}{\mu\left(\eta(p_{ij}) - 1\right) + 2\hat{\mu}}.
$$

On the other hand, from (7) we obtain

$$
p_{ij} = (c + m)\,\frac{\mu\eta(p_{ij})}{\mu\left(\eta(p_{ij}) - 1\right) + 1}. \tag{27}
$$

Equating and solving for m leads to the result stated in the Proposition. **Q.E.D.**

Proof of Proposition 4. Equating p_{ij} in (26) and p_{ii} from (7) leads to $1 - 2\hat{\mu}/\mu = (1-\mu)/(2-\mu)$, or $\hat{\mu} = \mu/(4-2\mu)$. For larger $\hat{\mu}$ we will have $p_{ij} < p_{ii}$, due to (26). **Q.E.D.**

Proof of Proposition 5. Substituting for CS and concentrating only on the off-net elements, we obtain

$$
W = 2\bar{\Pi}^* + CS = \frac{\mu}{2}\left(r_{ij} + v_{ij}\right) + const. \tag{28}
$$

Thus the socially optimal off-net price continues to be $p_{ij} = c$ even for general calling patterns, as should be expected. Equating to (27) and solving for a leads to the result in the text. The profit-maximizing access charge is equal to the socially optimal one if and

[17] It is easy to show that the sufficient second-order condition for a strict local maximum holds for a constant demand elasticity, which implies also that profits are quasi-concave in p_{ij}.

only if the relative weights in the objective functions (13) and (28) on r_{ij} and v_{ij} are equal, or if $\hat{\mu} = \mu/2$. **Q.E.D.**

References

Armstrong, M. "Network Interconnection in Telecommunications." *Economic Journal*, Vol. 108 (1998), pp. 545-564.

Armstrong, M. and Wright, J. "Mobile Call Termination." *Economic Journal*, Vol. 119 (2009), pp. 270-307.

Baron, D. "Morally Motivated Self-Regulation." *American Economic Review*, Vol. 100 (2010), pp. 1299-1329.

Birke, D. and Swann, P. "Network Effects and the Choice of Mobile Phone Operator." *Journal of Evolutionary Economics*, Vol. 16 (2006), pp. 65-84.

Calzada, J. and Valletti, T. "Network Competition and Entry Deterrence." *Economic Journal*, Vol. 118 (2008), pp. 1223-1244.

Cambini, C. and Valletti, T. "Information Creation and Competition in Telecommunications Networks." *Journal of Industrial Economics*, Vol. 56 (2008), pp. 707-728.

Dessein, W. "Network Competition in Nonlinear Prices." *RAND Journal of Economics*, Vol. 34 (2003), pp. 593-611.

Dessein, W. "Network Competition with heterogeneous customers and calling patterns." *Information Economics and Policy*, Vol. 16 (2004), pp. 323-345.

Farrell, J. and Klemperer, P. "Coordination and Lock-In: Competition with Switching Costs and Network Externalities." In M. Armstrong and R.H. Porter, eds., *Handbook of Industrial Organization*. Amsterdam: Elsevier, 2007.

Fjeldstad, Ø., Moen, E.R. and Riis, C. "Competition with Local Network Externalities." CEPR Discussion Paper 7778, 2010.

Gabrielsen, T.S. and Vagstad, S. "Why is On-net Traffic Cheaper than Off-net Traffic? Access Markup as a Collusive Device." *European Economic Review*, Vol. 52 (2008), pp. 99-115.

Gans, J. and King, S. "Using "Bill and Keep" Interconnect Arrangements to Soften Network Competition." *Economics Letters*, Vol. 71 (2001), pp. 413-421.

Genakos, C. and Valletti, T. "Testing the Waterbed Effect in Mobile Telephony." *Journal of the European Economic Association*, Vol. 9 (2011), pp. 1114-1142.

Hermalin, B. and Katz, M.L. "Customer or Complementor? Intercarrier Compensation with Two-Sided Benefits." *Journal of Economics & Management Strategy*, Vol. 20 (2011), pp. 378-408.

Hurkens, S. and Jeon, D.S. "Promoting Competition by Regulating Termination Charges." *International Journal of Industrial Organization*, Vol. 30 (2012), pp. 541-552.

Jeon, D.S., Laffont, J.-J. and Tirole, J. "On the "Receiver-Pays" Principle." *RAND Journal of Economics*, Vol. 35 (2004), pp. 85-110.

Jullien, B., Rey, P. and Sand-Zantman, W. "Mobile Call Termination Revisited." *International Journal of Industrial Organization*, forthcoming.

Katz, M.L. and Shapiro, C. "Network Externalities, Competition, and Compatibility." *American Economic Review*, Vol. 75 (1985), pp. 424-41.

Laffont, J.-J., Rey, P. and Tirole, J. "Network competition I: Overview and non-discriminatory pricing." *RAND Journal of Economics*, Vol. 29 (1998a), pp. 1-37.

Laffont, J.-J., Rey, P. and Tirole, J. "Network competition II: Price discrimination." *RAND Journal of Economics*, Vol. 29 (1998b), pp. 38-56.

Lopez A. and Rey, P. "Foreclosing Competition through Access Charges and Price Discrimination." Mimeo, 2012.

Ofcom. *Wholesale mobile voice call termination - Statement.* London: Office of Communications, 2011.

Shi, M., Yang, B. and Chiang, J. "A Dyad Model of Calling Behaviour with Tie Strength Dynamics." Mimeo, 2012.

Sobolewski M. and Czajkowski, M. "Network effects and preference heterogeneity in the case of mobile telecommunications markets." *Telecommunications Policy*, Vol. 36 (2012), pp. 197-211.