





# Nonequilibrium noise as a probe of pair-tunneling transport in the BCS–BEC crossover

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## Abstract

The detection of elementary carriers in transport phenomena is one of the most important keys to understand nontrivial properties of strongly correlated quantum matter. Here, we propose a method to identify the tunneling current carrier in strongly interacting fermions from nonequilibrium noise in the Bardeen–Cooper–Schrieffer to Bose–Einstein condensate crossover. The noise-to-current ratio, the Fano factor, can be a crucial probe for the current carrier. Bringing strongly correlated fermions into contact with a dilute reservoir produces a tunneling current in between. The associated Fano factor increases from one to two as the interaction becomes stronger, reflecting the fact that the dominant conduction channel changes from the quasiparticle tunneling to the pair tunneling.

**Keywords:** transport, nonequilibrium noise, strongly interacting fermions, BCS–BEC crossover

## Significance Statement

The anatomy of the elementary transport carriers involving strong correlations has been a long-standing issue in the fields of cold atoms as well as superconductors. We show that the Fano factor, the ratio between a current and its nonequilibrium noise, reflects information on anomalous pair-tunneling transport in strongly correlated superfluids and superconductors. The Fano factor changes from 1 to 2, according to whether the quasiparticle or the pair tunneling is dominant, and hence can be a direct probe for the nontrivial pair-tunneling current. Our result can be tested in cold atomic and condensed matter experiments.

Transport phenomena have contributed to the development of the fundamental physics in previous centuries. Various unconventional phenomena such as superfluidity and superconductivity were observed using transport measurements. However, clarifying the microscopic mechanism of the transport phenomena in strongly correlated systems remains challenging because of their complexities such as strong interactions, lattice geometries, as well as multiple degrees of freedom.

Recently, an ultracold atomic system has been regarded as a quantum simulator for strongly correlated many-body systems such as unconventional superconductors and nuclear systems, owing to its controllability of physical parameters (e.g. interparticle interactions and lattice structures) and its cleanliness (1, 2). In particular, state-of-the-art experiments for tunneling current have been conducted in strongly interacting Fermi gases (3–8). Moreover, thermoelectric transport has been demonstrated experimentally in an ultracold Fermi gas (9). A quantum point

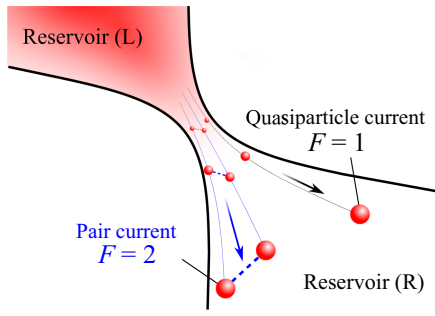
contact has also been implemented for atomic superfluid junctions (10). These experiments motivate us to study tunneling transport associated with the Josephson effect and Cooper-pair tunneling in the superfluid phase of the Bardeen–Cooper–Schrieffer (BCS) to Bose–Einstein condensate (BEC) crossover (11–19). Such a direction are recently referred to as *atomtronics* (20).

One crucial problem is to understand how strong correlations affect the conduction mechanism, which is necessary for future development of quantum-transport technology. Recently, several theoretical efforts have been paid to understand an anomalous tunneling current induced by pairing fluctuations in the normal phase (21–24), as observed in experiments (3–8). It is anticipated that such anomalous pair-tunneling currents can be induced by the nonlinear tunneling processes (21), tunneling of a closed-channel molecule (22), and the proximity effect associated with two-body interactions (25). However, regardless of

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**Fig. 1.** Strongly interacting quantum gases (reservoirs L and R) with a large chemical potential bias in between. The Fano factor  $F$  can be regarded as an indicator of the current carrier, i.e. quasiparticle current ( $F = 1$ ) and the pair current ( $F = 2$ ).

these different origins, the existence of the pair-tunneling current itself is still an important pending problem because it is difficult to distinguish quasiparticle- and pair-tunneling currents experimentally. In this sense, it is worth exploring clear evidence for anomalous pair currents in a strongly interacting Fermi gas.

For this purpose, measuring the Fano factor is promising, which is defined by a current and the associated nonequilibrium noise (26, 27). The Fano factor in the large-biased setup reflects the effective charge per elementary transport process regardless of system's detail. The most fascinating example is the detection of fractional charges in fractional quantum Hall systems (28, 29). The Fano factor has been used to determine the effective charge (or spin) in various physical systems such as superconductors (30, 31), Kondo quantum dots (32, 33), and magnetic junctions (34–37). Once the Fano factor is measured in strongly interacting Fermi gases, the existence of the pair-tunneling current will be revealed in an unbiased way.

In this study, we show that the Fano factor  $F$  can be used as a probe for the current carrier in the BCS–BEC crossover. Fig. 1 shows a schematic setup of the large-biased system. Using the many-body T-matrix approach (TMA) (38, 39), we numerically calculate the current and nonequilibrium noise within the Schwinger–Keldysh approach in the two-terminal tunneling junction under a large bias. We reveal how the Fano factor  $F$  changes in a strongly interacting regime, thereby reflecting the change of the dominant carrier. In particular, the change of  $F$  is a crucial evidence for the pair-tunneling current. Our result can be tested by cold-atom experiments for which the noise measurement has been theoretically proposed (40). Moreover, the Fano factor provides direct information of pair-fluctuation effects rather than other measurements such as spin susceptibility and photoemission spectra previously studied in this field (41). The current noise measurement can also be used to identify the carriers of the BCS–BEC crossover in condensed matter systems such as FeSe semimetal (42–45), lithium-intercalated layered nitrides (46, 47), magic-angle twisted trilayer graphene (48), and organic superconductor (49). Moreover, the noise measurement has recently been conducted in a copper oxide heterostructure (50, 51) and disordered superconductor (52).

In the following, we take  $\hbar = k_B = 1$  and consider a unit volume.

## Tunneling current and noise

We consider the Hamiltonian  $H = H_L + H_R + H_{1T} + H_{2T}$ . The reservoir Hamiltonian  $H_{j=L,R}$  is given by

$$H_j = \sum_{\mathbf{p},\sigma} \xi_{\mathbf{p},j} c_{\mathbf{p},\sigma}^\dagger c_{\mathbf{p},\sigma} + g \sum_{\mathbf{q}} P_{\mathbf{q},j}^\dagger P_{\mathbf{q},j}, \quad (1)$$

where  $\xi_{\mathbf{p},j} = p^2/(2m) - \mu_j$  denotes the kinetic energy measured from the chemical potential  $\mu_j$  and  $c_{\mathbf{p},\sigma,j}$  denotes the annihilation operator of a Fermi atom with momentum  $\mathbf{p}$  and the pseudospin  $\sigma = \uparrow, \downarrow$ . The second term in Eq. 1 denotes the attractive interaction with a contact-type coupling  $g$ , where  $P_{\mathbf{q},j} = \sum_{\mathbf{p}} c_{-\mathbf{p}+\mathbf{q}/2,\downarrow,j} c_{\mathbf{p}+\mathbf{q}/2,\uparrow,j}$  is the pair-annihilation operator and  $g$  is related to the scattering length  $a$  as  $m/4\pi a = (1/g) + \sum_{\mathbf{p}} (m/p^2)$  (39).

The one-body tunneling Hamiltonian,

$$H_{1T} = \sum_{\mathbf{p},\mathbf{k},\sigma} [t_{\mathbf{p},\mathbf{k}} c_{\mathbf{p},\sigma,L}^\dagger c_{\mathbf{k},\sigma,R} + \text{h.c.}], \quad (2)$$

is associated with the one-body potential barrier, where  $t_{\mathbf{p},\mathbf{k}}$  denotes its coupling strength. The two-body tunneling Hamiltonian reads

$$H_{2T} = \sum_{\mathbf{q},\mathbf{q}'} [w_{\mathbf{q},\mathbf{q}'} P_{\mathbf{q},L}^\dagger P_{\mathbf{q}',R} + \text{h.c.}], \quad (3)$$

where  $w_{\mathbf{q},\mathbf{q}'}$  is the two-body coupling strength, induced by the local interaction term in Eq. 1 combined with the one-body potential barrier (25). Such two-body tunneling processes can also be obtained within the multiple one-body tunneling processes in the nonlinear regime (17, 21, 24, 53). We note that regardless of their origins, these two-body tunnelings induce the pair-tunneling current. Similar tunneling effects have also been examined in one-dimensional few-body systems (54, 55). Here, we do not go into details on the origin of the one- and two-body tunneling, but rather investigate their possible consequence in observable quantities. However, we emphasize that the two-body tunneling term is necessary to describe the molecule tunneling in the deep BEC side (and therefore the entire crossover), where the pair tunneling induced by the higher-order one-body tunneling process is suppressed due to the reduced dissociation of molecules with the large binding energy (24). In Fig. S1, we estimate the tunneling couplings in the case of delta-function-like potential barrier (19, 56) based on Ref. (25).

Using the Schwinger–Keldysh approach, we evaluate the expectation values of the current operator  $\hat{I} = i[\hat{N}_L, H]$  ( $\hat{N}_j = \sum_{\mathbf{p},\sigma} c_{\mathbf{p},\sigma}^\dagger c_{\mathbf{p},\sigma}$  denotes the density operator in the  $j$ -reservoir) in the steady state at the lowest-order tunneling couplings by a sum of the one- and two-body contributions as  $I = I_{\text{qp}} + I_{\text{pair}}$ , where each component reads (25)

$$\begin{aligned} I_{\text{qp}} &= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sum_{\mathbf{p},\mathbf{k},\sigma} |t_{\mathbf{k},\mathbf{p}}|^2 \mathcal{A}_{\mathbf{k},L}(\omega) \mathcal{A}_{\mathbf{p},R}(\omega) \\ &\quad \times [f_L(\omega) - f_R(\omega)], \\ I_{\text{pair}} &= 2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sum_{\mathbf{q},\mathbf{q}'} |w_{\mathbf{q},\mathbf{q}'}|^2 \mathcal{B}_{\mathbf{q},L}(\omega) \mathcal{B}_{\mathbf{q}',R}(\omega) \\ &\quad \times [b_L(\omega) - b_R(\omega)]. \end{aligned} \quad (4)$$

In Eq. 4,  $\mathcal{A}_{\mathbf{k},j}(\omega)$  and  $\mathcal{B}_{\mathbf{q},j}(\omega)$  denote one- and two-particle spectral functions, respectively,  $f_j(\omega)$  and  $b_j(\omega)$  denote the Fermi and Bose distribution functions, and  $\mu_{b,j} = 2\mu_j$  denotes the bosonic-pair chemical potential in the  $j$ -reservoir. For the detection of the pair-tunneling current, it is crucial to consider the small tunneling coupling regime where the nonequilibrium noise reflects an effective particle number in tunneling process.<sup>a</sup>

We define the current noise as  $\mathcal{S}(t_1, t_2) = (1/2)\langle \hat{I}(t_1)\hat{I}(t_2) + \hat{I}(t_2)\hat{I}(t_1) \rangle$  (57–60) [see also, e.g. Ref. (36)]. For the steady-state transport with the time-translational symmetry, we assume that the

noise depends on  $t_1 - t_2$  as  $\bar{S}(t_1, t_2) \equiv \bar{S}(t_1 - t_2)$  (being independent of  $(t_1 + t_2)/2$ ). Its Fourier component reads

$$\bar{S}(\omega) = \frac{1}{\tau} \int_0^\tau dt_1 \int_0^\tau dt_2 e^{i\omega(t_1 - t_2)} \bar{S}(t_1 - t_2), \quad (5)$$

where  $\tau$  is the typical time scale for the noise measurement. Taking  $t = t_1 - t_2$  and  $\bar{S}(t) = \frac{1}{2}(\hat{I}(t)\hat{I}(0) + \hat{I}(0)\hat{I}(t))$ , we obtain the zero-frequency limit of the noise power  $S \equiv \bar{S}(\omega \rightarrow \eta)$  ( $\eta$  is an infinitesimally small number) as

$$S = \frac{1}{2} \int_{-\infty}^{\infty} dt (\hat{I}(t)\hat{I}(0) + \hat{I}(0)\hat{I}(t)), \quad (6)$$

where we considered the limit of  $\tau \rightarrow \infty$ . In this regard, we briefly note that  $\tau$  should be sufficiently longer than the transport time-scale  $\tau_0$ , where in the recent experiment  $\tau_0 = O(10^{-1})$  s is found (9). Similar to the calculation above, we can evaluate the current noise (Supplementary Material) as the sum of the two contributions:  $S = S_{\text{qp}} + S_{\text{pair}}$ , where

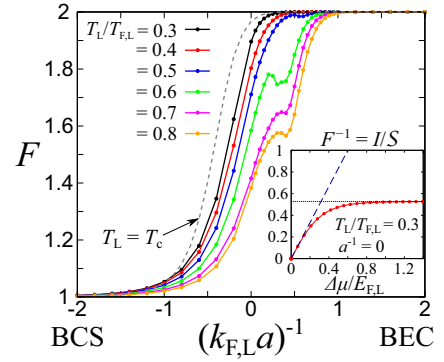
$$\begin{aligned} S_{\text{qp}} &= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sum_{\mathbf{p}, \mathbf{k}, \sigma} |t_{\mathbf{k}, \mathbf{p}}|^2 \mathcal{A}_{\mathbf{k}, \mathbf{L}}(\omega) \mathcal{A}_{\mathbf{p}, \mathbf{R}}(\omega) \\ &\quad \times [f_{\mathbf{L}}(\omega)[1 - f_{\mathbf{R}}(\omega)] + \{1 - f_{\mathbf{L}}(\omega)\}f_{\mathbf{R}}(\omega)], \\ S_{\text{pair}} &= 4 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sum_{\mathbf{q}, \mathbf{q}'} |w_{\mathbf{q}, \mathbf{q}'}|^2 \mathcal{B}_{\mathbf{q}, \mathbf{L}}(\omega) \mathcal{B}_{\mathbf{q}', \mathbf{R}}(\omega) \\ &\quad \times [b_{\mathbf{L}}(\omega)[1 + b_{\mathbf{R}}(\omega)] + b_{\mathbf{R}}(\omega)[1 + b_{\mathbf{L}}(\omega)]. \end{aligned} \quad (7)$$

The bias between the reservoirs is included in the distribution function and therefore Eq. 7 is valid for the case with the temperature bias (61). In the large chemical potential bias limit ( $\Delta\mu \equiv \mu_{\mathbf{L}} - \mu_{\mathbf{R}} \rightarrow \infty$ ), we can prove  $S_{\text{qp}}/I_{\text{qp}} = 1$  and  $S_{\text{pair}}/I_{\text{pair}} = 2$  without any further approximations (Supplementary Material). This motivates us to consider the Fano factor:

$$F = \frac{S}{I} = \frac{S_{\text{qp}} + S_{\text{pair}}}{I_{\text{qp}} + I_{\text{pair}}}. \quad (8)$$

The Fano factor  $F$  changes from 1 to 2, according to whether the quasiparticle or pair tunneling is dominant and hence, it is a useful probe for the current carrier. In particular, the Fano factor  $F$  becomes 1 and 2 in the BCS limit ( $a^{-1} \rightarrow -\infty$ ) and BEC limit ( $a^{-1} \rightarrow \infty$ ), respectively. Importantly, the deviation of  $F$  from 1 indicates a clear evidence of the pair-tunneling process yet to be not well understood in cold atomic systems (25). Therefore, the observation of  $F$  can be a crucial key for understanding transport phenomena in strongly interacting systems.

In this study, we consider the large bias regime (see Fig. 1) characterized by  $\mu_{\mathbf{L}} - \mu_{\mathbf{R}} \rightarrow \infty$  (Supplementary Material) (62) and the momentum-conserved tunneling processes as  $t_{\mathbf{p}, \mathbf{k}} = \mathcal{T}_1 \delta_{\mathbf{p}, \mathbf{k}}$  and  $w_{\mathbf{q}, \mathbf{q}'} = \mathcal{T}_2 \delta_{\mathbf{q}, \mathbf{q}'}$ , for simplicity. To see the qualitative behavior of  $F$ , we use the spectral functions  $\mathcal{A}_{\mathbf{k}, \mathbf{j}}(\omega) = -2 \text{Im} G_{\mathbf{k}, \mathbf{j}}(i\omega_n \rightarrow \omega - \mu_{\mathbf{j}} + i\eta)$  and  $\mathcal{B}_{\mathbf{q}, \mathbf{j}}(\omega) = -2 \text{Im} \mathcal{G}_{\mathbf{q}, \mathbf{j}}(iv_\ell \rightarrow \omega - \mu_{\mathbf{b}, \mathbf{j}} + i\eta)$  with an infinitesimal small number  $\eta$ , where thermal single- and two-particle propagators  $G_{\mathbf{k}, \mathbf{j}}(i\omega_n)$  and  $\mathcal{G}_{\mathbf{q}, \mathbf{j}}(iv_\ell)$  with fermion and boson Matsubara frequencies  $i\omega_n$  and  $iv_\ell$  are evaluated within the many-body TMA (63, 64) (see also Supplementary Material). We employ  $\eta = 10^{-2} E_{\text{F}, \mathbf{L}}$  in the numerical calculation to avoid the divergent behavior of the current associated with the momentum-conserved tunneling in the weak- and strong-coupling limits, where  $E_{\text{F}, \mathbf{L}} = (3\pi^2 N_{\mathbf{L}})^{2/3} / (2m)$  denotes the Fermi energy of the L reservoir with the number density  $N_{\mathbf{L}}$ . However, our result can be qualitatively unchanged by this treatment because the distribution functions play a key role in determining  $F$  rather than the detailed structures of tunneling junctions. Moreover,  $\mathcal{T}_2$  must be normalized to



**Fig. 2.** Fano factor  $F$ , associated with tunneling transport between two reservoirs, throughout the BCS–BEC crossover for various temperatures  $T_{\mathbf{L}}$  in the reservoir L. The reservoir R is almost vacuum. The ratio between tunneling couplings is given as  $\mathcal{T}_{2,\text{ren.}}/\mathcal{T}_1 = 1$ . For comparison, we plot the result at  $T_{\mathbf{L}} = T_{\text{c}}$  (dashed curve). Note that  $T_{\text{c}}$  changes in the range of  $0.02T_{\text{F}, \mathbf{L}} \lesssim T_{\text{c}} \lesssim 0.24T_{\text{F}, \mathbf{L}}$  depending on  $(k_{\text{F}, \mathbf{L}} a)^{-1}$ . The inset shows the bias ( $\Delta\mu$ ) dependence of  $F^{-1}$  at  $T_{\mathbf{L}}/T_{\text{F}, \mathbf{L}} = 0.3$  and  $a^{-1} = 0$ . The dashed and dotted lines represent the Onsager's relation  $F^{-1}(\Delta\mu \rightarrow 0) = \Delta\mu/2T$  (Supplementary Material) and the large bias limit, respectively.

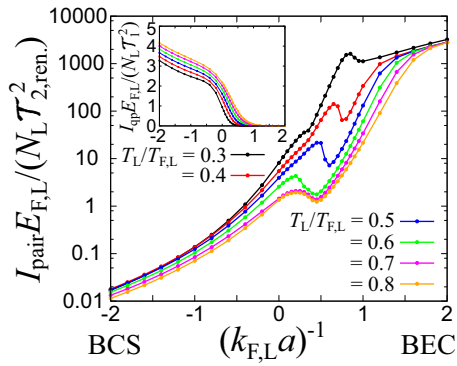
suppress the ultraviolet divergence in  $B_{\mathbf{q}, \mathbf{j}}(\omega)$ . For this purpose, we introduce the renormalized two-body tunneling coupling  $\mathcal{T}_{2,\text{ren.}} = (\Lambda^2 k_{\text{F}, \mathbf{L}} / 3\sqrt{2}\pi^2) \mathcal{T}_2$  where  $k_{\text{F}, \mathbf{L}} = \sqrt{2mE_{\text{F}, \mathbf{L}}}$  denotes the Fermi momentum. Such a divergence can also be avoided by introducing the form factor for the relative momentum  $\mathbf{p}$  in  $P_{\mathbf{q}, \mathbf{j}}$  (65). In this work, we take  $\Lambda = 100k_{\text{F}, \mathbf{L}}$  (39) in the practical calculation. This value is associated with the effective range  $r_{\text{eff}}$  as  $r_{\text{eff}} = 4/\pi\Lambda$  (39).

## Fano factor throughout the BCS–BEC crossover

Fig. 2 shows the Fano factor  $F$  as a function of the dimensionless interaction parameter  $(k_{\text{F}, \mathbf{L}} a)^{-1}$  in the entire BCS–BEC crossover regime above the superfluid critical temperature  $T_{\text{c}}$ . We considered  $\mathcal{T}_{2,\text{ren.}}/\mathcal{T}_1 = 1$ , and the reservoir R was regarded as almost vacuum ( $\mu_{\mathbf{L}} - \mu_{\mathbf{R}} \rightarrow \infty$ ) (Supplementary Material). As we showed in the inset of Fig. 2, the large-bias assumption can be justified when  $\Delta\mu$  is larger than a typical many-body scale in the reservoir (i.e.  $E_{\text{F}, \mathbf{L}}$ ). One can clearly see that  $F$  evolves from 1 to 2 with increasing the interaction strength in Fig. 2, indicating that the current carrier gradually changes from quasiparticles ( $F = 1$ ) to pairs ( $F = 2$ ). Such a behavior is universal in the sense that these asymptotic values do not depend on any details on the model parameters and structures of tunneling junctions. More explicitly, at the large bias limit, one can obtain (Supplementary Material)

$$F(\Delta\mu \rightarrow \infty) \rightarrow \frac{I_{\text{qp}} + 2I_{\text{pair}}}{I_{\text{qp}} + I_{\text{pair}}}, \quad (9)$$

where  $I_{\text{qp}}$  and  $I_{\text{pair}}$  denote the contributions of the quasiparticle and pair tunnelings, respectively. The Fano factor  $F$  approaches 1 and 2 in the quasiparticle-dominant ( $I_{\text{qp}} \gg I_{\text{pair}}$ ) and pair-dominant regimes ( $I_{\text{pair}} \gg I_{\text{qp}}$ ), respectively. Although the interaction dependence of the Fano factor  $F$  is deeply related to properties of the tunneling junctions and spectral functions of the carriers, one can find from Eq. 9 that  $F \rightarrow 1$  ( $F \rightarrow 2$ ) in the limit of  $a^{-1} \rightarrow -\infty$  ( $a^{-1} \rightarrow \infty$ ) regardless of the detailed properties of the system. Moreover,  $F = 2$  can be realized even above  $T_{\text{c}}$  because of strong interactions leading to the formation of preformed Cooper pairs in the BCS–BEC crossover. With increasing the temperature,  $F$  tends to be suppressed because thermal effects assist the



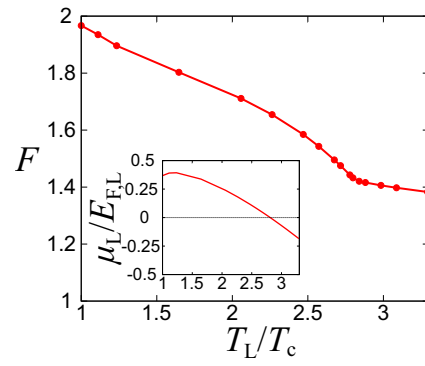
**Fig. 3.** Pair-tunneling current  $I_{\text{pair}}$  in the normal phase throughout the BCS–BEC crossover at different temperatures. The inset shows the quasiparticle current  $I_{\text{qp}}$  with the same horizontal axis  $(k_{F,L}a)^{-1}$ .

dissociation of pairs. Nevertheless, even at finite temperature,  $F$  approaches 2 with increasing the interaction because bound molecules are dominant in the deep BEC regime<sup>b</sup> where  $T_L \lesssim E_b$  [ $E_b = 1/(ma^2)$  is the two-body binding energy].

To see the detailed behavior of the Fano factor  $F$ , we plot  $I_{\text{qp}}$  and  $I_{\text{pair}}$  throughout the BCS–BEC crossover at different temperatures in Fig. 3. From the inset of Fig. 3, the quasiparticle current  $I_{\text{qp}}$  is exponentially suppressed with increasing the attractive interaction. This suppression (in particular, the rapid drop of  $I_{\text{qp}}$  at  $(k_{F,L}a)^{-1} \gtrsim -0.5$ ) is induced by the pairing fluctuation effect (39), i.e. the reduction of  $A_{\mathbf{k},L}(\omega)$  near  $|\mathbf{k}| = k_{F,L}$  and  $\omega = E_{F,L} (\simeq \mu_L)$  by the particle–hole coupling. We note that this fluctuation effects result in the pseudogap in the density of state near  $T_c$  (41). Finally,  $I_{\text{qp}}$  approaches zero in the BEC limit  $(k_{F,L}a)^{-1} \rightarrow \infty$  because of the formation of molecules with large binding energies. These results are qualitatively consistent with previous work (21, 24). On the other hand,  $I_{\text{pair}}$  drastically increases with increasing the interaction strength  $(k_{F,L}a)^{-1}$  as shown in Fig. 3. At the BCS side  $(k_{F,L}a)^{-1} < 0$  where the attraction is not strong to form a two-body bound state in vacuum, the contribution of  $I_{\text{pair}}$  can be regarded as the tunneling of the preformed Cooper pairs into the two-body continuum in the reservoir R. In the strong-coupling BEC regime  $(k_{F,L}a)^{-1} > 1$  and  $T_L/E_b \lesssim 1$ ,  $I_{\text{pair}}$  describes the tunneling transport of bound molecules across two reservoirs, because the two-body bound state exists in the reservoir R with the same coupling  $g$ . Such a tunneling current associated with weakly interacting molecular bosons becomes large due to their long lifetime and the Bose enhancement of low-energy distributions.

One can also see a dip-hump structure of  $I_{\text{pair}}$  in the intermediate regime. Here,  $\mu_L$  is close to zero and changes its sign, indicating that the dominant contribution changes from the preformed-pair transfer to the molecule-to-molecule transport across the junction. From the unitary limit  $(k_{F,L}a)^{-1} = 0$ , the preformed-pair transfer increases due to the overlap with the bound-state spectra in  $B_{\mathbf{q},R}(\omega)$  and eventually decreases because of the decrease in  $\mu_L$ . With increasing the interaction further, the inter-reservoir molecule-to-molecule transition emerges where the bound-state spectra in two reservoirs get close to each other in the energy axis  $\omega$ . Although these structures reflect the physical properties of the system, they also depend on the detailed setup of the tunneling junctions (e.g. the ratio between the tunneling couplings  $\mathcal{T}_{2,\text{ren.}}/T_1$ ) (Supplementary Material).

Fig. 4 shows the temperature dependence of the Fano factor  $F$  in the unitary limit  $(k_{F,L}a)^{-1} = 0$ . Because  $B_{\mathbf{q},R}(\omega)$  does not involve a bound molecule pole, the transfer of the preformed Cooper pairs in the reservoir L to the two-body continuum in the reservoir R



**Fig. 4.** Temperature dependence of the Fano factor  $F$  in the unitary limit [ $1/(k_{F,L}a) = 0$ ] with  $\mathcal{T}_{2,\text{ren.}}/T_1 = 1$ . The horizontal axis is taken as  $T_L/T_c$ , where  $T_c$  is the superfluid critical temperature. The inset shows the chemical potential  $\mu_L$  as a function of  $T_L/T_c$  for a given Fermi energy  $E_{F,L}$ .

can be anticipated in the unitary limit. One can see the enhancement of the Fano factor  $F$  at the low-temperature regime. In particular, the curvature of the Fano factor  $F$  is modified at  $T_L/T_c \simeq 2.8$ , where the sign of  $\mu_L$  changes from negative to positive one as the temperature decreases (see the inset of Fig. 4). Although the Fano factor depends on  $\mathcal{T}_{2,\text{ren.}}/T_1$  as shown in Fig. S2, the qualitative behavior, i.e. suppression of the pair-tunneling current due to increase of the temperature is unchanged regardless of the value of  $\mathcal{T}_{2,\text{ren.}}/T_1$ . For estimating the value of  $\mathcal{T}_{2,\text{ren.}}/T_1$  (which depends on the potential barrier and the interaction strength) in each experimental setup, see Ref. (25). In the Supplementary Material, we show that  $\mathcal{T}_{2,\text{ren.}}/T_1$  can be tuned and it is possible to realize  $\mathcal{T}_{2,\text{ren.}}/T_1 \simeq 1$  by adjusting the strength of the potential barrier as  $\mathcal{T}_{2,\text{ren.}}/T_1 \propto [1 + (V_0/E_{F,L})]^{-1} [1 + (V_0/E_{F,L})^2 (k_{F,L}\ell)^2]^{-1/2}$  for the potential barrier given by  $V = V_0\delta(x/\ell)$  perpendicular to the  $x$  axis ( $V_0$  and  $\ell$  are the strength and the characteristic length scale of the barrier). At a positive  $\mu_L$ , the pole of the preformed Cooper pairs gradually appears in  $B_{\mathbf{q},L}(\omega)$ . Thus, the behavior of the Fano factor  $F$  can be regarded as a signature of the preformed Cooper pairs. Because the preformed Cooper pairs play an important role in the pseudogap physics of ultracold Fermi gases (41), the Fano factor contributes to the further understanding of pairing pseudogaps in the BCS–BEC crossover regime. Incidentally, because TMA does not capture the self-energy shift in  $\Pi_{\mathbf{q},L}(\omega)$ , the curvature change of the Fano factor  $F$  may differ from the temperature where  $\mu_L = 0$  in actual experiments and in more sophisticated theoretical approaches (38, 39). To evaluate the spectral functions, the analytic continuation should be carefully performed in Monte Carlo simulations (66). We note that because TMA reproduces the second-order virial expansion (67), our result in the relatively high-temperature regime can give an accurate estimate of  $F$  for given tunnel couplings.

## Summary

In this study, we showed that the Fano factor (i.e. the noise-to-current ratio  $F = S/I$ ) can be a useful probe for current carriers in the BCS–BEC crossover at large-biased tunneling junctions. Using the many-body TMA, we demonstrated that the Fano factor  $F$  gradually changes from one to two as the interaction strength increases in the normal phase, indicating that the dominant current carrier changes from the quasiparticle ( $F = 1$ ) to the pair ( $F = 2$ ) along the BCS–BEC crossover. Our prediction can be tested by experiments and uncover nonequilibrium strong-coupling physics via transport measurements. While we have

focused on the large bias limit, such a situation can be achieved when the bias is larger than the many-body energy scale (i.e. Fermi energy of the dense reservoir). Furthermore, our result indicates that the noise measurement is useful for the study of the BCS–BEC crossover and pair-fluctuation effects in unconventional superconductors.

## Notes

- We note that the validity of the truncation with respect to the lowest-order tunneling coupling was confirmed in the recent experiment (7).
- Here, “BEC regime” is used for the regime where the two-body attraction is so strong that the associated superfluid state behaves like molecular BEC below  $T_c$  (38, 39). In this regard, the strongly attractive regime even above  $T_c$  is also referred as to the BEC regime for characterizing the interaction strength.
- We note that in this regime the numerical cost is large due to the overlap of Bose distribution function and sharp peaks in  $\mathcal{B}_{\mathbf{q},L,R}$ . We confirmed that the qualitative behavior is robust against the accuracy of the frequency integration.

## Supplementary material

Supplementary material is available at PNAS Nexus online.

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## Author contributions

M.M. and T.K. designed research, H.T., D.O., M.M., and T.K. performed the calculations and wrote the manuscript.

## Data availability

All data are included in the manuscript.

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