Optimal Design of
Morphing Structures

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Abstract

Morphing structures change their geometric configuration to achieve a wide range of performance goals. For morphing aircraft these include alleviating drag, or altering aerofoil lift. The design of structures capable of realising these goals is a highly multidisciplinary problem. Optimally morphing a compliant structure involves finding the distribution of actuation which best achieves a desired configuration change. In this work, the location and magnitude of discrete actuators are optimised, to minimise both aerodynamic and geometric objective functions. A range of optimisation methods, including differential and stochastic techniques, has been implemented to search optimally the large, nonlinear, and often discontinuous design spaces associated with such problems.

The optimal design of morphing systems is investigated through consideration of a morphing shock control bump and an adaptive leading edge. CFD is implemented to evaluate the aerodynamic performance of optimiser-controlled morphing structures. A bespoke grid-generation algorithm is developed, capable of producing a mesh for all possible geometries, with low levels of cell skewness and orthogonality at the fluid-structure boundaries. Structural compliance – a prerequisite for morphing – allows significant displacement of the structure to occur, but simultaneously enables the possibility of detrimental aeroelastic effects. Static aeroelasticity is catered for, at significant computational expense, via coupling of the structural and aerodynamic models within individual optimisation function evaluations. Morphing geometry is investigated to reduce computational design requirements, and provide an objective starting point for an aeroelastic optimisation. The requirements of morphing between aerodynamic shapes are evaluated using geometry-based objective functions. Displacements and curvatures are compared between an optimiser-controlled structure and the target morph, and the differences minimised to effect the required shape change. In addition to enabling optimal problem definition, these geometric objective functions allow conclusions on the feasibility of a morph to be drawn a priori.
Declaration

I hereby declare that all work presented within this thesis is my own, or has been referenced accordingly.

Signed: ............................................ ........

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Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nomenclature</td>
<td>4</td>
</tr>
<tr>
<td>List of Figures</td>
<td>6</td>
</tr>
<tr>
<td>List of Tables</td>
<td>12</td>
</tr>
<tr>
<td>1 Introduction: The Morphing Problem</td>
<td>13</td>
</tr>
<tr>
<td>2 Literature Review</td>
<td>15</td>
</tr>
<tr>
<td>2.1 Definition of a Morphing Structure</td>
<td>16</td>
</tr>
<tr>
<td>2.2 Morphing Applications</td>
<td>16</td>
</tr>
<tr>
<td>2.3 Morphing Aircraft</td>
<td>20</td>
</tr>
<tr>
<td>2.4 Materials and Actuation</td>
<td>26</td>
</tr>
<tr>
<td>2.5 Design of Morphing Structures</td>
<td>39</td>
</tr>
<tr>
<td>2.6 Optimisation</td>
<td>50</td>
</tr>
<tr>
<td>2.7 Summary and Thesis Objectives</td>
<td>56</td>
</tr>
<tr>
<td>3 Aeroelastic Morphing Design</td>
<td>60</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>60</td>
</tr>
<tr>
<td>3.2 Motivation</td>
<td>60</td>
</tr>
<tr>
<td>3.3 Shock Control Bump</td>
<td>63</td>
</tr>
<tr>
<td>3.4 Morphing SCB Design</td>
<td>68</td>
</tr>
<tr>
<td>3.5 Aerodynamic Modelling</td>
<td>69</td>
</tr>
<tr>
<td>3.6 Design</td>
<td>71</td>
</tr>
<tr>
<td>3.7 Optimisation</td>
<td>84</td>
</tr>
<tr>
<td>3.8 Results</td>
<td>85</td>
</tr>
</tbody>
</table>
# Nomenclature

## Greek Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
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<td>–</td>
</tr>
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<td>m</td>
</tr>
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</tr>
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<td>Curvature</td>
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</tr>
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<td>$\nu$</td>
<td>Poisson’s ratio</td>
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<tr>
<td>$\xi, \eta$</td>
<td>Computational space mesh axis</td>
<td>–</td>
</tr>
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<td>$\rho$</td>
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</tr>
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</tr>
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<td>Yield stress</td>
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</tr>
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<td>Von Mises stress for element $i$</td>
<td>Nm(^{-2})</td>
</tr>
<tr>
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<td>Interpolation blending functions</td>
<td>–</td>
</tr>
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<td>$\omega$</td>
<td>Over-relaxation factor</td>
<td>–</td>
</tr>
</tbody>
</table>

## Roman Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_i$</td>
<td>Boundary location</td>
<td>m</td>
</tr>
<tr>
<td>$c$</td>
<td>Chord length</td>
<td>m</td>
</tr>
<tr>
<td>$C(u)$</td>
<td>Location in Cartesian space of NURBS evaluated at $u$</td>
<td>–</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Two-dimensional drag coefficient</td>
<td>–</td>
</tr>
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</tr>
<tr>
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<td>Pressure coefficient</td>
<td>–</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus</td>
<td>Nm(^{-2})</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>-------</td>
</tr>
<tr>
<td>$f$</td>
<td>Drag-based objective function</td>
<td>–</td>
</tr>
<tr>
<td>$G_y, G_z, G_{\delta(s)}$</td>
<td>Displacement-based geometry fitting objective function</td>
<td>m</td>
</tr>
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</tr>
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</tr>
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</tr>
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</tr>
<tr>
<td>$K$</td>
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</tr>
<tr>
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<td>Shock control bump length</td>
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<td>m</td>
</tr>
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<td>m</td>
</tr>
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<td>Gaussian curvature-based structural objective function</td>
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</tr>
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<td>–</td>
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</tr>
<tr>
<td>$x, y, z$</td>
<td>Cartesian spatial coordinate system</td>
<td>m</td>
</tr>
</tbody>
</table>
List of Figures

1.1 Considerations in morphing-aircraft design .............................................. 14

2.1 Solar sail deployed via four bistable boom structures [1] ................................. 17
2.2 Reflector control via shape morphing [2] ....................................................... 18
2.3 (a) MEMS displacement multiplier attached to an electrostatic actuator [3], and (b) laser-machined force inverter [4]. .................................................... 19
2.4 Wing warping on an early Wright-brothers aircraft [5] .................................. 20
2.5 US Navy F-14 shown in its (a) unswept and (b) swept configurations [6] ......... 21
2.6 Morphing concepts: (a) NASA Dryden inflatable wing during flight testing [7]; and (b) telescopic wing in testing configuration [8]. ................................. 22
2.7 (a) Lockheed Martin folding-wing UAV concept, and (b) NextGen Batwing morphing UAV concept [9]. ................................................................. 23
2.8 MAV roll control achieved via wing warping [10] ........................................... 24
2.9 (a) Variform fuel-bladder actuated morphing wing concept [11], and (b) Buckle-wing morphing concept [12]. ......................................................... 25
2.10 Flexsys Inc. adaptive trailing edge [13]: (a) +10° and (b) −10° configurations. ...... 26
2.11 (a) Layup of LIPCA actuator, and (b) location on biomimetic trailing edge [14] ... 28
2.12 Application of post-buckled pre-compressed actuator to a morphing UAV for flight control [15] ................................................................................. 29
2.13 Shape-memory based multifunctional panel actuator: (a) schematic, and (b) experimental testing [16]. ................................................................. 30
2.14 Bio-inspired SMA actuator: (a) vertebrae core, SMA actuators and heating elements; (b) realised morphing aerofoil design [17]. ................................. 31
2.15 (a) Bistable panel, and (b) associated potential wells [18]. ............................ 32
2.16 Piezoelectric actuation of a bistable composite structure between its two stable states

[19] ................................................................. 33

2.17 Bistable winglet for enhanced takeoff [20] ....................................................... 34

2.18 (a) Compliant windscreen wiper – single-piece construction [3], (b) Compliant gripper [21] ................................................................. 35

2.19 (a) Living hinge mechanism for stress reduction, and (b) aerofoil with morphing trailing edge, achieved via structural compliance [22] ......................................................... 36

2.20 (a) High-frequency mechanical vortex generator, and (b) close-up image of 7 vortex generators in operation on a prototype wing [13] ......................................................... 37

2.21 (a) Corrugated wing skin, and (b) its orientation to resist spanwise loading, but allow large chordwise strains [23]. ................................................................. 38

2.22 Structural optimisation parameterisation techniques for optimising stiffness of a simply supported beam with centralised distributed loading [24]: (a) sizing optimisation, (b) material optimisation, (c) shape optimisation, and (d) topology optimisation. . . 40

2.23 Topology optimisation of a cantilever beam subject to a point load at its free end (adapted from [21]): density-based, and load-path based parameterisation techniques. 42

2.24 Morphing aerofoil leading edge designed via load-path topology optimisation [25]. Displayed in its (a) original and (b) actuated configurations. . . . 43

2.25 (a) Bezier-spline parameterisation for aerofoil shape control [26], (b) and (c) PAR-SEC parameterisation [27]. ................................................................. 46

2.26 Optimised geometry, starting from the B-737 wing: (a) root aerofoil, (b) tip aerofoil. 47

3.1 Qualitative effect of free-stream Mach number on a thin aerofoil at constant angle of attack [28] ................................................................. 61

3.2 (a) Transonic aerofoil with normal shock and (b) a transonic aerofoil with \( \lambda \)-shock structure ................................................................. 62

3.3 Definition of typical SCB parameters influencing shock control .......................... 63

3.4 (a) Default rounded bump and (b) schlieren visualisation of \( \lambda \)-shock [29]. .... 64

3.5 Effect of SCB location with respect to normal shock ................................. 66

3.6 (a) Contours of Mach number and (b) comparison of \( C_P \) distribution with experimental data for RAE2822 aerofoil case 9 [30] ................................................................. 70
3.7 (a) Contours of Mach number and (b) $C_P$ distribution for RAE2822 aerofoil $Ma = 0.75$, $Re = 6.5 \times 10^6$ and $\theta = 2.81^\circ$. .............................................................. 71
3.8 (a) RAE2822 aerofoil geometry and (b) decision variables for structural morphing. 72
3.9 Mesh block construction in OpenFOAM .............................................................. 74
3.10 Finite-difference stencil in computational space .................................................... 78
3.11 Mesh generation via (a) linear TFI and (b) elliptical refinement of initial mesh .... 79
3.12 Application of Neumann boundary conditions during elliptical refinement ....... 80
3.13 Contours of constant $\eta$ obtained via elliptical refinement with Neumann boundary conditions: (a) leading edge (b) example morphing shock control bump. .............. 81
3.14 Flow diagram detailing the aeroelastic optimisation process ............................. 82
3.15 Aeroelastically converged (a) contours of Mach number and (b) $C_P$ distribution, for the optimisation initial condition. .............................................................. 84
3.16 Distribution of (a) Mach number and (b) $C_P$ distribution after iteration 1; and Distribution of (c) Mach number and (d) $C_P$ distribution after iteration 2. .... 86
3.17 Variations in (a) pressure distribution and (b) structural deformation, between subsequent aeroelastic iterations. .............................................................. 89
4.1 Shock control bump parameterisation variables for default rounded bump [29] .... 95
4.2 (a) NURBS curve together with control polygon, and (b) basis functions. ........... 97
4.3 NURBS curve fitting optimisation .............................................................. 98
4.4 Effect of control point numbers on NURBS shape-fit accuracy ............................ 99
4.5 NURBS description of default rounded bump ................................................. 100
4.6 2D Morphing SCB FE model and allocation of decision variables ....................... 101
4.7 Effect of load width on $S_z$ for $n = 3$ .............................................................. 102
4.8 Curvature evaluation via osculating circle method ........................................ 104
4.9 Target curvature distribution for default rounded bump .................................... 105
4.10 (a) Convergence of maximum stress, and (b) maximum bump height and $S_y$ objective function, with the number of FE mesh elements. ................................. 107
4.11 Optimal values of $S_y$ for varying actuation complexity ($n = 2$ to 6) ............ 108
4.12 Convergence of gradient-based minimisation of $S_y$ for $n = 2$ ......................... 109
4.13 (a) Convergence of GA for varying population size, and (b) function evaluation count for different optimisation methods: for $n = 4$. ................................. 111
4.14 Optimal values of $S_κ$ for $n = 2$ to 6 ........................................ 113
4.15 GA child generation via crossover ................................................. 114
4.16 Optimal curvature fit from $S_κ$ minimisation .................................. 116
4.17 Effect of chordwise location of optimal morphing geometry on $C_L/C_D$ ................................................................. 118
4.18 Schematic of 2D morphing SCB demonstrator ............................... 120
4.19 Optimal morphing geometry demonstration model: (a) initial configuration and (b) deployed configuration. .......................... 121
4.20 Optimal two-dimensional morphing geometry .................................. 121
4.21 (a) Relation of in-plane strain to curvature, and (b) cross-sectional distribution of strain, for a beam section. ..................... 124
4.22 Von Mises stress distributions for (a) $S_y$-minimised optimal geometry and (b) $S_κ$-minimised optimal geometry; and (c) associated $κ$-distributions. ............................ 126

5.1 SCB geometry parametrisation [29] ................................................. 133
5.2 Default rounded bump NURBS: (a) spanwise, (b) streamwise and (c) 3D perspectives ......................................................... 135
5.3 Initial morphing configuration FE model, shown with $n = 1$ actuation point. ................................................................. 136
5.4 Comparison of deflected and target shapes using point projection ........................ ................................................................. 139
5.5 Fitting a small surface to a node and its surrounding nodes using linear regression ........................ ................................. 140
5.6 (a) Convergence of $σ_{max}$ with the number of mesh elements, and (b) the effect of mesh refinement on function evaluation time ........................................ 143
5.7 Convergence of (a) displacement-based objective function, and (b) the Gaussian curvature objective function, with the number of mesh elements. ................................. 143
5.8 Convergence of the Gaussian curvature objective function with the number of nearest neighbours used for surface generation during $K$ evaluation ................................. 144
5.9 Convergence of optimal objective functions with $n$: (a) optimal $S_z$ with corresponding $S_K$, and (b) Optimal $S_K$ with corresponding $S_z$. ................................. 146
5.10 $ΔK$ between initial and target morphs: (a) top, (b) spanwise, (c) streamwise and (d) 3D perspectives. ......................... 147
5.11 $ΔK$ achieved via the optimum actuation configuration for $n = 6$ during the Monte Carlo initialisation: (a) top, (b) spanwise, (c) streamwise and (d) 3D perspectives. ................................. 148
5.12 Objective-function improvement after gradient-based refinement ................................. 150
5.13 Evolution of NURBS bounding curve during gradient-based refinement for $n = 6$ ................................. 150
5.14 Schematic of model construction .......................................................... 151
5.15 Optimal morphing geometry demonstration model: (a) unactuated spanwise configuration (b) actuated spanwise configuration, (c) unactuated streamwise configuration and (d) actuated streamwise configuration. ................................................. 152
5.16 Comparison of demonstration-model displacements with FE data and target geometry: (a) spanwise, and (b) streamwise directions. ......................................................... 153

6.1 Leading-edge flap: (a) Conventional, and (b) Morphing. ......................... 157
6.2 NACA2421 geometry with example drooped leading edge ......................... 159
6.3 Decision-variable space for NURBS shape-fit optimisation to generate leading-edge geometry .............................................................. 160
6.4 Geometrical evaluations using arc-length parameterisation .......................... 162
6.5 NURBS descriptions of initial and target leading-edge geometries ................. 165
6.6 Curvature distributions of leading-edge geometries ................................ 166
6.7 Curvature changes between the initial and target morphing configurations .... 166
6.8 Effect of initial curvature on structural response ...................................... 168
6.9 Decision variables for leading-edge morph and n = 2 .............................. 170
6.10 Convergence of fully-refined (a) $S_\delta(s)$, and (b) $\sigma_{\text{max}}$ with actuation complexity. Note the legend for Fig. 6.10(b) is displayed on Fig. 6.10(a) for clarity. .......................... 174
6.11 Optimal morphing geometry for minimised $S_\delta(s)$ and n = 4, 8\% camber configuration 175
6.12 Optimal morphing geometry for minimised $S_\delta(s)$ and n = 4: (a) 12\% camber, and (b) 16\% camber configurations. ................................................................. 176
6.13 Error distribution for $S_\delta(s)$-minimised morphs .................................. 177
6.14 Stress distributions for $S_\delta(s)$-minimised leading-edge morphs: (a) 8\% camber, (b) 12\% camber and (c) 16\% camber. ................................................................. 178
6.15 Convergence of fully-refined (a) $S_\kappa(s)$, and (b) $\sigma_{\text{max}}$, with actuation complexity. Note the legend for Fig. 6.15(b) is displayed on Fig. 6.15(a) for clarity. ......................... 180
6.16 (a) Minimised $S_\kappa(s)$ distributions for 16\% camber configuration and n = 2 and 3, (b) associated morphing geometries. ...................................................... 181
6.17 Optimal 16\% camber morphs for minimised $S_\delta(s)$ and $S_\kappa(s)$ (n = 4): (a) curvature change, (b) morphing geometry. ...................................................... 183
6.18 Pareto plot of fully-refined optimal solutions ........................................... 184
6.19 Optimal 8 % camber morph for minimised $S_{\kappa(s)}$ and $n = 5$: (a) curvature change, (b) morphing geometry. .......................................................... 186

6.20 Optimal 12 % camber morph for minimised $S_{\kappa(s)}$ and $n = 4$: (a) curvature change, (b) morphing geometry. .......................................................... 187

6.21 Stress distributions for $S_{\kappa(s)}$-minimised leading-edge morphs: (a) 8 % camber, (b) 12 % camber and (c) 16 % camber configurations. ................................. 188
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Properties of smart materials [31]</td>
<td>27</td>
</tr>
<tr>
<td>3.1</td>
<td>Quantitative effects of mesh-refinement algorithm</td>
<td>76</td>
</tr>
<tr>
<td>3.2</td>
<td>Optimisation results – aerodynamic attributes</td>
<td>87</td>
</tr>
<tr>
<td>3.3</td>
<td>Optimisation results – structural attributes</td>
<td>88</td>
</tr>
<tr>
<td>3.4</td>
<td>Effects of mesh quality and density on aerodynamic properties</td>
<td>90</td>
</tr>
<tr>
<td>4.1</td>
<td>2D Parametrisation of <em>default rounded bump</em> centre line [29] (dimensions in mm unless otherwise stated)</td>
<td>95</td>
</tr>
<tr>
<td>4.2</td>
<td>Geometrical analysis of morphing geometry</td>
<td>125</td>
</tr>
<tr>
<td>4.3</td>
<td>FE analysis of morphing geometry</td>
<td>127</td>
</tr>
<tr>
<td>5.1</td>
<td>3D Parametrisation of <em>default rounded bump</em> [29] (dimensions in mm unless otherwise stated)</td>
<td>133</td>
</tr>
<tr>
<td>6.1</td>
<td>NURBS fit of leading-edge geometry</td>
<td>163</td>
</tr>
<tr>
<td>6.2</td>
<td>Structural stress predictions from morphing geometries</td>
<td>167</td>
</tr>
<tr>
<td>6.3</td>
<td>Initial Monte Carlo results for minimisation of $S_{\delta(s)}$ for 8% camber morph: (a) Monte Carlo displacements, and (b) calculated displacements</td>
<td>172</td>
</tr>
<tr>
<td>6.4</td>
<td>Optimisation data from minimisation of $S_{\delta(s)}$</td>
<td>173</td>
</tr>
<tr>
<td>6.5</td>
<td>Optimisation data from minimisation of $S_{\kappa(s)}$</td>
<td>179</td>
</tr>
<tr>
<td>6.6</td>
<td>Comparison of calculated, actual and predicted maximum stress levels for optimal morphs</td>
<td>185</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction:

The Morphing Problem

Morphing systems achieve variations in performance via compliance of all or part of a their structure. This offers several advantages over conventional hinged and jointed mechanisms, such as reduced weight, and the ability to maintain a continuous shape. These characteristics are highly desirable in the field of aerodynamics and morphing aircraft, however, it will be shown in a review of the literature that morphing has also been targeted for application in micro-electro-mechanical systems and spacecraft.

Despite exhibiting highly desirable properties, the design requirements of morphing systems are complex, and often conflicting, making their development and implementation relatively slow. From an aerodynamic perspective large geometrical changes are required to affect significantly the flow. However from a structural perspective, achieving large displacements via morphing is a challenge constrained by material limitations. With regard to morphing aircraft, new design methods are required, as conventional fixed-wing sizing routines no longer hold. Therefore all aspects of a morphing aircraft must be developed simultaneously, to ensure the desired holistic performance is achieved. This creates an expensive problem to solve, which has led to extensive use of optimisation to aid searching of the complex design space. Nonetheless the morphing-aircraft design engineer has many factors to consider – see Fig. 1.1. The target flight envelope will determine whether morphing will be used for flight control, or for gross shape changes leading to a multi-purpose aerial vehicle. Morphing of any kind requires flexibility of the structure, which engineers must provide without compromising the structural integrity or dynamic stability of the aircraft. Simultaneously material
and actuation requirements must remain within the limits of current technology, and not exceed the fatigue life, under repeated morphing transformations. This creates a complex series of design constraints for the morphing aircraft optimisation problem. However if they can be satisfied, the rewards from morphing are great, offering the opportunity to revolutionise aircraft performance.

This thesis investigates design methods for morphing aircraft structures, and looks objectively to reduce the complexity of the design space. Structural and aerodynamic perspectives are considered, together with the common link between these fields – the geometry. The thesis begins with a review of the current literature on morphing aircraft and other fields of application. Materials, actuation, and current design techniques are all investigated. Aspects where the literature is lacking are then defined, and become the subject of the remainder of the thesis. Morphing of an aerodynamic surface is explored, using high-fidelity aerodynamic and structural solvers to perform static aeroelastic analysis. An optimal structure is presented, along with the inherent problems associated with this type of analysis. The following chapters introduce the concept of shape morphing as a means of defining the geometry change. The problem of morphing between geometries of known aerodynamic performance is then investigated, in both two and three dimensions. Optimal morphs are realised in the form of demonstration models, suitable for wind-tunnel testing, as proofs of concept of these methods. The design of an adaptive leading-edge flap is then considered, utilising the idea of shape morphing to parameterise the problem, and simplify its solution. Finally conclusions are drawn on the subject of morphing aircraft, and continuing work required to develop the field suggested.
Chapter 2

Literature Review

morph /mɔːf/ • v. “change smoothly and gradually from one image to another...”

Concise Oxford English Dictionary [32].

The dictionary definition of morph [32] leads one to believe the phenomenon of morphing is limited to the virtual world of computer graphics. However, it is possible to realise engineering structures which “change smoothly and gradually from one configuration to another”, resulting in substantial changes in performance. Designing such structures is a complex and multidisciplinary task, and hence morphing structures are yet to be as comprehensively understood, or utilised, as their ‘rigid’ structure and mechanism counterparts. Advances in materials engineering are enabling larger deformations, and hence more pronounced morphs are becoming possible. Whilst modelling such materials poses new challenges for engineers, the applications are exciting and have led to renewed interest in the field of morphing structures, and morphing aircraft in particular.

Before investigating the design of morphing structures, it is first useful to summarise the current scientific literature on the subject. The review begins by defining morphing from an engineering perspective, discussing what qualifies a structure as ‘morphing’. Applications of morphing structures are then summarised, before concepts in the field of morphing aircraft are covered in more detail. A summary of enabling technologies follows, discussing materials and actuation methods making morphing possible, and displaying a range of conceptual applications. Finally design and modelling methods for adaptive structures are covered, presenting approaches from both structural and aerodynamic perspectives. A short summary is then given, including a discussion of the limitations in the literature, before the key objectives of the thesis are outlined.
2.1 Definition of a Morphing Structure

All engineering structures contain compliance, as no material is perfectly rigid. However they are traditionally designed with high stiffness in mind, using joints and mechanisms to enable kinematic behaviour [33]. Morphing structures use compliance to change shape, and achieve variations in performance by doing so. Geometry changes enable structures to perform optimally in multiple configurations, improving efficiency throughout their operating envelope. In addition to improved performance, compliant structures offer many attractive properties to the design engineer: reduced weight compared to conventional mechanisms; reduced system complexity; and reduced maintenance requirements [34, 35]. To showcase their benefits, it is useful to look at potential morphing applications.

2.2 Morphing Applications

This section investigates some of the current applications of morphing technology. The ability to present a smooth and continuous surface in multiple configurations, makes morphing structures ideal for interacting with fluid flow. This enables flow control in the fields of aircraft, marine vehicles, and wind turbines. Additionally, the ability to change shape without the use of conventional joints and hinges, is beneficial in the environment of space, and at length-scales where the production of mechanisms is impossible.

Morphing Space Structures

Due to size restrictions on launch vehicles, components such as solar-panel arrays and reflector dishes seldom occupy their operational configuration during transport [36]. Instead these components are folded to compact them during transit, and once in space, are ejected and actuated into their operating configuration [37]. Deployment of space structures via morphing is beneficial for several reasons. The ability to reduce weight is advantageous as the high costs of space transport are proportional to payload mass. The problem of outgassing – the accelerated evaporation of lubricating fluids caused by the vacuum of space – is solved by removal of conventional hinges and joints. System complexity is reduced, improving deployment reliability, and potentially reducing actuation energy.

An example application of morphing is the deployment of a solar sail – a propellantless spacecraft that achieves motion via the momentum exchange of photons. Their large size means they have to
2.2. Morphing Applications

Figure 2.1: Solar sail deployed via four bistable boom structures [1]

be stowed for transit, and deployed on arrival in space. The design in [1] developed a 5×5 m² sail. It was capable of being stowed inside a 3U Cubesat during transit, and deployed in a similar fashion to that shown in Fig. 2.1. Unfurling of the sail is achieved via four bistable composite booms, leading to a light and simple deployment system. The simplicity of the morphing deployment system enables minimisation of the structural mass – a crucial requirement of all spacecraft.

A second application of morphing in space is the control of a reflector or antenna, as displayed in Fig. 2.2. Composite [38] and membrane [39] reflectors have both been investigated in the literature. Membranes are lightweight relative to solid reflectors, and can be rolled or folded during transit, greatly reducing their volume [40]. Once in orbit they assume their parabolic structure via attachment to a mechanism, which stretches the membrane to the desired shape, or via inflation of a built-in chamber [41]. A class of adaptive antennas, capable of directivity (beam steering) and power density control (beam shaping), were developed in [42]. Actuation was achieved by attachment of polyvinylidene fluoride (PVDF) film to a metallised Mylar substrate, and subsequent application of an electric field. A theoretical force-deflection relationship was defined, and subsequently used to manipulate the far-field radiation patterns of an aperture antenna. Accuracy is of paramount importance when controlling a reflector surface, as the captured data is a function of the membrane configuration. The shape inaccuracies of inflatable reflector antennas were investi-
2.2. Morphing Applications

Chapter 2. Literature Review

Figure 2.2: Reflector control via shape morphing [2]

...gated in [43]. In the inflated configuration, shape inaccuracies occur due to geometrically nonlinear structural deformations, and cause the reflector shape to deviate from parabolic. Active control was applied, again using PVDF actuators, and the resulting deformations analysed via electronic speckle-pattern interferometry. It was concluded that whilst reflector-shape control was achieved, actuator technology was the limiting factor in obtaining the required parabolic configuration, and optimising system performance.

Whilst these concepts detail the benefits that morphing space structures can bring, further understanding of deployment characteristics and shape control are required before they can be used in practice. Nonetheless morphing systems are capable of out-performing conventional structures, enabling engineers to explore new and unique space missions.

Micro Electro-Mechanical Systems

Micro Electro-Mechanical Systems (MEMS) are sub-millimetre devices capable of performing mechanical tasks at extremely small length scales. Their size is limited by the manufacturability of their constituent components: gears, hinges, joints and other mechanical devices. Via structural compliance, a morphing MEMS is able to achieve the same properties as its mechanism counterpart [24], simultaneously eliminating mechanical problems such as backlash and wear. If successful, the
2.2. Morphing Applications

Figure 2.3: (a) MEMS displacement multiplier attached to an electrostatic actuator [3], and (b) laser-machined force inverter [4].

limiting factor on MEMS length scales is moved to that of the manufacturing process. Successful gripper systems have been constructed at the 100 $\mu$m length scale in [4], demonstrating the functionality of morphing. An example compliant stroke multiplier, capable of providing displacements of the order 20 $\mu$m, is displayed in Fig. 2.3(a). When combined with an electrostatic actuator, it was able to increase the force per unit area by a factor of 220 [3]. Based on elastic analysis, the monolithic compliant structure was designed to meet both kinematic and static stiffness requirements. The geometric advantage – the ratio between the input and output displacements – was between 12 and 20, depending on the frequency of operation. Targets of future work are to improve accuracy via tailoring of the structure’s natural frequency to a value outside the operating range. Nonetheless fatigue life was impressive, with one specimen being driven for $10^{10}$ cycles with no visible signs of wear.

The force-inverting/magnifying MEMS developed in [4], is shown in Fig. 2.3(b). The device is constructed on a rapid-prototyping machine, using laser micro machining. The structure, which is built from Silicon with Young's modulus 180 GPa, is 7 $\mu$m thick and 300 $\mu$m long. The device was seen to respond reliably to scaling, with similar behaviour exhibited for a 500 times increase in length scales. This provides heritage for producing morphing MEMS at even smaller length scales with progress in manufacturing technology.
Morphing aircraft structures are not a new idea: the Wright brothers utilised wing warping to achieve roll control of some of their early aircraft. With limited thrust, reducing aircraft mass was critical, eliminating the possibility of flight control through the use of heavy and complex mechanisms. Instead compliance of the structure was utilised, and cable actuation used to warp the wing, as shown in Fig. 2.4. The resulting rolling moment enabled flight control, with minimal weight penalty [44]. A more recent aircraft exhibiting a gross change in wing geometry was the US Navy’s F-14, displayed in Fig. 2.5. Whilst not strictly a morphing structure, as performance changes are not a result of structural compliance, it is noteworthy for its ability to perform a range of missions due to its constantly variable wing sweep – a property which could potentially be achieved via morphing. In the fully unswept configuration the aircraft is able to take off and land on an aircraft carrier, and perform agile manoeuvres during fighting. Conversely in its fully-swept high-speed dash configuration, the aircraft is capable of over Mach 2, allowing the F-14 to cover large distances quickly [6].

Despite the benefits demonstrated by these early concepts, morphing structures were unable to make their way into commercial aircraft. For certification and reliability purposes, engineers instead tended towards rigid structures. Current design conventions state that aircraft should have rigid wings, with control achieved through movable surfaces such as leading and trailing edge flaps. However advances in engineering are causing these conventions to be redefined. The
development of smart materials and improved understanding of CFRP structures, have led to new concepts for design and actuation of aircraft structures. Improved analysis techniques mean reliable structures can be designed and optimised relatively quickly, allowing engineers to explore the realms of feasibility in ways not previously possible. All of these factors have led to renewed interest in the field of morphing aircraft, and provided the necessary confidence required to invest in attaining the aerodynamic benefits they can bring. This has caused an explosion of literature on the subject, a full account of which is beyond the scope of the thesis. Instead a summary of key concepts is presented, together with information on the enabling technology and design methods. When considering morphing aircraft structures, the type of morphing is typically split into two categories [35]: morphs which occur in the plane of the wing itself (planform morphing), varying properties such as wing span, sweep and planform area; and those normal to it (out-of-plane morphing), where variations in camber, chord length, and aerofoil profile are achieved.

**Planform Morphing**

Planform morphing typically results in significantly affected performance; enabling aircraft with previously conflicting design requirements to be combined [9]. For example, high-altitude reconnaissance aircraft sacrifice manoeuvrability and speed to achieve cruise and loiter characteristics. Similarly, military fighter aircraft achieve manoeuvrability and high speed at the expense of efficiency [45]. Planform morphing facilitates creation of a multi-purpose vehicle, capable of operating optimally at both extremes [46]. Some current concepts, and other novel ideas for large-scale
2.3. Morphing Aircraft

Figure 2.6: Morphing concepts: (a) NASA Dryden inflatable wing during flight testing [7]; and (b) telescopic wing in testing configuration [8].

Functional inflatable wings have been used in aircraft applications since the 1950’s. An example of this type of aircraft is the Goodyear Inflatoplane (GA-468), developed by the US military. This aircraft could be dropped behind enemy lines for downed pilots to escape on [7]. In 2001 an inflatable UAV, developed by NASA Dryden, underwent test flights. This was designed as a gun-launched surveillance vehicle, utilising inflatable wings to ensure the concept could fit inside a capsule suitable for launching. In-flight deployment of the inflatable concept, which takes approximately 0.3 s, is shown in Fig. 2.6(a). Once deployed to their 5 ft wingspan, the wings are capable of sustaining the aerodynamic loads associated with UAV flight. Whilst wings such as these are unlikely to support the large aerodynamic loads found on commercial aircraft, the concept has helped produce new designs for more versatile UAVs. One such example is the telescopic spar concept, enabling aspect-ratio variation, shown in Fig. 2.6(b) [8]. The internal mechanism is constructed from three concentric aluminium tubes, decreasing in diameter and increasing in length. Pressurised deployment of these tubes translates the outer fibre-glass sections, achieving a variety of wingspan configurations. During testing the joints in the outer skin led to viscous drag, making the wing 25% less efficient than a single-piece wing of comparable size. The fully-deployed structure exhibited a lift to drag ratio of 10. Although much less than some conventional wings, the additional benefit of reduced high-speed drag in the partially-deployed state must also be considered.

Lockheed Martin investigated the design of a planform morphing aircraft in [45]. They developed a concept, displayed in Fig. 2.7(a), which morphed using two chordwise hinges to fold the wing.
Morphing Aircraft

Chapter 2. Literature Review

Figure 2.7: (a) Lockheed Martin folding-wing UAV concept, and (b) NextGen Batwing morphing UAV concept [9].

Morphing achieved an effective sweep change of 30°, without changing planform edge alignment. This results in a span increase of 71 %, increasing total wing area by 180 %. The effect of morphing is therefore a wetted area 23 % less in the folded configuration, whilst achieving a 52 % increase in L/D in the unfolded state. These properties would enable the concept to perform efficient high-speed low-altitude flight using the folded state to reduce drag and improve efficiency – ideal for dash characteristics. The concept could then morph to the unfolded configuration to improve range and endurance for high-lift and loiter situations – useful for surveillance applications. The folding-wing concept shows promise for delivering a multi-purpose vehicle capable of agility, speed and efficiency, relative to fixed wing counterparts.

Another concept capable of planform morphing is the Batwing UAV developed by NextGen Aeronautics [47]. The UAV morphs via two mechanisms: variable sweep angle, and variable chord length – as displayed in Fig. 2.7(b). To control the sweep angle, a series of linear actuators are fitted within the mechanical framework of the wing. Variations of 15° are achieved, with the added benefit of increased span as sweep angle is reduced. The chord length is altered by a mechanism mounted within the fuselage. This pulls the trailing edge towards the tail of the UAV, stretching chord length across the entire wing span. The chord-length variation is a function of sweep angle, with a potential increase in wing area of 14% with the wings swept back, and 84% with the wing swept forward [48]. These changes allow the craft – which is intended to be a high-altitude surveillance platform – to morph between three mission-specific configurations. A climb configuration for take off with the wings swept back and fully extended; a loiter configuration with wings retracted and swept forward; and a high-lift configuration with wings swept forward and extended to provide maximum lift for landing. The loiter configuration is tailored to provide maximum efficiency, increasing the UAV range, as this is the configuration in which it will spend the majority of its time.
2.3. Morphing Aircraft

Chapter 2. Literature Review

Figure 2.8: MAV roll control achieved via wing warping [10]

Out-of-Plane Morphing

Out-of-plane morphs typically result in smaller deflections than in-plane variations. Therefore instead of providing multi-purpose capabilities, they are targeted at aircraft control. Morphing structures replace the conventional control surfaces and mechanisms, such as ailerons and leading-edge flaps. This enables a smooth, continuous surface to be presented to the flow, delaying the transition to a turbulent boundary layer, and retarding the onset of separation. Methods that enable this control are variations in camber, wing twisting and wing bending. These methods are demonstrated by the following concepts from the literature.

A morphing micro air vehicle (MAV) using wing twisting for roll control is investigated in [10]. The MAV is constructed from a carbon composite frame with a plastic-membrane wing. To enable morphing, torque rods attached to electronic servos located inside the fuselage, are sewn into the membrane wing’s leading edge. Application of torque causes the wing to twist, creating a large displacement at the trailing edge, and wing twisting, as shown in Fig. 2.8. During flight tests wing warping was found to be sufficient to provide roll control of the aircraft. However, attachment of a rudder was required to give yaw control and ensure stability.

A variform morphing-wing concept is investigated in [11], which alters the shape of the wing as fuel is used. This enables tailoring of the L/D ratio to optimise efficiency as the amount of fuel is reduced during flight. Surveillance aircraft can experience weight variations of more than 50% during a typical mission. Therefore minimising drag over a wide range of lift scenarios is critical.
2.3. Morphing Aircraft

Figure 2.9: (a) Variform fuel-bladder actuated morphing wing concept [11], and (b) Buckle-wing morphing concept [12].

for maximum efficiency [13]. A proposed implementation method is to hold the fuel in bladder-like tanks which interact with the structure of the wing, as shown in Fig. 2.9(a) [11]. As fuel is used and less lift is required, the wing is actuated accordingly to minimise drag. Thus offering the opportunity to greatly increase the range of the vehicle as efficiency is optimised.

A buckle-wing concept capable of a new range of missions was investigated in [12]. Via morphing the concept can independently change wing loading, aspect ratio and wing-section shape during flight. The buckle wing, shown in Fig. 2.9, is comprised of two sections, and utilises bistability of the upper surface to enable morphing. In the fused configuration they act as a single aerodynamic entity. Once actuated via buckling of the upper surface, two lifting surfaces are presented to the air. Simulations show that via optimisation of the aerodynamic properties of both sections, two significantly different flying configurations can be attained. For example, in the split configuration the vehicle is agile and easily manoeuvred, whilst in the fused mode improved cruise and loiter characteristics could be achieved.

A variable-geometry trailing-edge flap, designed by Flexsys Inc, the US Air Force Research Labs (AFRL) and Lockheed Martin, is discussed in [13]. The goal was to develop a flap for a concept high-altitude long-endurance (HALE) UAV. Based on the required mission profile, flap deflections were designed in the range $-10^\circ$ to $+10^\circ$, including spanwise variations. This allowed optimisation of lift and drag, and wing loading respectively. Target aerofoil shapes were selected based on achieving the smoothest possible shape change given the required flap deflection. In wind tunnel smoke tests no separation occurred up to $+8^\circ$ deflection, with only minimal separation relative to conventional flaps occurring thereafter. On-going work is looking to develop full-scale aeroelastic simulation tools to predict, and tailor, the flutter response.
2.4 Materials and Actuation

A feature of all the aforementioned concepts is that during morphing they put a tremendous demand on the materials from which they are constructed. This makes it difficult to morph and simultaneously withstand the aerodynamic and aeroelastic loads associated with flight. In addition to this, fatigue loading of composite morphing structures is difficult to predict, and is likely to reduce the lifetime of components significantly. However, materials science and engineering are continually evolving and new materials with improved performance are constantly being developed. A range of materials have emerged which provide some of the key properties required for construction and actuation of morphing aircraft. Often referred to as smart materials, they are able to react to their surroundings in a predetermined manner through built-in sensors and actuators [49]. Smart actuation systems are of particular interest to morphing aircraft for their ability to reduce weight and power requirements. They include systems such as piezoelectricity, shape memory materials, multistability and structural compliance. A range of these enabling technologies is discussed below,

Figure 2.10: Flexsys Inc. adaptive trailing edge [13]: (a) $+10^\circ$ and (b) $-10^\circ$ configurations.

This selection of concepts demonstrates the feasibility of chordwise morphing, and the improvements in control and efficiency it can produce. When coupled with the large-scale changes achievable with planform morphing, the possibility of an efficient, multi-purpose vehicle, becomes evident. However engineers face a number of challenges before these systems become a reality. The following section investigates emerging technology, which it is thought will enable these systems to be realised.
2.4. Materials and Actuation

<table>
<thead>
<tr>
<th>Material</th>
<th>Max Strain (%)</th>
<th>Max Stress (MPa)</th>
<th>Elastic Energy Density (J/g)</th>
<th>Efficiency (%)</th>
<th>Bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piezoelectric</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ceramic PZT</td>
<td>0.2</td>
<td>110</td>
<td>0.013</td>
<td>≥ 90</td>
<td>Fast</td>
</tr>
<tr>
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<td>0.13</td>
<td>≥ 90</td>
<td>Fast</td>
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<tr>
<td>Polymer (PVDF)</td>
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<td>4.8</td>
<td>0.0013</td>
<td>n/a</td>
<td>Fast</td>
</tr>
<tr>
<td>Shape Memory Effect</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Alloy (NiTi)</td>
<td>5</td>
<td>200</td>
<td>15</td>
<td>≤ 10</td>
<td>Slow</td>
</tr>
<tr>
<td>Polymer</td>
<td>100</td>
<td>4</td>
<td>2</td>
<td>≤ 10</td>
<td>Slow</td>
</tr>
</tbody>
</table>

Table 2.1: Properties of smart materials [31]

highlighting their use with regard to the development of morphing structures.

**Piezoelectric**

A selection of ceramic materials, and some polymers, exhibit the phenomenon of piezoelectricity. Polarisation from an electric field or voltage causes alignment of dipoles within the crystal structure, resulting in mechanical strain [50]. Commonly used piezoelectric materials include: barium titanate, lead zirconate-titanate (PZT), lead titanate, and sodium-potassium niobate. The inverse piezoelectric effect – generation of an electric field from an applied strain – is also exhibited by these materials. Piezoelectric materials are able to achieve large stresses, and operate at high frequencies, however they are restricted to relatively small strains [51]. For example a randomly orientated crystalline sample of PZT is capable of a maximum strain of 0.2 % and a maximum stress of 110 MPa, but can be actuated at frequencies over 20 kHz [31].

RAINBOW and THUNDER are piezoelectric actuators developed at the NASA Langley Research Center [52]. Their design aim was to increase the maximum strain and improve the actuator stroke of piezoelectric materials, to enable actuation of morphing aircraft technologies. They consist of piezoelectric ceramic layers bonded to one or more non-piezoelectric layers. Due to elevated processing temperatures, internal stresses are created which significantly enhance the through-thickness displacement during actuation. During testing both devices were capable of load-free displacements in excess of 3 mm when exposed to an electric field of ±9 kV/cm, however noticeable displacement degradation was observed when the actuators were tested under load (20% – 50%).
2.4. Materials and Actuation

is thought that when fully developed these systems could provide actuation for a morphing aircraft, by arranging multiple actuators in optimum locations to effect a shape change.

A biomimetic wing section using a lightweight piezo-composite actuator (LIPCA) was designed and tested in [14]. A PZT wafer was embedded in a composite structure comprising glass/epoxy and carbon/epoxy layers, in an antisymmetric stacking sequence, as shown in Fig. 2.11(a). Two actuators were subsequently embedded, in series, on the upper surface of a morphing trailing edge device (Fig. 2.11(b)). A linear-elastic thermal analogy is used to estimate the actuated shape, modelled using FE. This showed good agreement with experimental deflections for voltages ≤ 150 V, however it under predicted deformation for input voltages above this value. Trailing-edge rotations of 5° were achieved for a 300 V input voltage, demonstrating that control of an aerial vehicle could be achieved using this actuator.

A new class of control actuators for morphing UAVs, comprising post-buckled pre-compressed (PBP) elements, are developed in [15]. The PBP actuator is constructed from two piezoelectric plates, sandwiching a structurally-stiff composite-plate core. Axial compression of the PBP is applied during curing, via a mismatch in coefficients of thermal expansion between layers. This provides pre-stressing of the structure close to the buckling limit. Asymmetric actuation compresses one side of the PBP, whilst placing the other in tension. This buckles the centre plate, enabling the PBP to achieve enhanced free deflections (by a factor of 4), with an energy conversion efficiency approaching 100 %. The PBP actuator was embedded in a morphing UAV with 1.4 m wingspan, enabling trailing-edge rotations of ±3° – as shown in Fig. 2.12. Roll-control authority was improved by 38 % relative to conventional servo actuators, while the rate-of-change of rolling moment was increased by a factor of 3.7. The solid-state actuators reduced the part count from 56 to 6 components, reducing system complexity. Furthermore, the low operating current allowed a
lighter battery and wiring system to be implemented, saving weight.

**Shape Memory Alloys**

Shape Memory Alloys (SMA) and Shape Memory Polymers (SMP), are materials which through heat-treatment and processing, memorise a particular shape or configuration. After undergoing large plastic strains, which can exceed 5%, the material will return to this memorised shape if heated above the transition temperature [50]. The memory effect is the result of a phase transformation between a low-modulus martensitic phase at low temperatures, and a high-modulus austenitic phase at high temperatures. Actuators take advantage of the memory effect, attaching the SMA such that returning to the memorised state causes a force to be exerted on the structure [53, 54].

Three commercially available SMAs dominate the literature: nickel titanium, copper zinc aluminium, and copper aluminium nickel. Nickel titanium, also known as Nitinol (NiTi), is generally favoured due to its greater ductility, increased recoverable motion, and stable transition temperatures. Furthermore it exhibits similar corrosion resistance to 300 series stainless steel, and can be heated electrically – a useful mechanism for shape recovery of morphing aircraft. Typical NiTi stiffness properties are $E_a = 75$ GPa and $E_m = 34$ GPa, and $\sigma_a = 440$ MPa and $\sigma_m = 100$ MPa, where the subscripts $a$ and $m$ denote the austenite and martensite phases respectively [55]. To ensure complete recovery, the allowable strain limit of the austenite phase is 0.83%. SMAs are well suited to deployment of space structures, as their slow release rates aid vibration mitigation [37].

Figure 2.12: Application of post-buckled pre-compressed actuator to a morphing UAV for flight control [15]
heating causes gradual actuation of a system such as a solar panel array, minimising shock loading on the remainder of the space craft. Drawbacks of SMA actuators with regard to morphing aircraft are their relatively slow operating frequency, and the fact that in their pure form, they are one-way only [31]. This is a particular problem when designing an actuator for the repeatable actuation of a morphing aircraft structure, however a number of concepts for overcoming this limitation are discussed below.

A method for enabling repeatable SMA actuation, is to couple the device with a mechanical means of restoring the SMA to its low-temperature martensitic state. Such biasing forces can be applied by springs, or partially buckled plates/beams [56]. However the efficiency of these actuators is compromised by the SMA having to work against the spring during actuation. To overcome these inefficiencies a multifunctional structural-panel actuator was developed in [16]. This consisted of two one-way effect SMA face sheets sandwiching a stainless steel truss core. The face sheets are heated individually to provide actuation. The core design forces the inactive face sheet to undergo martensitic deformation in tension when the opposing face sheet is heated. The cycle is completed via alternate face-sheet heating, creating an efficient, reversible SMA actuator. A prototype actuator is shown in Fig. 2.13(b), lifting a weight of 540 g a distance of 92 mm. The LCD display is connected to a thermocouple, outputting the temperature applied to the top face sheet (99.3 °C) during actuation. Further comments state that actuator performance characteristics, such as: actuation frequency, peak load capacity and structural stiffness, can be tailored via selection of appropriate: face sheet material, core thickness, and actuator length.
A vertebrae-actuator concept for camber control of a morphing aerofoil is presented in [17]. This is similar to the previous concept, however the core material is replaced with hinged bio-inspired vertebrae. This offers the advantage of minimal resistance to bending, and is relatively simple and inexpensive to manufacture. The individual vertebrae, shown in Fig. 2.14(a), are linked above and below with SMA face sheets. As before, actuation of the top face sheet causes deformation of the lower surface, which is subsequently heated to complete the actuation cycle.

In [55] the benefits of coupling a structure and actuator, both constructed from SMA with an intelligent control scheme, are demonstrated through the design of a compliant manipulator. This consisted of a super-elastic central column actuated by three SMA wires. The design was capable of control in three dimensions, and strains were achieved of 4%, with actuation speeds in the frequency range of 2 Hz. Although not suitable for wing morphing in its current state, the manipulator is thought capable of applications such as antenna positioning and an endoscopic tip.

An investigation into the thermo-mechanical response of NiTi wires exhibiting a two-way shape memory effect is investigated in [57]. Material modifications introduce dislocations and defects, causing the SMA to favour a certain microstructural orientation when actuated. The macroscopic manifestation of these modifications is an SMA capable of two-way actuation when cycled between two specific transformation temperatures. Dynamic mechanical analysis examined the influence of cold work on the quasi-static and fatigue behaviour, of SMA wires exhibiting the two-way shape memory effect.
2.4. Materials and Actuation

Chapter 2. Literature Review

Figure 2.15: (a) Bistable panel, and (b) associated potential wells [18].

Multistable Structures

Structures with multiple stable states are able to exist in several minimum-energy configurations [58]. An energy input is required to transition between different stable configurations, however once the new configuration is assumed, the structure will remain in this configuration indefinitely when the actuation is removed [18]. This property makes them an attractive concept for morphing aircraft systems, as large, permanent displacements can be achieved with minimal actuation energy. The most common aerospace materials exhibiting multistability are specially-processed composite laminate structures. Unsymmetric layup of laminates induces residual thermal stresses during curing [59], leading to geometrical nonlinearities [60] in the form of panel curvature, as shown in
2.4. Materials and Actuation

Fig. 2.15(a). If the panel is bent in order to remove this curvature, rather than returning to the original configuration, it will ‘snap-through’ to a second stable state. The structural potential energy associated with such a snap-through is sketched in Fig. 2.15(b). The two potential wells ($W_{e1}, W_{e2}$) represent the stable states, with the well denoted by the subscript 1 being more stable, relative to the well with subscript 2, as increased energy is required to cause snap-through ($E_1 < E_2$). Figure 2.15(b) is common for rectangular composite structures, and is indicative of their bistability, i.e. they have two potential wells and hence two stable configurations.

The snap-through and stable-state characteristics can be controlled, by tailoring of the thermal curing process [61] and material parameters. Bistable morphs can be achieved from either positive or negative Gaussian curvature changes, while the magnitude of the stable-state curvatures defines the relation between bending and stretching strain-energy densities [62]. For orthotropic materials, bistability occurs, specifically when the shear modulus is different to that of the isotropic value. A large shear modulus is particularly advantageous for bistable aircraft structures, as it gives a correspondingly large torsional stiffness. This helps to ‘lock’ the structure in a particular state, improving its resistance to external influences such as aerodynamic loading [63].

Whilst bistable structures can achieve large displacements, they require actuation to enable transition between stable states. In [19] a piezoelectric patch is bonded to a bistable composite plate, enabling one-way snap-through, as shown in Fig. 2.16. The process cannot be reversed in its present configuration, due to increased forces being required to return to the original state. Structural tailoring could facilitate two-way piezoelectric actuation, by careful construction of the stable-state energy levels. However the problem is complicated by the piezoelectric patches contributing significantly to the curvature and stiffness of the structure.
Bistable concepts were investigated for use in a morphing aerofoil section in [64]. Three morphing concepts were considered: an adaptive trailing edge, camber change, and chord length change. For the first case the trailing edge was constructed from two bistable composite plates, one forming the upper surface, the other the lower. After snap-through the trailing edge underwent a deflection much like that of conventional ailerons, but with the benefit of exhibiting a continuous surface at all times. The camber change was achieved by inserting a bistable composite plate along the chord of the aerofoil section, a distance 30% of the chord from the trailing edge. The actual camber change achieved is not mentioned, however it is noted that the aerodynamic loading on such a section would be greater than the force required to cause the snap-through. This makes it impossible for the increased camber formation to be achieved in practice without additional bracing. To shorten the chord a bistable plate was fitted inside the wing section in the spanwise direction. This was the most promising concept as the span wise orientation of the piezoelectric plate provided the capability of carrying spanwise bending loads whilst simultaneously actuating the system.

Bistable winglets were investigated to enhance lift-off capability in [20], a schematic of the concept is shown in Fig. 2.17. The winglet assumes a cambered configuration during takeoff to provide maximum lift. As the dynamic pressure increases during flight, the winglet loading increases, until it is of sufficient magnitude to cause snap-through to the winglet’s second stable configuration. This represents a more traditional winglet shape, designed to minimise drag during cruise conditions. The winglet was attached to a swept Zagi 12 wing section (30° leading-edge sweep), with baseline span of 0.6 m and root and tip chords of 0.326 m and 0.185 m respectively. Wind tunnel testing showed that lift augmentation was achieved by the initial cambered configuration, relative to the more traditional shape. However significant dynamic loading was experienced during snap-through, causing undesirable vibrations to be transmitted throughout the structure. Therefore it is thought that additional control and attenuation is required to enhance deployment of the bistable structure.
2.4. Materials and Actuation

Compliant Structures

Structural shape change without the use of conventional joints and mechanisms, requires compliance of some or all of the structure itself. Through careful material distribution, large global deformations can be achieved with small local strains, whilst simultaneously meeting structural integrity requirements. Examples of such structures are shown in Fig. 2.18. A compliant windscreen wiper is shown in Fig. 2.18(a), which through the use of structural compliance, is able to simplify the unit to a single component. A compliant gripper mechanism is shown in Fig. 2.18(b), which exhibits a gripping force of known magnitude when actuated via a predefined displacement. Conventional mechanisms suffer from problems such as backlash and wear, these reduce accuracy, and make system behaviour hard to predict over time. This is a particular problem when mechanisms are scaled down in size, and the accuracy levels tend to the length scale of the structure itself. Due to their lack of mechanisms, compliant structures scale well, enabling them to exhibit the same behaviour at different length scales. This has led to application of compliant structures in MEMS, which operate at length scales where the construction of conventional mechanisms is difficult. Localised living hinges, such as that depicted in Fig. 2.19, remove stress concentrations due to bending by tailoring of the hinge length and thickness. This ensures local strains are well below the yield strain, whilst allowing large rotational displacement of the members attached to the hinge. This approach is known as concentrated compliance, as relatively high localised straining of the structure is used in order to obtain large global deformations [4, 65].
2.4. Materials and Actuation

Morphing aerofoils designed utilising compliant structures are investigated in [22, 66, 25]. In [22] the design of an aerofoil is considered, capable of withstanding the associated aerodynamic loads, whilst achieving an aileron-like displacement of the trailing edge. A structure is designed with a wing-box-like section in the centre for bending and torsional stiffness, with a compliant mechanism filling the trailing-edge region to facilitate morphing. A single actuator, located at the site of the red double-pointed arrow in Fig. 2.19(b), is used to effect the shape change.

A second aerospace application of structural compliance is the design of a high-frequency mechanical vortex generator, as shown in [13]. A vortex-generating blade is caused to oscillate with fixed amplitude via attachment to a compliant structure. The system is designed to operate on subsonic and transonic wings, enabling control of separation through vortex generation. Figure 2.20(a) shows the compliant structure running at 150 Hz, whilst (b) shows an array of 7 vortex generators implemented on a prototype wing. The structures are capable of 5 mm amplitudes, running in the frequency range 30 – 150 Hz. The compliant structure acts as a motion amplifier for a voice-coil motor. During aerodynamic testing the device generated an oscillatory stream of boundary-layer embedded vortices. These proved effective in mitigating separation on the upper surface of a deflected flap, when a similar array of static vortex generators could not [67].

Morphing Skins

Typical aircraft wings on commercial aircraft consist of an aluminium skin, strengthened in the spanwise direction via spars, and in the chordwise direction via ribs. Whilst most wing loading is dealt with by these underlying structures, the wing skin is still required to take tensile, compressive and shear forces, and diffuse the aerodynamic pressure loads to the internal wing structure [68]. From an aerodynamic standpoint continuous surfaces are desired, to achieve the advantages asso-
2.4. Materials and Actuation

Figure 2.20: (a) High-frequency mechanical vortex generator, and (b) close-up image of 7 vortex generators in operation on a prototype wing [13].

Associated with morphing. If a morphing wing skin is manufactured from traditional materials, such as aerospace grade aluminium, the skin is likely to yield plastically, making the morph one way only. This is a problem encountered by large planform morphs, where the wing surface area changes dramatically. Morphing concepts so far have overcome this problem by using novel materials or concepts which enable large global deformations, whilst ensuring local strains are below material limitations. As aircraft get bigger, and wing loading increases, this becomes increasingly difficult. A range of skin materials and concepts for overcoming the problems of achieving large strains, whilst retaining the wing’s structural integrity, are discussed below.

Elastomers are a class of polymer with a low density of cross links, enabling high strains to be experienced, without permanent changes in shape occurring. Such materials include thermoplastic polyurethane, copolyester elastomer, and woven materials from elastic yarns [69]. They have a low tensile modulus, typically in the range 0.5–50 MPa, and are easily deformable up to 1000 % strains [68]. This is useful from a morphing perspective, as the forces required to actuate and sustain a morph will be relatively low. However elastomers have several drawbacks for use as a morphing wing skin. Their low Young’s moduli will make them susceptible to undesirable deformations under aerodynamic loads. Also, when chilled below their glass-transition temperature, elastomers become brittle – meaning the elastomer would have to be kept within strict operating conditions. Finally they exhibit viscoelastic properties and are susceptible to creep, which could develop problems with control of the morphing structure at different frequencies, and over large periods of time.
2.4. Materials and Actuation

Nonetheless, they are one of the only choices available to design engineers targeting planform morphs where large strains dictate material selection.

Out-of-plane morphs typically use the skin to provide structural stiffness to the wing, leading to a different set of wing-skin requirements. High stiffness in the spanwise direction is advantageous to aid bending stiffness of the wing, whilst low chordwise stiffness and high maximum strain are needed to facilitate morphing. A method to design the skin requirements from a morphing definition is developed in [70]. Camber morphing is used as an example, with the model predicting that a skin capable of 2 % strain is required to facilitate a camber change of $6^\circ$. However it is noted that it is not necessarily possible to reduce stiffness to produce the required strain response, and aerodynamic forces can have a significant effect on the skin, and wing performance.

A concept to achieve the required skin-stiffness properties is the use of corrugated skin [23, 71, 69]. Orientation of the folds in the spanwise direction gives the skin bending and torsional stiffness, whilst enabling high strains to occur in the chordwise direction (see Fig. 2.21(b)). Such ultra-anisotropic materials are investigated in [23]. CFRP corrugated structures produced excellent morphing stiffness properties, with spanwise to chordwise ratios of tensile and bending moduli of 4600 and 6800 respectively. However it is noted the corrugated surface is unlikely to perform well aerodynamically. To remedy the poor aerodynamic performance and further increase stiffness, an elastomer skin is used to fill the corrugations on the upper surface, and reinforcing is added in the form of CFRP rods (see Fig 2.21(a) lower). Addition of the elastomeric skin increases the chordwise...
bending and axial stiffness, causing the ratio of spanwise to chordwise tensile and bending moduli to change to 4200 and 8940 respectively. Although deformation of this reinforced skin would require increased actuation energy, the benefits of a smooth aerodynamic surface are obvious. Furthermore the addition of CFRP rods as stiffeners works well, increasing the spanwise stiffness properties without loss of flexibility in the chordwise direction.

### 2.5 Design of Morphing Structures

A factor that has not been considered thus far is how morphing structures are designed. Traditional engineering structures, such as aircraft wings, are designed to exhibit a desired stiffness response. This ensures that operational deflections will be small, and hence the aerodynamic and aeroelastic loading are well defined. Consequently structural engineers know that if they construct a wing of the required mass and stiffness, and which is the shape set out by aerodynamics engineers, the predicted lift and drag properties will be achieved. Well-developed sizing techniques for rigid-bodied aircraft exist, speeding up the early stages of the design process further [72]. Unfortunately such methods do not hold, or yet exist, for the design of morphing aircraft. The large changes in performance achievable affect the mass, stiffness and actuation requirements. This makes aircraft sizing difficult [9], as it is not intuitive which morphing configuration should dominate the design.

In the literature these difficulties have been overcome by reducing the problem to its constituent components. These involve designing the aircraft structure such that morphing shape changes can be achieved, or performing aerodynamic optimisation to find the optimum shapes that wings should morph between. A third topic investigates the aeroelastic interaction of the structure and aerodynamics, to ensure stability during flight. Splitting the design problem into individual disciplines has enabled production of a number of morphing aircraft concepts, as seen in Sec. 2.3. However it should be noted that the holistic performance must be considered before these systems can be realised on commercial aircraft. A summary of these design methods is presented below, to highlight the difficulties of an integrated-design approach.

### Structural Optimisation

Structural optimisation solves a material distribution problem subject to constraints [73]. For the majority of structural optimisation problems the aim is to achieve a structure which can support a given loading, whilst achieving another characteristic such as: minimum total weight, optimal centre
2.5. Design of Morphing Structures

Chapter 2. Literature Review

Figure 2.22: Structural optimisation parameterisation techniques for optimising stiffness of a simply supported beam with centralised distributed loading [24]: (a) sizing optimisation, (b) material optimisation, (c) shape optimisation, and (d) topology optimisation.

of mass location, or meeting a given density requirement. A number of possible parametrisation schemes exist to solve the optimisation problem [24], four of which are shown in Fig. 2.22 and summarised below. They are discussed with regard to maximising the stiffness of a simply supported beam under centralised distributed loading.

- **Sizing optimisation** - An initial structure is defined (normally consisting of truss elements) that is subjective to the designer, but is intuitive in shape and topology. The decision variables for optimisation are the truss member thicknesses. As these vary, the effect on the resulting structural response is found – the optimum solution is the structure that has the maximum stiffness whilst using the minimum amount of material.

- **Material optimisation** - This case is particularly suited to the design of composite structures. It is therefore assumed the desired component is built from a multi-layered fibre-reinforced composite. The decision variables for optimisation are the fibre and matrix material properties, the fibre orientation angle and the laminate thickness. The optimiser finds the combination of these parameters which satisfies the maximum stiffness and minimum weight criterion.

- **Shape optimisation** - An intuitive way to save weight in a structure is to remove internal
material from the constituent components. Depending on the loading conditions, removing material from the wrong place can cause stress concentrations which are undesirable. In shape optimisation, a number of holes are made within the component. The optimisation algorithm then varies the shape of these voids, in order to achieve the minimum stress level and mass, for the maximum structural stiffness.

In these three cases it is possible for the optimisation algorithm to vary the appropriate decision variables and find an optimal solution. However all three of these techniques require a subjective initial parameterisation to be defined by the designer. For example: the truss lattice needs definition in sizing optimisation, the component shape is required for material optimisation and the number of voids is required to begin the shape optimisation. This subjectivity means that the ‘optimal’ solution produced is only truly optimal if the initial problem was parametrised in an optimal way. A fourth technique has been developed to address this issue of subjectivity:

- **Topology optimisation** - This is a hybrid of sizing, shape, and if required by the designer, the material optimisation methods. Whilst being able to vary all of these parameters during solving, it is also able to alter the topology i.e. the number of voids within the structure. This enables the method to start with material filling the entire design space – an objective starting point – and remove material until the optimum topology has been found. Because the optimisation algorithm has complete freedom in the manner in which this topology is created, it can be said, providing the optimiser has converged to a suitable level, that a true optimum is found.

There are two methods commonly used in the literature for solving the fully defined structural topology optimisation problem: the density-based approach [74] and the load-path method [75]. The density-based optimisation method was one of the first approaches developed to use computers to optimise a topology for a given structure [76]. An example method of application is the SIMP technique [77]. The design domain is parametrised by a series of similar elements of uniform density, as shown in Fig. 2.23(left). An optimisation algorithm then operates on this parametrisation, altering the density of individual elements to produce a new topology. This new topology is analysed, typically via FEA, to assess its effectiveness at fulfilling the predefined problem objectives and constraints. The optimisation algorithm is then able to minimise/maximise the objective function, by searching the design space in an iterative fashion, until the optimum topology has been found [21]. Finally post-processing is performed to realise a structure from the optimal topology. Figure 2.23
2.5. Design of Morphing Structures

Chapter 2. Literature Review

Figure 2.23: Topology optimisation of a cantilever beam subject to a point load at its free end (adapted from [21]): density-based, and load-path based parameterisation techniques.

details the steps required to solve the topology optimisation problem of a cantilever beam subject to a point load at its free end. There are a number of difficulties with implementing the density-based approach, chiefly concerning the post-processing and manufacturability of the designs. During the optimisation process the density of elements is typically varied between $0 \leq \rho \leq 1$. This allows the optimal topology to be found, however a variable-density solution is often produced which is impossible to manufacture with current methods [78]. A number of techniques have been developed to overcome this issue, such as algorithms capable of taking the continuous-density structure and converting it into a discrete-density system, where material is either present or not [79]. Common filtering techniques are the Voigt bound, and the Hashin-Shtrikman bound [77]. These filtering techniques come at an increased computational cost, which, if a dense ground mesh is used – a requirement to generate a truly optimal topology – leads to a computationally expensive problem [80].

An alternative solution process is the load-path topology optimisation method. The design space is parametrised using a truss system, as opposed to elements of variable density [21]. This initial parameterisation is defined as the parent lattice or ground structure, and is the starting point for the topology optimisation (see Fig. 2.23 right). Rather than varying the density of the
2.5. Design of Morphing Structures

Chapter 2. Literature Review

(a)  

(b)  

Figure 2.24: Morphing aerofoil leading edge designed via load-path topology optimisation [25]. Displayed in its (a) original and (b) actuated configurations.

Truss elements, the optimisation algorithm varies their shape and whether they are present in the structure or absent. As before the resulting structural response is analysed using FEA to assess the structural objectives. The topology is then updated and re-analysed iteratively until an optimal solution is found [81]. Advantages of this method include no grey areas of variable density, and minimal post-processing due to the structure-like original parametrisation.

Example applications of this method for the design of compliant structures are shown in [21] for a compliant gripper mechanism and in [82] for a predefined morphing shape change between two curves. In [25] the design of a morphing leading edge is considered for the Alenia Aeronautica Sky-X UAV. A technique is developed to restrict path lengths in the parent lattice to those of minimal length. This improves the solution time by restricting searching of non-optimal solution space. The optimal morph, a combination of minimising shape-change error and mass, is shown in Fig. 2.24. A comparison with the density-based optimisation approach is performed, showing that a sub-optimal solution was produced when compared to the solution obtained via the load-path method.

A drawback of the load-path optimisation technique is that all topologies produced during the optimisation process are a subset of the original parent lattice. The complexity of the optimal topology is therefore bound by that of the parent lattice, and hence it is difficult to ensure the results obtained from this method are truly optimal. To overcome this a complex parent lattice
with fine parametrisation could be constructed. However this requires more detailed analysis during each optimisation function evaluation, leading to a computationally expensive problem. Instead the lattice generation is often the choice of a designer, and hence a degree of subjectivity is included in the solution from the outset.

To remove this subjectivity, it is required a priori to estimate the optimum number and locations of the output points for the structural optimisation using a systematic approach. Not only would this ensure that an optimal solution is produced, it may also offer the possibility to reduce the parametrisation, and hence the computational expense, of the optimisation process. A method for finding the optimal actuation locations is described in [83], however it is formulated as a feasibility study rather than a design initialisation tool. Chapters 4-6 of this thesis build on this work, and investigate the concept of shape morphing, to provide objective information about the required parameterisation during the early phases of structural design.

Aerodynamic Optimisation

Historically aerodynamic configurations were tested at multiple design points, and large libraries of data generated [84]. The geometry best matching the required characteristics could then be selected for a given application. Advances in computational power and modelling techniques have led to the design of individual geometries, which can optimally perform a specific task [85]. The research field focusing on optimisation of complex aerodynamic shapes is vast. Note that for the purposes of this literature review the problem is therefore limited to the optimisation of an aerofoil. Experimental data exists for many aerofoils, enabling designers to choose shapes with characteristics suited to their particular task. However through changes in geometry the performance of many existing aerofoils can be improved. Aerodynamic optimisation is the process of finding the geometry which provides the best aerodynamic properties. A typical problem optimises a set of decision variables governing the aerofoil shape through a predefined parameterisation scheme. The new shape’s aerodynamic properties are then evaluated, and used to update the decision variables, until the geometry has converged to an optimal solution. A brief summary of the constituent components follows this introduction, followed by discussion of a selection of the available solution methods.
Aerofoil Shape Parameterisation

The optimisation decision variables control the aerofoil shape through specification of coefficients in a predefined parameterisation. Aerofoils are typically built using a combination of shape functions, to ensure smooth shapes are produced. For example, the NACA parameterisation uses a chord-line description, together with independent thickness distributions for the upper and lower surfaces [84]. During aerodynamic optimisation it is desirable to have additional control over the shape, and hence methods with which to achieve greater localised shape control have been developed. A general example is to use splines to define the aerofoil surface, such as one of the Bezier curves in Fig. 2.25(a). The spline control-point locations become decision variables, and hence give the optimisation algorithm shape control of the aerofoil [26]. A major drawback of the spline method is the lack of a relation between control-point location and key aerofoil parameters such as leading-edge radius and maximum thickness. Therefore it is difficult to bound the design space, and to avoid production of sub-optimal discontinuous geometries, such as those with a sharp leading edge.

A second type of parameterisation is characterised by the PARSEC method shown in Fig. 2.25(b). Eleven variables are defined by the optimiser, which govern aerofoil attributes such as trailing-edge angle and thickness, the location of the maximum upper and lower surface thickness, and leading-edge radius [27]. The domain is separated into regions, as in Fig. 2.25(c), and the aerofoil coordinates in each region generated via the linear combination of shape functions according to

\[ Y_k = \sum_{n=1}^{6} a_{a,k} X_k^{n-1} \]  \hspace{1cm} (2.1)

where \( a_n \) are shape functions and \( X,Y \) are the aerofoil coordinates. The shape functions for each region are generated based on the optimiser-controlled coefficients, and continuity at the region boundaries. The physical relevance of the optimisation variables makes them easy to bound, narrowing the design space and speeding up optimisation convergence. A drawback of this method is the reduction in shape control achievable relative to the spline method. Therefore a final method of shape parameterisation exists, which combines the best features of both methods. PARSEC-type variables are used to generate spline control points, which control the camber and thickness profile of the aerofoil. This enables spline levels of shape control, whilst using variables with physical sense which improves the optimisation process. An example of this method is the Bezier-PARSEC BP3333 parameterisation [26, 86, 27]. Example thickness and camber profiles defined by the BP3333 method are shown in Fig. 2.25(a), which also displays the effect of a selection of optimiser-controlled PARSEC variables.
2.5. Design of Morphing Structures

Chapter 2. Literature Review

Figure 2.25: (a) Bezier-spline parameterisation for aerofoil shape control [26], (b) and (c) PARSEC parameterisation [27].

Single Point and Multipoint Optimisation

Once the aerofoil shape has been generated, the aerodynamic properties must be evaluated. Note that for the purposes of discussing the process of aerodynamic optimisation it is assumed that evaluation of the aerodynamic properties is both reliable, and performed to a suitable degree of accuracy, such that it does not have an effect on the optimisation process. The simplest optimisation case is single-point optimisation, where an aerofoil’s performance is optimised for a single set of flow conditions. Angle of attack, free-stream velocity, pressure, temperature and all other solver variables are held constant, whilst the geometry is varied. Properties such as lift and drag are evaluated according to a method suited to the flow regime, and used as metrics for optimisation.
Figure 2.26: Optimised geometry, starting from the B-737 wing: (a) root aerofoil, (b) tip aerofoil.

An example optimisation of this type was performed in [85] on a Boeing 737 wing. A Bezier spline using 11 optimiser-controlled control points was used to parameterise the geometry. The optimised aerofoil contours for minimum drag are displayed in Fig. 2.26. Case 1 refers to the single design point $C_L = 0.43$ and $M_\infty = 0.75$, and case 2 to the single design point $C_L = 0.43$ and $M_\infty = 0.8$. A drag reduction of 60.2 counts relative to an original 210.2 counts was achieved for case 1, and for case 2 a reduction from 308.8 to 157.3. The differences between the optimal geometries for the two configurations summarises the problem with single-point optimisation. The change in Mach number of the flow conditions results in different geometries at all locations on the aerofoil. This means that the optimal geometries are highly specific to their individual design points, and hence will not perform optimally for variations in the flow. This is undesirable as it is unlikely that an aircraft will be able to match exactly the required conditions over its entire flight envelope. Changing fuel loads are likely to result in altitude changes sufficient to perturb the flow properties away from a single design point. Also manufacturing tolerances may be unable to produce exactly the required shape without incurring prohibitive expense [87]. Therefore in practice a compromise is attained by performing multipoint optimisation. Rather than evaluating the aerofoil performance for a single set of flow conditions, several design points are analysed simultaneously. A single objective function is created according to the weighted sum of the individual aerodynamic components [88]. If the individual design points are generated in favourable positions, it is possible to interpolate the objective function between them, and hence gain a better understanding of the design space. Case 3 in Fig. 2.26 represents the optimised geometry for a multipoint optimisation. Three design points
Chapter 2. Literature Review

were used including the two defined previously, and a third of $C_L = 1.28$ and $M_\infty = 0.2$. This enables the geometry to be optimised for two cruise conditions – defining a window of operation – and a high-lift configuration, suitable for take-off and landing. The range of operating conditions result in a compromised geometry. Drag reductions of 50 and 130 counts are achieved for the case 1 and 2 design points respectively. This demonstrates the compromise on cruise performance enforced by ensuring the aerofoil performs well throughout the entire flight envelope. It also shows that if through morphing different aerofoil geometries can be produced, there are significant aerodynamic gains to be made by alleviating this compromise.

The problem is further complicated by uncertainty in flight conditions and manufacturing tolerances. There will be natural variations in atmospheric conditions during flight, and whilst the aircraft control systems will endeavor to attain cruise conditions, there will be times when the aerofoil is operating at off-design conditions [89]. There will also be limits on manufacturing tolerances, which will constrain the accuracy to which the optimised shape can be produced. The combination of these factors means that to achieve a robust design – even for a multipoint optimisation – it is desirable for the optimal performance to be consistent when subjected to small perturbations in the flow conditions. This is typically analysed via evaluation of the objective-function differentials with respect to the dominant flow variables such as velocity.

**Optimisation**

Optimisation of the shape parameterisation variables can be performed via several approaches. Stochastic methods have a number of advantages such as robustness and global-searching capabilities. A failed aerodynamic analysis due to a solver instability – which can easily occur when large geometry changes are instigated – will only result in the failure to perform a single function evaluation. The remainder of the population are likely to have been evaluated successfully, and will enable the algorithm to proceed unaffected [90]. The global searching capabilities also enable location of counter-intuitive solutions capable of performing optimally for all multipoint flow conditions [91]. This is of particular importance with conflicting design requirements, as a deterministic method is likely to find local minima close to the initial search condition. Deterministic methods do offer significant advantages in terms of optimisation performance, as improved convergence behaviour can be obtained through knowledge of solution gradients [90]. Furthermore the initial condition for optimisation is typically an aerofoil capable of performing optimally at one or several of the design points [92]. Therefore the path between the starting point and globally-optimal solution is likely
to be continuous and convex, making gradient-based methods ideally suited to the problem. A complete survey of methods capable of evaluating solution gradients with respect to perturbations in the decision variables is provided in [90]. These range from finite differences through to the continuous and discrete adjoint methods. Finite differences use small perturbations of the decision variables to evaluate the resulting objective-function gradient. Implementation is simplified due to the black-box treatment of the aerodynamic analysis, enabling optimisation with a range of CFD and panel codes. Their major drawback is large computational expense when large numbers of decision variables are used to parameterise the shape, as individual aerodynamic simulations must be performed for every single perturbation. This is a particular problem when optimising three-dimensional geometries with large numbers of shape parameters. The adjoint method offers a reduction in computational expense, by considering all objective-function gradients simultaneously. The system of governing equations is linearised with respect to each decision variable prior to solution [93]. Therefore the sensitivity of the solution to perturbations of all individual variables can be found from evaluation of a single set of flow calculations. This offers superior computation times relative to the finite differences method, however linearisation of the solver for complex flow conditions and of relevant turbulence models is difficult [90].

Morphing Aerodynamic Optimisation

Aerodynamic optimisation of morphing structures requires replacement of the shape parameterisation by a structural model with optimiser-controlled actuation [94]. The sensitivity analysis can then be performed to investigate the result of small perturbations in the actuation variables on the resulting structure, and subsequent flow properties. Computationally-efficient structural models are typically implemented, to reduce the expense of predicting structural displacements [34, 95]. This reduces the computational requirements of the sensitivity analysis, which are often dominated by aerodynamic calculations. A benefit of this type of analysis is that the aeroelasticity of the problem can be readily evaluated [96]; a summary of which follows below.

Aeroelastic Tailoring

Aeroelasticity is defined as the effect of elastic deformation on the airloads associated with normal operating flight [97]. It can have a profound effect on flight performance: handling, stability, and structural load distribution can all be affected. When considering the design of morphing structures,
it is typically required to increase their compliance to enable shape changes. This can make them susceptible to large deformations due to aerodynamic loads, which in turn can influence aircraft stability. For example in [98] the dynamic aeroelasticity of compliant aerofoils is investigated. A number of situations are identified in which the flutter boundary is reduced significantly with respect to rigid aerofoils. This shows that introducing compliance can lead to reduced stability, and that aeroelasticity is therefore an important consideration in the design of morphing aircraft. Not all aeroelastic effects are detrimental however. A study on the aeroelastic performance of MAV membrane wings in [99] showed that deformation due to aerodynamic loads is not necessarily a bad thing. Gust alleviation through gust washout, delayed stall, and increased lift were all achieved through consideration of structural properties such as membrane pre-tension and manufacturing methods. Although difficult to design for, significant gains in performance were predicted through large structural deformations.

Whilst aeroelasticity is an important consideration, it is beyond the scope of this literature review to cover the range of analysis techniques available. The work in this thesis is targeted at developing design tools for morphing structures, and not entire morphing aircraft. Therefore the design of morphing structures which can be implemented on current aircraft are investigated, such that the existing aeroelastic performance will not be compromised. Stability is investigated by ensuring the structure deforms to a stable configuration under actuation, and by assuming the aircraft’s dynamic stability is unaffected. This enables assessment of morphing performance in the early design phases, without incurring the computational expense associated with a full aeroelastic analysis.

2.6 Optimisation

As shown in Sec. 2.5, the design of morphing aircraft relies heavily on optimisation. A brief overview of optimisation methods is therefore included in this literature review for completeness, and as a reference for the remainder of the thesis.

A typical optimisation finds the set of decision variables $d$ that minimise an objective function $f(d)$, subject to predefined constraints and design-space bounds [73]. Expressed mathematically as

\[
\begin{align*}
\text{minimise} \quad & f(d) \\
\text{s.t.} \quad & g(d) = 0, \quad h(d) \leq 0, \\
& \text{and domain constraints} \quad d_l \leq d \leq d_u
\end{align*}
\]
2.6. Optimisation

where \( f(d) \) is the objective function quantifying the ability of an individual design at fulfilling its objective. Equality and inequality constraints are defined by \( g(d) \) and \( h(d) \), allowing the designer to restrict the design space such that unfeasible regions are not explored during optimisation. The individual design variables are bounded by \( d_u \) (upper) and \( d_l \) (lower), and the region \( d_l \leq d \leq d_u \) is known as the design domain \( \Omega \).

Many techniques exist to solve the fully-defined optimisation problem. These can be split into differential methods, evolutionary methods, and methods based on random number assignment. Differential methods search about an initial location via assessment of the objective-function gradient with respect to the decision variables. An optimal solution is found when a stationary point, either maxima or minima, is found. Differential methods perform well on problems with continuous, convex design space. However they are known to struggle when multiple local minima or maxima exist, or when design-space discontinuities are present. Evolutionary methods utilise patterns observed in nature to optimise a problem [100]. They include genetic algorithms (GA), which replicate the Darwinian theory of evolution and survival of the fittest [101]. Ant colony optimisation (ACO), where the ability of ants in the wild to discover the optimum path between their nest and a food source, is mimicked to locate promising areas of solution space [102, 103]. Particle swarm optimisation (PSO), where the design space is populated with solution ‘particles’, which traverse the design space comparing their current objective-function value with that of the particle cloud. This enables swarming, and hence better searching, of design-space regions with potential optima [104]. Finally, simulated annealing (SA), where an initial solution undergoes random permutations to minimise an objective function, mirroring the energy minimisation process during annealing in reality.

Whilst optimality is of interest to this work, the primary investigation is into the design of morphing aircraft. Therefore existing optimisation routines are made use of throughout. Due to problem development within the Matlab [105] computing environment, two built-in optimisation algorithms are utilised – \( fmincon \) and \( GA \) – along with a developed Monte Carlo method. This facilitates understanding of how the morphing problem responds to optimisation, allowing suggestions on solving future problems to be made. The three algorithms are discussed during application, but are also summarised here for clarity.
2.6. Optimisation

\textbf{fmincon}

An algorithm capable of minimising a multi-variable objective function, subject to linear and non-linear constraints. The function is a minimum when the first-order optimality criterion is satisfied. For unconstrained problems this means finding the location in design space such that the Jacobian of the objective function with respect to the decision variables is zero

\[ |\nabla_d f(d)|_{\infty} = 0 \tag{2.3} \]

where the infinity norm is taken to find the maximum absolute value of the vector of partial derivatives. If this value is close to zero, then the objective function is close to a minimum. For the constrained problem the first-order optimality criterion is set according to the Karush-Kuhn-Tucker (KKT) conditions, which modify Eqn. 2.3 to take into account constraints. The objective function is replaced with the Lagrangian \( L(d, \lambda) \)

\[ L(d, \lambda) = f(d) + \sum \lambda_g g(d) + \sum \lambda_h h(d) \tag{2.4} \]

where \( g \) and \( h \) are the aforementioned equality and inequality constraints, and \( \lambda \) is the vector of Lagrange multipliers made up from concatenation of \( \lambda_g \) and \( \lambda_h \). The first-order optimality condition therefore becomes

\[ |\nabla_d L(d, \lambda)|_{\infty} = |\nabla_d f(d) + \sum \lambda_g \nabla_d g(d) + \sum \lambda_h \nabla_d h(d)|_{\infty} = 0 \tag{2.5} \]

which is zero at a minimum. In order to optimise the problem the Lagrangian is used together with Eqn. 2.5 and the second-order differential

\[ H(d) = \nabla^2_d L(d, \lambda) = \nabla^2_d f(d) + \sum \lambda_g \nabla^2_d g(d) + \sum \lambda_h \nabla^2_d h(d) \tag{2.6} \]

where \( H(d) \) is known as the Hessian. For all results presented herein, the active-set algorithm is used to generate the Hessian, and subsequently create a line vector in solution space representing the optimal direction for searching. The starting point of the new iteration is then generated by taking a step of maximum possible length in the direction of this vector, which satisfies the constraints. The Hessian is estimated using a quasi-Newtonian updating method. This generates an estimate of \( H_{k+1} \) based on rank-one updates specified by gradient evaluations at the current iteration \((k)\), according to the BFGS method \([106]\), where

\[ H_{k+1} = H_k + \frac{q_k q_k^T}{s_k s_k} - \frac{H_k s_k^T s_k H_k}{s_k^T H_k s_k} \tag{2.7} \]
and

\[ s_k = d_{k+1} - d_k \]
\[ q_k = (\nabla f(d_{k+1}) + \sum \lambda \nabla g(d_{k+1})) - (\nabla f(d_k) + \sum \lambda \nabla g(d_k)) \]

and the initial guess of the Hessian is estimated as a scaled version of the identity matrix. A requirement within the active-set method is that \( H \) is positive definite, which requires that \( q_k^T s_k \) is positive. Therefore if this is not the case, the approximation is updated on an element-by-element basis until \( q_k^T s_k > 0 \). The full mathematics of this process are omitted here for brevity, however full details can be found in the Matlab user guide \[105\]. Once a new solution has been obtained, the process repeats itself until convergence. Due to the manner in which the Hessian is approximated, a single perturbation of each decision variable is required to calculate the partial derivatives of the objective function. Following construction of the Hessian, one more function evaluation is performed, which forms the starting point of the subsequent optimisation iteration. This means that if the vector of decision variables \( d \) has \( r \) terms, then a minimum of \( r + 1 \) function evaluations are required per iteration. This is a minimum requirement, as extra function evaluations may be performed to update the Hessian, or to ensure constraint violation does not occur.

Convergence criteria are set according to three user-defined values: \( \text{tolfun} \), \( \text{tolcon} \) and \( \text{tolX} \). For complex nonlinear optimisation problems, it may be difficult to achieve a value of exactly zero for the first-order optimality criteria in Eqn. 2.5. The value \( \text{tolfun} \) therefore replaces the zero, and is set such that when \( |\nabla_d L(d, \lambda)|_{\infty} \leq \text{tolfun} \), the solution is deemed optimal. The solver will also terminate when the objective-function improvement between subsequent iterations is less than \( \text{tolfun} \), as whilst this may not be a minimum, the algorithm has reached the limit of possible improvement. Such a situation is likely to arise when near a constraint boundary. During solution of the QP sub-problem, it is possible for the constraints to be violated, as this aids searching of the feasible design-space boundary. Constraint violation is also permitted in the optimal solution, providing it is below the user-specified value \( \text{tolcon} \). In the current work the constraints represent material limitations, such as yield stress. Because these are set values which cannot be changed, the value of \( \text{tolcon} \) is set to zero throughout. Finally \( \text{tolX} \) is used to limit the minimum step size taken during construction of the Hessian. As a minimum is approached the search step is likely to consistently reduce. Therefore when the step is less than \( \text{tolX} \), the solution is deemed at the location of the minimum, and hence optimal. This is useful when optimising engineering problems where variables represent quantities such as distance and force, as it allows matching of the step
2.6. Optimisation

size to manufacturing constraints. For example if the accuracy with which an actuator can be located is of the order millimetres, it is wasteful to optimise its location to the nearest micrometre.

Problem solution via \textit{fmincon} therefore requires: functions for calculating the objective function and constraint; lower and upper bounds on the design variables; a user-defined initial condition; and convergence tolerances. The development and specification of which are discussed with respect to individual problems during application.

\textbf{Genetic Algorithm}

A genetic algorithm replicates the survival-of-the-fittest phenomenon, utilised by species in nature who alter their attributes to thrive within their environment. Therefore, rather than operating from a single location in design space, many locations are evaluated simultaneously. Solutions which best fulfil the problem objectives are combined to create a new series of points at which to evaluate the design space. Each design point is referred to as an individual, and each set of individuals is known as a generation. Via careful selection of how individuals interact, it is possible for successive generations to improve the objective function, and hence optimise the problem.

To generate the initial population, a random number generator is used, creating individuals with design variables assigned via random weightings within the bounds $d_l$ and $d_u$. The objective function is calculated for each individual, and used to rank the population via rank-scaling. An individual with rank $r$ is assigned a value $k/\sqrt{r}$, where $k$ is set such that the sum of the values over the entire population is equal to the number of required parents. This retains population diversity relative to other approaches such as top-scaling, whilst simultaneously ensuring that the traits of the fittest individuals are passed on to increased numbers of children. Once ranked, the parents are selected according to a stochastic selection function. A line is created, with each individual corresponding to a section of the line, of length proportional to its rank-scaled value. The algorithm then moves along the line in steps of equal size; the number of which is defined by the number of required parents, with the first step generated via a random number between zero and the step size. At each step, the algorithm allocates a parent from the line section it lands on.

Once the parent solutions have been identified, reproduction occurs via elitism, crossover, and mutation. The best solutions from each generation are preserved through elitism, by copying them directly to the subsequent population. The variable \textit{elitecount} specifies the number of elite children; for the current work this is set to 10 \% of the population (rounded to an integer value where necessary). Crossover combines the vectors of decision variables (also known as chromosomes)
of two parents, to create a potentially better child solution. This process will be discussed in more detail during application of the GA. The number of individuals generated through crossover typically dominates the child population, and is set by the variable \textit{crossoverfraction}. For the current work this fraction is set at 0.8. Therefore for an example population of 20 individuals, 10\% of the child population will be generated through elitism, i.e. 2, and 80\% of the remaining 18 will be generated through crossover, i.e. $14.4 \approx 14$. The remaining 4 individuals will be created via the last reproduction mechanism – mutation. Mutation applies random variations to parent chromosomes to generate children with completely new properties. This retains diversity within the population, and helps ensure convergence to a global, rather than local, minimum. In the current work, mutations are added to each variable in the parent chromosome, through random selection of a location in the Gaussian distribution, scaled to fit bounded design space. Once the new population is constructed, the process repeats itself until convergence.

The algorithm terminates when an improvement in objective function is not achieved between subsequent generations. As with \textit{fmincon}, the variable \textit{tolfun} is used to set the tolerance for convergence. If an improvement of greater than \textit{tolfun} cannot be made, the generation is reported as a stall generation, and the reproduction process repeated. If a second stall generation is encountered – no objective-function improvement can be made via operating on the first stall population – the solution is deemed converged.

\textbf{Monte Carlo}

Monte Carlo methods are based on random number generation, and are named as a reference to the many casinos in the city, located in the principality of Monaco. Random numbers are generated and used as weightings to create decision variables from the upper and lower bounds. The method requires sufficient evaluation points to ensure optimality, however this number is typically much less than the number of points in a direct search. The random numbers can be generated according to a predefined distribution, such as normal or Gaussian [107]. For the implementation displayed here, no benefit is predicted from weighting decision-variable generation to the central region of the domain. Therefore a uniform distribution is implemented, using Matlab’s random number generator to create numbers within the region 0 and 1. It will be shown during application that the number of required function evaluations to ensure optimality, is approximately equal to the square of the number of design variables.
2.7 Summary and Thesis Objectives

This literature review began with a definition of what constitutes an adaptive, or morphing, structure – a system which changes shape through compliance of the structure itself, and achieves a significant change in performance from doing so. The benefits that can be achieved using adaptive structures were then demonstrated. Spacecraft take advantage of their reduced weight and maintenance schedules, whilst MEMs utilise the extremely small length scales at which compliant systems can be manufactured. Perhaps the most useful property of adaptive structures is that harnessed by morphing aircraft – the ability to maintain a continuous shape in multiple configurations. This enables improved aerodynamic properties such as reduced drag; as demonstrated by a number of concepts from the literature. Planform morphing enables multi-purpose vehicles to be constructed, capable of performing multiple missions with previously conflicting characteristics. Meanwhile out-of-plane morphing places less stress on the constituent components, achieving smaller deformations suitable for aerofoil alteration and aircraft control.

Enabling technologies for morphing structures were discussed in the form of piezoelectric materials, SMAs, compliant and multistable structures, and morphing skins. Smart material actuators can be embedded within morphing structures, contributing to structural stiffness whilst simultaneously facilitating morphing. Piezoelectric materials are capable of high forces and operating frequencies, but relatively small strains. This has led to their coupling with other materials to enhance their actuation characteristics. SMAs enable large forces and strains to be applied, but are relatively slow to respond. Further complications with SMAs are due to their heat-activated actuation. The operating envelope of the aircraft must be tailored to ensure the environmental conditions do not effect actuation. Bistable composites have shown promise for creating structures with significantly different geometrical configurations. However, snap-through creates a range of dynamic problems, and structures have difficulty withstanding typical wing loading without additional bracing.

Compliant structures have demonstrated the ability to provide large deformations, whilst retaining structural integrity, however designing such systems is complex and computationally expensive. A range of materials exist for use as morphing skins. Elastomers are favoured for high-strain applications, however these must be kept well above their glass transition temperature to ensure an elastic response. Corrugated composite structures have shown promise for out-of-plane morphing where stiffness requirements are ultra-anisotropic. Stiffening through the addition of composite rods, and filling of the aerodynamic surface with a smooth elastomer, have further enhanced prop-
Therefore whilst there are many promising methods and materials for morphing being developed, there are still many obstacles between the current state of the art, and fully-functioning commercial morphing aircraft of the future.

Design methods for morphing aircraft are typically biased towards either structural or aerodynamic analysis. Structural optimisation has the capability to create structures exhibiting significant shape changes, such as the adaptive leading edge shown in Fig. 2.24. However during optimisation little concern was given to the aerodynamics of the morphed shape. Conversely, aerodynamic optimisation is capable of optimising aerofoil geometry to minimise drag. Several sources even coupled simplified structural models to predict how this shape change could be effected. However, during investigation of materials and actuators for morphing, it was seen that nonlinear FE analysis was the only method which could predict accurately the response of complex structures and smart materials. A final design method used simplified models for both aerodynamics, and structural deformations, in order to assess morphing aeroelasticity. Whilst this does allow engineers a better understanding of the problem, it is unlikely these methods possess the fidelity to perform detailed design of morphing aircraft.

Thesis Objectives

The target of this work is to better understand the requirements of morphing aircraft, and to develop design methods accordingly. The key objectives are to:

- Develop high-fidelity aeroelastic morphing analysis tools;
- Develop feasibility tests for a generic morphing problem;
- Investigate optimisation requirements and define a protocol for design-space searching.

A high-fidelity analysis of both the structural and aerodynamic response, and how they interact, will enable a well-informed optimisation process to be carried out. This will create an accurate representation of the design space, enabling an optimal morphing solution to be found. A difficulty with structural optimisation was seen to be the subjectivity of the design-space parameterisation. Objectivity is fundamental to achieving a truly optimal solution, and ensures that potentially optimal, counter-intuitive locations of design space are searched. However objectivity typically comes at the cost of a large increase in computational requirements. This is highly undesirable if the end result is unfeasible. Aerodynamic optimisation is well established in terms of improving
design aspects such as aerofoil shape, using geometrical parameterisation techniques. However, there is little known about parameterising the shape changes using high-fidelity structural models, and the effect this will have on the optimisation process. Structural constraints due to factors such as material limitations will restrict the type of shape changes that are achievable, leading to difficulties in locating global optima. Therefore, like the structural morphing problem, ensuring a truly optimal morphing-aircraft structure is produced is likely to result in an expensive design-optimisation problem.

Therefore in addition to developing a high-fidelity optimisation framework, the thesis investigates ways in which the morphing design space can be reduced, to improve the efficiency and effectiveness of the optimisation process. The scientific literature on aerodynamic optimisation contains information on many geometrical changes which cause variations on aerodynamic performance – for example the Boeing 737 wing optimisation discussed in Sec. 2.5. However there is no provision for achieving these shape changes other than manufacturing a new wing. In order to reduce the design-space size, it is proposed to take advantage of these existing shapes of known aerodynamic performance, to bound a region of design space relating to a feasible morph. If via analysis of these geometries, it is possible to adjudge the structural feasibility of morphing subject to current material limitations, then it is known that a solution to the optimisation problem exists. This will not only allow the morphing effectiveness (in terms of aerodynamics) to be known, it will also provide an objective estimate to the optimal morphing structure, which can form the starting point for a full aeroelastic optimisation. An attempt to define morphing feasibility in this manner was made in [108], where independent components of a geometrical strain were computed for predefined morphing geometries. The current work looks to develop these methods further, to predict how the components of strain will interact, and hence provide a more conclusive feasibility analysis tool.

A final target of the work is to investigate the optimisation process, to discover the best way in which to solve the morphing optimisation problem. Better understanding of the design space will enable optimal designs, which model accurately the aerodynamic and structural response, to be produced more efficiently. This will reduce the time between design-phase iterations, accelerating the production of morphing aircraft.

In order to achieve these goals it is necessary to simplify the problem to its fundamental components. This will enable greater exploration of the key factors in the design of morphing-aircraft structures. Morphing structures are required to undergo a shape change to alter their aerodynamic
properties. From an aerodynamic perspective, the key structural property is the shape of its outer boundary. For example, an aerofoil profile governs its aerodynamic response, whilst the internal configuration has zero effect. Therefore in order to achieve variations in aerodynamic performance, it is this outer profile which must be altered during morphing. In this work, the morphing problem is simplified to morphing the wing-skin shape. In two dimensions this requires morphing between structures defined by curves, and in three dimensions surfaces. From a curve or surface representation, it is possible to use discrete actuation to alter the shape, and subsequently predict the aerodynamic response. If the outer skin can be actuated to achieve the desired response, then the morphing problem is feasible. The configuration of the internal structure and actuation can then be formulated as a subsequent design problem. The internal structure can utilise mechanisms to achieve the desired actuation, however the wing skin is required to remain continuous at all times to maintain aerodynamic flow – and hence must alter its shape via morphing. If the outer skin cannot be morphed to achieve the desired response, the modelling techniques can be used to predict the skin requirements necessary for morphing, and hence used to decide upon which technology covered in the literature is best suited to enabling the shape change.
Chapter 3

Aeroelastic Morphing Design

3.1 Introduction

In order to develop design methods for adaptive structures, it is useful to have a problem which, potentially, can be solved via morphing. Therefore, for the majority of the thesis the design of a morphing shock control bump is considered. This chapter begins by discussing the motivation for studying the problem, followed by a review of the current literature on the subject. The design of an adaptive bump is then considered in detail. An aeroelastic morphing design method is defined, incorporating aerodynamic and structural analysis. The problem is then optimised, and the resulting morphing structure presented. Finally conclusions are drawn on aeroelastic morphing design, and possible improvements suggested.

3.2 Motivation

There are many directives currently in place to improve the efficiency, and reduce the environmental impact of transport aircraft. For example, the Vision 2020 directive issued by the European commission, targets transport aircraft of the year 2020 to achieve: a 50 % cut in carbon dioxide emissions, per passenger per kilometre; and an 80 % reduction in nitrous oxide emissions [109]. Refining current designs is reaching a plateau in terms of improving aircraft aerodynamics, therefore engineers are looking to new methods to try to achieve these goals. A limiting factor on the flight speed and efficiency of many commercial aircraft is the presence of wave drag. As an aircraft passes the Mach divergence number, which is typically $Ma_D \approx 0.75$, a precipitous increase in drag is experienced, as shown in Fig. 3.1. This increase in drag does not subside until the free-stream
3.2. Motivation

Figure 3.1: Qualitative effect of free-stream Mach number on a thin aerofoil at constant angle of attack [28]

velocity exceeds $Ma = 1.3$, when the entire flow becomes supersonic. This limits the velocity of commercial aircraft to below $Ma_D$, in order to achieve acceptable levels of efficiency.

Wave drag occurs due to the presence of shock waves. When an aerofoil is subjected to a flow of $Ma > 0.7$, a region of supersonic flow can appear as air is accelerated over the upper surface of the aerofoil (see Fig. 3.2a). Before the free stream is reached at the trailing edge, this region of supersonic flow must return to a subsonic velocity. Apart from limited special cases [110], this supersonic flow can only transition back to a subsonic velocity via a normal shock. This shock is characterised by an abrupt increase in pressure, density and temperature, and decrease in Mach number [111]. The large pressure difference experienced whilst transitioning this shock region contributes to a precipitous increase in drag [72], buffeting [112], and boundary layer separation [113, 94]. However, if via flow control these wave-drag effects can be reduced, it is possible to create an aircraft which requires less power in the cruise regime, or that can fly faster for the same level of drag. This is advantageous: commercially, in the form of reduced flight times and fuel costs; and environmentally, as fewer harmful emissions will be produced.

The change in entropy across any shock is proportional to its strength, which is defined as the ratio of the upstream to downstream Mach numbers. The wave-drag contribution is proportional to this change in entropy, and hence the stronger the shock wave, the higher its contribution to drag. By instigating a bifurcated shock structure such as shown in Fig. 3.2(b) – also known as a \(\lambda\)-shock – it is possible to affect the entropy change across the shock such that the wave-drag increase is
3.2. Motivation

Figure 3.2: (a) Transonic aerofoil with normal shock and (b) a transonic aerofoil with $\lambda$-shock structure

avoided. The $\lambda$-shock structure consists of an oblique shock wave, upstream of the normal shock, which causes an initial increase in pressure and reduction in velocity. This causes the pressure difference across the final normal shock to be lower, improving the total pressure recovery, and reducing drag. The EUROSHOCK and EUROSHOCK-II projects investigated passive and active flow control devices for instigating the $\lambda$-shock structure on transonic aircraft wings [114]. The most promising flow control mechanism was found to be a contoured bump, also known as a shock control bump (SCB) [115, 116, 117, 118]. Whilst shock control can also be achieved via suction/blowing jets [119, 120, 121], this is not without a viscous penalty. However, whilst a fixed SCB offers the possibility to reduce wave drag, this benefit is compromised by their off-design performance. Although they can be designed to operate successfully for a given set of flow conditions, variations in these parameters can lead to increases in parasitic and viscous drag, and the onset of buffet, relative to an uncontrolled case. This makes it difficult to justify their inclusion on a commercial aircraft, despite the negligible weight penalty and lack of actuation energy associated with such devices. Morphing an SCB from an initially clean configuration, offers the chance to produce a system which can perform optimally in multiple configurations. Drag reduction in the transonic cruise regime is catered for, without incurring the off-design drag, or buffet, penalties associated with a permanent structure. This chapter investigates the structural and aerodynamic design of an adaptive SCB. In order to do this it is first useful to review the current literature on SCBs, to understand how they work, and the properties that must be replicated via morphing.
3.3 Shock Control Bump

A typical shock control bump (SCB) consists of a ramp, crest and tail, and has a streamwise centre-line profile such as that depicted in Fig. 3.3. The front shock leg is created by compression of the near wall flow, as it turns the angle at the initial ramp incline. Front shock-leg strength is governed by the ramp angle, which is a function of bump length, crest location, and bump height. The crest is a smooth region, often flat, where the final normal shock is intended to rest. The length of the $\lambda$-shock structure, which is defined by the ramp length and ramp angle, determines the height of the triple point (the point at which the shock bifurcates). This height determines the amount of the flow that travels through the bifurcated shock system, and hence is proportional to the improvement in pressure recovery. As the subsonic flow travels from the crest to the bump tail, it accelerates due to re-expansion. Careful design of the tail and tail angle are required to ensure that supersonic flow conditions are not encountered in this region. If designed with too steep an angle, or a sharp transition between crest and tail, it is possible for a second shock structure to be produced. This leads to a severe viscous drag penalty due to boundary-layer thickening, or in the worst case, shock-induced separation.

A range of three-dimensional shock control bumps were investigated in [29]. Different bump geometries were tested along with the effect of their position, relative to the reference location of the normal shock. The wave-drag reduction and ability to control buffet, were assessed using
3.3. Shock Control Bump

Figure 3.4: (a) Default rounded bump and (b) schlieren visualisation of $\lambda$-shock [29]

experimental and computational techniques. Schlieren visualisations were used to examine the experimental flows, allowing observation of the instigated $\lambda$-shock structure. A typical SCB from the study – the default rounded bump – together with its $\lambda$-shock schlieren image, is shown in Fig. 3.4. For all bump shapes the shock structure depended largely on the original shock position with respect to the bump. The optimum position was when the rear shock foot was located at the downstream end of the bump crest (as shown in Fig. 3.4(b)). Locating the bump too far downstream of the normal shock does not realise the full potential of the SCB, and the $\lambda$-shock structure is small in height, limiting the improvement in pressure recovery. Placing the SCB too far upstream relative to the shock causes a region of recompression on the bump tail. This often leads to a second region of supersonic flow, and a second normal shock, which has an adverse effect on the boundary layer and significantly increases viscous drag. The optimum upstream deflection angle (ramp angle) was found to be $4.9^\circ$, and the optimum bump height 9.7 mm, as this allowed an 86% reduction in total pressure loss. The $\lambda$-shock structure was found to decay slowly in the spanwise direction, a phenomenon that was overcome successfully by using several control bumps distributed in this direction. This collection of bumps incurs negligible viscous penalty, and, when significant wave drag is present, is able to achieve a total drag reduction of approximately 30%. Finally it is noted that care must be taken when optimising the shape of such a control bump on a transonic wing. Overall performance must be judged on both: the ability to alleviate the losses due to the shock at a single design point; and in terms of the effect on buffet and viscous drag over the entire flight plan.

The performance of an SCB subjected to downstream pressure variations – such as those associ-
3.3. Shock Control Bump

ated with buffet – is investigated in [122]. The effects of downstream pressure perturbations on the \( \lambda \)-shock structure are investigated experimentally, for \( \text{Ma}_{\infty} = 1.4 \) and 1.5, and pressure perturbation frequencies of 16–90 Hz. For different problem setups, the downstream pressure perturbations were found to cause oscillations in the chordwise shock location. The front shock leg increased in strength during movement upstream, and decreased when moving in the direction of the flow. These variations are related to the shock strength, and the subsequent effect this has on shock boundary layer interaction (SBLI), and/or shock-induced separation. The potential for separation when using a fixed SCB in an oscillating flow is significant, meaning that flight conditions causing pressure oscillations should be avoided.

The study in [116] investigates three-dimensional shock control bumps for drag reduction and buffet onset retardation for transonic swept wings. The influence of sweep was found to be small with respect to a two-dimensional aerofoil analysis, as similar results were found concerning the effect on the pressure distribution and boundary layer development. It is noted that the total drag reduction for the three-dimensional case is slightly lower than that predicted for the two-dimensional analysis, due to the lower contribution from wave drag to the overall value. Placing the control bump in the shock region proved its effectiveness at reducing the wave drag. Additionally, it was found that locating the SCB downstream of the shock caused a positive influence on the separated flow region, in the form of reduced downstream RMS pressure fluctuations (buffet control). This offers the possibility to reduce drag, or postpone buffet to higher lift coefficients.

The effects of finite-span two-dimensional shock control bumps, on infinitely swept and unswept wings, are investigated in [117]. The control bumps consisted of a luff side-step region, a linear crest of varying height in both the spanwise and streamwise directions, and a leeward step region. The control efficiency was found to increase significantly by optimising the spanwise contours; with crest height and SCB location (as shown in Fig. 3.5) being the dominant variables for facilitating this control. It was found that perturbations in the spanwise flow component limited the efficiency increase to 25 % for the swept case, with a much higher increase achieved for zero sweep. It was deemed that further investigations into the spanwise crest slope are required, to understand better the shock boundary layer interaction mechanisms [118].

A technique for evaluating the wave-drag reduction achieved via flow control was investigated in [123]. It is noted that determination of the wave-drag component in isolation is difficult from CFD and experimental data, and hence an analytical analysis tool is developed. An extension is made to a model first proposed in [124], estimating wave drag from a single normal shock on an aerofoil.
This extension allows prediction of wave-drag on a curved aerofoil with a $\lambda$-shock structure. Several additional assumptions are made in addition to those of the original model, including: the front shock leg is an oblique shock, and the rear shock leg can be approximated as a normal shock; there is a uniform pressure distribution in the $\lambda$-region; and the position of the main shock does not change as a result of control. An expression is formulated detailing the contributions to wave-drag from the two shocks, as functions of the: ramp angle; free-stream Mach number; surface Mach number immediately upstream of the oblique shock; length of the $\lambda$-region; and the aerofoil curvature at the shock site. A parametric study was performed on the DA LVA-1A aerofoil, for a fixed bump length of 20 % of chord, with crest located at 70 % of chord. The optimal ramp angle was found to be 2.5°, which corresponds to a bump height of 3 % of chord. Despite showing good agreement with CFD results in terms of predicting the wave-drag savings, the method was seen to break down when the flow in the $\lambda$-region was sub optimal. Such flow conditions arise when the shock is not located properly with respect to the bump, or if the bump exhibits geometry contributing to re-expansion in the crest-tail region. This method is therefore useful in terms of understanding the key bump parameters, however more detailed methods such as CFD are required to analyse fully the flow control capabilities of a bump with complex geometry.

The design of an adaptive bump for shock control – the target of this work – is discussed in [95]. It is claimed that structure-integrated-actuation systems are required to enable an adaptive SCB to support the associated aerodynamic loads, and provide the required levels of deformation to generate a typical bump shape. It is proposed to integrate the adaptive device into wing spoilers to leave the principal aircraft design unchanged, and maintain the structural integrity of the wings. A shape parameterisation scheme is introduced linking the actuation and control requirements,
which is subsequently optimised to find minimum actuation energy configurations. Unfortunately a lack of aerodynamic analysis means accurate levels of wave-drag reduction are absent, however an estimated possible drag reduction of 14% is stated.

A final point of consideration in the design of an SCB for a large commercial transonic aircraft, concerns the ever-changing conditions experienced during flight. As fuel is burnt, less lift is required, causing the aircraft consistently to reduce its effective angle of attack. Because the mechanical means to do this efficiently are limited, constant lift is achieved by flying at alternative altitudes, where air of different density is encountered. This reduces the amount of lift produced for the same flying configuration and equivalent velocities. Rather than continuously varying flight conditions, a series of fixed design points are used, which are optimised for efficiency over the flight envelope of the aircraft [125]. These changes in altitude will alter the shock properties, as they will influence the local Mach number and density, which contribute to the normal shock strength and location. The fact that the strength and location of the shock keep changing means that to function with optimal efficiency, the SCB location and shape needs to change too. An adaptive contour bump of variable height is one way in which to achieve this. If successful, it has been predicted that a reduction in Cash Operating Costs (COC) of 1.3% for a laminar-type transport aircraft, and up to 0.8% for an A3xx-type design [125], can be achieved.

Summary

In this review of the literature of SCBs, it has been shown that if designed correctly, SCBs have a positive influence on transonic flow around an aerofoil. Relative to an uncontrolled case with a single normal shock, SCBs are capable of reducing wave drag, controlling buffet, and reducing viscous drag. Two-dimensional bumps are seen to provide the largest reduction in wave drag, however they are difficult to implement in practice due to factors such as wing sweep, and spanwise variations in wing geometry. SCB performance is generally better on unswept wings, with the level of wave-drag reduction tending to that for two-dimensional flow. However, collections of three-dimensional bumps have successfully enabled shock control on swept wings, with spanwise variations in bump geometry being used to combat the oblique flow angle. Assuming optimal bump placement, the bump parameters governing the level of attainable wave-drag reduction are ramp angle, bump length, and crest height. Successful bumps were shown with crest heights of 1–3% of chord, and ramp angles in the range $2.5^\circ - 5^\circ$, and the crest located at approximately 70% of the bump length (in the streamwise direction).
Despite these benefits, if used at off-design conditions, SCBs can cause detrimental effects to these parameters (drag, buffet and separation), severely effecting aircraft performance. Optimal placement of the bump, relative to the original normal shock, is the single most important factor in achieving the required flow properties. Therefore, because shock location is constantly varying as fuel is burnt, it is impossible for a static bump to perform optimally throughout the flight envelope. Whilst a sub-optimal drag reduction could be tolerated, the off-design penalties such as buffet and shock-induced separation can not. This has prevented the application of contour bumps on current transport aircraft, and restricted their cruise speeds to values below \( M_{\text{D}} \). A morphing bump is an attractive means to overcome this problem [126]. The most advanced case would be to design an adaptive structure, capable of producing a range of bump geometries at various chordwise locations. This would allow wave-drag reduction throughout the flight envelope, and the original aerofoil geometry to be returned prior to take-off and landing. Unfortunately such a design is difficult to implement structurally, without compromising large regions of space within the aircraft wing. A further drawback is the weight penalty associated with actuating a large variety of morphed shapes.

A second application of an adaptive bump, is to limit flow control to be applied only when an aircraft deviates from its design conditions. Occasionally, keeping to schedule may require flying faster than the ‘design’ velocity. For example, if an aircraft is delayed or encounters bad weather, it will need to make up time. In doing so, a strong shock wave is likely to develop on the wing, and a drag penalty will be incurred, resulting in higher fuel burn. However, if the cost of burning extra fuel is less than that associated with arriving late and passengers missing connections etc., the pilot has no choice. If the pilot were able to deploy an adaptive bump whilst deviating from the design velocity, time could be made up without incurring the penalties associated with increased drag. The remainder of this chapter investigates the design of such a bump. Knowledge of exactly when the SCB will be deployed enables a well-defined operating point to be specified. The shock location will be fixed, along with the flow velocity and aircraft altitude. Hence a morphing bump can be defined to perform optimally in these conditions, without concern for its off-design behaviour.

### 3.4 Morphing SCB Design

The design of a morphing SCB is a complex multidisciplinary problem, requiring both structural and aerodynamic analysis [96]. This creates a large and nonlinear design space, with no analytical
representation. To find a design that minimises drag, but exhibits a sustainable level of deformation within the structure, requires optimisation. This is described formally as finding the distribution of actuation which achieves

$$\min(f) \quad \text{(drag-based objective function)}$$

s.t.

$$h \leq 0 \quad \text{(stress constraint)}$$

From this definition it can be seen that the objective function is generated from aerodynamic analysis, while the constraint applies to the structure, highlighting the multidisciplinary nature of the problem. The remainder of this chapter describes the synthesis of a compliant structure, capable of achieving a wave-drag reduction on the RAE2822 aerofoil, when subjected to a combination of pressure-load actuations. When actuation is removed, the structure returns to the RAE2822 aerofoil profile. Section 3.5 investigates CFD modelling of the transonic flow regime, highlighting the requirements of such aerodynamic analysis and providing validation of modelling techniques. This CFD analysis is then coupled to an optimiser-controlled structural model, capable of affecting the shape of the top surface of the rear half of the RAE2822 profile. Static aeroelasticity is considered via a weak FE/CFD coupling to ascertain the final bump configuration. A drag-based objective function is then minimised via a gradient-based local refinement, starting from an initial condition specified using existing SCB knowledge. Finally optimum results are presented along with concluding remarks on the process, and potential improvements to the method.

### 3.5 Aerodynamic Modelling

Modelling the transonic flow past an aerofoil is a complex process requiring a solver capable of dealing with both subsonic flow, and supersonic, compressible flow [127, 128, 129]. A high-fidelity approach is required to capture sufficiently the effect on the flow of small changes in geometry, such as introducing an SCB. To meet these requirements CFD is selected as the modelling tool throughout. Specifically the open-source CFD toolbox OpenFOAM[130] is employed, utilising the transonic solver `sonicFoam`. This is a transient code which solves the Reynolds Averaged Navier-Stokes (RANS) equations via the finite-volume method. The Spalart-Allamaras turbulence model [131], with standard coefficients and an eddy viscosity of $\nu = 0.14 \text{ m}^2\text{s}^{-1}$, is used to close the RANS equations throughout. As a validation of the solver, flow conditions are assigned to allow comparison with case 9 in the experimental data from [30]. The free-stream velocity is assigned
Figure 3.6: (a) Contours of Mach number and (b) comparison of $C_P$ distribution with experimental
data for RAE2822 aerofoil case 9 [30]

as $Ma = 0.73$, with a chord-based Reynolds number of $Re = 6.5 \times 10^6$. The aerofoil is given a
corrected angle of attack, to account for wind tunnel edge effects, of $\theta_{corr} = 2.81^\circ$. Figure 3.6(a)
shows contours of Mach number about the RAE2822 aerofoil, and (b) the comparison of pressure
coefficent $C_p$ obtained using $sonicFoam$ with experimental data from [30]. A clear shock structure
is visible on the suction surface of the aerofoil as the flow is accelerated and abruptly decelerated
before it reaches the trailing edge. Good agreement is found with the experimental data, both in
terms of the pressure distribution and the location of the shock wave returning the flow to subsonic
conditions. This result gives confidence in the solver’s ability to capture the fundamental aspects
of the problem, and allows progression to the optimisation stage of the design process.

Whilst this case can be modelled accurately using CFD analysis, the relatively weak normal
shock strength of approximately 1.14 does not lend itself to control. Shock strength is defined as
the ratio of the upstream Mach number to $Ma = 1$ for consistency. To increase shock strength
the free-stream velocity is set at $Ma = 0.75$, whilst keeping the Reynolds number constant at
$Re = 6.5 \times 10^6$. The result, detailed in Fig. 3.7, is a strong recompression shock of strength 1.33,
with a drag coefficient of $C_D = 0.0309$ due to the increase in wave drag. An increase of lift coefficient
to $C_L = 0.859$ is also encountered. This configuration forms the initial design condition, about which
an optimisation algorithm will operate to reduce the drag coefficient of the aerofoil. The following
Figure 3.7: (a) Contours of Mach number and (b) $C_P$ distribution for RAE2822 aerofoil $Ma = 0.75$, $Re = 6.5 \times 10^6$ and $\theta = 2.81^\circ$.

sections detail the structural morphing process, objective-function and constraint derivation, and optimisation.

### 3.6 Design

An optimiser-controlled morphing structure is generated to control the transonic flow. This section details the actuation method, how the displaced structure is meshed ready for CFD analysis, and its subsequent aeroelastic interaction with the flow.

#### Structural Modelling

Existing knowledge of the shock location on the RAE2822 transonic aerofoil is used to bound the design space. Rather than letting the entire aerofoil morph – which may be beneficial, but significantly increases the complexity of the optimisation – the morphing region of the aerofoil is limited to the aft half of the suction surface of the aerofoil, in the region $0.5 \leq \frac{x}{c} \leq 0.9$. This ensures that any morphing bump is deployed in a region where it can affect the normal shock, and potentially cause a drag reduction. This is beneficial due to the time-intensive aerodynamic analysis prohibiting searching of non-optimal design space. The decision variables for optimisation,
Figure 3.8: (a) RAE2822 aerofoil geometry and (b) decision variables for structural morphing.

displayed in Fig. 3.8, govern the magnitude and location of actuation applied to morph the structure. To respect the 2-D modelling approach, all loading and boundary conditions remain constant in the spanwise direction. Actuation is provided by pressure loads applied over a spanwise strip of constant 5 mm thickness. In this study, the number of actuation locations is limited to two in order to develop the method. Increasing the number of actuation locations will enhance the range of shapes that can be produced, and may allow larger deformations for the same maximum stress [132], perhaps leading to a more pronounced structure capable of greater drag reduction. However, this adds further complexity to an already costly optimisation process, and hence will be investigated in future studies. For the case shown here, the vector of decision variables is

\[
\begin{bmatrix}
p_1 \\
p_2 \\
\delta_1 \\
\delta_2 \\
B_1 \\
B_2
\end{bmatrix}
\]

(3.1)

where \(p_i\) denotes the magnitude of the pressure applied, \(\delta_i\) the chordwise location of the actuation on the pseudo-2D geometry, and \(B_1\) and \(B_2\) the boundaries of the morphing region. Nodes are created at these boundaries, and used to apply fully-fixed boundary conditions. Allowing the boundaries to move enables the best location of the SCB to be discovered during optimisation. The structure outside these bounding nodes is fixed in order to remove any detrimental aerodynamic effects of an elastic response outside the morphing region. The morphing section is intended to be part of the wing skin, and hence is assigned material properties of isotropic aerospace grade 7075 (T6 temper) aluminium (\(E = 72\) GPa, \(\nu = 0.33\)) and a thickness of 0.5 mm. The commercial FE package Sancef [133] is used to compute structural displacements resulting from actuation. The structure is modelled using second-order Mindlin shell elements, and a geometrically nonlinear, quasi-static
Chapter 3. Aeroelastic Morphing Design

3.6. Design

Analysis is performed.

Repeatable morphs are desired, it is therefore necessary to limit the maximum deformation to a value below the yield point of the material. To limit stress present in the morph, the von Mises equivalent stress [134] is calculated over each element according to

\[ \sigma_{i,eq} = \sqrt{\sigma_{i,1}^2 - \sigma_{i,1} \sigma_{i,2} + \sigma_{i,2}^2} \]  

(3.2)

where \( \sigma_{i,1} \) and \( \sigma_{i,2} \) are the principal in-plane stresses for element \( i \). The maximum is evaluated by taking the infinity norm of the vector containing these values, and forms the value to be constrained by the optimisation

\[ \sigma_{max} = \| \sigma_{eq} \|_{\infty} \]

\[ h = \sigma_{max} - \sigma_y (1 - k) \]  

(3.3)

\[ h \leq 0. \]  

(3.4)

For the results shown here, the safety factor \( k \) is set at 10%, and \( \sigma_y \) is assigned the yield strength of the selected grade of aluminium \( \sigma_y = 505 \) MPa.

CFD Grid Generation

Once a bump satisfying the constraint has been created, a CFD grid must be generated in order to evaluate its shock-control capabilities. Local refinement in the region of the bump, and at the aerofoil boundary is employed, as shown by the example mesh in Fig. 3.9. This increases solution accuracy, without incurring significant computational expense. Wall functions are avoided, due to the possibility of bump-induced separation [135]. A major difficulty with optimisation of this kind is ensuring the CFD is capturing the physics of the flow, and that the results are not simply a function of the mesh. When performing a single analysis, such as the validation case discussed previously, it is possible to vary the density and quality of the mesh, and observe the effect on the results. A mesh for which small perturbations in density and quality causes no change in flow characteristics is deemed converged, and the results reliable. Applying this technique during the automated optimisation procedure would be prohibitively expensive in terms of function evaluation time. Instead a systematic approach to mesh generation is adopted via a dedicated mesh generation algorithm. This algorithm is capable of generating a grid exhibiting the required density, local refinement, cell aspect ratio, and cell skewness, regardless of the bump shape produced by the morphing structure. A structured C-grid set up is chosen, of length 10 chord-lengths downstream.
to capture wake effects, and 7 chord-lengths above and below the aerofoil to ensure boundary effects are not encountered by the aerofoil. A multi-block structured grid is preferred for systematic generation, allowing treatment of complex geometries, tailoring of boundary regions to a required specification; and being well suited to parallelisation, enabling greater speed-up than with an unstructured grid [136]. Figure 3.9 shows the block structure generated in $(\xi, \eta)$ computational space. Blocks downstream of the aerofoil, such as block 1, are specified directly in the OpenFOAM case file. Blocks around the aerofoil, such as 2-4 in Fig. 3.9, are generated using the mesh refinement algorithm to ensure the required mesh quality is achieved. Due to the range of possible geometries the mesh generation algorithm must cater for, it is difficult to define a method for specifying directly the required grid. Direct generation of a grid exhibiting the required cell aspect ratio and orthogonality is likely to result in grid-folding and an unusable mesh. Instead an initial grid is generated of unsuitable quality, but exhibiting the required connectivity and density. This initial grid is then refined elliptically, until orthogonality conditions are met both at the aerofoil boundary and the interior of the mesh.
Initial Grid Generation

The most widely used algebraic techniques for surface grid generation from a prescribed boundary point distribution is the Transfinite Interpolation (TFI) method [136]. First described by Gordon and Hall [137], TFI utilises 1-D univariate interpolation in each of the computational-space coordinate directions. Boundary grid nodes are specified using the aerofoil coordinates together with the displaced bump geometry for the $\eta = 0$ internal boundary, and the C-grid extremum coordinates for the $\eta = 1$ outer boundary. The general form of the univariate interpolation functions $U$ and $V$ in dimensions $\xi$ and $\eta$ respectively are

$$U = \sum_{i=1}^{L} \sum_{n=0}^{P} \chi_n^i(\xi) \frac{\partial^m r(\xi_i, \eta)}{\partial \xi^n}$$

$$V = \sum_{j=1}^{M} \sum_{m=0}^{Q} \psi_m^j(\eta) \frac{\partial^m r(\xi, \eta_j)}{\partial \eta^m}$$

where $\chi$ and $\psi$ are blending functions, and $r$ stands for the position of a grid point in the physical (Cartesian) space. The specified boundary node locations are inserted to construct the univariate interpolation function. When $U$ and $V$ are known the internal grid is generated by the Boolean sum of the interpolation functions

$$r = U \oplus V$$

$$= U + V - UV$$

which guarantees all boundary curves are matched. The tensor product $UV$ is evaluated according to

$$UV = \sum_{i=1}^{L} \sum_{j=1}^{M} \sum_{n=0}^{P} \sum_{m=0}^{Q} \chi_n^i \psi_m^j \frac{\partial^m r(\xi_i, \eta_j)}{\partial \eta^m \partial \xi^n}.$$
### 3.6. Design

#### Chapter 3. Aeroelastic Morphing Design

<table>
<thead>
<tr>
<th>Mesh Generation Method</th>
<th>Max Non-Orthogonality</th>
<th>Mean non-Orthogonality</th>
<th>Max Skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear TFI</td>
<td>70.55°</td>
<td>18.33°</td>
<td>1.93</td>
</tr>
<tr>
<td>TFI + elliptical refinement</td>
<td>70.57°</td>
<td>13.40°</td>
<td>2.70</td>
</tr>
<tr>
<td>Neumann boundary conditions</td>
<td>23.30°</td>
<td>5.96°</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table 3.1: Quantitative effects of mesh-refinement algorithm

giving the interpolation functions \( U, V \) and \( UV \) as

\[
U = \sum_{i=1}^{2}\chi_{i}^{0}(\xi)\mathbf{r}(\xi, \eta)
\]

\[
V = \sum_{j=1}^{2}\psi_{j}^{0}(\eta)\mathbf{r}(\xi, \eta)
\]

\[
UV = \sum_{i=1}^{2}\sum_{j=1}^{2}\chi_{i}^{0}(\xi)\psi_{j}^{0}(\eta)\mathbf{r}(\xi, \eta)
\]

respectively. The coordinates in physical space are found by evaluating Eqn. 3.6, giving

\[
\mathbf{r}(\xi, \eta) = (1 - \xi)\mathbf{r}(0, \eta) + \xi\mathbf{r}(1, \eta) + (1 - \eta)\mathbf{r}(\xi, 0) + \eta\mathbf{r}(\xi, 1)
\]

\[
- (1 - \xi)(1 - \eta)\mathbf{r}(0, 0) - \eta(1 - \xi)\mathbf{r}(0, 1) - \xi(1 - \eta)\mathbf{r}(1, 0) - \xi\eta\mathbf{r}(1, 1)
\]

(3.10)

An example grid generated via linear TFI is shown in Fig. 3.11(a), with contours of constant \( \eta \) omitted for clarity. The linear aspect is clear, with straight grid lines of constant \( \xi \) intersecting the curved aerofoil boundary. This leads to high levels of non-orthogonality and cell skewness, as shown in Tab. 3.1.

#### Elliptical Refinement

The initial grid is improved elliptically based on the solution of a set of PDEs, which is known to produce a grid with smoothly-varying cell sizes and slopes of grid lines [138]. Orthogonality near walls and spacing at these boundaries can also be controlled. In 3-D the system of Poisson’s equations is used to define the unknown Cartesian grid coordinates. In 2-D this becomes

\[
\alpha \left( \frac{\partial^2 \mathbf{r}}{\partial \xi^2} + \frac{P}{\partial \xi} \right) - 2\beta \frac{\partial^2 \mathbf{r}}{\partial \xi \partial \eta} + \gamma \left( \frac{\partial^2 \mathbf{r}}{\partial \eta^2} + Q \frac{\partial \mathbf{r}}{\partial \eta} \right) = 0
\]

(3.11)
where as before \( \mathbf{r} = [x, y]^T \), \( P \) and \( Q \) are control functions, and \( \alpha, \beta \) and \( \gamma \) are known as the metric coefficients defined as

\[
\alpha = \left( \frac{\partial x}{\partial \eta} \right)^2 + \left( \frac{\partial y}{\partial \eta} \right)^2
\]
\[
\beta = \frac{\partial x}{\partial \xi}\frac{\partial x}{\partial \eta} + \frac{\partial y}{\partial \xi}\frac{\partial y}{\partial \eta}
\]
\[
\gamma = \left( \frac{\partial x}{\partial \xi} \right)^2 + \left( \frac{\partial y}{\partial \xi} \right)^2.
\]

(3.12)

It should be noted that when \( \beta = 0 \), the grid is orthogonal, however this cannot be specified \textit{a priori} as grid folding may occur. By setting \( P = Q = 0 \) in Eqn. 3.11, the problem collapses to the Laplace equation, which is known for its smoothing capabilities when operating on an algebraic grid. For example, a system of grid lines satisfying the Laplace equation familiar to aeronautical engineers, are the mutually orthogonal streamlines and equipotential lines present in potential flow [139]. A drawback of this assumption is relinquished control over interior node location via functions \( P \) and \( Q \). These control functions require careful selection [140], and are highly case dependent, making them an unsuitable method for applying boundary conditions during automated mesh generation. To retain boundary-control capabilities, Neumann boundary conditions are imposed and will be discussed shortly. With the following assumptions the governing equation becomes

\[
\alpha \left( \frac{\partial^2 \mathbf{r}}{\partial \xi^2} \right) - 2\beta \frac{\partial^2 \mathbf{r}}{\partial \xi \partial \eta} + \gamma \left( \frac{\partial^2 \mathbf{r}}{\partial \eta^2} \right) = 0
\]

(3.13)

which is discretised and solved using central finite-difference stencils. An example finite-difference stencil, centred at \( \mathbf{r}_{i,j} \) is shown in Fig. 3.10. By setting \( \Delta \xi = \Delta \eta = 1 \) the system reduces to

\[
\alpha (\mathbf{r}_{i+1,j} - 2\mathbf{r}_{i,j} + \mathbf{r}_{i-1,j}) - \frac{\beta}{2} (\mathbf{r}_{i+1,j+1} - \mathbf{r}_{i+1,j-1} - \mathbf{r}_{i-1,j+1} + \mathbf{r}_{i-1,j-1})
\]
\[
+ \gamma (\mathbf{r}_{i,j+1} - 2\mathbf{r}_{i,j} + \mathbf{r}_{i,j-1}) = 0
\]

(3.14)

and the metric coefficients to

\[
\alpha_{i,j} = \frac{1}{4} \left[ (x_{i+1,j} - x_{i,j})^2 + (y_{i+1,j} - y_{i,j})^2 \right]
\]
\[
\beta_{i,j} = \frac{1}{4} \left[ (x_{i+1,j} - x_{i,j}) (x_{i+1,j+1} - x_{i,j+1}) + (y_{i+1,j} - y_{i,j}) (y_{i+1,j+1} - y_{i,j+1}) \right]
\]
\[
\gamma_{i,j} = \frac{1}{4} \left[ (x_{i+1,j} - x_{i,j})^2 + (y_{i+1,j} - y_{i,j})^2 \right]
\]

(3.15)

An iterative Gauss-Seidel scheme is implemented to solve the discretised governing equation. Rearranging Eqn. 3.14 in terms of \( \mathbf{r}_{i,j} \) enables a new value of \( \mathbf{r}_{i,j} = \mathbf{r}_{i,j}^* \) to be obtained from the
surrounding grid points.

\[
\mathbf{r}_{i,j}^* = \frac{1}{2(\alpha + \gamma)} \left[ \alpha (\mathbf{r}_{i+1,j} + \mathbf{r}_{i-1,j}) + \gamma (\mathbf{r}_{i,j+1} + \mathbf{r}_{i,j-1}) - \frac{\beta}{2} (\mathbf{r}_{i+1,j+1} - \mathbf{r}_{i+1,j-1} - \mathbf{r}_{i-1,j+1} + \mathbf{r}_{i-1,j-1}) \right]
\]  
(3.16)

Considered over the entire grid, this offers the opportunity to iterate the grid towards the desired solution. Successive over relaxation (SOR) [141] is implemented in order to accelerate convergence to the final grid. The updated solution is found via a combination of \(\mathbf{r}_{i,j}^*\) and the current \(\mathbf{r}_{i,j}\)

\[
\mathbf{r}_{i,j}^{\text{updated}} = \omega \mathbf{r}_{i,j}^* + (1 - \omega) \mathbf{r}_{i,j}
\]  
(3.17)

where \(\omega\) is the relaxation factor and is assigned a value in the range \(1 < \omega < 2\) for over-relaxation. The extra weighting allows a larger step in the new solution direction to be taken, thus increasing the speed of convergence. Due to the generation of a starting point via TFI, good convergence behaviour is exhibited. Therefore an aggressive value of \(\omega = 1.8\) is selected here, helping to speed up mesh generation and reduce function evaluation time. Example grids generated using linear TFI, and linear TFI with elliptic refinement are shown in Fig. 3.11. Note that lines of constant \(\eta\) are omitted from these plots for clarity. The smoothing properties of elliptical refinement are evident at the interior of the domain, particularly in the region around the leading edge. However, a large degree of cell skewness is still present at the aerofoil boundary. A quantitative assessment of the different mesh generation stages is displayed in Tab. 3.1. The mean orthogonality is improved by \(27\%\) in comparison with linear TFI, however the maximum non-orthogonality is still relatively high, and the max cell skewness has been compromised during mesh smoothing. Whilst satisfying
3.6. Design

Chapter 3. Aeroelastic Morphing Design

Figure 3.11: Mesh generation via (a) linear TFI and (b) elliptical refinement of initial mesh

the Laplace equation over the interior of the domain, control of the grid at the boundary is lost, resulting in poor mesh quality in this region. Therefore the remainder of the section discusses generation of a boundary condition to regain grid-quality control at the aerofoil surface.

Neumann Orthogonality Boundary Condition

During the elliptical grid-refinement process, an additional step is included at the beginning of each iteration to enforce convergence to a grid with desirable properties at the intersection with the aerofoil boundary. This is achieved by imposing Neumann orthogonality boundary conditions [142] on Eqn. 3.13 before its solution. In order to specify gradients on the boundary, a body-fitted grid with gradient control is constructed as shown in Fig. 3.12. The surface gradient at location \( (x_{i,1}, y_{i,1}) \) is approximated using the central-difference scheme

\[
\left. \frac{dy}{dx} \right|_{i} \approx \frac{y_{i+1,1} - y_{i-1,1}}{x_{i+1,1} - x_{i-1,1}} \tag{3.18}
\]

and used to define the equation of the line passing through points \( (x_{i+1,1}, y_{i+1,1}) \) and \( (x_{i-1,1}, y_{i-1,1}) \). This is then used to evaluate the line with the inverse gradient \( -\left. \frac{dx}{dy} \right|_{i} \), which passes through the point \( (x_{i,2}, y_{i,2}) \). The updated \( x_{i,1} \) coordinate is given by the intersection of this line with the aerofoil boundary. The updated \( y_{i,1} \) coordinate is evaluated from the parametric representation of the aerofoil geometry at \( x_{i,1} \). Before each iteration of the discretised grid-generation process, each boundary node is ‘slid’ along the spline defining the aerofoil surface, to the location required
3.6. Design

Chapter 3. Aeroelastic Morphing Design

Figure 3.12: Application of Neumann boundary conditions during elliptical refinement

for boundary orthogonality. Fig. 3.12 shows this process for a single boundary node. Once the boundary conditions are specified, the elliptical mesh refinement is performed. The boundary condition cannot be imposed during refinement, and so the subsequent grid is unlikely to exhibit the required boundary properties. To overcome this, the required boundary node locations are updated at the beginning of each mesh-smoothing iteration, and the process repeated until convergence to a suitable grid is achieved. Guards are placed on either side of the node being operated on to ensure grid-line crossover does not occur.

Grid contours of constant $\xi$, generated using elliptical refinement with Neumann boundary conditions, are shown in Fig. 3.13. The algorithm is able to generate a mesh capable of retaining orthogonality and minimising the level of cell skewness, even in regions of large curvature change such as at the leading edge. Simultaneously a bump with geometry characteristic of an SCB can be meshed with good orthogonality at the aerofoil boundary. The quantitative improvement in mesh quality relative to elliptical refinement alone is shown in Tab. 3.1. Reduced levels of both mean and maximum mesh non-orthogonality are observed, along with a reduction of cell skewness to 0.3, representing a mesh which passes all of OpenFOAM’s $\text{checkMesh}$ grid checks. The mesh generation algorithm has demonstrated its capability to generate a grid of good quality, regardless of the exact geometrical configuration. It is now possible to proceed with CFD analysis, with the knowledge
3.6. Design

Chapter 3. Aeroelastic Morphing Design

Figure 3.13: Contours of constant $\eta$ obtained via elliptical refinement with Neumann boundary conditions: (a) leading edge (b) example morphing shock control bump.

that all results will be obtained on grids of comparable quality, and hence provide reliable data for comparison during optimisation.

Aeroelasticity

The aerodynamic properties of the actuated geometry subject to the aforementioned flow, are evaluated via CFD. The drag and lift coefficients, $C_D$ and $C_L$, are evaluated via summation of pressure and viscous forces over the aerofoil surface. For example

$$C_D = C_{D,\text{pressure}} + C_{D,\text{viscous}}$$

(3.19)

where $C_{D,\text{pressure}}$ contains the costly wave-drag component. Whilst it is hoped to reduce this component via deployment of a morphing SCB, due to boundary-layer thickening during alteration of the aerofoil geometry, it is likely that $C_{D,\text{viscous}}$ will be increased. As such there is a trade-off between reducing wave drag, and increasing viscous drag. It is therefore sensible to minimise $C_D$, the net combination of both components of drag. An additional concern is that in controlling the shock, the lift distribution is compromised, resulting in a reduction of lift. In a real-world situation this would require a different angle of attack be selected, altering the shock-control effectiveness. It is therefore useful to maximise the ratio of lift to drag during optimisation, to ensure the required lift distribution is preserved during attempted shock control. The objective function for minimisation
is therefore defined as

\[ f(p_1, p_2, \delta_1, \delta_2, B_1, B_2) = \frac{C_D}{C_L} \quad (3.20) \]

which is evaluated from the geometry prescribed by the FE output described previously. The structural displacements for the upper surface in the range \(0.5 \leq \frac{x}{c} \leq 0.9\) are recombined with the RAE2822 aerofoil geometry, and a CFD grid generated via the mesh generation algorithm. This is then subjected to a flow of \(Ma = 0.75, Re = 6.5 \times 10^6\), and angle of attack \(\theta = 2.81^\circ\), to match the test condition described in Sec. 3.5. In order to facilitate a shape change under realistic actuation forces, it is necessary to increase the flexibility of the structure. When considering a morphing system such as the wing skin in the current case, this means reducing the thickness and Young’s modulus. However in reducing the stiffness, the skin is likely to respond differently to the aerodynamic forces. Although the structural and aerodynamic aspects have until now been treated...
independently, it is necessary to investigate how they will interact in order to gain a complete understanding of how the morphing SCB will behave in practice. To compute the aeroelasticity of the problem, a weak coupling of the structural and aerodynamic solvers is implemented. This is deemed sufficient as both aspects are treated as quasi-steady during their individual analysis. A study of transient effects during bump deployment is beyond the scope of this work, and would prohibit the optimisation process due to the complexity of a single function evaluation.

The structural model is loaded with the optimiser-defined actuation, together with pressure loading representing the suction forces present on the upper surface of the wing, which, for the initial aeroelastic iteration, are directly proportional to the $C_P$ distribution shown in Fig. 3.7(b). During subsequent aeroelastic iterations the pressure forces are evaluated from CFD, and are a direct function of the bump geometry. The values applied are representative of atmospheric flight at an altitude of approximately 10000 m above sea level. The inside of the aerofoil is assumed to be at the free-stream pressure $P_\infty$, therefore the actual pressure on the elastic part of the wing is calculated as the difference $P_{local} - P_\infty$. An iterative process solves until aerodynamic pressure force variations are no longer significant enough to affect the structure, and hence affect the flow. Figure 3.14 details a single function evaluation, from input of decision variables, through to output of aerodynamic properties. Aeroelastic convergence is defined using current and previous values of the objective function $f$. The aeroelastic loop (shown within the dashed lines in Fig. 3.14) will iterate until

$$\left|1 - \frac{f(current)}{f(previous)}\right| \leq 1 \times 10^{-4}.$$  \hfill (3.21)

Due to the large computational expense of a single function evaluation, which is dominated by the aerodynamic analysis, a number of measures are taken to speed up the optimisation process. OpenFOAM is well suited to parallelisation, especially when working with a structured grid. Domain decomposition is supported, allowing separate processors to operate on specific sections of the grid using MPI [143]. Imperial College London’s High Performance Computing (HPC) facility – a cluster comprised of Intel Xeon quad-core processors running at 2.8 GHz, with 1600 MHz FSB and 16 GB of RAM – was utilised for parallelisation. For the results shown here 16 processors were employed achieving a speed up of approximately 20 times relative to a single-core desktop machine (2.33 GHz, with 1300 MHz FSB and 4 GB of RAM). Due to the structured grid generation, comparable numbers of cells are generated regardless of bump size or shape. Therefore to reduce solution time further, the flow is initialised to the developed condition for the solved test case. This reduces the computation time by approximately half, as fewer transient effects are experienced.
Despite these efforts, function evaluation times are still relatively costly, taking an average of 12 hours to achieve aeroelastic convergence (comprising 3 aeroelastic iterations at approximately 4 hours each). The time taken to complete an aeroelastic iteration (one loop in Fig. 3.14) is split between the individual analysis stages, with approximately: 75% spent on CFD analysis, 20% used during mesh production, and the remaining 5% containing the FE analysis, and passing of data between the subsequent stages.

### 3.7 Optimisation

The nonlinear solution space and interconnected decision variables require optimisation to find the actuation distribution which minimises drag. Existing knowledge of SCBs is used to generate a starting point about which to search the design space – a shock bump of length $l \approx 0.3c$, with crest located at the intersection of the normal shock and aerofoil boundary. Decision variables are selected creating the bump shown in Fig. 3.15(a). On an aerofoil with 1 m chord length, this represents a bump length of 285 mm, bump height of 8.02 mm and ramp angle of 3.19°. The converged (aeroelastically) pressure distribution and contours of Mach number are displayed in Fig. 3.15. A $\lambda$-shock region has been created, with the front shock leg anchored to the initial
incline of the bump, and the rear normal shock located a short distance upstream of the bump crest. From Fig. 3.15(b) a small reduction in pressure is achieved in the \( \lambda \)-region, which remains constant until the normal shock wave returns the flow to subsonic conditions. This two-stage reduction in pressure causes improved pressure recovery, reducing the pressure-drag contribution by 3.2\%. A small region of re-expansion occurs as the flow travels over the downstream crest of the bump, causing the spike in pressure coefficient in Fig. 3.15(b). Local velocities do not reach sonic conditions during this expansion, so no wave-drag penalty is encountered. However, boundary-layer thickening does occur, causing an increase in viscous drag of 6.5\% relative to the clean RAE2822 aerofoil at test conditions. The overall net change in drag is a 0.3\% reduction. However due to the plateau region created in the \( \lambda \)-shock, the lifting surface is effectively extended, causing an increase in lift, and hence an increase in the \( C_L/C_D \) ratio to 29.058.

Due to the successful drag reduction of the initial distribution of decision variables given above, it is deemed sensible to search locally about this design point to refine the shock-control capabilities further. Therefore a gradient-based optimisation algorithm is employed, specifically the \textit{fmincon} function implemented in Matlab [105], to execute a constrained minimisation. The decision variables are bounded in the regions

\[
0.5 \leq \frac{x_i}{c} \leq 0.9 \\
0.25 \text{ MPa} \leq P_i \leq 1.5 \text{ MPa} \\
0.5 \leq \frac{B_1}{c} \leq 0.6 \\
0.8 \leq \frac{B_2}{c} \leq 0.9.
\]

(3.22)

to limit the solution to the localised region about the initial condition, and to prevent searching of non-optimal design space.

### 3.8 Results

Due to the computationally intensive nature of this optimisation, two iterations of the gradient-based optimisation algorithm were performed. The problem was seen to respond well to optimisation, encouraging further investigation in the use of morphing structures for transonic shock control. Aerodynamic, structural and aeroelastic considerations of results are given separately, followed by general comments on meshing and the optimisation process.
3.8. Results

Chapter 3. Aeroelastic Morphing Design

Figure 3.16: Distribution of (a) Mach number and (b) $C_P$ distribution after iteration 1; and Distribution of (c) Mach number and (d) $C_P$ distribution after iteration 2.

Aerodynamic

Contours of Mach number and $C_P$ distribution for the optimal configuration after each optimisation iteration are displayed in Fig. 3.16. After the first iteration, a stronger front shock leg relative to the initial condition had been created, again anchored at the initial ramp incline. The stronger oblique shock causes the plateau of pressure to be lower, resulting in a smaller pressure difference
Table 3.2: Optimisation results – aerodynamic attributes

3.8. Results

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$C_L$</th>
<th>$C_{D,\text{pressure}}$</th>
<th>$C_{D,\text{viscous}}$</th>
<th>$C_D$</th>
<th>$C_L/C_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean (No bump)</td>
<td>0.859</td>
<td>0.0222</td>
<td>0.0087</td>
<td>0.0309</td>
<td>27.800</td>
</tr>
<tr>
<td>Initial Condition</td>
<td>0.895</td>
<td>0.0215</td>
<td>0.0093</td>
<td>0.0308</td>
<td>29.058</td>
</tr>
<tr>
<td>Iteration 1</td>
<td>0.896</td>
<td>0.0209</td>
<td>0.0091</td>
<td>0.0300</td>
<td>29.867</td>
</tr>
<tr>
<td>Iteration 2</td>
<td>0.905</td>
<td>0.0206</td>
<td>0.0090</td>
<td>0.0296</td>
<td>30.574</td>
</tr>
</tbody>
</table>

across the normal shock as the flow returns to subsonic. This increases total pressure recovery, and causes a reduction of $C_{D,\text{pressure}}$ of 5.9%. However, as with the initial condition, after travelling over the crest of the bump, re-expansion occurs, and whilst the flow remains subsonic, it does contribute to boundary-layer thickening. Hence an increase in viscous drag relative to the clean condition is experienced, which limits the net reduction in drag to 2.9%. Similar phenomena occur after the second iteration, however the increased ramp angle and bump height allow for a greater $C_D$ reduction of 4.2%. This information is summarised in Tab. 3.2. The lift coefficient, which was increased relative to the clean case for the initial condition, remains relatively stable during the first optimisation iteration, but is increased by 1% after the second. Observation of the $C_P$ distribution shows that flow control is limited to the shock region, leaving the remainder of the flow unaffected. For the second iteration, the plateau of pressure after the initial shock leg is extended, leading to the more pronounced $\lambda$-shape visible in Fig. 3.16(c).

Structural

As mentioned during Sec. 3.3, the design of a morphing SCB is a multidisciplinary task, and whilst bump performance is fundamentally an aerodynamic consideration, bump shape is governed by structural properties. The defining bump dimensions: height, length and ramp angle, are detailed in Tab. 3.3, along with $\sigma_{\text{max}}$ for each configuration. From the initial condition the optimiser consistently increased ramp angle: doing so first by reducing bump length, and second by increasing actuation loads. Ramp angle is seen to have a significant influence on front shock-leg strength, and hence drag reduction, with a larger ramp angle causing an increased pressure jump. Constraint violation is disallowed during optimisation, ensuring all configurations result in feasible morphs. After the second iteration, $\sigma_{\text{max}}$ has been increased close to the constraint, indicating that a bump height of 8.38 mm is approaching the limit of SCB height achievable with a morphing structure of this type. The default rounded bump is an existing SCB developed at the University of Cambridge,
Table 3.3: Optimisation results – structural attributes

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Bump Length (mm)</th>
<th>Bump Height (mm)</th>
<th>Ramp Angle (°)</th>
<th>$\sigma_{\text{max}}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Condition</td>
<td>285</td>
<td>8.02</td>
<td>3.19</td>
<td>429</td>
</tr>
<tr>
<td>Iteration 1</td>
<td>262</td>
<td>7.59</td>
<td>3.25</td>
<td>385</td>
</tr>
<tr>
<td>Iteration 2</td>
<td>266</td>
<td>8.38</td>
<td>3.8</td>
<td>447</td>
</tr>
<tr>
<td>default rounded bump</td>
<td>118</td>
<td>4.8</td>
<td>5</td>
<td>–</td>
</tr>
</tbody>
</table>

and has demonstrated shock-control capabilities during experimental testing [29]. The morphing bumps shown here exceed the default rounded bump in terms of height and length, due to their application on an aerofoil of chord length 1 m. It is interesting to see comparable values of ramp angle have been generated by the morphing structures, indicating potential for improved shock control with further optimisation.

A recurring phenomenon downstream of the normal shock, was re-expansion as the flow travelled over the bump crest and further downstream. This can become a limiting factor if the flow is re-accelerated to sonic conditions. A sharp corner increases the risk of re-expansion, and hence it is desirable to avoid significant negative changes of curvature. However when limited to two actuation locations a conflict of interests arises between achieving the maximum ramp angle, and maintaining smooth contours. Increasing the number of actuation points would allow smoother structures to be generated, which is likely to help eliminate the problem of downstream re-expansion.

**Aeroelasticity**

Static aeroelasticity was seen to have only a small contribution during function evaluations. The majority of cases were seen to require three complete iterations for aeroelastic convergence. The change in pressure distribution, and resulting structures for the three iterations required for aeroelastic convergence of optimisation iteration 1 are shown in Fig. 3.17. A difference in both the pressure distribution and structure are visible between the first and second iterations, however the second and third iterations yield identical results indicating aeroelastic convergence. After the first aeroelastic iteration, the pressure distribution is updated to that given by the bifurcated shock structure. Therefore a small geometrical change is encountered when updating the FE model, as the structure is now subject to a stepped transition in pressure, as opposed to the single discontinuity of the test condition. The updated geometry is given by the solid line in Fig. 3.17(b). Only
Chapter 3. Aeroelastic Morphing Design

3.8. Results

Figure 3.17: Variations in (a) pressure distribution and (b) structural deformation, between subsequent aeroelastic iterations.

the structure around the bump crest is visible, in order to provide scaling suitable to observe the differences between aeroelastic iterations. Due to the extension of the supersonic region caused by the $\lambda$-shock, the upstream surface and part of the crest are subject to a reduction in pressure relative to the test condition. The result is a small increase in bump height relative to the previous iteration. From Fig. 3.17(a) it is observed this small change in height does not affect the shock-control capabilities, however it does smooth the small re-expansion peak at the normal shock, present during the first CFD analysis. No significant change was observed in the second aeroelastic iteration, and hence convergence was achieved after the third loop.

Meshing

A final point of interest is verification that the solution is independent of the mesh density. Therefore grids with fewer and more cells were generated (using the aforementioned mesh generation algorithm), and the same problem solved via CFD. The resulting data is shown in Tab. 3.4. Case A was generated using the mesh generation algorithm, and exhibits the density specified during optimisation. Mesh B is of comparable quality, however contains increased numbers of grid cells. Mesh C is also of comparable quality, but contains a reduction of grid cells. Mesh D contains the same number of cells as mesh A, however the Neumann orthogonality was omitted during generation, and hence a mesh similar to to Fig. 3.11(b) is produced. The grid exhibits good orthogonality
3.8. Results

<table>
<thead>
<tr>
<th></th>
<th>Number of Cells</th>
<th>Max non-Orth.</th>
<th>Mean non-Orth.</th>
<th>Max Skewness</th>
<th>$C_L/C_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>77000</td>
<td>23.32</td>
<td>5.82</td>
<td>0.34</td>
<td>30.54</td>
</tr>
<tr>
<td>B</td>
<td>82000</td>
<td>23.33</td>
<td>6.03</td>
<td>0.27</td>
<td>30.59</td>
</tr>
<tr>
<td>C</td>
<td>73000</td>
<td>23.33</td>
<td>5.79</td>
<td>0.34</td>
<td>30.51</td>
</tr>
<tr>
<td>D</td>
<td>77000</td>
<td>67.86</td>
<td>12.63</td>
<td>2.69</td>
<td>25.34</td>
</tr>
</tbody>
</table>

Table 3.4: Effects of mesh quality and density on aerodynamic properties

properties at the interior of the domain, however a high level of skewness and non-orthogonality is present at the aerofoil boundary. Small changes in the objective of less than 0.2 % were observed for changes in mesh density, but with comparable grid quality. A 20 % reduction in the objective-function value was obtained on a mesh of equal density, but reduced quality. This serves to highlight the importance of boundary orthogonality in capturing viscous flow effects.

**Optimisation**

The complexity and computational cost of a single function evaluation makes this an extremely difficult optimisation to solve. Two iterations of a gradient-based algorithm have been carried out on an initial condition with advantageous properties. Despite numerous efforts to improve computation time – such as parallelisation and non-zero flow initialisation – this process still took approximately 9 days. In addition to this already costly process, it would be desirable to increase the complexity of actuation to allow generation of smoother bumps, hence complicating the process further. The shock-control capabilities depend on a number of factors, such as ramp angle, bump height, and bump length. All of which were, to a certain extent, recognised by the optimisation algorithm. It is noted however, that placing a bump of optimal shape in a non-optimal location can cause detrimental flow properties, and hence cause an optimisation algorithm to move away from this region of the design space. In the current parameterisation of the morphing bump, under a gradient-based optimisation algorithm the effect of translating the bump is lost. Because the beginning and end points are set by variables $B_1$ and $B_2$, the shape of the bump changes as they are varied individually. It may therefore be beneficial to keep $B_1$, but to set $B_2 = B_1 + \ell$, where $\ell$ is the length of the bump, and replaces $B_2$ in the vector of decision variables. In doing so, during the optimisation iteration, translation of the bump without modifying its shape would be facilitated, perhaps improving optimal location of the SCB.

The objective function $f = C_D/C_L$ was seen to be sensitive to small changes in pressure dis-
tribution, potentially giving problems with convergence of the aeroelastic loop during function evaluation. Also, it is seen that all configurations led to an increase in lift, as well as a reduction in drag. Although reductions in wave drag were achieved, increasing lift was not the objective of this work, and hence it may be beneficial to select a more drag-oriented objective function. Measuring drag via downstream pressure recovery relative to the uncontrolled case would facilitate this, however the lift force would need monitoring in order to ensure a suitable design.

3.9 Conclusions

The key findings of this chapter investigating aeroelastic morphing design are summarised as follows:

- Morphing SCB has been designed, capable of 4.2\% drag reduction on RAE2822 aerofoil.
- Bespoke meshing algorithm generated for grid production, ensuring orthogonality and acceptable levels of skewness.
- Drag to lift ratio objective function responded well to a gradient-based optimisation, recognising key SCB parameters such as crest height and ramp angle.
- Aeroelastic fluid-structure interaction caused small structural deformations, which led to a smoothing of the re-expansion zone.

The optimal design of a morphing structure capable of reducing the effects of wave drag on the RAE2822 aerofoil in transonic flow has been demonstrated. A constrained minimisation has yielded a reduction in drag of 4.2\%, whilst retaining a comparable level of lift. Hence the Mach divergence number of the aerofoil has been raised, allowing the potential for increased flight speed with no increase in drag in the deployed configuration, and zero off-design penalty due to morphing capabilities.

The aerodynamic properties were evaluated using OpenFOAM’s \textit{sonicFoam} solver. Good comparison with experimental data in terms of pressure distribution, lift and drag properties was observed. Solutions were seen to be dependent on grid quality, and hence a dedicated mesh generation algorithm was developed to ensure results were not a function of the mesh during optimisation. A two-stage process was used: linear TFI to generate a grid exhibiting the required connectivity; followed by elliptical refinement including Neumann boundary conditions at the aerofoil surface for smoothing. The minimised solution was invariant under increased or reduced grid density, highlighting the necessity, and success, of the mesh generation process.
3.9. Conclusions

Conflicts of interest arose between achieving maximum bump height (and thus ramp angle), and obtaining a smooth transition between the crest and slope regions. Increasing the number of actuation locations is likely to improve both aspects, but at the expense of increased optimisation time. Despite this, morphing structures have been shown to create SCBs geometrically similar to those previously observed in the literature, although with the addition of key morphing functionality. Further optimisation of these structures is likely to increase their shock-control effectiveness, which, when combined with their zero off-design penalty, demonstrates a strong incentive for continued investigation of morphing SCBs.

Further Work

The aeroelastic optimisation has been solved, catering for fluid-structure interaction, and the integrity of the CFD solution has been preserved via a dedicated, but costly, mesh generation process. During optimisation it was seen that via deployment of the SCB, the aerofoil’s lift was altered at the same time as controlling drag. As mentioned in Sec. 3.3, a change of lift will require a change in effective angle of attack to remain at constant altitude. This change in effective angle of attack is likely to compromise the performance of the morphing SCB. Furthermore, this is for the case where the lift distribution is the same, but simply increased in magnitude. During deployment of the morphing SCB it was seen that the normal shock location moved downstream, and increased the size of the suction region on the aerofoil’s upper surface. This would have a significant effect on the aerofoil’s pitching moment, and hence aircraft performance. To ensure the aerofoil’s lift distribution (or moment distribution) is not compromised during SCB deployment, it is proposed to constrain $C_L$ so that the lift cannot change significantly during optimisation. This will restrict improvements in the objective function to come solely from reductions in drag, however the optimisation design space is likely to be more difficult to search due to the stringent constraints.

A second technique for overcoming this problem would be to apply the morphing bump to control the shock wave boundary layer interactions on a non-lifting surface. An example case would be the shock waves encountered on a scram-jet inlet. Undesirable shock interactions within the engine isolator can lead to flow separation and engine unstart. Using contoured bumps it is possible to control the reflected shocks, and improve engine efficiency [144]. A morphing bump would enable tailored performance levels over a larger operating envelope relative to static contours, and the subsequent design problem would not require simultaneous consideration of lift.

Whilst aeroelasticity must be considered in the final design stages of a morphing structure, it was
3.9. Conclusions

not seen to have a significant effect on the morphed shape. This is because the variations in pressure loading are two orders of magnitude less than those providing structural actuation. Therefore whilst aeroelasticity is crucial in terms of final SCB performance, it contributes a significant computation-time penalty to a single function evaluation during the early phases of design. It is thought that a better design route would omit the aeroelastic considerations until the final stages of the design process. This will speed up searching of the design space, and allow reintroduction of aeroelastic analysis when promising regions of solution space have been located.

Even with these improvements to the optimisation process, a subjective starting point is still required for initialisation of the search. This must be provided without any prior knowledge of the problem, which in terms of a morphing SCB, is lacking in the literature and is not intuitive. The similarity between the optimal morphing geometry and the default rounded bump – an existing static SCB in the literature – was noted during discussion of the results. It is therefore suggested to utilise successful SCB designs from the literature to reduce computation time. A more efficient design process could be obtained by attempting to morph to these specific geometries, prior to inclusion of aerodynamic and aeroelastic effects within the modelling stages of optimisation. This would provide an objective starting point for a final optimisation, and reduce the computational expense due to narrowing of the design space. This concept is considered in subsequent chapters: shape morphing of a shock control bump is investigated in two and three dimensions, followed by extension of the analysis to the design of a morphing leading-edge flap.
Chapter 4

2D Shape Morphing

4.1 Introduction

Structural optimisation via aerodynamic analysis is complicated greatly by material limitations and aeroelasticity. It is noted however, that there are many shapes whose aerodynamic properties are known – for example the range of NACA aerofoils [145]. Therefore, as a preliminary design step, it is possible to remove the aerodynamics from the morphing optimisation, and focus purely on the structural problem of changing from one predefined geometry to another. Once a good representation of the required structural morph has been obtained, it can be used as an objective starting point for a secondary aeroelastic optimisation. Not only will this remove subjectivity, it will also narrow the design space and avoid searching of sub-optimal regions.

This chapter investigates morphing of the default rounded bump, a successful SCB from the literature. A NURBS description of the morphing geometry is produced, along with a structural model for optimisation. Two objective functions quantifying morphing accuracy are developed, together with a constraint to maintain structural integrity. The reduced computation times of geometry-based objective functions facilitate exploration of the optimisation problem. Gradient-based and evolutionary methods are used to optimise the morphing problem, and the characteristics of each method, and their resulting morphing structures, are discussed. The optimal morphs are assessed according to the analysis in the previous chapter, and their ability to provide an objective starting point for an aeroelastic optimisation investigated. Finally the chapter is concluded by examining the concept of designing morphing geometry, in an effort to ensure morphing feasibility from an early stage in the design process.
4.2 Morphing Geometry Generation

As with the study outlined in Chap. 3, to find the optimal distribution of actuation to morph requires optimisation. In order to perform a geometry-based optimisation, a continuous representation of the morphing geometry is required. This involves specification of the initial structural shape, and the target shape that it is desired to be morphed to. Due to their versatility, and capability to represent almost all known curves [146], NURBS are used to define the geometry. Their ability to represent a variety of shapes means the techniques can be extended to a generic morphing problem, provided a NURBS description of the initial and target geometries can be obtained. It is interesting to investigate shape morphing of a shock control bump similar to that analysed in the previous chapter. The target morphing geometry is therefore defined by work carried out at the University of Cambridge on SCB performance [29]. The profile along the spanwise centre-line of the default rounded bump – shown in Fig. 3.4 – is chosen due to its benchmark status in the study. In the study a range of bump shapes are generated according to the parameterisation in Fig. 4.1; the values for the default rounded bump are detailed in Tab. 4.1. The target morphing geometry is generated via a NURBS-fitting optimisation, with range data provided by this parameterisation. Before demonstrating the curve-fitting procedure, it is first useful to understand the construction of a NURBS curve.

![Shock control bump parameterisation variables for default rounded bump](image)

Figure 4.1: Shock control bump parameterisation variables for default rounded bump [29]

<table>
<thead>
<tr>
<th>Bump Shape</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$b$</th>
<th>$c$</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$l_B$</th>
<th>$w_B$</th>
<th>$h_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default rounded bump</td>
<td>5°</td>
<td>12°</td>
<td>50</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>10</td>
<td>118</td>
<td>60</td>
<td>4.8</td>
</tr>
</tbody>
</table>

Table 4.1: 2D Parametrisation of default rounded bump centre line [29] (dimensions in mm unless otherwise stated)
4.2. Morphing Geometry Generation

NURBS

Non-Uniform Rational B-splines (NURBS) are a parametric representation of curves [147]. The same methodology can be extended to surfaces, and will be considered in the following chapter. They are most commonly described by the summation

\[
C(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u) w_i P_i}{\sum_{i=0}^{n} N_{i,p}(u) w_i} \quad a \leq u \leq b \tag{4.1}
\]

in which \( C(u) \) are the Cartesian coordinates of the curve, \( u \) is the location in knot-vector space, \( P_i \) the location of the \( i \)th control point, \( w_i \) the weight of the control point, and \( N_{i,p}(u) \) the basis function of degree \( p \) [148]. The control points are a series of \( n \) points, often described in Cartesian space, which influence the overall shape of the curve. If the control points are connected with a straight line in the order in which they have weight, the control polygon is produced. NURBS exhibit the strong convex hull property, which ensures the NURBS curve is always contained within the control polygon. The knot vector defines where, and for how long, each control point influences the shape of the curve. A *periodic uniform* knot vector has knots which are evenly spaced relative to one another, including the first and final knots. An *open uniform* knot vector has evenly spaced knots as before, however the first and final knots exhibit multiplicity – repetition of a knot – equal to the order of the NURBS. This ensures that the NURBS passes through the first and last control points. A *periodic non-uniform* knot vector has terms which are equally spaced, however they can exhibit multiplicity up to the order of the NURBS. A *non-uniform* knot vector has unequally spaced knots, and can also have multiplicity of any term in the knot vector. The non-uniform properties of the knot vector are what make a rational b-spline a *Non-Uniform Rational B-Spline* (NURBS). Once the knot vector is known, it is possible to create the set of basis functions from which the NURBS is constructed. It is common to scale the knot vector such that the first term is 0 and the last term is 1. This is advantageous as it causes Eqn. 4.1 to simplify to

\[
C(u) = \sum_{i=0}^{n} R_{i,p}(u) P_i \tag{4.2}
\]

in which \( R_{i,p}(u) \) are the rational basis functions. There are several methods for constructing the basis functions, here the following recurrence relation has been used [149]. Given a knot vector \( U \), where \( U = u_0, \ldots, u_m \) and \( u_i \leq u_{i+1} \), and where \( n \) is the number of control points and \( m = n - 1 \), the
4.2. Morphing Geometry Generation

Chapter 4. 2D Shape Morphing

The nth degree basis functions are generated according to

\[ N_{i,0}(u) = \begin{cases} 1 & \text{if } u_i \leq u \leq u_{i+1} \\ 0 & \text{otherwise} \end{cases} \]

\[ N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u) \]  

(4.3)

where the quotient \( \frac{0}{0} \) is, by convention, treated as zero. The order of the NURBS \( q \) and degree of the basis functions \( p \) are always related in the form \( q = p + 1 \). Therefore to produce a third-order NURBS, second-degree basis functions are required. An example NURBS curve is shown in Fig. 4.2. It is created using five control points, and an open uniform knot vector \( U \), where

\[ U = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 1 & 1 & 1 \end{bmatrix} \]

generating rational second-order basis functions. This causes a third-order NURBS to be produced – note that although a uniform knot vector is used, the curve is referred to as a NURBS throughout to avoid confusion. The NURBS, its control points and control polygon, are shown in Fig. 4.2(a); whilst (b) details the basis functions. The strong convex hull property is evident here, as only three basis functions have non-zero value at any one time, relating to the influence of three consecutive control points. Multiplicity of the first and last terms in the knot vector, equal to the order of the curve, causes the NURBS to pass through the first and final control points.
Shape-Fitting Optimisation

Curve fitting is performed via optimisation. Due to their formulation, there are a number of ways in which optimiser-controlled shape variations of NURBS can be achieved. For example: control-point location, knot distribution, and control-point weighting, all contribute to the overall NURBS geometry. Whilst it might seem straightforward to use the point-cloud data as control points, and include multiplicity to ensure the NURBS passes through each location, this would still generate an optimisation problem. Due to the nonlinear relationship between knot-vector space and arc length, it would be difficult to specify the required knots for the corresponding point-cloud locations. Therefore it is likely that closed regions of the curve will occur, as the NURBS is forced to pass through the control points. To produce an accurate shape fit, and an open curve, the knot locations would require optimisation. Definition of an objective function is difficult as the reference data has been included in the NURBS description, and hence a tangent-based argument would be required, which is elusive for a generic shape fit. Therefore, an open uniform knot vector is used, and the control-point locations altered, to enable optimiser-controlled shape variations. This facilitates comparison with the point-cloud data to quantify the shape-fit effectiveness. The shape-fit optimisation is shown schematically in Fig. 4.3. The control-point locations are fixed in $x$-space, and the $y$-coordinates are the decision variables controlling the curve shape. To form an objective function for minimisation, the NURBS height ($y$-coordinate) is compared with that of the point-cloud data at like $x$-locations. These differences are then combined according to the...
4.2. Morphing Geometry Generation

Chapter 4. 2D Shape Morphing

![Graph](image)

Figure 4.4: Effect of control point numbers on NURBS shape-fit accuracy

The root-mean-squared error

\[
G_y = \sqrt{\frac{\sum_{i=1}^{p} (y_i,NURBS - y_i,Cloud)^2}{p}}
\]  

(4.4)

where \( p \) is the length of the vector containing the point-cloud \( x \)-coordinates, and \( G_y \) the shape-error objective function for minimisation.

Although the bump length is known from the original parameterisation, the length of the morphing geometry is extended to allow freedom during the structural optimisation to relieve stress and improve the shape change. The NURBS curve length is therefore extended approximately half a bump length both fore and aft relative to the original parameterisation. The target data consists of 119 locations evaluated from the default rounded bump parameterisation. An objective starting point is set by assigning all control-point heights to zero, and \( G_y \) subsequently minimised using the gradient-based approach. The effect of control-point numbers on the optimal values of \( G_y \) is explored in Fig. 4.4(a), whilst (b) shows the shape-fit capabilities at regions of high curvature change. If too few control points are used, the optimised NURBS cannot capture the required shape, therefore increasing the number of control points is beneficial. However, as the number of control points is increased beyond the number of target point locations, the NURBS response is undesirable. Oversampling causes the point cloud to be fitted well, however the regions between the comparison points have large errors which go undetected by the objective function, as seen
4.3 Structural Modelling and Objective Functions

A structural optimisation is performed to find the optimum distribution of actuation which achieves the required shape change. Analytical methods are unable to cope with the large, and geometrically nonlinear, deflections of the structure. The structural problem is therefore solved using the commercial FE package Samcef [133]. A base model defining the initial morphing configuration is created according to Fig. 4.6. Actuation is defined by input from an optimisation algorithm, which can be varied to alter the shape of the structure. The displaced nodes are then used to calculate objective functions, quantifying the success of a particular morph. The objective functions will be dealt with in the following section, after a description of the FE model, its parameters, and synthesis with the optimisation algorithm.

The initial geometry is assigned material properties of aerospace grade 7075 (T6 temper) aluminium of $E = 72$ GPa and $\nu = 0.33$, and a uniform thickness of 0.4 mm. It is modelled using
4.3. Structural Modelling and Objective Functions

Chapter 4. 2D Shape Morphing

![Diagram of 2D Morphing SCB FE model and allocation of decision variables](image)

Figure 4.6: 2D Morphing SCB FE model and allocation of decision variables

Mindlin shell elements, of unit depth in the $z$-direction. Actuation consists of forces applied over a uniform area, to apply a pressure to the structure. Forces are applied as opposed to displacements, as a large displacement applied in close proximity to a small one, will cause high stress in the region between actuators. However, two pressure forces can coexist in close proximity, and may improve the shape-capture process by doing so. Actuation is optimiser-controlled via definition of the force magnitude $P_i$, and location $x_i$, where $i$ refers to the actuator number. Figure 4.6 displays the decision variables for two actuators. Separate optimisations with varying numbers of actuators are performed, to investigate the relationship between morphing accuracy and actuation complexity. Boundary conditions are applied to fix the geometry in space. Fully-fixed boundary conditions are applied at the bounding nodes at either end of the morphing geometry. As mentioned during geometry generation, the initial and target NURBS are increased in length relative to the original parameterisation. This allows the location of these bounding nodes to vary during optimisation, and hence control the location and stress state in the resulting morphing structure. The upstream bound is defined by $B_u$ and the downstream node by $B_d$, as shown in Fig. 4.6. The vector of decision variables for optimisation changes length according to the number of actuators, however it takes the general form

$$[B_u \ B_d \ x_1 \ P_1 \ x_i \ P_i \ x_n \ P_n ]$$ (4.5)
where the aforementioned variables are bound to the following intervals during optimisation

\[-0.05 \text{ m} \leq B_u \leq 0 \text{ m}\]
\[0.118 \text{ m} \leq B_d \leq 0.170 \text{ m}\]
\[0 \text{ m} \leq x_i \leq 0.118 \text{ m}\]
\[0 \text{ kN} \leq P_i \leq 7.5 \text{ kN}\]  

which are chosen to give freedom of mobility to all actuation points and bounds, and the required forces are those achievable with current actuator technology. As previously mentioned, actuation forces are applied over an area of fixed width in the spanwise $z$-direction. The effect of loading area on the optimal objective function is shown in Fig. 4.7. Load width influences the morph only at the two extremes. For loading-area sizes of $\leq 0.002$, stress concentrations are experienced which limit the shape-fitting capabilities. For larger loading areas, better distribution of stress is observed, however actuation locations are spread further apart, and hence cannot apply loading at the required locations. Therefore in the remainder of the chapter, a constant loading size of width $0.005 \text{ m}$ is selected, representing a trade-off between these two extremes.
Objective Functions

To enable optimisation, an objective function detailing the morphing quality of a given distribution of decision variables is required. This section discusses generation of two objective functions for this purpose, and the effect they have on the morphing optimisation.

Displacement-Based Objective

The first objective is evaluated in the same manner as the shape-fitting objective function used during geometry generation. By applying an assignment of decision variables, the displaced structure can be found. The displaced structure is then compared at like locations, with the NURBS defining the target morphing geometry. Due to the initial configuration being discretised during the structural analysis, the nodal locations in $x$-space are used as the comparison points. The target NURBS is evaluated at these $x$-locations giving the target height. This is then compared with the height attained by the displaced structure. The error over the entire structure is then combined according to

$$S_y = \sqrt{\frac{\sum_{i=1}^{p} (y_{NURBS}(x) - y_{FE}(x))^2}{p}}$$

where $p$ is the number of nodes in the FE model, and $S_y$ is the displacement-based shape-fit objective function.

Curvature-Based Objective

The second objective function is based on comparison of curvature. By minimising the difference in curvature between the target NURBS, and the actuated FE model, it is possible to ensure that the structure is bending accurately to the required shape. Curvature $\kappa$ is defined as the second-order differential of displacement with respect to arc length $s$ [150]

$$\kappa = \frac{d^2y}{ds^2} = \frac{d^2y}{dx^2}$$

however it is often approximated via differentiation in Cartesian space for simplicity. In this work it is not possible to make the assumption of small displacements, as the shape changes in morphing structures are by definition large. Moreover, to ensure that a generic process is created, it is desirable to calculate the exact curvature for a morph which does not begin in an initially flat configuration. It is difficult to evaluate $\kappa$ from differentiation, because NURBS are by definition only $C_1$ continuous at
the knots; therefore a geometrical technique is applied to calculate $\kappa$ numerically. Several methods exist for curvature calculation such as those demonstrated in [151] and [152]. During this analysis the osculating circle method is used, which estimates the curvature via generation of a circle with the same radius of curvature as a localised region of the curve. The inverse of this radius gives the local curvature. To construct the osculating circle, two points are evaluated from the NURBS description, either side of the point in question on the curve – as in Fig. 4.8. Using these three points it is possible to solve the general equation of a circle in Cartesian form

$$r^2 = (x - \alpha)^2 + (y - \beta)^2 = \left(\frac{1}{\kappa}\right)^2$$

$$\kappa = \frac{1}{r}$$

(4.9)

Figure 4.8: Curvature evaluation via osculating circle method

where $(\alpha, \beta)$ is the circle centre, and $r$ the radius. This radius is known as the radius of curvature, the inverse of which defines the curvature $\kappa$. Note that if the circle centre is above the curve (in Cartesian coordinates) then the curvature is denoted positive. The change in curvature between the two morphing configurations, $\Delta \kappa(x)$, can now be found as the difference in curvature at comparable locations in $x$-space, again the nodal locations are used for convenience. The target NURBS
4.3. Structural Modelling and Objective Functions

Figure 4.9: Target curvature distribution for default rounded bump

The curvature distribution $\kappa_{NURBS}(x)$ is found according to this method, and plotted in Fig. 4.9. Four regions of curvature are observed, linked by regions of zero curvature. Good comparison with the default rounded bump parameterisation is achieved in terms of curvature location and magnitude, however there is overshoot of the curvature magnitude at the transition points. This is due to the NURBS control points being located to ensure the correct curve location, and could be corrected through the use of additional data in the generation of the shape-fit objective function.

The curvature distribution of the structural model $\kappa_{FE}(x)$ is also found, again according to the osculating circle method. The differences at like $x$-locations are compared, and combined according the root-mean-squared error

$$S_\kappa = \sqrt{\frac{\sum_{i=1}^{p}(\kappa_{NURBS}(x) - \kappa_{FE}(x))^2}{p}}$$

in which $p$ is the number of nodes in the FE model, and $S_\kappa$ the curvature-based objective function for minimisation. Due to inaccuracies in the curvature distribution of the target NURBS – shown in Fig. 4.9 – for the remainder of the chapter $\kappa_{NURBS}$ is replaced with the distribution taken directly from the default rounded bump parameterisation according to Fig. 4.1 and Tab. 4.1. This ensures that the optimisation process is not compromised by the curvature error of the target NURBS.
4.3. Structural Modelling and Objective Functions

Constraint

Whilst achieving the desired shape change is the primary target of this work, it is also the intention to produce feasible designs, which can be constructed from standard engineering materials. The optimisation is therefore constrained to elastic morphs, by ensuring the maximum stress is below the yield strength of the material. The selected grade of aluminium has yield strength $\sigma_y = 505$ MPa, however to provide a margin of safety, a factor of $k = 10\%$ is incorporated according to

$$\sigma_{\text{max}} \leq \sigma_y (1 - k)$$

$$h = \sigma_{\text{max}} - \sigma_y (1 - k)$$

(4.11)

where $h$ is the constraint. When the maximum stress is greater than 454.5 MPa, $h$ becomes positive and the constraint is violated, indicating an unfeasible design.

Model Convergence

The constrained minimisation problem is now ready for optimisation. To ensure accurate and repeatable analysis is performed, convergence of the various modelling stages must be achieved. Reliable modelling of the structure is dependent on a suitable FE mesh being produced. The objective functions also make use of nodal locations, and hence these too are functions of the distribution and density of the FE mesh. For the case of the shock control bump, the initial morphing geometry is defined by a flat configuration. This is meshed easily, in a highly-repeatable manner during all optimisation function evaluations. Therefore the variable governing mesh quality is density, as the rectangular configuration means orthogonal elements are guaranteed. Increasing the density is a technique to improve the accuracy of the FE solution, but is done so at the expense of computation time. In the context of a morphing design optimisation, reduced computation times are desirable, due to solving repeatedly the structural problem. Therefore a mesh convergence study is performed to investigate the function evaluation time and solution accuracy trade-off. An example distribution of design variables is kept constant, whilst variations in mesh density are applied. The response of maximum stress, $S_y$, and maximum crest height, are shown in Fig. 4.10. Convergence of all variables is seen for meshes containing 8000 elements or more, and hence this is selected as the mesh density for use throughout the remainder of the chapter. Although slight changes of crest height occur for increasing density beyond this value, these are below 10 $\mu$m. In Chap. 3 it was
Figure 4.10: (a) Convergence of maximum stress, and (b) maximum bump height and $S_y$ objective function, with the number of FE mesh elements.

seen changes in bump height of this order, do not affect significantly aerodynamic performance, and are therefore deemed acceptable. With this level of accuracy in the nodal locations of the FE model, the objective functions $S_y$ and $S_\kappa$ can be evaluated to four significant figures. This is useful information as it enables a bound on the optimisation convergence criteria to be set. There is no benefit in looking for improvements in the objective function of less than $1 \times 10^{-7}$, i.e. improvements of less than 0.02%. Doing so would exploit the error present in the FE solution below this accuracy, ultimately incurring computational expense without a gain in morphing quality.

### 4.4 Optimisation

Reduced function evaluation times relative to the full aeroelastic optimisation enable exploration of the morphing optimisation process in detail. The constrained minimisation problem is solved for $n = 2$ to 6, and the optimum values of the objective functions $S_y$ and $S_\kappa$, as defined in Eqns. 4.7 and 4.10, are displayed in Figs. 4.11 and 4.14 respectively. To examine how the problem responds to optimisation, it is interesting to consider minimisation of the individual objective functions with respect to the different algorithms used.
4.4. Optimisation

Chapter 4. 2D Shape Morphing

\[ S_y \] – \textit{fmincon} Optimisation

The differential algorithm was implemented with the aforementioned bounds. In the interest of objectivity, the actuators were initialised to apply zero force, and were distributed evenly throughout the design domain. Maximum and minimum search-step sizes were set to facilitate steps of 10 mm and 0.2 mm in the spacial design variables, and 1 MPa and 0.02 MPa in the loading variables. Despite a relatively large maximum step-size with respect to the domain, actuator crossover did not occur during optimisation. This led to the zig-zag response of \( S_y \) with \( n \) for the gradient-based algorithm. For even numbers of actuation locations, the objective starting point caused actuation locations to be initialised in favourable locations with respect to the bump crest. This meant that two actuators were able to locate and actuate the bump crest successfully. During optimisation, loads applied at other locations did not improve \( S_y \), and hence remained at their initialised value. Therefore, even when \( n > 2 \), only two actuators provided forces leading to structural displacements. Figure 4.12 shows the convergence behaviour of the objective function and constraint with optimiser iterations for \( n = 2 \). Note that the constraint only becomes active when violated, hence a
value of zero is reported if a feasible design is produced. The zero-loading objective function is \( S_y = 2.1 \times 10^{-3} \) m, which corresponds to an average shape-morphing error of 2.1 mm at each comparison point. A large improvement is encountered after the first iteration, which is then steadily reduced until iteration 6. On the 7th iteration the constraint becomes active, as the objective-function improvement was achieved at the expense of exceeding the maximum allowable stress. For the remaining iterations the constraint and objective function oscillate as the optimiser locates the boundary of the feasible design-space. The optimal objective function is \( S_y = 0.53 \times 10^{-3} \) m, indicating the average morphing error is 0.53 mm at each comparison location. However, in reality this error is centralised in the region of the SCB crest, which will be demonstrated in Sec. 4.6 during examination of the optimal design.

Figure 4.12: Convergence of gradient-based minimisation of \( S_y \) for \( n = 2 \)
4.4. Optimisation

*S_y – GA Optimisation*

The GA is implemented on the problem with the same bounds. The optimal values of *S_y* for varying numbers of individuals are shown in Fig. 4.11. The factor governing GA performance was population size. To investigate its effect, the population size was set as the number of decision variables raised to sequential powers. The problem was therefore solved for \((2 + 2n)^i\) individuals for \(i = 1\) to \(3\), and for the number of generations required for convergence. For \(i = 1\) there was insufficient diversity in the initial population to produce results in the range of those achieved with the gradient-based algorithm, these values are therefore omitted from Fig. 4.11. For \(i = 2\) the results were improved, and for \(n = 2\) a value of \(S_y = 6.3734 \times 10^{-4}\) m was achieved. When considered relative to the zero-loading objective function, this represents an error reduction of \(69.7\%\), compared to \(75.1\%\) obtained with the gradient-based algorithm. As \(n\) increases, the GA predicts similar levels of optimality, except for \(n = 6\) where the optimum value of \(S_y = 7.2374 \times 10^{-4}\) m was found. When \(i = 3\) the GA was able to predict the optimal morph with similar accuracy to the gradient-based method. However, it did not suffer from conditioning problems during initialisation, and was therefore able to achieve improvements in \(S_y\) for \(n = 3\) and \(5\). The optimal decision variables are similar to those obtained with the gradient-based algorithm, and enable an error reduction of approximately \(74.5\%\) for \(n = 2\) to \(5\). As for \(i = 2\), the optimal value of \(S_y\) for \(n = 6\) is predicted less accurately, achieving an error reduction relative to the zero-loading objective function of \(73.8\%\). It is thought the reduced searching ability is a consequence of the increased number of decision variables for \(n = 6\).

During optimisation it was observed that relatively few generations were performed. This means that generation of new individuals through crossover reproduction did not lead to an improvement in the objective function. A plot detailing the objective-function development with optimisation iterations is shown in Fig. 4.13(a). The results shown are for \(n = 4\), and are representative of those obtained for all values of \(n\). Figure 4.13(a) shows that the number of generations for varying population size ranges from 3 to 5. For \(i = 1\), 2 and 3, the objective-function improvements between the first and final generations are \(2.9\%\), \(1.5\%\) and \(0.18\%\) respectively. This shows that little improvement in the objective function was obtained through crossover and mutation, and the fitness characteristics of the initial population were preserved through elitism. The predominant searching mode during optimisation was therefore due to the random initialisation of decision variables. Whilst results comparable with those achieved using the gradient-based algorithm were found, the number of function evaluations was greatly increased, making optimisation expensive.
The number of function evaluations required for convergence of all optimisation methods, for \( n = 4 \), is shown in Fig. 4.13(b). The efficiency of the gradient-based method is clear, with only 85 function evaluations required over 7 iterations to achieve convergence. The number of function evaluations completed whilst using the GA increases with population size. Whilst the number of function evaluations completed for \( i = 1 \) (273) was comparable with the gradient-based method, optimality was compromised, as shown in Fig. 4.13(a). The population defined when \( i = 2 \) represents a trade-off between optimality and computational expense. The objective-function fitness is approaching the level achieved using the gradient-based algorithm, and 1482 function evaluations were performed. For \( i = 3 \) the optimal solution was of comparable fitness with the gradient-based method for \( n = 2, 4 \) and 6, and exceeded it for \( n = 3 \) and 5. However, this consistency came at significant computational expense, as 5184 function evaluations were required for convergence when \( n = 4 \). During GA optimisations, constraint violation is prohibited. This means that during generation of the initial population, if an unfeasible solution is produced, it is discounted, and another generated in its place.
Likewise during reproduction if an unfeasible child solution is created. This zero-tolerance approach means that location of the feasible design-space boundary is difficult, and hence the optimiser is forced to operate without knowledge of its existence. In the gradient-based optimisation it was observed that the problem is constraint-bound, meaning that the optimal solution is limited by the stress constraint. The lack of tools within the GA for searching up to this boundary, led to a large population size being required to guarantee optimality of the final solution.

To summarise, both optimisation methods exhibit desirable features. The gradient-based method was able to search the design domain close to the boundary due to knowledge of the objective function and constraint gradients. It was able to do so relatively cheaply, however subjectivity in the initial conditions caused local minima to be found for $n = 3$ and 5. The GA is objective due to random initialisation of the design variables. However, achieving the same level of optimality as the gradient-based method incurred significant computational expense. By coupling the two methods in series, it is possible to harness the objectivity of the GA, and the efficient searching of the gradient-based method. The optimal results obtained with the GA for $i = 2$ are used as the initial condition for gradient-based optimisations over the full range of $n$. The resulting objective-function values are plotted in Fig. 4.11. Optimal solutions are obtained for $n = 2$ to 5, whilst a slight penalty was encountered for $n = 6$. This shows that: the use of the GA has overcome the conditioning problems encountered when using the gradient-based method individually; and that from this objective starting point, $f_{\text{mincon}}$ was able to locate the edge of the feasible design space, and find the optimal solution.

\textbf{$S_y$ – Actuation complexity ($n$)}

A relatively small improvement of the objective function is observed, for both optimisers, as the number of actuators is increased. Whilst it was expected that an increased number of actuators would lead to a reduction in stress for a given morph, stress concentrations were seen to limit SCB height. On the upper surface the locations of maximum stress were at the loading sites around the bump crest, whilst on the lower surface the maximum stresses were found at the fixed boundaries. Using extra actuators did not avoid these stress concentrations, and so the objective function did not improve with increasing $n$. The optimal morph found during these optimisations is therefore defined as that achieved with 2 actuators. A maximum crest height of 4.02 mm was achieved, which is 83.8\% of the target default rounded bump.
4.4. Optimisation

Chapter 4. 2D Shape Morphing

The same methods and bounds were applied to minimise $S_\kappa$. For the gradient-based method the optimal fitness appears to converge with increasing actuation complexity, as shown in Fig. 4.14. However, inspection of the optimal decision variables shows that, at most, two actuators were applied at any one time. The other forces remained at their initialised zero-load value. The optimal solution was found for $n = 6$, but employed only one actuator. It was found when the model was parameterised using six actuators, as this facilitated a starting point which avoided local minima during optimisation. The optimum objective function was $S_\kappa = 4.51$ m$^{-1}$, which is a reduction of 21% relative to the zero-loading objective function $S_\kappa = 5.69$ m$^{-1}$.

$S_\kappa - \textit{fmincon}$ Optimisation

GA performance was again governed by population size (see Fig. 4.14). The population was parameterised according to the number of decision variables, set by $(2 + 2n)^i$ for $i = 1$ to 3. The optimised morphs for $i = 1$ were unable to achieve a reduction of the objective function relative
4.4. Optimisation

Chapter 4. 2D Shape Morphing

Figure 4.15: GA child generation via crossover

Figure 4.15: GA child generation via crossover

to the zero-loading configuration, and hence are omitted from Fig. 4.14 for clarity. For \( i = 2 \), the optimal values were approaching those found with the gradient-based method for \( n = 2 \) to 4. For \( n = 5 \) and 6 however, insufficient population diversity was present to match the optimality achieved previously. For \( i = 3 \) the solutions were improved, and exceeded the differential algorithm for \( n = 2 \) and 3. As observed during minimisation of \( S_y \), the GA was unable to achieve significant reductions in \( S_x \) relative to the initial population. Therefore the predominant searching method of the GA was through the diversity in the initial population. This explains the improved performance for \( i = 3 \), however it is observed that insufficient diversity is present to match the performance of the gradient-based method for larger values of \( n \).

To investigate why this is the case, the operation of the GA must be examined in more detail. As shown in Fig. 4.13, the problem with the GA is that the solution is not improved during generation of the new population. Instead the optimal fitness is obtained during the initial creation, and preserved through elitism. Figure 4.15 details the crossover process for two example parent chromosomes. A range of crossover processes can be employed to generate a child solution from
two parent chromosomes. A child generated via the method known as double-point crossover is shown in Fig. 4.15(c). Double-point crossover selects two places at which to alter the parent from which a particular term in the chromosome is selected. For example, in Fig. 4.15, the two parents are

\[
\begin{align*}
  r_X &= [B_{ax} B_{dx} P_{1x} x_{1x} P_{2x} x_{2x}] \\
  r_Y &= [B_{ay} B_{dy} P_{1y} x_{1y} P_{2y} x_{2y}]
\end{align*}
\]

where the bold terms represent the design variables selected for the child solution. A schematic of the resulting morph is depicted in Fig. 4.15(c), which is a poor solution relative to the parents. Allowing actuators to move around the entire domain gives them freedom to occupy a different order in the chromosome. Whilst this ensures objectivity during searching, it makes generation of child solutions through crossover difficult. It may be possible to sort the chromosome during crossover, so that actuators closest to one another in design space are used to generate the child chromosome. Whilst this may preserve the parent qualities to an extent, it introduces subjectivity into the optimisation algorithm, and, the problem persists if two actuators are far from one another. Another option is to use a different crossover technique, such as the weighted averaging of the decision variables from both parents. This approach was employed in the current work in an attempt to combat poor GA performance. To generate the child decision variable, a line is constructed between the two parent variables. The new variable is then constructed a short distance along this line from the fittest parent. For example, if parent \( r_Y \) from the above example is the fitter of the two, the child chromosome \( r_c \) is given by

\[
r_c = r_X + R(r_Y - r_X)
\]

where the value of \( R \) determines the weighting, and for all optimisations was assigned the value \( R = 1.2 \). This causes a small step to be taken in the direction of the less-fit parent, thus enabling design-space searching whilst preserving the properties of the fitter parent. Unfortunately, in practice even this crossover method was seen to struggle, with the maximum objective-function reduction achieved via GA operation being the 2.9\% found during minimisation of \( S_y \) for \( n = 4 \).

Despite the poor performance achieved through crossover, the GA was still able to operate at comparable levels with the gradient-based optimiser, due to the diversity of the initial population. This ability to search the entire domain is advantageous when compared to the localised searching performed by the differential algorithm, as it encourages convergence to a global minimum. Therefore the optimal results achieved for \( i = 2 \) are used as starting points for a subsequent
4.4. Optimisation

Chapter 4. 2D Shape Morphing

Figure 4.16: Optimal curvature fit from $S_\kappa$ minimisation

gradient-based refinement. Figure 4.14 shows that this combination of optimisers outperforms the other methods for all values of $n$, except $n = 6$ for which it matches the objective function achieved via the differential method alone.

$S_\kappa – Actuation Complexity (n)$

Despite there being no optimal solution utilising more than two actuation points, it is interesting to note that the optimal value of $S_\kappa$ converged with increasing $n$. To investigate why this happens, the target curvature distribution, and that for the minimised value of $S_\kappa$, are plotted in Fig. 4.16. During minimisation of $S_\kappa$ it was observed that the maximum reduction relative to the zero-loading objective function was 21%. This means that the error with respect to the target shape has only been reduced by approximately a fifth. As seen previously, the constraint is active and limits any further improvement in morphing. Inspection of the required curvature distribution, and that achieved through optimisation of $S_\kappa$ provides an explanation as to why achieving this morph is difficult. The actuator loading is unable to provide the required curvature distribution. At the site of the single actuator, a smooth peak is developed to match the region of maximum negative curvature on the target distribution. As a reaction to this force, two regions of opposite-sign curvature are developed at the upper and downstream bounds. The fully-fixed boundary conditions here cause a curvature distribution reminiscent of that for a cantilever beam under a point load at the free end. Both bounding nodes are located such that these curvature peaks overlap the target curvature distribution in these regions. Hence the bump length remains fixed at 118 mm throughout optimisation. Any attempts to increase the length of the morphing geometry in the
hope of increasing crest height, are met with an increase in the curvature-fit error $S_{\kappa}$. The use of a single actuation point means that a flat bump crest cannot be achieved, which may have significant implications in terms of SCB shock control: this will be investigated in the following section.

**Summary**

The key findings from exploration of the shape-morphing optimisation problem are:

- The gradient-based optimisation is able to locate stress-constrained optimal morphs on the feasible design-space boundary, but is prone to locating local minima;
- The GA is able to search the entire design space, however to achieve optimality large numbers of individuals are required;
- A combination of the two methods is seen to be the most efficient manner in which to achieve optimal solutions for all values of $n$;
- Minimisation of $S_y$ achieved the maximum crest height of 4.02 mm using two actuators.
- Minimisation of $S_{\kappa}$ achieved a bump height of only 2.44 mm, however the bump length of the target geometry was respected.

The relatively low computational expense of the geometry-based objective functions has allowed exploration of the optimisation process for morphing structures. Gradient-based algorithms were seen to perform well when initialised to favourable starting points, however they were prone to finding local minima if not. The GA was unable to achieve objective-function reductions through crossover, however good searching characteristics were achieved with large population sizes. The computational-cost penalty was avoided by using a population size equal to the square of the number of decision variables, followed by a gradient-based refinement of the optimal solution. The differential algorithm is able to take advantage of this objective starting point, and, via knowledge of objective-function and constraint differentials, find the global minimum of the actively constrained optimisation problem.

The remainder of this chapter investigates the properties of the optimal morphing geometries. The aerodynamic and aeroelastic attributes are assessed according to the analysis in Chap. 3, and the structural modelling techniques are validated through realisation of the $S_y$-minimised morph in a mechanical demonstration model. Finally the geometry of the optimal and target morphs is investigated, with the aim of assessing and predicting morphing feasibility.
4.5 Aerodynamic Analysis of Optimal Morphs

A target of this chapter is to provide an objective starting point for an aeroelastic optimisation. The optimal designs obtained in this chapter via minimisation of the two objective functions $S_z$ and $S_\kappa$, are therefore evaluated in terms of their shock-control effectiveness. The optimal morphing structures are created on the upper surface of the RAE2822 aerofoil, and subjected to the same flow conditions used in Chap. 3. The initial condition of the morphs in this chapter was a flat plate, differing from the curved surface of the aerofoil. However in this analysis, the section on the rear half of the upper surface is treated as linear, and the same actuation and bounds optimised in the current chapter applied. To ensure the optimum performance of the shape-morphed bumps is recorded, their geometry is fixed, whilst their location along the surface of the wing is varied, and the aeroelastic function evaluation performed as in the previous chapter. The result of this chordwise sweep is shown in Fig. 4.17. The chordwise location of the initial ramp incline is plotted on the abscissa with the ordinate detailing the lift-to-drag ratio. Despite the reduced crest height relative to the bumps in the previous chapter, correct positioning of the shape-morphed designs
4.6 Demonstration Model Construction

produces comparable improvements in $C_L/C_D$. Optimal lift-to-drag ratios of 30.76 and 30.53 were achieved for the $S_y$ and $S_\kappa$ minimised geometry respectively. This shows that the ramp angle is the governing factor of bump performance, as despite their height differences, their ramp angles are comparable, leading to their similar operational performance. The increased height of the $S_y$-minimised SCB facilitates greater drag reduction than the $S_\kappa$-minimised SCB, and hence it is deemed the optimal design overall. The use of a single actuator in the $S_\kappa$-minimised SCB creates a small crest region, causing SCB performance to peak and then drop away sharply. Conversely the $S_y$-minimised bump has a longer crest, meaning that SCB performance is optimal for a range of chordwise locations. This is likely to be beneficial during operation, as SCB performance was seen to be strongly related to chordwise location of the normal shock wave.

This chordwise sweeping of the design space highlights the difficulty of the full aeroelastic optimisation problem. Optimising SCB geometry at the same time as finding its optimal location would be computationally expensive. Finding the optimal shape morph, and subsequently investigating the effect of chordwise location, has enabled a promising region of design space to be highlighted with minimal computational expenditure. The optimal morphing geometry is therefore defined as that achieved during $S_y$-minimisation, with the initial ramp incline located at $x/c = 0.6125$.

4.6 Demonstration Model Construction

To validate the structural analysis performed during optimisation, a physical demonstration model of the optimal morph is constructed. A morphing skin of 0.4 mm 7075 (T6 temper) aluminium is bonded to two aluminium blocks to apply the fixed boundary conditions, the length of which is specified by the optimal configuration. These components are then bolted to a base plate housing screw actuators to apply displacements, as shown in Fig. 4.18. Displacements are applied via line actuators, held in place via guide rails during loading. The actuators have constant width of 5 mm, and lightly-sanded corners to ensure no stress concentrations occur at their edges. Flat-topped actuators are deemed appropriate, as the curvature of the FE model at the upstream and downstream loading sites is $\kappa = 2.4 \text{ m}^{-1}$ and $13.02 \text{ m}^{-1}$ respectively. These values translate to radii of curvature of $r_\kappa = 0.417 \text{ m}$ and $0.077 \text{ m}$, which, when considered over a 5 mm section, are approximately flat.

Photographs of the demonstration model in operation are displayed in Fig. 4.19 in its (a) original, and (b) deployed, configurations. The bump is orientated in the same manner as shown
4.6. Demonstration Model Construction

Figure 4.18: Schematic of 2D morphing SCB demonstrator

previously: such that the flow would travel from left to right over its surface. Measurements of the displaced demonstration model are taken from these images, and converted to actual displacements via scaling. This enables comparison of the displaced demonstration-model shape with the FE model and target geometry, as seen in Fig. 4.20. As expected there is little variation between the demonstration-model measurements and the FE model. This shows that the structural analysis performed during optimisation is an accurate reflection of how such a model would behave in practice. The boundary conditions are captured via bonding the skin to the supports, however slight differences in the morphed shape occur at the site of the actuators. It is thought that the forces applied via the line actuators varies from the pressure applied in the FE model. In particular the contact made by the upstream actuator is not perpendicular to the morphing skin, and causes increased local bump height. Removal of all actuation causes the structure to return to the original flat configuration, demonstrating the repeatability of the morph, and further validating the structural analysis performed during optimisation.
4.6. Demonstration Model Construction

Figure 4.19: Optimal morphing geometry demonstration model: (a) initial configuration and (b) deployed configuration.

When compared to the default rounded bump NURBS, it is seen that the initial ramp and bump crest are positioned in similar locations on the morphing SCB. However the bump tail terminates at a location further downstream, and has reduced gradient relative to the target shape. The positions

![Diagram](image)

Figure 4.20: Optimal two-dimensional morphing geometry
of the bounding nodes have therefore located to increase overall bump length during optimisation. This increases the length of the morphing section, which in turn allows a larger length change to be actuated, which increases the maximum crest height. The gradient reduction of the tail may well alter SCB performance relative to that of the default rounded bump, however the increased bump height is likely to counter this, with improved drag reduction.

4.7 Analysis of Morphing Geometry

During the structural optimisation it was observed that the target shape of the default rounded bump was unattainable using the specified material. Fortunately, the morphing bump achieved a good representation of the target geometry, and hence fulfilled its function as an initial condition for an aeroelastic optimisation. This does, however, pose the question of what should be done if the achievable morph is not a good representation of the target shape. Respecting the specified material limitations means that extra deformation of the structure to improve the morph is not an option. It is therefore useful to investigate the features of the shape change which prohibit morphing. If this can be achieved through inspection of the morphing geometry, then it will enable the feasibility of the morph to be judged \textit{a priori}. Not only will this remove unnecessary analysis, it may also offer a technique for tailoring of the initial and target geometry to ensure morphing feasibility.

Using geometry to assess the feasibility of a morph defined by initial and target geometry was investigated in [108]. A morph was deemed feasible if the maximum geometrical bending strain or axial strain, did not individually violate the material limitations. This section begins with individual analysis of these two components of strain.

Change in Curve Length

A change in length between the initial and target morphing configurations will impart membrane strain. By definition the streamwise length of the initial morphing configuration is $L_i = 240$ mm. By evaluating the NURBS curve defining the target shape at locations along its length, it is possible to generate a piecewise-linear version of the geometry. Providing the discretisation of the NURBS is sufficiently fine, the length of the NURBS defining the morphing geometry $L_m$, can be found via summation of the individual piecewise-linear components. Therefore

\[
L_m = \sum_{i=1}^{n} \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}
\]  

(4.13)
where the points \((x_i, y_i)\) and \((x_{i+1}, y_{i+1})\) are the curve coordinates, evaluated at consecutive locations in arc-length space \(s\). By assuming the morphed geometry is equal to the length of the bump in the \(x\)-direction, that is \(L_B = L_i - 122 = 118\) mm, the strain of the morphed geometry due to the increase in length, can be found according to

\[
\epsilon_{\Delta L} = \frac{L_m - L_i}{L_B} = 0.508\%.
\]  

(4.14)

This value can now be compared to the theoretical maximum strain allowable for the specified material,

\[
\epsilon_y = \frac{\sigma_y}{E} = \frac{505\text{ MPa}}{72\text{ GPa}} = 0.701\%.
\]  

(4.15)

where \(E\) is the Young’s modulus, and \(\sigma_{\text{max}}\) the yield stress. This shows that the in-plane strain induced by the change in length is within the limits of the material. However, there are additional strains present in the morph due to bending, which can be considered via the geometrical quantity curvature.

### Change in Curvature

Using Euler-Bernoulli beam theory, the relationship between displacements and a range of loading cases can be determined [134]. This is based on finding the bending moment, or shear force, distribution along the length of a slender beam, and integrating with appropriate boundary conditions. To perform the analysis it is assumed that cross-sections must remain plane, and the resulting deflections are small. For slender beams, these assumptions hold due to the lack of significant shear stresses, and hence the results perform well when compared to experiments [153]. Due to the lack of shear, the bending stiffness arises due to compressive and tensile stiffness in the plane of the beam. Here the cross-section is assumed constant, and hence the curvature \(\kappa\) is directly proportional to the bending moment according to

\[
M = EI\kappa = EI\frac{d^2y}{dx^2}
\]  

(4.16)

where for an initially straight beam the curvature is often approximated by the second-order differential of displacement \(d^2y/dx^2\). However, the curvature calculated according to the osculating circle method is the exact value, which enables replacements of the approximate differential with the exact one

\[
M = EI\kappa = EI\frac{d^2y}{ds^2}
\]  

(4.17)

where \(s\) is the distance along the curve. When calculated from this exact equation, the shape of the deflected curve is known as the elastica [150]. It is now useful to consider bending of a small
4.7. Analysis of Morphing Geometry

Chapter 4. 2D Shape Morphing

Figure 4.21: (a) Relation of in-plane strain to curvature, and (b) cross-sectional distribution of strain, for a beam section.

section of beam described by this expression, such as that in Fig. 4.21(a). The radius of curvature $r$ is the reciprocal of curvature $\kappa$, and the neutral axis, the line along which the section length $l$ does not change, passes through the centroid of the section at $t/2$. The new length of the section’s upper surface as a result of bending is therefore defined as $l + \delta l$, where

$$ l = \frac{\theta}{\kappa} \quad \text{and} \quad l + \delta l = \theta \left( \frac{1}{\kappa} + y \right) $$

and the strain due to bending $\epsilon_{\kappa}$ is therefore

$$ \epsilon_{\kappa} = \frac{\delta l}{l} = \frac{\theta y}{l} = \kappa y $$

$$ \epsilon_{\kappa} = \kappa y \tag{4.18} $$

which is a maximum for $y = t/2$. This is easily developed into the more commonly referred to relation for predicting maximum stress

$$ \sigma = E \epsilon_{\kappa} = E \kappa y $$

$$ M = EI \kappa \quad \frac{M}{I} = E \kappa $$

$$ \sigma = \frac{My}{I} \tag{4.19} $$

from the bending moment $M$, and the distance from the neutral axis $y$. The thickness is a key parameter in defining the bending strain, as this places material further from the neutral axis, increasing bending resistance. Assuming a constant thickness of $t = 0.4$ mm, and an inextensional
4.7. Analysis of Morphing Geometry

Chapter 4. 2D Shape Morphing

<table>
<thead>
<tr>
<th>Geometry</th>
<th>$\epsilon_{\Delta L}$ (%)</th>
<th>$\epsilon_\kappa$ (%)</th>
<th>Total Strain ($\epsilon_{\Delta L} + \epsilon_\kappa$) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min $S_y$, $n = 2$</td>
<td>0.173</td>
<td>0.422</td>
<td>0.595</td>
</tr>
<tr>
<td>Min $S_\kappa$, $n = 1$</td>
<td>0.105</td>
<td>0.415</td>
<td>0.520</td>
</tr>
<tr>
<td>default rounded bump</td>
<td>0.508</td>
<td>0.416</td>
<td>0.924</td>
</tr>
</tbody>
</table>

Table 4.2: Geometrical analysis of morphing geometry

neutral axis, the maximum curvature change achievable in the elastic range is

$$\kappa_{\text{max}} = \frac{0.00701}{t/2} = 35.05 \text{ m}^{-1} \tag{4.20}$$

which is greater than the maximum curvature change between the initial and target morphing configurations of the default rounded bump ($\Delta \kappa = 20.8 \text{ m}^{-1}$).

**Superposition**

Individual analysis of these components of strain for the default rounded bump ($\epsilon_{\Delta L} = 0.508 \%$ and $\epsilon_\kappa = 0.416 \%$) would lead to the conclusion that the morph is feasible, as both values are below the yield strain $\epsilon_y = 0.701 \%$. However, it was proven earlier in Sec. 4.4 that this is not the case.

Examining the stress (or strain) distribution for one of the optimal morphs found previously, it is seen that stress concentrations occur at the loading sites (Fig. 4.22(a)&(b)). However, between these loading sites, the stress and strain are approximately constant. It is therefore assumed that the stress throughout the structure is due to in-plane extension, i.e. change in length, and is distributed uniformly throughout the structure. This means that the stress concentrations are caused by regions of localised curvature change, i.e. stresses due to bending. By assuming the problem is linear, the two components of strain can be combined via superposition. These assumptions enable prediction of the maximum strain present in a morph, simply via analysis of the initial and target morphing geometries. The optimal geometries from minimisation of $S_y$ and $S_\kappa$ are used to validate these assumptions. The strain due to change in curve length $\epsilon_{\Delta L}$, and the maximum strain due to bending $\epsilon_\kappa$, are evaluated according to Eqn. 4.14 and 4.18 respectively. The results are shown in Tab. 4.2, for the optimal morphs for the two objective functions, whilst the calculated values from the FE models are displayed in Tab. 4.3. For both shape-morphing cases, the total strain is dominated by the contribution of strain due to bending. It is seen that the strain experienced in the FE model is represented well using the superposition of the geometrical components. For the $S_y$-minimised morph, the maximum strain is underestimated by 4.2 %, and for the $S_\kappa$-minimised
Figure 4.22: Von Mises stress distributions for (a) $S_y$-minimised optimal geometry and (b) $S_\kappa$-minimised optimal geometry; and (c) associated $\kappa$-distributions.
4.7. Analysis of Morphing Geometry

<table>
<thead>
<tr>
<th>FE model</th>
<th>Upper surface $\epsilon_{\text{max}}$ (%)</th>
<th>Mid-plane $\epsilon$ (%)</th>
<th>Lower surface $\epsilon_{\text{max}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min $S_y$, $n = 2$</td>
<td>0.423</td>
<td>0.169</td>
<td>0.620</td>
</tr>
<tr>
<td>Min $S_\kappa$, $n = 1$</td>
<td>0.329</td>
<td>0.103</td>
<td>0.554</td>
</tr>
</tbody>
</table>

Table 4.3: FE analysis of morphing geometry

morph by 6.5 %. The lack of shear and bending-extension coupling in the geometrical analysis is thought to be the reason these values are consistently underestimated. Comparison of the structural curvature response – shown in Fig. 4.22(c) – with the associated stress distribution, shows that the two quantities are related closely. The stress levels on the upper, middle and lower surfaces are seen to follow the curvature distributions, with the side in tension contributing the maximum stress.

This analysis can now be extended to the case of the default rounded bump. Geometrical analysis indicates that to morph exactly the default rounded bump, a strain of 0.924 % would be experienced. Therefore a material of equivalent stiffness and safety factor (10 %), would require a yield strength of 728.9 MPa to enable morphing. At present, such a material that is suitable for use as a wing skin does not exist, making morphing of the default rounded bump in this manner impossible.
4.8 Conclusions

During this chapter, shape morphing of a two-dimensional structure has been investigated. Initial and target geometries were prescribed, and the actuation required to effect the structural shape change between the two sought. The key findings are summarised as follows, and discussed in detail below:

- NURBS computational geometry used to represent the initial and target geometries.
- Displacement and curvature-based objective functions developed to assess shape-morphing accuracy of structures actuated via optimiser-controlled loads and boundary conditions.
- Optimal results found using a GA with $(2 + 2n)^2$ individuals for initialisation, followed by gradient-based refinement.
- Morphing SCB problem is constraint bound, limiting crest height to 83.8% of the target.
- Aeroelastic analysis of optimal shape-morphed structures showed net drag reductions of over 3%.
- Structural analysis validated through construction of physical model realising the optimal morph, good correlation with numerical models was observed.
- Geometric analysis developed and applied to optimal and target morphs. Prediction of the magnitude and location of the maximum stress was achieved within 6.5% using only geometrical considerations. Demonstrated infeasibility of morphing the default rounded bump using current materials.

Morphing geometry was generated via fitting a NURBS to data generated from the parameterisation of the default rounded bump. It was seen that NURBS are well suited to this task as their parametric makeup enables a large range of shapes to be produced. The quality of the NURBS geometry was therefore seen to be a function of the amount of data to be fit. Specifically, increased sampling is required in range data around regions of large curvature, to ensure the NURBS responds correctly. Starting from an objective, linear, configuration, a gradient-based optimiser performed shape-fitting to a high level of accuracy. The continuous convex solution space enabled good convergence behaviour for prescribed geometry.

Structural shape-morphing was performed via minimisation of two objective functions, calculated from the displacements of an optimiser-controlled structure. A displacement-based objective
function compared the height of the actuated structure to that of the target geometry, and a curvature-based objective function compared curvatures at like locations. Structural displacements were calculated using a nonlinear, quasi-static, FE model employing Mindlin shell elements. Actuation location, magnitude, and bounding locations were optimiser-controlled, and a stress constraint was applied to ensure production of repeatable elastic morphs.

Reduced function evaluation times relative to the full aeroelastic analysis in the previous chapter allowed exploration of the optimisation process in detail. Gradient-based optimisations were able to locate the boundary of the feasible design space, and discovered that the problem was constraint bound. Their knowledge of the objective-function and constraint differentials enabled optimum solutions, which lie on the feasible design-space boundary, to be found. Objective initialisation was ensured by uniform distribution of actuators within the design space, and initialising all actuation forces to zero. Whilst this performed well for even numbers of actuators, when odd numbers were used the solution was seen to converge to sub-optimal local minima. An evolutionary method was tested in the form of a GA. Performance was seen to be proportional to population size for both objective functions. Populations of \((2+2n)^3\) individuals were seen to replicate the optimality of the gradient-based methods, however significant computational expense was incurred due to the large number of required function evaluations. Investigation of GA operation showed a lack of objective-function improvement through crossover. The random ordering of the actuators within the design space made crossover difficult, despite the use of a heuristic approach, which generated a child chromosome a short distance in design space from the best-fitting parent. Good GA performance was therefore due to favourable regions of design space being located during creation of the initial population. The suggested optimisation method for the shape-morphing problem is therefore: a Monte Carlo search, containing approximately \(r^2\) function evaluations (where \(r\) is the number of decision variables), to create an objective starting point; followed by a gradient-based refinement of the most promising design-space regions.

The \(S_p\)-minimised geometry utilised two actuators, and had a crest height of 4.02 mm, 83.8 % of the target NURBS height. The objective function enabled extension of the bump length to achieve maximum crest height, whilst respecting the stress constraint. The \(S_\kappa\)-minimised geometry used a single actuator and had a crest height of 2.44 mm. Bump length was kept constant at 118 mm, which limited the maximum crest height achievable. However, any movement of the bounding nodes during optimisation resulted in an increase in \(S_\kappa\), which was not countered by the increase in bump height. Both objective functions were seen to obtain little benefit from increased actuation
4.8. Conclusions

complexity, as localised stress concentrations at the morphing extremities were unavoidable through the use of additional actuation.

The two optimal designs are assessed aerodynamically, to test their ability at initialising a subsequent aeroelastic optimisation. The morphing structures are recreated on the RAE2822 aerofoil, and subjected to the same flow conditions described in Chap. 3. To ensure maximum performance is recorded, a chordwise sweep of the optimal geometry is executed. The use of two actuators on the $S_y$-minimised geometry produced a longer crest, which reduced the sensitivity of the morphing SCB to chordwise location. The $S_\kappa$-minimised SCB peaked sharply in performance, achieving a drag reduction of 3.1% relative to 3.6% by the $S_y$-minimised morphing geometry. It is therefore concluded that the displacement-based objective function provides the best starting point for a full aeroelastic optimisation, with the initial ramp incline located at $\frac{x}{c} = 0.6125$.

To verify the structural analysis performed during optimisation, a physical demonstration model was constructed of the $S_y$-minimised geometry. Repeatable elastic morphing was achieved, and the loading and boundary conditions were replicated well. Photographs were taken, and scaled measurements used to record the demonstration-model displacements. These correlated well with the FE data, giving validation to the employed modelling techniques, and to the concept of a morphing SCB.

The chapter concluded by investigating the contribution morphing geometry can play in the design of morphing structures. Changes in length and curvature between the initial geometry and optimal solutions are assessed, and used to predict the structural stress distribution. Linear superposition of the two geometrical strain components predicted the maximum stress present in the $S_y$-minimised morph to within 4.2%, and the $S_\kappa$-minimised morph to within 6.5%. Furthermore it enabled accurate prediction of stress concentrations, both in terms of location and magnitude. In the structural optimisation it was observed that the default rounded bump could not be morphed accurately using conventional materials and the available actuation. Performing the aforementioned geometrical analysis on the default rounded bump geometry shows that the morphing-skin material would require at least a 16.7% higher yield stress to enable accurate morphing without violating the material constraint. This geometrical analysis relies on the assumptions of beam theory, and hence lacks the accuracy of a full FE solution. However, the computational expense is extremely low, meaning it could be used to help characterise the design space during the early phases of morphing-structure design. For example, during generation of possible morphing geometries, this analysis could help ensure that a feasible morph is proposed. The typical curvature response
4.8. Conclusions

of different actuation types will detail the level of local curvature control that can be obtained during morphing, and hence will enable increased understanding of morphing feasibility during the conceptual design phases. Finally knowledge of stress-concentration locations may allow localised tailoring of the structure. For example, regions where large bending moments are experienced could be parameterised with variable-thickness elements controlled by the optimiser. It would be expensive to parameterise the entire structure in this manner, so the geometrical analysis would provide objective information, enabling reduced optimisation times and a refined, but objective, initial condition.
Chapter 5

3D Shape Morphing

5.1 Introduction

The previous chapter introduced the concept of two-dimensional shape morphing. Focusing on geometry allowed reduced computation times, morphing feasibility to be estimated \textit{a priori}, and estimations on optimum actuation locations and areas of maximum stress to be discovered. This in turn defined an objective starting point, and reduced design space, for a subsequent aeroelastic optimisation. This chapter extends the concept of shape morphing to three dimensions, to investigate whether the same benefits can be achieved. The chapter begins by introducing example morphing geometry, and creating a definition suitable for morphing analysis. A model is then set up for optimisation using FE to compute displacements from optimiser-defined actuation. Two objective functions are investigated to quantify morphing effectiveness, and their behaviour discussed with respect to the FE mesh. A two-stage optimisation strategy is employed to tackle the highly nonlinear and interconnected problem, and results are presented detailing the optimal solution. Finally post-processing is performed, and the optimal solution realised in the form of a physical demonstration model.

5.2 Morphing Geometry Generation

As in the previous chapter, a computational description of both the initial and target geometry is required to perform shape-morphing analysis. Again, NURBS definitions are chosen for their versatility; and for consistency, the target geometry is the \textit{default rounded bump} SCB [29].

A NURBS surface is constructed in a similar manner to a NURBS curve. A three-dimensional
5.2. Morphing Geometry Generation

Chapter 5. 3D Shape Morphing

![Figure 5.1: SCB geometry parametrisation [29]](image)

Table 5.1: 3D Parametrisation of default rounded bump [29] (dimensions in mm unless otherwise stated)

<table>
<thead>
<tr>
<th>Bump Shape</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$b$</th>
<th>$c$</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$l_B$</th>
<th>$w_B$</th>
<th>$h_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default rounded bump</td>
<td>5°</td>
<td>12°</td>
<td>50</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>10</td>
<td>118</td>
<td>60</td>
<td>4.8</td>
</tr>
</tbody>
</table>

The net of control points $P_{i,j}$ is used, as opposed to a series of planar control points defining a control polygon. Two perpendicular knot vectors $U$ and $V$, are used to specify how the control net influences the surface, again through the construction of basis functions. If rational basis functions $R_{i,j}(u,v)$ are constructed according to Eqn. 4.3, the surface coordinates $S(u,v)$ at the location $(u,v)$ in knot-vector space, are expressed as

$$S(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} R_{i,j}(u,v) P_{i,j} \quad (5.1)$$

where $m$ and $n$ define the order of the basis functions, and hence the order of $S(u,v)$.

The default rounded bump SCB geometry again forms the target morphing configuration. In the previous chapter, only the profile of the bump centre-line in the streamwise direction was morphed, however here a morph of the full three-dimensional shape is sought. The various bump shapes, tested at the University of Cambridge in the referenced study, are parameterised according to Fig. 5.1. The individual parameters are again chosen for the default rounded bump, which are
5.2. Morphing Geometry Generation

Chapter 5. 3D Shape Morphing

detailed in Tab. 5.1. In addition to this data, the following relations are also required to constrain the geometry: \( a_1 = e_1, a_4 = e_3, e_1 = w_1/2, e_2 = w_2/2, e_3 = w_3/2, r_1 = a_1/\tan \theta_1, r_2 = a_2'/\tan \theta_1, r_3 = a_3'/\tan \theta_2, r_4 = a_4'/\tan \theta_2, \) where \( a_2' = a_2/(1 + \cos \theta_1), \) and \( a_3' = a_3/(1 + \cos \theta_1). \) Finally, the flanks of the bump are defined using cosine functions, to ensure smooth junctions to the surrounding geometry. This parameterisation enables a point cloud defining the bump shape to be produced. An optimisation is then performed to fit a NURBS surface to the point-cloud data. To allow freedom in the structural morphing problem, the NURBS surface is extended half a bump length fore and aft in the streamwise direction, and a bump width in the spanwise direction. Symmetry is observed along the SCB centre line in the streamwise direction. Therefore the morphing geometry is restricted to one half of the SCB to reduce computational cost: both during geometry generation, and the subsequent structural optimisation. A 2nd order NURBS surface is created with 49 control points in the streamwise \((u)\) direction, and 19 in the spanwise \((v)\) direction of the bump. A uniformly-weighted knot vector is used to generate basis functions, with equally spaced knots in both the \(u\) and \(v\) directions. The control points are fixed, except for a rectangular region bounded by \(0 \leq x \leq 120\) and \(0 \leq y \leq 30,\) where the bump is known to have non-zero height. Here the control points are fixed in their \(x,y\) locations, but are assigned a \(z\)-value from an optimiser-controlled decision variable. It follows that there are 7 control points in the spanwise direction, and 25 in the streamwise direction, giving a total of 175 decision variables for optimisation. The decision variables are bound in the region \(-0.01 \leq z \leq 0.01,\) giving freedom to create the desired shape. The control points located on the line of symmetry are constrained to lie on the target geometry, the height of which is known from the parameterisation. Due to multiplicity in the knot vector, the NURBS will pass through these control points, removing the need to optimise their location.

An objective function is required to quantify the NURBS’s ability to represent the target shape. The resulting NURBS from a given configuration of decision variables is evaluated at the same \(x,y\) locations defined in the target point cloud. The difference between the target height and the actual height of the NURBS is then evaluated, and combined over the entire geometry according to

\[
G_z = \sqrt{\sum_{i=1}^{p} (z_{i,NURBS} - z_{i,Cloud})^2} / p
\]  

(5.2)

where \(z_{i,Cloud}\) is the target location and \(z_{i,NURBS}\) is the evaluated NURBS height at the same \(x,y\) location. When \(G_z = 0,\) the point cloud has been represented exactly by the NURBS. The optimised target NURBS achieved a value of \(G_z = 3.54 \times 10^{-5}\) m, and is displayed in Fig. 5.2. This NURBS
5.3 Structural Modelling and Objective Functions

A morph between two arbitrary surfaces, in the most general case, requires continuous actuation from an infinite number of points over the entire surface. In reality this is extremely complex to implement. It is therefore desirable to reduce this in a systematic manner, to enable a morph using a discrete number of actuation locations. Individual morphs for \( n \) actuation points are considered, and their ability to create the desired shape change with the minimum stress is quantified. Due to the large design space and interconnected decision variables, optimisation is required to find the optimum locations for a given number of actuators [132]. The optimisation is defined as

\[
\min (S_z, S_K) \quad \text{(shape change error)}
\]

\[
\min (n) \quad \text{(number of actuation points)}
\]

s.t.

\[
0 < \delta_{xyz} < \Omega_{xyz} \quad \text{(domain boundary)}
\]

\[
h \leq 0 \quad \text{(strain constraint)}
\]

Figure 5.2: Default rounded bump NURBS: (a) spanwise, (b) streamwise and (c) 3D perspectives.

forms the target geometry definition during the remainder of the chapter.
5.3. Structural Modelling and Objective Functions

Figure 5.3: Initial morphing configuration FE model, shown with \( n = 1 \) actuation point.

The derivation of the decision variables, objective functions \((S_z \text{ and } S_K)\), and constraint \((h)\), will be dealt with in the following sections, but first the three-dimensional structural model and its synthesis with the optimiser is introduced.

**Structural Modelling**

The target morph exhibits large displacements, and is likely to require multiple actuators, meaning analytical methods cannot model accurately the structural response. Therefore nonlinear FE analysis is implemented to predict the structural displacements resulting from optimiser-controlled loading. The geometry prescribed for the FE analysis is that of the initial morphing configuration – a flat plate \( 0.24 \times 0.09 \) m. The morphing SCB design is to be constructed from aerospace grade 7075 (T6 temper) aluminium sheet. Therefore a Young’s modulus of \( E = 72 \) GPa, Poisson’s ratio of \( \nu = 0.33 \), and uniform thickness of \( t = 0.4 \) mm are assigned. Actuation is provided by forces in the \( z \)-direction, applied at \( n \) locations, over a circular area of diameter \( d = 5 \) mm. Forces are favoured over displacements, due to the high-stress state caused by applying a large and small displacement in close proximity to one another. The magnitude of the force applied at the \( i \)th actuation location is specified through the variable \( P_i \). This force is centred at \((x_i, y_i)\), as displayed in Fig. 5.3. Only \((x_i, y_i)\) values are required, as there is a unique \( z_i \) value for each \((x_i, y_i)\) combination, which is
evaluated from the NURBS description of the initial morphing configuration. The bounds enforced on the spatial locations and loads are the same for all actuation points and are as follows

\[
\begin{align*}
0 \text{ m} & \leq x_i \leq 0.12 \text{ m} \\
0.05 \text{ m} & \leq y_i \leq 0.09 \text{ m} \\
0.2 \text{ kN} & \leq P_{i,z} \leq 2 \text{ kN}
\end{align*}
\] (5.3)

The actuation locations are limited to \((x_i, y_i)\) values where the target geometry is known to have non-zero height. The load bounds ensure that all actuators have an effect on the structure – avoiding the scenario where zero force is applied – but limit the actuation forces to those achievable with current actuator technology.

Fully-fixed boundary conditions are used around the bump perimeter, as it is thought the morphing SCB will be incorporated into a wing panel. A rectangular plate is unlikely to provide the optimal boundary to achieve a morph to the target shape. Therefore it is sensible to allow the outer boundary conditions to vary within the optimisation. In order to do this an optimiser-controlled NURBS curve is used to form the boundary, as shown in Fig. 5.3. A second-order NURBS curve, constructed using four control points and a knot vector of uniform weighting \([0 0 0 0.5 1 1 1]\), is deemed capable of capturing any benefits arising from a contoured boundary. The NURBS curve is constructed using six optimisation decision variables. Due to the symmetry of the design domain, two variables are required to set the distance along the centre-line of the start \((C_s)\), and end \((C_e)\), of the bounding curve. Additionally, two pairs of decision variables, \((C_{1,x}, C_{1,y})\) and \((C_{2,x}, C_{2,y})\), are used to locate the NURBS control points. The bounds imposed on these variables during optimisation are

\[
\begin{align*}
-0.05 \text{ m} & \leq C_s \leq 0.01 \text{ m} \\
-0.05 \text{ m} & \leq C_{1,x} \leq 0.05 \text{ m} \\
0.07 \text{ m} & \leq C_{2,x} \leq 0.17 \text{ m} \\
0.01 \text{ m} & \leq C_{1,y} \text{ & } C_{2,y} \leq 0.08 \text{ m} \\
0.11 \text{ m} & \leq C_e \leq 0.17 \text{ m}
\end{align*}
\] (5.4)

which are again governed by the dimensions of the initial morphing geometry. Note the specification of different bounds on \(C_{1,x}\) and \(C_{2,x}\) in order to stop the two control points becoming coincident and voiding the NURBS generation. Combining the loading decision variables with those defining
the boundary curve, the array of decision variables assigned during optimisation is

\[
[C_s \ C_{1,x} \ C_{1,y} \ C_{2,x} \ C_{2,y} \ C_e \ x_i \ y_i \ P_{iz}]
\]  

(5.5)

Once assigned, the Samcef [133] commercial FE package is used to compute the resulting structural displacements. A quasi-static nonlinear solver with Newton-Raphson automatic time stepping is used. After completion, the deflected geometry is exported to the Matlab [105] computing environment, where objective functions are evaluated to assess the quality of an individual morph.

**Objective Function Generation**

This section describes the derivation of two geometry-based objective functions for minimisation. To build on the previous chapter’s work on two-dimensional shape morphing, these are based on displacement and curvature comparisons.

**S_z – Displacement Comparison**

To represent the computed deflections as an objective function it is necessary to compare them to those of the target morphing shape. Discrete comparison points are projected onto both surfaces and distances from a relative datum plane are calculated and compared. For convenience the \((x, y)\) coordinates of each node are used to project a point onto the target NURBS surface, as shown in Fig. 5.4. Once projected, the difference in \(z\)-coordinate of the actual and projected geometries is combined according to the root-mean-squared error

\[
S_z = \sqrt{\frac{\sum_{i=1}^{p} (z_{i,proj} - z_{i,FE})^2}{p}}
\]

(5.6)

where for the \(i\)th comparison point located at \((x_i, y_i)\), \(z_{i,proj}\) is the distance from the datum plane to the target surface, \(z_{i,FE}\) the corresponding value for the deformed surface, and \(p\) the total number of comparison points (the number of nodes).

**S_K – Gaussian Curvature Comparison**

The motivation for this second objective function is the knowledge that aerodynamically-clean shapes – smooth surfaces without sharp edges or spikes – are often advantageous when a structure is to be placed in a flow. Therefore the second objective function uses Gaussian curvature \(K\) as
5.3 Structural Modelling and Objective Functions

Figure 5.4: Comparison of deflected and target shapes using point projection

its argument, which is a measure of how a surface is bent in three dimensions [154]. Minimising the difference in Gaussian curvature $\Delta K$ at like locations on the target geometry and actuated FE model, will ensure they are both curved in the same manner, and that the target morph is achieved. In order to formulate the objective function, it is first necessary to discuss the generation of a Gauss map – evaluation of the $K$-distribution over a surface.

The calculation of $K$ for a surface with a continuous analytical description is relatively straightforward, and is evaluated through differential geometry [155]. However, because the versatility of NURBS is required to describe the initial and target morphing configurations, it is unlikely that an analytical surface can be constructed to fit the entire data exactly. Therefore it is necessary to calculate local values of $K$, from a series of discrete data points lying on the surface. Fortunately this is not a new problem – the conversion of discrete data to a smooth surface is a task often encountered in image processing and object recognition. For example, this allows the response of an automated robot to be produced according to terrain it is passing over, or a three-dimensional
5.3. Structural Modelling and Objective Functions

Chapter 5. 3D Shape Morphing

Figure 5.5: Fitting a small surface to a node and its surrounding nodes using linear regression model to be created from a range of sensor data on which surgeons could learn from prior to surgery [156]. A summary of the basic techniques for calculation of $K$ from range data is shown in [157]. The generic problem is confined to a graph surface representation, where a surface is described in two dimensions $(u, v)$, by its height above a reference plane $f(u, v)$. Such a surface is known as a Monge patch, and has coordinates of the form $(u, v, f(u, v))$. The Gaussian curvature $K$ of the patch, is evaluated from differential geometry [155], giving

$$K = \frac{f_{uu}f_{vv} - f_{uv}^2}{(1 + f_u^2 + f_v^2)}$$

where $f_u$ and $f_{uu}$ are the first and second-order partial derivatives with respect to $u$, respectively.

There are several other methods for $K$ calculation, which do not rely on surface fitting and evaluation of partial derivatives. These include: tensor voting [158], three-dimensional mesh generation [159], the method of surface normals [160], and normal vector voting [161]. These are particularly useful when range data is noisy, and local surface representations are difficult to produce [157].

The connectivity of the FE mesh, and NURBS parameterisation of the target geometry, make the current problem suited to a surface-fitting approach. Although the range-data curvature is complex when considered as a whole, locally, the surface is modelled accurately using second-order polynomials. The problem is therefore reduced to obtaining localised descriptions of the surfaces. Once a local representation has been found, its first and second-order derivatives can be combined to calculate the local value of $K$. The process can then be repeated over the entire surface to
generate the Gauss map. Several surface-fitting options exist, such as the method of orthogonal polynomials – a surface fitting technique that uses a series of separable convolution operations to generate orthogonal curves which form a surface [156]. In the current work the surface fit is performed using linear regression, following the method demonstrated in [162]. The range data is approximated using a Monge patch, constructed from a second-order surface of the form

\[ f(u, v) = b_{20}u^2 + b_{02}v^2 + b_{11}uv + b_{10}u + b_{01}v + b_{00} \]  

(5.8)

where the coefficients \( b_{ij} \) are found via regression [163]. To fit complex data, it is typical to choose the origin at the point of investigation so that \( f(u, v) = 0 \), and the tangent plane is orientated in the \((u, v)\) plane such that \( f'(u, v) = 0 \). The first and second-order partial derivatives with respect to \( u \) and \( v \) are

\[
  f_u = 2b_{20}u + b_{11}v + b_{10} \\
  f_v = 2b_{02}v + b_{11}u + b_{01} \\
  f_{uu} = 2b_{20} \\
  f_{vv} = 2b_{02} \\
  f_{uv} = b_{11}
\]  

(5.9)

which are combined according to Eqn. 5.7 to calculate \( K \). The surface-fitting process is shown in Fig. 5.5, for a series of nodes in the FE mesh. Once a Gauss map has been generated for the FE data, like locations are projected onto the target morphing geometry, and the process repeated to obtain a Gauss-map for the target morphing configuration. The difference in \( K \) at like locations on the two maps is then computed, and summed over the entire domain according to the root-mean-squared error

\[
  S_K = \sqrt{\frac{1}{p} \sum_{i=1}^{p} (K_{i,\text{Target}} - K_{i,\text{FE}})^2}
\]  

(5.11)

where \( S_K \) is the Gaussian curvature-based objective function for minimisation.

**Actuation Complexity**

As in Chap. 4, the effect of actuation complexity is investigated. Increasing the number of actuators is likely to increase the accuracy of the morph, but also the complexity of the system. The number of actuation locations \( n \), is varied during optimisation to discover the trade-off between morphing accuracy and complexity. Because this value is also a decision variable within the optimisation, it is difficult to minimise this value together with the shape-fitting objective functions simultaneously. It is therefore necessary to perform multiple optimisations with sequentially increasing values of \( n \), and measure the benefit of increasing the complexity of actuation. Here the number of actuation points is investigated in the range \( 1 \leq n \leq 8 \).
5.3. Structural Modelling and Objective Functions

Constraint

To ensure repeatable morphs, the constraint used in previous chapters is implemented; its definition is repeated here for clarity. The von Mises stress is calculated for every element in the FE model according to

\[ \sigma_{eq} = \sqrt{\left(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\right)\frac{1}{2}} \]  

(5.12)

where \( \sigma_1 \) and \( \sigma_2 \) are an element’s principal stresses. The maximum value is found by taking the infinity norm of these values

\[ \sigma_{max} = \|\sigma_{eq}\|_\infty \]  

(5.13)

The optimisation algorithm requires the constraint be of the form \( h \leq 0 \). Therefore to limit the maximum allowable stress to be below the yield stress, the constraint becomes

\[ h = \sigma_{max} - \sigma_y(1 - k) \]  

\[ h \leq 0 \]  

(5.14)

where \( \sigma_y \) is the yield stress of 7075 (T6 temper) aluminium \( \sigma_y = 505 \text{ MPa} \) [50], and \( k \) is a safety factor of 10%. Hence \( \sigma_{max} \leq 454.5 \text{ MPa} \) in order to satisfy the constraint.

Convergence

In order to produce an optimal design it is necessary to ensure convergence of the individual steps in the solution process. Specifically: convergence of the structural solver, convergence of the objective function, and convergence of the optimisation itself. Convergence of the optimiser is defined by the tolerances allocated in the solver properties, and is discussed in the optimisation section. Convergence of the FE solver is investigated through a mesh refinement study, the results of which are displayed in Fig. 5.6. A user-defined SCB, representative of a typical optimisation function evaluation, was created as a benchmark case for convergence tests. It uses 6 actuation points to achieve a bump height of 1.4 mm, and \( \sigma_{max} \approx 375 \text{ MPa} \). The constraint value is used to measure convergence of the FE model with respect to the number of mesh elements. Figure 5.6(a) shows that \( \sigma_{max} \) for the upper and lower surfaces plateaus for meshes containing \( \geq 30000 \) elements. This corresponds to an average function evaluation time of 1000 seconds, as shown in Fig. 5.6(b).

The objective functions must also be independent of the FE mesh. This will ensure that the actuation distribution is optimised, and not the scenario which generates a mesh producing an
5.3. Structural Modelling and Objective Functions

Figure 5.6: (a) Convergence of $\sigma_{\text{max}}$ with the number of mesh elements, and (b) the effect of mesh refinement on function evaluation time.

Figure 5.7: Convergence of (a) displacement-based objective function, and (b) the Gaussian curvature objective function, with the number of mesh elements.
optimal result. Therefore the effect of mesh refinement on both $S_z$ and $S_K$ is investigated, and displayed in Fig. 5.7. The objective functions are calculated from structural displacements, and hence converges with behaviour similar to that of the maximum stress. This means that a mesh of $\geq 30000$ elements will ensure reliable objective-function values are passed to the optimiser.

Due to the manner in which the Gaussian curvature is calculated – fitting a surface to a point and its surrounding nodes, then calculating and combining its derivatives – refining the mesh will influence the data made available to fit the surface, and hence calculate $S_K$. Increasing mesh density will increase the number of nodes in localised regions. Therefore when the algorithm searches for a point’s nearest nodes during the surface-fitting process, for refined meshes, the nearest neighbours will be closer to the point of investigation. This should give a more accurate reflection of what is happening geometrically at a localised region of the surface. In order to verify this, the mesh refinement was fixed at the maximum level (approximately 50000 cells), and the number of nearest neighbours used in $K$ calculations varied. The resulting convergence curve is plotted in Fig. 5.8. The maximum objective-function value recorded increases as the number of nodes used to fit the surface is reduced. For large numbers of nodes the surface is smoothed, causing the value of $S_K$ to tend towards a stable value. Therefore the most accurate representation of $S_K$ is found using small numbers of nearest neighbours. Although this value increases for less than 20 nearest neighbours

![Figure 5.8](image.png)

Figure 5.8: Convergence of the Gaussian curvature objective function with the number of nearest neighbours used for surface generation during $K$ evaluation
which is due to the node-fitting limitations of a second-order surface – consistency between subsequent function evaluations is achieved by fixing the number of nearest neighbours throughout optimisation. This value is set at 10, as using fewer than this encounters conditioning problems during objective-function generation.

5.4 Optimisation

Nonlinear and discontinuous design space make optimisation complex. Additionally it is difficult to specify an objective starting point without prior knowledge of the problem. Therefore a Monte Carlo approach is used for a preliminary search of the design space. This provides an objective starting point, from which a gradient-based algorithm is employed to refine the solution.

Preliminary Optimisation

Matlab’s random number function is used to generate a vector of random numbers equal in length to the vector of decision variables. These operate on the bounding interval to generate a vector of decision variables for optimisation. The decision-variable bounds are set to prevent searching of known non-optimal design space. As previously mentioned, the actuation locations are limited to \((x_i, y_i)\) values where the target geometry is known to have non-zero height, and the forces applied \(0.2 \text{kN} \leq P_i \leq 2 \text{kN}\) (see Eqn. 5.3). This ensures that all actuation points have an effect on the geometry simultaneously, and limits the required forces to those achievable in a demonstration model. The NURBS control points have the freedom to move around the entire geometry, but are ordered to ensure an open curve is produced. However different bounds are set on \(C_{1,x}\) and \(C_{2,x}\) in order to stop the two control points becoming coincident and voiding the NURBS generation. The problem is optimised for sequential numbers of actuation points in the range \(1 \leq n \leq 8\). In the previous chapter it was observed that \(r^2\) individuals were required to provide sufficient diversity to characterise the design space. The same rule of scaling is therefore applied during the Monte Carlo optimisation, with the number of function evaluations for \(n\) actuation points given by \(f_{\text{eval}} = (3n + 6)^2\).

The improvement of the optimal objective-function values with \(n\) is shown in Fig. 5.9: (a) shows the optimal values of \(S_z\) with the corresponding value of \(S_K\); and (b) the optimal value of \(S_K\) with the corresponding value of \(S_z\). Note that both objective functions are normalised against the zero-loading objective function, meaning an objective function of less than 1 shows an improved morph.
5.4. Optimisation

Chapter 5. 3D Shape Morphing

Figure 5.9: Convergence of optimal objective functions with $n$: (a) optimal $S_z$ with corresponding $S_K$, and (b) Optimal $S_K$ with corresponding $S_z$.

over the initial configuration. The solid line in Fig. 5.9(a) shows $S_z$ decreasing as the number of actuation points is increased. However the corresponding curvature-based objective function, shown with the dashed line, appears unaffected by actuation complexity. Figure 5.9(b) displays the optimal values of $S_K$ (solid line) together with their corresponding $S_z$ values (dashed line). Again it is observed that the curvature-based argument offers little variation throughout the Monte Carlo search of the design space. Improvements of $\approx 2\%$ are observed for $n = 3$ and 5, however there is no significant change in $S_K$ with the number of actuations points.

To investigate why negligible variation of $S_K$ is observed, a Gauss map comparing the change in $K$ between the initial and target morphing configurations is evaluated, and plotted in Fig. 5.10. For the target geometry, the $K$ distribution is dominated by several discontinuities of curvature. There are two positive peaks, which correspond to the domed regions at the top of the bump, and four negative peaks defining saddle curves where the bump transitions to the flat regions at its outer edge. Along the line of symmetry, the surface has zero curvature in the spanwise direction, leading to a value of $K = 0$ at this location as expected. The $K$-distribution achieved via actuation – shown in Fig. 5.11 – contains similar peaks of Gaussian curvature, however they are much lower in magnitude than those of the target shape. Furthermore, significant $K$-variations are limited to regions local to the actuators. This shows that the structure resists changes in $K$, and that achieving $K$ changes in regions even a short distance from an actuator is difficult.
The relationship between Gaussian curvature and in-plane strain is developed in [154]. A small surface is considered in \( x, y \) coordinates, and the Gaussian curvature imparted from surface strain derived through analysis of the angular defect at the vertex and its associated area. The individual strains \( \varepsilon_x, \varepsilon_y \) and \( \gamma_{xy} \), have individual \( K \) contributions of

\[
K = -\frac{\partial^2\varepsilon_x}{\partial y^2} \text{ (due to } \varepsilon_x) \quad K = -\frac{\partial^2\varepsilon_y}{\partial x^2} \text{ (due to } \varepsilon_y) \quad K = \frac{\partial^2\gamma_{xy}}{\partial y \partial x} \text{ (due to } \gamma_{xy}) \quad (5.15)
\]

Figure 5.10: \( \Delta K \) between initial and target morphs: (a) top, (b) spanwise, (c) streamwise and (d) 3D perspectives.
5.4. Optimisation

Chapter 5. 3D Shape Morphing

Figure 5.11: $\Delta K$ achieved via the optimum actuation configuration for $n = 6$ during the Monte Carlo initialisation: (a) top, (b) spanwise, (c) streamwise and (d) 3D perspectives.

which can be combined via superposition to give the total Gaussian curvature imparted

$$
\Delta K = -\frac{\partial^2 \varepsilon_y}{\partial x^2} + \frac{\partial \gamma_{xy}}{\partial x \partial y} - \frac{\partial^2 \varepsilon_y}{\partial x^2}
$$

(5.16)

which applies to shallow, initially-developable shells. If $\Delta K = 0$, then the above relationship becomes the plane-strain compatibility condition. Whilst this relationship is unlikely to hold for a generic case containing large strains, it does provide insight into the relationship between plane deformations and changes in Gaussian curvature. Creating dome and saddle regions will always impart in-plane strains on the surface, the magnitude of which will depend on the principal radii.
of curvature, and any other extensions or contractions applied to the surface. It was seen during investigation of two-dimensional shape morphing, that morphs containing large in-plane strains are difficult to achieve due to material limitations. This problem is compounded when considered in three-dimensions, as changes of Gaussian curvature (non-developable morphs) contain extra strain components, further limiting the available deformation for a given stress level. This is reflected in the 2.12 mm crest height of the three-dimensional morphing SCB, compared to that of 4.02 mm achieved with the two-dimensional optimisation. Furthermore, it is seen that discrete actuators are limited to applying localised changes in Gaussian curvature, and hence localised stress concentrations occur close to actuation points. Actuating over a large surface area, for example via an increase in air pressure, may increase the global Gaussian curvature change available. However, the maximum change in $K$, and hence the ability to create a complex shape such as the default rounded bump, will always be constrained by material limitations.

### Final Optimisation

The gradient-based algorithm is used to search locally about the results obtained during the Monte Carlo optimisation. From the preliminary domain search, $n = 6$ was found to be the optimum trade-off between actuation complexity and $S_z$ minimisation, however for completeness all values of $n$ are refined. The bounds and constraint on the optimisation are the same as in the previous Monte Carlo search. The objective-function improvement due to the gradient-based refinement is displayed in Fig. 5.12. For the $n = 1$ and 2 cases, the Monte Carlo search produced comparable levels of optimality with the gradient-based method. However for $n \geq 3$ improvements were observed. As seen in the previous chapter, the gradient-based optimisation allows searching of the feasible design-space boundary. This enables the optimum objective function to be discovered for the stress-constrained problem. The optimal solution is chosen for $n = 6$; this has two actuators located on the axis of symmetry, which enables reduced actuation complexity during realisation of the model in practice. This morph exhibits a maximum crest height of 2.12 mm. The effect of the variable boundary curve is displayed in Fig. 5.13. Its evolution throughout the gradient-based refinement is displayed for $n = 6$. In the spanwise direction the curve bounds the morphing geometry in a position close to where the target geometry begins to have height. However the morphing region is extended in the streamwise direction to enable a larger morphing length, and hence larger displacements to occur for like strains. This results in a higher bump crest, and better fitting of the objective function.
Figure 5.12: Objective-function improvement after gradient-based refinement

Figure 5.13: Evolution of NURBS bounding curve during gradient-based refinement for $n = 6$
5.5 Demonstration Model Construction

A physical model of the optimal three-dimensional morph was constructed to demonstrate functionality of the morphing design process. A schematic of the three-dimensional bump assembly is shown in Fig. 5.14. A morphing skin, of the required 7075 (T6 temper) aluminium and thickness 0.4 mm, is attached to a base plate to apply the optimised displacements and boundary conditions. Displacements are provided via screw actuators, in the form of M5 and M10 bolts. To protect against indentation of the morphing skin, these are given flat tops and an edge radius equal to the depth of the thread. These actuators are screwed into a base plate of 5 mm thick 6061 aluminium, to provide the actuation displacements. To apply the complex curvature of the optimised bounding curve, a negative of the curve was cut from a sheet of 2 mm 6061 aluminium. To capture the curve exactly, a CNC machine with an input file generated from the optimal NURBS curve was utilised. This negative was bonded to both the base plate and the skin material to provide the necessary clamping conditions present in the optimised FE model. To ensure a good bond was achieved around the entire curve, the surfaces were degreased and then abraded to provide a key for the adhesive. Araldite 2015 was used for its aluminium-aluminium bond shear strength of 16 MPa.

![Figure 5.14: Schematic of model construction](image-url)
5.5. Demonstration Model Construction

Figure 5.15: Optimal morphing geometry demonstration model: (a) unactuated spanwise configuration (b) actuated spanwise configuration, (c) unactuated streamwise configuration and (d) actuated streamwise configuration.

[164]. A final degreasing was performed before glue was applied, and the parts placed in a vacuum table to remove any air from the bonded joint. Non-stick tape was used to ensure glue did not spread onto the skin material and consequently affect its stiffness.

Spanwise and streamwise images of the model in its original and deployed configurations are shown in Fig. 5.15. A fluorescent back drop is used to reflect light off the bump flanks and ramp to enable visualisation. Scaled measurements of these images are used to evaluate the displacements of the demonstration model. This enables comparison with the FE model, and validation of the structural analysis performed during optimisation. These measurements, together with the FE model displacements, and target data, are plotted in Fig. 5.16. The demonstration model captures the FE displacements well, particularly in the spanwise direction (Fig. 5.16(a)). Difficulty in replicating the spanwise discretisation during measuring, led to small errors in crest height. However the stepped crest was captured, along with a good representation of the morphed structure. Coupled with the fact that elastic, repeatable, morphs were performed, this is sufficient to validate the
structural modelling process.

As with the two-dimensional optimal morph in the previous chapter, it is observed that the optimiser increases the size of the morphing region to improve the displacement-based objective function. Extending the bump length and width relative to the target geometry enables a greater bump height for comparable levels of stress. Although the shape fit is penalised via structural displacement outside the footprint of the target shape, this is offset by the objective-function improvement achieved from increased bump height. This serves to highlight the problems arising from using a displacement-based objective function, when only a partial morph is feasible. The bump shape is changed fundamentally, which may be critical in terms of its aerodynamic performance.

Figure 5.16: Comparison of demonstration-model displacements with FE data and target geometry: (a) spanwise, and (b) streamwise directions.
Moreover, this is not recognised by the optimiser, as the objective function is unable to quantify the local morphing properties.

## 5.6 Conclusions

This chapter investigated shape morphing from an initial flat configuration to the three-dimensional *default rounded bump*. The key findings are summarised as follows, and discussed in detail below:

- NURBS surfaces generated representing the initial and target geometries. Symmetry recognised to reduce modelling requirements.

- Displacement and Gaussian curvature-based objective functions developed to assess morphing accuracy. Stress constraint applied to ensure production of repeatable, elastic morphs.

- Structural shape control provided through discrete actuators, with the location and magnitude of applied forces assigned through optimiser-controlled variables. Variable boundary conditions around the bump perimeter were achieved using an optimiser-controlled NURBS curve, which responded well to optimisation.

- Two-stage optimisation process implemented to locate optimal morphing configurations: Monte Carlo initialisation with \((6+3n)^2\) function evaluations, followed by gradient-based refinement.

- During morphing large changes of Gaussian curvature are difficult to sustain due to associated in-plane strains violating material constraints.

- Structural modelling validated through construction of demonstration model detailing the optimal morph. Elastic morphing was exhibited, and the displacements correlated well with FE model predictions.

A NURBS description of the target geometry was created through optimisation of a geometrical shape-fit objective, evaluated on point-cloud data reproduced from a study at the University of Cambridge. Structural morphing of this shape change was then performed using an optimiser-controlled morphing structure. A flat plate modelled using FE analysis was subjected to actuation, the location and magnitude of which was specified via an optimisation algorithm. Additional shape control was provided via application of fixed boundary conditions around the bump perimeter, using a NURBS curve whose control points were located through allocation of optimiser decision variables.
5.6. Conclusions

Two objective functions are defined to quantify shape-morphing effectiveness. The first evaluates the height of the morphing structure above a reference plane, and compares this displaced value with that at the same location on the target NURBS. Comparisons over the entire structure are then combined according to a root-mean-squared error. The second evaluates the difference in Gaussian curvature at the same comparison points on the structure and target surface. Feasible morphs are ensured through a stress constraint limiting the maximum von Mises stress present in the structure.

An initial Monte Carlo search of the design domain was performed, evaluating both objective functions over $(6 + 3n)^2$ function evaluations. This highlighted promising areas of design space for the displacement-based objective function, however negligible improvements in the Gaussian curvature-based objective were discovered. Inspection of the Gauss map of the target morph shows that a complex curvature distribution must be imparted in order to morph to the default rounded bump geometry. The Gauss map of a typical function evaluation is also investigated, and the maximum $\Delta K$ achievable with the prescribed actuation is seen to be an order of magnitude less than the target. During optimisation, stress concentrations are experienced at the actuation sites, with the remainder of the structure experiencing comparatively low stress levels. This behaviour is mimicked in the distribution of Gaussian curvature change, showing that changes in Gaussian curvature are associated with in-plane strains, and hence large changes of curvature in orthogonal directions are unlikely to be achieved via elastic means for a continuous shell.

The optimal displacement-based solutions obtained through the Monte Carlo search were refined using a gradient-based minimisation. Six actuators were seen to provide the best trade-off between morphing accuracy and actuation complexity. The NURBS curve defining structural boundary conditions was seen to perform well, and evolved throughout the refinement process to enable optimal shape fit. The morphing geometry was extended in both the downstream and spanwise directions, as this enabled a higher crest to be produced. Although the shape-fit is penalised via structural displacement outside the footprint of the target shape, this is offset by the objective-function improvement achieved from increased bump height. This serves to highlight the problems arising from using a displacement-based objective function, when only a partial morph is feasible. The bump shape is changed fundamentally, which may be critical in terms of its aerodynamic performance. Moreover, this is not recognised by the optimiser, as the objective function is unable to quantify the local morphing properties.
5.6. Conclusions

To validate the structural analysis performed during optimisation, a demonstration model of the optimal design was constructed. Measurements from photographs were taken, and scaled to enable comparison with displacements from the FE model and target data. The construction method was able to capture the bump boundary well, and the response to actuation was modelled correctly. This confirms the optimality of this structure, and the fact that it is on the limit of the feasible design space.

In summary a successful morphing structure has been designed through shape optimisation. The optimal morph was constrained via material limitations, and hence was unable to achieve the target shape accurately. Three-dimensional effects limited the morphing crest height to approximately half that of the optimal two-dimensional morph of Chap. 4. It is seen that the target geometry exhibits large changes in Gaussian curvature relative to the initial configuration, and the stress constraint limits those achieved via morphing to be over an order of magnitude less. It follows that any morph requiring large changes of Gaussian curvature will require strains which exceed those achievable with current material limitations. Even small changes in Gaussian curvature are likely to result in high stress levels. It is therefore suggested that morphs involving large changes in Gaussian curvature should be avoided. Instead designers should, where possible, look to take advantage of developable morphs, where the change in Gaussian curvature between initial and target configurations is negligible. This will enable structural deformations to occur without high levels of strain. In turn this will enable large structural displacements, which are likely to enable the performance variations sought with morphing aircraft.
Chapter 6

Design of Morphing Geometry

During the previous two chapters conclusions on the relevance of geometry to the design of morphing structures have been drawn. This work was carried out with reference to a morphing shock control bump. This chapter implements the previous findings in order to aid the design of a morphing leading-edge flap, and demonstrate the applicability of the developed techniques to other morphing structures.

6.1 Introduction: Morphing Leading-Edge Flap

A leading-edge flap is a high-lift device, capable of increasing the stall angle of an aerofoil. A typical leading-edge flap is shown in Fig 6.1(a), with a morphing version depicted in (b). The device achieves an increase in stall angle via drooping of the leading edge. This increases camber on the upper surface of the aerofoil, which has been shown to be a major factor in determining maximum lift [72]. Leading-edge flaps are typically used for improving the transonic manoeuvring performance of high-speed fighters, which need thin wings for supersonic flight. A drawback of

![Diagram of leading-edge flap](a) Continuous upper surface

![Diagram of morphing leading-edge flap](b) Compliant structure

Figure 6.1: Leading-edge flap: (a) Conventional, and (b) Morphing.
the conventional leading-edge flap is the joint that is present between the hinged section and the main wing. It is known that slotted systems such as leading-edge slats, are a dominant source of airframe noise during approach [165]. However whilst slot-less systems, such as a leading-edge flap, do not incur the same noise penalty, the gap present between the rotated leading edge and the main wing produces undesirable aerodynamic effects. Even in its contracted configuration a small gap remains, which provides sufficient disturbance to the flow to cause transition to turbulence directly after the gap [166]. This contradicts the design requirements of aircraft wings capable of meeting future noise and emissions requirements. Natural laminar flow is likely to be harnessed to help meet future targets, and therefore to transition to turbulence at the end of the leading-edge region is highly undesirable. Morphing structures allow a leading-edge flap to be deployed without breaking the continuity of the aerofoil surface. Thus enabling design of a wing with a high-lift leading-edge device, and natural laminar flow.

The remainder of the chapter investigates the design of a morphing leading-edge flap. It begins with definition of the morphing geometry with developability in mind. A structural optimisation is then performed to find the actuation requirements of the system. Finally conclusions are drawn about the morphing procedure, and the remaining stages of the design process discussed.

### 6.2 Morphing Geometry Generation

The design of a morphing leading edge is investigated for implementation on the NACA2421 aerofoil; as used by the Alenia Aeronautica Sky-X UAV following the work presented in [25], which is displayed in Fig. 2.24. The first quarter-chord forms the initial configuration for the morph, the geometry of which is generated from the NACA parameterisation. The target configuration is achieved via increasing the camber of this initial parameterisation, as seen in Fig. 6.2.

**Initial Morphing Configuration**

A NACA four-digit aerofoil is constructed according to the parameterisation in [84]. The upper and lower surface coordinates are evaluated according to

\[
\begin{align*}
  x_U &= x - y_t \sin \theta \\
  y_U &= y_c + y_t \cos \theta \\
  x_L &= x + y_t \sin \theta \\
  y_L &= y_c - y_t \cos \theta
\end{align*}
\]

where \(x_U\) and \(y_U\) are the upper surface coordinates, \(x_L\) and \(y_L\) are the lower surface coordinates, \(x\) and \(y_c\) are the chord lengths, \(y_t\) is the camber, and \(\theta\) is the angle of rotation.

(6.1)
6.2. Morphing Geometry Generation

Chapter 6. Design of Morphing Geometry

Figure 6.2: NACA2421 geometry with example drooped leading edge

where $\theta = \tan^{-1} \frac{dy_c}{dx}$. The thickness distribution $y_t$, and the mean-line shape $y_c$, are given by

$$y_t = \frac{t}{0.2} \left( 0.2969x^{0.5} - 0.126x - 0.3516x^2 + 0.28430x^3 - 0.10150x^4 \right)$$

$$y_c = \begin{cases} 
\frac{m}{p^2}(2px - x^2) & \text{forward of maximum ordinate} \\
\frac{m}{(1-p)^2} \left[ (1-2p) + 2px - x^2 \right] & \text{aft of maximum ordinate} 
\end{cases}$$

(6.2)

where $t$ is the maximum thickness expressed as a percentage of chord, $m$ is the maximum mean-line ordinate expressed as a fraction of chord, and $p$ is the chordwise location at which $m$ occurs. A continuous description of the geometry is advantageous, and hence a NURBS is fit to this data.

A fourth-order NURBS is used to ensure accurate capture of the curvature contribution of the $x^4$ term in the above parameterisation. The NURBS is created via optimisation, with the NURBS control points defined according to Fig. 6.3. The decision variables controlling the shape of the NURBS can no longer be parameterised according to their height above a datum plane. Due to the approximately circular leading-edge geometry, the NURBS control points are defined as fractions of radial distance of the circle centred at $(0.125c, 0)$, as shown in Fig. 6.3. Radial lines are constructed at equal angles, the size of which depends on the number of NURBS control points used. The optimisation decision variables vary in the range $0.25 \leq \Omega \leq 1$, and control the location of the NURBS control points on the radial lines. A value of 1 represents the outer circle, with
0 being located at the circle centre. The parameterisation method removes a degree of freedom relative to defining a pair of \((x, y)\) coordinates for each control point. This speeds up optimisation convergence, and ensures that an open curve is produced.

To fit the NACA2421 data a gradient-based minimisation is performed using the previously defined \(G_y\) objective function (Eqn. 4.4). Inspection of the geometry shows that the neither the curve-fitting process nor the morphing geometry, can be parameterised by a \(y = f(x)\) type approach, as it is no longer a one-to-one mapping. When considering geometries of this kind it is sensible to parameterise the geometry according to the arc length \((s)\) of the curve. A unique location in Cartesian space \(P(x, y)\) can therefore be found by evaluating the NURBS at a given \(N(s)\) location

\[
P(x, y) = N(s) \tag{6.3}
\]

and likewise, through a numerical step, the process can be repeated to find \(N(S)\) for a given \(P(x, y)\). This is important when an objective function for shape fitting is considered. Previously,
6.2. Morphing Geometry Generation

the NURBS height and that of the target point-cloud data were compared at common $x$-values. Due to the leading-edge geometry, this is no longer possible. Instead it is required to compare points at proportional distances along the initial and target shapes. To facilitate this, the design space is populated densely by evaluating the NACA2421 over a fine discretisation. If these points are generated in order, beginning at the top surface of the wing at the quarter-chord, and travelling over the leading edge to the underside extremity of the target geometry, they can be collected into an array

$$D(x, y) = \begin{bmatrix} x_1 & x_2 & x_\ldots & x_q \\ y_1 & y_2 & y_\ldots & y_q \end{bmatrix}$$

(6.4)

where $q$ is the total number of points generated from the NACA2421 description, $(x_1, y_1)$ is at the quarter chord on the upper surface and $(x_q, y_q)$ is at the quarter chord on the lower surface. The distances between consecutive coordinates are evaluated via Pythagoras’ theorem, and summed according to

$$L \approx \sum_{i=1}^{q-1} \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}$$

(6.5)

to give an approximation of the curve length for the overall configuration. Although an approximation, the value of $L$ converges to a constant value providing the number of point-cloud data $q$ is high enough. For the results shown here a uniform distribution of 5000 points is evaluated from the NACA description in Eqn. 6.1. This not only gives an accurate estimation of the initial morphing configuration length, but also provides a detailed set of range data with which to generate a NURBS.

The length of the initial morphing configuration is used to normalise the locations of comparison points during objective-function evaluation. Whilst fitting the NURBS to the target data, the NURBS is likely to change length, and hence simply comparing points at given arc lengths would be inaccurate. Instead the location of the point in $s$-space is normalised against the NURBS length, to provide worthwhile comparison. For example, the point located 40% along the length of the target data is compared with the point 40% along the length of the NURBS produced during optimisation. The local shape-fit error is found by constructing and evaluating the length of the vector $r$, which defines the distance between the two points in Cartesian space (as shown in Fig. 6.4). To construct the objective function, the NURBS created by the optimiser is evaluated at equivalent $s$-locations present in the point-cloud description, and the individual values of $r$ calculated according to Pythagoras’ theorem

$$r_i = \sqrt{(P_i,\text{cloud}(x) - P_i,\text{NURBS}(x))^2 + (P_i,\text{cloud}(y) - P_i,\text{NURBS}(y))^2}$$

(6.6)
6.2. Morphing Geometry Generation

Figure 6.4: Geometrical evaluations using arc-length parameterisation

which are combined via summation into the objective function $G_{\delta(s)}$ for minimisation

$$G_{\delta(s)} = \sqrt{\sum_{i=1}^{p} \frac{r_i^2}{p}}. \quad (6.7)$$

A gradient-based optimisation algorithm is used to minimise $G_{\delta(s)}$. The decision variables are initialised to their maximum values, causing the NURBS to take approximately the form of the outer circle shown in Fig. 6.3.

The optimal fit of the NACA2421 leading-edge geometry, optimised for varying numbers of NURBS control points, is shown in Tab. 6.1. Increasing the number of NURBS control points causes the shape-fit accuracy to improve. The average error is reduced to approximately 0.2 mm using 10 NURBS control points, and 6 $\mu$m using 50. This improvement in accuracy is reflected in the NURBS length, which converges with increasing numbers of control points. Whilst accuracy is important, increasing the number of NURBS control points adds complexity to the optimisation.
6.2. Morphing Geometry Generation

Process, which is undesirable. Therefore due to NURBS length converging to the nearest micron for 30 control points, this is the number used during generation of all NURBS in the remainder of this chapter.

## Target Morphing Configuration

The target geometry for optimisation is constructed in a similar manner: range data is created, followed by a NURBS fit to provide the required continuous geometry description. To create the range data, the NACA parameterisation is used with an increased camber value to provide the drooped leading edge. In previous chapters the importance of a developable morph has been demonstrated. To ensure the two-dimensional morphing geometry is developable, it must have the same length as the initial NACA2421 leading edge. Because the arc-length cannot be specified in the NACA parameterisation, optimisation is required to generate a cambered leading edge of the required length. A gradient-based optimisation is performed, where the NACA parameters are the design variables. The camber is set by increasing the value of $m$ in the range $8\% \leq m \leq 16\%$, and varying the value of $c$, $t$ and $p$ during optimisation. Once a new geometry has been created, the chordwise location at which the thickness is the same as the original aerofoil at the quarter-chord is found. The length of the section in front of this point is then evaluated, and compared to the initial length according to

$$\Delta L = \frac{L_{init} - L_{morph}}{L_{init}}$$

where $\Delta L$ is minimised during optimisation. Hence a cambered leading edge is created with minimal difference in length with respect to the original aerofoil leading edge. The method performed well enabling definition of three geometries with values of 8%, 12% and 16% camber, and the required length evaluated according to Eqn. 6.5.

<table>
<thead>
<tr>
<th>NURBS Control Points</th>
<th>$G_\delta(s)$ (m)</th>
<th>$L$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$2.057 \times 10^{-4}$</td>
<td>0.580456</td>
</tr>
<tr>
<td>20</td>
<td>$2.425 \times 10^{-5}$</td>
<td>0.580520</td>
</tr>
<tr>
<td>30</td>
<td>$9.973 \times 10^{-5}$</td>
<td>0.580515</td>
</tr>
<tr>
<td>40</td>
<td>$6.665 \times 10^{-6}$</td>
<td>0.580515</td>
</tr>
<tr>
<td>50</td>
<td>$5.913 \times 10^{-6}$</td>
<td>0.580515</td>
</tr>
</tbody>
</table>

Table 6.1: NURBS fit of leading-edge geometry
6.2. Morphing Geometry Generation

Point-cloud data is generated for the different camber configurations with the optimised values of \( c, p \) and \( t \). Using this point-cloud data, NURBS are fitted using the same technique shown in Fig. 6.3. However, unlike during generation of the initial morphing geometry, three constraints are applied during optimisation. The first, \( h_L \), ensures the change in length between the initial and target configurations is less than 50 \( \mu m \)

\[
h_L = \frac{L_{\text{initial}} - L_{\text{target}}}{L_{\text{initial}}} - 8.6 \times 10^{-5} \quad h_L \leq 0 \tag{6.9}
\]

Specifying minimal change in length between the two morphing geometries ensures the morph will be developable. Therefore, it can be assumed the only structural resistance to morphing will be due to bending, as negligible change in length of the neutral axis, and consequently membrane strain, will occur. This is an important constraint for the leading-edge morph, because the curvature of the original structure will make applying forces to stretch or compress the neutral axis difficult. A buckling tendency will dominate the structural response in compression, and a straightening of the structure in tension. In the absence of axial loads, the strains experienced during morphing will be entirely due to bending. It was seen in previous chapters that it is possible to estimate the bending strain from the change in curvature between morphing configurations. This idea is used to bound the design space during generation of the target morphing geometry, thus ensuring a feasible morph is specified. The curvature change at like locations in \( s \)-space of the initial and target morphing geometry, is found according to Fig. 6.4, with the maximum change denoted by \( \kappa_{\text{max}} \). This value is limited to be below the equivalent yield strain, as specified according to the definition in Chap. 4

\[
E \sigma_y = \epsilon_y = y \kappa_y
\]

\[
\epsilon_\kappa = 0.701 \% \quad \kappa_y = 35.05 \text{ m}^{-1}
\]

\[
h_\kappa = \kappa_{\text{max}} - \kappa_y (1 - k). \tag{6.10}
\]

The safety factor is increased to \( k = 0.4 \), to reduce the maximum strain and improve the fatigue life of the morphing system. Finally, in an effort to ensure compatibility with the remainder of the NACA2421 geometry, the curvature of the NURBS at the outer extremities is constrained to be within 1.0 \( \text{m}^{-1} \) of the original leading-edge geometry. This continuity constraint is defined as

\[
h_c = \Delta \kappa - 1.0 \tag{6.11}
\]

where \( h_c \) is defined as the maximum of

\[
\Delta \kappa(0) = |\kappa_{\text{NURBS}}(0) - \kappa_{\text{NACA2421}}(0)|
\]

\[
\Delta \kappa(1) = |\kappa_{\text{NURBS}}(1) - \kappa_{\text{NACA2421}}(1)| \tag{6.12}
\]
6.2. Morphing Geometry Generation

where $\kappa(0)$ refers to the curvature at the start of the leading-edge geometry, and $\kappa(1)$ to that at the end. Therefore the optimisation to find the target morphing configuration is defined formally as

$$\min(G_{\delta(s)}) \quad \text{(shape-change error)}$$

s.t.

$$h_L \leq 0 \quad \text{(length-change constraint)}$$
$$h_\kappa \leq 0 \quad \text{(curvature-change constraint)}$$
$$h_c \leq 0 \quad \text{(boundary-curvature continuity constraint)}$$

The starting point for optimisation is specified as the initial morphing geometry as it is known to satisfy both constraints. A gradient-based algorithm is used to operate on this starting point to produce NURBS for the three values of camber, the details of which are displayed in Tab. 6.2. The target geometries for the three configurations are displayed in Fig. 6.5, whilst their curvature distributions are displayed in Fig. 6.6. A similar change in curvature is required for all configurations, however the maximum increases with camber. As the leading edge rotates downwards the location of the maximum curvature changes, as seen in Fig. 6.6 Using superposition of the strain
6.2. Morphing Geometry Generation

Figure 6.6: Curvature distributions of leading-edge geometries

Figure 6.7: Curvature changes between the initial and target morphing configurations
Table 6.2: Structural stress predictions from morphing geometries

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Maximum $\Delta \kappa$ (m$^{-1}$)</th>
<th>$\Delta L$ (m)</th>
<th>Estimated $\sigma_{\text{max}}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 %</td>
<td>10.07</td>
<td>$1.5 \times 10^{-5}$</td>
<td>146.09</td>
</tr>
<tr>
<td>12 %</td>
<td>14.35</td>
<td>$5.2 \times 10^{-5}$</td>
<td>210.38</td>
</tr>
<tr>
<td>16 %</td>
<td>17.22</td>
<td>0</td>
<td>247.97</td>
</tr>
</tbody>
</table>

imparted due to changes in length and curvature, it is possible to predict the stress distribution that will be experienced during morphing. The maximum stress will will be located at the site of maximum curvature change, which is approximately 60 % of the distance along the morphing geometry from the upper surface extremity. These values are plotted in Tab. 6.2 for the three morphing configurations.

**Effect of Initial Curvature**

Assuming a structural response based on the elastica in Eqn. 4.17, it is possible to estimate the maximum stress and strain experienced during morphing. The initial curvature of the structure could have a significant impact on these predictions, if the structure ceases to behave according to the assumptions of elastica and beam theory analysis [150] (see Sec. 4.7). The essential difference between pure bending of straight and curved beams is explained best using the extreme case of a sandwich beam. The beam is assumed to have an initial radius of curvature $R$, flange separation distance $H$, and flange thickness $t$, as shown in Fig. 6.8. If a straight beam is loaded with moments of opposite sense but equal magnitude at either end, then the flanges will be put into tension and compression. If the same tensile and compressive forces are applied to an initially curved beam, the same loading is experienced by the flanges, but the core now undergoes a compressive radial force as shown in Fig. 6.8. Because the problem is elastic these stresses result in small changes in length of the flanges, and a small change in thickness of the core. The contributions of these strains are considered separately to assess the effect of an initially curved structure. In [154] it is shown that the reduction in core thickness, in the absence of strain in the flanges, will lead to a change in curvature of the beam relative to the original configuration, which is denoted $\Delta \kappa_2$. Conversely, in the absence of strain in the core, the change in length of the flanges will lead to a change in curvature $\Delta \kappa_1$, as it would in the straight beam. For the sandwich beam illustrated it can be shown
that the ratio of these changes in curvature is given by

$$\frac{\Delta \kappa_2}{\Delta \kappa_1} = \left( \frac{E}{E'} \right) \left( \frac{TH}{2R^2} \right)$$

(6.13)

where \( T \) is the thickness of the flanges, \( H \) is the separation of the flanges, \( R \) is the original radius of curvature of the curved beam, and \( E \) and \( E' \) are the Young’s modulus of the flanges and core respectively. If the ratio \( \Delta \kappa_2/\Delta \kappa_1 \) is sufficiently small, it can be assumed that the effect of compressibility in the core is negligible, and hence the response will be governed by the flanges as in the straight beam. For a curved beam of solid cross-section, the through-thickness compressive stress varies continuously across the thickness. The corresponding value of the curvature ratio is therefore given by

$$\frac{\Delta \kappa_2}{\Delta \kappa_1} = \frac{1}{12} \left( \frac{t}{R} \right)^2$$

(6.14)

where \( R \) is the initial radius of curvature and \( t \) the thickness of the beam. The maximum curvature of the NACA2421 leading edge is taken from Fig. 6.6, and results in a value of \( R = 1/21.32 = 0.047 \) m. Together with the thickness \( t = 0.4 \) mm this gives \( \Delta \kappa_2/\Delta \kappa_1 = 6.06 \times 10^{-6} \), which is \( \ll 1 \). This demonstrates that the contribution of \( \Delta \kappa_2 \) is negligible, and that the treating of the initially curved beam in the same manner as a straight one is valid.
6.3 Structural Optimisation

The structural optimisation resembles closely that performed for the morphing SCB, however due to the arc-length parameterisation there are some fundamental differences. Material properties match those used previously, and are assigned $E = 72$ GPa and $\nu = 0.33$. A pseudo two-dimensional FE model is created using the NURBS description of the initial geometry, and meshed with Mindlin shell elements. The use of a surface enables application of aerodynamic pressure loading during subsequent design phases. Actuation is applied using locations defined according to their arc-length distance, normalised by the length of the initial morphing configuration $s_i = s/L_{\text{initial}}$. Due to the curvature of the structure, and the multi-directional morphing requirements, displacements are preferred over forces for actuation. For each actuator, orthogonal components of displacement are defined by $\delta_{x,i}$ and $\delta_{y,i}$ as in Fig. 6.9. Variable fixed boundaries are omitted from optimisation, as both initial and target geometries exhibit the same behaviour at their extremities, removing the possibility of improving the solution by allowing these locations to vary. The vector of decision variables governing actuation is therefore

$$[s_1 \ \delta_{x,1} \ \delta_{y,1} \ s_2 \ \delta_{x,2} \ \delta_{y,2} \ ... \ s_n \ \delta_{x,n} \ \delta_{y,n}] \quad (6.15)$$

where $n$ is the total number of actuators. Because the three target morphing geometries require different displacements, the problem-specific bounds

$$\begin{align*}
0.00 \text{ m} & \leq \delta_x \leq 0.01 \text{ m} \\
-0.04 \text{ m} & \leq \delta_y \leq 0.00 \text{ m} \\
0.00 \text{ m} & \leq \delta_x \leq 0.02 \text{ m} \\
-0.09 \text{ m} & \leq \delta_y \leq 0.00 \text{ m} \\
0.00 \text{ m} & \leq \delta_x \leq 0.03 \text{ m} \\
-0.11 \text{ m} & \leq \delta_y \leq 0.00 \text{ m}
\end{align*}$$

are enforced on the displacement variables. Due to normalisation of the arc-length location by the initial morphing geometry length, the actuator locations are bounded in the region $0 < s/L_{\text{init}} < 1$, where 0 represents the upper extremity of the geometry, and 1 the lower extremity.

**Objective Functions and Constraint**

The displacement-based objective function used during geometry generation (Eqn. 6.7) is utilised to provide a zeroth-order objective function for structural optimisation. The FE nodes are used as
6.3. Structural Optimisation

Chapter 6. Design of Morphing Geometry

Figure 6.9: Decision variables for leading-edge morph and \( n = 2 \)

comparison points for convenience. Like locations in \( s \)-space are projected onto the target NURBS to calculate the vector \( \delta \) (replacing \( r \) in the geometry-fitting formulation), and evaluate \( S_{\delta(s)} \)

\[
\delta_i = \sqrt{(P_{i,FE}(x) - P_{i,NURBS}(x))^2 + (P_{i,FE}(y) - P_{i,NURBS}(y))^2}
\]

\[
S_{\delta(s)} = \sqrt{\frac{\sum_{i=1}^{p} \delta_i^2}{p}} \tag{6.17}
\]

where \( p \) is the number of nodes in the FE model. The curvature-based objective function is also modified for minimisation. The FE node locations are projected onto the target NURBS as in construction of \( S_{\delta(s)} \). This allows the curvature to be evaluated using the osculating circle method described in Chap. 4, and compared at like \( s/L_{init} \) locations as shown in Fig. 6.4. The difference between the two configurations for all points is then combined via the root-mean-squared error

\[
S_{\kappa(s)} = \sqrt{\frac{\sum_{i=1}^{p} (\kappa_{i,NURBS} - \kappa_{i,FE})^2}{p}} \tag{6.18}
\]
where the subscript $i$ refers to like values of $s/L$ for the NURBS and FE model. To reduce $S_{\kappa(s)}$ to zero for the 8%, 12% and 16% camber configurations, the curvature distributions in Fig. 6.6 must be effected via actuation. This means imparting the changes in curvature displayed in Fig. 6.7.

The same structural constraint used in previous chapters is implemented to ensure repeatable elastic morphs are produced. Therefore the maximum von Mises stress is constrained to be below the yield strength of aerospace grade aluminium $\sigma_y = 505$ MPa including a safety factor of $k = 10\%$

$$\sigma_{max} \leq \sigma_y (1 - k)$$

$$h = \sigma_{max} - \sigma_y (1 - k) \quad h \leq 0$$

(6.19)

where $h$ is the stress constraint. When the maximum stress is greater than 454.5 MPa, $h$ becomes positive and the constraint is violated, indicating an unfeasible design.

6.4 Optimisation

The structural shape-fit objective functions are minimised using the developed protocol: a Monte Carlo initialisation, followed by a gradient-based refinement. The two objective functions are discussed separately for clarity.

$S_{\delta(s)}$ Minimisation

The results of a Monte Carlo search using $(3n)^2$ function evaluations for the 8% camber geometry, and $n = 1$ to 4, are shown in Tab. 6.3(a). Application of displacements in two dimensions at multiple locations, often leads to violation of the stress constraint. Actuators in consecutive locations in arc-length space are able to move in opposite directions, which applies a large stress to the adjoining structure. For a single actuator this was not a problem, as only 24 function evaluations were required to achieve $(3n)^2$ feasible designs. For values of $n \geq 2$ however, the total number of function evaluations greatly exceeded $(3n)^2$. For example, for $n = 4$, 2826 function evaluations were required to generate 144 feasible individuals. As an initialisation step this is undesirable, as a large amount of time is wasted searching unfeasible and sub-optimal design space. Furthermore, the fitness of the optimal solutions deteriorates for increasing $n$. Inspection of the feasible solutions shows that in general the applied displacements are small, and hence do not generate large stresses. This typically results in a morph which is close to the initial geometry in shape, failing to highlight promising areas of design space for subsequent refinement.
6.4. Optimisation

Table 6.3: Initial Monte Carlo results for minimisation of $S_{\delta(s)}$ for 8\% camber morph: (a) Monte Carlo displacements, and (b) calculated displacements.

This problem is solved by using the morphing geometry to bound the design space. Due to tailoring of the initial and target geometry to the same length, it can be assumed that an actuator located at $s = 0.5L_{\text{initial}}$ on the initial configuration will be displaced to the point $s = 0.5L_{\text{target}}$ on the target configuration. Hence for a given location in arc-length space, the displacements required to morph exactly to the target shape can be evaluated from the NURBS descriptions. This reduces the vector of decision variables to

$$r = [s_1 \ s_2 \ s_3 \ldots \ s_n] \quad (6.20)$$

for $n$ actuators. Re-running the Monte Carlo optimisation using calculated displacements yields the results in Tab. 6.3(b). The number of constraint violations is reduced to zero for all cases, considerably improving optimisation times. Furthermore the optimal objective functions are improved by an order of magnitude for $n = 2$, and two orders of magnitude for $n = 3$ and 4. Thus providing superior locations in design space for initialising subsequent optimisations. Once the actuation locations have been initialised, they are refined using a gradient-based optimisation – still making use of calculated displacements – to improve further their locations. A final refinement step is then performed utilising optimiser-controlled displacements. This three-stage optimisation process
6.4. Optimisation

Table 6.4: Optimisation data from minimisation of $S_{\delta(s)}$

<table>
<thead>
<tr>
<th>n</th>
<th>Optimisation Method</th>
<th>8 % camber</th>
<th>12 % camber</th>
<th>16 % camber</th>
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is executed for all values of $n$, with the results of the individual optimisation steps displayed in Tab. 6.4. Convergence of the fully-refined objective functions with actuation complexity is shown in Fig. 6.10. The camber increase of the target shape reduces the accuracy of the optimal morph, and increases the maximum stress experienced during structural shape change. The optimal morph improves with the number of actuators for all leading-edge configurations. Increased actuation enables better fitting of the structure to the target shape, but achieves this typically at the expense of increased stress. Four actuators is the optimum number for achieving all three morphs. Comparable objective functions are achieved for $n = 3$ to 5, however there is a reduction in $\sigma_{\text{max}}$ for $n = 4$ relative to $n = 3$, and hence four actuators is deemed the optimal amount. The optimal morph for the 8 % camber configuration is displayed in Fig. 6.11. The morphed geometry is displayed in blue, and a range of target data-points are shown in red. The actuator locations on the initial and target
geometry are shown in green. Two actuators are located at the leading-edge tip, with the remaining actuation located near the domain boundaries. This enabled the leading-edge tip to be translated to the target configuration. The optimal morphs for the 12% and 16% camber configurations are displayed in Fig. 6.12(a) and (b) respectively. Similar actuation distributions are visible for both morphs, with two actuators used at the leading-edge tip. One actuator is located on the upper surface as seen for the 8% camber morph, whilst on the lower surface the final actuator is located much nearer to the leading-edge tip.

The well-behaved convergence curves in Fig. 6.10 indicate that global optima have been obtained using the three-stage optimisation process. The maximum stress encountered for the 8% camber morph ($n = 4$) was well below the constraint value. This meant the constraint was inactive during optimisation, and the optimiser was free to minimise the objective function. This was not the case for the 12% and 16% configurations, whose optimal solutions were constraint limited. Nonetheless good convergence behaviour is observed, indicating that the optimal feasible morphs have been found.

As expected, structural morphing becomes increasingly difficult as the target deformations become greater. To compare this for the three cases, the shape-morphing error is plotted in arc-length space in Fig. 6.13. The 8% camber morph achieved an average error of 0.2 mm, which is...
6.4. Optimisation

Chapter 6. Design of Morphing Geometry

Figure 6.11: Optimal morphing geometry for minimised $S_{\delta(s)}$ and $n = 4$, 8% camber configuration

reflected in the error plot. Although there are regions on the upper surface before the leading-edge tip where the error is greater than this value, the average is brought down by the sub 10 $\mu$m error over the last 40% of the morphing geometry. The 12% and 16% camber geometries were unable to match this level of morphing accuracy. The 12% camber morph exhibited sub-millimetre errors for the majority of its length, however a region of consistently poor shape fit existed at the extremity on the lower surface. A similar error distribution, but with increased magnitude, is observed for the 16% camber configuration. The optimal morphing geometry for these cases is displayed in Fig. 6.12, and shows that the areas of consistent error exist due to a lack of actuation. Conversely the 8% camber morph, which exhibits excellent shape-fit accuracy in this region, has an actuator located close to the lower surface extremity. To understand why the optimisation algorithm did not locate actuators in this region for the 12% and 16% camber morphs, the stress distributions resulting from morphing the three configurations are displayed in Fig. 6.14. The intention of this chapter was to design developable morphing geometry, and hence to impose negligible extension of the neutral axis during structural shape changes. Figure 6.14(a) shows that for the 8% geometry only a small region of length change was imparted by actuation, and that the majority of the shape

175
Figure 6.12: Optimal morphing geometry for minimised $S_{\delta(s)}$ and $n = 4$: (a) 12% camber, and (b) 16% camber configurations.
change was achieved via bending. Two regions of peak stress occur, which are approximately equal in magnitude. The first is located in the region of the leading-edge tip, where bending was required to translate the location of maximum curvature in arc-length space. The second occurs at the lower extremity of the morphing geometry, where the fully-fixed boundary conditions are applied. The magnitude of both stress concentrations was approximately half that required to violate the constraint, and hence did not affect optimiser operation. For the increased camber cases larger stress concentrations were experienced, requiring activation of the constraint during optimisation to ensure a feasible result. For both cases, the maximum stress locations were again located at the leading-edge tip, and the model extremity on the lower surface. Both areas exhibited stress levels approximately 90% of the constraint limit. The stress levels in the region of the leading-edge are contributed to via extension of the geometry coupled with bending. Whilst the stress concentration at the domain boundary is a result of applying fully-fixed boundary conditions. The optimiser finds it worthwhile to apply axial strain to enable better shape-morphing of the leading-edge region. Actuation is moved away from the lower-surface domain boundary however, as improving the shape-fit here would result in a larger reaction stress from the structural boundary conditions, and subsequently violation of the stress constraint. This increase in stress is also visible at the upper surface domain extremity for the 12% and 16% camber morphs, however it is of less concern due to the direction of morphing being away from this location.
6.4. Optimisation

Figure 6.14: Stress distributions for $S_{\delta(s)}$-minimised leading-edge morphs: (a) 8% camber, (b) 12% camber and (c) 16% camber.
6.4. Optimisation

Chapter 6. Design of Morphing Geometry

<table>
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<th>12 % camber</th>
<th>16 % camber</th>
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Table 6.5: Optimisation data from minimisation of \( S_{\kappa(s)} \)

\( S_{\kappa(s)} \) Minimisation

The curvature-based objective is minimised using the same three-stage optimisation process; the results of the individual stages are displayed in Tab. 6.5. Convergence of the fully-refined objective-functions and associated constraint values are displayed in Fig. 6.15. Unlike the displacement-based objective function, the curvature fit is not seen to improve with actuation complexity. For the 8 \% camber configuration \( S_{\kappa(s)} \) is approximately constant for \( n \geq 2 \). Whilst for the 12 \% and 16 \% camber configurations, the value of \( S_{\kappa(s)} \) is a minimum for \( n = 2 \), and is penalised for \( n > 2 \).

To understand why this behaviour is exhibited, the curvature distribution from the \( S_{\kappa(s)} \)-optimal geometry is examined. The most pronounced drop-off of \( S_{\kappa(s)} \) with \( n \) is for the 16 \% camber morph, and hence the curvature changes imparted from these morphs are displayed in Fig. 6.16(a). The \( n = 2 \) morph achieves a good representation of the target, offering the required curvature
6.4. Optimisation

Figure 6.15: Convergence of fully-refined (a) $S_{\kappa(s)}$, and (b) $\sigma_{\text{max}}$, with actuation complexity. Note the legend for Fig. 6.15(b) is displayed on Fig. 6.15(a) for clarity.

change around the leading-edge tip, but having minimal effect at the extremities. Because all morphs require a similar type of curvature change to be effected, this explains why two actuators coupled with the structural bending response, provided optimal objective functions. The additional actuation of the $n = 3$ case causes an increase in $S_{\kappa(s)}$. Although a similar level of curvature response is achieved around the region of the leading-edge tip, the objective function is penalised by regions of error at the structural boundaries. This behaviour is seen to become more pronounced with further increases in actuation complexity for the 16% camber morph. For the 8% and 12% camber morphs, these regions of error are generated, however they are offset by improvements in the curvature-fit around the leading-edge tip. This causes the objective function to stabilise to a consistent value for $n \geq 3$. The fact that the objective function gets worse with increased actuation is counter intuitive, therefore the resulting morphing geometries are investigated in Fig. 6.16(b).

Although exhibiting the desired curvature response, this shows that the optimal morph for $n = 2$ is far from achieving the target morph. Conversely the morph with three actuators, which has a poorer value of $S_{\kappa(s)}$, is seen to achieve a better likeness of the morphing geometry. This suggests that $S_{\kappa(s)}$ is not a useful tool in the design of morphing structures, as minimising its value produces structures with shapes different to that of the target geometry.

However it is known that by definition curvature is the second differential of displacement, and
Figure 6.16: (a) Minimised $S_{\kappa(s)}$ distributions for 16 % camber configuration and $n = 2$ and 3, (b) associated morphing geometries.
6.4. Optimisation

Chapter 6. Design of Morphing Geometry

hence the two properties should be closely linked. A problem arises with the leading-edge morphs due to the behaviour of the geometry at the boundaries with the original aerofoil. Because the geometries were generated with curvature continuity in mind, and not tangential continuity, matching the entire curvature response is analogous to integrating with incorrect boundary conditions. For all target geometries, morphs using two actuators are able to achieve the minimum value of $S_\kappa(s)$, because they can effect the curvature change at the leading-edge tip, but do not influence the curvature response at the extremities. Due to the initial optimisation phases, additional actuation is forced to translate to the target geometry, and hence results in a more accurate morph in terms of $S_\delta(s)$, but which still exhibits the desired curvature response along the majority of its length.

As the short-comings of the curvature-based optimal morphs have been uncovered through analysis of their displacements, it follows that examining the curvature of the displacement-based morphs will be insightful. Therefore the curvature distribution of the optimal solution found during minimisation of $S_\delta(s)$ for $n = 4$, is plotted in Fig. 6.17(a). The curvature response shows significant errors at both extremities, with several discontinuities also present in the region of the leading-edge tip. It is interesting to compare the curvature response with the stress distribution for the same configuration in Fig. 6.14(c). Whilst the stress plot shows absolute values, it is clear that they are related closely to the curvature distribution. Although this morph achieved the minimum value of $S_\delta(s)$, with an average morphing error of 1.55 mm, the large number of curvature discontinuities are likely to make it aerodynamically unfriendly. Also plotted in Fig. 6.17 is the curvature distribution and morphing geometry of the $S_\kappa(s)$-minimised morph for $n = 4$. This exhibits a smoother curvature response, with a better fit at the leading-edge tip. The regions of error exist at the structural extremities, however the curvature change is spread over a longer arc-length distance compared to the $S_\delta(s)$-minimised solution, which reduces the magnitude of the discontinuity and hence the stress experienced in this region. Figure 6.17(b) shows that the $S_\kappa(s)$-minimised morph is able to achieve similar morphing accuracy in terms of displacement, recording an average morphing error of 2.98 mm. The final consideration is the maximum stress encountered during morphing. The $S_\delta(s)$-minimised morph experiences a maximum stress of 402.09 MPa, which is located at the site of extension near the leading-edge tip, however a similar stress level is found at the boundary on the lower surface. The $S_\kappa(s)$-minimised morph exhibits a maximum stress of 271.36 MPa, which is located at the change of curvature at the leading-edge tip. The maximum curvature change is $-18.9 \text{ m}^{-1}$, and whilst this is slightly more than the target of $-17.22 \text{ m}^{-1}$, it predicts a stress of 272.16 MPa according to the elastica analysis (Eqn. 4.17). This explains the increase in stress.
Figure 6.17: Optimal 16 % camber morphs for minimised $S_{\delta(s)}$ and $S_{\kappa(s)}$ ($n = 4$): (a) curvature change, (b) morphing geometry.
To summarise the findings, it was first thought that the curvature-based approach lacks meaning for low numbers of actuators, and if the geometry is not created appropriately. However for increased actuation complexity it provided morphs with desirable curvature and stress distributions. The displacement-based objective enables good shape-fitting, however it is prone to creating structures with severe discontinuities of curvature, and high stress concentrations. To investigate further the relationship between the two objectives, and which provides the optimal morph, a Pareto plot of the optimal solutions and their corresponding non-optimised objective-functions, is displayed in Fig. 6.18. The $S_{\delta(s)}$-minimised solutions are plotted with square markers, whilst the $S_{\kappa(s)}$-minimised solutions are shown with circles. The filled markers represent the $n = 1$ solutions, with the markers joined by a line in order of ascending $n$. If the solutions for $n = 1$ are ignored, then a clear Pareto front is visible for all three morphing configurations. It is clear that each
objective function can be minimised at the expense of the other, and that the overall fit decreases with the camber of the morphing configuration. It was shown above that minimisation of $S_\kappa(s)$ produced morphs with more desirable stress and curvature distributions, and hence solutions with better $S_\kappa(s)$ are favoured when selecting an optimal morph. For all cases lines of $S_\kappa(s) = -200S_\delta(s)$ – which are 145° to the positive $S_\delta(s)$-axis – are approximately tangential to the arc of the Pareto front at its point closest to the origin. Therefore the optimal solutions are those which lie on the line $S_\kappa(s) = -200S_\delta(s)$, and have the minimum axes intercepts. For the 8 % camber morph this is the $S_\kappa(s)$-minimised morph for $n = 5$; and for both the 12 % and 16 % camber configurations, the $S_\kappa(s)$-minimised morph for $n = 4$. The optimal morphing geometry for the 16 % camber morph was displayed together with the resulting change in curvature in Fig. 6.17. The optimal geometry and associated curvature change for the 8 % and 12 % camber morphs are displayed in Fig. 6.19 & 6.20 respectively.

The stress distributions for all three optimal morphs are displayed in Fig. 6.21. The extremely low stress levels in the mid-plane of the structure shows that negligible strain of the neutral axis has occurred, and hence all morphs are developable. This is confirmed by the fact that all three stress distribution plots are directly proportional to the absolute distributions of the changes in curvature. By using the relationship

$$\sigma = E\epsilon = Ey$$  \hspace{1cm} (6.21)

where $E = 72$ GPa and $y = 0.2$ mm, the maximum stresses for the individual configurations are calculated and displayed in Tab. 6.6. As expected, the calculated values match closely those of the FE model. Furthermore if these values are compared to those predicted during generation of the geometry in Tab. 6.2, it is seen that a good correlation is found. These values are included in the ‘$\sigma_{max}$ (Predicted)’ column of Tab. 6.6 for comparison purposes. For the 12 % and 16 % camber

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Table 6.6: Comparison of calculated, actual and predicted maximum stress levels for optimal morphs
6.4. Optimisation

Figure 6.19: Optimal 8% camber morph for minimised $S_{\kappa(s)}$ and $n = 5$: (a) curvature change, (b) morphing geometry.
Figure 6.20: Optimal 12% camber morph for minimised $S_{\kappa(s)}$ and $n = 4$: (a) curvature change, (b) morphing geometry.
Figure 6.21: Stress distributions for $S_{k(s)}$-minimised leading-edge morphs: (a) 8 \% camber, (b) 12 \% camber and (c) 16 \% camber configurations.
morphs the predictions of maximum stress made during geometry generation were within 9% of those experienced in the FE model; and for the 8% camber morph, a difference of only 1.7% was observed. Examining the \( \Delta \kappa \) plot for the 8% camber morph shows that the accuracy of this prediction is a result of achieving accurately the target change in curvature at the leading-edge tip. The optimal morphs for the 12% and 16% camber configurations were unable to achieve exactly the response in this region, resulting in a slight increase of the maximum \( \Delta \kappa \) of the structure relative to that of the specified morphing geometries – see Fig. 6.20(a) & 6.17(a). However, from Tab. 6.6, it is seen that the maximum stress experienced as a result of this increased \( \Delta \kappa \) is captured well by Eqn. 6.21. To summarise, it has been shown that minimisation of \( S_{\kappa(s)} \) has produced Pareto-optimal morphs, which are developable, and have maximum stress levels which are a direct result of the maximum curvature change imparted during morphing.
6.5 Conclusions

This chapter has investigated the design of a morphing leading-edge flap, for application on an aerofoil with the NACA2421 profile. This is a slot-free system with a continuous upper surface, which has potential to enable natural laminar flow in its original configuration. In its actuated state it will enable high lift due to increased camber, and stall to be postponed to larger angles of attack. The key findings are summarised as follows, and discussed in detail below:

- Target geometry was ‘designed’ with morphing in mind, through constraints on curvature continuity, maximum curvature change, and overall length change.

- Displacement and curvature-based objective functions were reparameterised into arc-length space, along with decision variables controlling actuation.

- The previously-defined optimisation protocol was adapted to a three-stage method. Displacements calculated from the morphing geometry were utilised during the Monte Carlo initialisation step, followed by a two-stage gradient-based refinement. This facilitated a design-space reduction, and improved optimal fitness.

- Displacement-based optimisation achieved morphs with sub-millimetre levels of accuracy. However, stress levels were close to the constraint value, and curvature discontinuities were encountered.

- Curvature-based optimisation produced developable morphs with smooth variations in curvature. Stress levels were directly proportional to the change in curvature imparted through actuation, and significantly lower than the material limit.

- Geometrical constraints are capable of producing developable morphing geometry, and could be incorporated into aerodynamic optimisation with minimal computational cost.

Building on the work of previous chapters, initial and target geometries are defined, targeted at achieving a developable morph. The first quarter-chord of the NACA2421 aerofoil is designated as the initial morphing configuration. The NACA parameterisation is then used to generate target geometries of increased camber, but which exhibit the same length, and curvature at the extremities, as the initial geometry. Continuous NURBS descriptions of the target geometries are created using data from the optimised NACA parameterisations.
6.5. Conclusions

A structural optimisation is performed to find the optimum actuation distribution to achieve elastic morphing. Displacements are favoured over forces due to the nature of the structure, which is expressed according to an arc-length parameterisation rather than the Cartesian approach used previously. This avoids problems encountered due to the structure being a many-to-one mapping if expressed on Cartesian axes. A quasi two-dimensional FE model is defined using the NURBS of the NACA2421 leading edge. Optimiser-controlled actuation is defined in terms of arc-length location and orthogonal components of displacement. Two objective functions are defined based on displacements and curvature respectively, which are a reparameterisation into arc-length space of those used in Chap. 4.

These objective functions are minimised using a three-stage optimisation process based on the protocol defined previously. Due to the large design domain of the displacement variables, during an initial Monte Carlo optimisation these values are calculated to ensure feasible solutions. The applied displacements are assigned by assuming actuators will move between like positions in arc-length space during the morphing transformation. Once the actuator locations have been initialised their position is refined using a gradient-based optimisation – still using calculated displacements. A final gradient-based refinement is then performed, where the optimiser has full control over actuator locations and applied displacements.

Minimisation of the displacement-based objective function enabled average errors of 0.2 mm, 0.68 mm and 1.55 mm to be achieved for the 8 %, 12 % and 16 % camber configurations respectively. However, for the 12 % and 16 % camber morphs, it was noted that the error was concentrated near the domain extremity on the lower surface, close to where the structural boundary conditions are applied. The structural stress was a maximum in this region, and the constraint prevented further objective-function reductions, as any shape-fit improvements were met with an increase in stress, violating the constraint. As expected, increasing actuation complexity resulted in improved morphing accuracy, with $n = 4$ being deemed best in the trade-off between morphing accuracy and actuation complexity. The curvature-based objective appeared to be minimised for $n = 2$, as this enabled the optimum curvature fit. Inspection of the resulting geometries showed that whilst the required curvature change had been captured accurately, due to a lack of tangential continuity between the original morphing geometry and the NACA2421 aerofoil, the resulting structures did not match the target geometries. The problem was reduced for increased actuation complexity due to the nature of the calculated displacements in the three-stage optimisation process. To find the truly optimal morph for the leading-edge problems, both objective functions are evaluated for
all optimal morphs, and displayed on a Pareto plot. Pareto fronts are visible for the three target morphs, with the curvature-based objective function providing optimal morphs. The optimal number of actuators ranges between 4 and 5. The curvature objective produced a smoother curvature response relative to the displacement-based minimisation. This resulted in a stress distribution which was directly proportional to the change in curvature, and was closely related to the target curvature change specified by the morphing geometries. This enabled prediction of the maximum stress due to morphing \textit{a priori}. For the 12 \% and 16 \% camber configurations, the geometry was able to predict the maximum morphing stress to within 9 \%; and this value was reduced to within 2 \% for the less-demanding 8 \% camber morph.

The morphing leading-edge problem has demonstrated the benefits of developable morphs, and has investigated ways in which to define and actuate them. By specifying zero length change between morphing configurations, the maximum stress due to morphing can be predicted on the assumption that all stresses will be a result of bending, and hence the change in curvature. Unless the curvature response can be captured exactly, it is likely that additional stress concentrations will occur in regions of high curvature change, and may cause the maximum stress to exceed this predicted value. The maximum stress resulting from the change in curvature between initial and target geometries is therefore a lower bound for the morphing problem. Achieving the required change in curvature accurately will result in this being the actual stress experienced, as was seen for the 8 \% camber optimal morph. This is an extremely useful analysis tool in the design of morphing structures, as a lower bound on the magnitude and location of the maximum stress can be determined from the initial and target geometry, and enable a designer to assess the feasibility of morphing without performing a structural analysis.

Improvements in the specification of morphing geometry could be achieved through enforcing tangential continuity with any connecting geometry, in addition to the curvature-continuity and zero length-change constraints. However it should be noted that specification of the curvature behaviour through tangent allocation may confine the design unnecessarily, and lead to difficulties in morphing – after all it is difficult to ‘draw’ a structural response. Instead it is recommended to use the curvature-based objective function to optimise morphing actuation, but to monitor simultaneously the displacement-based objective function to ensure Pareto-optimal solutions are found.
Chapter 7

Conclusions

This chapter concludes the thesis investigating the design of morphing aircraft structures. It begins with a summary of those preceding it, followed by a discussion of the research contribution made by the contained work. Finally, future avenues of investigation in the field are suggested.

7.1 Thesis Summary

A review of the literature showed that the research field of morphing structures is vast. In particular, morphing structures are well suited to aircraft design. The design of structures that can change their shape to perform optimally in multiple environments is a complex problem requiring multidisciplinary analysis. However, if these goals can be achieved, then the reward will be a huge step forward in terms of aircraft performance and efficiency. A range of smart materials such as piezoelectrics and shape memory alloys are being developed with morphing in mind, to help deal with the huge demands placed on structures during morphing, and to simultaneously provide actuation. Other novel concepts such as compliant structures, bistable composites and corrugated skins are also being developed to address the unorthodox design requirements of morphing systems. Even with these technologies, and the advances in modern computing power, the design of morphing structures is still a complex task. The use of optimisation algorithms is common, due to the large design space and interconnected decision variables of a typical morphing structure. This has led to simplification of the problem to either a structural or aerodynamic optimisation, in order to enable research to be performed in a reasonable time frame. Structural optimisation problems centre around topology optimisation – distributing material in locations where it enables a structure to best perform its required task.
The aeroelasticity of morphing structures was investigated with respect to a morphing shock control bump – a geometrical device capable of shock control in the transonic regime. An optimiser-controlled morphing structure was used to deploy a bump on the RAE2822 aerofoil, which is known for its transonic shock properties. Structural deformations were calculated using nonlinear FE, and the aerodynamic response evaluated via solution of the RANS equations using CFD. Static aeroelasticity was catered for via a weak coupling of the aerodynamic and structural analyses. The CFD solution was strongly grid dependent, in particular to high levels of cell skewness and non-orthogonality at the aerofoil wall. Therefore a dedicated mesh generation algorithm was developed to ensure meshes of comparable, high, quality were generated for all optimisation iterations. An optimal design was produced providing a drag reduction of 4.2%. Whilst this is less than that achieved by fixed SCBs in the literature, the morphing SCB has the added functionality of zero penalty at off-design conditions when actuation is removed.

The complexity and high computational cost of optimising the aeroelastic morphing problem, led to investigation of ways in which to reduce the computational expense of designing morphing structures. The concept of shape morphing was introduced, where it was sought to find the actuation required to actuate from one predefined geometrical configuration to another. An existing shock control bump geometry is used as the target, morphing from an initially flat configuration. A continuous description of the geometry is generated using NURBS optimisation, which enables formulation of two structural shape-fit objective functions: one based on structural displacements, the second on comparisons of curvature. Nonlinear FE models were generated with optimiser-controlled actuation and boundary conditions, to find the optimal manner in which to actuate the target SCB shape. Optimisation was performed using a genetic algorithm and a differential method. Freedom in the ordering of actuator decision variables – a requirement for objectivity – saw the genetic algorithm struggle to improve individuals between subsequent generations. The differential algorithm was able to provide solutions of superior optimality relative to the genetic algorithm, however it was prone to locating local minima if initialised to unfavourable design points. A combination of the two algorithms was seen to provide optimal results for all cases. The geometry-defined morph was seen to be strongly constrained by the stress limits imposed to ensure repeatable elastic morphs are produced. The optimal morph was only able to achieve partially the target shape, with the maximum morphing SCB height only reaching 80% of the target value. Nonetheless, aeroelastic analysis of the optimal solutions exhibited comparable levels of drag reduction with those achieved in the aeroelastic optimisation, justifying the technique as valid for specification
of an aeroelastic-optimisation initial condition. To verify the structural analysis performed during optimisation, a physical demonstration model was manufactured. The resulting displacements were a close reflection of the comparable FE model, giving heritage analysing morphing structures in this manner. Finally the initial and target geometry was investigated to find the aspects which made morphing exactly the shape difficult. Two components of geometrical strain were identified: one due to the length change between the two configurations, and one due to the change in curvature. Furthermore, superposition of the stresses resulting from the two components was able to predict the maximum stress of the two optimal solutions to within 6.5%. This introduces the concept of evaluating morphing feasibility through assessment of only the geometry.

The idea of three-dimensional shape morphing was investigated, in the hope that the geometrical analysis could be extended to surfaces. Again initial and target morphing geometries were defined, and the actuation distribution with which to best effect the shape change sought. Discrete actuation was applied to replicate screw-actuators, the location and magnitude of which were assigned via optimisation decision variables. An optimiser-controlled NURBS curve was used to apply fixed boundary conditions around the perimeter of the morphing region. A two-stage optimisation process was employed: utilising a Monte Carlo search to initialise the problem, followed by a gradient-based refinement of the most promising regions of design space. The optimum configuration was seen to employ six actuators, and the bounding curve was shaped to provide the required bump width, but increased the length of the morphing region enabling a higher bump crest. Again a demonstration model was constructed, the displacements of which correlated well with the optimal FE model, validating the structural modelling applied during optimisation. The displacement-based objective provided optimal solutions, however a second objective function based on Gaussian curvature – a measure of how a surface is bent in two dimensions – was seen to perform badly. Comparison of the target Gaussian curvature distribution with that achieved by a typical optimiser iteration, showed that significant improvements in the Gaussian curvature objective function were impossible whilst respecting the material constraint. It follows that any change in Gaussian curvature will be difficult to achieve with a continuous shell structure such as an aircraft skin. The large in-plane strain that will always accompany a significant change in Gaussian curvature will exceed current material limitations. It is therefore sensible wherever possible to define morphs which are developable, and hence can exhibit large deformations with relatively small in-plane strains. In addition to allowing greater changes in aerodynamic performance, structural actuation will require less energy.

The thesis concludes by examining the design of ‘morphable’ geometry, to aid in the synthesis
of an adaptive leading edge. The original morphing configuration is specified by the first quarter chord of the NACA2421 aerofoil. Three target geometries are then generated with increased camber (of 8 %, 12 % and 16 %). Constraints on length, maximum curvature, and curvature continuity at the boundaries, are employed to ensure developability. The NACA parameterisation is used to generate these geometries, to ensure aerofoil-like shapes are produced. NURBS are fit to the three leading-edge NACA parameterisations to provide continuous data for analysis during optimisation. The effect of morphing from an initially curved structure is shown to be negligible for the prescribed target geometries, and hence the changes in curvature are used to predict the maximum stresses that will result from morphing. A structural model was defined using the NACA2421 NURBS, with optimiser-controlled actuation specified through arc-length location and two orthogonal components of displacement. An adaptation of the two-stage optimisation method used previously was implemented. During an initial Monte Carlo search, displacements were calculated by assuming actuators would move between like locations in arc-length space on the initial and target geometries. This ensured feasibility of the Monte Carlo solutions, and improved optimisation times. Two gradient-based refinement steps were then performed: the first utilising these computed displacements to refine the actuator location in arc-length space; followed by a full refinement, where both the actuator locations and displacements were optimiser-controlled. The same process was repeated for both displacement and curvature-based objective functions. The displacement-based objective was able to achieve average nodal errors of 0.2 to 1.55 mm, with optimisation typically constrained via stress concentrations at the model extremity on the lower surface. The curvature-based objective function was minimised using two actuators for all target morphs. Inspection of the morphing geometry showed that whilst the target curvature distributions had been achieved, a significant difference existed between the target and actuated morphing geometries. It is thought that this difference is due to a lack of tangential continuity with the connecting geometry at the domain boundaries. Nonetheless for increased actuation complexity the objective was seen to produce morphs which had a desirable curvature distribution at the leading-edge tip, and matched the target shape well. Furthermore, due to curvature analysis during optimisation, the optimiser was prevented from creating structures with sharp changes in curvature or localised length changes. This causes developable morphs to be produced, with smooth variations in curvature. This removes stress concentrations relative to the displacement-based objective, and means the maximum stress is solely due to structural bending. This enabled prediction of the stress distribution from analysis of the morphed structure’s curvature to within 1.4 %. The maximum stress was experienced at
the location of maximum curvature change at the leading-edge tip. For the 8% case, where this region of curvature change was captured exactly, the FE model’s maximum stress is predicted to within 1.7% by the value calculated solely from the morphing geometry. The 12% and 16% camber morphs exhibited small increases in curvature at the leading-edge tip, to enable a better fit of the overall distribution. The morphing stress predicted through consideration of the morphing geometry is therefore underestimated by 9% for these cases. It is concluded that a lower bound on the maximum morphing stress can be estimated from the target geometry. This is a powerful tool for assessing morphing feasibility during the early stages of design.

7.2 Research Contributions

The thesis contributes the following to the research field of morphing aircraft and adaptive structures:

- Developed a framework for performing high-fidelity, static aeroelastic, optimisation of morphing structures: demonstrated feasibility of transonic shock control via deployment of a morphing shock control bump.

- Investigated the concept of shape morphing in two and three dimensions: defined an optimisation procedure for performing stress-constrained minimisation of displacement and curvature-based objective functions.

- Demonstrated a range of optimiser-controlled structures: applying forces and displacements, variable boundary conditions using NURBS, parameterised in both Cartesian and arc-length space, and in both two and three dimensions.

- Introduced the concept of designing morphing geometry: specification of developable morphs enabled conclusions on morphing feasibility to be drawn \textit{a priori}.

Designing morphing structures through optimisation of their aeroelastic response is vital to ensure stability of the structure in operation. The developed framework enabled gradient-based single-point optimisation of the lift to drag ratio, accounting for static aeroelasticity. Whilst the investigated morphing shock control bump responded well to optimisation, the aerodynamic improvement was comparable with that provided via shape morphing in Chap. 4. Aeroelasticity was not seen to contribute significantly to the final shape assumed by the morphing geometry, due to
an order of magnitude difference between the actuation forces and aerodynamic loads. Significant computational expense is incurred performing this analysis. Coupling the finite-difference style sensitivity analysis with aeroelasticity, results in a large number of function evaluations per optimiser iteration. This problem would be compounded with the inclusion of uncertainty and/or multipoint aerodynamic analysis. In the literature review it was seen that adjoint methods are often employed to overcome the large computational cost of calculating the problem sensitivity via finite differences. For the morphing problem, this would require linearisation of both the aerodynamic code, and the structural model from which the displacements are produced. Whilst this would provide a powerful analysis tool for morphing structures, it is beyond the scope of this thesis.

Due to the large computational expense of performing an aeroelastic optimisation, the concept of shape morphing was introduced. This investigated the requirements of morphing from one predefined shape to another. Aerodynamic loading, which was seen in the first chapter to have negligible effect on the morphing structure, is omitted from this preliminary design stage, but can be re-introduced in a subsequent phase once promising morphs have been identified. To respect industry practice the computational geometry was described by NURBS – the standard language of many CAD/CAE packages. This provides a continuous description of the geometry during analysis, and enables direct exportation of optimised geometries to subsequent engineering analysis or manufacturing tools. A range of morphing structures were considered, requiring parameterisation of optimisation variables, and structures, in both Cartesian coordinates and arc-length space. For non-developable structures, forces were the preferred method of application, due to their ability to coexist in close proximity without creating a large stress in the adjoining structure. For the developable leading-edge morphs, displacement actuation was applied. Using displacements calculated from the morphing geometry aided convergence, by improving the fitness of initial Monte Carlo solutions. For the two and three-dimensional shock control bump morphs, all designs were stress limited due to active constraints. For these cases the displacement-based objective function performed best, allowing relaxation of the structural boundaries to maximise morphing displacements. It is observed that simultaneously applying both in-plane and bending stresses results in violation of the material constraints, when morphs exhibiting significant displacements are targeted. This problem is magnified in three dimensions when changes of Gaussian curvature are sought. Changes of Gaussian curvature provided via discrete actuation were limited, and resulted in areas of high stress at the site of actuation, but had negligible impact on the curvature response elsewhere. This is due to the large in-plane strains that are required to enable a significant change of Gaussian
7.2. Research Contributions

curvature. Morphs exhibiting these changes will require materials with extremely high yield strains to facilitate morphing, and will require large actuation forces to adapt and hold the structure in its morphed configuration. The research suggests that such morphs should be avoided for these reasons, and instead the focus where possible should be on morphs with zero change in Gaussian curvature. Such morphs are termed developable, and result in zero strain of the structure’s neutral axis during shape change. Large changes in geometry can be exhibited by developable structures, which can be actuated by relatively small actuation forces. Furthermore, these changes can be achieved using existing, certified, materials and actuation methods. This makes them the ideal shape-morphing mechanism for morphing aircraft.

The concept of designing morphing geometry is useful in bridging the gap between structural and aerodynamic morphing analysis. It was seen in a review of the literature that there is a tendency to focus on how to achieve a structure capable of large displacements, whilst neglecting aerodynamics. This has generated a plethora of adaptive concepts, many of which lack the heritage of proven aerodynamic performance. By modelling the wing-skin geometry – the only part of the structure that must be morphed – it is possible to define shapes of known aerodynamic performance. Furthermore, it has been shown that by simple geometrical analysis, conclusions on the feasibility of morphing between two predefined shapes can be drawn a priori. This not only allows assessment of existing aerodynamic shapes, but also allows definition of a geometrical constraint for an exclusive aerodynamic optimisation. The maximum length and curvature change can be limited during generation of shapes for aerodynamic analysis, ensuring that the optimised geometry is a feasible morph. Once feasible morphing geometry has been defined, it recommended to use a curvature-based shape-fitting objective function to optimise actuation. It was demonstrated that this will preserve the developability of the morph, and minimise the maximum stress. The knowledge of optimal actuation locations, and the displacements they must apply, is extremely valuable in a subsequent structural optimisation. For example, the actuation locations can define output points in the parent-lattice parameterisation of a load-path topology optimisation problem. Fixing of the number and location of output points required to effect the morph a priori, bounds the complexity of the parent lattice, and removes degrees of freedom from the topology optimisation. This objective design-space reduction will improve convergence and reduce the computation time of the subsequent structural optimisation.
7.3 Future Work

The concept of shape morphing has been shown to be a useful analysis tool in the preliminary design phases of morphing structures. To develop this work further it would be useful to apply it to the design of a range of adaptive systems. For example, consideration of a morphing reflector for space applications, or the design of a morphing shock control device for a scram-jet inlet. It would also be interesting to consider the design of an adaptive aerofoil, where both the entire upper and lower surfaces are able to morph. Developability and feasibility could be specified through the techniques in Chap. 6, and a morph between existing high-lift and low-drag configurations investigated. The geometric nature of the analysis would allow constraints to be enforced during generation of the morphing geometry, to ensure a wing stiffener such as a wing box is accommodated and exhibits the required connectivity with the wing skin. Estimations of the required actuation energy and structural mass could also be made, enabling rapid assessment of design feasibility.

During the thesis material selection has been limited to isotropic metals, to aid development of design techniques. This provides a straightforward approach for structural modelling, and analysis of the maximum stress. Additional shape control could be achieved via tailoring of local structural attributes, such as variations in thickness and stiffness. Specifically, CFRPs will allow creation of materials with directional properties, which may aid morphing whilst retaining structural integrity. Tow-steering of fibres, and variations in fibre volume fraction will enable the local Poisson’s ratio and Young’s modulus to change, which may enable stress alleviation and tailoring of the curvature response. In order to use such materials the optimisation constraint will require a composites-based derivation as opposed to a von Mises criterion. A first-ply failure technique is likely to be suitable to ensure no damage occurs in the structure and the material remains in the elastic range. Added design variables such as ply angle and stacking sequence would increase the available design space, the opposite of the target of much of this work. It is advised to include these variables with caution, particularly on a new problem where little or nothing is known about the feasibility of morphing, as the increased design space may lead to sub-optimal local minima being found. Information from the geometrical analysis could be utilised during the structural parameterisation. It was seen that the areas of maximum curvature change were also the locations where maximum stresses occurred. Therefore the curvature distribution could be used to assign regions of variable thickness to high-stress locations, in order to reduce the maximum stress due to bending.

Chapter 5 concluded that morphs which exhibit zero change in Gaussian curvature are desirable.
However, they are unlikely to be achieved during implementation of three-dimensional morphing structures in practice. Wing bending in the spanwise direction will instigate a curvature change, which, if coupled with chordwise morphing, will result in non-zero Gaussian curvature. The effect of uncertainty in the level of spanwise bending on chordwise morphing should therefore be evaluated. This may enable definition of a bound on the level of chordwise curvature change, which respects the strain present under extreme aerodynamic loads such as gusts. Such a problem was encountered in [166], in the design of a smart-droop leading edge. The morphing structure was able to deform as predicted in ground tests in the absence of wing-bending loads. However, when forces simulating 2 g loading were applied to bend the wing in the spanwise direction, a stress concentration was experienced at the leading edge, close to the region of maximum chordwise curvature change. If in the early phases of design the imparted Gaussian curvature change due to wing bending had been analysed using geometrical techniques, this stress concentration could have been avoided.

A significant contribution of this work is its ability to bridge the gap between aerodynamic and structural design of adaptive structures. The highly multidisciplinary field of morphing structures, and morphing aircraft in particular, requires unification of these individual disciplines to enable production of commercially-viable morphing systems. The common language between all fields is the geometry, and hence this should be exploited to enable information to be exchanged during all stages of the design process. It is hoped therefore that the techniques presented in this thesis will enable increased communication between aerodynamicists and structural engineers, accelerating the development and production of morphing systems in the future.
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