THE MECHANISM OF MIXING:

VORTICULAR MOTION and its APPLICATION

Thesis submitted for the degree of Doctor of Philosophy in the University of London by G.C. Shipp, B.A. (Cantab.) of Imperial College (London) Chemical Engineering Department.

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### BIBLIOGRAPHY
INTRODUCTION: PART I

The Unit Operation of Mixing
(a) The Unit Operation of Mixing

The unit operation of mixing is of practical importance throughout the whole of chemical industry, being employed, at some stage, in most industrial processes. In addition to the straightforward mixing of materials - solids, liquids and gases - in their various combinations, this operation is employed for

1. The promotion or control of chemical reactions;
2. The dissolution or suspension of solids in liquids;
3. The preparation of emulsions, foams, mists, smokes, etc., and
4. The improvement of mass and heat transfer processes.

This short list gives an idea of the wide scope of the operation and justifies the attention which is being paid to it.

(b) The Definition of Mixing

It is essential, if any serious attempt is to be made to understand the subject of mixing that what is meant by the term "mixing" should be clear. The literature does not help a great deal in this respect, due to the fact that the operation is looked upon by some from the economic and by others from the statistical point of view.

Mixing is considered complete in the former case when particles of the initially separate components are in such relative positions that a desired result is obtained, whereas in the latter case mixing is considered complete only when the particles of each component are evenly distributed throughout the whole volume of
material. This statement may be looked upon as a generalised
definition of mixing. It should be pointed out that "complete"
mixing is not, in the general run of processes, "perfect" mixing and
indeed in some cases "perfect" mixing may be undesirable.

Asquith (53, 56, 57) refers to "The process of mixing, which
in general may be defined as a means whereby a heterogeneous mass is
rendered sufficiently less heterogeneous for the purpose in hand".
This definition is limited in that it does not refer to the economic
aspect and gives no indication of perfect mixing.

Hinson's definition (54) is better: "It may be defined," he says, "as the unit operation of which particles of the components
of a mass of materials are put into such space relation to one another
that a desired result may be obtained in a minimum time with the least
expenditure of energy". This leaves the question of the "desired
result" to the operator of the process. In addition the "minimum time"
may be irreconcilable with the "least expenditure of energy".

In the literature appears this definition of perfect mixing (90): "In all cases two or more materials existing either separately or in an
unevenly mixed condition are, by mixing, to be put into such a condition
that each particle of any one material lies as nearly adjacent as possible
to a particle of each of the other materials". This definition cannot
be applied to the emulsifying of two immiscible liquids, since what is
meant by a "particle" of the material has not been specified. Stable,
uniform emulsions can be prepared over a very large range of "particle"
sizes.
There also exists some confusion as to the various terms used within the general scope of mixing operations. The term "kneading" has a special application to doughy masses and indicates a stretching and tearing action. "Whipping" has a special application to milk and cream and is always carried out at high speed, being a beating together of the fluids, resulting in the production of a fine-grain emulsion. "Blending" is usually applied to the mixing of miscible liquids, of liquid hydrocarbons, for example, to form a petrol, though it may also be used for the mixing of solids. The terms "agitation", "stirring" and "mixing" are very frequently used interchangeably.

Normally "agitation" and "stirring" would be used to refer to the creation of motions within a homogeneous phase, usually a liquid, and "mixing" to the agitation or stirring of more than one phase, not necessarily liquid. In this thesis the word "mixing" is to be construed to cover all these various operations.

(c) The Classification of Mixing Equipment

In the past there has been little investigation of the principles underlying the operation of different types of mixing equipment, and workable mixers for given processes have been designed without knowledge of these principles. In consequence there are many types of mixers in use in industry, but they can generally be assigned to one or other of five classes, namely:-
(1) Flow mixers;
(2) Rotating element mixers;
(3) Helical mixers;
(4) Turbine mixers, and
(5) Tumbling mixers.

In the first group are, for example, injectors, orifice columns, spray towers and packed towers. In the rotating element group are the paddle, gate and anchor mixers and the Sigma-bladed kneading machines. Also in this group are the rotating pan mixers, where although it is the pan and charge which rotate about a stationary arm the arm is rotating relative to the charge.

The helical classification contains the many variations of the simple propeller and helical-ribbon mixers for solids. In the next group are the turbines, straight and curved-bladed, single and multiple, with or without stationary deflecting rings. The last group includes the tumbling barrel, or drum, double cone, mushroom and rotating cube mixers, principally for solids.

Recent articles by Valentine and Maclean (90) and Smith (62, 63) present a picture of the mixing equipment at present available.

(d) The Classification of Mixing Systems

All mixing systems may be classified into seven categories, namely, (a) Gas-gas (b) Gas-liquid (c) Gas-solid (d) Liquid-liquid (e) Liquid-solid (f) Solid-solid, and (g) Gas-solid-liquid.
There is little literature dealing with the system (a) Gas-gas. A paper by Chilton and Genereaux (47) reported investigations in one particular case and indicated methods of improving mixing. The mixing of gases in open-hearth furnaces has also been investigated.

Very little quantitative information is available on the systems (b) Gas-liquid and (f) Solid-solid. References may be found in papers by Cooper, Fernstrom and Miller (55) on gas-liquid contacting, and Maitra and Coulson (60, 64) on solid-solid systems.

The system (c) Gas-solid is generally regarded as being within the province of drying, but it has other applications, such as the fluidisation of powdered catalysts. The system (g) Gas-liquid-solid finds application in, for example, the catalytic hydrogenation of oils.

The two systems which remain, (d) Liquid-liquid and (e) Liquid-solid comprise nearly the whole of the published work on mixing. The related operations of emulsification and colloidal suspension are not normally included under the heading of mixing and are, in consequence, not dealt with here.

The more specialised problems of agitation in heat transfer apparatus (35-45) and autoclaves and very high speed stirring (26-34) have also been omitted from this survey, although a number of references to papers dealing with them are included in the bibliography.

At the present time there is a great deal of research and development in progress on agitation. This work is divisible into two main groups, one concerned with industrial applications, and the other
concerned with more fundamental work.

The first group has for its objects the design of new, or the adoption of existing, equipment for specific purposes and the extension of the fields of utility and increase of efficiency of equipment already in production.

The second group, research into the fundamental aspects of mixing, is the main concern of this review. In this group fall the suggestion and examination of methods of measuring agitator efficiency, sampling, methods of power measurement and so on. These will be discussed with particular reference to processes involving a liquid phase.

(e) The Selection of Mixing Equipment

In order to select a mixer for a process a suitable specification must be available. The details required in this specification will vary from one process to another, but certain basic requirements may be laid down.

Information must be available about the viscosity of the materials to be mixed, that is, whether they are of Newtonian or non-Newtonian character. This is important from the point of view of impeller speed. The specific gravities of the various components must be known since this property affects power requirements and, if any tendency to settle is evident, the type of mixing element.

Other physical properties of the materials, both before and
during mixing, may need to be considered, such as ease of wetting, particle size and adhesiveness. The surface tensions of liquids may be important or, in the case of emulsions, the interfacial tension. The temperature effect of the addition should also be known. The relative proportions of the ingredients, corrosion characteristics and order of addition to the mix should also be stated in the specification.

If, now, the procedure be known (that is, whether large batches, small batches or continuous working is required) a suitable mixer with appropriate materials of construction and a matching power unit can safely be recommended.

The selection of mixers is dealt with fully by Valentine and Maclean (90).

(f) The Flow of Fluids

None of the early work on fluid flow (up to about 1920) has any very direct application to the process of mixing. The special subject of the flow between concentric rotating cylinders will be dealt with in detail later. This type of flow was investigated by Taylor and used in a mixing system by Black.

The literature pertaining to the flow in mixing vessels is of a purely qualitative character, the motion (with the exception of that between cylinders) having proved too difficult for attack by the normal methods of hydrodynamics.

It has been noted that visual evidence of turbulence
(or placidity) does not necessarily mean good (or bad) mixing (13). This is particularly noticeable with slow-speed paddles, where there is usually very little evidence of turbulence although mixing may be good.

For a given impeller three types of motion have been described (7), "passive", "curvilinear" and "turbulent", depending on the impeller speed, but no attempt has been made to connect the type of flow with efficiency, nor to define the flow in any but the vaguest terms.

In other experiments (49) it has been found that sand is not suspended uniformly in water by a paddle. This is due to the fact that there are three regions of flow in the mixing vessel; one with a vertical velocity component, one without and one with a centrifugal component. Their dependence upon paddle shape and speed have not been investigated.

The drag on a mixing vessel due to a rotating impeller in the contained fluid has been measured (17), but no attempt was made to connect the drag with the type of flow. Similarly a method has been developed for measuring the displacement capacities of impellers in mixing vessels (59), but the primary concern was the amount of flow and not the type.

Simultaneous flow of fluid and mass transfer have, in general, been dealt with by analogy with previously-derived heat transfer equations (9), attacking the problem by dimensional analysis.

More recently attention has been given to the effect of
baffles. It has been found that increasing the number, width or depth of baffles increases the power drawn by the impeller (at constant speed) until a maximum is reached. At this maximum power the flow is said to be "fully-baffled" since further baffling does not alter the power input (23).

A discussion of the possible types of flow behind baffles (vortex rolls or vortex streets) has recently been published. In the same paper the effect, on the flow, of offsetting the baffles was considered (65).

In an appendix to this thesis the flow in a propeller-agitated liquid is briefly discussed and illustrated by photographs.

(g) Criteria of Mixing

Since serious study of the operation of mixing commenced, in 1922 (13) a number of criteria which give an indication of mixer performance have been suggested. Such criteria are essential for the comparison of agitators. The methods employed may be considered under five headings, namely:

1. Variation of rates of reaction and solution with agitation;
2. The dispersion of electrolytes in water;
3. The distribution of insoluble solids in liquids;
4. Liquid-liquid extraction and
5. Density variations in a mixture of liquids.

In addition there are various other methods which will be
mentioned, briefly, after the discussion of those set out above.

Rates of Reaction and Solution (1)

Fick (1) in 1855, enunciated a law relating to the diffusion of dissolved substances through liquids. He stated that the amount of solute, $dw$, which diffuses across an area $A$, in a time $dt$, through a thickness $dx$ at right angles to the plane of $A$, is given by

$$\frac{dw}{dt} = -D \cdot A \cdot \frac{dc}{dx} \quad \cdots \cdots \cdots (1)$$

where $c$ is the concentration and $D$ the diffusivity constant. Constant temperature is assumed throughout. Fick verified this expression for diffusion upwards against gravity.

Fick's law was not concerned with the process of solution, but only with the diffusion of a solid in solution. Noyes and Whitney (2) first studied dissolution by rotating cylindrical stocks of benzoic acid and lead chloride in water. They postulated a film of saturated solution in contact with the solid and assumed the rate of solution to depend on the diffusion of this saturated film into the bulk of liquid.

For finite concentrations Fick's law becomes

$$\frac{dw}{dt} = -D \cdot A \cdot \frac{\Delta c}{\Delta x} \quad \cdots \cdots \cdots (2)$$

where $\Delta x$, the film thickness, depends upon the intensity of agitation. At constant agitation, if the amount of solid dissolved is very small, this may be rewritten, since the area is now constant

$$\frac{dw}{dt} = K_1 (c_s - c) \quad \cdots \cdots \cdots (3)$$
where $c_s$ is the saturation concentration and $c$ the concentration in the main bulk of liquid. Rewriting,

$$\frac{dc}{dt} = k_2 (c_s - c) \tag{4}$$

which on integration between $c$ and $c_0$ (the initial concentration) gives

$$k_2 t = \log(c_s - c_0) - \log(c_s - c) \tag{5}$$

Noyes and Whitney verified this equation by demonstrating the constancy of $k_2$ within the limitations imposed.

Murphree (4) deduced an equation connecting reaction rate and crystal dimensions, followed by Hixson and Crowell (7) who introduced the "cube-root law" of crystal dissolution.

We have seen that

$$\frac{dw}{dt} = -k_2^A (c_s - c) \tag{6}$$

which may be rewritten as

$$\sqrt[3]{\frac{dw}{dt}} = -k_3 w^{2/3} (w_s - w_o + w) \tag{7}$$

where $w_o$ is the original weight of crystals, $w$ the weight at time $t$ and $w_s$ the weight needed to saturate the solution of volume $V$. This equation assumes that agitation is the same at all faces of the crystal, that no breaking occurs, that the surface area is proportional to the two-thirds power of the weight and that the agitation is sufficient to create a constant concentration in the bulk of the liquid.

This equation yields, on integration, the cube-root law, an expression too complicated for use in mixing systems. With certain simplifying assumptions various useful forms may be obtained, which are
discussed later.

This treatment has been verified and extended by a number of workers (8, 9, 10) and used in comparing various systems of agitation.

The Dispersion of Brine in Water (2)

The first reported work on agitation was that of Wood, Whittemore and Badger in 1922 (13). The system they investigated was a wooden paddle rotating in a tank of some 600 gallons capacity.

After unsuccessfully attempting to use the distribution of methyl violet (estimated colorimetrically) and hydrochloric acid (estimated by volumetric analysis) as criteria of agitation they developed a conductivity method. The liquids used were water and a strong brine.

The paddle was started and brine introduced into the bottom of the tank full of water. Samples of the liquids were drawn off through a number of conductivity cells arranged in series. Initially all the cells were arranged to have the same resistance and across each was a voltmeter. Mixing was assumed complete when the voltage drops across the cells again became equal.

The time taken to reach this state was taken as the measure of the intensity of agitation. These workers discovered that the simple paddle was much more efficient than usually believed.
The Distribution of Sand in Water (3)

White (48, 49, 51) determined the distribution of sand in water using a simple paddle agitator rotating at constant speed. He found, as had Wood and others, that the paddle was surprisingly effective.

Using special sampling devices White and his collaborators determined lines of equal concentration of sand in their tank and discovered the variation of concentration with distance of the paddle from the bottom of the vessel. They noticed that classification of the sand occurred and subsequently suggested that a paddle may be used as a classifier (50). Mixing experiments required closely-sized sand. They also concluded that particles of solid were not necessarily associated with definite filaments of liquid.

Hixson and Tenney (16) modified this approach and based their criterion on the evenness of suspension, using a so-called "mixing index". This index is defined as

\[
\% \text{ mixed} = \frac{S}{S_o} \times 100 \quad \text{if the liquid is in excess and}
\]

\[
\% \text{ mixed} = \frac{l}{l_o} \times 100 \quad \text{if the solid is in excess.}
\]

In these definitions

\(S = \% \) of solid by weight in the sample

\(l = " \) liquid " " " " "

\(S_o = " \) solid " " " " whole mixture

\(l_o = " \) liquid " " " " "

If representative and reproducible samples can be taken this
method yields an average mixing index which can be used for comparison purposes. A somewhat similar index has been used by Miller and Mann (21) for mixtures of immiscible liquids.

The Extraction of a Dye from One Liquid by another Immiscible with it. (4)

This method has been used, in very small-scale laboratory equipment, by Yates and Watson (13). The two liquids used were paraffin and distilled water containing acetic acid and methyl red.

The two liquids were agitated and the dye (25 mg/litre at the start) was transferred to the paraffin. The arbitrary end-point selected was when 10 mg/litre of the dye remained in the water phase. This end-point was ascertained by stopping the agitator at different times, in successive experiments, so that the amount of dye remaining after the various times of agitation could be measured colorimetrically. A graph was drawn connecting time and remaining dye and the arbitrary end-point found by interpolation. This time was expressed in terms of a transfer velocity and its variation with agitator speed and vessel size determined.

More recently a similar method has been used by Hixson and Smith (61), using the system water-iodine-carbon tetrachloride.

Observations of Density Variations by the Method of Striae. (5)

This method has been used by Dodd (46) in measuring the time required to mix systems of two miscible liquids. When the two liquids were imperfectly mixed there were density variations throughout the
bulk of liquid. These variations were viewed as striae when light
was projected through them by a suitable optical arrangement. The
time at which the striations disappeared was taken as the point of
complete mixing.

Miscellaneous Methods

The methods discussed shortly above serve to give an idea
of the lines of thought taken in mixing research, but they do not
comprise the whole of mixing criteria. Among others used, or
suggested, are:

1. Rate of heat transfer through agitated liquids;
2. Adsorption of a dye in solution by a solid in suspension;
3. Slow-motion photography;
4. Bleaching of oils by Fullers' earth;
5. Impeller pumping capacity;
6. Power measurements in specified conditions and
   radioactive tracer elements.

Others have been employed in this research and will be
described later.

(h) Methods of Sampling

The majority of the mixing criteria discussed in the previous
section require that there should be accurate methods of sampling.
The sampler used should not disturb the flow pattern when inserted,
should be instantaneous in operation and should not interfere with
subsequent mixing. In practice only a compromise can be found.
Standardisation of sampling methods for liquid fuels has been effected by the Institute of Petroleum and a useful summary of these methods is given in "Fuel Testing" (92).

Using a large mixing vessel (a steel cylinder, 52" in diameter, of 500 gallons capacity) White and his co-workers (48) determined the distribution of sand in water. Samples were obtained through glass tubes of 5 mm. diameter, 26" in length, which could be pushed up to this length into the tank. The points of entry were spaced at 4" intervals vertically up the side of the tank. Each glass tube was fitted with a rubber tube and clip.

To take a sample the clip was released and liquid drawn off for 15 seconds, being rejected. A measured volume was then drawn off, filtered and the weight of sand determined. A somewhat similar system was used by Wood, Whitemore and Badger.

Harrison and Tenney (16) report a quite different method which has the merits of simplicity, easy use and a wide field of utility. The sampler consisted of a long glass tube inside which moved a rod with a conical bung at one end and a leather washer at a suitable distance from it. The glass tube was drawn back to the level of the washer and the sampler inserted into the liquid. When the disturbance had settled down the glass tube was quickly thrust down, trapping the liquid between the washer and the bung. The sampler was then removed for analysis of the contents.

The entrapped volume could be varied by altering the relative
positions of the washer and bung, or the diameter of the glass tube. Extension rods could be fitted to the bottom to enable reproducible samples to be taken at specified distances from the vessel bottom. These workers chose a number of sampling points and averaged results.

In small-scale experiments Hixson and Baum obtained satisfactory results using pipettes with large ends. Previously they had used a much more complicated method, withdrawing samples through three glass tubes, connected to a common vessel, by applying a vacuum.

The sampling of solids is, in general, more difficult than that of liquids. The procedures for sampling coal and coke in bulk are set out in British Standard Specifications and summarised in "Fuel Testing". An interesting laboratory method has been introduced by Maitra (60).

(j) Measurement of Power

In the past four main methods have been employed in measuring power input to mixers, namely:

(1) The measurement of electrical power supplied to the agitator at load and no-load conditions;

(2) The direct measurement of power supplied to the driving shaft by the motor;

(3) The measurement of the torque produced on the agitator vessel and
The measurement of the torque produced on the motor.

The first method was employed in early work. Wood and others (13) and Hixson and Wilkens (8) used it, but found it unsatisfactory. Stoops and Lovell (20) found that they were unable to correlate power inputs as measured electrically with data obtained using a Prony brake.

Among the disadvantages of this method are that with small units the net power input may be of the order of the meter inaccuracies, gear efficiency varies with speed and load, circuit losses are unknown and variations in supply voltage may be troublesome. With very accurate meters some reproducible results are claimed, but, in general, this method should be used only on a large-scale equipment with large net power inputs.

The second method is now a standard laboratory technique. White and others (14, 15) "broke" the driving shaft, connected rods to the top and bottom portions and reconnected with a calibrated spring. This was improved upon by Stoops and Lovell, who used disks instead of rods and measured spring extension by an electrical make-and-break device. The tendency is now towards simpler devices, readings being taken stroboscopically.

In the third category are the "torque-tables" in which the torque transmitted to the containing vessel is balanced by a restraining couple. Hixson and Luedke (17) and Taylor (32) report work carried out with this type of equipment. The main
drawback is that for even small mixing vessels the torque-table needs to be large and robust. This means more friction in the table bearings and a larger moment of inertia. Consequently, the experimental error tends to be large.

The torque table has been criticised (56, 57) on the ground that only the horizontal component of the transmitted torque is measured, neglecting the vertical component, which is neither constant nor a fixed proportion of the total.

Torque tables are also used for power measurements in the fourth category. In this case the motor is free to rotate and the restraining couple is measured as before. The same disadvantages apply so that this method is suitable only for small motors.

(k) Results and Correlations Achieved

The results of mixing research fall into two main groups. The first group connects intensity of agitation with liquid properties, vessel and impeller dimensions and impeller speed, while the second connects power consumption with vessel and impeller dimensions and so-called Reynolds numbers of the flow.

Some results of a qualitative nature have been mentioned above. In this section a few of a more quantitative nature will be noted, in order to give an idea of general trends and tendencies in this field.

Milligan and Reid (35) measured reaction velocities for the
ethylolation of benzene and the hydrogenation of cotton-seed oil, finding them to be directly proportional to stirrer speed. Later work (6) showed two other types of reaction, where the reaction velocity was proportional to the stirrer speed after a certain speed had been reached and where stirring had no effect. The experimental data were covered by equations of the type

\[ k = k_0 + K_N \]  
\[ k = K_N(N - N_0) \]  
\[ k = k_0 \]

for the three classes. In these equations

- \( k \) = reaction rate,
- \( k_0 \) = reaction rate without agitation,
- \( N \) = stirrer speed,
- \( N_0 \) = some critical stirrer speed.

Hixson and Crowell (7) have verified the general and various special cases of their cube-root law of crystal dissolution. The general case is

\[ \frac{K}{t} = \frac{V}{F^2} \left\{ \sqrt[3]{\frac{\alpha_{m} - \frac{1}{3} \pi \frac{2}{3} \left( 3 \cdot \frac{2}{3} \pi \left( F - h \right) \right)}{3F^2 + (2hF)(2j - F)}} + i \cdot 1.513 \frac{\log \left( F + h \right)^2 \left( F^2 - F^2 - F^2 - h^2 \right)}{F(j + h)^2 \left( F^2 - F^2 - h^2 \right)} \right\} \]

where

- \( f = \left( \frac{W_s - W_o}{W_o} \right)^{\frac{1}{3}} \)
- \( h = \frac{W_o}{j^{1/3}} \)
- \( j = \frac{1}{h^{1/3}} \)

When there is just enough solute present to saturate the solution (i.e., \( f = 0 \)) we have

\[ K \frac{t}{t} = \frac{V}{j^{1/2} - \frac{1}{h^{1/2}}}; \]  

\[ \text{----------(12)} \]
when the concentration change in the solution is negligible we have

\[ \text{Kg} = (h-j) \]  \hspace{1cm} (13)

It should be noted that equations (12) and (13) cannot be obtained by substitution in (11). Hixson and his co-workers have extended this treatment and it is now an accepted method for the evaluation of mixing intensity.

Hixson and Baum, working from an approximate integrated form of the cube-root law, arrived at expressions for mass transfer in geometrically similar agitation systems. For turbine agitators two states of flow were recognised and the expressions took the form

\[ \frac{k_d}{D} = K_{i0} \left( \frac{N d^2 \rho}{\mu} \right)^{1.62} \left( \frac{\mu}{\rho D} \right)^{0.5} \]  \hspace{1cm} (14)

at Reynolds numbers greater than \( 6.7 \times 10^4 \) and

\[ \frac{k_d}{D} = K_{i1} \left( \frac{N d^2 \rho}{\mu} \right)^{1.4} \left( \frac{\mu}{\rho D} \right)^{0.5} \]  \hspace{1cm} (15)

at Reynolds numbers below this value. In this work the Reynolds number was defined as \( \frac{N d^2 \rho}{\mu} \). In these equations

- \( d \) = vessel diameter
- \( \rho \) = solvent density
- \( \mu \) = solvent absolute viscosity

For a propeller agitator the same workers obtained

\[ \frac{k_d}{D} = K_{i2} \left( \frac{N d^2 \rho}{\mu} \right)^{1.0} \left( \frac{\mu}{\rho D} \right)^{0.5} \]  \hspace{1cm} (16)

and no critical Reynolds number was observed.

The first attempt to connect power requirements with tank
and impeller dimensions and liquid properties was made by White, Brenner, Phillips and Morrison (15). The problem was attacked by dimensional analysis, a method followed by most subsequent workers in the field. The result obtained for a paddle was

\[ P = K_{13} L^{2.72} \mu^{0.14} N^{2.86} \rho^{0.86} d^{1.1} W^{0.3} H^{0.6} \quad \ldots \quad (17) \]

where \( P \) = power input,
\( L \) = paddle diameter,
\( W \) = paddle width,
and \( H \) = liquid depth.

Some evidence of a critical point dividing two types of flow was found and accordingly the accuracy of the expression should not be taken for granted in all conditions.

Hixson and Luedeker (17) measured the power lost as wall friction in agitation systems by a torque table method. For propellers in the turbulent region they obtained

\[ P = K_{14} \rho^{0.79} \mu^{0.21} N^{2.79} d^{3.58} (d+4H) \sin \theta \quad \ldots \quad (18) \]

In this equation

\[ \theta = (1.13y - 12) \]

\( y \) = stirrer pitch, degrees.

The criterion of similarity was taken as \( \frac{Nd^{2}\rho}{\mu} \).

The work of Yates and Watson was a distinct advance in that they deduced an efficiency for their apparatus. Power measurements indicated that the power was proportional to about the third power of the speed of the impeller. In addition the logarithm of the mass
transfer rate was found to be proportional to the speed. By combining these two sets of measurements a plot of transfer rate/power (i.e. efficiency) against speed was obtained, which showed a decided optimum speed for their apparatus (fig.1).

For turbine-type agitators the expressions below are reported in the turbulent range.

\[ P = K_{15} N^{2.77} d^{0.54} \rho^{0.77} c^{1.33} \] .................................. (19)

\[ P = K_{16} N^{2.83} d^{0.76} \rho^{0.53} c^{1.12} \] .................................. (20)

Stoops and Lovell for a marine-type propeller obtained the equation

\[ P = K_{17} N^{2.18} d^{0.51} \rho^{0.52} c^{2.37} \] .................................. (21)

No critical Reynolds number \( \left( \frac{N L^2 \rho}{\mu} \right) \) was observed.

Miller and Mann (21) in experiments in single-phase liquid system reported the expression

\[ P = K_{18} N^{2.78} d^{0.78} c^{0.22} L^{4.57} \] .................................. (22)

The impeller used was a two-bladed flat paddle.

Mack and Kroll (23) with a simple flat paddle in "fully-baffled" conditions expressed their results in the form

\[ P_g = K_{19} N^3 L^5 \] .................................. (23)

where \( g \) is the acceleration due to gravity. Following on this Mack and Harriner (12) correlated the performances of dissimilar radial-type impellers and dissimilar liquid dimensions, in fully baffled conditions, by the equation
FIG. 1. STIRRER EFFICIENCY CURVE.
where $\psi$ is some function and $t$ is the time required to complete the reaction studied.

Recent articles by Rushton and others (24.) report extensive experiments on power consumption. The data are plotted as the variation of a "power group" or "power number", defined as $P_g/p\nu^{1.5}$, with the Reynolds number of the flow, defined as $\rho N L^2/\mu$. Typical curves are shown in figure 2, bearing out the work of Mack and Kroll (see earlier) on the effect of baffles. Below Reynolds numbers of about 500 baffling has no effect on the power drawn. Above this value increasing the number of baffles increases the power consumption until a maximum is reached. This maximum power corresponds to the "fully baffled" conditions discussed earlier and in this condition power consumption is proportioned to the cube of the impeller speed.

All the work reported above has been carried out with conventional impellers - paddles, propellers and turbines. The power requirements of liquids contained between a stationary outer and a rotating concentric inner cylinder have been measured by Black. These results will be dealt with more fully later, but they are quoted here for comparison with those above. The equations refer to three different flow systems examined.

$$P = k_{20} \rho N^{2.5} \nu^{0.768}$$ ..........................(25)

$$P = k_{21} \rho N^{2.5} \nu^{0.768}$$ ..........................(26)

$$P = k_{22} \rho N^{2.5} \nu^{4/3}$$ ..........................(27)
FIG. 2. GENERAL POWER CORRELATION.
In these expressions \( \nu \) is the kinematic viscosity. The important point to note is that the index of \( N \) is relatively low; power input rises less steeply with speed than with conventional agitators.

\( \text{L) Summary and Criticism} \)

The subject reviewed in the preceding pages is one which overlaps many others. Fluid motion, velocity of chemical reaction, power measurement, properties of materials and others are all integral parts of the subject of mixing. Even with the limitation effected by omitting emulsions, colloidal suspensions, solid-solid mixing, etc., it has not been possible to discuss the subject exhaustively.

Mixing is a very important operation but has not, until recent years, been widely studied. This has resulted in the production of a variety of mixers of widely differing efficiencies, particularly those designed specifically for solids and pastes. On the other hand, research into liquid agitation has led to a much better understanding of the underlying principles and for most duties a paddle, propeller or turbine can be safely recommended.

The approach to liquid agitation from the viewpoint of fluid motion has proved very difficult. It is usually possible to say, qualitatively, whether a given motion will give good or bad mixing, but at the present time little more can be said for even the simplest systems. Visual evidence of turbulence or placidity may be very misleading, particularly with slow-speed paddles.
Nevertheless, some general conclusions may be drawn. Power consumed is proportional, roughly, to the cube of the impeller speed, varies as a low power of the viscosity (about 0.2) and a power nearly unity of the density (about 0.8). This is to be expected on theoretical grounds. In streamline flow the power should be proportional to $N^3\mu$ and in turbulent flow to $N^3\rho$. Actual values between these limits are to be expected.

Few of the researches mentioned combine performance and power consumption. Yates and Watson showed that their equipment had a definite optimum speed for high efficiency, but although theirs seems to be an obvious method of attack very little has been done in this direction. How this optimum varies with impeller and vessel size is unknown, although these workers showed that it did vary. What is equally important and unknown, is how, for a given apparatus, it varies for different processes.

There is a tendency to "modify" standard dimensionless groups in the study of liquid agitation and these groups thereby lose their physical significance. In the modified Reynolds number, for example, the velocity and linear dimension to be used are matters of choice. In agitated heat transfer equipment where viscosity and density also vary the meaning of the "Reynolds number" becomes even more obscure.

The recent work on the function of baffles in agitated liquids has led to a much clearer conception of their correct employment and the connection between size and number of baffles and power consumption.
has been clearly demonstrated. There has been no work, however, on their effect on the type of flow. Indeed, there is very little published work dealing with the flow of liquids through conventional agitators, even without baffles, and although one sees in the literature diagrams purporting to show the flow through various agitators they have no firm experimental basis. More work in this direction is definitely required.

The position with respect to solid-solid mixing is much worse. There are no generally accepted methods for evaluating a degree of mixing, due to the fact that relatively little fundamental work has been done in the subject. There are no data available as to the net power consumption of solids mixers (i.e. power actually put into the material system, as opposed to bearing and other losses) although in some cases (for example, mixing drums) these would appear to be relatively easy to obtain.

In some drums for solids mixing, with or without a liquid present, the shells are not smooth but have, at intervals around the circumference, shelves or pegs to raise the materials and allow them to fall. The best size for these shelves or pegs, their effect on flow, power consumption and efficiency are unknown. Design of such pieces or equipment is completely by rule of thumb.

Summarising, it may be said that enough is known of liquid agitation to design equipment effectively, although much remains to be done to complete our knowledge of the process, but that very little is known of processes involving solids and that much research will be
necessary before design, in this field, can be based upon scientific principles.
INTRODUCTION: Part II

The Motion of Fluid between Rotating Concentric Cylinders with no Flow Parallel to the Axis
Introduction. It has been brought out, in the foregoing pages, that the study of mixing problems in which the continuous phase is a liquid has been complicated by lack of knowledge of the flow pattern and hence the inability to describe, or reproduce it, accurately. Black (93), in an attempt to simplify this problem, sought a means of obtaining a known, reproducible, fluid pattern and decided to study the motion of a liquid contained between two concentric cylinders, the inner rotating, the outer stationary. This system had been studied extensively, both theoretically and experimentally and suited the purpose admirably.

Since the present research is a development and extension of this, it will be necessary first to describe, briefly, previous work on this system and, secondly, to describe the experiments and conclusions of Black on the fluid system and on its ability to suspend solid particles.

Flow past solid boundaries. The flow of fluids past solid boundaries has been widely investigated. Experimental work seemed to indicate that steady motion existed at low fluid velocities but that eddying flow could be obtained at high velocities. The work of Reynolds, for example, pointed to this conclusion. Rayleigh (72) has shown that the slow, steady motion of a viscous fluid (or the steady motion of a very viscous fluid) is stable.

Many attempts were made to decide, on theoretical grounds,
when fluid motion would be stable or unstable. Kelvin, Rayleigh and others worked on the stability of a fluid contained between infinite parallel planes, but all attempts to calculate the relative speed of the planes at which instability would occur were unsuccessful.

Orr (70), as reported by Taylor (75), determined, in particular cases, the highest speed of flow at which all small disturbances in the flow initially decreased, that is, the highest flow at which the flow must be stable. No indication was given of the speed at which the flow would become unstable and in this respect his results were only of a negative character.

For example, Orr deduced that in a circular pipe the flow would always be stable at Reynolds numbers below 180, whereas experiment showed the flow to be stable below a Reynolds number of 2100 and sometimes stable up to values much greater than this.

More recently Hopf (73) and Heisenberg (76) have attacked the problem, the latter arriving at the conclusion that turbulence would occur at Reynolds numbers in the region of 1000.

Flow between concentric rotating cylinders. Attention was next turned to the case of a fluid contained between two coaxial rotating cylinders. Mallock (66) in 1888 determined the viscosity of water by rotating the outer of two concentric cylinders with water contained in the annular space and measuring the torque required to stop the inner one rotating. His results agreed well with those of Poiseuille.

Later Couette (67) and Mallock (68, 69) used similar types
of apparatus in experiments on the flow in the annular space. Both workers found, with only the outer cylinder rotating, that at low speeds the torque transmitted to the inner cylinder was a linear function of the speed, but that once a certain speed had been passed the torque increased more quickly, being proportional to something greater than the first power of the speed. They concluded that, at this point, "instability" set in. That is, at low speeds of the outer cylinder the motion was stable, at high speeds unstable. With only the inner cylinder rotating Mallock found that the torque transmitted to the outer cylinder was proportional, throughout the whole range investigated, to the 1.8 power of the speed. That is, with the inner cylinder rotating the flow was always unstable.

Rayleigh studied the stability of an inviscid fluid contained between two coaxial rotating cylinders, assuming perfect slipping at the boundary. He concluded that if the motion were confined to two dimensions it would be stable if the fluid were initially flowing steadily with the same distribution of velocity that a viscous fluid would have in the same system. When the disturbances were assumed symmetrical about the axis Rayleigh's conclusion was that if the initial flow of the inviscid fluid were the same as that of a viscous fluid in steady motion it would be unstable if the two cylinders were rotating in opposite directions. If they were rotating in the same direction the motion would be stable or unstable according to whether \( \Omega_2 R_2^2 \) was greater or less than \( \Omega_1 R_1^2 \). Here \( \Omega_1 \) and \( \Omega_2 \) refer to the angular velocities and \( R_1 \) and \( R_2 \) to the radii of the inner and outer cylinders respectively.
In an attempt to clear up these differences between theory and experiment, Taylor (74) constructed a rough apparatus in which the two cylinders could be rotated separately. His results appeared to show that Rayleigh's criterion for stability \( \Omega_1 \xi_2 > \Omega_1 \xi_1 \) was approximately true for a viscous liquid, but that his conclusion that with cylinders rotating in opposite directions the flow would always be unstable was not valid. These experiments also indicated that the type of disturbance produced when instability occurred was a symmetrical one.

The work of Taylor. Following the work referred to above Taylor undertook the complicated problem of calculating the possible symmetrical disturbances of a viscous fluid contained between concentric rotating cylinders. He showed that at low speeds of rotation of the cylinders, in the same or opposite directions, the motion of the fluid was two-dimensional, particles of the fluid rotating in circles concentric with the cylinders. At a critical speed this two-dimensional flow was replaced by a symmetrical three-dimensional motion, the type depending on whether the cylinders were rotating in the same or opposite directions.

When the cylinders were rotating in the same direction the motion took the form of a series of vortex rings, one upon the other, each occupying a square annular compartment, adjacent rings rotating in opposite directions. This is shown in figure 3A.

With the cylinders rotating in opposite directions the motion consisted of two series of vortex rings, one pair upon the other, the rings in the pair being side by side, each concentric with the cylinders and adjacent vortices both parallel and perpendicular to the
FIG. 3A. VORTEX STREAMLINES: $\gamma + \omega e$.

FIG. 3B. VORTEX STREAMLINES: $\gamma - \omega e$. 
axis rotating in opposite directions. This is shown in figure 3B.

By introducing filaments of coloured liquid into the apparatus Taylor was able to verify his predictions of both the form of the unstable motion and its dimensions and also to verify predicted values of the critical speeds, within a limited range. Agreement between theory and experiment was good.

The previous results of Mallock and Couette were shown to be erroneous and Taylor commented upon possible sources of error in their apparatus.

When the cylinders were rotating in the same direction the motion was stable or unstable according as the velocity of the inner cylinder \( \Omega_1 \) was less or greater than that given by the expression

\[
\frac{\pi^2 \nu^2 (R_1 + R_2)}{2 \Omega_1 \nu^2 (1 - \frac{R_2}{R_1})} = C \cdot \cos \gamma \left( \frac{1 + \gamma}{1 - \gamma} - 0.652 \frac{c}{R_1} \right) + 0.057 \gamma \left( \frac{1 + \gamma}{1 - \gamma} - 0.652 \frac{c}{R_1} \right)^{-1}
\]

\[\text{(28)}\]

The symbols not previously defined are

\[\gamma = \frac{\Omega_2}{\Omega_1}\]
\[c = R_2 - R_1\]

In this equation the term \( 0.652 \frac{a}{R_1} \) is a correction for the fact that \( \frac{a}{R_1} \) is not small. It must be borne in mind that Taylor's treatment was for cylinders of infinite length with diameters very nearly equal. No similar simple expression was found for the case when the cylinders were rotating in opposite directions but the equation
above can be expected to hold for negative values of $\gamma$ until the second term on the right hand side becomes comparable with the first.

Taylor also showed that Rayleigh's criterion held, with the additional qualification that for instability $\Omega$, must be greater than the value deduced from the equation.

Later Taylor (81) measured the distribution of velocity and temperature in the annular space, but since the work was carried out at high cylinder speeds it is possible that the flow was turbulent and not vortical. A further paper (32) dealt with measurements of the torque transmitted from one cylinder to the other through the liquid.

The confirmatory work of Lewis and others. Lewis (78) set out to submit Taylor's theory to a more searching test, in a greater variety of conditions. His apparatus is also described in an earlier paper (77).

With the outer cylinder fixed Lewis found that the expression for the critical speed held up to values of $\frac{3}{R_1}$ as high as 0.71, over a range of viscosities. It held so accurately indeed that Lewis suggested this type of apparatus as a method of measuring viscosity. As $\frac{2}{R_1}$ increased the spacing of the vortices varied within increasingly wide limits.

A further discovery of Lewis's was that if, after the vortices had been formed, the inner cylinder speed were reduced, then the vortices disappeared, but at a lower speed than that at which they had
been formed. These upper and lower critical speeds tended to become equal as the ratio $\frac{a}{R_1}$ decreased. Both Taylor and Lewis put their data in the form of stability diagrams, an example of which is attached (figure 4).

Lewis also carried out some rough experiments with air as the fluid (at atmospheric pressure) and showed that the motion and critical speed were as predicted. It must be noted that Taylor's original treatment was for incompressible viscous fluids of Newtonian character. The effects of compressibility and non-Newtonian properties are unknown.

Taylor's original treatment of his problem was solved by obtaining approximate results. Synge (85) gave a proof of stability, without approximation, for the particular case when $\Omega_2 \frac{R_2^2}{R_1^2} > \Omega_1 \frac{R_1^2}{R_2^2} > 0$.

Meksyn (86) has examined the whole problem and given solutions, based on a different mathematical technique, which agree very well with the work of Taylor and Lewis.

More recently a theory for the hydrodynamics of non-Newtonian liquids has been presented and two special cases investigated, one of which was that of the motion of liquid between rotating cylinders. This paper (88) need not concern us here however. The motion of fluids between eccentric rotating cylinders has recently been investigated (87).

(b) The Theories of Taylor and Meksyn. These two theories form the
FIG. 4. STABILITY DIAGRAM.
basis on which the apparatus used in the present research has been designed. It is proposed, therefore, to consider them briefly in order to bring out the difficulties peculiar to this problem and the assumptions involved. The nomenclature of the original papers has been altered somewhat to fit into the general scheme adopted in this thesis.

The equations of steady motion. Let \( X \) be the velocity at any point in an incompressible viscous fluid in steady motion between two infinitely long concentric cylinders of radii \( R_1 \) and \( R_2 \) \((R_2 > R_1)\) rotating at angular velocities \( \Omega_1 \) and \( \Omega_2 \). See figure 5.

If \( r \) is the distance of a point from the axis then it is known that

\[
X = Yr + \frac{Z}{r}
\]

where \( Y \) and \( Z \) are constants which are connected with the angular velocities by the relations

\[
\Omega_1 = Y + \frac{Z}{R_1^2}
\]

\[
\Omega_2 = Y + \frac{Z}{R_2^2}
\]

From these equations it can be shown that

\[
Y = \frac{R_1^2 \Omega_1 - R_2^2 \Omega_2}{R_1^2 - R_2^2} = \frac{\Omega_1 \left(1 - \frac{R_2^2}{R_1^2}\right)}{1 - \frac{R_2^2}{R_1^2}}
\]

\[
Z = \frac{R_1^2 \Omega_1 (1 - Y)}{1 - \frac{R_2^2}{R_1^2}}
\]

where \( Y = \frac{\Omega_1}{\Omega_1} \)

Specification of the symmetrical disturbance. Let \( v_y, X + v_x, v_z \)
FIG. 5. DISTURBED VELOCITIES.
be the components of velocity of the disturbed motion. Velocity $V_y$ is the component in an axial plane and perpendicular to the axis. $V_x$ is the component perpendicular to the meridian plane and to the axis, that is, in the direction of the undisturbed motion. Velocity $V_z$ is the component parallel to the axis.

Assume that $V_x$, $V_y$, $V_z$ are small compared with $X$ and that the disturbance is symmetrical - that is, they are functions of $r$, $z$ and $t$ only. $Z$ is the co-ordinate parallel to the axis and $t$ is the time.

The equations of the disturbed motion. Neglecting terms containing products or squares of $V_x$, $V_y$, $V_z$ the equations of the disturbed motion may be written

$$\frac{1}{\rho} \frac{\partial \rho}{\partial r} - \frac{X^2}{r} = - \frac{\partial V_y}{\partial t} + 2 \left( \frac{r^2}{r^2} \right) V_x + \nu \left( \frac{V_x^2}{r^2} + \frac{\partial^2 V_y}{\partial z^2} - \frac{V_y}{r^2} \right) \quad (35)$$

$$O = - \frac{\partial V_x}{\partial t} - 2 V_y V_x + \nu \left( \frac{\partial^2 V_x}{\partial z^2} + \frac{\partial^2 V_z}{\partial r^2} - \frac{V_x}{r^2} \right) \quad (36)$$

$$\frac{1}{\rho} \frac{\partial \rho}{\partial z} = - \frac{\partial V_z}{\partial t} + \nu \left( \frac{\partial^2 V_z}{\partial r^2} + \frac{\partial^2 V_z}{\partial z^2} \right) \quad (37)$$

where $\rho$ represents pressure, $\rho$ density and $\nu$ kinematic viscosity. $\nabla^2$ represents the operator $\frac{\partial^2}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r}$. The equation of continuity is

$$O = \frac{\partial V_y}{\partial r} + \frac{V_y}{r} + \frac{\partial V_z}{\partial z} \quad (38)$$

The six boundary conditions to be satisfied are

$$V_x = V_y = V_z = 0 \text{ at } r = r_1 \text{ and } r_2 \quad (39)$$
Assume as a solution

\[
\begin{align*}
V_x &= V_{x_1} \cdot (\omega \lambda z, e^{it}) \\
V_y &= V_{y_1} \cdot (\omega \lambda z, e^{it}) \\
V_z &= V_{z_1} \cdot (\omega \lambda z, e^{it})
\end{align*}
\]

where \( V_{x_1}, V_{y_1}, \) and \( V_{z_1} \) are functions of \( r \) only.

Eliminating \( p \) between equations (35) and (37) equations (35) (36) (37) and (38) reduce to

\[
\begin{align*}
\frac{V_{y_1}}{r} + \frac{\partial V_{y_1}}{\partial r} + \lambda V_{y_1} &= 0 \\
\nu \left( \frac{\partial^2 V_{y_1}}{\partial r^2} - \frac{\nu^2}{r^2} - \lambda^2 - \frac{\xi}{\nu} \right) V_{y_1} &= 2 \nu V_{y_1} \\
\frac{\nu^2}{\lambda} \frac{\partial}{\partial r} \left( \frac{\partial V_{z_1}}{\partial r} - \frac{\nu^2}{r^2} - \xi \right) V_{z_1} &= -2 \left( \nu + \frac{\xi}{r^2} \right) V_{x_1} - \nu \left( \frac{\partial^2 V_{x_1}}{\partial r^2} - \frac{\nu^2}{r^2} - \lambda^2 - \frac{\xi}{\nu} \right) V_{y_1}
\end{align*}
\]

The new boundary conditions are

\[
V_{x_1} = V_{y_1} = V_{z_1} = 0 \text{ at } r = r_1 \text{ and } r_2
\]

The fact that there are no terms containing \( z \) in these equations shows that the normal modes of disturbance are simple harmonic with respect to \( z \), the wavelength being \( 2\pi/\lambda \). The quantity \( \xi \) determines the rate of increase of a normal disturbance. If \( \xi \) is positive the disturbance increases and the motion is unstable. If \( \xi \) is negative the disturbance decreases and the motion is stable. If \( \xi \) is zero the motion is neutral. It will be seen from the way \( \xi \) enters into the equations that it cannot be imaginary or complex unless \( V_{x_1}, V_{y_1} \) and \( V_{z_1} \) are complex.

Up to this point the reasoning of Taylor and Mekyn is the same. Their methods of solution of these equations are quite different.

The former expanded the velocities in orthogonal Bessel functions of
order zero and unity. This method leads to a solution for both cases, when the cylinders rotate in the same or opposite directions. The formulae obtained, however, are rather complicated and the resulting equation takes the form of an infinite determinant equated to zero, giving the values of critical $\Omega_1$ and $\lambda$ for any values of $R_1, R_2$ and $\gamma$. To evaluate this determinant the assumption is introduced that the annular space is small, that is that $R_1$ is approximately equal to $R_2$.

Taylor was able to deduce an approximate form of the solution for the case when $\gamma$ is zero or positive. This has been referred to above. He was unable to do this for the case when $\gamma$ is negative. In the latter case results must be obtained from the unapproximated solution, a very long process.

The method of solution employed by Mekayn was an asymptotic expansion of the velocities in inverse powers of a large parameter. The final formulae are very simple but the solution is valid only for the case where the cylinders rotate in the same direction (or the outer stationary).

Additionally it was found that for a fixed critical $\Omega_1$ there is only a limited number of possible wavelengths and that probably only one could be observed. It was also shown that there are no solutions for complex or imaginary $\epsilon$.

Mekayn's formula is

\[ \frac{4\Omega_1^2}{\nu^2 \lambda^+} \frac{(1 - \frac{R^2}{R_1^2})}{(1 - \frac{R_2}{R_1^2})} \left\{ 1 + \frac{3}{2} \frac{\beta}{\alpha} (i - \gamma) \right\} = -17.57 \% \]

\[ \cdots \cdots (45) \]
where \( \lambda \alpha = \kappa \) ............(46)

\[ R_2 - R_1 = 2\alpha = a \] ............(47)

\[ R_2 + R_1 = 2\alpha \] ............(48)

This equation again involves the assumption that \( R_2 = R_1 \).

In later papers Mekayn was able to extend his solution to cover the case when \( \gamma \) is negative. There is an important difference, mathematically, between the two cases.

It is necessary to find (using Mekayn's technique) the asymptotic integrals of a certain linear differential equation. In the case where the cylinders rotate in opposite directions these integrals become infinite within the range under consideration, approximately at the point where the mean rotational velocity is zero. The asymptotic expansions change their form in passing through this point. It is therefore necessary to find the law of transformation of these integrals. This required a rather extensive mathematical investigation and formed the subject of the two later papers referred to.

Mekayn was then able to deduce equations giving the critical speed and vortex spacing for negative values of \( \gamma \). The equations are

\[
4 \alpha \beta^3 = \frac{8 \Omega \alpha (1 - \frac{R_2}{R_1})^2}{\nu^2 \lambda^4 (\frac{R_2}{R_1} - 1)^2 R_2} \] ............(49)

where

\[ \lambda \alpha = \frac{\pi}{1.5 + \beta} \] ............(50)

\[ \alpha_1 = R_2 - R_1 \] ............(51)

and

\[ R_2 = \frac{(1 - \gamma) \frac{R_2}{\nu\alpha}}{(1 - \frac{\gamma R_2^2}{\nu}) \nu} \] ............(52)
In these equations \( R_0 \) is the radius where the mean velocity \( X \) vanishes and \( 2\pi/\lambda \) is the wavelength parallel to the axis of the cylinders.

(c) **Discussion.** An interesting point arises from Lewis's observation that there are "upper" and "lower" critical speeds for the production of vortices from simple circular motion and vice versa. Taylor's theory indicates that exactly at the critical speed the vortices are formed infinitely slowly. To observe them therefore the speed must be greater than the theoretical to an extent determined by the experimental conditions. In exactly the same way the reversion to the simple motion would also occur infinitely slowly at the critical speed and a lower critical value would be obtained experimentally. The formation speeds of Lewis and Taylor (and also those reported here) are, in general, higher than the predicted values. Lewis's reversion speeds are lower than the predicted and further from it than the upper formation speed.

Account must also be taken of errors in the formulae. Taylor has verified that his expression for the critical speed is in error to an extent less than 1% at a value of \( a/R_1 \) of \( \frac{1}{3} \). It will be shown below that Meksyn's results up to a value of \( a/R_1 \) of \( \frac{1}{3} \) agree well with Taylor's and hence may be taken as correct.

At higher values of \( a/R_1 \) the position is not so clear. Lewis has used Taylor's formula successfully up to an \( a/R_1 \) ratio of 0.71 and the present work indicates that both Taylor's and Meksyn's
formulae hold good up to 0.81, but beyond this little can be said.
There are definite points at which these formulae break down, but how far these points are beyond their range of use is impossible to say.
Taylor's formula becomes indeterminate when \( (1 - 0.652 \frac{a}{R_1}) \) becomes zero, at an \( \frac{a}{R_1} \) ratio of 1.53. Mekayn's formula tends to infinity as \( \frac{a}{R_1} \) tends to the value 4.

It is interesting to see how and where calculations based on these two theories diverge. This can be done simply for the case when the outer cylinder is stationary. With this condition the formulae may be arranged as

\[
\frac{2 \pi^2 R_1^4}{\pi^4 \rho^2} = \frac{R_1^2 (2 R_1 + a)}{a^3} \left\{ c \cdot c \cdot 5 \left( 1 - 0.65 \frac{a}{R_1} \right) + c \cdot c \cdot c \cdot 6 \left( 1 - 0.65 \frac{a}{R_1} \right) \right\}^{-1} \quad \ldots \ldots (53)
\]

and

\[
\frac{2 \pi^2 R_1^4}{\pi^4 \rho^2} = \frac{2 R_1^2 (17.576) (2 R_1 + a)^2}{c^3 (4 R_1 - a)} \quad \ldots \ldots (54)
\]

It should be noted that both sides of each equation are dimensionless and that the value of the right hand side depends on the value of \( \frac{a}{R_1} \) only. The left hand side may be used, therefore, as a comparison group for both theories and for experimental work. Results are shown on Graph 1.

Taylor's and Mekayn's values do not begin to diverge until \( \frac{a}{R_1} \) is about 0.8, which is the limit to which experiments yielding numerical results have been taken so far. It would be interesting to see which is the more correct curve from 0.8 to 1.4. From 1.4 obviously Taylor's curve must be considered valueless.

The fact that these curves have vanishing points does not mean
GRAPH 1. COMPARISON OF VORTEX THEORIES.
that vortices do not exist. Later in this thesis systems will be described where vortex rings were obtained at \( \frac{a}{R_1} = 2.33 \) (between the two vanishing points) at \( \frac{a}{R_1} = 4.00 \) (exactly at a vanishing point) and at \( \frac{a}{R_1} = 7.50 \) (considerably beyond the range of both theories). These rings were of precisely the same type as at low values.

On the same graph have been plotted the points due to Taylor, Lewis and the author. Agreement is good, but it is obvious that past 0.8 one, or both, of the formulae, must be discarded.

Study of vortex wavelengths reported by Lewis yields the fact that of 59 measurements 7 are above the theoretical (wavelength = \( 2a \)) 4 are in exact agreement and 48 are below. This, taken in conjunction with the author’s measurements (to be discussed later) leads to the conclusion that the theoretical wavelength in fact represents a maximum value rather than a mean. The error involved is such, of the order of a few per cent, that it can be neglected. On the other hand Taylor’s values for wavelength when \( \gamma \) is greater than zero are all high. Lewis does not report values for this case. It would seem possible, therefore, that there is some connection between wavelength and positive \( \gamma \) which does not appear in either theory.

In connection with this last point it should be made clear that there is no reason to suppose that the wavelength is exactly \( 2a \). In Taylor’s analysis the wavelength is \( 2a \) divided by a quantity whose square can be shown, graphically, to be approximately 1 and almost certainly lying between 0.98 and 1.02. A similar calculation arises
in Meksyn's case. There it is necessary to find, graphically, a parameter corresponding to the minimum of a very shallow curve. This parameter is then cubed in the final formula \((-2.60)^3 = -17.576\). The author has shown, by calculation, that the minimum lies between -2.59 and -2.61 and so the error involved is not great. The wavelength relation corresponding to the value -2.60 is not, as Meksyn quotes, \(\lambda \omega = \pi\) but \(\lambda \alpha = 3.141596\). This difference is quite insignificant numerically.

There is another theory, due to Goldstein, a special case of which covers much the same ground as those discussed above. This will be dealt with in the next section.
INTRODUCTION: Part III

The Motion of Fluid between Rotating Concentric

Cylinders with Flow Parallel to the Axis
The work reported above was not concerned with systems with a fluid velocity parallel to the axis. The more difficult problem, where there is such a velocity, has been attacked however.

Goldstein (79), in 1931, reported that he had investigated, theoretically, the stability of such a system, but his results were not published until several years later. Meanwhile Cornish (80), in 1933, investigated a similar problem experimentally.

In Cornish's experiments the annular clearances were very fine and the ratio of the clearances to the rotor radius very small. Two sets of apparatus were used, where the mean radius of the clearance was 6.00 and 10.00 cm. The ratio \( \frac{a}{R_1} \) was varied from 0.0059 to 0.0036. The inner cylinder only rotated. Water was led into a chamber at one end of the apparatus, flowed through the clearance and out of a similar chamber at the other end. Pressures were measured in these chambers.

Cornish measured the pressure drop in the apparatus and the angular velocity of the rotor. These he correlated by plotting the group \( \frac{\partial p}{\partial z} \frac{u^2}{4\nu} \), where \( \frac{\partial p}{\partial z} \) represents the pressure drop per unit length of the clearance and \( v \) the mean velocity of axial flow, against \( (R_1 + R_2)\Omega / 2\nu \). He found that the resulting curves had three well-defined regions. Up to a given point \( \frac{\partial p}{\partial z} \frac{u^2}{4\nu} \) was constant, but beyond this the resistance to flow increased, at first very rapidly and then more slowly. Figure 6A shows the type of result obtained.
**FIG. 6A.** FLOW RESISTANCE.

**FIG. 6B.** CRITICAL SPEED.

**FIG. 6C.** CRITICAL SPEED CORRELATION.
Because of experimental difficulties it was not found possible to measure the critical angular velocity directly. This was done, graphically, by finding the values of $\frac{\partial^2 \theta}{\partial z^2} + \frac{2}{\nu} \frac{\partial \theta}{\partial z}$ for a number of values of $\frac{\Omega v}{\nu}$ near the critical at constant Reynolds number. (See Figure 68). Cornish gave these results in the form of a plot of $a. (R_1 + R_2) \frac{\Omega_1 / \nu}{v}$ against Reynolds number. He found that as the linear flow $(v)$ increased the critical angular velocity increased, as indicated in Figure 66.

In 1937 (83) Goldstein's mathematical analysis was published. He investigated the stability of flow of an incompressible viscous fluid, under the influence of a pressure gradient parallel to the axis, along a narrow annular space between two infinitely long coaxial rotating cylinders. The assumptions were that the disturbance was symmetrical about the axis and periodic along it.

Goldstein found that the stability of the fluid depended on the group

$$\frac{2 \Omega^2 \alpha^3 (1+\gamma) \left[ (R_1^2 - \gamma (R_1 + a)^2) \right]}{\pi^4 + \nu^2 (2R_1 + a)}$$

When the value of this group was above a certain critical, at a given Reynolds number, the flow was unstable, when below the critical it was stable. Goldstein calculated the critical value of this group for a series of Reynolds numbers including zero. The group reduces to

$$\frac{2 \Omega^2 \alpha^3 R_1^2}{\pi^4 \nu^2 (2R_1 + a)}$$

when the outer cylinder is stationary and is referred to as $G$ subsequently. For each Reynolds number a wavelength of the disturbed motion was calculated.

Goldstein compared his variations of $G$ with Re with results supplied to him by Cornish, found them to be in complete disagreement.
and concluded that Cornish's experiments did not show the first onset of instability.

The calculated values of $G$ were only rough. With a Reynolds number of zero, that is with no flow, Goldstein's value differed by only $2\%$ from that calculated on the basis of Taylor's theory. The error was thought to increase with increasing Reynolds number, but due to the great labour involved in computation the subject of errors was only briefly touched upon in Goldstein's paper. The values of Reynolds numbers (defined as $\frac{\nu}{\nu}$) were very low, the highest being about 26.

Later, in 1938, a paper on this subject was published by Fage (34). In his apparatus the outer cylinder was fixed and the inner rotated. Pressures were measured at each end of a test length. The fluid was water.

For each Reynolds number two measurements were made. The pressure drop per unit length ($\frac{\Delta p}{L}$) was measured first with the rotor still and then at constant angular velocity. Values of the group $G\nu/\nu^{2} + \frac{\partial p}{\partial z}$ were calculated for each case and their ratio determined. Values of this ratio at various Reynolds numbers were plotted against $\nu, \frac{\omega R}{\nu}$. See figure 7A. In the first expression $Q$ represents the quantity of fluid flowing per unit time.

The ratio was constant (equal to unity) at low values of angular velocity but at higher values it decreased, slowly at first and then more quickly. Fage took as his critical point for the onset of instability the point where the curve first left the constant value.
FIG. 7A. FAGE'S PRESSURE DROP CURVE.

\[
\frac{Q \mu / R_2^4}{\frac{dP}{dz}^2} = \left( \frac{Q \mu / R_2^4}{\frac{dP}{dz}^2} \right)_{1}
\]

\[
\frac{\nu a}{2 \nu} = 6.90.
\]

\[
\frac{\nu a}{2 \nu} = 129.
\]

FIG. 7B. FAGE'S STABILITY CURVE.

\[
\Omega R_1 a \left( \frac{2R_1 + 2R_2}{a} \right) \frac{1}{\nu}.
\]

Taylor (theory).
The resulting graph of critical angular velocity against Reynolds number (figure 73) extrapolates approximately to Taylor's value at zero flow. The critical rotation increases with increasing Reynolds number. Since only one apparatus was used there is no significance in the fact that \( R_1, R_2 \) and \( \sigma \) were included in the correlation.

Page's work, except at its extrapolation to zero flow, disagrees with that of Goldstein and Cornish. At zero flow Page and Goldstein agree with Taylor's well-substantiated criterion, but Cornish does not. Very few data are reported by the experimental workers, making the drawing of conclusions difficult, especially since they used the indirect method of pressure measurement to infer the onset of instability.

More recently Meksyn's method of solution of Taylor's problem has been extended by Massoud (Q4) to deal with Goldstein's case. Some agreement with Goldstein has been demonstrated at very low Reynolds numbers.

(b) The Theories of Goldstein and Massoud.

The steps in these two theories are the same as in Taylor's and Meksyn's case. The treatment is more general.

The equations of the undisturbed motion are very similar to those of the simpler case, but here they are expressed in cylindrical polar co-ordinates \( (r \theta z) \). The comparable components of velocity in the steady motion are, employing the
same symbols as before, \( \theta \), \( X \) and \( \nu \) in the directions \( r, \theta \) and \( z \) increasing respectively.

The disturbance is specified exactly as before. It is specified as small, so that squares and products may be neglected and as symmetrical, so that the disturbance is independent of \( \theta \). The components of velocity now become \( V_r, X + v_z, \) \( Y + v_z \), where \( V_r, v_z \) are the components of the small disturbance. This allows the equations of the disturbed motion to be written.

The solution is then assumed. The small disturbances are taken to vary as \( e^{\pm i \xi t} \) and then, with the further assumption that the disturbances may be analysed into constituents periodic along the axis, the assumed solutions may be written:

\[
\begin{align*}
V_r &= V_{r0} e^{(ct + \lambda z)} \\
v_z &= v_{z0} e^{(ct + \lambda z)} \\
v_z &= v_{z0} e^{(ct + \lambda z)}
\end{align*}
\]

Then, as before, for stability of motion the imaginary part of \( \xi \) must be positive and for instability negative. The critical flow is that for which \( \xi \) is real (the imaginary part zero) and the disturbance is a wave travelling parallel to the axis. The frequency is \( \xi/2\pi \), the wavelength \( 2\pi/\lambda \).

Goldstein then develops the solution by Fourier series and drastic approximations are found necessary to obtain numerical results. As mentioned previously the motion, at any given Reynolds number, is stable or unstable according as the group \( 2\gamma c^3(1+\gamma)(R_z^2 - \gamma R_s^2)/\nu^4 v^2(2R_s \gamma) \) is less or greater than a critical value. The critical value when
when $Re = 0$ agrees well with Taylor and Meksyn, it rises at first with increase in Reynolds number and then falls sharply.

Values of wavelength may also be deduced. The predicted wavelength falls steadily from $2.04a$ at $Re = 0$ (good agreement with Taylor and Meksyn) to $0.38a$ at $Re = 25.8k$. The range of Goldstein's results is therefore low.

Massoud develops the solution, after Meksyn, by an asymptotic expansion of the velocities in inverse powers of a large parameter. When $\varepsilon$ is zero Massoud's problem reduces to that of Meksyn and his results are corroborated. When $\varepsilon$ is not zero two cases have been calculated, at Reynolds numbers of $\frac{1}{2}$ and $1$. How these compare with Goldstein's results will be discussed below.

(c) Discussion

When the outer cylinder is stationary and there is no axial flow, Goldstein's formula for the critical speed reduces to

$$G_1 = \frac{2ae^2}{\pi^2 \beta^2 (2R_1 + \alpha)} = 17.78$$

In similar circumstances Taylor's and Meksyn's formulae reduce to the forms

$$G (\text{Taylor}) = \left[0.4571(1 - 0.652 \frac{a}{R}) + c_4 \frac{a}{R} (1 - 0.652 \frac{a}{R})^{-1/2} \right]$$

$$G (\text{Meksyn}) = 17.576 \left( \frac{R_1}{R} \right)$$

Now the formulae of Taylor and Meksyn can be used at values of $\frac{a}{R_1}$ which are not small. No correction has been introduced by Goldstein to allow for this and so the three formulae can be compared directly only when $\frac{a}{R_1}$
can be neglected. The values of these groups then become
\[
\begin{align*}
G. (\text{Goldstein}) &= 17.70 \\
G. (\text{Taylor}) &= 17.34 \\
G. (\text{Meksyn}) &= 17.576 \\
\end{align*}
\]
.. (59)

The agreement is very good, especially when it is remembered that three different methods of solution have been employed each requiring more or less severe approximations to obtain numerical results.

In the course of their analysis Goldstein and Massoud evaluate a group which, in our notation, is \( \frac{C_0}{\pi \nu} \), where \( c \) is the wave-velocity of the vortices. The numerical values appear below.

\[
\begin{align*}
\text{Goldstein} \\
\begin{cases}
(Re = 0) & 5.17 & 10.41 & 15.50 & 20.67 & 25.84 \\
\frac{C_0}{\pi \nu} = 0 & 1.91 & 3.88 & 6.04 & 8.49 & 11.46
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\text{Massoud} \\
\begin{cases}
(Re = 0) & \frac{1}{2} & 1 \\
\frac{C_0}{\pi \nu} = 0 & 0.206 & 0.4113
\end{cases}
\end{align*}
\]

Goldstein's first three values give, almost exactly, a straight line when plotted against Reynolds number. Massoud plotted his points similarly and found that they fell very close to Goldstein's line, concluding that their results thus compared well. See figure 8.

This agreement is deceptive, however. It will be found that Massoud's points are in error (taking Goldstein's as correct) by about 10%. This may not appear serious. However, the vortex wave-velocity, \( c \), may be evaluated as \( \frac{C_0}{\pi \nu} \sqrt{\frac{Re}{2}} \) and in this the error becomes more pronounced. The wave-velocity calculated on the basis of Goldstein's figures varies from \( 1.16 \nu \) at \( Re = 5.17 \) to \( 1.39 \nu \) at \( Re = 25.84 \), while Massoud's figures give \( 1.29 \nu \). Goldstein's figure at Massoud's
FIG. 8. COMPARISON OF GOLDSTEIN'S AND MASSOUD'S THEORIES.
Reynolds numbers would therefore be less than $1.16 \times V$, as compared with $1.29 \times V$. It seems curious that the vortex disturbance should travel more quickly than the fluid in which it exists and any deviations from the fluid velocity would be very noticeable in visual observation. The excess vortex velocity is twice as great in Massoud's case as in Goldstein's and such a difference is very important indeed. This question of wave-velocity will be dealt with more fully later, when experimental results will be compared with these figures.

A comparison of the results of Cornish, Goldstein and Fage shows that they are in almost complete disagreement. Their results (all calculated in terms of the group $G$ for convenience) have been plotted in graphs 2 and 3. Fage and Goldstein show some little agreement since they both extrapolate approximately to Taylor's value at no flow. Cornish's value at no flow is much higher. Cornish and Fage both indicate a rise in angular velocity with Reynolds number except for Fage's first two points. Goldstein indicates a fall. It is, in fact, impossible to reconcile these sets of results.

It seems probable that the type of disturbance investigated by Cornish is not that envisaged by Goldstein and Fage. In the region below the critical speed, where the resistance to flow is constant, Cornish shows that this resistance agrees very well with that calculated for flat plates. Since his $\frac{8}{R_1}$ is very small it would not be unexpected that, in fact, his apparatus behaves as two shearing flat plates, rather than as co-axial cylinders. Work by Taylor indicates that the effect of flat plates will be reached when $\frac{8}{R_1}$ is of the
GRAPH 2. RESULTS OF CORNISH, GOLDSTEIN, FAGE.
GRAPH 3. RESULTS OF CORNISH, GOLDSTEIN, FAGE.

$\text{log } Re. (Re = \frac{va}{\nu})$. 

Graph showing results of Cornish, Goldstein, and Fage with respective points on the graph for each.
order of 0.001. Cornish's values, it will be remembered, varied from 0.0059 to 0.0036. The conclusion is that a different type of disturbance was being investigated.

The reliability of Cornish's work may also be questioned. Results were not obtained directly because accurate speed control of his D.C. motor was impossible and the temperature rise in the annulus rapid. From Cornish's remarks it would appear that the very sharp temperature gradient in the annulus would be sufficient to modify drastically any regular vortical motion which might possibly form. In addition the clearance was so fine that accurate centring must have been extremely difficult.

The main criticism of Fage's work is that his method of deducing the critical angular velocity is inherently inaccurate. It is very difficult (see figure 7A) to find the point of departure of the shallow curve from the straight line on the data provided. His results indicate that the angular velocity at which the vortical instability sets in is the same at a Reynolds number of 300 as at 0. This seems most unlikely. Taylor describes how a slight, accidental, component of velocity was sufficient to cause a spiral instability to form, replacing the rings. At the critical speed a slight decrease in angular velocity is sufficient to alter the flow. It seems unlikely, therefore, that liquid flowing at a Reynolds number of 300 should have no effect on the critical angular velocity.

It is not certain how reliable Goldstein's computations are. It is stated that the error in $G$ is about $2\frac{1}{4}$% at a Reynolds number of zero and about $5\%$ at 10.34. The error increases with $Re$ and may
account for the sharp fall in $G$ at $Re = 25.34$.

This problem has been investigated experimentally by the author, using a visual method to determine the onset of unstable flow. Experiment shows that the assumptions on which the theories are based are not valid and that no prediction of critical angular velocity can be made. The predicted wave-velocities are, in general, incorrect, although at very low Reynolds numbers Goldstein's figures are approximately correct. Due to the many types of flow possible, pressure measurements, alone, as used by Page and Cornish, are unreliable.
INTRODUCTION: Part IV

The Work of Black
(a) Description of the Apparatus

It has previously been noted that Black, in the search for a defineable flow system investigated Taylor's problem experimentally from the point of view of the mixing process. The equipment of Taylor and Lewis was built for precise observation and measurement, that of Black for less accurate measurement, but use as a piece of small-scale mixing equipment. Since this apparatus, with slight modifications which will be mentioned later, was used in the present research, a description of it and an appreciation of previous work carried out with it is necessary.

The diagram (figure 9) shows the main features of the mixing unit. It should be noted that the ratio of the annular space to inner radius was high (about 0.4) and that the ratio of annular space to depth of liquid was also fairly high (about 0.09 throughout). The fixed outer cylinder was of glass (diameter 6.2") and the rotating inner cylinder brass (diameter 4.36").

For power measurements Black used a type of dynamometer similar to that used by Stoops and Lovell. This consisted of two disks, one driven by the motor, one driving the inner cylinder, connected by a spiral spring, the extension of which was measured by an electrical make-and-break device across the two disks (see Plate 1).

The mixing unit was immersed in a constant temperature bath, in order that the liquid properties should be known accurately. The driving motor was a synchronous A.C. type, speed control of the
FIG. 9. BLACK'S MIXING UNIT.
Plate 1. GENERAL VIEW OF VERTICAL AGITATOR
inner cylinder being effected by change of pulleys. The cylinder speed was measured by counting the number of makes and breaks per minute in the electrical circuit, across the dynamometer, using headphones.

The sampling device used in the solid-liquid systems was similar to that used by Hixson and Tenney. A rule was fixed outside the glass cylinder for measuring the positions of vortex boundaries in the agitated fluid. For a more detailed description of the auxiliary equipment and for the various experimental techniques involved the original thesis should be consulted.

(b) Fluid Patterns in Single Liquids

The apparatus was filled with liquid (glycerol-water mixtures or a light transformer oil) to a standard depth of ten inches for all the experiments. For the observation of fluid patterns a little aluminium powder was added. In suspensions the solid particles acted as tracers.

Black found that on starting up the apparatus the fluid pattern consisted of 9, 11, 13 or 15 vortex rings. Fifteen vortices were only rarely seen, but the other three systems occurred very frequently, despite the fact that theory would predict eleven vortices in this case. Even numbers of vortices were never formed and each system, once produced, was stable.

The sizes of the vortices varied from top to bottom of the liquid. No single vortex could be selected as representative of the
system and Black was unable to rationalise this behaviour in any way. However, a given fluid pattern was found to be reproducible under the same conditions of $\frac{N}{\nu}$ in different experiments.

With the exception of the work reported in the next paragraph all experiments on the fluid pattern and power consumption were carried out in the range about $3 - 30$ for $\frac{N}{\nu}$, where $N$ is measured in r.p.m. and $\nu$ in centistokes. The critical for the apparatus was $\frac{N}{\nu} = 0.622$, in these units, being calculated on Taylor's theory.

At very high values of $\frac{N}{\nu}$ (up to 400 times the critical) spontaneous and continuous changes of fluid pattern were observed, various numbers of vortices being produced in sequence. The motion at these high values of $\frac{N}{\nu}$ was very ragged but definitely vorticular. Complete turbulence was never obtained. At these high speeds even numbers of vortices were observed as well as odd. Even numbers of vortices were obtained artificially at low $\frac{N}{\nu}$ by "removing" the top vortex of an odd number, by means of an obstruction (a glass rod for example) which, on withdrawing, allowed the expansion of the next vortex to the free surface.

(c) **Fluid Patterns in Suspensions**

When light, smooth particles of lycopodium powder were suspended in the liquid the fluid patterns were observed to be exactly as for pure liquids. The solid particles did not appear in the outer regions of the vortices until quite high concentrations
of solid were in suspension. At no concentration was sedimentation observed.

Suspensions of quartz were next examined. The particles were heavy, hard and jagged. Again it was noted that solid particles did not appear at the outside of the vortices until quite high concentrations were reached. The behaviour of these suspensions was, however, quite different from that of the lycopodium suspensions.

Even numbers of vortices were observed in quartz suspensions at low values of \( \frac{1}{2} \) and were found to be quite stable. In addition it was possible to saturate the system with quartz. For every particle size a critical speed existed dividing two phenomena. Below this critical value a certain maximum of material was held in suspension, while further addition caused complete sedimentation. Above this critical speed material could be added until the vortex system began to break down.

The complete sedimentation was observed to occur when the bottom clearance between the two cylinders became full of solid. Above the critical speed the clearance could not be filled with solid before the vortices were destroyed. The distribution of solid was observed to be uneven, increasing from top to bottom, for both types of suspension.

(d) Power Measurements

Having discovered that various fluid patterns were possible at the same speed in the same depth of fluid, it was found necessary
carefully to standardise conditions for power measurement. Ten inches of liquid was the standard depth used. The power input to this liquid was measured and the liquid siphoned off to a two inch depth. The power consumed in this two inches was then determined. The difference gave the net power input to eight inches of liquid in vorticular motion, which, it was found, depended on whether the original ten inches had contained 9, 11 or 13 vortices.

The effects of fluid density and viscosity and speed of rotation were deduced. When the number of vortices was 9 or 13 the resultant correlations were

\[ P = K_{20} \rho N^{2.5} \theta^{0.768} \]  \hspace{1cm} (60)

\[ P = K_{21} \rho N^{2.5} \theta^{0.768} \]  \hspace{1cm} (61)

which differed from the corresponding correlation for 11 vortices, which was

\[ P = K_{22} \rho N^{2.5} \theta^{2/3} \]  \hspace{1cm} (62)

The power consumption of lycopodium suspensions, up to 17% lycopodium by volume, could be correlated substantially as for pure liquids. Beyond this concentration and for quartz suspensions no similar expression was found, but qualitative conclusions were drawn as to the effect of speed of rotation and concentration.

(e) Discussion

It is evident that the apparatus used had severe limitations and in particular end effects dominated the fluid pattern. The main consequence of this is that although the system is easily
reproducible it cannot be scaled up or down, since the theory
developed by Taylor and others does not apply rigorously. As used
by Black, therefore, the system only partially fulfilled its object
of producing a defineable field of flow. No measurements were made
of the critical speed at which the vortical motion set in.

The power measurements are not of general value. The effect
of variation of cylinder sizes was not deduced. In addition, Black
did not deal with a definite number of vortices but a fractional
number corresponding to ten inches of vortical motion containing
9, 11 or 13 vortices minus two inches of unspecified motion. The net
power inputs were small and errors in measurement appreciable.

It would appear, from Black's observations, that the ability
of the system to suspend solid particles depended, to a large extent,
on the magnitude of the bottom end effect, which imparted a vertical
component of force to the system. This was presumably aided by a
centrifugal effect in the bottom clearance which was insufficient to
keep the clearance free of solid at speeds below a certain critical.
Hence followed destruction of the vertical component and sedimentation.
Above this critical value presumably, the speed was sufficient to keep
the clearance free, retain the vertical energising effect and keep the
particles suspended.

Equipment used for any further experiments in this field
should not be greatly different from that envisaged in the theoretical
work so that the theory may be used for design purposes. In addition
it is undesirable, in fundamental work, for equipment to introduce
large end-effects for the suspension of solid particles.
EXPERIMENTAL WORK: PART I

Experiments with Black's Apparatus on Single-phase Systems
(a) The Mode of Formation of Vortices and the Critical Speed

Taylor's theory predicts that for this apparatus vortices will be formed at \( \frac{N}{\nu} = 0.622 \). Black's observations were confined to values of \( \frac{N}{\nu} \) greater than about 3 (where \( N \) is r.p.m. and \( \nu \) is in centistokes). Experiments were carried out to compare the experimental and theoretical critical values and to examine qualitatively the effects of the bottom clearance and the free surface on the mode of formation of the rings.

For these experiments the A.C. motor was replaced by a D.C. type and a suitable resistance included in the circuit, so that for each pulley arrangement between motor and reduction gear a range of speeds could be obtained. The dynamometer scale and pointer were removed, but the electrical contact system across the dynamometer and the headphones were retained for use as a speed counter. Rotor speeds up to 250 r.p.m. - the maximum obtainable without undue vibration of the apparatus - could be counted accurately by this means. The apparatus is shown in Plate 1.

The mixing vessel was filled to a depth of 20 cms. with liquid (glycerol-water mixtures or an oil) containing a small amount of aluminium powder to show the flow lines and the inner cylinder started at low speed. The speed was increased at the rate of 1 r.p.m. every two or three minutes until vortices just formed. This formation speed was noted. The speed was further increased until the whole depth of liquid was moving in vorticular motion. This speed was also noted. The temperature of the liquid was
taken immediately before and after each experiment at both top and bottom of the vessel. Differences between these four temperatures were negligible. The viscosity of the liquid was measured using a Redwood No.1 viscometer (see Appendix A).

The vortices were always formed first at the bottom of the vessel and always in pairs, with the exception of the vortex ring at the free surface. At the "lower" formation speed there was a pair of vortices at the bottom of the vessel with simple circular motion above it. As the speed was increased another pair of vortices formed on top of the pair already present. This continued until finally the single vortex at the surface was formed, the total number of vortices always being odd. The rotor speed at which the vortex system was completed was considerably higher than that at which the first pair appeared. There is not, therefore, a definite "critical" formation speed for vortices in this apparatus, a result not unexpected when end-effects are taken into account.

The pairs of rings, as they were successively formed, were formed almost exactly in the positions they occupied when the whole depth of liquid contained vortices. When the vortex motion was complete some readjustment of boundaries occurred, taking only a few seconds. When the rotor speed was maintained between the upper and lower critical speeds the combination of vortical and circular motion persisted indefinitely.

It was also noted that at a speed of one or two r.p.m. below that at which a vortex pair appeared the boundaries corresponding to
these vortices were visible as dark lines. While the motion was definitely not vorticular the boundaries of the pair of vortices about to be formed were visible. When the speed was held constant these lines remained. On reducing the rotor speed the vortices reverted to the undisturbed flow in reverse order.

Calculation showed that the critical value of corresponding to the lower formation speed was considerably lower than that predicted by Taylor's theory, while the upper critical value was considerably higher. The mean critical value agreed reasonably well with the theoretical value.

Typical data are:  

<table>
<thead>
<tr>
<th></th>
<th>Viscosity (μ)</th>
<th>90.2</th>
<th>63.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower speed (N, rpm)</td>
<td>53</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>Upper speed</td>
<td>65</td>
<td>46</td>
<td></td>
</tr>
<tr>
<td>( \text{N/V} ) lower</td>
<td>0.588</td>
<td>0.597</td>
<td></td>
</tr>
<tr>
<td>( \text{N/V} ) upper</td>
<td>0.720</td>
<td>0.723</td>
<td></td>
</tr>
<tr>
<td>( \text{N/V} ) mean</td>
<td>0.655</td>
<td>0.660</td>
<td></td>
</tr>
<tr>
<td>( \text{N/V} ) Taylor</td>
<td>0.622</td>
<td>0.622</td>
<td></td>
</tr>
</tbody>
</table>

(b) Vortex Size and Configuration

Black investigated this problem and concluded that although definite conditions tended to produce a definite vortex pattern no particular vortex pair could be selected for comparison with theory. In addition he could find no relation between the position of a vortex in a system and its size.

In the present work the apparatus was filled with liquid
containing a small amount of aluminium powder to show the flow lines. On starting the inner cylinder, which rotated at constant speed, vortices were formed. The heights of the vortex boundaries from the bottom were measured, using a scale attached to the outside of the glass cylinder.

The number of vortices was now altered by inserting a rod into the top vortex and altering the flow lines to allow a new system to be established. The diagrams in figure 10 show how an odd-numbered system, which was always obtained on starting, could be altered to the even-system with one more or one fewer vortices. The systems produced in this way were stable. On agitating the top pair of an even-system the odd-system with one vortex fewer was usually obtained, but occasionally the odd-system containing one vortex more resulted. The boundary positions of these new systems were measured as before. In this way the various configurations possible for a given depth of liquid were discovered. This procedure was carried out using various depths of liquid.

Where the depth of liquid was high a large number of systems was produced. For example, in 25 cms. the systems containing 9, 10, 11, 12, 13 vortices were easily produced. Plates 2 - 8 show the systems 7, 8, 9, 10, 11, 12, 13 vortices in 25 cms. of liquid. Since the annular space was 2.27 cms. 11 vortices would be expected in the former case and 10 in the latter. In the former case 11 vortices were usually obtained on starting, whereas in the latter, since even numbers are only obtained by modifying the flow pattern, 9 and 11 vortices were both readily formed on starting. All these systems, once formed, were stable.
FIG. 10. ALTERATION OF FLOW PATTERNS.
Plate 2. 7 VORTICES
Plate 6. \textbf{VORICICES}
Plate 7. 12 VORTICES
Where the depth of liquid was low the number of possible vortex systems was reduced. For example, in 9.3 cms. of liquid only two systems were obtained, containing 3 and 5 vortices. Theory predicts 4 in this case. Below 4.1 cms. only one system (a single vortex) was obtained. In suitable conditions a single vortex pair could be produced. The direction of rotation of the bottom vortex was always the same (downwards at the outer cylinder) so that when even numbers of vortices were obtained it was the surface vortex which rotated in the unusual direction.

Immediately the inner cylinder started to rotate vortices were formed, but occasionally the boundaries took several seconds to adjust themselves, the number of vortices remaining constant. Once this settled system had been formed the vortex sizes were independent of rotor speed provided it exceeded the critical value. This appears to disagree with Black's observations.

The vortex sizes were plotted against the number of the vortex counting from the top of the system. The graphs show the vortex systems lying in definite families. In graph 4A both the surface and bottom vortices are larger than the remainder, the size of which decreases from top to bottom in approximately linear fashion. The difference in size between the first and second vortices increases as the average size throughout the system increases (that is, as the graphs are displaced upwards). On the other hand, the difference in size between the bottom vortex and the one above it decreases.

This top and bottom end effect is apparent also in graph 4B.
**GRAPH 4A. VORTEX PATTERNS.**

**GRAPH 4B. VORTEX PATTERNS.**
where are plotted the vortex sizes in systems containing 3 vortices. Where the average size is greater than the annular space the surface vortex is the biggest. Where the average size is approximately the same as the annular space the surface and bottom vortices are approximately equal. Where the average size is less than the annular space the bottom vortex is the largest. The slope of the left hand line increases and that of the right hand line decreases as the graphs are displaced upwards. The bottom line is for a single vortex and the slope shows that it is dominated by end-effect. No vortex could be produced in a liquid depth less than in this case.

For the larger numbers of vortices a different graphical representation was employed, that of wavelength (pair size) against number (graphs 5, 6 and 7). The surface effect is still noticeable but the end-effect is now masked. This is because the bottom pair consists of the large bottom vortex and the smallest single vortex in the whole system. These graphs also fall into families.

Occasionally systems were obtained where alternate boundaries were oscillating, although the surface and pair-boundaries were level and well-defined. These oscillations were generally greater in amplitude towards the bottom of the liquid. Types were found where the oscillations decreased in amplitude with time (giving rise to stable systems) where they were neutral (continuing unchanged indefinitely) and where the amplitude increased with time (eventually causing a change in the number of vortices to a stable system). Similar effects are described by Black, but in his case they occurred at very high values
GRAPH 5A. VORTEX PATTERNS.

GRAPH 5B. VORTEX PATTERNS.
GRAPH 6A. VORTEX PATTERNS.

GRAPH 6B. VORTEX PATTERNS.
GRAPH 7. VORTEX PATTERNS.
of \% \% was much lower, of the order of 10.

On a few occasions spiral systems were produced which persisted only momentarily before being replaced by the normal rings. These spirals persisted longer at low speeds than at high and were similar to those described by Taylor and Lewis.

(c) **A Continuous Flow System**

Later in the course of this work this apparatus was adapted for continuous operation - liquid being added to the surface and withdrawn through an outlet connected to the base plate. This was undertaken after a flowing system had been investigated in another piece of equipment, which will be described later. With low rates of flow the vortices were normal and well-defined. As the flow rate increased the entry and exit disturbances created turbulence and destroyed the top and bottom vortices. As the flow increased further these disturbances were such as to destroy the vortices completely. It was concluded that for continuous operation the apparatus would need to be considerably longer in order to damp out the entry and exit disturbances.

(d) **Discussion**

The experiments described above have little fundamental significance, because of the type of apparatus used. It departed from the theoretical conditions since the ratio of annular space to rotor radius was high \( \frac{B}{R} = 0.41 \) and the ratio of depth of liquid to annular space was low (about 11 at its maximum). The
work had some significance from the point of view of design.

Although Taylor's value of the critical speed does not agree with the experimental values it is near enough to be used for design purposes. This is clearly indicated in graph 1.

The vortex sizes are not as would be expected on theoretical grounds, but this is important from the point of view of mixing equipment. Later work shows that when the ratio of liquid depth to annulus becomes greater than about 15 better agreement with theory results.

For a given depth of liquid many stable vortex systems may be possible. The stability of the boundaries is well illustrated in the plates, which were taken with a ten-second exposure. Some unstable systems are formed, but these revert to stable ones. In any case they may be equally good for mixing purposes. The flow system resulted in destruction of the vortices and it appears that the ratio of depth of liquid to annular space must be high if stable vortical motion is to be obtained.
EXPERIMENTAL WORK: Part II

Experiments with Black's Apparatus on Two-liquid Systems
(a) Fluid Motion in the Two Liquid Layers

The apparatus was filled with two immiscible liquids, the lower layer being water or a water-glycerol mixture, the upper layer a mixture of various oils. In some experiments the lower layer was the oil phase, made denser by the addition of carbon tetrachloride. Each layer was 10 cm. in depth.

On increasing the rotor speed from zero vortices were formed in the water phase (four in number, evenly spaced) but not in the oil phase. The vortex motion/circular motion boundary was the liquid-liquid interface, which was well-defined. As the speed was raised through that corresponding to the critical speed for the oil phase vortices were formed, three or five in number. The top oil vortex was a single one so that the interface was a boundary between vortex pairs.

Increase in speed to about 100 r.p.m. served only to increase the circulation within the vortices, there being no tendency to mix. Beyond 100 r.p.m. an occasional globule of water appeared in the oil phase and vice versa, but the interface was still clear.

(b) The Liquid-liquid Interface

The interface, which up to a speed of about 100 r.p.m. had been well-defined and flat, developed ripples, which increased in amplitude as the speed increased further. At about 160 r.p.m. (the speed varying with the liquids used) the number of dispersed globules increased rapidly and with no further increase in speed the
two phases formed a coarse emulsion. During this period the now indistinct interface travelled downwards and the water was transferred to the emulsion immediately above it.

The oil vortex immediately above the interface was the first to become roughly emulsified. There was relatively slow mass transfer between this vortex and those above it, the emulsion being distributed upwards. The result, in a few minutes, was a nearly-uniform emulsion. Continued agitation at the same speed caused slight rearrangement of the emulsion resulting in a more uniform system. Complete uniformity was never attained, there being a slight increase in globule size from top to bottom of the emulsion. There seemed to be no change in globule size after about thirty minutes agitation.

(c) **Vortex Configurations in Emulsions**

The motion in the two liquids and in the emulsion, both during and after the mixing period, was vortical. The most usual number of vortex rings in the emulsion was 7, but on some occasions 6, 8, and 9 were observed. The general character of the motion, circulation and spacing were as for single liquids. The even numbers of vortices were a new feature. These did not occur in single-phase systems or lycopodium suspensions, but Black observed them in quartz suspensions.

The emulsion vortices were stable at all speeds above their formation speed up to the maximum obtainable with the apparatus (250 r.p.m.). The even-numbered emulsions were usually
obtained at speeds near the maximum and reduction of speed caused a reversion to the odd configuration, a new vortex being produced at the surface.

As the speed was reduced below the formation speed the vortices became progressively less vigorous until the system reverted to simple circular motion, starting with the bottom pair (the largest globule size) and working up. At lower rotor speeds settlement of the emulsion occurred.

(d) The Mixing Curve

To investigate more thoroughly the dependence of mixing on the rotor speed the procedure described below was adopted. The mixing vessel was set up with thermostatic control and the rotor started at a constant speed. Agitation was continued until a dynamic equilibrium was reached between mixed and unmixed liquid, this being ascertained by recording the position of the emulsion/water interface until it was constant for 15 minutes. Rotor speed readings were taken regularly throughout the experiment. The difference between the amount of water remaining at the equilibrium position and that at the beginning (10 cms.) gave a "mixing index" for this speed. This experiment was repeated for a number of speeds and a graph of mixing index against speed constructed.

Examples of the results are given in graphs 8 and 9. In the first case there is a sharp gradient in the mixing curve at the point where the interface disrupts, the speed being 160 - 165 r.p.m. Above this speed mixing is complete. Graph 9 shows a stable partly-mixed state. There is still a definite speed at which the water/oil interface
GRAPH 8. MIXING CURVE.
Graph 9. Mixing Curve.
breaks but mixing is not complete until the speed is raised to over 200 r.p.m. Between the "upper" and "lower" critical speeds stable systems are set up with a new emulsion/water interface. A few experiments showed this method to be unsuitable for measuring the breaking speed for a large number of systems because of the ambiguity of the results and the lengthy procedure involved.

In order to obtain the mixing curves more quickly the experiments were made continuous. The rotor was adjusted to about 100 r.p.m. and the interface allowed to settle. The speed was then increased at the rate of about 2 r.p.m. per minute until the speed was just below the breaking speed (found roughly in an earlier experiment) and no mixing had occurred. From this point the speed was increased at the rate of 1 r.p.m. every three or four minutes, readings of rotor speed and interface level being taken continuously. This was continued until the liquids were completely mixed. An example of the type of result obtained is given in graph 10. The stable partly-mixed systems were not obtained and the breaking speed may be taken fairly accurately from the graph.

(a) Correlation of Interfacial Breaking Speed with Liquid Properties

The interface breaks when the forces tending to disrupt it are greater, by an infinitesimally small amount than those tending to restore it. Exactly at the critical point these forces may be equated. The only disrupting force would be some function of the angular velocity of the rotor. The restoring forces would be proportional to (a) the difference in density between the two phases, and (b) the interfacial
Rotor speed (rpm)

Graph 10. Mixing Curve.
tension. A viscosity function must also be included.

Harrison, in 1903 (71), considered the problem of the stability of two stationary superposed fluids, of infinite extent, when subjected to a wave-like disturbance at the interface. His conclusion was that the modulus of decay of the disturbance would be proportional to the reciprocal of the square root of the kinematic viscosity. The present problem is similar, in some respects, to Harrison's.

Two viscosities are involved, of the upper and lower liquids and the system investigated here does not appear to be symmetrical - that is, the viscosity function may not be the same for each liquid. Symmetry was shown in several experiments by adding carbon tetrachloride to the upper layer to reverse the positions of the phases. In these cases the mixing occurred as before and the interface moved upwards, that is, in the reverse direction. No further investigation was made of this effect.

It may then be written, that at the critical point

\[ \psi(N) = K \sigma (\rho_o - \rho_i) \psi(\nu_i^{0.5}, \nu_o^{0.5})^{-1} \quad \ldots \ldots \ldots \ldots (63) \]

where \( \sigma \) is the interfacial tension, \( \gamma \) and \( \psi \) are functions and the subscripts \( o \) and \( i \) refer to the water and oil phases respectively.

Consideration of the experimental results shows that the correlation is, in fact

\[ N = K \sigma (\rho_o - \rho_i)(\gamma_i^{0.5} + \gamma_o^{0.5})^{-1} \quad \ldots \ldots \ldots \ldots (64) \]

This equation represents, as plotted on graph 11, a straight line
\[ \frac{(p - p_0) \cdot 10^4}{\sqrt{\frac{1}{\rho_1} + \frac{1}{\rho_2}}} \]  

Interfacial breaking speed (rpm.)

GRAPH II. INTERFACE INSTABILITY CORRELATION.
passing through the experimental points. On producing the graph until the right hand side of the equation is zero it is seen that mixing can never occur, in this apparatus, below a speed of about 85 r.p.m.

Experiments were carried out to check this point. The right hand side of this equation was made zero by adjusting the two liquids to equal density. It could not be done by reducing the interfacial tension to zero, since spontaneous emulsification would occur, nor by increasing viscosity, since vortices would not be formed and at infinite viscosity, of course, no flow would result. The experiments gave the points marked on the graph, in good agreement with the equation.

(f) Emulsification as a Criterion of Agitation

Following on the work reported above it was decided to investigate the rate of mixing above the critical speed at which the liquid-liquid interface becomes unstable. With a given system of liquids there is only one variable, the rotor speed and so far this apparatus the agitation intensity can be characterised by the rotor speed and the time needed to carry out a given amount of mixing. As mixing progresses in this apparatus the emulsion/water interface travels downwards until the whole system is emulsified. It was decided to use, as a criterion of mixing, the time taken for the interface to travel a certain distance down the system.

Several sets of experiments were carried out. In some
the liquid system consisted of 10 cms. of water and 10 cms. of oil
and here the time was taken for the emulsion to reach the 2 cms. level,
that is, the time taken for 80% of the water to be incorporated in the
emulsion. In other sets of experiments 13 cms. of each phase was used
and the time taken for the emulsion layer to reach the 3 cms. level,
that is, the time for 10 cms. of water to be incorporated in the
emulsion. Times were measured from the moment of starting the rotor.
Rotor speeds were measured continuously during each run. For each
set of experiments many runs were carried out at different speeds.

During the mixing time it was observed that both even and odd
numbers of vortices were readily produced in the emulsion layer, changes
occurring spontaneously as the emulsion depth increased. When the
number was even it was the surface vortex which rotated in the unusual
direction. As the speed of rotation increased the apparent stability
of the emulsion vortices increased and their rate of settlement, after
agitation, decreased.

The results were plotted simply as time of agitation required
to produce the standard amount of mixing against the rotor speed for the
individual experiments. All the graphs obtained show the same
characteristics.

As the speed of agitation increases the time decreases
(graph 12). The curve tends to a mixing time of infinity near the
interfacial breaking speed and to a mixing time of zero at high rotor
speeds. In some cases as the speed passes the 200 r.p.m. mark the
mixing time increases with increasing speed. This effect is due to
Graph 12A. Mixing Time.

Graph 12B. Mixing Time.
centrifugal force. Large globules are thrown from the emulsion layer and their presence in the water layer delays the formation of the next emulsion vortex. A logarithmic plot (graph 13) yields a straight line, but is of doubtful use.

The accuracy of these curves is low. The speeds given are correct to 2 r.p.m. but the times may be up to 15 seconds in error. In addition, stable partly-mixed systems are formed where mixing is delayed. This difficulty, to judge from the results obtained, is not as great as might have been expected.

It is possible to deduce an optimum speed for this apparatus from the mixing-time curves. Since the range of speeds has been covered from the breaking speed to the point where increased speed means increased mixing time it is obvious that a power/speed curve for this operation will show a minimum. Black has shown that the power consumed is proportional to the 2.5 power of the speed when density, viscosity and fluid pattern are constant. This may be assumed in any one group of experiments represented by one graph.

If, therefore, the group (time \( \times \) speed \(^{2.5} \)) that is, a group proportional to the total work, be plotted against speed, the curve will show a minimum corresponding to the optimum speed.

Graph 14 shows that this is the case, the optimum speed being 193 r.p.m. This speed is the optimum for emulsifying one pair of liquids. Whether it would be the same for other liquids, or another type of agitation, is a point requiring further investigation. The work/speed graph is similar to that of Yates and Watson discussed.
GRAPH 13. MIXING TIME.

log. of mixing time (secs).

log. of rotor speed (rpm).
Graph 14. Optimum Speed.
earlier (figure 1). These workers plot efficiency (in effect the reciprocal of work) to get a maximum corresponding to the optimum speed.

One important point arises from this work, that for emulsification this apparatus is inefficient. The rotor has to be run at high speed to break the interface, which is unnecessary in conventional mixers. Emulsification takes place only at and near the interface, which means that the bulk of the liquid is being agitated at high speed possibly unnecessarily, since a lower speed may be sufficient merely to retain droplets in suspension.

(g) Emulsion Settling Rate as a Criterion of Agitation

It has been noted above that the settling rates of the emulsions produced in this apparatus varied with rotor speed and time of agitation. It was also observed that after agitation had been carried out for some time no further change in the emulsion could be detected by eye. That is, the minimum particle size and hence the minimum settling rate had been reached. Some work was carried out to ascertain how much agitation was needed to reach this optimum condition and also to find out if the settling rate could be used as a criterion of agitation.

The apparatus was filled with water (0-13 cms.) and an oil mixture (13-26 cms.) the theostat and pulleys arranged to give a constant rotor speed in the mixing range and the rotor switched on for a definite length of time. On switching off the rotor a stop-watch was started and the course of the settling observed.
On settling, both oil and water separated from the emulsion. The emulsion/water interface was usually very ragged, but after some time the oil/emulsion interface became a well-defined horizontal line. The time was recorded for this line to reach the 14 cm. mark, that is, one cm. above the complete separation line. This breaking time was plotted against mixing time for each constant speed. Graph 15 shows the type of result obtained.

The graph indicates that for this particular system of liquids at one speed the minimum settling rate was reached after some 15 minutes. For times of agitation above 15 minutes the settling time remained constant. Further experiments indicated that the minimum settling rate decreased (that is, the constant value of settling time increased) as the rotor speed increased.

The accuracy of measurement in these experiments was poor, because the oil/emulsion interface was occasionally ill-defined and the overall settling time was low. Percentage errors were, accordingly, rather high. For later experiments mixtures were chosen with the following properties:

1. The interfacial breaking speed was low, so that a wider range of rotor speeds could be used;
2. The oil/emulsion settled down to a well-defined line quickly and a complete settling curve was recorded;
3. Long overall settling time.

The apparatus was filled with 10 cms. of each phase, the rotor switched on and the speed raised uniformly, during a period of
Graph 15. Emulsion settling rate.
one minute, to the value selected for the set of experiments. As the rotor was switched off at the end of the mixing period the stopwatch was started and the position of the oil/emulsion interface noted after definite intervals of time. A settling rate curve was plotted from these data (graph 16) and from it was deduced the time taken for 90% of the oil to separate. This criterion was quite arbitrary.

This procedure was repeated for a number of agitation times for each speed and for a number of speeds. Results are shown in graph 17. The curves tend to a minimum settling rate (maximum settling time) but this minimum does not vary directly with speed. It will be noticed that the curve for 207 r.p.m. lies between those for 179 and 157 r.p.m. Again, prolonged agitation at the highest speed (238 r.p.m.) causes an increase in settling rate (see graph 18). It is clear, however, that there is a definite time of agitation, depending on the conditions, beyond which further agitation merely serves to maintain the dispersed particles in suspension.

(h) The Maintenance of Emulsions

Energy is expended in two ways when this apparatus is used as an emulsifier. Energy is used for (a) the production of the globules and (b) the suspension of the globules. A few experiments were carried out to discover to what extent the vortical motion would maintain emulsions as distinct from forming them.

A system was chosen (containing 13 cms. of each phase) with an interfacial breaking speed of 165 r.p.m. This system was agitated for half an hour at a rotor speed of 200 r.p.m., that is, until the
GRAPH 16. SETTLING RATE.

- ○ 3 mins. agitation.
- ▲ 25
GRAPH 17. EMULSION SETTLING RATE.
238 rpm.

**GRAPH 18. EMULSION SETTLING RATE.**
emulsion was uniform and moving in regular vorticular motion. The rotor speed was then reduced slowly (2 or 3 r.p.m. every minute) and observation made of the state of the vorticular motion and the settlement of the emulsion (graph 19).

As the speed decreased to the interfacial breaking speed the vortices remained stable and no settling occurred. At about 148 r.p.m. the vorticular motion reverted to simple circular motion from the bottom upwards disappearing until at about 115 r.p.m. the vortices had completely disappeared. At this speed the bottom 6 - 7 cms. of the emulsion became stationary, water separated quickly and the emulsion began revolving again. The rotor was kept as near as possible to this speed and complete settling took place.

Settling, in this case, was due to the disappearance of the vorticular motion. It can be shown that settling may occur when the motion is vorticular and the circulation intense.

A slow-settling emulsion (water-carbon tetrachloride-linseed oil) was made by hand shaking, the mixing unit filled and the rotor set at a fixed speed. At definite intervals of time samples of the emulsion were withdrawn, using a 50 ml. pipette and the settling rate measured. Agitation was continued and samples taken over periods up to 120 minutes.

Samples were always taken from the same place and the settling rates measured by filling standard tubes and timing settlement through an arbitrary distance of fall of the oil/emulsion interface. Four rotor speeds were used and the results collected in graphical form.
GRAPH 19. SETTLEMENT DURING MIXING.
At the lowest speed agitation up to 30 minutes results in a decreased settling rate but beyond this time increases it. At the next higher speed (109 r.p.m.) the settling rate is substantially constant while at 160 r.p.m. a constant increase in settling rate is apparent. At the highest speed (183 r.p.m.) the fall in settling time (increase in settling rate) is sharp.

This apparatus, in this instance, is tending to separate the constituents of the emulsion and this means that the variation of emulsion settling rate cannot be used as a criterion of agitation intensity.

(j) Liquid-liquid Extraction as a Criterion of Agitation

It has been shown above and will be confirmed later that the agitation within the vortex rings is very intense and that there are no dead spaces in the system. However, the important point is not that such a mixing system will do a given duty, but whether it will do it for less total expenditure of work than other systems.

It was decided to compare the efficiency of the vortex rings directly with a conventional agitator. A propeller was selected as being the simplest to use and as a criterion of mixing it was decided to use a liquid-liquid extraction process.

Below work with the vorticular agitator alone is described. Details of the results obtained with the propeller are given in Appendix G, only the final power figure being quoted here.

The method was as follows. A mixture of paraffin and carbon
GRAPH 20. SETTLEMENT DURING MIXING.
tetrachloride was made up to a density slightly greater than that of water, a known amount of benzoic acid dissolved in it and a 10 cm. layer put in the agitator. This was carefully covered with a 10 cm. layer of water. The rotor was switched on to a constant speed in the mixing range. At the end of two minutes the rotor was stopped, the liquids allowed to separate and a sample of the water layer removed. After a further two minutes the machine was restarted, a further two minutes agitation carried out and another sample taken. This was continued until equilibrium conditions of mass transfer were reached. Samples were removed by pipette and acid determined by volumetric analysis (Appendix D). From these figures a graph of percentage saturation against time was plotted (graph 21).

This procedure was repeated at other rotor speeds and a percentage saturation/time curve constructed for each. The time of agitation required for the water phase to reach 70% of the saturation value was selected as an arbitrary criterion, a value being deduced from each extraction/time curve. A plot of this standard-extraction-time against rotor speed yields (graph 22A) a curve of the same type as that obtained for standard-emulsion-time against rotor speed (graph 12).

As before the optimum speed was then obtained by plotting a power group (time x speed \(^2\)) against rotor speed and finding the speed for which the total work consumed was a minimum. The optimum speed (graph 22B) for this system was about 155 r.p.m. which differs considerably from the optimum speed (about 195 r.p.m.) for the emulsification system used previously.
GRAPH 2IA. EXTRACTION RATE.

GRAPH 2IB. EXTRACTION RATE.
GRAPH 22A. EXTRACTION TIME.

GRAPH 22B. OPTIMUM SPEED.
Black gives, for this apparatus, a power consumption graph, the variables being viscosity, density, speed and fluid pattern. The oil phase used here consisted of 290 mls. of carbon tetrachloride and 710 ml.s of paraffin, the water phase of 1000 mls. of water. For this system a mean density of 1.00 gr/ml and a mean viscosity of 1.15 centistokes were assumed. Of the three power relations given by Black the one giving the largest power requirement was used and on inserting the values \( \rho = 1.00, \ \nu = 1.15, \ N = 155 \) the power requirement was seen to be 2.60 ft-lbs/min. No correction was necessary for liquid depth since the same was used in each case. At 155 r.p.m. the total agitation time (graph 22A) was about 1\( \frac{1}{2} \) minutes, giving a figure of 3.25 ft-lbs for the total work.

The work figure obtained using a propeller, for the same amount of extraction, using the same volumes of the same liquids, was 3.02 ft-lbs. The two mixing systems require, therefore, approximately the same total expenditure of work.

(k) Extraction Across the Liquid-liquid Interface

The extraction as measured at speeds greater than that at which the interface breaks is a measure of the mass transfer to and from dispersed droplets agitated in vortex rings. This type of experiment gives no idea of the rate of mass transfer across a vortex when no globules are present.

It might be argued that mass transfer across a vortex would be rapid due to the intense agitation and circulation within it. On the other hand the streamlines in a vortex ring move in closed nearly-
circular paths at velocities different from adjacent streamlines and show no tendency to mix with one another. In this case it might be expected that mass transfer would occur only by diffusion normal to the streamlines and would be a slow process. A short programme of experiments was carried out to elucidate these points.

The bottom liquid layer (0 - 10 cms.) was water, the upper (10-15 cms.) was paraffin containing a known, dissolved, amount of benzoic acid. This depth of paraffin gave two vortices and thus reduced any errors which might occur due to mass transfer resistances at inter-vortex boundaries. It was not found possible to obtain a single stable paraffin vortex in a reasonable depth of liquid.

The lower (water) layer was put into the apparatus and the rotor run for several minutes to remove air bubbles. The level was adjusted to 10 cms., the upper paraffin layer added carefully and the rotor switched on. Rotor speed readings were taken continuously during the run.

At a given time the rotor was stopped, most of the bottom layer run off through a pipe fitted to the baseplate and the remainder of the water layer and the paraffin layer run into a separating funnel. The layers were separated and the paraffin titrated. Since all the paraffin was recovered large samples could be used for the titrations, giving increased accuracy.

The procedure was repeated for a number of rotor speeds and times of agitation. Results were expressed as the percentage of original acid transferred from the paraffin phase and these percentages
were plotted against agitation time (graph 23). The change in extraction rate with rotor speed is negligible.

At a later stage one set of experiments was carried out in this apparatus and another set, for comparison, carried out in a stoppered, unagitated bottles, the procedure being a slight improvement on that described. The results appear in graph 24. Again it will be observed that, in the first 45 minutes at least, agitation does not result in greatly increased extraction.

(1) Discussion

The experimental work reported in this section has a bearing on both practical and theoretical problems. On the one hand it shows the advantages and disadvantages of vortical motion as a means of mixing and on the other it connects directly with theoretical work on interface stability.

The vortical motion in each of the two liquid layers was similar to that in a single phase system. The lower layer was modified by the upper in that even-systems were possible, that is, the "surface-effect" has disappeared. The upper layer still showed the surface-effect and an odd-system but the bottom-effect, the very large bottom vortex, had disappeared. The vortex arrangement was usually such that the interface coincided with a boundary between pairs, but on several occasions arrangements were observed where the interface coincided with a boundary between the two vortices of a pair. In this case one vortex pair consisted of two dissimilar vortices in two liquids.
GRAPH 23. EXTRACTION RATE.
GRAPH 24. EXTRACTION RATE.

0 30 r.p.m.
Δ no agitation.
The liquid-liquid interface was not destroyed by the vortices, but by a wave-motion of increasing amplitude. There was never any tendency for a vortex to be set up across the interface, which would have resulted in mixing. When the upper layer was too shallow for the formation of a single vortex the liquid was not taken into the vortex immediately below it in the lower layer. The motion in this shallow upper layer remained, apparently, circular. The vortices tended to be single-phase vortices.

The shape of the mixing curve is a direct result of the existence of a critical breaking speed for the interface and the tendency for the vortices to remain single phase vortices. It would be expected that when the interface breaks mixing would continue until all the contents had been emulsified. The only vertical components of force, however, are those within the vortices. Consequently the dispersed droplets cannot be removed quickly from the interface, which gradually changes from a liquid-liquid to an emulsion-liquid interface. The breaking speed of the interface has now changed and a stable condition reached which can only be made unstable by exceeding the new critical speed. When the apparatus is run continuously and the speed is raised continuously this new interface has less opportunity to form.

The mixing curves obtained are similar to those reported by Hixson and Tenney for the suspension of sand (see figure 11). It would appear that there is a similar viscosity effect also, since as viscosity decreases mixing becomes more difficult.

The precise part the vortices play in the disruption of the interface is not clear. For very viscous liquids the interfacial
FIG. II. SAND SUSPENSION BY PROPELLER.
breaking speed would be low, while the vortex formation speed would be high. Under these conditions it might be possible to destroy the interface with no vortices present. It may be that the vortices are an incidental, rather than an indispensable, part of this process.

Some experimental work has been reported by Keulegan on the mixing of streams of miscible fluids (89) flowing one over the other. There is a general similarity in mechanism of mixing. The interface, in both cases, took up a wave-like motion before mixing occurred, droplets of liquid being thrown from the crests of the waves, into the other phase.

Fine-grained emulsions cannot be produced in this apparatus, nor maintained, since there may be a separating effect. For each system there is an optimum speed of production for minimum work, a maximum time of agitation beyond which particle size is not decreased and a minimum speed for maintenance of the emulsion below which settling occurs.

It should be made clear what has been meant when it has been said that the settling rate has been increased by agitation. This refers to the rate of settlement after agitation has ceased. Consider, for example, an emulsion which settles a given amount in 30 minutes and which, after 30 minutes agitation, then settles the same amount in 25 minutes. Here it would be said that agitation has increased the settling rate. It might also be argued that since the total settling time after agitation is 55 minutes (30 + 25) the settling rate has been retarded.

Settling rate is not a good criterion of agitation. In this
the author agrees with Olney and Carlson (22) who attempted, without success, to use it.

The vortex formation speed is considerably higher for emulsions than it is for either of the components. Since the similarity is this means that the apparent kinematic viscosity of the emulsion is considerably greater than that of the components. This deduction is not justified, however, since the theory is for a homogeneous fluid.

Liquid-liquid extraction may be used as a criterion of agitation and it has been shown, using this method, that the minimum power required to do a given duty is about the same for a propeller and a vorticular system. This shows high efficiency for the vorticular system, since it was being used at far from the best conditions. Extraction was carried out for two minutes at a time and in this period most of the power is utilised for breaking the interface rather than for mixing. Mixing occurred almost immediately in the propeller system. The real efficiency of the vortices for liquid-liquid extraction must, therefore, be considerably greater than that of the propeller system.

Below the interfacial breaking speed the extraction rate across the liquid-liquid interface varies little with speed and is very near the rate with no agitation. This means that the interfacial film resistances and thicknesses are the same. This suggests that the vortex streamlines do not penetrate close enough to the interface to interfere with the resisting film, that is, that the rotating section of the vortex does not occupy the whole of the annular compartment.

At the boundary between two vortices the conditions are as indicated in the diagram (figure 12). Taylor has shown, in his original
Motion approximates to parallel streams at A-B

FIG. 12. FLOW AT BOUNDARY.
work, that where $\psi$ represents the Stokes' stream function (a measure of the velocity of the vortex streamlines) $\psi_2 > \psi_1 > \psi_0 = 0$.

Across the interface, therefore, since the streamlines in the two vortices are moving in the same direction, the velocity gradient is low. This system approximates to two liquids, flowing parallel with low velocity and with low velocity gradient across the interface. Mass transfer across such an interface would, therefore, be due mainly to diffusion, showing little variation with rotor velocity.

This argument may be extended to the case where energy is transferred from the inner cylinder to the outer (or vice versa) or from the centre of a vortex to a cylinder.

The transfer meets three resistances (a) the inner cylinder film (b) the main body of the vortex and (c) the outer cylinder film. The process is controlled by (a) and (c) when vortices are present. All three are important when vortices are not present, that is when the rotor speed is low. Since (a) and (c) are substantially independent of rotor speed the process in these two cases is diffusion-controlled, slow and does not vary much with rotor speed except at the point where vortices are formed. This is illustrated in figure 13A.

The resistances (a) and (c) can be reduced by axial flow of liquid (i.e. flow parallel to the cylinder axis) and it will be shown later (Part III) that such flow does not destroy the vortical motion until a critical velocity has been exceeded. A flow of liquid, therefore, increases the transfer process until the point is reached where the vortices are destroyed, when resistance (b) increases and the overall
A. At constant or zero axial flow rate.

B. At constant speed above the critical.

FIG. 13. VARIATION OF TRANSFER RATE.
transfer rate drops. Further increases in flow rate reduce (a) (b) and (c) due to normal turbulence, increasing transfer but at a lower rate than before (see figure 13B).
EXPERIMENTAL WORK: PART III

Experiments in the Horizontal Long-tube Agitator

with a Single Liquid
All the apparatus which has been built so far for the observation of the vortex rings has been vertical, giving horizontal vortex rings. Since the theories of Taylor and Meekyn take no account of the force of gravity there is no reason to assume that a horizontal apparatus, or one inclined at any angle, should not produce rings. A horizontal apparatus would be more convenient to operate, since it could be sited at bench level. The length of the apparatus could also be increased.

In addition no work at all has been carried out on the observation of the motion when the fluid is flowing through the apparatus. The experiments of both Fage and Cornish (see earlier) were on pressure drop only. Since the results of these two workers disagree and there are differences between the theories of Massoud and Goldstein the position is far from clear.

It will be remembered, too, that the resistance measurements of Couette and Mallock could not be reconciled with the visual observations of Taylor, which leads one to doubt whether the equally indirect pressure measurements of Fage and Cornish are reliable.

It was decided, therefore, to investigate the motion of a liquid contained between a rotating inner and stationary outer cylinder under the influence of a pressure gradient parallel to the axis. The apparatus was constructed to fit the following requirements:

-91-
(i) Size and shape convenient for bench operation;
(ii) Capacity sufficient for use as a piece of small-scale mixing equipment, should the fluid motion prove suitable;
(iii) The ratio of annular space to rotor radius to be variable between fairly wide limits;
(iv) The ratio of rotor length to annular space to be fairly high to reduce or eliminate end effects;
(v) Suitable for visual observation of the flow;
(vi) Suitable for experiments under both axial-flow and no-flow conditions;
(vii) Wide range of rotor speeds;
(viii) Usual devices for filling and emptying, provision of thermometer pockets, etc.

The apparatus is shown in figure 14 and plate 9. The author would like to acknowledge the assistance of the late Mr. M.P.T. Stephens of this department in the design, construction and subsequent modification of the equipment.

(b) Description of the Horizontal Long-tube Agitator

The outer cylinder of this apparatus was of glass, 2' 6" in length and 1\(\frac{5}{8}\)" internal diameter. It was supported at each end in a screwed liquid-tight joint. The brass flanges holding this glass cylinder were bolted to standard 1\(\frac{5}{8}\)" brass T-pieces. To one T-piece was bolted a brass end-plate fitted with a ball race and to the other an assembly, also in brass, containing a ball race and stuffing-box.
FIG. 14. DETAILS OF HORIZONTAL AGITATOR.
Plate 2. HORIZONTAL AGITATOR
The rotor, of brass, was supported by these two bearings. The free end of the rotor passed through the stuffing-box and was fitted with a pulley which was driven, by V-belt through reduction pulleys, by a resistance-controlled D.C. motor. Gaskets were of rubberised canvas and packings graphited asbestos.

The T-pieces were fitted, through bushes, with \( \frac{1}{2} \)" inlet and outlet pipes connected to a centrifugal pump, driven by an A.C. motor, and a liquid storage tank. A control valve was inserted on the inlet line. The whole apparatus was supported on a rigid aluminium frame. Plate 9 shows the apparatus as used for demonstration experiments.

When run continuously the apparatus was filled by means of the pump and liquid circulated for some time before commencing experiments in order to eliminate air from the apparatus. When run batchwise the fittings to the T-pieces were disconnected and the liquid was poured in directly, air being removed in this case by tilting the apparatus. Emptying was carried out by running off through draincocks, fitted in the bottom of the T-pieces, to trays placed under the supporting frame.

The liquids used for fluid motion investigations were oils and oil mixtures, of various kinematic viscosities, containing small amounts of aluminium powder as a tracer.

(c) Preliminary Description of the Fluid Motion

It was at once obvious, with no axial flow, that vortex
rings were formed in the long-tube apparatus which were exactly similar to those observed in the vertical apparatus. At low rotor speeds the motion was a simple circular one which at higher rotor speeds was replaced by vortex rings. The motion when the fluid was flowing axially was also very similar to that observed previously. At low rates of flow the vortices were in discrete rings moving bodily along the tube. As the flow rate increased the vortex wave-velocity increased until a spiral type of flow supervened and eventually, at higher flow rates, the vortices were completely destroyed.

The agitation within the whole system was uniform and intense. There were no dead spaces. No end-effects were apparent except at very high flow rates. The detailed results and observations appear below.

(a) The Vortex Formation Speed with no Axial Flow

The vortices formed with no axial flow were exactly of the type described by Taylor and Lewis. It was therefore decided to check their formation speeds against theory for variations in rotor size, kinematic viscosity and, within a limited range, outer cylinder size.

The apparatus was filled with oil, the oil temperature taken and the rotor speed raised slowly until vortices just appeared. Owing to slight irregularities in the glass cylinder they did not appear all along the tube at the same time. This rotor speed was noted, using a small hand tachometer. The speed was further raised
until the visible length of the tube was just completely full of vortices. This speed was noted. The rotor speed was then slowly reduced until a portion of simple circular motion appeared, the tachometer reading taken and the speed further reduced until the last vortices just disappeared. This last speed was also noted and the oil temperature taken.

The mean of these four speeds was taken as the critical speed and the mean of the two temperatures used for deriving the kinematic viscosity of the oil. The ratio of critical speed to kinematic viscosity was calculated and compared with values deduced from Taylor's theory. The results are summarised below.

<table>
<thead>
<tr>
<th>Ratio $\frac{a}{R_1}$</th>
<th>$\frac{N}{D}$ (exp)</th>
<th>$\frac{N}{D}$ Taylor</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.24</td>
<td>14.9</td>
<td>13.7</td>
<td>9</td>
</tr>
<tr>
<td>0.36</td>
<td>13.6</td>
<td>11.0</td>
<td>24</td>
</tr>
<tr>
<td>0.36</td>
<td>12.7</td>
<td>11.0</td>
<td>15</td>
</tr>
<tr>
<td>0.53</td>
<td>9.1</td>
<td>8.70</td>
<td>5</td>
</tr>
<tr>
<td>0.75</td>
<td>8.2</td>
<td>7.93</td>
<td>4</td>
</tr>
<tr>
<td>0.81</td>
<td>6.8</td>
<td>7.40</td>
<td>8</td>
</tr>
</tbody>
</table>

These values have been discussed in a general manner, with respect to theory, in Introduction II (c).

Agreement is better at large values of $\frac{a}{R_1}$ than it is at small values. This is in disagreement with both Taylor and Lewis and is also not to be expected on theoretical grounds. The reason is simple however. The glass cylinders were used as supplied by the
manufacturers and were not exactly uniform. In consequence the differences in the size of the annular space throughout the length were greater, relatively, for small annuli than for large. It is this error which predominates in the experimental values.

The diameters of the tubes were derived from the total volume divided by total length and whereas a small length of relatively large bore would have little effect on this diameter it would have a disproportionately greater effect on the experimental critical speed. In the circumstances the agreement with theory must be considered fairly good.

The difference between the two values quoted for a ratio of \( \frac{a}{R_1} \) of 0.36 was probably due to slight off-centring in one or both cases.

Graph 25 shows a direct comparison of these values with those calculated from the theories of Taylor and Meksyn and may be looked upon as a portion of graph 1 showing greater detail. Values calculated from Goldstein's theory are also included. Taylor's and Meksyn's values are in fairly good agreement with experimental results, but those of Goldstein are all very low. This would be expected since Goldstein made no correction for cases where \( \frac{a}{R_1} \) was not small. As \( \frac{a}{R_1} \) decreases Goldstein's values approach the experimental values.

(e) The Vortex Wavelength with no Axial Flow

Theory predicts that at the critical speed, with only the
GRAPH 25. FORMATION SPEEDS: COMPARISON OF RESULTS.
inner cylinder rotating, the vortex spacing will be equal to the annular space, that is, that the pair size, or wavelength, will be equal to twice the annular space. Previous workers have shown this to be approximately correct for horizontal rings at the critical point but no work has been carried out in the region above it. Experiments are reported below which check the theory for the apparatus used here. The behaviour of the rings at speeds above the critical is also noted.

The apparatus was filled with oil, the rotor speed adjusted to be very slightly above the critical and measurements made of vortex size. This was done by counting the number of vortices in the visible length of the tube, a correction being made for the short lengths at each end for fractions of rings which were not included. Mean results are tabulated below.

<table>
<thead>
<tr>
<th>$\frac{\bar{a}}{R_1}$</th>
<th>Theoretical</th>
<th>Experimental</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.36</td>
<td>1.03</td>
<td>1.04</td>
<td>1</td>
</tr>
<tr>
<td>0.53</td>
<td>1.35</td>
<td>1.33</td>
<td>1(\frac{1}{2})</td>
</tr>
<tr>
<td>0.75</td>
<td>1.67</td>
<td>1.62</td>
<td>3</td>
</tr>
<tr>
<td>0.81</td>
<td>1.79</td>
<td>1.65</td>
<td>8</td>
</tr>
</tbody>
</table>

Agreement with theory is good, becoming closer as the ratio $\frac{\bar{a}}{R_1}$ decreases, that is, as conditions approach theoretical ones. The percentage difference between experimental results and theoretical predictions increases sharply for the small change in $\frac{\bar{a}}{R_1}$ from 0.75 to 0.81.

Once the vortices had been formed it was possible to observe
the effect of increasing speed on them. If the speed was increased with a set of vortices already in the tube there was no change in size. If the rotor was stopped, the liquid allowed to come to rest and the rotor then started at a higher speed the vortices were smaller than previously. This observation applied to vortices formed at low values of \( \frac{a}{R_1} \) but not at large. The figures below show the type of result obtained.

\[
\frac{a}{R_1} = 0.53 \quad 2a = 1.35 \text{ cm.}
\]

<table>
<thead>
<tr>
<th>Speed (r.p.m.)</th>
<th>Wavelength (cms.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>168</td>
<td>1.26</td>
</tr>
<tr>
<td>213</td>
<td>1.19</td>
</tr>
<tr>
<td>266</td>
<td>1.18</td>
</tr>
<tr>
<td>343</td>
<td>1.19</td>
</tr>
<tr>
<td>437</td>
<td>1.15</td>
</tr>
<tr>
<td>527</td>
<td>1.12</td>
</tr>
<tr>
<td>542</td>
<td>1.13</td>
</tr>
<tr>
<td>571</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Here the decrease in size is not regular but is most definite, the maximum deviation from the mean (1.17 cm) being 8%. The maximum deviation from the theoretical value is 1%. The figures below are for a larger value of \( \frac{a}{R_1} \).

\[
\frac{a}{R_1} = 0.81 \quad 2a = 1.79 \text{ cms.}
\]
<table>
<thead>
<tr>
<th>Speed (rpm)</th>
<th>Wavelength (cms)</th>
<th>Speed</th>
<th>Wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>271</td>
<td>1.71</td>
<td>364</td>
<td>1.67</td>
</tr>
<tr>
<td>313</td>
<td>1.69</td>
<td>371</td>
<td>1.69</td>
</tr>
<tr>
<td>315</td>
<td>1.59</td>
<td>399</td>
<td>1.69</td>
</tr>
<tr>
<td>321</td>
<td>1.59</td>
<td>401</td>
<td>1.61</td>
</tr>
<tr>
<td>327</td>
<td>1.63</td>
<td>414</td>
<td>1.63</td>
</tr>
<tr>
<td>339</td>
<td>1.61</td>
<td>445</td>
<td>1.65</td>
</tr>
<tr>
<td>350</td>
<td>1.65</td>
<td>445</td>
<td>1.67</td>
</tr>
<tr>
<td>351</td>
<td>1.65</td>
<td>447</td>
<td>1.65</td>
</tr>
<tr>
<td>359</td>
<td>1.63</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In this case it is impossible to say whether there is a decrease or not. The variation is less marked than when \( \frac{S}{R_1} = 0.53 \), the maximum deviation from the mean (1.65cm) being only 4%. In no case however is the actual wavelength greater than the predicted value.

The wavelength also depends on the previous history of the system. For example, in some experiments with axial flow a spiral was formed and the axial flow then stopped. The resulting vortex rings were considerably larger than the rings which would normally have been produced at that rotor speed. Size increases up to 10% were observed. Occasionally rings so formed were larger than the theoretical values.

In general, however, the theory is in good agreement with experiment at and near the critical point at values of \( \frac{S}{R_1} \) of 0.75 and below.
With no axial flow the fluid pattern was the same as that described by Taylor, Lewis, Black and the author earlier in this work. Plates 10, 11, 12 show this type of motion for three different rotor sizes in the same glass cylinder.

When there was flow down the annulus there were four possible primary types of flow: flowing rings, flowing spirals, standing spirals and turbulent flow. Secondary types of flow were made up of two or more of these together in the apparatus.

The flowing ring system is shown in plate 13. A comparison with earlier plates shows that this motion is of the same type as the standing rings. The vortex spacing is approximately equal to the annular space. As the linear flow of liquid increased these rings travelled more quickly down the tube and lost some of their rigidity. Plate 14 shows this condition and it should be noted that the wavelength remains approximately constant.

As the flow further increased a spiral type of instability set in, which is shown in plate 15. This motion was well-defined and its surfaces of separation as rigid as those of the rings. It will be noted that the leading vortex of the pair is larger than the trailing vortex, a fact noted by Taylor in quite different circumstances. A similar motion will be described later when a conical apparatus is used. The general motion is that of a double-threaded screw, each thread being a vortex-roll, the threads being unequal in size and rotating in opposite directions, the whole
Plate 11. STATIONARY RINGS
Plate 12. STATIONARY RINGS
moving in the direction of linear flow.

With the smallest annulus it was found possible, over a small range of flow-rates, to obtain a standing spiral, that is, a spiral of exactly the same type as before, but with no forward motion of the threads. This type was exceedingly well-defined, but tended to revert to separate flowing rings. Plate 15, is in fact, a photograph of this type.

At high rates of flow it was found possible to destroy the vortex system altogether. Plate 16 shows a spiral motion where the flow rate was very nearly sufficient to destroy it. As the rotor speed increased the flow rate necessary to destroy the vorticular motion increased. This forms the basis of a separate section below.

As the liquid flow rate was reduced from the turbulent zone the reverse of that described above occurred. First an unstable spiral was formed, then a stable one, followed by a flowing ring system which became, at zero flow, a standing ring system.

It should be pointed out that the terms "streamline" (for the state below the value of the critical speed at zero flow) and "turbulent" (for the state where the liquid flow rate is too great for vortices to exist) are used here without any reference to a critical Reynolds number. The term "vorticular" is used to describe the flow when rings or spirals are present.

Occasionally two or more of the types of motion described above co-existed. This occurred fairly frequently with the standing
Plate 16. VORTEX SPIRAL NEARLY DESTROYED
spirals, where a portion of the tube was sometimes occupied by flowing rings, one type being slowly replaced by the other. Occasionally other combined types were seen, for example, where from a central portion of standing or slowly flowing rings two spirals commenced, one on each side of the central portion, moving in opposite directions. This type was most peculiar in that one of these two spirals was, of course, moving against the direction of axial flow.

(g) The Wave-velocity of the Flowing Vortices

It was found possible to follow, accurately, a single vortex boundary from one end of the tube to the other for both moving rings and spirals. This could be done most accurately for slow moving rings, least accurately for the high-velocity unstable spirals near the onset of turbulence.

Experiments were performed where the linear velocity of the vortices and of the liquid were measured. The former was measured by timing (with a 1/5 second stop-watch) a vortex boundary from one end of the glass tube to the other (some 70 cms.). The latter was deduced from readings of volumetric flow per unit time and the cross-sectional area of the annulus.

The type of result obtained is shown in graphs 26 and 27. It will be seen that the flow may be divided into three zones. In the first, or linear, zone the linear velocity of the vortex is directly proportional to the linear velocity of the liquid. In the second, or acceleration, zone the vortex velocity is greater than the liquid velocity, is increasing at a greater rate and tends to infinity. The third zone
GRAPH 26: VORTEX WAVE VELOCITY.
GRAPH 27. VORTEX WAVE VELOCITY.
is the turbulent zone, where the vortices have been destroyed. In
the first zone were the flowing rings, in the second the spirals.
There was some overlap between them.

Closer examination, see graphs 28 and 29, shows that in the
linear range the vortices were travelling at a greater velocity than
the liquid in which they were being formed. The ratio of vortex
wave-velocity to liquid flow rate increases as the ratio $\frac{B}{R_1}$ decreases.

The graphs also show that increased rotor speed increases both the
linear and acceleration zones, as would be anticipated.

(h) The Onset of Turbulence

Attention was next given to the onset of turbulence. The
point at which this occurred was reasonably sharp and the graphs
already referred to showed that the linear flow needed to produce
turbulence increased as rotor speed increased. It was, therefore,
decided to proceed along the same lines as before in order to correlate
the position of the turbulence point with the two cylinder diameters,
rotor speed and liquid properties. This has been successfully done.

The method of attack was as follows. The rotor speed was
adjusted to a desired value, the vortices allowed to settle down, the
pump started and the inlet valve adjusted to give a very small flow.
The valve was then slowly opened until the whole of the vortical
motion was just destroyed. At this point the discharge from the
apparatus was switched from the pump reservoir (a circulating system
being used) to a measuring vessel (8 litres capacity) and the time
GRAPH 28. LINEAR RANGE.

slope = 1.15.
A. \( \frac{\alpha}{\kappa_i} = 0.36 \).

slope = 1.05
B. \( \frac{\alpha}{\kappa_i} = 0.81 \).
GRAPH 29. LINEAR RANGE. ($\alpha / \kappa = 0.53$)
needed to fill it observed. The liquid temperature was taken and the kinematic viscosity deduced. From these figures were calculated the group \( \frac{N}{V} \) for the experiment (r.p.m./sec) and the linear velocity of the liquid along the tube (\( v \), in cms/sec).

Graph 30 shows the type of result obtained, where a representative number of points has been plotted. It will be seen that although there is a fair scatter of points, they are well represented by a straight line. At low values of \( \frac{N}{V} \), near the critical for no axial flow, the fluid motion was very prone to disturbance in the spiral state just before the onset of turbulence. This led to greater uncertainty as to the position of the turbulence point.

This is reflected in graph 31, where \( \frac{a}{R_1} \) is 0.36 and even more so in graph 30, where \( \frac{a}{R_1} \) is larger. As \( \frac{a}{R_1} \) decreases the rigidity of the lines of separation increases and this effect is not noticed - see graphs 32 (\( \frac{a}{R_1} = 0.24 \)) and 33 (\( \frac{a}{R_1} = 0.21 \)).

In all, four systems were investigated using three outer cylinders, three inner cylinders and a range of kinematic viscosities and rotor speeds.

It will be noticed that as the Taylor critical values increase in the four systems the slope of the resultant straight line increases (graph 34). Also the curves, when projected, cut the vertical axis near the Taylor critical value. These points suggest that these curves can be replaced by one if account be taken of the Taylor constant. This has been done in graph 35, where three of the sets of data are correlated.
Graph 30. \( \frac{a}{h} = 0.53 \)
GRAPH 31. ($\frac{a}{R_L} = 0.36$)
GRAPH 32. \( \frac{\Delta}{\varphi} = 0.24 \)
GRAPH 33. \( \frac{a}{K_1} = 0.21 \).
\[
\frac{\rho}{R_1} = 0.21, \quad \frac{N}{N_{\text{crit}}} = 16.8 \\
\frac{\rho}{R_1} = 0.24, \quad \frac{N}{N_{\text{crit}}} = 13.6 \\
\frac{\rho}{R_1} = 0.36, \quad \frac{N}{N_{\text{crit}}} = 11.0 \\
\frac{\rho}{R_1} = 0.63, \quad \frac{N}{N_{\text{crit}}} = 8.70
\]
well. The fourth set is low and the slope slightly different, suggesting that an incorrect value of $\frac{N}{D}$ crit. has been used. This line corresponds to the smallest annulus, where a small error in the outer cylinder radius could easily alter $\frac{N}{D}$ crit. to this extent.

(j) Discussion

In the experiments reported above a visual method has been used to investigate the motion of a liquid contained between two concentric cylinders, the inner rotating, with flow parallel to the axis. This method is at once more dependable and wider in scope than pressure measurements only. Indirect pressure measurements have the disadvantage that instability of motion has to be inferred, and no idea of the type of instability can be obtained.

In the present work many types of flow have been observed and other types, in other pieces of equipment, will be discussed later. In general the flow within these fluid systems must be so complex as to preclude any theoretical approach. Where several types may exist, together or separately, in the same conditions, the problem appears even more complex. However, points of agreement can be found between these results and the theoretical work discussed earlier in this thesis.

The speed of formation of the standing rings has been shown to agree well with theory. The results obtained here are high, but this is to be expected since at the critical speed the vortices form infinitely slowly. To view them, therefore, the
the rotor speed must be greater than the theoretical critical. The agreement is good up to values of \( \frac{a}{R_1} \) of 0.31, the largest for which measurements have been made.

The wavelength of the standing rings also agrees with theory, but with some modifications. The difference increases as \( \frac{a}{R_1} \) increases and the wavelength is never greater than the theoretical. This can be shown to be general. The values of Taylor and Lewis (as deduced from their papers) are all lower than the theoretical, as are those for all systems dealt with in this thesis where there has been no axial flow and where the number has not been altered by modifying the flow lines with a rod. This does not apply to vortices exhibiting surface or end-effects. It appears that the theoretical wavelength is, in fact, a maximum value, for the systems mentioned above.

Systems have been produced artificially, by modifying the flow lines with a glass rod, in the vertical agitator where the size was greater than the theoretical and rings produced from spirals also exhibit this. A further qualification is that increase of rotor speed at low values of \( \frac{a}{R_1} \) reduces the wavelength.

To estimate vortex size, therefore, it is necessary to know the values of \( a, \frac{a}{R_1} \), the ratio of liquid depth to annulus, the values of \( \frac{N}{\nu} \) and \( \frac{N}{\nu_{\text{crit}}} \), and details of the origin of the vortex rings.

It has been noted that increase of rotor speed does not appear to affect the size of vortices where \( \frac{a}{R_1} \) is large. This means that these vortices are more persistent, though less well-defined,
than those at low values of \( \frac{a}{R_1} \), a hypothesis supported by the fact that Lewis was able to destroy vortices by excessive rotor speed at low values of \( \frac{a}{R_1} \) but not at high.

Goldstein's deduction that wavelength is approximately equal to twice the annular space at and near a Reynolds number of zero is shown to be true, but the results obtained here are in complete disagreement with his prediction that the wavelength will decrease sharply as the Reynolds number is increased. This sharp reduction in his figures may be due to his drastic approximation. Massoud's wavelength figures are also about twice the annular space, but his Reynolds numbers are very low.

The subject of vortex wave-velocities is a very interesting one. In the acceleration zone little work could be done with this apparatus. The spirals are most stable at low values of \( \frac{a}{R_1} \) and it is in these conditions that the present apparatus is least useful, due to errors in centring the two cylinders. In the linear zone however some very interesting observations were made.

The vortex wave-velocity is greater than the fluid velocity and in the linear zone their ratio is constant. The ratio of wave-velocity to fluid velocity decreases as \( \frac{a}{R_1} \) increases. For the smallest value of \( \frac{a}{R_1} \) used a connection is possible with Goldstein's theory. Goldstein's work applies to small annular spaces and hence it is to be expected that only the smallest one used experimentally would be suitable for comparison. It is found for this that the wave-velocity
in the linear zone is 1.15 times as great as the fluid velocity. (Graph 28A).

In the course of his analysis Goldstein evaluates a quantity which, in our notation, is \( \frac{c_a}{\nu} \), where \( C \) is the wave-velocity. Since the Reynolds number is defined as \( \frac{V_a}{D} \), it is obvious that the ratio \( \frac{C}{V} \), the velocity ratio, can be found from \( \pi \times \frac{c_a}{\nu} / \text{Re} \). The results appear below.

\[
\begin{array}{cccccc}
\text{Re} &=& 5.17 & 10.34 & 15.50 & 20.67 & 25.84 \\
\frac{c_a}{\nu} &=& 1.91 & 3.33 & 6.04 & 8.49 & 11.46 \\
\frac{C}{V} &=& 1.16 & 1.18 & 1.22 & 1.29 & 1.39
\end{array}
\]

This evidently represents an acceleration zone, which is, in fact, not obtained at low Reynolds numbers. Errors in Goldstein's figures may be due to the drastic approximation which was employed, an error which increases with increasing Reynolds number. Hence it may be expected that extrapolation of the velocity ratio figures to a Reynolds number of zero would give a more nearly correct answer. This is so. The extrapolated value (see graph 36) is 1.15, which agrees very well with the experimental value. Massoud's values, on the other hand, are much too high and some error in analysis must be suspected.

The work on turbulence throws more light on the work of Massoud and Goldstein discussed earlier. It has been shown that just before destruction by increased flow at constant speed, or just after formation by increased rotor speed at constant axial flow, the fluid motion is a spiral vorticular type. This motion is asymmetrical. However, both Massoud and Goldstein assume that the flow is a symmetrical
Graph 36. Goldstein's Wave-Velocities.
one and their results must necessarily be incorrect. These workers, therefore, can give no criterion for the onset of this type of motion. At zero Reynolds number (that is, with no axial flow) their assumption of symmetry is a correct one and so their results are close to those of Meekyn and Taylor.

On the other hand at relatively low flow rates and at speeds somewhat above that needed for the onset of vorticular motion there is a linear zone where the flow is in discrete rings. Hence theoretical deductions based on assumptions of symmetry might be expected to apply, since the flow is symmetrical. Thus it would be expected that predictions of wavelength and wave-velocity would be correct. This too, is the case. Both workers give values of about one for the ratio of wavelength to twice the annular space. Goldstein's values at higher Reynolds numbers are much too low but this may be due to excessive approximation.

Cornish's work has already been commented upon and since there is really no connection between his work and the present research no more will be said. The work of Page, on the other hand, shows interesting resemblances to the present work. From graph 35 it must be inferred that, since the curve cuts the horizontal axis positively, the vortex flow at the Taylor critical possesses some stability to liquid flow. Page, graphs 2 and 3, shows this much more clearly.

The explanation is a simple one, showing similarity to the observations of Lewis which have been discussed above. The "critical speed" measured here is the "lower critical speed" relative to the rotor.
The motion has been allowed to set in and then destroyed by a high flow of liquid. At the critical liquid flow the vortices would be destroyed infinitely slowly. The results obtained therefore would, as carried out here, mean a too-high flow rate. The reverse method (fixing an axial flow rate and increasing rotor speed until the onset of vortices) would give an "upper critical speed" with respect to the rotor and an intercept on the vertical axis. The "ideal" line would pass through the origin. The author's criticism of Fage's experimental work has appeared above (Part III).

The stability diagram constructed by the present work allows prediction of the state of flow of liquid between coaxial cylinders (streamline, vorticular or turbulent) to be made for variations in cylinder diameters, liquid properties, fluid velocity and rotor speed for a rotating inner and stationary outer cylinder. It can, however, be combined with the Taylor stability diagram to give a three-dimensional general stability diagram for a flow system where both cylinders rotate, in the same or opposite directions (figure 15).

The data at present available allow only a line diagram to be given. The complete general stability diagram would show a surface of separation between vorticular and non-vorticular flow, but to complete it more work similar to that dealt with here would need to be performed with both cylinders rotating. This surface of separation would itself be bounded by a line beyond which the flow would be "turbulent" in the usual sense.

Finally it should be pointed out, since the work was undertaken with this in mind, that this vorticular flow system has great
FIG. 15. GENERAL STABILITY DIAGRAM.
potentialities for agitation processes. The circulation is regular and intense, dead spaces non-existent and the waste of energy in forming useless eddies negligible. Its use in agitation processes is investigated below.
EXPERIMENTAL WORK: PART IV

Miscellaneous Experiments with the Long-tube Agitator
(a) Introduction

Black states that when solid was added to a liquid in vortical motion in his apparatus a portion was retained in each vortex and a portion deposited on the bottom. With two liquids the vortices adjust themselves so that a vortex boundary coincides with the interface below the critical mixing speed. In the first case dispersion is not uniform and in the second a relatively high speed is required to produce an emulsion.

It seemed that the horizontal agitator should be an improvement upon the vertical one for both these operations. The vertical rings produced in the horizontal apparatus would, it was thought, cut through the liquid-liquid interface. In a similar way solids would be swept off the bottom of the outer cylinder. Accordingly it was decided to use the horizontal agitator for these processes in the hope that the efficiency of the method would be enhanced.

(b) Solid-liquid Suspension and Dissolution

The apparatus was filled with a suspension of barium hydroxide, in saturated aqueous solution, and the particles allowed to settle. Sufficient solid was added just to cover the bottom of the glass tube continuously. The rotor was then started (at about 40 r.p.m.) to produce vortices in the solution. The vortices, which were very well defined and visible due to the very fine particles in suspension, showed no tendency to remove settled solid at this speed. Indeed the speed was raised above 500 r.p.m. (about 50 times the critical for vortex ring formation) before the solid was suspended.
Similar experiments were performed with benzoic acid suspensions, the solid rising to the top of the glass cylinder. The same result was obtained, namely, that the settled solid was removable only at high speed.

The agitator was then arranged so that water could be circulated through the annular space, a feed-hole for solid being left in a Y-piece fitted at the inlet end. The rotor was started, the water flow adjusted and barium hydroxide introduced into the feed hole. The solid remained in suspension and showed no tendency to settle while vortices were present. The same result was obtained with benzoic acid, even when added in fairly large amounts.

These experiments showed that vortices do not remove settled solids readily, that solids, once suspended, remain in suspension and that continuous operation is both possible and desirable. It should be observed that the vortices containing solid were perfectly stable and that agitation was, judged visually, intense.

It was then decided to go into the matter in more detail, operating the equipment as a continuous solid-liquid agitator. The feed end was arranged to take two supply inlets, one for water and one for a suspension of benzoic acid in water. The exit end was fitted with a filtering arrangement and sampling device.

Water was allowed to run through the apparatus and the rotor started so that the motion was vorticular. The suspension was then added to the water stream, both flow rates being measured directly from the reservoirs. The concentration of the filtered exit solution was
determined by volumetric analysis.

It was found that, provided that the motion was vortical, the exit solution was so nearly saturated that no differences in solution rate could be measured for different speeds. This means that the apparatus was too long for this operation, showing the vortical system to agitate solids in liquids very effectively. It will be recalled that there is a definite minimum hold-up time (or maximum flow rate) in this apparatus for a given speed. As the rotor speed is decreased, to give less effective agitation, the minimum hold-up time increases, to give more effective agitation. These two effects neutralise one another with the result that the exit solution shows no variation from its saturated condition.

(c) Liquid-liquid Agitation

Attention was next turned to liquid-liquid agitation. The bottom half of the annulus was filled with coloured water and the top half with paraffin.

At low speeds of rotation (about 40 r.p.m.) the interface remained firm, showing small irregularities locally. This speed would have been sufficient to produce vortex rings in either of the liquids separately. As the rotor speed was increased the interface became more and more irregular and drops of both phases were formed. At speeds of about 500 r.p.m. the interface was completely destroyed, the liquids having been mixed to a coarse dispersion.

By following individual droplets of coloured water it was
seen that the motion was not vorticular, the droplets moving in
circular paths. Prolonged agitation served only to reduce the drop
size slowly; no vortices were seen. The same result was obtained
for two liquids of very nearly equal density (water and a paraffin-
carbon tetrachloride mixture).

Bearing in mind the fact that continuous operation was
found to be preferable in the case of the solid-liquid system it was
decided to run the apparatus continuously as a liquid-liquid extractor.
Into the Y-piece at the feed end, paraffin (containing dissolved
benzoic acid) and water were supplied. These were agitated in the
annulus and samples of the mixture withdrawn. The samples were
allowed to settle into two layers, which they did quickly and both
phases were then titrated for acid.

A very low flow rate was chosen (volumes of the two phases
being kept equal) and the inner cylinder rotated at constant speed
(550 r.p.m.). Samples were withdrawn and the flow rates increased.
After allowing time to reach equilibrium more samples were taken and
the flow rates again raised. This procedure was repeated until
sufficient flow rates were used to enable a graph of extraction at
constant rotor speed against flow rate to be constructed (graph 37A).

These experiments showed that extraction increased as the
time of hold-up increased, a result in accordance with expectations.
No vortices were observed.

When these experiments were repeated at a rotor speed of
GRAPH 37A. LIQUID-LIQUID EXTRACTION.

GRAPH 37B. LIQUID-LIQUID EXTRACTION.
620 r.p.m. there was a very slight, indefinite indication of vortical motion. In this case it was found that saturation conditions had been reached at every flow rate (graph 37B). The increase in speed from 550 to 620 r.p.m. has an effect considerably greater than would be expected and suggests that the motion was, in fact, vortically.

(d) Agitation of Carbon Black Suspensions

It was thought that the shear inside a vortex might be sufficient to reduce in size large agglomerates of particles such as are found in the pigments used in paint manufacture. It was decided to use the long-tube agitator (set up vertically for these experiments) for the agitation of carbon black suspensions, which were then subjected to particle size analysis.

A suspension was made up of 10 mls. of loosely-packed carbon black in a mixture of 250 mls. of pale boiled linseed oil and 250 mls. of white spirit. The settling rate of this suspension was low enough for it to be introduced into the apparatus with very little settling. The suspension was agitated gently to uniformity and a sample withdrawn. The remainder was placed in the agitator and the rotor set at a constant speed.

Readings were taken of time from start and rotor speed. Every 20 minutes the rotor was stopped, 150 mls. of suspension run off, a sample taken and the 150 mls. replaced in the agitator. Rotor and stop-watch were then restarted.
One ml. of each sample was diluted with 109 mls. of white spirit and the settling rate measured in a photo-extinction sedimentometer, against distilled water as a standard (see Appendix F). The results are shown in graph 38 and indicate that the relatively large flocs of carbon black, which were visible in the sample before agitation, were reduced in the apparatus, but that the main bulk of solid was unaffected.

(e) Discussion

It is not easy to lift solid material from the vessel bottom (in both vertical and horizontal apparatus) but material already in the vortices stays there. The dissolution of a solid in a liquid is very rapid.

The horizontal apparatus is less efficient than the vertical for emulsification. On no occasion was an emulsion obtained which compared with those formed in the vertical agitator. Vortices were formed perfectly in emulsions prepared beforehand.

The work with carbon black suspensions shows that the intravorticular agitation is sufficient to reduce the larger particles in the suspension resulting in a more even size distribution. Apparatus of this type therefore might have some use for a final homogenising of paint suspensions and the blending of colour.
GRAPH 38. CARBON BLACK SETTLING RATES.
EXPERIMENTAL WORK: PART V

Vorticular Motion in Systems other than
the Double-Cylindrical System
(a) Introduction

All previous work on the vortex rings and the work reported so far in this thesis has been concerned with the flow between concentric cylinders and a small annular gap. The largest ratio of $\frac{a}{R_1}$ so far used (0.81) occurred in work with the long-tube agitator. It has been pointed out earlier that flow in the concentric cylinder system was chosen for investigation by Taylor because the mathematical analysis was less difficult than for other cases and experimental verification of the theory straightforward. Similarly the assumption that $\frac{a}{R_1}$ was relatively small was brought in to simplify the numerical solutions.

There is, on the basis of previous work, no reason to suppose that vortex rings should not be formed in systems other than those so far used. The formulae due to Taylor and Meksyn cannot be applied at high values of $\frac{a}{R_1}$, but this is no indication that vortices will not be formed. Accordingly it was decided to construct various simple pieces of apparatus to see in what conditions vortex rings would, or would not, be formed.

(b) The Double-Cone Apparatus

The apparatus used is shown in figure 16. The outer cone was of glass, the inside diameters of the ends being 1½ inches and 2½ inches. The height was 12 inches. The inner cone, of duralumin, had end diameters of ½ inch and 2 inches and a height of 12 inches.
FIG. 16. DOUBLE CONE AGITATOR.
The glass cone was held down, by a threaded brass ring packed with graphited asbestos, on a neoprene gasket seated in a groove in the brass base-plate. This base-plate was attached to the bench. The duralumin cone rotated in two ball-bearings, one seated in the base-plate and the other, at the top end of the cone, held by a rigid clamp. The cone was driven by pulleys from a high-speed D.C. motor controlled by suitable resistances. Provision was made for filling and emptying the annulus. Lubricating oil containing traces of aluminium powder was used as the liquid. As set up for these experiments the ratio \( \frac{A}{R_1} \) varied from 0.66 at the top to 0.25 at the bottom of the liquid. The annular space was \( \frac{1}{2} \) inch throughout.

It was found that many vortex configurations were possible, none of them very stable. There was always a tendency to change from one to the other. In certain circumstances a static set of rings was obtained very similar to those previously described. This system is shown in plate 17. It will be noticed that the vortices are very similar to one another in size, decreasing slightly from top to bottom, all very nearly equal to the annular space. The top vortex was a single one, rotating in the "normal" direction - downwards at the outer cone. This system was not obtained very frequently. Usually a portion of the liquid was in a spiral vortex motion as shown in plate 18. In this type of flow the boundary between the two motions was very sharp. The spiral sometimes occupied the whole of the liquid, sometimes the top or bottom half, very occasionally an isolated inch or so in the middle of the system. In the photograph there are 7\( \frac{1}{2} \) vortex pairs above the spiral and 2 below.

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Plate 17. VORTICES IN THE DOUBLE CONE APPARATUS
The mixed ring and spiral motion showed several points of interest. The size of the rings was greater than when spirals were absent. In addition the two vortex-spirals in the spiral-pair were unequal, the upper being considerably smaller than the lower. This can be seen in plate 13 and may be compared with plate 15.

The most usual system obtained was a system of rings similar to those shown in plate 17 but with a regular vertical displacement. The whole system moved downwards, the boundaries remaining perfectly rigid and distinct, vortices being created at the surface and destroyed at the bottom. Once formed this system usually remained indefinitely, the vertical displacement velocity remaining constant. No attempt was made to connect displacement velocity with rotor speed and no measurements were made of vortex formation speeds.

It was found that neither slight off-centring of the cones, nor slight wobbling of the rotor had much effect on the stability of the vortices. It was not found possible to produce turbulence by using a very high rotor speed (up to 1000 r.p.m.).

The Cone and Cylinder Apparatus

This apparatus was exactly the same as the double-cone described above except that the rotor was a cylinder of \( \frac{3}{4} \) inch diameter. In this case the ratio \( \frac{\frac{3}{4}}{R_1} \) varied from 0.66 at the top (as before) to 2.33 at the bottom, the latter value being three times as large as in any previous apparatus.
The resultant vortex motion was much more stable than with the conical rotor. The vortices were very similar to the double-cylindrical type, having well-defined stationary boundaries and the size increasing from top to bottom (see plate 19). There was no tendency for spiral formation.

At low speeds the number of vortices was odd, the top vortex being a single one with the "normal" direction of rotation for the surface vortex. At high speeds (above about 600 r.p.m.) the top vortex split into a pair, the surface vortex now rotating in the unusual direction, that is, upwards at the outer glass cone. On reducing the speed the single vortex reappeared. Thus at high speeds the even configuration was more stable than the odd, there being sufficient disturbing forces in the apparatus to promote the change from one system to the other.

The pair size divided by the annular space at the middle of the pair should, according to the theories for the double-cylindrical system be equal to 2. This ratio has been plotted against the number of vortices counting from the top (graph 39). Nowhere is this ratio as great as 2 but it tends to this value at the surface. The ratio decreases linearly from top to bottom of the system and there appears to be a definite connection between wavelength and agitator dimensions.

(d) Systems where \( \frac{a}{R_1} \) is very large

The ratio \( \frac{a}{R_1} \) was taken up to 2.33 in the cone and cylinder apparatus. Two other simple pieces of apparatus were constructed to discover whether or not larger values of \( \frac{a}{R_1} \) would cause breakdown of
Plate 19. VORTICES IN CONE AND CYLINDER APPARATUS
GRAPH 39. VORTEX WAVELENGTHS.
In the first apparatus (figure 17) the outer cylinder was of glass of 4.02 cms. internal diameter and the inner cylinder a brass rod of 0.80 cms. diameter. The depth of liquid was about 29 cms. The ratio \( \frac{a}{R_1} \) was, therefore, 4.0. The inner cylinder was supported on two cork bearings and was driven directly by a 12 volt D.C. motor controlled by a rheostat.

It was at once obvious that the vortex rings formed were stable and the boundaries rigid. Within a test depth of 29.3 cms. several configurations were found possible. The most usual system was of 23 vortices, that is, of 11 pairs and the top single vortex. The other systems obtained were 21 (10 pairs + 1) and 19 (9 pairs + 1) vortices. Plate 20 is a photograph of vortices in this apparatus. The vortices were very similar to those obtained in Black’s apparatus. The number was always odd, the top vortex always rotated in the same, normal, direction and the bottom pair was large compared with the remainder.

Observations of the sizes of the vortices in these three systems yielded some interesting points. For example, with 10 and 11 pairs (plus the surface vortex) the pair size was very nearly constant throughout the liquid, with the exception of the surface vortex and the bottom pair, whereas with the 9 ring system the central 4 pairs were similar in size but the second and seventh were very large. The effect was a symmetrical one (graph 40A).
FIG. 17. APPARATUS WHERE $\frac{a}{R_1} = 4.0$. 
Plate 20. VORTICES AT $\frac{\theta}{R_1} = 4.0$
GRAPH 40A. VORTEX WAVELENGTHS.

GRAPH 40B. VORTEX WAVELENGTHS.
Frequently the boundaries within the pairs were found to oscillate, sometimes violently. This did not affect the boundaries between pairs, which remained distinct and stable.

In the second apparatus the outer cylinder was of glass of internal diameter 6.90 cms. The inner cylinder, bearings, driving arrangements, etc., were as before. In this case the value of the ratio \( \frac{b}{R_1} \) was 7.5, that is, some ten times the largest previous value outside this research.

In this apparatus stable vortices were again formed. In a liquid depth of 36.6 cms, two systems were produced containing 15 vortices (7 pairs and a surface vortex) and 13 vortices (6 pairs and a surface vortex). The 15 vortex system was the less stable and changed spontaneously to the greater number. The 15 vortex system was quite stable, but the rings were not so regular as those previously described.

The vortex sizes showed the same characteristics as before. The bottom vortex was always very large and the vortex pairs (with the exception of the bottom pair and the surface vortex) were very nearly equal in size. This is shown in graph 40B. Photographs are attached (plates 21 and 22) showing two configurations of vortices in this apparatus. In both these agitators the mean vortex pair size was less than twice the annular space.

(c) A System where \( \frac{b}{R_1} \) Changes Sharply

In the cone and cylinder and double-cone apparatus the
Plate 21. 11 VORTICES AT $\frac{a}{R_1} = 7.5$
Plate 22. 13 VORTICES AT $\frac{a}{R_1} = 7.5$
ratio $\frac{a}{R_1}$ changed in value, but did so slowly and continuously. The apparatus described below was designed to show the effect of sharp changes in this ratio on the vortex configuration.

The apparatus is shown in figure 13A. The outer cylinder was of glass with an inside diameter of 4.02 cms. The compound inner cylinder consisted of a wooden cylinder 4.9 cms. long and 2.3 cms. diameter on a brass rod of 0.30 cms. diameter. The relative positions of the wood and brass cylinders are shown in the figure. The general arrangement was as described under (d). The ratio $\frac{a}{R_1}$ was 4.0 for the large annular space and 0.75 for the small. The test liquid was a lubricating oil containing aluminium powder.

Here vortices were formed varying considerably in size throughout the apparatus. The number was odd with a single vortex at the surface. The configuration at low speeds was as shown in figure 13B and plate 23. In the central portion (in the narrow annulus) two small vortex pairs were formed. Immediately above and below this were vortex pairs containing one small and one very large vortex. The remaining vortices were very similar to those shown in plate 20.

The black-line pair boundaries were extremely clear and firm. The small vortices were the most vigorous and appeared to be formed first. Occasionally one or three pairs of vortices formed in the small annulus, but always changed spontaneously in a few seconds to two pairs. The sizes of both large and small vortices were such as would be expected from the geometry of the system and the position of
pair / pair

brass cylinder
0.8 cm. diam.

wood cylinder
2.3 cm. diam.

glass cylinder
4.02 cm. I.D.

12.5 cm.

4.9 cm.

10.0 cm.

FIG. 18A. DETAILS OF COMPOUND ROTOR.

8 pairs and surface vortex

FIG. 18B. VORTICES: LOW

FIG. 18C. HIGH
Plate 23. VORTICES WITH COMPOUND ROTOR
the two small pairs coincided with the position of the narrow gap. These points are illustrated in graph 41B.

As the rotor speed increased the two centre vortex pairs remained the same but the pairs immediately above and below were enlarged at the expense of the top and bottom vortices. This is clearly shown in graph 41A. At greatly increased speeds the flow character changed radically. The two small central pairs remained, but the two vortex pairs above and the two below were replaced by a vorticular type of motion with a spiral surface of separation. This phenomenon is shown in figure 18C and plate 24. The general agitation in this case was intense but the turbulence was still ordered. It was not found possible to destroy the vortices at high rotor speeds (about 1000 r.p.m.).

(f) Discussion

These experiments show that vortex rings can be produced in a great variety of conditions in roughly-made equipment. They can be produced at very high ratios of $\frac{A}{R_1}$ and in systems where this ratio is not constant. In these cases the motion of individual particles is much more complicated than in the simple two-cylinder apparatus. The motion envisaged in Taylor's work is a complex three-dimensional spiral but the individual particles are confined to one vortex. In some of the apparatus used here the particles do not remain in one vortex. Such motions are equally intense however.

In the compound cylinder apparatus two types of motion
**GRAPH 41A. VORTEX WAVELENGTHS.**

**GRAPH 41B. VORTEX WAVELENGTHS.**
Plate 24. VORTICES WITH COMPOUND ROTOR - HIGH SPEED
were present — one the larger vortices with the less intense circulation, the other the small vortices with very intense circulation. In some applications the small volume of very intense agitation may be sufficient (e.g. for dissolution or emulsification) while the larger volume serves for general agitation (e.g. maintenance of the emulsions produced, or of solids). Such an apparatus could be run as a continuous mixer.

Energetic vortices can be produced at large ratios of $\frac{R}{R_1}$ (space/rotor radius). This means that large volumes of liquid can be effectively agitated using small rotors, although of course in this case the theoretical work cannot be used for design purposes.
EXPERIMENTAL WORK: PART VI

Miscellaneous Experiments
It was noticed early in the work with the long-tube agitator that a slight degree of eccentricity did not affect the stable appearance of the vortices. As the degree of eccentricity increased the vortices began to wobble and lose their firm character (figure 19 A1). As the inner cylinder was off-centred still further another set of "rings" was set up (figure 19 A2) and the original rings seemed to regain their stability. The second set of "rings" was a sluggish, ill-defined motion in the large annular space. These rings rotated, at the outer glass cylinder, in a direction opposite to that of the rotor.

More unusual was the motion observed when the eccentricity was such that the maximum annular clearance was some ten times the minimum. The brass cylinder of the vertical apparatus was rotated in a glass vessel of 12" diameter. The wobble of the rotor, without its bottom bearing, was such that the maximum annular clearance could not be taken much below an inch. In this case sections of vortex rings were formed of good stability and rigid appearance. The vortex rings formed at AA' (figure 19B) were very stable, the size being approximately equal to the annular space. The rings at AB and A'B' were less stable, while the area BCB' contained only irregularly turbulent motion.

These ring sections persisted over a wide range of settings of the outer cylinder and a wide range of speeds. The agitation within the area AA' was intense, that within the main bulk
I firm rings

less stable rings

...irregular vortex A

2. system

FIG. 19. EFFECT OF ECCENTRICITY.
moderate to poor depending on the conditions. Slight wobble of the rotor had little effect on the vortex stability. The effect was also small when the cylinders were slightly skew.

(b) Vortices in Concentrated Suspensions

The vertical apparatus was filled with a light oil, the rotor speed adjusted to produce vortices and a small quantity of Fuller's earth added. More solid was added and the speed raised, if necessary, so that the suspension remained in vorticular motion. The addition of solid and any necessary increase in speed was carried on until the maximum speed of the rotor had been reached (about 200 r.p.m.). Inspection showed that at this speed the suspension, still in vorticular motion, was more or less homogeneous.

At the point at which the experiment was concluded the suspension contained about 35% by volume (about 50% by weight) of solid and was a loose paste. The apparent viscosity in these conditions was \( \frac{200}{0.622} \), or about 320 centistokes. There was no sign of breakdown of the motion, though it was much less violent than in a liquid of lower viscosity.

(c) Vortex to Vortex Mass Transfer

Mass transfer across a liquid-liquid interface that is also a vortex boundary is slow. It can be shown that mass transfer across a vortex boundary where there is no interface (a homogeneous liquid) is also slow.
The vertical agitator was filled to a depth of about ten inches with water containing a small amount of hydrochloric acid and sufficient thymol blue to give the dilute solution a definite colour. Small pellets of sodium hydroxide were deposited on the bottom of the vessel and the rotor started at constant speed.

The caustic soda dissolved gradually in the bottom vortex and the indicator colour changed in that vortex. After a gap of a few seconds the blue colour began to appear in the second vortex and over a period gradually grew more intense. Colour then appeared in the next vortex. The colour change took place stepwise and it took several minutes for any trace of colour to appear in the top vortex. As the rotor speed decreased the time increased.

Similar experiments were carried out in the horizontal apparatus, where over a hundred vortices were involved in place of the eleven or so in the vertical agitator. The apparatus was filled with clean oil, the rotor set at several hundred r.p.m. and then to one end oil containing aluminium powder was added. The powder was then transferred from one end of the apparatus to the other. The rate of transfer was very slow. Transfer from one end to the other (some 60-70 cms.) took several hours and on no occasion was powder added in this way uniformly distributed after agitation.

(d) Batch Agitation with Circulation

One of the drawbacks of a batch process, such as is carried out in the vertical agitator, is the slowness of mass transfer from one vortex to another in single phase systems. This has been
overcome, in the long-tube agitator, by running the apparatus continuously. Although a continuous process may be desirable it is not always possible and some compromise between batch and continuous processes may be necessary.

An apparatus (figure 20) was constructed for this purpose. The hollow rotor (a length of glass tubing) was attached to a propeller shaft and the whole assembly immersed in the liquid to be agitated. The rotor clearance from the vessel bottom and the submergence were made as small as possible. The thin cork layer which served to connect rotor and shaft contained several large holes to allow easy movement of the liquid. As the rotor revolved, the propeller, in this case upthrusting, circulated the liquid. Good vortex rings were obtained in the annulus, which were generally stationary. Spirals and flowing vortices were seldom seen.

The system was stable over a fair range of speeds. Agitation within the rings was intense and the circulation sufficient to raise small solids from the vessel bottom. It is suggested that more flexibility could be obtained by independent rotation of propeller and rotor.

(c) Discussion

It is obvious that equipment using the vortex ring principle need not be accurately constructed.

The vortex ring sections may repay further investigation.
FIG. 20. BATCH-CIRCULATION SYSTEM.
It is conceivable that enough agitation could be accomplished in a small ring section to agitate a large batch effectively. This would mean that a small rotor could agitate a large tank.

It would appear that any solid-liquid system can be dealt with where the liquid is the continuous medium. Very high viscosity suspensions could be dealt with by designing equipment where the critical $N/\sqrt{D}$ is very low, thus requiring low speeds for agitation.

The slowness of vortex to vortex mass transfer in homogeneous liquids means that batch agitation is impracticable, except for the maintenance of suspensions (and some emulsions) over long periods. The alternatives are continuous operation and batch-circulation systems and although both of these are definitely practicable much development work would be necessary to arrive at the best designs for such equipment.
The object of this research has been the examination of vortical motion with a view to its possible use as an agitation system. In order to do this the literature pertaining to the mixing process and to vortical motion has been critically reviewed. Experiments have been performed to test theoretical deductions and to ascertain in what conditions vortices can, or cannot, be formed.

The various mixing systems have been enumerated and the work devoted to each reviewed. The bulk of the published work refers to the liquid-liquid and solid-liquid systems and most of the literature survey has been devoted to them. There is very little published work on the solid-solid system, though of recent years a new approach has been made by regarding the system as a shuffling of particles and dealing with it statistically.

The many types of mixer have been classified into a few groups, showing that although there are large numbers of designs of mixers, the principles on which they work are few in number. The tendency in recent years has been to reduce the number of mixers to a few standard models.

Research work has been reviewed with particular emphasis on the two main problems, the assessment of mixer performance and the measurement of power and its correlation with impeller and liquid characteristics. More recently attention has been given to flow patterns and the effect of baffles - the centre of attention has been moved from the impeller to the liquid. The work on fully-baffled
conditions has thrown much light of earlier power measurements and Rushton's work on power numbers has shown how power input varies with baffling. The connection between baffling, flow patterns and performance has not yet been made.

It is now well-recognised that the baffling of an agitated fluid has an important bearing on the power drawn, but no explanation has been put forward for it. It is possible to explain it, qualitatively, by making use of the laminar film concept (91).

Between a turbulent stream of fluid and the wall containing it there exists a laminar layer (where the flow is streamline) and a buffer layer (where the flow is neither completely streamline nor completely turbulent). This is shown diagrammatically in figure 21. As usually envisaged the wall is stationary and the fluid is in motion. The same idea may be extended to cover the case of an impeller rotating in a liquid. At the surface of the rotating element there is a laminar film and between this and the agitated contents a buffer layer. The transmission of power from the impeller to the fluid depends, it is assumed, upon the thickness of the laminar film.

At low impeller velocities the film will be of significant thickness and possibly in contact with a fluid in laminar motion. The power transmitted will be low. At higher impeller velocities the film will be reduced in thickness and as it becomes progressively thinner the power drawn will increase. When the impeller is moving so rapidly that the film has been reduced to insignificant thickness the power has reached its maximum. For a given angular velocity the peripheral
FIG. 21. FLOW AT A SOLID BOUNDARY.
speed will be higher for a large impeller than for a small, the film thickness will be less and in consequence more power will be transmitted.

Consider the case of an impeller rotating in a fluid at such a speed that there is a significant laminar film. On inserting a baffle the flow-lines at the vessel wall are broken up and the motion of the bulk of fluid becomes more turbulent. This, in turn, increases the turbulence in the buffer layer and diminishes the thickness of the laminar layer, resulting in increased power consumption. This will be repeated as additional baffles are added until the laminar film may be neglected. Power consumption is now at its maximum and further baffling has no effect.

It has been noted earlier that in the laminar region the power consumption depends on $\mu N^2$ and in the turbulent region on $\rho N^3$. The mechanism outlined suggests why unbaffled agitators usually give a power figure falling between these values. It also suggests that the Reynolds number used in power correlations is only of value in so far as it indicates the condition of the flow at the surface of the impeller. Reynolds numbers involving the diameter of the containing vessel can have no general significance.

It does not follow because two agitation systems are fully-baffled that their states of flow are similar and can therefore be neglected in assessing mixer performance. The power consumption (according to the ideas put forward here) depends solely on the thickness of the laminar film at the impeller, whereas the performance in carrying out a particular process depends upon the flow throughout
the whole bulk. These two are usually, but not necessarily, related.

No experimental work has been reported on the flow behind baffles in agitated vessels, although a paper has appeared which makes suggestions about the type of flow (65). In this paper, of which the writer was a co-author, it was suggested that the flow would be a vortex roll for baffles at the wall and a vortex-street for offset baffles, the difference in position possibly leading to some difference in power consumption because of the different types of flow. Such vortex rolls have been observed in liquids behind the fixed shelves of a rotating drum (figure 22). The flow in this case is much simpler than with a rotating impeller and the vortex roll is very stable. With a rotating impeller the vertical components might be such as to mask, or destroy, the vortical motion.

The early work on the flow of liquid between coaxial rotating cylinders has been reviewed and the work of various authors on vortical motion has been compared graphically. The graphical method shows that the present work agrees with the theories of Taylor and Meksyn and indicates where these theories will diverge.

The corresponding work with axial flow of liquid has also been reviewed, the work of Cornish and Page criticised and suggestions made as to their sources of error. It is felt that their methods were unreliable and that in any case Cornish's work approximated to flow between shearing flat plates and that there were no vortices.

The theories of Goldstein and Massoul have been discussed
FIG. 22. VORTEX BEHIND BAFFLE IN DRUM.
and their assumption of symmetry of the motion shown to be incorrect. These workers can, therefore, give no indication of formation speeds of vortices where there is flow along the axis of the rotating cylinders. In certain conditions, where there is symmetrical flow, their figures for the wave-velocity ratio (ratio of vortex velocity/liquid velocity) may be correct and when corrections have been made for Goldstein's approximations his figure agrees with that obtained experimentally. Massoud's wave-velocity figure is badly in error and there must be some error in his analysis.

At zero Reynolds number of the axial flow their assumption of symmetry is correct and they should agree with Taylor and Meksyn. It has been shown, graphically, that this agreement is only for low values of $\frac{\alpha}{R_1}$. Goldstein made no correction for large values of $\frac{\alpha}{R_1}$ and his results differ from those of Taylor and Meksyn, which agree with the experimental work.

The work of Black, of which the present is a continuation, has been reviewed. Work with his apparatus shows that, contrary to his report, predictions can be made about the size of vortices and the number to be expected in a given depth of liquid. The vortex formation constant has been found to vary from top to bottom of his apparatus, but the mean value agrees reasonably well with that calculated according to Taylor's theory.

Using Black's apparatus it has been shown that emulsions can be produced, a definite rotor speed being needed to disrupt the liquid-liquid interface. The variation of this critical interfacial breaking
speed has been correlated with density, viscosity and interfacial
tension. It has been shown that rate of emulsification and rate
of extraction can be used as criteria of agitation, but that
emulsion settling rates are unreliable. Below the interfacial
breaking speed the rate of extraction is substantially-independent
of rotor speed. The reasons for this have been discussed with
reference to Taylor's work on the form of the vortex streamlines.

A horizontal vortical agitator was built and with no flow
parallel to the axis it was shown that the wavelength and formation
speed were as predicted by theory.

With axial flow it was found that there were three zones of
motion - a linear zone, where the vortex wave-velocity increased
linearly with flow rate, an acceleration zone, where the wave-velocity
increased at a greater rate than the liquid flow and a turbulent zone,
where the vortices were destroyed by excessive flow. The various
types of ring and spiral motion have been photographed and described.

In the linear zone the wave-velocity ratio increases with
decreasing \( \frac{S}{R_i} \). By extrapolating Goldstein's values of the wave-
velocity to zero Reynolds number (to overcome his errors due to
excessive approximation) it was shown to give good agreement with the
experimental value. Since his analysis assumes \( \frac{S}{R_i} \) is very small
only one of the experimental values could be used for comparison.

A definite liquid flow rate was required to destroy the
vortical motion for any given liquid, apparatus and rotor speed.
The variation of this flow rate was correlated with apparatus dimensions,
rotor speed and liquid viscosity, from which data a stability
diagram was constructed. This diagram was combined with the Taylor
stability diagram to give a general three-dimensional stability
diagram, which covers the general case for axial flow with both
cylinders rotating in either direction. The diagram at present is
in line form only. Further work will be required to fill in the
full surface diagram.

A number of miscellaneous experiments have been carried
out to test possible application of this method of agitation. It
was shown to be suitable for solid-liquid mixing and single liquid
agitation (as in heat transfer).

Experiments where the ratio \( \frac{R}{R_1} \) varies continuously,
changes sharply and where this ratio is very large indicate that
ergetic vortices can be produced easily in a variety of conditions.
Further experiments showed that the rotor need not be centrally
situated. Batch-circulation systems (where a batch of liquid is
circulated through a region of vorticular motion using a propeller)
may be useful.

The general conclusion reached is that vorticular agitation
can be used successfully in liquid and solid-liquid agitation systems
(run batchwise or continuously) and that sufficient data are now
available for design purposes.
APPENDIX A

The Measurement of Kinematic Viscosity

Where pure liquids were used kinematic viscosities were taken from published data. Most of the liquids used were mixtures and their viscosities were measured in a Redwood No. 1 Viscometer.

The procedure adopted in the use of this viscometer was as laid down in "Fuel Testing" (92). Usually four-six runs were made and the mean result used. The results took the form of time, in seconds, required to fill a 50 ml. flask with the test liquid.

Using these time readings the kinematic viscosities of the liquids were calculated from

$$\nu = 0.264t - \frac{120}{t}$$

for values of $t$ from 40 to 85 seconds and

$$\nu = 0.247t - \frac{65}{t}$$

for values from 85 to 2000 seconds. In these equations $\nu$ is in centistokes.

These formulae are based on results of work carried out at the N.F.L. at 70°F. At that temperature they are correct to ±1%. The maximum error in the final viscosity figure is probably about 5% for the lowest viscosities.
The Measurement of Density

The densities of the various liquids used in the investigations were measured by a method designed to give about \( \frac{1}{2} \) accuracy simply and quickly. Relatively large amounts (hundreds of mls.) of the liquids were always available.

A 250 ml. measuring flask was weighed (a) empty (b) filled with water and (c) filled with the liquid under test. The usual precautions were taken in cleaning. Weights were taken to the nearest decigram.

All densities were expressed relative to water at the same temperature. All measurements were made at room temperature, variation 15 - 20°C.
The Measurement of Interfacial Tension

The Donnan drop-number method was employed. In this the volume of a drop of one liquid formed in the liquid against which the interfacial tension is desired is deduced by counting the number of drops formed from a given volume. The drops must be formed slowly.

The interfacial tension is deduced by finding the drop number for a system of known interfacial tension. If the shapes of the drops are the same in both cases then

\[
\frac{V (\rho \omega - \rho)}{V' (\rho \omega - \rho')} = \frac{\sigma}{\sigma'}
\]

where

- \( V \) = drop volume
- \( \rho \omega \) = water phase density
- \( \rho \) = oil phase density
- \( \sigma \) = interfacial tension

and the dashed symbols refer to the second (standard) system.

The volumes \( V \) and \( V' \) are inversely proportional to the respective drop numbers. If the densities are known, then \( \sigma \) can be evaluated when \( \sigma' \) is taken from tables (benzene/water).

In this work, ten mls. of liquid, or 100 drops, whichever was the larger volume, was used. Each drop was formed in 10 seconds or longer.

The greatest error in drop number measurements was probably
Appendix C (cont.)

2 or 3%. The greatest error in the density difference was probably 5%. Since no corrections were made for the shape of the drops the maximum error on the tension figure is likely to be about 10%.
APPENDIX D

The Determination of Benzoic Acid in Solution

In the extraction experiments solutions of benzoic acid in water (or paraffin) have been agitated with pure paraffin (or water) and the resultant solutions sampled and the acid determined.

Wherever possible (it was usually possible) 50 ml. samples were taken and each determination carried out three times, the mean being taken.

The direct titration of benzoic acid solutions (oil phase) against alkali is a slow process of doubtful accuracy due to endpoint difficulties. Accordingly here a back-titration method was adopted.

To the 50 ml. sample excess caustic soda was added - 25 mls. of a standard solution about 0.1 Normal. The excess alkali was then back-titrated in the usual manner against standard hydrochloric acid, also about 0.1 Normal. The indicator used was thymol blue. The required acid equivalent of the solution was found by difference.

The colour change was blue-yellow but after a few trial titrations it was found possible to titrate to a constant green to the nearest drop.

Water-phase titrations were simple and accurate. Oil-phase titrations were less accurate, but good results were obtained when adequate time was allowed for agitating and settling during the
Appendix D (cont.)

determination. The results were probably correct to 0.1 ml., this being about 5% on the smallest titrations. Errors were, of course, greater when differences in titrations were used as measures of mass transfer but mass balances usually allowed a check on the accuracy of this work.
The Conical Agitators as Viscometers

Lewis first suggested the use of his apparatus as a viscometer, but this suggestion has never been taken up. The operation, in theory, would be simple. A small amount of liquid containing a tracer would be placed in the apparatus and the rotor speed raised until vortex rings were just formed. The critical value of $\Omega/\nu$ could be found either from theory or from calibration with a known liquid.

This method has certain drawbacks however. The apparatus would need to be very accurately constructed. Lewis's paper illustrates the practical difficulties in construction. The gradual increase of rotor speed to the critical value would call for very accurate speed control of the driving mechanism over a fairly long period. During a long determination temperature gradients would be set up.

The cone and cylinder apparatus overcomes several of these difficulties. As the scale of the apparatus and the annular space are somewhat larger the accuracy of construction of the equipment need not be so great.

The operation would be as follows. The rotor speed would be adjusted to be between the critical speeds at the low value of $S/R_1$ at the top and the high value of $S/R_1$ at the bottom. Vortices would form at the bottom and not at the top. Experiments have shown that
this circular motion-vortex motion boundary is quite sharp. The critical value of \( \frac{\Omega}{\nu} \) could be calculated for the position of this boundary for small values of \( \frac{a}{R_1} \), or could be found by direct calibration for large values. Alternatively, the speed could be adjusted to bring the circular motion-vortex motion boundary to a definite position where the critical constant is known.

One difficulty would be the provision of a reasonably accurate glass cone. It is suggested - it has not been investigated - that a method of overcoming this would be to rotate a metal cone in a glass (or other transparent material) cylinder, where the same range of \( \frac{a}{R_1} \) could be arranged.

More investigation is required of possible cone and cylinder systems before they can be considered seriously for use in viscometry.
APPENDIX F

The Photo-electric Sedimentometer

A sketch of this apparatus is shown in the figure. Light from a filament lamp is brought on to an adjustable slit as a parallel beam by a lens and through this to a photo-electric cell connected to a millivoltmeter or galvanometer. This optical system is fixed to a platform pivoted at one end so that light can be projected through either of two sample cells.

The cells are cubes, of plane glass, complete with cover plates. One cell contains a standard liquid (in this case distilled water) and the other the suspension under test.

The lamp is switched on and the beam projected through the standard cell. The width of the slit is adjusted to give full scale deflection on the galvanometer. The beam is then projected through the test cell, the suspension being diluted to give a meter deflection comparable with the standard.

The suspension is agitated, the stimer removed and the meter reading taken. At fixed periods of time the slit is adjusted to give the same deflection as before through the standard and then the test cell value taken. In this way all readings are taken relative to a standard reading.

As the suspension settles the meter reading increases, less light being intercepted by solids. At zero time the larger the meter reading the larger the mean particle size, assuming each sample to
PHOTO-ELECTRIC SEDIMENTOMETER.
Appendix F (cont.)

contain the same volume of solids. The greater the slope of the meter reading/time curve the greater the settling rate. The results are purely comparative.
APPENDIX C

Work with a Propeller Agitator

The vorticuler agitator was compared directly with a propeller for a liquid-liquid extraction system. The methods of measurement and the arbitrary level of extraction chosen for comparison have been discussed earlier.

In this case speeds were measured using a stroboscope and power at load and no load conditions was measured using an accurate voltmeter and ammeter. A power-speed curve was drawn for load and no load conditions and the difference between them, at a given speed, taken as the net power input at that speed. These net power inputs gave a straight line when plotted against the propeller speed on logarithmic co-ordinates, the value of the slope (2.93) agreeing with that obtained by many other workers.

The optimum speed and minimum power were arrived at as described for the vorticuler system. The optimum speed was found to be 200 r.p.m. and the minimum power 3.02 ft.lbs. The figure shows the dimensions of the propeller system, the volumes of the liquids being the same as used in the other apparatus.

It has been mentioned repeatedly that there are no dead-spaces in the vorticuler system and that no energy is wasted in useless eddies. This is not the case with the propeller agitator. Photographs of several types of motion were taken to illustrate this.

In plate 25 for example an air vortex is drawn down the shaft.
DETAILS OF PROPELLER.
Appendix G (cont.)

Should the speed be sufficient to bring this air vortex to the propeller itself the power loss jumps sharply as air is dispersed (plate 26). This air vortex can be reduced by off-centring the propeller (plate 27) but if the speed increases again the power wastage again increases due to air dispersion (plate 28).

This air is not uniformly dispersed so if, in fact, the propeller were intended to be dispersing air it would be inefficient. It is inefficient also for dispersing liquid. Plate 29 shows that although one liquid is being dispersed in the other (by means of a vortex extending to the propeller) very few droplets reach the bottom of the vessel.

There is a dead space at the bottom, in exactly the same way as there is a vortex at the top. This is shown in plate 30, where although the propeller is rotating at several hundred r.p.m. the "vortex" of the lower liquid is rotating very slowly and remains unmixed.

The conclusion to be drawn is that a propeller-agitated mixture will not be perfectly homogeneous and that to reduce dead-spaces it will be necessary to position the propeller correctly in a suitably shaped agitation vessel. The literature pertaining to this subject has been reviewed in the introduction to this thesis.
Plate 30. BOTTOM DEAD-SPACE
APPENDIX H
Nomenclature

**Capitals.**

- **A** = Area
- **C** = Concentration
- **D** = Diffusivity
- **G** = Goldstein's group \( \frac{2 \Omega \alpha^2 (1+\gamma)(R_1^2 - \gamma R_2^2)}{A^2 \nu^2 (2R_1 + a)} \)
- **H** = Liquid depth
- **K** = Constant
- **L** = Impeller diameter
- **N** = Agitator speed
- **P** = Power
- **Q** = Volume per unit time
- **R** = Cylinder radius (inner = 2, outer)
- **S** = Percentage solid in a mixture
- **V** = Volume
- **W** = Impeller width
- **X** = Velocity in the \( x \) direction
- **Y** = Flow constant defined as \( \frac{R_1^3 \Omega_1 - R_2^3 \Omega_2}{R_1^3 - R_2^3} \)
- **Z** = Flow constant defined as \( \frac{R_1^3 R_2^3 \Omega_1 (1-\gamma)}{R_2^3 - R_1^3} \)

**Lower case.**

- **a** = annular space \( (R_2 - R_1) \)
- **c** = vortex wave-velocity
- **d** = vessel diameter
- **f** = \( (\bar{w}_o - w_o)^{1/3} \)
- **g** = acceleration due to gravity
- **h** = \( \bar{w}_o^{1/3} \)
- **i** = \( \sqrt{-1} \)
\[ j = w^3 \]

\[ k = \text{reaction velocity constant; mass transfer coeff.} \]

\[ l = \text{percentage liquid in a mixture.} \]

\[ p = \text{pressure.} \]

\[ r = \text{radius.} \]

\[ t = \text{time.} \]

\[ v = \text{velocity.} \]

\[ w = \text{weight.} \]

\[ x = \text{length.} \]

\[ y = \text{stirrer pitch (degrees).} \]

**Greek Capitals.**

\[ \lambda \ (\text{lambda}) = \text{wavelength.} \]

\[ \phi \ (\text{phi}) = \text{a function.} \]

\[ \psi \ (\text{psi}) = \text{a function.} \]

\[ \Omega \ (\text{omega}) = \text{angular velocity (1, inner cylinder; 2, outer).} \]

**Greek lower case.**

\[ \alpha \ (\text{alpha}) = \text{constant.} \]

\[ \gamma \ (\text{gamma}) = \text{ratio of angular velocities} \left( \frac{\Omega_2}{\Omega_1} \right) \]

\[ \varepsilon \ (\text{epsilon}) = \text{quantity determining stability in hydrodynamical equations.} \]

\[ \lambda \ (\text{lambda}) = \text{a function of wavelength} \left( \lambda = \frac{2\pi}{\lambda} \right) \]

\[ \mu \ (\text{mu}) = \text{absolute viscosity.} \]

\[ \nu \ (\text{nu}) = \text{kinematic viscosity.} \]

\[ \rho \ (\text{rho}) = \text{density.} \]

\[ \sigma \ (\text{sigma}) = \text{interfacial tension.} \]

**Additional.**

\[ \text{Re} = \text{Reynolds number} \]

\[ x, y, z = \text{rectangular co-ordinates.} \]

\[ r, \theta, z = \text{cylindrical polar co-ordinates.} \]

\[ \frac{df}{dz} = \text{pressure drop, per unit length, in } z \text{ direction.} \]
### Rates of Reaction and Solution

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### Power Measurement and Correlation

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