Simplifying Reservoir Models by Flow Regime

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DECLARATION

I declare that the work presented in this thesis is my own. Where other sources of information have been used they have been indicated and acknowledged.

______________________________
Bilal Rashid

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This study focuses on the interaction between geological heterogeneity and the reservoir processes which govern fluid flow in porous media. We have developed and tested a measure of heterogeneity which uses the coefficient of variation of the vorticity of the flow field to quantify the impact of geological uncertainty on oil recovery.

We go on to explore the vorticity formulation of the equations of motion in porous media as a basis for understanding reservoir dynamics, particularly in the presence of heterogeneity and density differences. We derive dimensionless numbers to quantify the relative importance of viscosity and density differences, molecular diffusion, dispersion, and permeability heterogeneity on reservoir flow behaviour. This approach is used to develop an objective measure of the impact of permeability heterogeneity on reservoir performance, which we have compared with traditional heterogeneity indices and shown how it may be used for realistic 2D and 3D geological models.

We have used our heterogeneity index, and the dimensionless numbers to analyse the impact of heterogeneity, buoyancy effects, mobility ratio and dispersion on breakthrough time and recovery for first contact miscible gas injection processes using geologically realistic reservoir models. We find that the new heterogeneity number, in conjunction with these dimensionless numbers, provides meaningful results for real non-linear reservoir flows.

We present phase diagrams which show how reservoir performance depends on mobility ratio, viscous-gravity ratio, and heterogeneity. We have proposed that the phase diagram, and a comparison of these dimensionless numbers can be used to identify the key factors which control recovery, thus assisting the engineer in determining appropriate enhanced oil recovery (EOR) techniques, without resort to detailed flow simulation. This will enable a quick, and more robust, evaluation of the impact of geological uncertainty in the field.
Some ideas and figures have appeared previously in the following publications:

   – (Rashid et al. a)

   – (Rashid et al. b)


The ideas in this thesis were presented at the following conferences:


I can’t wait for the oil wells to run dry,  
for the last gob of black, sticky muck to  
 come oozing out of some remote well.  
Then the glory of sail will return.

—Tristan Jones

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**NOMENCLATURE**

- **A** area, [L^2]
- **c** concentration
- **C_v** coefficient of variation
- **D** symmetric dispersion tensor, [L^2t^{-1}]
- **D_0** molecular diffusion constant, [L^2t^{-1}]
- **E** effective viscosity ratio
- **f_d** fractional flow of the displacing phase
- **F** electrical resistivity factor
- **g** acceleration due to gravity, [Lt^2]
- **H** thickness, [L]
- **H_K** Koval heterogeneity factor
- **H_{cv}, H_{sv}** curvature/shear vorticity heterogeneity indices
- **H_v, H_s** vorticity/shear-strain rate heterogeneity indices
- **J** velocity-gradient tensor, [t^{-1}]
- **k** permeability, [L^2]
- **k_h, k_v** vertical/horizontal permeability, [L^2]
- **k_r** relative permeability, any subscripts refer to phase
- **K** Koval factor
- **K_{l}, K_{t}** longitudinal/transverse dispersion, [L^2t^{-1}]
- **L** length, [L]
- **L_c** Lorenz coefficient
- **M** mobility ratio
- **N_c** capillary-viscous ratio
- **N_g** gravity-viscous ratio
- **N_{TD}** transverse-dispersion number
- **NX, NY, NZ** number of grid blocks in the x/y/z directions
- **P** pressure, [mL^{-1}t^{-2}]
- **P_c** capillary pressure, [mL^{-1}t^{-2}]
- **P_e** Peclet number
- **q** flux, [L^3t^{-1}]
- **R_{ca}, R_d** Craig’s aspect ratio/gravity number
- **S_{or}** residual oil saturation
- **S_w** saturation of water
- **S_{wc}** connate water saturation
- **t_c** characteristic time, [t]
- **t_D** dimensionless time, [t]
- **v** flux per unit cross-sectional area, [Lt^{-1}]
\( v_T \) total velocity, [Lt\(^{-1}\)]
\( V_d \) displacement velocity, [Lt\(^{-1}\)]
\( V_{dp} \) Dykstra-Parsons coefficient

\( \alpha \) angle of tilt
\( \alpha_l, \alpha_t \) longitudinal/transverse dispersivity, [L]
\( \dot{\gamma} \) shear-strain rate, [t\(^{-1}\)]
\( \Delta \) difference operator
\( \Delta \rho \) density difference, [mL\(^{-3}\)]
\( \theta \) wetting angle
\( \lambda \) mobility, [1/(mL\(^{-1}\)t\(^{-1}\)])
\( \mu \) viscosity, [mL\(^{-1}\)t\(^{-1}\)]
\( \rho \) density, [mL\(^{-3}\)]
\( \sigma \) interfacial tension, [mt\(^{-2}\)]
\( \phi \) porosity
\( \Omega \) vorticity, [t\(^{-1}\)]
\( \nabla \) differential operator, [L\(^{-1}\)]

**Subscripts**

- \( b_t \) breakthrough
- \( d \) displacing phase
- \( g \) gas phase
- \( o \) oleic phase
- \( w \) water phase
- \( x, y, z \) spatial co-ordinates

**ACRONYMS**

- PVI pore volumes injected
- EOR enhanced oil recovery
- VE vertical equilibrium
- FCM first contact miscible
- BOR breakthrough oil recovery
- PV pore volumes
INTRODUCTION

Reservoir simulation of oil and gas formations has been the main-stay of field development since the 1930s when petroleum engineering became a discipline. In those days simulation was restricted to either physical experiments in the laboratory or simplified analytical methods, such as the one-dimensional Buckley-Leverett solutions, which could be used to estimate recovery. The advent of the digital computer in the 60s introduced the ability to solve the large sets of partial-differential equations that describe flow-behaviour in heterogeneous porous media.

The construction of these reservoir models has changed little in the fifty years since their introduction. A discretised static-model of the subsurface is created and populated by a number of geological properties, such as permeability and porosity, and fluid distributions, such as water-saturation, gas-concentration, and hydrocarbon volume. This static model is combined with a fluid-model, conservation laws, and the momentum equation to create a set of finite-difference equations that are numerically solved to predict behaviour.

Given such a discretised model, it is clear that there will be some length-scale which will not be resolved by the model. In some cases this may be because of limitations in processing power, whilst in others it may be because that level of detail is unnecessary. Generally improvements in computing power encourage modellers to increase the levels of geological complexity in their models, so it is important when constructing a reservoir model to understand

A. the appropriate level of geological complexity that needs to be modelled,

b. and how the uncertainty in geology maps on to an uncertainty in reservoir performance.

The construction of an appropriate model requires an understanding of the fluid-fluid and fluid-rock processes in the subsurface, which in the ideal case means the model need only contain the heterogeneity to the scale that it impacts on the recovery process deployed in the reservoir. An assessment of the impact of uncertainty on future behaviour requires the detailed numerical simulation of a statistically-significant number of reservoir models; this requires considerable processing power and as models become increasingly complex this will remain a difficult proposition for the foreseeable future.
SUBSURFACE UNCERTAINTY

A large oil field may have a volume of $10^{10}$ m$^3$, whilst the only direct measurement of rock properties is from small pieces of core taken from wells with a typical volume of approximately $10^{-4}$ m$^3$. This means that measurements are only available for about $10^{-14}$ of the reservoir. Geostatistics (De Marsily et al. 1998, 2005) and detailed seismic surveys are used to infer the properties of the oil-bearing formation. This is the primary source of uncertainty in future predictions of reservoir behaviour.

The vast majority of these petrolierous formations are found in sedimentary basins. Oil is normally found within the matrix porosity of these formations so good reservoir rocks must be both porous and permeable. Reservoirs are therefore, predominantly, sandstones or carbonates. The nature of the depositional and diagenetic processes that formed these rocks is a strong control on the complexity and type of heterogeneities present.

Studies of these sedimentary deposits have shown that variations in rock properties exist at all scales of observation with partial correlation over these scales, and all these scales have the potential to affect oil recovery (Emanuel et al. 1989; Kjonsvik et al. 1994; Perez and Chopra 1997). Fig. 1.1 shows the type of heterogeneities found at different scales in a shoreface-shelf sandstone reservoir, and relates them to their respective geological, depositional or diagenetic processes.

Vertical and lateral permeability and porosity gradients are often the result of changes in grain-size and sorting, and may vary by whole orders of magnitude (Gerritsen and Durlofsky 2005). Cementing and stratification of low permeability fine-grained sediments gives rise to local baffles to flow (Sharp et al. 2003). Large-scale vertical changes in rock type (stratigraphy) due to changes in the depositional environment may create major barriers to flow or enhanced flow channels, whilst the essentially stochastic nature of deposition mean sedimentation is rarely uniform over reservoir distances (Warren and Price 1961).

To represent the effects of all these heterogeneities in a typical reservoir would require a simulation model with billions of grid-cells. Despite the significant advances of the last decade, detailed two- and three-phase flow simulations on fine-scale models are still constrained by limitations in memory and processing power. Static geological models have a typical resolution of the order of tens of metres, whilst simulation models are coarser still, with a typical horizontal resolution of a hundred metres and a vertical resolution of a few metres.

Much recent work has shown that different depositional environments and displacement processes affect recovery in different ways (Weber 1982; Giordano et al. 1985; Tyler et al. 1994; Tidwell and Wilson 2000; Coll et al. 2001; Henson et al. 2002; Jackson et al. 2005; Choi et al. 2011). A study of major sandstone and carbonate reservoirs in Texas (Fig. 1.2) showed that recovery efficiency can vary from less than 5% for a mud-rich submarine fan reservoir under a solution gas drive to almost 80% for a waterflood through a wave-dominated deltaic reservoir (Tyler et al. 1994).
Figure 1.1: The different scales of heterogeneity within shoreface-shelf sandstone reservoirs [from Sech et al. (2009)]: (A) The regional scale - 10s of kilometres, (B) The Reservoir Scale - kilometres, (C) Simulation Grid Block scale - 100 metres, (D) Fine Scale - 10s of cm, (E) Mineral Scale - mm.

Clearly the flow mechanism is a significant control on recovery and extensive research has gone into understanding the processes involved in secondary recovery mechanisms. For instance, viscous-dominated flow in a layered-heterogeneous reservoir will tend to cause channelling of the displacing fluid within high permeability layers and hence reduce recovery (Greenkorn and Haselow 1988). An adverse viscosity ratio will exacerbate this effect as even small fluctuations of permeability or con-
centration will result in the formation of viscous fingers which will channel through the oil giving early breakthrough (Homsy 1987; Houseworth 1991; Waggoner et al. 1992; Araktingi and Orr Jr 1993). Differences in fluid density in conjunction with heterogeneity may induce vertical flow in the reservoir, changing the flow pattern (Pande 1992) whilst diffusive and dispersive processes, and capillary pressure for immiscible flow, may become significant in some reservoirs (Lake and Hirasaki 1981; Fayers and Muggeridge 1990; Tungdumrongsub and Muggeridge 2010).

As an example of the level of understanding of the physics, it is possible to accurately predict stable and unstable miscible flow in heterogeneous media, both in detail and on average (Davies et al. 1991). So the current absurdity is that whilst we are able to compute the effects of permeability heterogeneity on single-phase flow extremely well, and to a lesser extent on two- and three-phase flow, the nature and description of that very heterogeneity is uncertain.

Ideally the impact of this uncertainty should be evaluated by performing multiphase flow simulations through multiple geological realisations of the reservoir’s heterogeneity distribution. However there are practical limitations to the number of simulations and the amount of complexity that may be modelled. The traditional approaches to evaluate uncertainty is to either, reduce simulation time by coarsening
the models, use semi-analytical methods, or to identify a small number of representative realisations to be used for multiphase flow simulation. We would like to be able to rank each realisation without recourse to such detailed simulation.

Therefore, what we require is a framework that enables engineers to rapidly evaluate the uncertainty in reservoir behaviour so that informed and objective decisions may be made regarding the economic viability and further development of heterogeneous reservoirs. Numerically assessing the importance and effect of permeability heterogeneity is a vital component of this framework.

In this thesis we have set out to address a subset of this problem. Our objectives are:

• to assess how accurate existing measures of heterogeneity are for different sedimentary formations and recovery mechanisms,

• to develop an objective and improved method of quantifying the level of heterogeneity in a reservoir in terms of its impact on flow;

• to introduce a unified mathematical framework that allows engineers to assess the impact of permeability heterogeneity on recovery in the presence of other competing forces and processes, with a view to building an appropriate simulation model.

**Structure of This Thesis**

This thesis may be divided into two sections, the first presents the background to the problem and the mathematical formalism, whilst the second focusses on our methods, results and conclusions.

In Chapter 2 we discuss the physical processes in a reservoir and their interaction with reservoir heterogeneity. This is used to provide the background to an analysis of the work, to date, on assessing the relative importance of different reservoir processes and estimating the impact of heterogeneity on recovery. We demonstrate the need for an objective measure of the impact of heterogeneity on recovery and the requirement that there be a unified mathematical framework for the design of enhanced oil recovery (EOR) processes in heterogeneous reservoirs.

Chapter 3 introduces the vorticity formalism as a basis for understanding reservoir dynamics. We discuss vorticity in porous media and introduce a new measure of permeability heterogeneity. The numerical simulation methods and the geological models used are discussed in Chapter 4.

Chapter 5 investigates the widely used Dykstra-Parsons coefficient and the newer dynamic Lorenz coefficient in their ability to rank reservoir realisations in terms of performance without detailed flow simulation. We compare these results with those using our heterogeneity indices for miscible and immiscible displacements at various mobility ratios.
Chapter 6 uses the vorticity based approach to understand the effect of dispersion, gravity, and heterogeneity on recovery in miscible EOR processes. Chapter 7 details a preliminary investigation of vorticity in 3D reservoir models and demonstrates how vorticity may be estimated for models gridded using corner point geometry. Chapter 8 discusses the decomposition of the vorticity field into different components which allows us to compare reservoir realisations with different well configurations in an improved way. This chapter provides indications on how vorticity may be calculated using streamlines. Chapter 9 concludes this thesis with a summary of our key findings and the future areas of research that must be explored.
Displacements in porous media on the Darcy scale\(^1\) are affected by viscosity differences, density differences, permeability and porosity, capillary pressure \(P_c\), and physical dispersion/molecular diffusion (Muskat 1937).

The effects of these processes and subsurface features on the spatial distribution and temporal evolution of reservoir fluids, the time to breakthrough of the injected fluid, and on the recovery of oil is critical to the design of a secondary or tertiary oil recovery process.

### 2.1 Reservoir Processes—Homogeneous Porous Media

Consider the injection of a fluid of viscosity \(\mu_d\) and density \(\rho_d\) into a homogeneous porous medium containing oil of viscosity \(\mu_o\) and density \(\rho_o\).

**Viscosity Differences** In miscible displacements where the displacing fluid is more viscous than the oil\(^2\), i.e. \(\mu_o/\mu_d \leq 1\), there is a sharp front, perpendicular to the direction of mean-flow, between the two fluids, the orientation of which is maintained throughout the displacement process. This is because the oil is able to move just as fast as the displacing fluid and this process results in most of the oil being recovered. A similar piston-like displacement is seen in immiscible displacements when the mobility ratio at the shock front is less than or equal to one; such a process may be described by Buckley-Leverett theory (Buckley and Leverett 1942).

When the injected fluid is less viscous than the oil then small perturbations at the front will lead to the growth of channels of faster-moving fluid which finger through the oil, resulting in the early breakthrough of fluid at the producer and pockets of bypassed oil (e.g. Blackwell et al. (1959); Heller (1966); Homsy (1987); Christie (1989), and many others). This fingering process is a result of the sensitivity of the pressure field in the presence of viscosity differences to small perturbations at the fluid-oil interface. A comprehensive review of viscous fingering in homogeneous media appears in Homsy (1987).

Viscous fingering in an oil field presents challenges to the design of EOR processes, in particular miscible gas and WAG (water alternating gas) injection schemes. When computing power limited the number of cells in simulation models to a few hun-

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\(^1\) The ‘Darcy scale’ refers to the macroscopic scale—as opposed to the molecular or microscopic scales—at which the individual particles/molecules that make up the porous medium are replaced by a representative continuum to which we can assign various Darcy scale parameters such as permeability, porosity and on which we can use macroscopic laws to understand the average behaviour of the fluid and rock.

\(^2\) In the absence of density differences and diffusion/dispersion
dred then empirical methods such as those of Koval (1963) and Todd and Longstaff (1972) were used to estimate its effect on recovery. Numerical simulation brought its own problems; fingering occurs on all length scales and in the absence of physical dispersion/molecular diffusion the equations allow infinite growth rates for short wavelength modes. This means that solutions are heavily dependent on grid refinement, where numerical dispersion may stabilise or limit the growth rate of fingers unless physical dispersion/diffusion are explicitly included (Christie and Bond 1987; Christie 1989). In simulation, viscous fingering is still often neglected as fingers are generally too narrow to be resolved in field-scale models.

### Density Differences

When one fluid displaces another, or when one fluid finds itself above a less dense fluid the interface between the fluids is prone to Rayleigh-Taylor instabilities (Rayleigh 1882). For miscible fluids the formation of a boundary layer between the fluids will cause the instabilities. For immiscible fluids perturbations to the interface will result in the less dense fluid forming sets of Rayleigh-Taylor fingers that will penetrate into the fluid below. This instability is regularly studied in other disciplines, for instance the intricate Rayleigh-Taylor fingers seen in the Crab nebula are one of the clearest examples of the phenomenon (Hester et al. 1996). Fig. 2.1 shows how a small perturbation to the front leads to Rayleigh-Taylor style inter-fingering of fluids.³

Where a lower density fluid is beside a more dense fluid this process will lead to segregation of the fluids within the reservoir. In the case of an oil reservoir this is likely to happen when a lower density fluid such as a gas or solvent is injected into oil. The displacement front moving perpendicular to \( g \) will often result in the formation of a finger of faster moving fluid—a gravity override— that arrives prematurely at the production well.

When viscosity differences combine with a displacement-front velocity sufficiently low to allow gravity segregation to occur, recovery is much reduced (Dietz 1953; Fayers and Muggeridge 1990). An average frontal advance of less than 0.5 m/day is typical of many reservoirs and is sufficient to allow gravity segregation to occur in most cases, although as this depends on \( k_v/k_h \) and \( \Delta \rho \), this will be most evident in gas-oil displacements rather than water-oil. Koval (1963) suggested that fluid velocities greater than approximately 5 m/day are needed to reduce the effect of density differences in a reservoir.

Where the displacement front is moving parallel to \( g \), and \( \Delta \rho \) is such that the heavier fluid is beneath the lighter, then density differences will generally tend to reduce the rate of growth of perturbations to the front due to viscous instabilities (Dumore 1964).

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³ The simulations were carried out using the Athena3D FORTRAN code from the Virginia Institute of Theoretical Astronomy [www.astro.virginia.edu/VITA/athena.php](http://www.astro.virginia.edu/VITA/athena.php) and [https://trac.princeton.edu/Athena/](https://trac.princeton.edu/Athena/)
In most cases the transition between viscous-dominated and gravity-dominated flow is governed by the flow rate and the relative direction of the mean flow-vector and gravity. The transition between viscous dominated and gravity dominated flow and the effects of viscous fingering, reservoir tilt and permeability anisotropy were studied by, amongst others, Crane et al. (1963) and Fayers and Muggeridge (1990).

**Dispersion and Capillary Pressure**  
Physical dispersion, diffusion (Perkins and Johnston 1963) and capillary forces (Lake 1989; Chaouche et al. 1993; Duijn et al. 1995) are three basic processes that have the ability to change the character of viscous and gravity-induced instabilities. When two miscible fluids are in contact, the initial sharp front between them will gradually become diffuse as they mix. This can act to reduce the difference in viscosity across the front and so dampen the formation of viscous fingers. As diffusion does not depend on the velocity of the fluids it is generally treated as an isotropic process. If, however, at the same time this displacement front

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**Figure 2.1:** Simulation of the Rayleigh-Taylor instability showing a density plot of the fluids in time. (a) At \( t = 0 \) there is a higher density fluid resting above a low density fluid. A random perturbation to the fluid velocity is introduced at the interface and the density maps shown at some time after the start of simulation (b) & (c). The fluids are miscible, gravity is acting downwards and \( \Delta \rho = 1 \). [from www.astro.virginia.edu/VITA/ATHENA/rt.html] using the Athena3D Code (Gardiner and Stone 2008)
is moving then there will be additional mixing due to microscopic variations in fluid velocity; this we call dispersion [the relative importance of diffusion and dispersion on fluid flow in porous media has been recently studied by Jha et al. (2008)].

Dispersion may be separated into two components, a longitudinal component, \( K_L \), parallel to mean fluid velocity and a transverse component, \( K_T \), perpendicular to flow velocity. In general \( K_L \) and \( K_T \) have opposing effects on the growth of viscous fingers. The longitudinal component will lead to the spreading of the fluid front, thus contributing to the growth rate of viscous fingers, whilst the transverse component may cause individual fingers to merge, reducing the number of fingers and so improving areal sweep.

For immiscible fluids the surface tension between fluids, described by \( P_c \), can both increase the effects of viscous fingering by spreading the front or stabilise fingering by causing transverse mixing of fluids and the merging of different viscous fingers. All these processes have long time constants and require low velocities to be effective.

Some of these adverse effects can be reduced by appropriate choices of fluid velocity and well location. At high displacement velocities the effects of density differences and diffusion may be reduced. Viscous fingering may be stabilised by gravity (Hill 1952; Dumore 1964), physical diffusion/dispersion (Heller 1966; Christie and Bond 1987; Riaz and Meiburg 2003, 2004) or capillary effects (Perkins and Johnston 1963; Daripa and Pasa 2008).

### 2.1.1 Dimensionless Flow-Regime Numbers

The relative importance of viscous, capillary, gravity and dispersive effects in homogeneous reservoirs is usually characterised in terms of dimensionless numbers obtained through dimensional or inspectional analysis of the equations of motion and mass conservation. These numbers indicate the prevailing flow regime in the reservoir [see Dietz (1953); Fayers and Muggeridge (1990); Shook et al. (1992); Li and Lake (1995); Novakovic (2002)], although the majority of these numbers depend upon reservoir permeability.

The motivation behind flow-regime dimensionless numbers is rooted in the quest for valid comparisons between laboratory-scale experiments and field-scale behaviour. Whilst single-phase flow could be numerically simulated in the third quarter of the 20th century, complex multi-phase flow problems were still investigated in the laboratory. The features of such flow, when subjected to gravity and capillary effects, are often related non-linearly to the dimensions of the system, flow rates, and the fluid and porous media properties. When the relevant scaling laws are obeyed, one may be confident that experimental results may be extrapolated to field scale behaviour.

Dimensionless numbers are also used to determine the dominant flow-regime in the reservoir (Rapoport 1955; Craig et al. 1957; Lake 1989; Fayers and Muggeridge 1990; Christie 1989) and so understand which process to mitigate against (or exploit)
when designing a recovery process. Some studies have correlated these indices with performance indicators to understand how reservoir behaviour changes under different flow-regimes.

The problem of generating valid scaling laws has, historically, been approached in two different ways. Dimensional Analysis was the traditional method, and was detailed methodically by Buckingham (1914). This was developed and supplemented by Inspectional Analysis, formalised by Ruark (1935).

**Dimensional analysis** is predicated on the assumption that the complete set of variables that describe a system are known (Macagno 1971). These variables are then combined by trial-and-error, or experience, to form independent dimensionless groups. The variables in each group have clear physical meanings, but the interpretation of these dimensionless groups in terms of physical forces is less obvious (Geertsma et al. 1956). This approach, whilst with merit does not guarantee that the scaling groups are true scaling parameters (Shook et al. 1992).

**Inspectional analysis** relies on a mathematical analysis of the equations that describe the phenomenon, which allows a physical interpretation to be easily placed on the individual groups. The great advantage is that such groups may be formed after the application of any approximations relevant to the specific problem, so reducing the number of groups and providing a clearer, less cluttered, picture of the physical forces important in a particular reservoir or model.

The importance of using a dimensionally scaled model was shown in a paper on the design of two dimensionally-scaled models of part of an idealised oil field by Leverett et al. (1942). They emphasised the importance of correctly taking into account boundary effects (Geertsma et al. 1956) which, in unscaled models, can play a disproportionately large role in laboratory results compared to field results. More experiments with unconsolidated sand packs were done by Engelberts and Klinkenberg (1951) and Croes and Schwarz (1955) to look at the effects of viscous fingering, density differences, and capillary pressure on water-floods (Rapoport and Leas 1953). These experiments used sand-packs (or in the case of Engelbert, sand with pyrite) and, on the Darcy-scale, may be considered homogeneous. All these studies used dimensional analysis and physical intuition to choose the variables pertinent to the problem and then construct scaling groups.

This approach was shown by Rapoport (1955) to, occasionally, result in more pertinent variables than absolutely necessary; in the case of the study of Leverett et al. (1942), who implied that acceleration was a pertinent variable, he showed using inspectional analysis that for laminar flows governed by Darcy’s law this was not a relevant variable; in the case of porosity his analysis showed that it should be considered a scalable parameter.

Hence, Rapoport (1955) was the first study to set out a comprehensive set of scaling laws based on an inspectional analysis of immiscible two-phase fluid displacements in three-dimensions. He took into account gravity and capillary effects and generated three basic scaling laws that would guarantee that two models would behave in a
2.1 RESERVOIR PROCESSES—HOMOGENEOUS POROUS MEDIA

similar way with respect to saturation distributions between the model and the field, the relationship between fluid injected and oil recovery, and the relative pressure drop. The three basic scaling laws involve a viscous-gravity ratio, a viscous-capillary ratio and a capillary-interfacial tension ratio. The mathematical analysis assumed that the relative permeability and capillary pressure functions were independent of rate and viscosity but, unlike Leverett et al. (1942) and Engelberts and Klinkenberg (1951), allowed porosity to be scaled between models.

Geertsma et al. (1956) in his review of the theory of dimensional scaling suggested that even inspectional analysis, when used in isolation, may not lead to the correct number of dimensionless groups. He suggested using inspectional analysis and then using the rule put forward by Buckingham (1914)—that the number of dimensionless groups is equal to the total number of variables minus the number of fundamental dimensions (these are those of mass, length, and time)—to supplement the scaling groups for a more complete analysis. They extended previous work by generating similarity groups for miscible fluid displacements, by replacing the groups with $P_c$ with time constants involving a diffusion constant; and the injection of hot-water, by introducing a simple thermal-balance equation and temperature-dependent fluid viscosities and densities.

Using the analysis of Engelberts and Klinkenberg (1951) they found that there were 12 groups with 19 parameters needed to describe an immiscible two-phase flood. By Buckingham’s rule this should give $19 - 3 = 16$ similarity groups which meant that the number of groups from inspectional analysis was four-short. One of these groups was the Reynolds’ number which did not appear through analysis of the equations because inertial effects were neglected in the equations of motion. In practice, the Reynolds’ number may be neglected on the Darcy scale as fluid-flow is often in the creeping flow-regime. The other, it was suggested, was a group that refers to the scaling of the average pore diameter and may be suitably accounted for by a group of the form $L/\sqrt{k}$, where $L$ is the length of the reservoir and $k$ the absolute permeability.

The scaling group $L/\sqrt{k}$ is difficult to use to scale real experiments. For instance, if $L$ were to be reduced by a factor $10^2$ then $k$ would have to be $10^4$ times lower, a condition not easy to satisfy. However if $L/\sqrt{k}$ is very large, i.e. the average diameter of the pore spaces are much smaller than the characteristic length of the reservoir, then this group may be ignored (Geertsma et al. 1956). Indeed there is no need to introduce the group and then justify deleting it. This group does not appear in an inspectional analysis of the equations because Darcy’s law itself is not applicable on the pore-scale, or when inertial-forces are significant.

In the studies discussed so far it was assumed that relative permeability curves, capillary pressure curves, and viscosity ratios were the same in both the experimental

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Note the difference in terminology. Geertsma et al. (1956) make the distinction between independent variable groups (dimensionless length/time etc.), dependent groups which are those which can be measured during an experiment (pore volumes injected, recovery in PV) and similarity groups which are independent constant groups whose value is known. The ‘scaling laws’ of Rapoport are similarity groups.
model and the real case. Conversely, using the scaling criteria the absolute permeability of the model must be higher than in the real-case if the model is to mimic the reservoir. Consider Darcy’s law for an immiscible fluid,

\[ v = -\frac{k_r(S_w)k}{\mu_w} \nabla P, \tag{2.1} \]

where for convenience we have ignored capillary pressure. If \( k \) is different in the model then it is likely that the relative permeability and capillary pressure curves will be different. If at the same time the viscosity ratio is maintained then the mobility \( (k_r/\mu_w) \) may be different, potentially giving completely different flow behaviour.

A method to allow different relative permeability and capillary pressure curves between the model and the real-case was introduced by Perkins and Collins (1960). They suggested replacing the viscosity ratio, as a scaling group, with the mobility ratio; introducing a new dimensionless saturation and new relative permeability and capillary pressure curves that must be kept consistent between models (as a function of dimensionless saturation). This work allowed the use of scaled relative permeability and capillary pressure curves between model and prototype.

Despite this work on scaling-groups most studies disagreed on the precise number of scaling-groups that were absolutely necessary and the variables that were pertinent to the problem. A rigorous procedure to apply inspectional analysis and derive the minimum number of dimensionless groups for a system was introduced by Shook et al. (1992) who constructed the complete set of groups for the immiscible displacement of oil by water in a tilted anisotropic homogeneous porous medium.

They used linear algebra to systematically reduce the ten non-independent dimensionless groups to 6 independent groups necessary for the problem. The groups were described as:

A. an effective aspect ratio;

B. a dip angle group;

C. a density number;

D. an end-point mobility ratio;

E. a buoyancy number;

F. a global capillary number.

If these six groups and the relative permeability and capillary curves were matched the laboratory-scale models could be scaled correctly. Further analysis showed that in most cases the density number had little effect on breakthrough oil recovery (BOR) so the required number of scaling groups for this system was reduced to five. Shook et al’s approach was shown to be particularly useful as it allows simplifying assumptions about the flow to be incorporated in the derivation of the scaling groups. For
instance, they showed that by assuming vertical equilibrium (VE) the effective-aspect ratio was eliminated, as a scaling group. This meant that

1. the scaling group that governed the approach to VE was identified,

2. it was shown that the recovery profile was independent of the aspect ratio, i.e. VE dominated displacement.

Numerical simulation could then be used to determine the critical value(s) of the effective-aspect ratio for which VE would be a reasonable assumption. This was the first use of a dimensionless scaling group and numerical simulation to understand the reservoir-flow conditions in which simplifying assumptions may be made about the nature of the flow.

Whilst these five groups are able to completely describe a two-phase immiscible fluid system subject to constant-rate boundary conditions (conditions that are plausible for an oil reservoir) and the usual assumptions made under Darcy’s law, it is the case that different fluids may require a different approach and additional scaling groups. CO₂ for instance is both compressible and may mix miscibly with oil. Both compressibility and the miscibility of CO₂ are pressure-dependent so a constant-pressure boundary condition is better suited to this problem. The replacement of constant-rate by constant-pressure boundary conditions required, as shown by [Wood et al. (2008)](#), changes to the global-capillary and buoyancy numbers and the introduction of two additional injection and production pressure groups. By ignoring heterogeneity they suggested ten dimensionless groups to scale CO₂ flooding, however, differences in viscosity and density depend on ∆P/P. In most reservoirs ∆P ≪ P, so density differences within the reservoir should be small.

Shook’s approach, and the recent applications to CO₂ and other fluids, is still the basis for dimensionless numbers in reservoir engineering. However, as discussed so far, these numbers are only adequate for scaling laboratory, or numerical, experiments of homogeneous reservoirs.

We have discussed the development of flow-regime dimensionless numbers in a broad context. These numbers may be used, empirically, to determine the dominance of one process over another on a reservoir-wide scale, however, real reservoirs are heterogeneous. The dimensionless numbers introduced are, with the exception of M, dependent on permeability, k, and permeability heterogeneity is a significant control on recovery. It is rare that an oil reservoir is entirely gravity- or viscous-dominated; viscous-gravity numbers depend on k so gravity or viscous effects may dominate in different parts of the reservoir. Heterogeneity leads to complications when the effect of the various dimensionless ratios on recovery is considered.

In Table 2.1 we list the dimensionless ratios that may be used to characterise the relative strengths of the physical processes in a homogeneous reservoir for two-phase flow.
Table 2.1: The form of dimensionless numbers used in reservoir engineering

<table>
<thead>
<tr>
<th>Number</th>
<th>Form</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mobility Ratio</td>
<td>$\frac{\lambda_d}{\lambda_o}$</td>
<td>Fluid-fluid interactions</td>
</tr>
<tr>
<td>Capillary-Viscous</td>
<td>$\frac{F_{\text{capillary}}}{F_{\text{viscous}}}$</td>
<td>Fluid-rock interactions</td>
</tr>
<tr>
<td>Gravity-Viscous</td>
<td>$\frac{F_{\text{gravity}}}{F_{\text{viscous}}}$</td>
<td>The Buoyancy/Viscous ratio</td>
</tr>
<tr>
<td>Gravity-Capillary</td>
<td>$\frac{F_{\text{gravity}}}{F_{\text{capillary}}}$</td>
<td>The Gravity/Capillary ratio</td>
</tr>
<tr>
<td>Inertial-Viscous</td>
<td>$\frac{F_{\text{inertial}}}{F_{\text{viscous}}}$</td>
<td>Reynolds Number</td>
</tr>
<tr>
<td>Adveective-Diffusive</td>
<td>Rate of advection Rate of diffusion</td>
<td>Péclet Number</td>
</tr>
</tbody>
</table>

Longitudinal/Transverse Dispersion Numbers, see [Lake and Hirasaki (1981)](#).

Now we consider, in turn, the mobility ratio, the gravity-viscous ratio, the capillary-viscous ratio, the capillary-gravity ratio, and the dispersion number in terms of their effect on reservoir performance.

**Mobility/Viscosity Ratio ($M$)** The likelihood of viscous instabilities at the fluid-oil interface, and so the extent of viscous fingering of the fluids, can be determined by calculating the mobility/viscosity ratio at the interface of the fluids.

The mobility ratio, [Muskat (1937)](#), is defined as

$$M = \frac{\lambda_d}{\lambda_o},$$  \hspace{1cm} (2.2)

where $\lambda$ is the mobility and the subscripts $d$ and $o$ refer to the displacing phase and oil, respectively. The mobility of the $i^{th}$ phase is defined as

$$\lambda_i = \begin{cases} 
\frac{k_{ri}(S_i)}{\mu_i} & \text{for immiscible fluids,} \\
\frac{1}{\mu_i} & \text{for miscible fluids,} 
\end{cases}$$ \hspace{1cm} (2.3)

where $\mu_i$, $k_{ri}$, and $S_i$ is the viscosity, relative permeability, and volume fraction (saturation) of the $i^{th}$ phase. Note that $k_{ri}$ is a function of $S_i$.

The effect of changing the mobility ratio of reservoir fluids was discussed earlier and was experimentally studied by [Blackwell et al. (1959)](#) and numerically studied by, amongst others, [Christie (1989)](#). In the absence of heterogeneity or other processes that may accentuate/dampen the growth of viscous fingers, $M > 1$ at the interface is sufficient and necessary to initiate the growth of viscous fingers. Conversely when $M < 1$ or is unity then the growth of viscous instabilities is unlikely and the flood-front remains stable.
2.1 Reservoir Processes—Homogeneous Porous Media

**Gravity-Viscous Ratios** In most reservoirs two of the most significant processes that compete with each other are the effect on displacement of differences in density between fluids and the effect of the pressure gradient across a reservoir. The main controls on the transition from viscous to gravity dominated flow are the flow-rates, controlled by the pressure-gradients, the relative direction of flow and gravity, and the density difference between fluids. For injection perpendicular to gravity, for instance, the larger the flow-rate the less time there is for fluids to segregate; the displacement in this case will be viscous-dominated. Conversely, the smaller the flow-rate the more likely it is that reservoir fluids will segregate, leading to the development of a gravity tongue/finger that rapidly arrives at the prediction well. This leads to poor recovery, and low sweep. Alternatively, if gas is injected at the top of a core in a vertical core-flood experiment (or water at the bottom), recovery may be improved by injecting slowly, this reduces the growth rate of viscous fingers and results in a more stable displacement front. Other influences that govern the transition can be geometric, (aspect ratio and reservoir dip), or geological (permeability anisotropy).

The literature contains a large number of dimensionless ratios to try and quantify the relative strength of these two processes. Table 2.2 lists the various numbers and comments on their evolution. The general form of most of these numbers is the same. It is a ratio of the vertical (hydrostatic gradient) to horizontal (viscous) pressure gradients. The first derivation that may be interpreted as a gravity number, or more accurately a buoyancy number, for incompressible immiscible two-phase flow, was by Engelberts and Klinkenberg (1951). Rapoport (1955) and Geertsma et al. (1956) (for miscible floods) placed his derivation on a firm footing through inspectional analysis of the equations.

The effect of gravity on recovery was specifically studied by Craig et al. (1957), who used the previously introduced scaling numbers and identified the effective-aspect ratio, $R_a$, when multiplied by the viscous-gravity number $R_d$, (both defined in Table 2.2) as having the dimensional form of a ratio of pressure differences. Fig. 2.2a shows how the viscous-gravity ratio captures the effect of $M$ on efficiency. As $R_aR_d$ increases, breakthrough oil recovery (BOR) increases, but once $M > 10$ it appears to make little difference what $R_aR_d$ is, for homogeneous reservoirs. They also correlated $R_aR_d$ with BOR, for a viscous-stable displacement through a layered-heterogeneous porous medium, in Fig. 2.2b. In the absence of other processes they showed that for large values of $N_g$ (i.e. when gravity dominate) sweep efficiency does depend on the ordering of high and low permeability layers.

The transition between viscous-fingering and gravity-dominated flow was studied by Fayers and Muggeridge (1990) by applying corrections to Dietz theory, and taking into account reservoir tilt, permeability anisotropy, reservoir-aspect ratio, and mobility ratio. They found that for stable ($M < 1$) and conditionally-stable ($M = 1$) displacements, $k_v/k_h > 0.1$ had little effect on recovery. When $M < 1$, breakthrough is generally premature; when this was combined with $k_v/k_h < 0.1$ then breakthrough was delayed as fluid segregation was inhibited.
2.1 Reservoir Processes—Homogeneous Porous Media

(a) Sweep efficiency against $R_d R_d$ at various $M$

(b) The effect of different two-layer heterogeneous models on the effect of gravity on the sweep efficiency of a reservoir, $M = 0.745$

Figure 2.2: Figures 2 & 4 from [Craig et al. (1957)]
2.1 RESERVOIR PROCESSES—HOMOGENEOUS POROUS MEDIA

Table 2.2: Gravity-Viscous Ratios in Reservoir Engineering

<table>
<thead>
<tr>
<th>Definition</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_g = \Delta \rho g \frac{k_h}{\nu_T \mu_w} )</td>
<td>The ratio of the hydrostatic pressure force to flow potential. Immiscible two-phase incompressible flow. Relative permeability and capillary pressure curves were not scaled.</td>
</tr>
<tr>
<td>( N_g = \Delta \rho g \frac{k_h}{\nu_T \mu_w} )</td>
<td>Derived from above above however ( g ) was not considered a pertinent variable and was discarded. This number is not dimensionless. Immiscible two-phase incompressible flow.</td>
</tr>
<tr>
<td>( N_g = \Delta \rho g \frac{k_h}{\nu_T \mu_w} )</td>
<td>Derived using inspectional analysis for both miscible and immiscible displacements. These two groups combine and reduce to the dimensionless number of Croes and Schwarz (1955).</td>
</tr>
<tr>
<td>( \rho_w g \frac{k_h}{\nu_T \mu_w} ), ( N_g = \rho_w \frac{g}{\nu_T} )</td>
<td>This takes into account anisotropy in a reservoir. The authors suggest that for linear systems the effective aspect ratio ( R_a = (L/H) \sqrt{k_v/k_h} ) should be multiplied by ( R_d = \frac{\nu_T}{\Delta \rho g} \frac{\nu_T}{\mu_w} ). This then reduces to ( N_g = (R_3 R_d)^{-1} ), which is similar to the earlier numbers except for the dependence on aspect ratio.</td>
</tr>
<tr>
<td>( \rho_w g \frac{k_h}{\nu_T \mu_w} )</td>
<td>Similar to the number of Craig et al. (1957) but they identified the aspect ratio as a control when the displacement is gravity dominated. The greater the aspect ratio the smaller the impact of gravity. This was combined with the first scaling method for relative permeability and capillary pressure curves.</td>
</tr>
<tr>
<td>( \rho_w g \frac{k_h}{\nu_T \mu_w} )</td>
<td>Derived for a water/pil/gas systems where the gas and oil are miscible. ( \nu_T ) is the segregation velocity.</td>
</tr>
<tr>
<td>( \rho_w g \frac{k_h}{\nu_T \mu_w} )</td>
<td>The Lake gravity number. This is for horizontal models (as were the earlier numbers) but its use of ( k_v ) suggests it is a ratio of characteristic times rather than forces. A ratio of the time taken for fluid to travel across the reservoir versus the time taken for fluid to travel from the top to the bottom of the reservoir. This was introduced to scale surfactant-CO₂ injection.</td>
</tr>
<tr>
<td>( \rho_w g \frac{k_h}{\nu_T \mu_w} )</td>
<td>Use for miscible solvent floods.</td>
</tr>
<tr>
<td>( \frac{L}{\nu_T} = 2 \left( \frac{\nu_T}{\Delta \rho g} \frac{L}{H} \right) )</td>
<td>Lake (1989) Derived by considering stability criteria for tilted reservoirs when viscous fingering may be stabilised by gravitational effects for miscible fluids. ( \alpha ) is the angle of tilt from the vertical.</td>
</tr>
<tr>
<td>( \rho_w g \cos \alpha \frac{k_h}{\nu_T \mu_w} )</td>
<td>This was rigorously derived for two-phase immiscible displacements in tilted reservoirs.</td>
</tr>
<tr>
<td>( \rho_w g \frac{k_h}{\nu_T \mu_w} )</td>
<td>This was based on the first discussion of the use of time-constant ratios rather than the traditional force-ratios used in most indices.</td>
</tr>
<tr>
<td>( G = \frac{g(\rho_w - \rho_o)}{\nu_T} \frac{k_h}{\mu_w} )</td>
<td>Derived using the vorticity equation and used for 3D Q5-spot models</td>
</tr>
</tbody>
</table>

\^ The original group in the study is a viscous-gravity ratio but for consistency and to allow an easier comparison we have written it as a gravity-viscous ratio.

The viscous-gravity ratio of Wellington and Vinegar (1985) is different from the earlier numbers in that they first introduced a ratio of characteristic times rather than a ratio of pressure gradients. Peters et al. (1998) investigated the behaviour of
N_{g}, which was a ratio of vertical to horizontal forces, and N_{g}^*, which was a ratio of the time taken for fluid to move horizontally vs. the time taken for fluid to move vertically under buoyancy alone. They found that when \( M > 1 \) \( N_{g}^* \) is the number that best describes recovery, whilst when \( M < 1 \) the traditional ratio of forces was more suitable.

Consider the two forms,

\[
N_{g} = \Delta \rho g \frac{k_{h}}{v_{T} \mu_{d}} \frac{H}{L},
\]

\[
N_{g}^* = \Delta \rho g \frac{k_{v}}{v_{T} \mu_{d}} \frac{L}{H} = \frac{t_{h}}{t_{v}} \left\{ \begin{array}{l} t_{h} = \frac{L}{v_{T}}; \\ t_{v} = \frac{\mu_{d} H}{\Delta \rho g k_{v}}. \end{array} \right. \]

\[
(2.4)
\]

\[
(2.5)
\]

\( N_{g} \) will, on account of \( H/L \), be small for reservoirs that are long and thin (\( L \gg H \)); this suggests that the displacements are viscous-dominated and that gravity effects are small. In contrast, for the same reservoirs, \( N_{g}^* \) will be large suggesting that gravity has more time to act and that the flow will be gravity-dominated. This presented a problem to Peters et al. (1998); was it actually true that recovery was better in a long thin reservoir compared to a short thick one? Experiments with a horizontal two-layer heterogeneous model showed that this depends on the mobility ratio. For the same injection rate, when \( M < 1 \), a short thick reservoir gives the best recovery, and when \( M > 1 \), a long thin reservoir gives better recovery as it allows time for gravity-segregation to occur and so disrupt the effect of viscous fingering on flow.

In more recent work, Riaz and Meiburg (2003) studied the same problem from the point of view of the creation and destruction of fluid vorticity, an approach used commonly in other applications of fluid-dynamics. By taking the curl of Darcy’s law in its dimensionless form, they derived an expression of vorticity that identified the different processes that create vorticity,

\[
\Omega = R \nabla c \times \mathbf{v} + \frac{C}{\mu} \nabla z \times \nabla c.
\]

\[
(2.6)
\]

where \( R = \ln(\mu_{d}/\mu_{o}) \) is the mobility number and \( G = g(\rho_{d} - \rho_{o}) k_{h} / |v| \mu_{d} \) is the gravity number. \( c \) is the concentration of the displacing phase in the oil phase, and \( z \) is the vertical direction. Using the vorticity equation they showed that the difference between 2D and 3D quarter-spot models, as opposed to rectilinear displacements, is that even in the absence of density differences, the interaction between the vertical and horizontal modes leads to a much lower recovery in the 3D cases. With density differences 3D models non-linear displacements need to be modelled in detail, gravity was associated more strongly with horizontal vorticity, whilst viscous effects were related to both horizontal and vertical vorticity components. Further work with randomly distributed permeability models was undertaken by Riaz and Meiburg (2004).
CAPILLARY-VISCOS RATIO S  Interfacial tension between two immiscible fluids is another process that may alter an immiscible displacement. It can have a significant effect on the nature of the displacement at low fluid velocities. When oil imbibes from an area of high permeability to an area of low permeability it can, depending on the direction of this imbibition, smoothen the front and reduce the negative effect of viscous fingering (and channelling). The capillary effect is often stronger in one direction than the other. If capillary effects are stronger parallel to flow then they tend to reduce oil recovery, whilst if perpendicular to flow then they can sometimes improve recovery by reducing the number of fingers, disrupting gravity segregation and reducing the effect of channelling through high permeability layers.

When viscous forces dominate water tends to channel into the higher permeability layers whilst bypassing the lower permeability lenses. When capillary forces dominate (at low injection rates) water will first channel through the high permeability zones at which point it will come into contact with oil-filled low permeability regions. The pressure differential at the boundaries, caused by capillary-pressure differences will mean it will imibe into the low permeability layers, thus forcing oil into the high permeability layers. The effect of this is that water will move fastest through the low permeability layers leading to early breakthrough (Coll et al. 2001; Stephen et al. 2001).

In Table 2.3 we list the Darcy-scale capillary numbers in use. Shook et al. (1992) showed that capillary effects are negligible when $N_{pc} < 0.1$ and an importance influence on flow when $N_{pc} > 5$. In the three real oil fields studied by Shook et al. (1992) $N_{pc}$ was of the order of 0.1.

It is of note that the viscous-capillary ratios discussed are all proportional to $\sqrt{k}$. Pore-scale viscous-capillary numbers, which depend on pore throat size, are proportional to $k$.

CAPILLARY-GRAVITY RATIOS  Capillary pressure effects, if they act transverse to average fluid flow (or in the direction of $g$) can induce cross-flow. This is rarely of significance on the field-scale so numerical simulations of oil reservoirs do not consider this effect; but in one application it may be significant.

<table>
<thead>
<tr>
<th>Table 2.3: Capillary-Viscous Ratios in Reservoir Engineering</th>
</tr>
</thead>
<tbody>
<tr>
<td>Definition</td>
</tr>
<tr>
<td>$N_c = \frac{1}{\mu_w \nu_T} \frac{\sqrt{k_h \phi}}{L} \cos \theta$</td>
</tr>
<tr>
<td>$N_c = \frac{k_h k_r w}{\nu_T \mu_w} \frac{\phi}{k_h} \frac{1}{L}$</td>
</tr>
<tr>
<td>$N_{pc} = \frac{\lambda_o}{\nu_T} \frac{\sqrt{k_h \phi}}{L} \sigma$</td>
</tr>
</tbody>
</table>
Table 2.4: Capillary-Gravity ratios in Reservoir Engineering

<table>
<thead>
<tr>
<th>Definition</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{DB} = \frac{\Delta \rho}{\sigma} k$</td>
<td>Pore-scale gravity-capillary ratio.</td>
</tr>
<tr>
<td>$N_{CG} = \frac{P_c}{\Delta \rho g L}$</td>
<td>Macroscopic scale capillary-gravity number</td>
</tr>
</tbody>
</table>

Gas-floods and gravity-drainage through cores are carried out in the laboratory to determine gas-oil relative permeability curves. On these small scales (with cores approximately two to six inches long) capillary effects need to be investigated, and if necessary scaled correctly otherwise it may have a disproportionate effect on recovery (Singh et al. 2001).

**Dispersions**

On the microscopic scale two fluids that are miscible will mix with each other from small velocity variations within pore spaces—dispersion (or convective dispersion to be precise)—and from molecular (Fickian) diffusion. The ratio of diffusion to advection in fluid transport is usually characterised in terms of the Peclet number. In the case, for instance, of aerodynamics the small velocity variations may be due to turbulence. In homogeneous porous media it is due to the microscopic heterogeneities on the pore-scale.

For fluids that are immiscible the dispersive process is due to the interfacial tension (capillary pressure effects) discussed earlier. In the rest of this section we will deal with miscible fluids.

On the reservoir scale these microscopic phenomena are traditionally described by a single macroscopic dispersion coefficient that includes both the effects of molecular diffusion and physical dispersion (convective component). However most early scaling studies discussed in Table 2.2 for miscible displacements were at low flow rates when dispersion is dominated by diffusion, so most of the early scaling numbers have a single ‘diffusion’ coefficient.

Blackwell (1962), Pozzi and Blackwell (1963), and Perkins and Johnston (1963) designed laboratory experiments and investigated the effect and nature of dispersion on recovery. When miscible fluids are in contact, the sharp interface between the fluids, as discussed, becomes diffuse. If the fluids are moving at the same time, as in the case of miscible-gas or solvent injection, there will be additional mixing called dispersion. It is convenient to separate this dispersion into longitudinal—in the direction of mean flow—and transverse—perpendicular to mean flow—components. In most cases these two components are not the same.

Transverse dispersion will reduce the tendency for viscous/gravity instabilities to develop into viscous/gravity fingering, and in the case of gravity segregation will hinder the development of a gravity over- or under-ride. Longitudinal dispersion will accentuate the formation of these fingers and lead to premature breakthrough and low recovery.
Longitudinal and transverse dispersion may be characterised as (Perkins and Johnston 1963):

\[ K_l = \frac{D_0}{F\phi} + \alpha_l v \]  
\[ K_t = \frac{D_0}{F\phi} + \alpha_t v \]  

where \( D_0 \) is the molecular diffusion coefficient, \( \phi \) is the porosity, \( F \) is an electrical resistivity factor dependent on the characteristics of the pore spaces, and \( \alpha_l \) and \( \alpha_t \) are the longitudinal and transverse dispersivity factors.

Dispersion, like capillary pressure effects, is clearly affected by the mean flow velocity. At low flow rates longitudinal diffusion is more important than the velocity-dependent convective dispersion. At higher flow rates it is convective dispersion that dominates. This is particularly important to consider in laboratory scaling as it is necessary to guarantee that dispersion has the same relative importance in both the model and in the real-case.

At reservoir flow-rates it is transverse dispersion that is likely to be significant, especially in horizontal reservoirs (Pozzi and Blackwell 1963). Their interactions may be studied by considering the transverse dispersion number introduced by Lake and Hirasaki (1981).

The concept of a velocity dependent ‘dispersion’ phenomenon came from Taylor (1953) who showed, in a phenomenon now known as Taylor’s diffusion that transverse molecular diffusion and longitudinal fluid flow in a pore gives rise to a longitudinal diffusion phenomenon under certain conditions. In a fully miscible displacement in a layered heterogeneous model, Lake and Hirasaki (1981) showed that high enough levels of transverse dispersion meant the displacement would behave as a single layered medium with increased longitudinal dispersion. This effect was captured in \( N_{TD} \), a transverse dispersion number:

\[ N_{TD} = 14 \frac{L}{H} \frac{K_{t2}}{H v_1} \]  

where \( L \) and \( H \) are the longitudinal and transverse model lengths, \( K_{t2} \) is defined as in Eq. (2.8) for layer 2, \( v \) is the interstitial velocity in layer 1, and the proportionality constant 14 was calculated by Lake and Hirasaki (1981) to be necessary to achieve the correct match to numerical simulation.

In the absence of any viscosity or density differences the displacement was dominated by permeability heterogeneity when \( N_{TD} < 0.2 \), and dominated by dispersion when \( N_{TD} > 5 \) as far as production profiles were concerned. Put another way permeability heterogeneity was most important when the transverse dispersion number was small, as expected. A neat summary of the effect of dispersion on fluid distribution and recovery for such a simple system is in Fig. 2.3 from Lake and Hirasaki (1981). Detailed numerical simulations by Tungdumrongsub and Muggeridge (2010).
Figure 2.3: Schematic illustration of Taylor’s dispersion in two-layer porous media [from Lake and Hirasaki (1981)] showing the effect of a change of $N_{TD}$ on the effluent histories and their manifestation in the distribution of fluids.
have shown that for a two-layer system and FCM displacement an increase in permeability contrast has a diminishing effect on recovery as the transverse dispersion number increases (i.e. transverse dispersion reduces the effect of channelling and of viscous fingering).

They found that at early times \([1 \text{ PV}]\) and in the absence of effects other than diffusion a permeability contrast of \(< 10\) between the layers made the effect of viscosity/mobility ratio on recovery negligible. At later times dispersion was a significant control on recovery, and that at large values of \(N_{TD}\) the differences in viscosity, permeability or the value of the viscous-gravity ratio did not have a significant influence on recovery.

This work may be extended to homogeneous multi-layered systems following Lake and Hirasaki (1981), but for more complex heterogeneities more research is needed.

2.1.2 Summary

We have discussed many of the Darcy-scale processes that affect recovery in a homogeneous reservoir, however,

A. reservoirs are heterogeneous in permeability and porosity,

B. permeability in most reservoirs is anisotropic,

C. the dimensionless numbers discussed generally depend on permeability,

D. permeability heterogeneity is a significant control on recovery,

E. and the current work on dispersion and \(N_{TD}\) has only considered a few simple multi-layer heterogeneous models.

Therefore, how do we account for the fact that \(k\) may vary in a reservoir, when we calculate these dimensionless numbers? How do we, independently, characterise the effect of heterogeneity on recovery?

This requires a single dimensionless number for permeability heterogeneity and/or a number for permeability anisotropy. This would be useful for:

A. the scaling of heterogeneous models from the laboratory to the field (e.g. core-flood experiments and the calculation of relative permeability/capillary pressure curves);

B. the design of an appropriate recovery mechanism to identify which processes need to be mitigated for to improve recovery;

C. reservoir modelling, especially because reservoir heterogeneity is uncertain so a method to assess the likely impact of heterogeneity on recovery without detailed numerical simulation is necessary to construct a robust depletion plan and identify any risks due to heterogeneity;
D. geological modelling of the reservoir to determine how detailed a model has to be to capture the key flow-processes that are affecting recovery.

2.2 RESERVOIR PROCESSES—HETEROGENEOUS POROUS MEDIA

There are two approaches to this problem, and the choice of approach depends on the application.

One way of including reservoir heterogeneity is to calculate the traditional dimensionless flow-regime numbers (e.g. the viscous-gravity ratio) in locally-homogeneous areas of the reservoir, subject to a careful consideration of the boundary conditions of such a region \( [\text{Coll et al. 2001}; \text{Stephen et al. 2001}] \). In most cases such a region may be identified as a single reservoir grid-block.

Heterogeneous reservoir properties will result in different regions of a reservoir being dominated by different physical forces. The concept of local dimensionless numbers was developed independently by \( [\text{Coll et al. 2001}; \text{Stephen et al. 2001}] \) but for a purpose different to that of previous investigators. Viscous-gravity and capillary-viscous ratios for each fine grid-block were calculated from a flow simulation leading to a force-regime map of the reservoir. The criteria to determine which force was dominant in a particular region was developed by \( [\text{Coll 1998}] \) by analysing the contribution to the fractional flow due to each particular term. This allowed the rapid identification of the dominant flow regime in different parts of the reservoir and showed, for the first time, the effect of heterogeneity-flow interactions. This method may be useful for designing an EOR process or to generate coarse-grids and upscale geological models to simulation models.

\( [\text{Stephen et al. 2001}] \) extended the work of \( [\text{Coll 1998}] \) by developing a method to rigorously determine the link between dimensionless numbers and flow regime from the equations describing two-phase flow. The relative magnitudes of viscous, gravity and capillary processes were determined for each grid cell and plotted on a ternary diagram. The position of each cell on the ternary program was used as an indicator of the prevalent force. This method removed the need to use empirically derived criteria applied to dimensionless numbers to determine flow regime.

Another approach is to calculate a global heterogeneity number and determine, much like for flow-regime numbers, when heterogeneity is worth ‘worrying about’. There are various such numbers in the literature.

2.2.1 Heterogeneity Indices

The heterogeneity indices in the literature may be divided into two groups, static indices and dynamic indices.
These tend to consider the statistics of the permeability and porosity fields in the absence of flow. A heterogeneity index can simply be the mean, variance or autocorrelation of the permeability fields correlated with some measure of reservoir performance.

Since variation in permeability is important, the simplest of these measures is simply the coefficient of variation of the permeability field, which is the ratio of the variance of the data to the expectation. For log-normal distributions of permeability this coefficient is independent of the mean and may be used to compare the levels of heterogeneity between different reservoirs and, could potentially, be used to scale reservoir-scale heterogeneities with lab-scale core analysis.

Unfortunately the interaction between geological heterogeneity and fluids depends on the length scales and correlation lengths of the permeability. For instance, varying the correlation length and the coefficient of variation of the permeability field affected the development of viscous fingering. Moissis and Wheeler found that the number of viscous fingers were a decreasing function of the coefficient of variation, the correlation length, and the viscosity ratio. In fact they concluded that for unstable two-phase displacements the variation of permeability was more important than its spatial distribution (and hence, presumably, correlation).

\( C_v \) itself is a derivative of the heterogeneity index of Dykstra and Parsons, which they developed to study water-flood performance in layered reservoirs. Their model did not allow cross-flow between layers and so assumed piston-like displacement within layers. In its original formulation, \( V_{dp} \) was defined as

\[
V_{dp} = \frac{k_{50} - k_{16}}{k_{50}},
\]

where \( k_{50} \) is the median permeability and \( k_{16} \) is the 16th percentile permeability. \( V_{dp} \) is 0 for a homogeneous reservoir and 1 for a very heterogeneous reservoir (i.e. \( k_{50} >> k_{16} \)).

To calculate \( V_{dp} \) the permeability for each layer is ranked in order of increasing value, probabilities are assigned to each data point and a log-probability plot is constructed. A line-of-best-fit on this plot generates the necessary percentile probabilities. The line-of-best-fit may be interpreted as an equivalent reservoir with a log-normal permeability distribution. Crucially this index does not require the reservoir permeabilities themselves to be log-normally distributed.

This graphical method assigns a greater weight to median values of \( k \) and so the higher permeability layers dominate the predicted behaviour. For viscous-unstable displacements this is suitable but for viscous-stable models the lower permeability layers are just as important as the more permeable layers.

The sensitivity of \( V_{dp} \) to oil recovery, and its reliance on sample size, was investigated by Jensen and Lake who cited modelling studies showing reservoir performance is insensitive to the value of \( V_{dp} \) when \( V_{dp} < 0.5 \) but extremely sensitive.
for the more heterogeneous models when \( V_{dp} > 0.7 \) (Lake and Jensen 1991). For instance, for an increase in \( V_{dp} \) from 0.2 to 0.3, the fractional oil recovery decreases by 5\%, whilst an increase in \( V_{dp} \) from 0.8 to 0.9 decreases recovery by 33\%. They found that a range of permeability distributions with quite different performance characteristics had the same value of \( V_{dp} \). The assumption of no cross-flow also presents difficulties when studying reservoirs with a large \( k_v/k_h \) ratio (Warren and Cosgrove 1964).

A recent development has been by Maschio and Schiozer (2003) who used the Dykstra-Parsons coefficient as a measure of local heterogeneity rather than a field-scale measure, and combined it with the minimum and maximum permeability limits within a coarse grid block to calculate effective permeability. They report marginally better upscaling performance compared to the traditional single-phase pressure solver method.

Zhou et al. (1997) identified the fact that to characterise the effect of heterogeneity on recovery a correlation length is needed in addition to \( V_{dp} \), which by its nature has no spatial information.

A more efficient and accurate estimator of \( V_{dp} \) for log-normally distributed—but non-layered—permeability fields, was suggested by Jensen and Currie (1990) as,

\[
V_{dp} = 1 - e^{-\sigma_k},
\]

where \( \sigma_k \) is given by,

\[
\sigma_k = \left[ 1 + \frac{1}{4(n-1)} \right] \times \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (\ln k_i - \ln \bar{k})},
\]

where \( n \) is the number of samples. For large \( n \), \( \sigma_k \) is approximately the standard deviation of \( \ln k \).

The Lorenz coefficient \( (L_c) \) of Schmalz and Rahme (1950) tries to take into consideration the spatial arrangement of a layered-reservoir by using the flow-capacity of a layer within the reservoir and its thickness to estimate recovery.

To calculate \( L_c \) the layers are ranked in order of decreasing permeability. The cumulative flow capacity \( F_m \) and cumulative thickness \( H_m \) for the \( m \)th layer is calculated and normalised. \( F_m \) is plotted against \( H_m \) to construct the Lorenz-curve, which for a homogeneous system is a straight line. As heterogeneity increases (i.e. as variation increases) the plot will deviate from a straight line. \( L_c \) is defined from the curve as,

\[
L_c = \frac{\text{Area above the curve}}{\text{Area below the curve}}.
\]

The Lorenz coefficient is able to take into consideration variable-thickness layers and variable rock porosity (Jensen 2000) and so appears to be more discriminating
than $V_{dp}$, however Warren and Cosgrove (1964), in Fig. 2.4 showed that for a log-normal distribution of permeability $L_c$ and $V_{dp}$ are closely related.

All the measures discussed so far, $C_v$, $V_{dp}$, or $L_c$, are suitable either for completely uncorrelated or completely layered-systems. They are unable to account for systems that have more complex spatial distributions of permeability heterogeneity within the reservoir.

There are a small number of measures (Lake and Jensen 1991) that do attempt to consider the distribution of permeabilities within a reservoir but these tend to be specific to depositional systems and have not been explicitly related to reservoir performance. Polasek and Hutchinson (1967) introduced an index which involved quantifying the amount and location of shales within a sandstone body. Alpay (1972) used a gamma-ray log to develop a sand index that would quantify the permeability of the sand, and subsequently used the variability in the sand index to provide some insight into reservoir performance.

Pirson (1958) is mentioned by Lake and Jensen (1991) as having discussed three more measures, a coefficient that measures the level of stratification in a reservoir, the degree of lensing and a coefficient of thinning. Little information is available on any of these measures and they have not been explicitly related to reservoir performance.
2.2 RESERVOIR PROCESSES—HETEROGENEOUS POROUS MEDIA

Most recently [Henson et al. (2002)] suggested two heterogeneity indices, a heterogeneity index to capture the effect of lateral- and another to capture the effect of vertical-heterogeneities. These were calculated by taking the ratio of the mean dimensions of genetic units with the relevant length scale (inter-well for lateral indices and reservoir thickness for vertical indices). The choice of the genetic unit to include was to be based on geological intuition. The correlations calculated with recovery factor were useful but the systems studied had binary distributions of permeability and it is unclear how to apply this approach for more complex sedimentary systems.

For the scaling of laboratory experiments, both the more-complex measures and the simple statistical measures of permeability distribution may not be as useful as, perhaps, expected. If they are to be calculated using core-samples and then related to reservoir performance then they must depend on the sample size used. **Fig. 2.5** from [Jensen and Lake (1988)], shows the error in the fractional oil recovered for a unit-mobility displacement in a reservoir model with $V_{dp} = 0.8$ and $L_c = 0.745$ calculated ‘on the basis of the standard error range of each measure’ and for a range of data set sizes. The asymmetry is noteworthy. When the coefficients overestimate fractional oil recovery, all have similar errors, but when they underestimate recovery, the Dykstra-Parsons coefficient is the least accurate.

![Figure 2.5: Performance of Heterogeneity measures](image)

**Figure 2.5:** The effect on predicted oil recovery of sample size for $V_{dp}$ and $L_c$ for layered systems (adapted from Fig. 14 of [Jensen and Lake (1988)])

**Dynamic indices** These indices use numerical flow-simulation to evaluate the interaction of flow with heterogeneity and so may be able to take into consideration well patterns, fluid properties, and production mechanisms. The [Koval (1963)] Het-
heterogeneity Factor and the field scale dispersivity proposed by [Arya et al. (1988)] are examples of dynamic indices.

Koval’s Heterogeneity Factor, $H_K$, is the most widely used of these measures. It attempts to capture the effect of heterogeneity on viscous fingering during miscible flooding, but it requires detailed flow-simulation and so is unsuitable for rapidly ranking different reservoir realizations. If we define $K$ as,

$$K = H_K E,$$  \hspace{1cm} (2.14)

where $H_K$ is the Koval heterogeneity factor, and $E$ is an effective viscosity ratio, calculated as

$$E = \left( 0.78 + 0.22 \left( \frac{\mu_o}{\mu_d} \right)^{\frac{1}{4}} \right)^4 .$$  \hspace{1cm} (2.15)

This suggests that the composition of the effective displacing fluid may be taken as 78% oil and 22% the injected solvent. This was calculated based on the experiments of [Blackwell (1960)]. The form of $E$ shown here is based on the traditional quarter-power mixing rules used in refinery calculations ([Koval (1963)].

Whilst $E$ is easily calculated, $H_K$, the heterogeneity factor, requires either the simulation of a unit-mobility-ratio displacement, or the use of an empirically fitted correlation between $V_{dp}$ and $H_K$. By analogy with the fractional flow equation of [Buckley and Leverett (1942)] an equivalent function for miscible floods as a function of $K$ is

$$f_d = \frac{1}{1 + \frac{1}{K} \frac{1 - S_d}{S_a}} .$$  \hspace{1cm} (2.16)

Using this and the frontal advance formula from Buckley-Leverett theory,

$$\frac{\partial x}{\partial t} \bigg|_{S_a} = \frac{q}{\phi A} \frac{\partial f_d}{\partial S_d} ,$$  \hspace{1cm} (2.17)

we can calculate the time to breakthrough, in dimensionless units as,

$$t_{D|bt} = \frac{1}{K} .$$  \hspace{1cm} (2.18)

If we simulate a unit-mobility displacement through the reservoir ($E = 1$), or inject a matched-viscosity matched-density miscible fluid into a core, then (2.18) may be used to calculate $K$, and so $H_K$.

The method of calculation implies that all rock properties, not just macroscopic channelling, but longitudinal dispersion is accounted for in $H_K$. A number of sandstone cores were tested by Koval at viscosity ratios from 1 to 30, $H_K$ was found to be constant across experiments at all these ratios which confirmed that $H_K$ was only dependent on the rock properties, suggesting that it was dependent only on the rock.
For a uniform layered model with a log-normal distribution $H_K$ is related to $V_{dp}$ empirically by (Paul et al. 1982)

$$\log H_K = \frac{V_{dp}}{(1 - V_{dp})^{1/5}}.$$  \hspace{1cm} (2.19)

![Figure 2.6: The Koval heterogeneity factor $H_K$ and the Dykstra-Parsons coefficient for a layered heterogeneous medium (Adapted from Paul et al. (1982)).](image)

This relationship is shown graphically in Fig. 2.6 and suggests that in his experiments physical dispersion was relatively low compared to the effects of channelling. It must be noted that for non-layered reservoirs, those with high levels of dispersion, or those without log-normal permeability distributions it would be more accurate to calculate $H_K$ directly using numerical simulation rather than this empirical fit.

The dynamic Lorenz coefficient is a dynamic measure, introduced by Shook and Mitchell (2009), by extending the traditional Lorenz coefficient to include flow information by exploiting the time-of-flight of streamlines and their volumetric flow rates. This only requires a single phase pressure solve to calculate and thus overcomes the computational issues associated with Koval’s factor.

Streamlines were generated from source to sink and their time-of-flight and volumetric flow rates recorded. This information was used to generate flow-capacity diagrams in a similar fashion to the static Lorenz coefficient of Schmalz and Rahme (1950) discussed earlier.
Other streamline based dynamic measures [e.g. Idrobo et al. (2000); Ates et al. (2003)] can include the coefficient of variation of the time-of-flight of streamlines, and their variation, which may be interpreted as an indicator of breakthrough time. Drawing on the work of Shook (2005), Shook and Mitchell (2009) obtained the sweep efficiency at breakthrough and at 1 PVI for a large number of synthetic earth models and concluded that the dynamic Lorenz coefficient was the most robust measure of dynamic heterogeneity.

Whilst these streamline methods are quick and efficient, they are derived from single-phase flow simulations, and cannot deal with compressible fluids, viscosity and density differences, and dispersive effects which will lead to time-varying streamlines [e.g. Thiele et al. (1996)]. So it is not clear that rankings derived from these measures will still apply in adverse viscosity ratio displacements, such as miscible gas flooding.

Percolation theory (Andrade et al. 2000; King et al. 2002) provides another approach to estimating the time to breakthrough and the connectivity of sand bodies in a reservoir by considering the probability that a pair of wells separated by some distance are connected. Whilst the method is easy to apply it has not been tested with a large number of different geological realizations, and can present difficulties for models where permeability varies gradually.

### 2.2.2 Heterogeneity Indices & Flow-Regime Dimensionless Numbers

None of the measures discussed so far have been used for scaling experiments and/or combined with the earlier flow-regime dimensionless numbers.

A general method to scale fluid-flow through heterogeneous media for immiscible fluid displacements was presented by Li and Lake (1995). Heterogeneity was characterized using ideas from geostatistics and image representation and four heterogeneity scaling groups were identified; a global heterogeneity number—in this case a derivative of the Dykstra-Parsons coefficient—which as seen in the previous section takes no account of correlation lengths in the reservoir; an effective correlation length to measure the statistical size of correlated regions; another number to represent the strength of correlation with these correlated regions; and a local heterogeneity number.

The scaling groups were tested by calculating breakthrough oil recovery for reservoirs with different values of the heterogeneity scaling groups. Their findings are in keeping with what we would expect. In the absence of density differences and capillary forces oil recovery is independent of realisation when global heterogeneity is small. With small correlation lengths but large spread in permeability (large $V_{dp}$) recovery depends on the realisation, and when both the variation in permeability is large and the permeability has significant correlation on the inter-well scale then heterogeneity is a significant control on recovery. Whilst this would suggest that the
scaling groups are not sufficient in most cases they found that matching correlation lengths gave similar saturation profiles and averaging across realisations gave statistically meaningful results.

The effect on the production profile curves was found to be time-dependent. Recovery was found to be generally independent of heterogeneity for the line-drive models before water breakthrough, whilst post breakthrough heterogeneity had more influence. Local and global heterogeneities were found to cause fingering and dispersive flows (associated with microscopic heterogeneity) whilst large correlation lengths were associated with channelling. Breakthrough oil recovery was found to increase as local heterogeneity increased, presumably related to the dispersive effects of small-scale heterogeneities, and found to decrease as correlation lengths increased. Breakthrough oil recovery was also found to depend on the aspect ratio of the reservoir.

Sorbie et al. (1990) used an alternative approach, that of numerical simulation. He suggested exercising caution when applying traditional scaling ideas to heterogeneity in that we cannot build small scale models that can represent reservoir level heterogeneity and the numerical value of many of these dimensionless numbers (e.g. the viscous to gravity ratio) do not tell us the exact contribution of gravity to final recovery, unless they are calibrated with numerical simulation. Simple layered-permeability models were tested but, for the first time, the permeability within each layer was varied to try and understand the role of sub-layer heterogeneities. It was found that the most sensitive measure of these sub-layer heterogeneities was the cumulative recovery from each layer (or for more complex systems the cumulative recovery from each identifiable channel).

Much of the work on combining flow-regime dimensionless numbers and heterogeneity indices to understand reservoir processes was brought together by Zhou et al. (1997), who took experimental results and data from numerical simulations for miscible and immiscible floods to identify the transition from one flow-regime to another. This work led to a flow-regime diagram for layered-heterogeneous models shown in Fig. 2.7 that can allow one to determine which flow-regime is dominant in a particular reservoir to inform the design an \textit{EOR} process. We envisage a similar diagram for complex heterogeneous porous media, provided a suitable dimensionless measure of heterogeneity can be found.

**Summary** The current status is that there is no single heterogeneity number that can be used with other flow-regime dimensionless numbers to capture, in a similar way as, for instance, the viscous-gravity ratio does for those two processes, the effect of heterogeneity on recovery. Some success has been possible with the methods of Li and Lake (1995) and Sorbie et al. (1990) but both methods rely either on multiple heterogeneity numbers (involving statistical properties of the static permeability field) or full-scale simulation to quantify the effect of heterogeneity on recovery. This makes them unsuitable for ranking reservoir realisations to understand uncertainty, and to design \textit{EOR} processes.
Therefore, despite this intricate link between flow-regime and heterogeneity, there is still no unified mathematical framework to determine under which flow-conditions reservoir heterogeneity becomes more significant than these other factors.

All of the heterogeneity measures discussed are essentially heuristic. The authors propose a plausible measure and then compare it with some measure of performance in a range of geological models for one or more secondary recovery or EOR processes (e.g. the time to solvent or water breakthrough, recovery at 1 PVI or the recovery factor).

These existing measures are unsatisfactory for a number of reasons:

A. Recovery and breakthrough time depend very strongly on the boundary conditions (well arrangement etc.) and geological heterogeneity. Static measures are unable to take this into account.

B. Some require detailed multiphase flow-simulation (e.g. Koval’s $K_H$ number). This type of simulation is precisely the type of simulation that is most difficult for large fine-scale heterogeneous models.

C. Many measures, in particular those derived from the Dykstra-Parsons coefficient, make simplifying assumptions about the nature of the heterogeneity which makes them inappropriate for use in adverse viscosity displacements or in sediments with more-complex facies distributions.
There is no clear method of combining these numbers with flow-regime dimensionless numbers to evaluate reservoir model behaviour.

To accurately and objectively rank reservoirs in terms of sweep and recovery, a general measure of heterogeneity must be able to quantify the difference in reservoir performance in the presence of heterogeneity with that observed in a homogeneous reservoir for the chosen well pattern, flow rates, density contrast, and viscosity ratio. If it is to be used for this purpose it must fulfil these criteria:

1. Correlate strongly with the time to breakthrough or recovery for realizations ranging from extremely heterogeneous to completely homogeneous.

2. The ranking of realizations must be preserved for adverse viscosity ratio miscible/immiscible floods.

3. Must not require any detailed multiphase flow simulation.

4. Must be useful to scale experiments from the laboratory to the field.
VORTICITY DYNAMICS IN POROUS MEDIA

In this chapter we introduce the vorticity formulation of the equations of motion as a basis for understanding reservoir dynamics.

We use this formulation to introduce five dimensionless numbers of which four represent the effects of viscous, gravity, capillary, and dispersive processes on flow and one represents the effect of heterogeneity on flow.

3.1 VORTICITY IN FLUID DYNAMICS

The concept of vorticity, $\Omega = \nabla \times \mathbf{v}$, in fluid dynamics has provided a useful qualitative and quantitative tool to explain many of the phenomena to come out of the Navier-Stokes equations of flow; phenomena from, the apparently simple puffing out of a candle, the complex combination of behaviours of a sail in motion through the water, the breathtaking richness of fluid motion seen, for instance, in Fig. 2.1, to the hydrodynamics of galaxies.

The physical interpretation of this number may be seen by dropping a particle of dust into a flow-field. The angular rotation of this particle about its rigid axis is then, given by, $\omega = |\Omega|/2$. So vorticity is a measure of the local rotation in a velocity-field. As well as providing an intuitive and physical explanation, the vorticity field is often more economical to define than the velocity field by its ability to express how real solutions for fluid-flow differ from the potential-flow calculations we can make, in many cases, analytically.

However, the concept is only really useful in the case of fluids where the density is constant, i.e. $\nabla \cdot \mathbf{v} = 0$. For these cases, vorticity may be treated as a conserved quantity, in that flow-fields that have little initial vorticity tend to maintain that state, whilst flow-fields that start with large vorticity tend to finish with large values of vorticity.

In porous media, Darcy’s law is the equivalent macroscopic equation of motion. As most porous media are heterogeneous in their local properties it is difficult to see how vorticity can be treated as a conserved quantity.
3.2 VORTICITY AND DARCY’S LAW

The general three-dimensional equation of motion for fluids in porous media, neglecting inertial and gravitational forces, and capillary effects, is

\[ \mathbf{v} = -\frac{k}{\mu} \nabla P, \tag{3.1} \]

where \( \mathbf{v} \) is the flux per unit cross-sectional area, \( k \) is the permeability tensor, \( \mu \) is the dynamic viscosity of the fluid, and \( P \) is the scalar pressure field. Let us consider three types of porous media, for which \( k \) will take different forms.

**Homogeneous isotropic porous media**: In such a porous medium \( k = k \) which is a constant. If the medium is saturated with a homogeneous fluid then \( \mu \) is a constant, as well, and Eq. (3.1) may be written as,

\[ \mathbf{v} = -\nabla \left( \frac{k \mu}{\mu} P \right) = -\nabla \phi, \tag{3.2} \]

where \( \phi \) is the velocity potential which satisfies

\[ \nabla \times \nabla \phi = 0; \quad \Omega = 0. \tag{3.3} \]

The resultant velocity field is described as potential and irrotational. There is no vorticity. The solutions for the velocity potential are solutions to Laplace’s equation (Batchelor 1967). For simple boundary conditions analytical solutions may be found (Morel-Seytoux 1966, 1965), whilst for more realistic geometries, numerical solutions may be calculated with little computational effort.

**Heterogeneous isotropic porous media**: In this case the permeability is a spatially-variable scalar, \( k = k(x, y, z) \), and the fluids are homogeneous (i.e. \( \mu = \text{constant} \)). Taking the curl of Eq. (3.1) with these assumptions suggests,

\[ \nabla \times \mathbf{v} = -\frac{k}{\mu} \nabla \times \nabla P + \frac{1}{\mu} \nabla P \times \nabla k \]

\[ = \nabla \ln k \times \mathbf{v}, \tag{3.4} \]

which shows, in contrast to the homogeneous case, that the flow is rotational except when the permeability gradient is parallel to the average fluid velocity (i.e. \( \nabla \ln k \parallel \mathbf{v} \)).

**Heterogeneous (anisotropic) porous media**: For such a medium permeability is a tensor quantity, \( k = k(x, y, z) \), and Eqs. (3.2) and (3.4) do not hold.
3.2 Vorticity and Darcy’s Law

Consider, for simplicity, the case when \( \mathbf{k} \) is a rank-two diagonal tensor in 2D,

\[
\mathbf{k} = \begin{pmatrix}
k_x \\ k_y \\ k_z
\end{pmatrix},
\]

and the reservoir is two-dimensional \((v_z = 0)\), and oriented horizontally with \( \mathbf{g} = \mathbf{g} \hat{\mathbf{k}} \) and \( v_z = 0 \). Taking the curl of Eq. (3.1), the Darcy velocity, and ignoring diffusion/dispersion, gives,

\[
(\nabla \times \mathbf{v})_z = -\frac{1}{\mu} \left( \frac{\partial k_y}{\partial x} \frac{\partial P}{\partial y} - \frac{\partial k_x}{\partial y} \frac{\partial P}{\partial x} \right) + \frac{\partial^2 P}{\partial x \partial y} (k_y - k_x),
\]

\( (3.6) \)

\[
(\nabla \times \mathbf{v})_z = \left( v_y \frac{\partial \ln k_y}{\partial x} - v_x \frac{\partial \ln k_x}{\partial y} \right) + \frac{\partial^2 P}{\partial x \partial y} (k_y - k_x).
\]

\( (3.7) \)

It is difficult to simplify this expression as was done for Eq. (3.4) but the vorticity now depends on terms that are second-order derivatives of \( P \). When the velocity is perpendicular to the permeability gradient, vorticity is maximised.

What this has shown is that the presence of any permeability heterogeneity creates vorticity in the flow (Kapoor 1997), and in general vorticity is maximised in the presence of large permeability gradients that are perpendicular to high flow rates. We can relate this macroscopic fluid behaviour to recovery in oil reservoirs. During EOR fluid is injected into a reservoir to sweep out the remaining oil. Let us assume that there are no differences between the injected fluid and the oil. In the absence of any permeability heterogeneity, \( \Omega = 0 \), and oil recovery is maximised. For instance, in the case of a linear displacement, 1 pore volume of injected fluid will recover all of the oil.

In the presence of permeability heterogeneity, which takes the form of a higher permeability streak, in a region of the reservoir, \( \Omega \) will be non-zero, so there will be higher flow-rates than elsewhere creating a preferential flow-path for the injected fluid. This will result in an early breakthrough of the injected fluid, and result in regions of the reservoir either being completely unswept or swept much later in the process, so reducing oil recovery.

Thus the conditions that are likely to affect the ideal recovery of oil in reservoirs require that \( \Omega \neq 0 \) whilst ideal conditions require \( \Omega = 0 \).

The form in Eq. (3.4) is discussed by Bear (1988) and was first derived by White and Horne (1987) who used this property to motivate the use of full-tensor permeabilities to upscale permeability heterogeneity from fine-scale models to coarse-scale models. Mahani and Muggeridge (2005) and Mahani et al. (2009) used the ability of \( \Omega \) to identify regions of maximum cross-flow to automate coarse-grid generation and guide the upscaling of reservoir models for flow simulation.
3.3 VORTICITY AND THE GROWTH-RATE OF INSTABILITIES

The identification of vorticity with the angular velocity of a rigid body makes it clear that the unit of vorticity is $1/\text{time}$, i.e. $\| \Omega \| = T^{-1}$. This suggests that vorticity, in perturbation analysis, can represent a growth rate.

Using this interpretation of vorticity as a growth-rate, Heller (1966) was able to study the development of instabilities at the boundary of two miscible fluids in a homogeneous reservoir. We now follow his approach.

Consider a miscible displacement process in an isotropic heterogeneous porous medium, $k = k(x, y, z)$, where we include the effects of gravity and dispersion. The displacing fluid is a solvent of lower viscosity and density that is fully miscible with the fluid in the reservoir. The state of the system is described by the Darcy velocity $v$, and $c$, the concentration of solvent in the fluid. The Darcy velocity is given by,

$$v = -\frac{k}{\mu} (\nabla P + \rho g), \quad (3.8)$$

where $g$ is the acceleration due to gravity and $\rho$ is the density of the fluid. We assume $v$ is independent of solvent concentration and that the fluid density is taken to be independent of pressure so

$$\nabla \cdot v = 0. \quad (3.9)$$

The time-evolution and spatial distribution of $c$ is described by the empirical relationship,

$$\frac{\partial c}{\partial t} = -\frac{v}{\phi} \cdot \nabla c + \nabla \cdot D \cdot \nabla c, \quad (3.10)$$

where $\phi$ is the porosity (which may be a scalar field) and $D$ is the symmetric dispersion tensor. Eqs. (3.8), (3.9) and (3.10) together describe the system in full.

Now consider the family of moving isoconcentration surfaces. In the absence of any heterogeneities in the fluid and/or in the porous media these surfaces will move across the reservoir unchanged. In the presence of heterogeneities these surfaces will deform (see Fig. 3.1). Imagine a particular parcel of fluid on one of these surfaces. The rate of change of concentration of that parcel as it travels with velocity $\text{d}r/\text{d}t$ is given by the Lagrangian derivative of the concentration,

$$\frac{Dc}{Dt} = \frac{\partial c}{\partial t} + \nabla c \cdot \frac{\text{d}r}{\text{d}t}. \quad (3.11)$$

By defining a ‘displacement’ velocity $V_d = \text{d}r/\text{d}t$ which satisfies Eq. (3.10) such that the total derivative vanishes, $Dc/Dt = 0$, we can confine the parcel of fluid to one isoconcentration surface. Using $V_d$ we can write down the velocity of the
Imagine a flood front moving from left to right

Homogeneous Porous Media
Saturation front is uniform and perpendicular to motion

Heterogeneous Porous Media
Saturation front is perturbed by permeability differences

Figure 3.1: An illustration of the perturbations to a fluid-front as it passes through a heterogeneous medium

isoconcentration surfaces in terms of the driving forces to which they respond (Heller 1966) as,

\[
V_d = \frac{v}{\phi} - \frac{(\nabla \cdot D \cdot \nabla c) \nabla c}{|\nabla c|^2}.
\]  

(3.12)

If we reduce the reservoir down to this system of moving isoconcentration surfaces then the changes in their velocity are sufficient to account for changes to the recovery of oil and the production curves from that expected in a uniform, homogeneous reservoir. Note that in the absence of fluid and permeability heterogeneities this reduces to the form of the expression in Eq. (3.2) and we recover the irrotational nature of the velocity field.

Taking the curl of Eq. (3.12) gives the vorticity of the displacement velocity; this describes the rate of deformation, in units of 1/time, occurring at any point along a chosen isoconcentration surface,

\[
\nabla \times V_d = \frac{1}{\phi} \left[ (\ln M) v + \frac{k \Delta \rho g}{\mu} - \nabla \left( \frac{\nabla \cdot D \cdot \nabla c}{|\nabla c|^2} \right) \right] \times \nabla c \\
+ \nabla \left[ \frac{\ln k}{\phi} \right] \times v,
\]  

(3.13)

where \( M = \mu_o/\mu_d \) is the ratio of the oil viscosity \( \mu_o \) to the viscosity of the displacing fluid \( \mu_d \) and we have assumed that the mixture viscosity is given by \( \mu = \mu_o \exp^{-(\ln M)c} \). As we have assumed the two fluids are miscible, the density of the mixture is the volume-average of the densities of the constituent fluids given by \( \rho = c \rho_d + (1-c) \rho_o \) and \( \Delta \rho = \rho_o - \rho_d \).
This expression neatly summarises the different factors contributing to the initial formation of perturbations to an isoconcentration surface and their initial, linear growth rates. We note that perturbations will only grow if:

a. the Darcy velocity is not parallel to the local concentration gradient and \( M > 1 \);

b. gravity is not parallel to the local concentration gradient, in which case there is a tendency for fluids to segregate;

c. there is curvature of the local concentration gradient e.g. there is an incipient viscous finger, in which case diffusion and dispersion will tend to damp out that curvature;

d. a permeability gradient perpendicular to the velocity (as described above for a single fluid system).

We also note that for a smooth interface initially perpendicular to the Darcy velocity the initial perturbations will be driven by the permeability heterogeneity, thus demonstrating that viscous fingering has to be initiated by concentration perturbations driven by heterogeneities. Heller (1966) used the first three of the four terms in Eq. (3.13), in conjunction with perturbation analysis, to analytically determine the criteria for stability of miscible displacement processes in porous media in the absence of heterogeneities. He observed, amongst other results, that in the absence of gravity there is a maximum growth rate for a particular wave-number that depends upon the level of transverse dispersion. This has also been observed by later workers including Christie and Bond (1987). More recently Camhi et al. (2000), Ruith and Meiburg (2000) and Riaz and Meiburg (2002, 2003, 2004) also used a numerical model based on vorticity to study the growth rate of instabilities in miscible floods.

Eq. (3.13) may be used to evaluate the relative importance of the different factors influencing the growth of perturbations, at least initially, by comparing the size and sign of the different terms. This is better achieved by using a characteristic time to non-dimensionalise the rate of deformation. An appropriate characteristic time would seem to be the time taken for a displacement front to cross the system,

\[
\tau_c = \frac{L}{|v|},
\]

where \( L \) is the system length. Eq. (3.13) becomes,

\[
L \frac{\nabla \times V_d}{|v|} = H \times \frac{v}{|v|} + \frac{1}{\phi} \left[ \frac{R}{|v|} + \frac{G}{|g|} - P_e \frac{\nabla}{l \frac{\nabla^2 c}{|\nabla c|^2}} \right] \times L \nabla c
\]

where we have neglected dispersion so we can describe the initial rate of rotation in terms of the following scalar dimensionless numbers:
A. a viscous instability number related to the mobility ratio,

\[ R = \ln M \]  \hspace{1cm} (3.16)

B. a gravity number (cf. Crane et al. 1963),

\[ G = \frac{k |g|}{\mu |v|} \Delta \rho \]  \hspace{1cm} (3.17)

c. a Péclet number,

\[ Pe = \frac{D_m}{l |v|} \]  \hspace{1cm} (3.18)

where \( l \) is a characteristic length, typically taken as the average grain size in the porous medium of interest but in this case is the characteristic length of heterogeneities, and \( D_m \) is the molecular diffusion coefficient;

d. a heterogeneity number (assuming the porosity \( \phi \) is constant),

\[ |H| = L \left| \nabla \frac{\ln k}{\phi} \right| \approx \frac{L}{k \phi} \nabla k \]  \hspace{1cm} (3.19)

Compare these with those numbers discussed in §2.1.1. The first three of these flow-regime dimensionless numbers, the mobility ratio, the gravity-viscous ratio, and some form of diffusion/dispersion number\(^1\), are already in common use in the reservoir engineering community. However, in most of these studies, including that of Heller, the form of these numbers was derived based on the assumption that permeability does not vary in the reservoir.

The main difference here is that in order to determine the relative importance of the different mechanisms affecting the interface between solvent and the oil we should take into account the orientation of the Darcy velocity to the concentration gradient and the gradient of the permeability distribution. Taking this into account gives rise to the four dimensionless numbers in Eqs. (3.16), (3.17), (3.18), and (3.19), which, together, represent the effects of viscous, gravity and dispersive processes for miscible displacements in heterogeneous porous media.

\( |H| \) is new and provides a measure of the importance of heterogeneity in determining the rate of deformation of an interface. Since permeability varies spatially in a heterogeneous reservoir, i.e. \( k = k(x, y, z) \), and \( k \) appears in both \( G \) and \( |H| \), strictly speaking both these numbers also vary in a reservoir. This is at-odds with the attempt to calculate a single field-wide number, whether a gravity or a heterogeneity number, to quantify the effects of these processes on average field behaviour. This suggests that a statistical treatment of these scalar-fields is required.

\(^1\) Strictly this number is a diffusion coefficient rather than a dispersion number. Dispersion would be represented by a velocity dependent tensor.
To provide some insight into the derivation of such a global heterogeneity number we will consider some simple analytically tractable cases.

### 3.4 Analytical Evaluation of Vorticity

Consider the model described in Fig. 3.2 composed of \( n \) layers, with permeabilities \( k_{x1}...k_{xn} \), represented in regular Cartesian geometry. The individual layers are isolated from one another so there is no pressure communication between them (i.e. \( k_{y} = 0 \), except in the the upstream injection face). Injection is along the left-hand face and production along the right-hand face under constant pressure boundary conditions. We ignore gravity, capillary, diffusive and dispersive effects.

The vorticity for such a case is given by Eq. (3.7). If \( k_{y} = 0 \) and \( k_{x} = k_{x}(x) \) then,

\[
\Omega_{z} = -\frac{\partial \ln k_{x}}{\partial y} \cdot v_{x}, \\
= -\frac{\partial \ln k_{x}}{\partial y} \left[ -k_{x} \frac{\partial P}{\partial x} \right], \\
= -N \frac{\partial k_{x}}{\partial y},
\]

where \( N \) is equal to \( \mu^{-1} \frac{\partial P}{\partial x} \). Since we have assumed that there are no viscosity differences and that there is a constant-pressure boundary condition in force, \( N \) is constant in the reservoir. For this model vorticity can be evaluated analytically and the method of Stiles (1949) may be used to calculate the time to water breakthrough (or the recovery of oil at water breakthrough) assuming linear relative permeabilities and a connate water saturation of 0.

The Stiles method assumes a linear system with no cross-flow or segregation in layers and piston-like displacement in each layer. It is used to generate effective relative permeability curves that are used to predict breakthrough time for the reservoir.

Consider three cases (where \( a \) and \( b \), if present, are constants).
1. $k_x = ay$: A linear function of $y$ (reservoir height). Substituting this into Eq. (3.22) gives

$$\Omega_z \propto a,$$

which implies that vorticity in the reservoir is constant. Using the method of Stiles the time to breakthrough [or breakthrough oil recovery (BOR)] is 0.5. As $a$ changes both permeability and vorticity will vary, however [BOR] will not change.

2. $k = a/y$: An inverse function of height. The vorticity is given by,

$$\Omega_z \propto -\frac{a}{y^2}.$$

An inverse relationship implies a difference of several magnitude in permeability between the least and most permeable layers. For this case breakthrough is relatively early, after injecting 0.16 PVI. As $a$ changes the time to breakthrough is constant, whilst the vorticity depends on $1/y^2$.

3. $k = be^{ay}$: An exponential function of height. Substituting for this in Eq. (3.22) suggests

$$\Omega_z = ab e^{ay}.$$

The vorticity in this case is of the same functional form as the permeability field suggesting the statistics of both permeability and vorticity will be similar. In this form the variation in vorticity represents the variation in the permeability field and we recover the Dykstra-Parsons coefficient. [BOR] depends on the coefficient $a$ (if $b = 1$). When $a = 1$, [BOR] is 0.05, and as $a \to 0$, [BOR] $\to 1$.

The implications of these three cases are:

A. Vorticity is a signed quantity and depends on the ‘sense’ of ‘rotation’ in the field. For instance it is quite clear, because of the sign of $\Omega_z$ that in the case when $k = a/y$ the velocities at the bottom will be larger than the velocities at the top, so a particle placed in the flow-field will rotate in an anticlockwise direction. In the case when $k = be^{a}$ the direction of change of velocity will change so rotation will be in the opposite direction.

B. An exponential change in $k$ is more heterogeneous than other cases so this will give an earlier than normal breakthrough.

C. Where the permeability field is discrete (there are a finite number of distinct layers, as we have assumed in the Stiles calculation) there will be a step change in permeability at the boundary. At these boundaries, the derivative of the permeability is undefined. This may be a problem in that vorticity at the boundary will also be undefined, however in a real reservoir it is unlikely that $k_y = 0$ and
permeability gradients tend to be finite. In fine-scale models the typical vertical resolution is meters, and on this scale permeability gradients are likely to be finite.

So far we have considered a simple layered model of a reservoir for which it is possible to analytically calculate vorticity fields and production profiles. For more complex heterogeneities it will be necessary to use numerical methods to calculate the velocity and vorticity fields.

3.5 shear and vorticity—an eulerian approach

In the analysis of vorticity so far, we have considered the dynamical history of a material element of fluid on a surface of isoconcentration. We will now take an approach that does not rely on any specific knowledge of the motion of an interface, or a priori knowledge, of the driving forces in the porous medium.

We will relate the changes in the velocity field over small distances to the perturbations of the fluid-fluid interface discussed earlier.

We consider a time-invariant velocity field. Let the Darcy velocity vector at a location \( r(x, y, z) \) be represented by \( \mathbf{v}(v_x, v_y, v_z) \). Using a Taylor-series approximation, the velocity at a nearby position \( r + \delta r \), may be estimated, to first order, as [see for example Pozridikis (1997); Batchelor (1967)],

\[
\mathbf{v}(r + \delta r) = \mathbf{v}(r) + \delta \mathbf{v},
\]

\[
= \mathbf{v}(r) + \mathbf{J} \cdot \delta r,
\]

where, for rectangular co-ordinates, the relative change in velocity \( \delta \mathbf{v} \) may be expressed as the second-order velocity gradient tensor \( \mathbf{J} \),

\[
\mathbf{J} = \begin{pmatrix}
\frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\
\frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\
\frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z}
\end{pmatrix},
\]

where each gradient in this tensor is evaluated at \( r \).

For incompressible, single-phase flow the only reasons why velocity will change with location will be well pattern (e.g. velocity decreases with distance from a well) or reservoir heterogeneity (changes in permeability). These velocity changes will result in a perturbation to the fluid interface. To understand the relative velocity \( \delta \mathbf{v} \),
3.5 SHEAR AND VORTICITY—AN EUCLERIAN APPROACH

Geometrically, we can decompose $J$ into symmetric and anti-symmetric components. For brevity we reduce the problem down to two-dimensions and decompose $\delta v$,

\[
\delta v = \delta v_s + \delta v_a, \quad (3.29)
\]

\[
= \frac{1}{2} \left[ \begin{pmatrix}
\frac{2}{\delta v_x}{\delta y} + \frac{\delta v_y}{\delta x} \\
\frac{\delta v_y}{\delta y} - \frac{\delta v_x}{\delta x}
\end{pmatrix} + \begin{pmatrix}
0 \\
\frac{\delta v_y}{\delta y} - \frac{\delta v_x}{\delta x}
\end{pmatrix} \right]. \quad (3.30)
\]

Both $\delta v_s$ and $\delta v_a$ contribute to the relative velocity $\delta v$ in different ways. Consider the two cases in turn.

$\delta v_s$ is a rank-two symmetric tensor. By diagonalising this tensor [see Batchelor (1967)] it is clear that this represents a pure straining motion where the rates-of-strain (the eigenvalues) are in the direction of its principal axes (defined by the corresponding eigenvectors). The diagonalised matrix is,

\[
\delta v_s \text{ diagonalised} = \frac{1}{2} \left( \begin{pmatrix}
\frac{\delta v_x}{\delta x} + \frac{\delta v_y}{\delta y}
\end{pmatrix} - a \\
0
\end{pmatrix} - a \begin{pmatrix}
\frac{\delta v_x}{\delta x} + \frac{\delta v_y}{\delta y}
\end{pmatrix} + a \\
0
\right), \quad (3.31)
\]

where,

\[
a = \left[ \left( \frac{\delta v_x}{\delta x} - \frac{\delta v_y}{\delta y} \right)^2 + \left( \frac{\delta v_x}{\delta y} + \frac{\delta v_y}{\delta x} \right)^2 \right]^{\frac{1}{2}}. \quad (3.32)
\]

The eigenvectors—the directions in which the principal rates-of-strain act—for $\delta v_s$ are,

\[
\begin{pmatrix}
\frac{\delta v_x}{\delta x} - \frac{\delta v_y}{\delta y} \\
\frac{\delta v_y}{\delta x} + \frac{\delta v_x}{\delta y}
\end{pmatrix} - a \quad \text{and} \quad \begin{pmatrix}
\frac{\delta v_x}{\delta x} - \frac{\delta v_y}{\delta y} \\
\frac{\delta v_y}{\delta x} + \frac{\delta v_x}{\delta y}
\end{pmatrix} + a. \quad (3.33)
\]

Considered another way, $\delta v_s$ represents the transformation of a packet of fluid from a sphere to an ellipse whose principal diameters remain the same but with a rate of extension given by the coefficients of the diagonalised tensor. In the case of an incompressible fluid the volume of the packet of fluid remains constant.

$\delta v_a$ is the corresponding rank-two antisymmetric tensor which, in 3D, has three independent components. It may be written as,

\[
\delta v_a = \frac{1}{2} \begin{pmatrix}
0 & -\Omega_z & \Omega_y \\
\Omega_z & 0 & -\Omega_x \\
-\Omega_y & \Omega_x & 0
\end{pmatrix}. \quad (3.34)
\]
where $\Omega_i$ may be identified as the $i^{th}$ component of the vorticity $\Omega = \nabla \times \mathbf{v}$. So $\delta v_a$ may be interpreted as the relative velocity produced in the vicinity of a point where a rigid-body would rotate with an angular velocity $1/2 |\Omega|$.

The motion of a material-element of fluid can, therefore, be broken down into three components:

A. A translation, where the packet keeps its shape but moves with velocity $\mathbf{v}$;

B. A deformation along its principal axes, described by the rate-of-strain tensor$^2$;

C. A rotation, described by the vorticity, and characterised by the local rotation of a rigid-body with velocity $(1/2) |\Omega|$.

In the context of porous media both deformation and rotation will be the result of fluid or medium heterogeneities, and both deformation and rotation involve the same derivatives of the velocity field.

In summary, we have used Heller’s analysis to show $\Omega$ in terms of the driving processes operating in the reservoir and show how it characterises the perturbations to the fluid interface in the miscible case.

Using this analysis we have shown that $\Omega$ and the shear-strain rate together represent the changes a packet of fluid undergoes as it moves through the medium.

Conceptually it is easier to imagine how the shear-strain rate may affect fluid recovery from an oil reservoir. Imagine a displacement front between two matched-viscosity and density fluids that are fully miscible, shown in Fig. 3.3. This front is initially planar and perpendicular to the average flow direction. In the presence of a higher permeability in one region compared to another the front will be perturbed, so that part of the front in the region of higher permeability will move ahead of that in the region of lower permeability. This will lead to the premature breakthrough of the displacing fluid at the reservoir boundary compared with the homogeneous case. The larger the permeability difference the greater the velocity in the more permeable layer, so the larger the shear-strain rate, and the earlier the breakthrough. Conversely, the greater the shear-strain rate the earlier the breakthrough.

So far this analysis has not referred to Darcy’s law or the equations of flow in porous media. From Eq. (3.30) we define the shear-strain rate as

$$\dot{\gamma} = \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}, \quad (3.35)$$

and, by following the derivation of the vorticity of two-phase flow by Mahani et al. (2009), we can derive a similar expression for $\dot{\gamma}$.

The two-phase extension to Darcy’s law gives the velocity of the $i^{th}$ phase as,

$$v_i = -\frac{k_{ri}}{\mu_i} \left( \nabla p + \rho_i g \right), \quad (3.36)$$

$^2$ In the compressible case this will also account for the change in volume of a fluid-packet.
3.5 SHEAR AND VORTICITY—AN EULERIAN APPROACH

Figure 3.3: Schematic of a displacement front for a matched viscosity FCM displacement from left to right. (a) shows a smooth flat front for the homogeneous case; the time to breakthrough is $1 \frac{PVI}{P}$, the average velocity is uniform and the shear-strain rate is 0 throughout the model. (b) shows a two-layer heterogeneous model. The velocity varies over the grid and the shear-strain rate at the boundary of the two permeable layers is non-zero.

where $k_{ri}$ is the relative permeability of the $i^{th}$ phase. For two-phase flow $i$ is $d$ for the displacing fluid, and $o$ for oil. We can define the mobility of a phase as,

$$\lambda_i = \frac{k_{ri}}{\mu_i}, \quad (3.37)$$

and $f_d$ as the ratio of the mobility of the displacing phase to the total mobility. By noticing that $f_o = 1 - f_d$ we may write the total velocity $v_T(v_{xT}, v_{yT})$ as,

$$v_T = - k\lambda_T \left( \nabla P + [f_d \Delta \rho + \rho_o g] \right). \quad (3.38)$$

Substituting this into Eq. (3.35) gives,

$$\dot{\gamma} = \frac{v_{xT}}{k} \frac{1}{\partial y} + \frac{v_{yT}}{k} \frac{1}{\partial x} \left( \frac{1}{\lambda_T} \frac{\partial \lambda_T}{\partial y} + \frac{1}{\lambda_T} \frac{\partial \lambda_T}{\partial x} \right)$$

$$+ \frac{v_{xT}}{\lambda_T} \frac{\partial \lambda_T}{\partial y} + \frac{v_{yT}}{\lambda_T} \frac{\partial \lambda_T}{\partial x}$$

$$- k\lambda_T \Delta \rho \left( g_y \frac{\partial f_d}{\partial x} + g_x \frac{\partial f_d}{\partial y} \right). \quad (3.39)$$

By assuming $\lambda_T = \lambda_T(f_o)$ this becomes,

$$\dot{\gamma} = \left[ \frac{\partial \ln k}{\partial y} v_{xT} + \frac{\partial \ln k}{\partial x} v_{yT} \right]$$

$$+ \frac{1}{\lambda_T} \frac{\partial \lambda_T}{\partial S_d} \left[ \frac{\partial S_d}{\partial y} v_{xT} + \frac{\partial S_d}{\partial x} v_{yT} \right]$$

$$- \frac{\partial f_d}{\partial S_d} k\lambda_T \Delta \rho \left[ g_y \frac{\partial S_d}{\partial x} + g_x \frac{\partial S_d}{\partial y} \right], \quad (3.40)$$

where $S_d$ is the saturation of the displacing phase.

It can be seen that the first set of grouped terms are directly related to how permeability changes perturb the flow field and are independent of fluid saturation. The individual terms in this first set are maximized in the presence of large velocities perpendicular to the permeability gradient. The second and third sets of terms de-
pend on the total mobility and saturation of the displacing phase. The third set of
terms also depends on the gradient of fractional flow with saturation, the saturation
gradient and acceleration due to gravity.

The terms in the second, mobility dependent, set are maximised in the presence
of large saturation gradients perpendicular to the velocities whilst the gravity depen-
dent terms, in the third set, are large in the presence of large saturation gradients
perpendicular to $g$. The former is only likely to occur when there is either viscous
fingerling or channelling. The latter will occur in the presence of large density differ-
ences or channelling.

Thus, we assert that the first, heterogeneity term, can be used to conveniently quan-
tify how heterogeneity impacts both single and multiphase flows. Assuming that
multi-phase flow does not significantly impact the total velocity field then this term
is easily calculated from the solution of the single phase Darcy’s law and so requires
only a single pressure solve.

3.6 NUMERICAL EVALUATION OF VORTICITY AND SHEAR

In the examples discussed in §3.4 vorticity was calculated directly from the expres-
sions for the permeability field and Darcy’s law. For reservoir models vorticity may
be evaluated either by solving the vorticity equations directly, or by first numerically
estimating the velocity field and then estimating velocity gradients.

In reservoir models this is typically done by constructing a grid and solving the
equations of motion for fluid-flow in porous media for pressure and velocity. In the
case of FCM fluids subject to viscous, gravity, and diffusive/dispersive processes these
are Eqs. (3.8), (3.9), and (3.10).

For a 2D reservoir model in Cartesian co-ordinates, where $v = v(x, y)$ is known,
the only non-zero component of vorticity is

$$
\Omega_z = \left( \nabla \times v \right)_z = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}.
$$

(3.41)

For the discrete case this is,

$$
\Omega_z = \frac{\Delta v_y}{\Delta x} - \frac{\Delta v_x}{\Delta y}.
$$

(3.42)

In the finite-volume formulation used by most commercial simulators Fig. 3.4 shows
the form of the velocity-output from a simulation, where the grid block is located at
$(x_{ij}, y_{ij})$. 
The velocities $v_x$ and $v_y$ at the centre of the grid-block are approximated by,

\[
v_x(i, j) = \frac{1}{2} [v_x(i + 1/2, j) + v_x(i - 1/2, j)] \tag{3.43}
\]

\[
v_y(i, j) = \frac{1}{2} [v_y(i, j + 1/2) + v_y(i, j - 1/2)] \tag{3.44}
\]

The vorticity $\Omega_z$ at each grid-block is given by the central-finite-difference approximation,

\[
\Omega_z(i, j) = \frac{1}{2\Delta x} [v_y(i + 1, j) - v_y(i - 1, j)] - \frac{1}{2\Delta y} [v_x(i, j + 1) - v_x(i, j - 1)],
\tag{3.45}
\]

where $\Delta x$ and $\Delta y$ are the width and length of each grid cell.

We will now reconsider the reservoir models discussed in §3.4 but relax the condition that $k_y$ be zero. The permeability field will be treated as isotropic, with the same boundary conditions used earlier. In Fig. 3.5 we look at three cases, when $k(x, y) = y$, $1/y$ and $\exp(y/4)$. All of these cases behave very differently in terms of breakthrough time and ultimate recovery.

1. **$k = y$: A linear function of $y$ (reservoir height).** Substituting for $k$ into Eq. (3.4) gives

\[
\Omega_z = -\frac{v_x}{y} \tag{3.46}
\]

The vorticity map shows the highest values in the bottom left quadrant, where fluid is forced upwards from the less permeable layers to the more permeable. It is because permeability increases upwards that vorticity is negative, signifying a clockwise rotation of particles were they suspended in the flow. Away from the injection well vorticity becomes constant as the cross-flow between layers decreases, and more closely resembles what we could compute for the $k_y = 0$ case in Eq. (3.23). Where vorticity does vary, these variations are due to differences in fluid velocity as the permeability gradient is constant throughout the
model [see Eq. (3.4)]. The displacing fluid breaks through into the production well after 0.6 PVI.

2. \( k = 1/y \) : An inverse function of height. With an inverse relationship there is a difference in permeability between the upper and lower layers of several orders of magnitude, which results in rapid breakthrough (0.18 PVI), with the highest flux in the bottom 10% of the reservoir. Most of the solvent moves rapidly through these layers, which means the rate of increase of solvent-cut increases in time. Substituting \( k(x, y) = 1/y \) into Eq. (3.4) gives

\[
\Omega_z = \frac{v_x}{y}. \hspace{1cm} (3.47)
\]

The analytical form for vorticity is similar to the linear case but the vorticity map shows a stronger dependence on permeability. The highest permeability region has very high levels of vorticity, consistent with this region of rapid flow, the histogram of vorticity shows more variation, in-line with the earlier breakthrough.

3. \( k = e^{y/4} \) : An exponential function of height. Substituting for this in Eq. (3.4) gives

\[
\Omega_z = -\frac{v_x}{4}. \hspace{1cm} (3.48)
\]

This is a different form to the previous two cases and suggests that in this case the statistics of the velocity field will show the same features as the statistics of the vorticity field. Breakthrough time is very early (0.15 PVI) consistent with the large differences in permeability and the larger variations in vorticity.

Traditionally it is the statistics of the permeability distribution that was used to quantify heterogeneity, although in some cases, and particularly when considering upscaling, the velocity field was used (Durlofsky et al. 1997). This was especially effective when the upscaled model was used to predict breakthrough time as large velocity differences in a model are to be found when there are high permeability channels.

The vorticity field, by virtue of Eq. (3.4), combines both the statistical properties of the permeability field and the dynamic velocity field, explicitly. We have also shown in §3.3 that it may be used to quantify the growth-rate of instabilities to a displacement front as the front passes through a heterogeneous region. As the difference between a homogeneous model and a heterogeneous model is the growth of instabilities it may be that vorticity is a suitable candidate for a more robust measure of heterogeneity.
Figure 3.5: Vorticity for some heterogeneous reservoir models, showing the concentration (saturation of the displacing phase, $S_d$) at 0.75 PVI, the probability and spatial distribution of vorticity, and the fraction of the displacing phase at the production well as a function of pore volumes injected (the solvent cut)
Conclusions

To construct a single heterogeneity index from vorticity, permeability, or any field variable requires identifying the property of the variable that is the biggest control on reservoir performance. In the case of permeability heterogeneity it is clear that both variation and correlation length are significant, however there is no simple way of combining this into a single measure.

Vorticity, at any point in the reservoir, represents, to first order, the growth-rate of instabilities of a displacement front. The lower the growth-rate the more likely it is that the displacement will be viscous-dominated rather than heterogeneity dominated. Conversely, the higher the growth-rate, the more likely it is that the reservoir is heterogeneity dominated.

As a first attempt we may treat vorticity statistically by calculating some mean or standard deviation and attempting to relate it to recovery. To allow some level of comparison between different reservoirs we will proceed by asserting that the coefficient of variation (the standard deviation normalised by the mean) may be a measure of the effect of heterogeneity on recovery.

In the next section we look at some simple cases to understand the distribution of vorticity in a reservoir and what it may tell us, restricting ourselves to permeability heterogeneity and single-phase flow. In later chapters we show how the coefficient of variation of vorticity can be used to calculate a heterogeneity index, as well as how vorticity can be decomposed into different components which may allow a better treatment of reservoirs with complex boundary conditions.
Figure 3.6: Vorticity map and concentration distributions, (a) & (b), at 0.75 PVI for a line-drive and pseudo-Q5-spot homogeneous models showing the small but non-zero vorticity field for the Q5-spot model. (c) shows the earlier breakthrough in the Q5-spot model.
3.7 VORTICITY IN SOME SIMPLE RESERVOIR MODELS

Let us return to the curl of the single-phase Darcy’s law [Eq. (3.4)] and treat k as a scalar field. Figs. 3.6 and 3.7 shows the vorticity fields from numerical simulation for some simple cases. The results are from a matched viscosity flow simulation with uniform and constant injection along the left face and production along the right face (for most) cases. As this is a two-component 1-phase flood the effects of dispersion, diffusion, gravity, and capillary forcing have all been ignored. We show the permeability map, the distribution of concentrations at 0.75 PVI, the numerically calculated vorticity field, the fraction of solvent in the production well (the solvent cut) and the probability distribution of Ωz. The vorticity probability distribution is included because, as discussed we propose treating the variation of the vorticity field as a basic measure of the negative effects of heterogeneity on recovery.

For the homogeneous cases shown in Fig. 3.6, the vorticity field must analytically be zero. This is the domain of potential flow. For the first case with the line drive this is in fact the case but for the pseudo-Q5 well pattern where we inject in the bottom left and produce from the right this is not the case. The source of this ‘vorticity’ which is not a feature of the real velocity field are errors in the numerical calculation because of the grid not being aligned with the direction of bulk flow – the grid orientation error. To assess the impact of heterogeneity we should minimise this error as much as possible. As it is the values of vorticity due to the grid orientation are small - it varies from −0.05 to 0.05. As comparison for the heterogeneous cases vorticity is 2 to 3 orders of magnitude larger so we expect the errors in the calculation of the heterogeneity index to be minimal.

With respect to vorticity the homogeneous models raise three points to be noted.

1. The location of the wells has an impact on the time to breakthrough. For the pseudo-Q5 spot well pattern it is 0.7 PVI.

2. The vorticity for all homogeneous models is always zero.

3. This means that despite different breakthrough times any heterogeneity index calculated from the vorticity field will be meaningless. This is not because vorticity is unable to capture the effect of different boundary conditions on flow – we show later that it can – but it is because of the explicit dependence on the permeability gradient.

So for homogeneous reservoirs – or almost homogeneous reservoirs – a statistical measure of the variation in the vorticity field will not provide robust results.

For heterogeneous reservoir models the hope is that vorticity does capture the effect of heterogeneity on recovery, as we will show in the next section. Fig. 3.7 shows a two-layer reservoir model with a permeability contrast of 1:10 and 1:100 respectively. For the lower contrast model vorticity is −83 between the two layers and 0 elsewhere and for the higher contrast it is −100 and 0 elsewhere. The breakthrough time is
similar in both cases but with the increase in permeability contrast the absolute value of vorticity has increased. The probability distribution and the coefficients of variation are similar.

The lens model is included to emphasise that the vorticity field is able to identify critical regions of flow. In the two-layer model the rotation at the boundary between layers is negative, fluid is moving from the low permeability region at the bottom to the high permeability region (an anti-clockwise rotation). In the lens model the rotation about the upper and lower boundaries differs in sign representing the curving of the streamlines into the higher permeability region. The boundaries most strongly identified by vorticity are those where the velocity field is perpendicular to a large gradient in permeability. These are regions that contribute significantly to channelling of fluids and are relevant channel boundaries for this particular well pattern. If injection was along the bottom face and production of the top it would be the vertical boundaries that would be most emphasised. It was this ability to identify the important boundaries of a channel that were used successfully by Mahani et al. (2005, 2009) to generate coarse grids.

This identification of channels is a useful feature of vorticity and easily understood in the context of well defined channels as is often found in fluvial systems.
Figure 3.7: Vorticity for two, layered, models and a lens permeability model, showing the concentration distribution at 0.75 PVI, the vorticity map and histograms, and the solvent cut.
3.8 A Statistical Measure of Vorticity

As shown in the previous section vorticity (or the rate of shear deformation) may vary over the reservoir as they depend upon both permeability and total velocity. We have calculated both using the single-phase velocity field determined by the numerical simulator. To calculate a single heterogeneity index for the entire reservoir we determine the coefficient of variation, $C_v$, of vorticity (or the shear-strain rate), and define heterogeneity indices $H_v$ and $H_s$ as,

$$ H_v = \frac{1}{C_v(|\Omega|)}, \quad (3.49) $$

$$ H_s = \frac{1}{C_v(|\dot{\gamma}|)}, \quad (3.50) $$

where

$$ C_v = \frac{\text{mean}}{\text{standard deviation}}. \quad (3.51) $$

This means that the heterogeneity indices

1. tend to 0 as the reservoir becomes more heterogeneous, and

2. they have an upper bound of $\infty$ as the reservoir becomes more homogeneous.

Realistically, there are two implications.

1. For a completely homogeneous reservoir, both vorticity and the shear-strain rate will be zero throughout the reservoir, so $C_v$ will be zero and $H_v/H_s$ will be undefined. The breakthrough time in such a reservoir will be governed by the location of the wells but for the type of line-drive models described in this chapter this will tend to $1/PVI$.

2. For an extremely heterogeneous reservoir, the properties of $C_v$ will mean there is a lower limit for the $1/C_v$ that is greater than zero. Consider the possibility when, for a reservoir of $N$ grid blocks, $N - 1$ vertices have zero vorticity, and one vertex is non-zero. In this case $C_v$ has a maximum value given by $\sqrt{N-1}$ (Schumann and Mostert, 1949; Katsnelson and Kotz, 1957). In the context of a reservoir the lowest recovery and earliest breakthrough time will involve an extremely permeable layer, one grid block wide bridging the shortest gap between production and injection wells (inter-well distance). Consider the vorticity map on a grid, size $NX \times NY$, with the injection well completed in the first column of grid blocks at $IX = 1$ and the production well in the last column of grid blocks at $IX = NX$. The number of vertices at which the vorticity is calculated is $(NX - 1) \times (NY - 1)$. For $NX = 220$, the vorticity will be non-zero at $2 \times (NX - 1)$ vertices, in this case 438 vertices. Thus $C_v$ will have a maximum at $\sqrt{[(NX - 1)(NY - 1)](2[NX - 1]) - 1}$ and $H_v$ will therefore have a lower
bound of $1/\sqrt{(NY-1)(2-1)}$. For the particular case of $NX = 220$, $NY = 60$ and $N = 13200$, $H_v$ will have a lower bound of 0.19.

Ideally a heterogeneity index will be normalised such that the most homogeneous reservoir will have a $H_v = 1$ (or 0) whilst the most heterogeneous will be $H_v = 0$, however the properties of $C_v$ make this difficult. In this study we will define the heterogeneity index using $C_v$ however, we can envisage an index normalised between 0 and 1 like the Dykstra-Parsons coefficient.
METHODS

In this chapter we describe the fluid and geological models we have used in this study. We also detail the numerical simulation tools and schemes we have used, and the algorithms to calculate the dimensionless flow-regime numbers and heterogeneity indices.

4.1 NUMERICAL FLOW SIMULATION

We used numerical simulation to calculate the flow-field, water/solvent saturations/concentrations and production profiles for the reservoir models we used. The majority of the simulations were carried out using MISTRESS, a finite-difference, high resolution, dimensionless code written and used at BP Research during the 90’s (Christie and Bond 1987; Christie 1989; Barley 1992).

The numerical scheme used can resolve the detailed behaviour of miscible and immiscible displacements using an implicit pressure explicit saturation (IMPES) solution scheme using a flux corrected transport algorithm (FCT). The mathematical model used makes the following assumptions:

- Two-phase Darcy flow;
- Three components—water, oil, solvent;
- All components are incompressible;
- Oil and solvent first contact miscible (FCM) - quarter power mixing rule for viscosities \( \mu = \left[ \frac{c}{\mu_s^{1/4}} + \frac{(1-c)}{\mu_o^{1/4}} \right]^{-4} \);
- Ideal mixing of the oil and solvent densities gives the phase density as \( \rho_o = c \rho_s + (1-c) \rho_o \);
- Oil and solvent mix through diffusion instead of dispersion;
- Porosity is assumed constant;
- Physical dispersion, capillary pressure and gravity effects are modelled.

The traditional fluid-flow equations are first re-written using the total velocity formulation of Peaceman (1977) to give an elliptic pressure equation and two hyperbolic conservation laws; the equations solved are described in detail by Christie and Bond (1987) and Christie (1989). In this section we have summarised the details and methods from Barley (1992).
The pressure equation is given by,
\[
\nabla \cdot [(\lambda(c) + \lambda_w) \nabla P] = \nabla \cdot [\lambda(c) \rho(c) + \lambda_w \rho_w] \mathbf{g} \\
- \frac{1}{2} \nabla \cdot [(\lambda(c) - \lambda_w) \nabla P_c] \\
- [q_o + q_w],
\]
where \(\lambda(c)\), \(\rho(c)\), and \(\mu(c)\) are defined earlier, \(q_o\) and \(q_w\) are the oil and water phase source/sink terms, and the average pressure \(P\) and the capillary pressure \(P_c(S_w)\) are defined as,
\[
P = \frac{1}{2} (P_o + P_w), \tag{4.2}
\]
\[
P_c(S_w) = P_o - P_w. \tag{4.3}
\]
The conservation equations are formulated using the total velocity \(v_t = v_o + v_w\), where,
\[
v_t = -\lambda_t (\nabla P_t - \rho_t \mathbf{g}). \tag{4.4}
\]
The mobility term is defined as,
\[
\lambda_i = \frac{k k_{ri}}{\mu_i}. \tag{4.5}
\]
This leads to the fractional flow equation,
\[
v_w = f_w v_t - \Psi(S, c) \nabla S - \lambda(c) f_w [\rho_w - \rho(c)] \mathbf{g}, \tag{4.6}
\]
where \(f_w\) is the fractional flow of water,
\[
f_w = \frac{1}{1 + \lambda(c)/\lambda_w}, \tag{4.7}
\]
and \(\Psi(S, c)\) describes the diffusive effects associated with capillary pressure,
\[
\Psi = -\frac{\lambda \lambda_w}{\lambda + \lambda_w} \frac{dP_c}{ds}. \tag{4.8}
\]
The system of equations solved by the simulator are the pressure equation [Eq. (4.1)], and using the fractional flow formulation [Eq. (4.6)], the two conservation laws,
\[
\phi \frac{\partial S_w}{\partial t} + \nabla \cdot v_w = q_w, \tag{4.9}
\]
\[
\phi \frac{\partial (c S_o)}{\partial t} + \nabla \cdot c v_o = \nabla \cdot S_o D \cdot \nabla c + c q_o, \tag{4.10}
\]
where \( c \) is the concentration of solvent, \( S_i \) is the saturation of the \( i^{th} \) phase and \( D \) is the dispersion tensor. \( D \) is a diagonal dispersion tensor with two diffusion components, \( D_{11} \) and \( D_{22} \). They are assumed constant throughout the simulation and are responsible for providing a physical cut-off length below which all instabilities are damped out.

The injection well is modelled as a source of constant strength, i.e. constant volumetric injection rate. The flow-rate out of each grid block which is part of a well is defined as,

\[
Q_{\text{out}} = -\frac{\text{PI}}{\Delta x \Delta y} (P_k - \text{BHP}_k), \quad (4.11)
\]

where \( \text{PI} \) is the productivity index of the well and \( \text{BHP}_k \) is the bottom hole pressure of the \( k^{th} \) grid block in the well. Both these can be specified before the simulation.

The equations were non-dimensionalised using system-characteristic values for length, time, velocity and pressure, before being solved on a regular \( NX \times NY \) grid. The pressure equation is solved implicitly using a five-point operator; this leads to \( N = NX \times NY \) equations that need to be solved simultaneously. The matrix formulation of these equations requires the inversion of the pentadiagonal \( N \times N \) matrix of source term coefficients and takes significant computational time. The method used is the Incomplete Choleski Conjugate Gradient (ICCG), chosen as it reduces memory usage by exploiting the nature of the sparse matrix, is based on the Conjugate Gradient (CG) method so is fast, and is easy to vectorise. Note that this method is particularly good when \( NX = NY \) and appears to fail when the dispersion coefficients are too high.

The conservation laws are solved using an explicit method that uses a block-centered variation of the Flux Corrected Transport (FCT) algorithm to reduce the problems due to a global application of a second-order scheme. First-order schemes tend to smear sharp fronts whilst second order schemes introduce spurious oscillations in regions that have sharp gradients. The hybrid FCT scheme is first-order in regions where the high-order scheme will result in spurious results and high order in regions where the flow is smooth.

Viscous fingering was initiated in simulations with a small randomly varying solvent concentration along the injection well to mimic instabilities due to small scale, low level heterogeneity.

The simulator has been validated by comparing its predictions with the results of various laboratory experiments investigating miscible flows in glass bead-packs (Christie and Bond 1987; Fayers and Newley 1988; Christie 1989; Christie et al. 1990; Muggeridge et al. 2002, 2005) and in a well-characterized sandstone slab (Davies et al. 1991).

For the purposes of this study we have modified the output routines in the MISTRESS code to provide easy visualisation of the relevant fields using the VTK format. The calculation of vorticity, shear and other derivatives of the velocity field were im-
4.2 ROCK PROPERTIES: HETEROGENEITY MODELS

When introducing or testing heterogeneity indices it is often tempting to use either very simple reservoir models or a range of synthetic permeability distributions in an effort to give statistical significance to the results. In many early studies numerical simulation was in its infancy and it was difficult to generate and simulate flow through a large number of realistic reservoir models in a reasonable amount of time.

In this study we have used some very idealised geological models to understand how heterogeneity affects recovery. These simple models have already been discussed in earlier chapters. To verify and test the indices we then used geological models extracted/constructed from two realistic reservoir models, described below.

4.2.1 SPE10 Model 2

Model 2 from the SPE 10th Comparative Solutions Project (Christie and Blunt [2001]) was derived from a model originally generated for the PUNQ project (Floris et al. [2001]). It was designed with the intention to compare upscaling and upgridding techniques for a waterflood. It is notoriously difficult to upscale and sufficiently large to make classical pseudoisation techniques very difficult.

The model represents part of a Brent sequence described on a regular Cartesian grid with dimensions of $368 \times 675 \times 52$ (m) or $1200 \times 2200 \times 170$ (ft) exactly. The fine scale model (see Fig. 4.1) has $60 \times 220 \times 85$ cells with cell dimensions of approximately $6 \times 3 \times 0.6$ (m) [$20 \times 10 \times 2$ (ft)]. The top 35 layers are part of the Tarbert formation representing a prograding near shore environment and are relatively less
heterogeneous than other layers. The bottom 50 layers represent the fluvial Upper Ness sequence.

The porosity varies from 0 to 0.5, the permeability in the x and y directions, \((k_h)\), from 0 to \(2 \times 10^4\) mD, and \(k_v\) from 0 to \(6 \times 10^3\) mD. As the main purpose of this study is to look at permeability heterogeneity we have used only the geological properties from Model 2 and disregarded fluid distributions, fluid properties and the well locations specified in the original project.

It was chosen as it represents a real geological formation, the geometry is Cartesian and so possible to use with MISTRESS and some layers are known to be difficult to upscale, thus presenting a challenge for both upscaling and assessing performance without detailed simulation.

This model was the source for most of the 2D and 3D reservoir models used in this study.

**2D Reservoir Models**  Using SPE10 Model 2 we have constructed two sets of 2D models. Each of the 85 horizontal layers \((x – y\) plane) became a \(220 \times 60\) grid block model with a grid-cell aspect ratio of 1 : 2, which is the same resolution as the original model. Besides being the original resolution this was chosen to ensure that dispersion was physical rather than numerical. This set of 2D layers is Model A. In addition to these layers we extracted 60 vertical cross-sections \((z – x\) plane) consisting of \(85 \times 85\) grid blocks with a grid-cell aspect ratio of 1 : 10. This aspect ratio was chosen to mimic a long thin reservoir, and so both the aspect ratio and the number of grid blocks is different from the original SPE10 Model 2. The set of these vertical cross-sections is Model B and cross-sections from this model were used for some of the results presented in Chapter 6 where we consider the influence of gravity on flow.

We have used the original \(k_h\) for Model A and \(k_v\) for Model B. For simplicity we have set porosity to a constant value of 0.2 as porosity variations, within a typical reservoir, are small compared to permeability variations.

Together these models provided a set of heterogeneous models that behave very differently in terms of breakthrough time and recovery. For instance, layer 9 from Model A represents the Tarbert formation, shown in Fig. 4.2a which is relatively homogeneous with respect to oil recovery, with any ‘channels’ of higher permeability oriented in the y direction, whilst layer 59 from the Upper-Ness formation (Fig. 4.2b) shows a tortuous high permeability channel with considerable amounts of difficult-to-sweep oil.

The models were all initially fully saturated with oil. Throughout the study reference to cross-sections, layers or reservoir realisations generally refer to 2D layers from this model, and the terms are used interchangeably.
4.2 ROCK PROPERTIES: HETEROGENEITY MODELS

It was difficult to construct a large number of realistic 3D reservoir realisations so SPE10 Model 2 was used to construct 16 separate realisations where we chose the geological properties that we needed from Model 2. These models are discussed in more detail in Chapter 7.

4.2.2 Other Models

The Stanford V model was used to provide an extra set of geological realisations that were not dependent on the geological simulation done for the PUNQ project. The Stanford V dataset is a synthetic model of a clastic reservoir with meandering fluvial channels (Mao and Journel 1999). It is designed on a regular Cartesian grid of $100 \times 130 \times 30$ grid-cells with a horizontal grid size of $25 \times 25$ (m). The thickness of each layer varies. We have simplified the model by setting porosity to be constant and by extracting each horizontal layer and treating it as one realisation of the reservoir, which provided 30 realisations to test.
4.3 Fluid Properties and Well Controls

All simulations assume that the pressure and viscosity of the fluids is independent of temperature and pressure. For most reservoirs the temperature variation within a reservoir is small enough to ignore temperature dependence. The pressure variation is generally small compared with the average total pressure so compressibility is negligible. Miscible fluids (such as gas/solvent and oil) are treated as fully miscible at first contact. Water is treated as an immiscible phase. The relative permeability curves use the Corey model with simulations using a Corey exponent of 2, \( S_{or} = S_{wc} = 0 \), and \( k_{rw}(1 - S_{or}) = k_{ro}(S_{wc}) = 1 \). Capillary pressure curves were not modelled, even for immiscible displacements.

In calculations of heterogeneity indices, unless otherwise stated, we inject at a constant-rate. For 2D models where the injection well is perforated in multiple grid blocks fluid is injected uniformly into each block. In cases where injection is at a constant pressure, the pressure is defined at the bottom of each well and the injection rate into each grid block depends on the permeability of that grid block and the bottom hole pressure of the well.

4.4 Calculation of Vorticity/Shear-Strain Rate

We calculated the vorticity and shear-strain rate on the grid using the face-centred velocity field output from the reservoir simulator. The finite-difference method used is detailed in §3.6.

4.5 Calculation of Traditional Heterogeneity Indices

The Dykstra-Parsons Coefficient \( V_{dp} \) was calculated using the static permeability data sets. As \( V_{dp} \) was originally defined for layered reservoirs and was calculated graphically, it was not suitable for the reservoir models we have used, which are not explicitly layered. We have used the improved approximation suggested by Jensen and Currie (1990) using Eq. (2.11) where, for a large number of grid blocks (large \( n \)) \( \sigma_k \) [Eq. (2.12)] is approximately the standard deviation of \( \ln k \) for a log-normal permeability distribution. This is suitable for the realisations in Models A & B from SPE Model 2 as \( k \) is log-normally distributed [Fig. 4.1b].

The Dynamic Lorenz Coefficient \( L_c \) was introduced by Shook and Mitchell (2009) as a streamline based extension of the static Lorenz coefficient of Schmalz and Rahme (1950). To calculate \( L_c \) we used the volumetric flow rate and times of flight.
of streamlines to generate a dynamic flow capacity plot \((F - \Phi)\), analogously to the static Lorenz coefficient. \(L_c\) is then given by

\[
L_c = 2 \left[ \int_0^1 F \cdot d\Phi - \frac{1}{2} \right]
\] (4.12)

where \(F\) and \(\Phi\) are,

\[
F_i = \sum_{r=1}^{i} q_r / \sum_{j=1}^{n} q_j, \quad (4.13)
\]

\[
\Phi_i = \sum_{r=1}^{i} V_{pr} / \sum_{j=1}^{n} V_{pj}, \quad (4.14)
\]

Streamsim 3DSL (Batycky et al. 1997; Thiele et al. 2010) was the simulator used. A single-phase flow simulation was carried out by injecting a tracer at constant-rate into the injection well. To calculate \(L_c\) we used the following procedure [detailed in Shook and Mitchell (2009)]:

a. Perform a single-phase pressure solve and generate streamlines in tracer mode;

b. Calculate streamline volumetric flow rates \((q_i)\) and time of flights \((\tau_i)\) of streamlines;

c. Calculate the pore volume of each streamline as \(V_{pi} = q_i \tau_i\) (Datta-Gupta 2000; Datta-Gupta and King 1995; Hastings et al. 2003);

d. Rank streamline by descending velocity and calculate \(F\) and \(\Phi\) using Eqs. (4.13) and (4.14).
HETEROGENEITY INDICES FOR 2D MODELS

We introduced the vorticity formalism in Chapter 3 to quantify the perturbations due to fluid or rock heterogeneities on a fluid front as it passed through a porous medium. We also presented a more physically intuitive approach, by analysing, mathematically, the changes experienced by a material element of fluid as it moves through a flow-field. This led to the introduction of vorticity, \( \Omega \), and the shear-strain rate, \( \dot{\gamma} \), of the velocity field as measures of the perturbations to a fluid displacement front.

We suggested that a single index, the reciprocal of the coefficient of variation (\( C_v \)) of these two fields, calculating using the single-phase velocity field, may capture the impact of heterogeneity on breakthrough time and recovery at 1\% recovery (1PVI).

In this chapter we show how \( H_v \) and \( H_s \), the heterogeneity indices based on vorticity and shear-strain rate, may be used to rank reservoirs in terms of breakthrough time and recovery at 1\% recovery for

1. a matched-viscosity FCM fluid displacement,

2. and models where there are viscosity differences, for both FCM and immiscible displacements.

We ignore the influences of gravity, diffusion, dispersion, and capillary processes in this chapter. Some of these processes will be reconsidered in Chapter 6.

We have compared these indices with the behaviour of the Dykstra-Parsons coefficient, \( V_{dp} \), and the dynamic Lorenz coefficient, \( L_c \), for the same cases. A linear-displacement model is analysed in most of this chapter (i.e. a line-drive well arrangement), however, we do conduct a preliminary investigation of the pseudo-Q5 spot well arrangement.

The purpose of these tests was to show that the statistical properties of the shear-strain and vorticity distributions, obtained from 1-phase flow, accurately reflect the perturbations and subsequent growth of fingers, even in the presence of differences in fluid viscosity or other fluid heterogeneities.

A number of other possible measures of heterogeneity from the data we had generated from streamline simulations were tested: the coefficient of variation of the time-of-flight of streamlines; the normalised difference between the streamlines with the largest and smallest velocities. These were found to be unsatisfactory. The results are in Appendix B.
Table 5.1: Model properties and parameter variations used in this chapter.

<table>
<thead>
<tr>
<th>Constants</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid size</td>
<td>2200 × 1200 ft. (220 × 60 cells)</td>
</tr>
<tr>
<td>φ, Porosity</td>
<td>0.2</td>
</tr>
<tr>
<td>Δρ, Density difference</td>
<td>0.0</td>
</tr>
<tr>
<td>Miscible fluids</td>
<td>Quarter-power mixing rule for viscosities</td>
</tr>
<tr>
<td>Immiscible fluids</td>
<td>$k_{rw} = S_w^2$</td>
</tr>
<tr>
<td></td>
<td>$k_{ro} = (1 - S_w^2)$</td>
</tr>
<tr>
<td></td>
<td>$S_{or} = 1$</td>
</tr>
<tr>
<td></td>
<td>$S_{wi} = 0$</td>
</tr>
<tr>
<td>Parameter Variations</td>
<td></td>
</tr>
<tr>
<td>Viscosity ratio, $\mu_o/\mu_d$</td>
<td>1, 10, 100</td>
</tr>
<tr>
<td>Permeability Heterogeneity</td>
<td>Layers 1 − 85 from Model 2</td>
</tr>
<tr>
<td>Boundary Conditions</td>
<td></td>
</tr>
<tr>
<td>Line-drive</td>
<td></td>
</tr>
<tr>
<td>Pseudo-Q5 spot</td>
<td></td>
</tr>
</tbody>
</table>

5.1 **Reservoir Model Setup**

In this chapter we have used the 85 layers from Model A (§4.2.1). Each layer is treated as a separate 2D geological realisation of the reservoir. We have used a line-drive reservoir model, shown in Fig. 5.1 in this chapter, except for §5.5, where to analyse the effect of well pattern we use a pseudo-Q5 spot pattern. We use the time to solvent breakthrough in pore-volumes of fluid injected ($t_{D|bt}$), and the recovery of oil after 1 PVI to assess the behaviour of a particular realisation. A summary of the reservoir properties used and those varied is provided in Table 5.1.

5.2 **Unit-Mobility FCM Fluid Displacements**

5.2.1 *The Dykstra-Parsons and Dynamic Lorenz Coefficients*

An overview of the range of breakthrough times, recovery, and the Dykstra-Parsons coefficients for each layer from Model A is presented in Fig. 5.2. In these layers, breakthrough is between 0.1 and 0.6 PVI whilst recovery after injecting 1 PV of fluid is between 0.4 and 0.8 PV of oil. The models which exhibit the earliest breakthrough are those that represent the Upper-Ness formation and typically feature narrow high permeability channels correlated in the x direction (the direction of mean fluid flow) through which fluid rapidly travels to arrive at the producer. The Tarbert formation layers are more strongly correlated perpendicular to the direction of flow (the y direction), however, in both cases, layers from the same formation show considerable variation in breakthrough time. Despite this, the behaviour of $V_{dp}$ is different, for the
5.2 UNIT-MOBILITY FCM FLUID DISPLACEMENTS

Figure 5.1: Layer 59 from SPE 10 Model 2 showing the reservoir model with constant rate injection uniformly along the left hand face and production along the right hand face.

Tarbert layers $V_{dp}$ varies considerably ($0.8 - 0.95$), whilst for the highly correlated models $V_{dp}$ shows very little variation around 0.95.

Thus, for all these models the variation in permeability is greater than 0.8, so these models are relatively heterogeneous as far as $V_{dp}$ is concerned. Compare $V_{dp}$ with breakthrough time for the Upper-Ness formation layers, there appears to be a very weak correlation between the two, with $V_{dp}$ insensitive to changes in breakthrough time from 0.1 to 0.5 [PVI] for these models. It is these highly correlated models for which [Waggoner et al.] (1992), using numerical simulation, suggested combining $V_{dp}$ with a measure of correlation in the direction of flow for a more accurate indication of model performance.

Conversely, the dynamic Lorenz Coefficient (Fig. 5.3), $L_c$, is a slightly better estimator of performance. There is some variability in $L_c$ for the Tarbert formation layers that does reflect some of the differences in performance seen between layers.

In order to understand whether these indices may be used to rank reservoir realisations in terms of performance or enable us to predict breakthrough time we need to understand the correlation with actual breakthrough times from numerical simulation. We have compared the value of $V_{dp}$ and $L_c$ against breakthrough time (Fig. 5.4) and several observations may be made about these traditional measures.

- $L_c$ and $V_{dp}$ show poor sensitivity for the most heterogeneous cases. $V_{dp}$ increases by 1% (0.97 to 0.98) whilst breakthrough is delayed by 47%.

- The correlation between breakthrough time and $L_c$ ($R^2 = 0.67$) and $V_{dp}$ ($R^2 = 0.60$) is non-linear over the entire range of breakthrough times tested. In particular $V_{dp}$ shows almost no correlation for the mildly heterogeneous layers of the Tarbert formation. $L_c$ has improved sensitivity, but the correlation is still weak, especially for the Upper-Ness formation layers.
Figure 5.2: The values of the Dykstra-Parsons coefficient $V_{dp}$, and the breakthrough time for a unit-mobility FCM displacement without gravity or dispersion effects. Note how the Dykstra-Parsons coefficient fails to capture the full variation in performance (breakthrough time varies from 0.1 to 0.6 PV). Continuous lines are used for breakthrough time and recovery merely to guide the eye.

Figure 5.3: The dynamic $Lc$, and the breakthrough time for a unit-mobility FCM displacement.
Figure 5.4: $V_{dp}$ and $L_c$ as functions of breakthrough time for a single phase line drive. $V_{dp}$ is insensitive to change in breakthrough time for the more heterogeneous Upper-Ness formation, whilst $L_c$ shows almost no correlation for these layers. The Tarbert formation layers show a trend with increasing $V_{dp}$ and $L_c$ but the correlation is weak.
5.2.2 Vorticity and Shear-Strain Rate Based Indices

The vorticity and shear-strain rate based heterogeneity indices demonstrate a strong correlation with both breakthrough time \((R^2 > 0.88)\), Fig. 5.5 and recovery at 1 PVT \((R^2 > 0.77)\), Fig. 5.6. There is a good discrimination between the most heterogeneous models as well as the more homogeneous ones. As both indices increase (i.e. tend towards a more homogeneous medium), breakthrough time and recovery increase proportionally. This linear relationship allows both a ranking of models in terms of performance, and may provide one of the tools necessary to predict breakthrough time without a full numerical simulation. This suggests that the coefficient of variations of the vorticity and shear-strain rate fields are adequately capturing the effect of permeability gradients on unit-mobility FCM displacements.

The calculations for these layers show some discrepancies in breakthrough time, particularly for the most heterogeneous models, when \(H_s\) and \(H_v\) are close to zero. This anomaly is amplified when longer-term oil recovery is calculated. Some discrepancy is to be expected, as both indices are determined from the coefficient of variation of changes in the velocity field around each grid node whereas breakthrough time and sweep are determined by the motion of the front through the velocity field over time, which can only be determined precisely by performing a full numerical simulation. These anomalous layers are typically those which show better recovery than would be expected from the values of the heterogeneity indices, and appear to reflect those cases where there are large local changes in the velocity field (resulting in a high coefficient of variation of shear/vorticity) in the model, but their influence on the displacement front cancels out as it progresses through the system. This limitation can be demonstrated by a checkboard permeability model where, as the fluid front progresses through the system it is perturbed, but the perturbations tend to cancel out over time, which results in a relatively smooth interface. However the variations in the vorticity field are non-zero and \(H_v\) provides an unduly pessimistic indication of performance.

To see the differences between the new indices and \(V_{dp}\) and \(L_c\) it is useful to compare the permeability distributions for two layers, that both behave identically in terms of fluid breakthrough time, layers 12 and 59 shown in Fig. 5.7. Both layers have the same value for \(H_v\), however their permeability distributions differ. Layer 59 consists of a very clear set of channels leading from the injector to the producer with a large variation in permeability, whilst layer 12 is more homogeneous in character, with a smaller contrast in permeability between the channel and the background. Their Dykstra-Parsons and dynamic Lorenz coefficients are different, and so do not capture their identical behaviour in terms of performance. This is primarily because both coefficients assume the reservoir is layered and that permeability variations are directly related to performance.
Figure 5.5: The new heterogeneity indices as functions of breakthrough time for a single phase line drive using Model A layers.
For single-phase displacements through geologically realistic models we find the new indices show the strongest correlation with performance of the measures of heterogeneity tested. $H_v$ rapidly captures what is not visible either by observing the spatial arrangement of permeability or by a statistical analysis of the static permeability data. For this set of models we find the dynamic $L_c$ is not a robust measure of the impact of heterogeneity on flow [cf. Shook and Mitchell (2009)]. Similarly, the Dykstra-Parsons coefficient, by virtue of the assumptions that underpin it, confirms the findings of Jensen and Currie (1990) of poor sensitivity when $V_{dp} > 0.7$. From our results, for the same delay in breakthrough, $H_s$ increases by 75% (0.28 to 0.6), whilst $V_{dp}$ increases by 1% (0.97-0.98).

Figure 5.6: $H_s$ and $H_v$ as a function of recovery at 1PVI

Figure 5.7: Permeability distributions for Layers 12 & 59 from Model A
5.3  THE RELATIONSHIP BETWEEN $H_v$ AND $H_s$, AND THEIR BEHAVIOUR UNDER GRID REFINEMENT

We showed in Fig. 5.5 that both $H_v$ and $H_s$ provided similar rankings of the reservoir models we tested. This is expected if we consider how they are calculated for 2D models, $H_v$ is calculated using $\Omega_z$, Eq. (3.41), and $H_s$ is calculated using $\gamma$, Eq. (3.35). Both involve the same gradients of the velocity field, $\partial v_y/\partial x$ and $\partial v_x/\partial y$.

In Fig. 5.8 we show a cross-plot of $H_s$ and $H_v$ for these models. We have already identified that both $H_v$ and $H_s$ provide a similar ranking of the models we have tested, this plot confirms that they are very similar. This suggests that in our models $\partial v_y/\partial x$ is likely to be much larger than $\partial v_x/\partial y$. An analysis of the ratio $\frac{\partial v_x}{\partial y} / \frac{\partial v_y}{\partial x}$ for each layer from Model A indicates that in over 60% of grid blocks this is the case, in fact in more than a third of the reservoir $\frac{\partial v_x}{\partial y}$ is two orders of magnitude greater than $\frac{\partial v_y}{\partial x}$. This is likely to be because these layers are longer than they are thick, and because the channels in these layers are oriented in the x direction.

This also explains the difference between the correlations for the Tarbert and Upper-Ness formation layers shown in Fig. 5.8. We found that for the Tarbert layers, on average, over 70% of grid blocks had $\frac{\partial v_x}{\partial y} / \frac{\partial v_y}{\partial x} > 1$ whilst for the Upper-Ness layers this fell to 60%. This explains why for the Tarbert layers $H_v$ and $H_s$ are more similar, and is due to the fact that the Tarbert layers are more homogeneous, and so have smaller variations in velocity in the x direction.

We end this section by presenting a comparison between $H_v$ calculated for the Model A layers for two grids, the standard $220 \times 60$ and a more refined model with $440 \times 60$ grid blocks. This was done to investigate the sensitivity of $H_v$ to grid resolu-
tion, particularly in light of the calculations presented in §3.8, which showed that the coefficient of variation has a maximum which depends on the size of the grid.

Fig. 5.9 shows $H_v$ for the base model against $H_v$ for the refined model. We found the ranking was preserved, however the refined models have a smaller value for $H_v$. This is because $C_v$ has an upper limit based on the number of grid blocks in the direction of flow. By doubling this we would expect the estimate for the heterogeneity index to be lower. Further refinement to a $660 \times 180$ grid block model for layer 59 showed that $H_v$ had converged.

In addition to this difference there appear to be differences, akin to those seen in Fig. 5.8, between the two formations. The values of $H_v$ appear to have converged for the Tarbert models at this resolution, however for models with $H_v < 0.5$ there is a larger difference. The refined model resolves the flow in the $y$ direction more accurately which reflects in a more accurate calculation of the velocity gradients. It is the more heterogeneous models where variations in velocity is most important so it is for these that there is the biggest difference between the coarse and fine models.

![Figure 5.9: Cross-plot of $H_v$ for the base case models against $H_v$ for the refined model with four times as many grid blocks.](image)

5.4 Ranking ability for more-realistic fluids

We have shown that $H_v$ and $H_s$ are linearly correlated with breakthrough time for a unit-mobility displacement. This suggests that they can act as predictors of breakthrough time. In the model setup for this chapter we treat all 85 realisations of Model A as realisations of the same reservoir. For a reservoir uncertainty study we often need to rank realisations in order of performance so that the lowest and highest performing models may be chosen without performing detailed simulation.
In real reservoirs there will be viscosity differences between the injected fluid and the oil. In the presence of viscosity differences, and in particular when the injected fluid is less viscous than the oil, any viscous fingering of the fluid may alter the breakthrough time of the fluid and the ranking. This will manifest itself as a time-varying velocity field.

When low viscosity gas/solvent displaces oil miscibly, the viscosity difference across the front leads to instabilities in the absence of heterogeneity. The source of this instability is the sensitivity of the displacement front to small variations in the pressure, which leads to the development of fingers of solvent that grow into the oil. $M > 1$ is sufficient to trigger viscous fingering leading to earlier than usual breakthrough. When there are heterogeneities then the growth of these fingers may be enhanced or suppressed by differences in permeability heterogeneity. Large correlation lengths of the permeability field results in flow channelling through these regions. When $M > 1$ then the adverse affects of channelling are exacerbated by the effects of viscous fingering which results in more of the fluid flowing through these channels.

Fig. 5.10 shows the effect of viscosity differences for both homogeneous and heterogeneous reservoirs. In the homogeneous case viscous fingering leads to early breakthrough because of the growth of fingers of faster moving fluid in the reservoir. In the heterogeneous case as the viscosity ratio increases more of the fluid flows through the high permeability channels which means less of the reservoir is swept. We have to be

![Figure 5.10](image-url)
especially careful when simulating viscous fingering to ensure the simulations have converged to the correct value. For the homogeneous case we can show it has by comparing the simulated behaviour with that calculated using Koval’s model [Eqs. (2.16) and (2.18)]. For an \( M = 100 \) miscible displacement Koval predicts breakthrough in 0.21 PVI. This agrees with our simulated breakthrough time for the model shown in Fig. 5.10.

To verify the ranking we simulated FCM displacements with three viscosity ratios (1, 10 and 100) and immiscible displacements with two viscosity ratios (1 and 10) in a homogeneous model as well as with each realisation from Model 2. Fig. 5.11 shows the correlation of \( H_s \) and \( H_v \) with breakthrough time for miscible fluid displacements with \( M = 1, 10 \) and 100. For \( M > 1 \) the time to breakthrough of the fluid decreased in all models compared to the single-phase cases, however there was still a good linear correlation between breakthrough time and the two indices. The gradient of the regression line ranges from 0.65 for \( M = 1 \) to approximately 0.18 for \( M = 100 \) for both indices, confirming that a change in heterogeneity index correlates with a correspondingly smaller change in time to breakthrough for larger mobility ratios. This suggests that performance was less sensitive to heterogeneity and more influenced by viscous fingering for very unstable displacements.

The greater scatter seen for more adverse viscosity ratios is attributed to the fact that viscous fingering is inherently a random process. This is particularly apparent in a homogeneous model and for very adverse viscosity ratios, where slightly different initial conditions can result in very different fingering patterns and thus different times to breakthrough.

Unlike displacement by a miscible fluid—and particularly the ideal FCM case we have studied—the behaviour of an immiscible displacement is controlled by the relative permeability model used as well as the difference in viscosity. Therefore we investigated the ability of \( H_s \) to rank breakthrough time and recovery at 1 PVI for immiscible displacements.

Fig. 5.12 shows \( H_s \) and breakthrough time for immiscible fluids and demonstrates a linear relationship between the two. This correlation between \( H_s \) and breakthrough time was preserved when the mobility ratio was changed from \( M = 1 \) to \( M = 10 \), again with the gradient decreasing as the mobility ratio increased. Compared with the miscible models, these appear to be more sensitive to heterogeneity, as shown by the gradient of the regression line. For the \( M = 1 \) immiscible case the gradient was approximately 30% greater than the corresponding miscible case whilst for \( M = 10 \) the gradient is approximately 20% greater.

The results for the miscible models were compared with the behaviour of the dynamic Lorenz coefficient for different viscosity fluids in Fig. 5.11c. As \( M \) increases the nature of the correlation does not change but, as seen for \( H_s \), the gradient does.

To summarise these results, Table 5.2 lists the \( R^2 \) coefficient of determination for a least-squares linear fit to each set of data at different viscosity/mobility ratios for \( V_{dp}, L_c, H_v, \) and \( H_s \). The value of this coefficient is a crude estimate of how good
a predictor we may construct from this data. As expected, the $R^2$ coefficient for $H_v$ and $H_s$ is consistently greater than for the other indices, and crucially, for the time to breakthrough, the ‘goodness-of-fit’ does not decrease.

![Graph](image)

Figure 5.11: The behaviour of $H_s$, $H_v$, and the dynamic $L_c$ for simulation models at viscosity ratios of 1, 10 and 100 for miscible fluids.
The behaviour at late times during the reservoir (recovery at $1 \text{ PVI}$) is more affected by large viscosity/mobility differences, reflecting the significant reduction in areal sweep due to large $M$. This is reflected in the strength of the correlation between the heterogeneity indices and the recovery at $1 \text{ PVI}$. $R^2$ decreases by $25 - 27\%$ for $H_v/H_s$, but by over $70\%$ for the dynamic Lorenz coefficient, indicating that the new indices perform well even at late times.

Table 5.2: $R^2$ coefficients of determination for the FCM line-drive reservoir models

<table>
<thead>
<tr>
<th>Breakthrough</th>
<th>$V_d$</th>
<th>$L_c$</th>
<th>$H_s$</th>
<th>$H_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M = 1$</td>
<td>0.60</td>
<td>0.67</td>
<td>0.88</td>
<td>0.90</td>
</tr>
<tr>
<td>$M = 10$</td>
<td>0.38</td>
<td>0.43</td>
<td>0.75</td>
<td>0.78</td>
</tr>
<tr>
<td>$M = 100$</td>
<td>0.40</td>
<td>0.47</td>
<td>0.76</td>
<td>0.78</td>
</tr>
<tr>
<td>Recovery at $1 \text{ PVI}$</td>
<td>$M = 1$</td>
<td>0.66</td>
<td>0.75</td>
<td>0.77</td>
</tr>
<tr>
<td>$M = 10$</td>
<td>0.34</td>
<td>0.39</td>
<td>0.65</td>
<td>0.69</td>
</tr>
<tr>
<td>$M = 100$</td>
<td>0.17</td>
<td>0.21</td>
<td>0.56</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Normalisation of results with respect to viscosity/mobility ratio

We have shown that the ranking of reservoir realisations in order of performance is preserved for different viscosity/mobility differences. This suggests that it may be possible to predict the breakthrough time for the heterogeneous cases when $M = 1$ given:

A. the breakthrough time when $M = 1$ for the heterogeneous case;
b. the breakthrough time at the required $M$ for the equivalent homogeneous reservoir.

One method of doing this is to use Eq. (2.18) and Eq. (2.15), Koval’s effective viscosity rate, to rescale the unit-mobility breakthrough times. $E = 1$ for a unit-mobility displacement, 1.88 for $M = 10$, and 4.74 for $M = 100$. This will translate into an equivalent change in gradient for $H_{vr}$ from 0.65 ($M = 1$) to 0.34 ($M = 10$) and 0.13 ($M = 100$). For $M = 10$ this calculated gradient is similar to that shown in Fig. 5.11a however for $M = 100$ Koval’s prediction overestimates the change in gradient by approximately 30%. It is likely that the Koval model is not adequately representing the mixing in these heterogeneous models with small-scale permeability variations in addition to the highly correlated channel areas. Fayers et al. (1992) found that for these models a fractional flow formulation, such as that of Koval, was not a suitable method on account of the dispersive broadening of the displacement front. The Todd and Longstaff approach was found to be a more accurate model, however this will need to be the subject of more detailed analyses.

For our purposes we have taken the simpler and more naive approach of using the breakthrough time calculated from the equivalent homogeneous models. In the miscible models this is calculated by running simulations for the homogeneous case with $M = 10$ and $M = 100$ and triggering viscous fingering randomly. As this is a random process an average of ten runs was taken. For the immiscible cases the process was repeated but with the specific relative permeability curves used for the heterogeneous cases. For the $M = 1$ FCM case breakthrough time is $1 \text{ PVI}$. This is not the case for the immiscible models, and we can calculate those breakthrough times for stable and unit-mobility displacements using Buckley-Leverett theory and for adverse mobility displacements using the compositional viscous fingering theory of Blunt.
and Christie (1994); Blunt et al. (1994). We have again adopted the more expedient approach of using numerical simulation to calculate the breakthrough times.

To show that this scaling is possible we normalised the breakthrough times for each realisation at various $M$ with the breakthrough time for the equivalent homogeneous case,

$$t_{D}^{n}_{bt}(M) = \frac{t_{D}^{n}_{bt}(M)}{t_{D}^{h}_{bt}(M)} \quad (5.1)$$

where $t_{D}^{n}_{bt}$ is the normalised breakthrough time and $t_{D}^{h}_{bt}(M)$ is the homogeneous breakthrough time at the relevant value of $M$.

Fig. 5.13 shows the same results as Figs. 5.11b and 5.12 but instead of plotting breakthrough time against $H_{s}$ we have plotted the normalised breakthrough time against $H_{s}$.

The normalised results show that all the data points, even in the cases when $M$ was greater than 1, may be reduced to the $M=1$ results by dividing by the homogeneous breakthrough times. This suggests that given the breakthrough time for an equivalent homogeneous reservoir at $M>1$ and the breakthrough time at $M=1$ for the heterogeneous case we can calculate the breakthrough time for the heterogeneous $M>1$ case.

### 5.5 IMPACT OF WELL PATTERN

In addition to fluid heterogeneity, a measure of heterogeneity should also account for different well patterns and boundary conditions. Even in homogeneous reservoirs different well-patterns will result in different breakthrough times and recovery profiles. This effect will tend to be exacerbated in heterogeneous reservoirs, so a carefully designed well pattern is important to maximise recovery. For simplicity, this chapter has initially focussed on testing different heterogeneities with a line-drive well arrangement. The nature of the shear-strain rate (and the vorticity) is that for a line drive in a homogeneous reservoir model, the shear-strain rate is zero.

For the more-realistic quarter-five-spot well arrangement, the shear-strain rate in the homogeneous case will not be zero because the principal direction of flow is not aligned with a grid face and flow diverges away from the injection well before converging towards the production well.

As we have argued that a heterogeneity index is one that compares the flow field in the heterogeneous case with that in the absence of heterogeneity we propose removing the effect of shear due to the well pattern by subtracting the homogeneous shear rate field from the heterogeneous one before calculating $H_{s}$. This involves an additional single phase pressure solve to calculate the velocity field and hence the shear rate for the homogeneous case, however as noted above this calculation is very quick compared with simulating a full miscible or immiscible displacement.
Here, we consider a well arrangement where we inject at constant rate into the bottom left corner of the reservoir model and produce from the top right. Strictly, this is not a quarter-five-spot well pattern as the length of the reservoir is greater than its width.

Fig. 5.14a shows the correlation between breakthrough time and $H_s$. The relationship is broadly similar to that seen in Fig. 5.5b for the line-drive model. The Upper-Ness formation layers are more easily ranked whilst most of the scatter is confined to the relatively homogeneous Tarbert formation. With this well arrangement recovery appears to be more sensitive to heterogeneity, as quantified by $H_s$, than with the line-drive well arrangement. However, in this case the calculation of $H_s$ is affected by the fact that the principal flow direction is not in line with the grid. This is a limitation of all uniform or non-uniform rectilinear grids using a 5-point stencil and so applies to all homogeneity/heterogeneity indices calculated in a similar way.

Compare this relationship with that in Fig. 5.14b which shows breakthrough time as a function of the dynamic Lorenz coefficient. $L_c$ is calculated using streamline time-of-flight distributions and would not be expected to have the limitation due to
grid orientation. Nevertheless, the relationship is less good, it is difficult to rank the more heterogeneous layers based on \( L_c \), whilst the less heterogeneous layers show a large degree of scatter.

The effect of the scatter related to the homogeneous Tarbert formation may be seen in the \( R^2 \) coefficients calculated for this well pattern in Table 5.3. At early time \( H_s \) (\( R^2 = 0.57 \)) has a weaker correlation than \( L_c \) (\( R^2 = 0.61 \)) for \( M = 1 \). Unfortunately the properties of the coefficient of determination mean that small differences like this may not be statistically significant. Nonetheless visual inspection of Figs. 5.14a and 5.14b shows that the homogeneity index derived from shear-rate is better able to distinguish performance between different models.

Table 5.3: \( R^2 \) coefficients of determination for the pseudo-Q5 reservoir models.

<table>
<thead>
<tr>
<th></th>
<th>( V_{dp} )</th>
<th>( L_c )</th>
<th>( H_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breakthrough</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( M = 1 )</td>
<td>0.58</td>
<td>0.61</td>
<td>0.57</td>
</tr>
<tr>
<td>( M = 10 )</td>
<td>0.40</td>
<td>0.40</td>
<td>0.45</td>
</tr>
<tr>
<td>( M = 100 )</td>
<td>0.45</td>
<td>0.45</td>
<td>0.46</td>
</tr>
<tr>
<td>Recovery at [ PVII ]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( M = 1 )</td>
<td>0.38</td>
<td>0.37</td>
<td>0.48</td>
</tr>
<tr>
<td>( M = 10 )</td>
<td>0.23</td>
<td>0.27</td>
<td>0.37</td>
</tr>
<tr>
<td>( M = 100 )</td>
<td>0.19</td>
<td>0.26</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Contrast these values for the \( R^2 \) coefficient with those in Table 5.4 where we have tabulated the coefficient of determination for only the Upper-Ness layers. All the values of the coefficients are very low and thus differences may well be statistically insignificant. Despite this it can be seen that at both early (breakthrough time) and late times (recovery at \[ \text{PVII} \]) those correlation coefficients obtained from the shear based homogeneity index are considerably higher than for either the dynamic Lorenz coefficient or the Dykstra-Parsons coefficient.

A comparison of Figs. 5.14a & 5.14b and Figs. 5.14c & 5.14d confirms that for the Upper-Ness layers the engineer will find the homogeneity index more useful than the Lorenz coefficient to rank these layers. As noted earlier the dynamic Lorenz coefficient does a very poor job at distinguishing between the expected recovery and breakthrough time in the Upper-Ness although it is better for models from the Tarbert formation.

At values of \( M > 1 \) (Figs. 5.14c & 5.14d) the gradient of the linear regression line, as expected, decreases suggesting the system is less sensitive to permeability heterogeneity and more sensitive to viscous fingering effects, for both the shear heterogeneity index and the Lorenz coefficient. However, where the more heterogeneous layers for \( H_s \) may still be ranked, the Lorenz coefficient shows even poorer discrimination between these layers.

Whilst we will explore the impact of well patterns on \( H_s / H_v \) in more detail later, there is no doubt that the current indices (\( V_{dp} \) in particular) are either unable to capture any dynamic information, or, despite taking into account the velocity field,
are unable to capture the impact of heterogeneity on flow reliably for either the linear injection pattern or the quarter-five spot ($L_c$).

Table 5.4: $R^2$ coefficients of determination for the pseudo-Q5 spot but only for layers 36-85, so the more heterogeneous Upper-Ness formation layers.

<table>
<thead>
<tr>
<th></th>
<th>$V_{dp}$</th>
<th>$L_c$</th>
<th>$H_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breakthrough</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M = 1$</td>
<td>0.20</td>
<td>0.13</td>
<td>0.27</td>
</tr>
<tr>
<td>$M = 10$</td>
<td>0.17</td>
<td>0.09</td>
<td>0.27</td>
</tr>
<tr>
<td>$M = 100$</td>
<td>0.14</td>
<td>0.07</td>
<td>0.27</td>
</tr>
<tr>
<td>Recovery at 1 PVI</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M = 1$</td>
<td>0.08</td>
<td>0.00</td>
<td>0.21</td>
</tr>
<tr>
<td>$M = 10$</td>
<td>0.13</td>
<td>0.00</td>
<td>0.20</td>
</tr>
<tr>
<td>$M = 100$</td>
<td>0.09</td>
<td>0.12</td>
<td>0.35</td>
</tr>
</tbody>
</table>

5.6 CONCLUSIONS

In this chapter we have shown that for the simple two-dimensional reservoir models,

A. $H_s$, the heterogeneity index based on the coefficient of variation of the shear-strain rate ($\dot{\gamma}$) is able to rank the different reservoir realisations in terms of both their breakthrough time and recovery at 1 PVI;

B. the ranking is better at early times than at late times;

C. the traditional static measure of heterogeneity $V_{dp}$ is unable to discriminate between the mildly heterogeneous models of the Tarbert formation and is insensitive to the realisations representing the Upper-Ness formation;

D. the dynamic Lorenz coefficient $L_c$, in a similar way, is unable to rank realisations in terms of performance and has problems discriminating between models of the Upper-Ness formation;

E. this effect is more noticeable for the most heterogeneous layers for which even a crude ranking is not possible with current indices;

F. the ranking as established by $H_s$ holds even in the presence of differences in fluid viscosity and for both miscible and immiscible fluids with a Corey coefficient of 2.

As we showed in Chapter 3, the shear-strain rate is easier to understand, physically, in terms of field performance than vorticity so $H_s$ was used to show the value of this approach. We showed in §5.3 that $H_v$ is indeed closely linked to $H_s$ and the two may be used, in most cases, interchangeably.

In future chapters reference will only be made to $H_v$ and all future calculations refer to $H_v$ and vorticity rather than the shear-strain rate.
In Chapter 5 we evaluated the effects of heterogeneity and viscosity differences on miscible and immiscible displacements. However, in many situations density differences and diffusion/ dispersion may become significant influences on recovery, and the relative importance of these forces and processes needs to be evaluated. In this chapter we use $H_v$, in conjunction with other dimensionless numbers, to analyse the relative impact of heterogeneity, buoyance effects, mobility ratio, and dispersion on breakthrough time and recovery at $1_{PVI}$ during miscible gas injection.

6.1 INTRODUCTION

In Chapter 3 we evaluated the curl of the displacement velocity as [Eq. (3.15)]

$$L \frac{\nabla \times \mathbf{V}_d}{|\mathbf{v}|} = H \frac{\mathbf{v}}{|\mathbf{v}|} + \frac{1}{\psi} \left[ R \frac{\mathbf{v}}{|\mathbf{v}|} + G \frac{\mathbf{g}}{\mu} - P_e \nabla \left( \frac{1}{|\nabla c|^2} \right) \right] \times L \nabla c. \quad (6.1)$$

We used this to introduce four dimensionless numbers to describe the initial rate of rotations at the fluid front. For this work we consider a viscosity ratio, a gravity- viscous number, a dispersion number and the vorticity based heterogeneity number, $H_v$, all defined as follows:

M  The Mobility / Viscosity ratio. For miscible fluids $M = \mu_o/\mu_d$. This is varied from 1 to 100.

$$R = \ln M \quad (6.2)$$

G  The gravity-viscous number of Crane et al. (1963),

$$G = \frac{k |\mathbf{g}|}{\mu |\mathbf{v}|} \Delta \rho, \quad (6.3)$$

where the symbols have their usual definitions. This definition is acceptable for a homogeneous reservoir however, in this study, we have used realistic reservoir models where permeability is a scalar field variable. To accommodate this we have replaced $k$ with the effective permeability of the reservoir model in the direction of mean flow ($k_{eff}$). The effective permeability was calculated using the pressure-solver method (Begg et al. 1989) detailed in Appendix A. The velocity $\mathbf{v}$ is the mean of the total
velocity calculated from the volumetric injection rate divided by the cross-sectional area.

\[ N_{TD} \] The transverse dispersion number as defined by Lake and Hirasaki (1981) for tracer flow. This was tested most recently by Tungdumrongsub and Muggeridge (2010) for more realistic fluid properties. The original definition was for reservoirs where the permeability model is layered. For heterogeneous models we have had to modify \( N_{TD} \) to

\[ N_{TD} = 14 \frac{D_m}{H|v|} \frac{k_{eff}}{k_{max}} \frac{L}{H}, \]  

(6.4)

where, as previously, \( k_{eff} \) is the effective permeability of the reservoir in the direction of mean flow, \( k_{max} \) is the highest value of permeability in the reservoir, and \( H \) is the width of the reservoir perpendicular to flow. In this analysis we have assumed that flow is affected only by diffusion, and have thus ignored the effects of convective dispersion.

Inspection of this number shows that it is actually obtained by taking the ratio of the Peclet number to the heterogeneity number,

\[ N_{TD} = \frac{D_m}{H|v|} \frac{k}{L} \frac{1}{\nabla k L}. \]  

(6.5)

\( H_v \) This heterogeneity number is calculated as described in §4.4. For the models used in this study with grids constructed out of \( 220 \times 60 \) and \( 85 \times 85 \) cells, \( H_v \) will have a lower bound of 0.19 and 0.16 respectively. This represents the physical minimum of the time to breakthrough.

We have calculated the recovery profiles for fluids of different viscosities, densities and dispersivities and for different values of \( H_v \) to understand those flow conditions when it is important that heterogeneity be modelled in detail.

6.2 Model Setup

The results in this chapter are based on simulations of those reservoir models used previously in Chapter 5 but also on sectors extracted from the SPE 10 Model 2 realisations. To avoid confusion we have labelled the two sets of reservoir realisations Model A, and Model B.

Model A consists of the 85 horizontal cross-sections/realisations from SPE 10 Model 2 introduced previously, with a grid cell aspect ratio of 1:2. Model B consists of sectors extracted from the vertical 2D cross-sections from SPE 10 Model 2. Unlike Model A these have 85\times85 grid blocks and a grid cell aspect ratio of 1 : 10. This was designed to mimic a long thin reservoir.
Table 6.1: Model properties and parameter variations used in this study.

<table>
<thead>
<tr>
<th>Constants &amp; Grid size</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal cross-sections (A)</td>
<td>2200 × 1200 ft. (220 × 60 cells)</td>
</tr>
<tr>
<td>Vertical cross-sections (B)</td>
<td>170 × 1700 ft. (85 × 85 cells)</td>
</tr>
<tr>
<td>( \phi ), Porosity</td>
<td>0.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter Variations</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_o/\mu_s ), Viscosity ratio</td>
<td>1 − 100</td>
</tr>
<tr>
<td>( G ), Gravity-Viscous ratio</td>
<td>0 − 2000</td>
</tr>
<tr>
<td>( N_{TD} ), Transverse dispersion coefficient</td>
<td>0 − 1 \times 10^{-2}</td>
</tr>
<tr>
<td>( H_v ), Permeability heterogeneity</td>
<td>0.2 − 0.85</td>
</tr>
<tr>
<td>( k_v/k_h )</td>
<td>1</td>
</tr>
</tbody>
</table>

The models were initialised with an oil saturation of one. A constant injection rate was imposed at the injection well, and fluid was injected uniformly along the entire upstream face. A constant flow rate was assumed at the production well and no-flow boundary conditions were imposed on the remaining sides. The mean flow direction was left to right. The properties of the basic simulation model and the range of the various dimensionless numbers examined are listed in Table 6.1.

6.3 Results

Mobility Ratio and Heterogeneity  The contribution of viscous instabilities to the initial rate of rotation is seen in Eq. (6.1) where we have found that it scales as \( \ln M \). This suggests that the mobility ratio can have a significant impact on recovery.

When lower viscosity solvent displaces oil, the viscosity difference across the front leads to instabilities, which in the homogeneous case will lead to the solvent fingering through the oil. For highly heterogeneous reservoirs the displacement is generally heterogeneity dominated at moderate mobility ratios although any adverse mobility ratio will exacerbate the effect of channelling. What we mean by this is that for these heterogeneity dominated cases it is heterogeneity which controls the location and the number of ‘fingers’ of fluid. In those cases where the mobility ratio dominates over heterogeneity (i.e. \( M >> H_v \)) the growth of fingers, their position, and their number is more random. In most cases very large mobility ratios are required to overcome the effect of geological heterogeneity (Waggoner et al., 1992). Both these effects will lead to a premature breakthrough of fluid and low areal sweep (Homsy, 1987).

We showed in Fig. 5.11a for \( M > 1 \) that both the breakthrough time and the sensitivity to \( H_v \) were found to decrease, suggesting that in Eq. (6.1) the viscosity term has become more significant compared with the heterogeneity term. A measure of this sensitivity is the gradient of the regression line. This gradient decreases from 0.65 (\( M=1 \)) to 0.32 (\( M=10 \)) to 0.19 (\( M=100 \)), which is consistent with the \( \ln M \) dependence of the magnitude of the viscous instability term.
Figure 6.1: The impact of mobility ratio ($M$) and heterogeneity ($H_v$) on time to breakthrough, for layers 1, 9, 12, 59, 79 from Model $A$, $N_{TD} = 0$, $G = 0$.

More specifically, we showed that for values of $M \gg 10$ breakthrough time only varies from 0.1 to 0.2 [PVI] between the most heterogeneous and least heterogeneous cases, suggesting that in this regime the levels of heterogeneity examined here were not a significant control on performance, and that the prevailing flow regime is viscous dominated.

In Fig. 6.1 we show the sensitivity of the breakthrough time to $M$ for a few layers from the most homogeneous (Layer 9, $H_v = 0.82$) to the most heterogeneous (Layer 79, $H_v = 0.31$). We find that for relatively heterogeneous models ($H_v < 0.5$) mobilities ratio from 1 to a 100 have a small impact on breakthrough time. Conversely, when $H_v > 0.5$ (e.g. Layer 9), breakthrough time can be halved as the $M$ increases form 1 to 20. The permeability maps for all these layers are in Fig. 6.2

**Dispersion and Heterogeneity** We used the transverse dispersion number ($N_{TD}$) defined in Eq. (6.4) to characterise the effect of dispersion on recovery at an adverse mobility ratio. At large values of $N_{TD}$ the contribution to the vorticity of the displacement velocity [Eq. (6.1)] is such as to reduce the instabilities due to geological heterogeneity and viscosity and density differences.

It is clear from a physical understanding of the processes in homogeneous reservoirs that are viscous dominated, transverse dispersion will cause solvent fingers to merge, and thus act to reduce the adverse effects of an adverse mobility ratio. In heterogeneous reservoirs, in addition, we would expect the effects of channelling to be reduced in a similar process. This may be seen in Fig. 6.3 where we show that as $N_{TD}$ increases the perturbations due to both heterogeneity and mobility ratio are
Figure 6.2: Permeability map for Layers 1, 9, 12, 59 & 79 from Model A

smoothed, reducing both the impact and number of high velocity fluid channels. When $N_{TD} > 0.01$ [Fig. 6.3c] the concentration distribution begins to resemble that of a homogeneous dispersion dominated displacement, an effect quite clearly seen in the effluent history shown in Fig. 6.3d.

In adverse mobility ($M = 10$) displacements in heterogeneous reservoirs we found as $N_{TD}$ is increased, breakthrough time is delayed, and the models become less sensitivity to heterogeneity (Fig. 6.4b). For instance, at $N_{TD} = 0$, breakthrough time is in the range $0.1 - 0.35$ PVI whilst at $N_{TD} > 10^{-4}$ the breakthrough range is $0.25 - 0.35$. It is the most heterogeneous models that show the biggest change in performance as dispersion increases, in line with our expectation that dispersion will reduce the number of high-velocity channels in the model. This confirms our view that $H_v$ is capturing those aspects of heterogeneity which contribute to early breakthrough. This tendency for breakthrough time to converge can be seen most clearly in Fig. 6.4a where we plot $N_{TD}$ as a function of breakthrough time for realisations with $H_v$ from 0.24 to 0.79. For values of $N_{TD} > 10^{-4}$ there is a rapid convergence to a breakthrough time of $0.3$ PVI. As $N_{TD} \to 10^{-2}$ any variation is further restricted to $\pm 15\%$ around 0.3.

This variability in breakthrough time reflects the influence of fluid channelling at early times. We found that the effect of dispersion at late times (e.g. after injecting 1 pore volume) is more pronounced. We can see in Fig. 6.4c a plot of $H_v$ as a function of oil recovery at 1 PVI at different values of $N_{TD}$. What is clear is that at $N_{TD} > 1 \times 10^{-4}$ recovery is almost completely dominated by dispersion irrespective of reservoir
Figure 6.3: The effect of dispersion on flow in the presence of heterogeneity at an adverse mobility ratio, \( M = 10, G = 0, H_v = 0.54 \).
6.3 Results

(a) Breakthrough time as a function of $N_{TD}$ for various layers from Model A

(b) Breakthrough time as a function of $H_v$

(c) Recovery at 1 pore volume injected as a function of $N_{TD}$

Figure 6.4: The impact on reservoir performance of $N_{TD}$ and $H_v$ at $M = 10$, $G = 0$. 
heterogeneity. This is because the mean location of the displacement front increases $\propto t^{1}$ (advection) whilst the instabilities (or dampening) due to dispersion grow as $t^{\alpha}$, where $0.5 < \alpha < 1$. For normal (Fickian) dispersion, (the asymptotic behaviour of the dispersion coefficient), $\alpha = 0.5$, for anomalous dispersion $\alpha > 0.5$. In most porous media as the displacement proceeds there is a transition from anomalous to normal dispersion but there are indications that in some very heterogeneous models flow may be persistently anomalous (Glimm et al. 1993; Berkowitz and Scher 1995; Sahimi 2012). In either case dispersive effects are generally seen over longer time-scales, so dispersion has had less time to act at breakthrough.

The level of heterogeneity plays a significant role, at early times, on the extent to which dispersion has a favourable influence on recovery. For $H_v > 0.6$ dispersion has relatively little impact on breakthrough time, with a maximum delay in breakthrough of about 50% (0.1 pore volumes (PV)). For the most heterogeneous cases, $H_v < 0.4$, breakthrough time can be delayed by as much as 200% (from 0.1 to 0.3 PV).

In general, dispersive effects delay the time to breakthrough and improve recovery. Specifically, for $N_{TD} > 1 \times 10^{-4}$ the contribution of the heterogeneity term to the vorticity of the displacement velocity is negligible.

**Gravity and Heterogeneity** The interaction between gravity, heterogeneity, and flow is complex and discussed in more detail in Chapter 2. In the absence of heterogeneity and with no density difference between the oil and gas, an adverse mobility ratio will result in the formation of a multitude of viscous fingers. As density contrast increases, viscous forces that drive viscous instabilities compete with buoyancy forces that attempt to create a gravity tongue. These multiple fingers are reduced to a single rapidly growing finger (Craig et al. 1957; Crane et al. 1963). The growth of this single finger of the displacing fluid (gravity tongue) is concomitant with reduced sweep recovery. The presence of heterogeneity alters the dynamics of these competing forces. We measure the relative importance of the viscous and gravitational forces using $G$, and measure heterogeneity using $H_v$.

The influence of heterogeneity on gravity dominated reservoir flows depends upon the density difference between the displacing fluid and the oil and the nature of the heterogeneity e.g. work by Tchelepi and Orr Jr. (1994), Coll et al. (2001), and Riaz and Meiburg (2003, 2004) has shown that the impact of layer ordering on gravity dominated displacements may be significant. If the displacing fluid is less dense than the oil then it will tend to rise to the top of the reservoir. Figs. 6.5 illustrates this transition from viscous to gravity dominated flow by showing the solvent distribution at 0.5 PV. As $G$ increases from 0 to 45, sweep efficiency initially improves as a result of increased vertical flow in the reservoir. Further increases in density contrast result in the formation of a gravity override, and at $G = 45$ the injected gas bypasses much of the oil. At this point the gravity term has become the dominant contributor to $\nabla \times \mathbf{V}_d$. A similar effect was seen by Crane et al. (1963) in homogeneous reservoirs in
the context of viscous fingering, and by [Tchelepi and Orr Jr] (1994) in heterogeneous reservoirs.

The vertical permeability gradient is important where there are density differences. For instance, the formation of a gravity over-ride will be accelerated if the upper regions of the reservoir are more permeable (but this will depend on the relative magnitude of $H_v$ and $G$). In contrast if the upper layer is less permeable than the development of a gravity over-ride will be delayed. Fig. 6.6 demonstrates this favourable effect of heterogeneity on recovery. For the more homogeneous reservoir (Fig. 6.6c $H_v = 0.57$) the rapid growth of the gravity override is unhindered. For the more heterogeneous reservoirs (Fig. 6.6a $H_v = 0.34$) heterogeneity disrupts vertical flow and hinders the lateral movement of the solvent making it difficult for buoyancy forces to segregate the fluids. This results in better recovery.

To understand how these processes interact to affect breakthrough time we have used both horizontal (Model A) and vertical (Model B) cross-sections from SPE10 Model 2. The vertical cross sections ($H_v = 0.34 - 0.57$) are more geologically realistic but have a narrower range of heterogeneity which necessitated the use of the horizontal cross-sections (orientated vertically) to examine the impact of a wider range of heterogeneity ($H_v = 0.25 - 0.85$) on gravitational segregation. In all simulations we have injected miscible gas into the reservoir to displace the oil.
Figure 6.6: Concentration and permeability maps showing the impact of layering on sweep and recovery when gravity dominates (vertical exaggeration ×3), NTD = 0, M = 10.

Figs. 6.7a and 6.7c show the effect on breakthrough time of Hv for G = 0 to G = 2000 for vertical and horizontal cross sections. The rapid development of the gravity tongue in the presence of a large density contrast is reflected in the reduced sensitivity of breakthrough time to heterogeneity for all cross-sections. To emphasise the difference between the impact of gravity at early time that leads to premature breakthrough and the impact of heterogeneity in the presence of gravity that can result in improved recovery we considered recovery when 1 pore volume of fluid had been injected.

Fig. 6.7b confirms our view that as heterogeneity increases (Hv decreases), recovery improves, and that performance is more sensitive to density contrast at late times than at early times. It is also very sensitive to heterogeneity. For Hv > 0.5, even small differences in density lead to a reduction in breakthrough time and recovery of up to 80%. Contrast this with Hv < 0.5 where recovery is reduced by as little as 20% and breakthrough time by 40%, reflecting the accepted view that the growth of a fast moving gravity tongue is hindered in the presence of large differences in permeability.

In general, G > 100 appears to reduce both the time to breakthrough and recovery at 1 PVI, the extent of which is determined by Hv. For Hv > 0.5 recovery is drastically reduced whilst a decrease in Hv in the presence of gravity results in improved recovery.
Figure 6.7: Performance at various G, for an M = 10 displacement, $N_{TD} = 0$. 

(a) Breakthrough time as a function of G for Vertical Cross Sections

(b) Recovery at 1PVI as a function of G for vertical cross sections

(c) Breakthrough time as a function of G for horizontal cross sections

(d) Breakthrough time as a function of G for various $H_v$
We have brought together the most important results of this chapter in the form of a phase diagram which shows breakthrough time as a function of $H_v$ and $G$ in Fig. 6.8.

At values of $G$ close to 1 breakthrough time depends on the heterogeneity index, the more heterogeneous the reservoir the earlier the breakthrough. We find that at intermediate values of $G$ there is a delay in breakthrough for the most heterogeneous reservoirs as the fluid front is smoothened by the influence of gravity which disrupts the flow of fluid through the high velocity channels. This delay in breakthrough time is not seen for the homogeneous models ($H_v > 0.5$) which do not have the same narrow high permeability channels. We find that when $G$ is greatest it is the most homogeneous models which, on average, have the lowest recovery and earliest breakthrough.

Putting this into context, we design enhanced oil recovery processes to mitigate some or all of the effects discussed in this chapter. The effective mobility ratio is often modified to provide a more favourable displacement e.g. by using polymers or water alternating gas injection (Christensen et al. 2001; Willhite and Seright 2011), or for viscous oils using steam injection (Ambastha 2008). The adverse effects of heterogeneity may be reduced by injecting polymer gels to block high permeability channels or areas that have already been swept (Liang et al. 1993), or by reducing the interfacial tension between fluids using surfactants. More radical changes may be made by changing well patterns to improve sweep.

To obtain a quantitative understanding and successfully mitigate against some of these effects we need to quantify these processes. In this chapter we have shown that Heller’s vorticity based formalism and the vorticity based heterogeneity index, $H_v$, may be used, in conjunction with the modified flow-regime dimensionless numbers $M$, $G$, and $N_{TD}$ to determine when flow is gravity, viscous, dispersion, or heterogeneity dominated. This demonstrates that the dimensionless numbers determined by the perturbation approach, which is strictly, only valid for linear systems, nonetheless provides meaningful results for real reservoir flows (which can be extremely non-linear, particularly in the presence of heterogeneity). In particular, the growth rate of perturbations to a solvent front due to heterogeneity may be modified as the front moves through the reservoir. Our analysis suggests that these further perturbations, for the realistic heterogeneous cases tested, are adequately captured by the new measure of heterogeneity, $H_v$.

Specific findings are that for $M \gg 10$ ($\ln M > 1$) viscous fingering and channelling of fluids results in early breakthrough such that the finer details of permeability heterogeneity are relatively unimportant. For more realistic cases where $M \approx 10$, heterogeneity is still a significant control on recovery. Similarly, when the transverse dispersion coefficient $N_{TD} > 0.001$ there is only a difference in time to solvent breakthrough of between $0.1$ and $0.2$ pore volumes injected between the most heterogeneous ($H_v \approx 0.2$) and least heterogeneous models ($H_v \approx 0.8$). We find, as others have,
that the effect of gravity on recovery is more complex. At early time \( G > 100 \) leads to reduced dependence of breakthrough time on heterogeneity, with breakthrough time ranging from 0.05 to 0.1 pore volumes injected for the most and least heterogeneous cases. At later times the dependence of performance on \( H_v \) is reversed. As \( H_v \) increases, recovery at 1 pore volume injected decreases and becomes more sensitive to heterogeneity even for \( G \gg 10 \). In this particular case for reservoirs with \( H_v < 0.4 \) (i.e. very heterogeneous) almost 40\% of the oil is recovered after injecting 1 pore volume of solvent, whilst for \( H_v > 0.5 \) recovery is only 10 to 20\%.

The range of values for \( M, G, \) and \( N_{TD} \) we have tested cover the broad range of reservoir conditions that may be found in most conventional fields. \( M \), for instance, was varied from 1 to 100. This would cover most water-floods and gas injection into conventional oils, however, it is likely that some EOR schemes for heavy oils may fall outside this range. There are cases where gas-oil viscosity ratios of over 500 are present, with correspondingly large differences in gas and oil densities.

Dispersion is only important on scales much smaller than the inter-well scale, so if we neglect dispersion we can imagine the relative impact of \( G, M, H_v \) for a given reservoir (and as a function of different realisations of the geological model) as forming a 3-dimensional parameter space, where only a subset of the space will give the best recovery (Fig. 6.8). EOR processes may then be designed to avoid these regions.

In this chapter we have only investigated 2D geological models from SPE10 Model 2 rather than constructing our own geostatistical models. In many cases purely statistical models of permeability heterogeneity are unrealistic and too homogeneous for the purposes of this study. The advantage with the models we have used is that they are based on real reservoir data and represent two different depositional environments. Their behaviour is also extremely heterogeneous, marked by low recovery and very early breakthrough, which made them ideal subjects for the testing of heterogeneity indices.

We have also modified the transverse dispersion and the gravity-viscous numbers so that they could be used for heterogeneous reservoirs, by introducing the effective permeability of the reservoir. A more thorough investigation into the most appropriate measure of heterogeneity is required as it is possible that much of the scatter seen in our results is due to the use of \( k_{eff} \). Ideally, we would like to unify the heterogeneity index, \( H_v \), in a consistent way, with those dimensionless numbers that depend on a single measure of heterogeneity.
6.4 Discussion and Conclusions

Figure 6.8: A surface plot of breakthrough time as a function of $H_v$ and $G$ for an adverse mobility $M = 10$ miscible displacement process. This is an interpolated surface using smoothed data from approximately 250 simulations of layers from Model A and using fluids of different densities.

Figure 6.9: 3D surface plot of breakthrough time as a function of $H_v$ and $G$. 
EXTENSION TO THREE DIMENSIONS

In many reservoir simulation studies used to evaluate the efficiency of the displacement processes we have discussed, simple 2D orthogonal models are appropriate, however full-field models tend to be non-orthogonal and 3D, so that the finer scale details of complex geology may be captured. In this chapter we investigate the possibility of a vorticity-based heterogeneity index for 3D models.

7.1 VORTICITY IN 3D

The theoretical basis for the vorticity based heterogeneity index introduced in Chapter 3 relied on an analysis of the growth rate of perturbations to the displacement front. No specific assumption was made about the two- or three-dimensional nature of the velocity field. However, when a fluid is confined to a plane (the 2D case), the vorticity vector is orthogonal to that plane. Its direction is constant whilst its magnitude may vary over the reservoir. In the 2D case this means that the only non-zero component of vorticity is $\Omega_z$.

The curl of a velocity field, defined in 3D is,

$$\Omega = \begin{pmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{pmatrix} = \begin{pmatrix} \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} \\ \frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y} \\ \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \end{pmatrix}.$$  \hspace{1cm} (7.1)

This means that there are now three non-zero components of vorticity, and so both the direction and the magnitude of vorticity may vary over the reservoir. The physical interpretation of this is that the axis of rotation (and so the plane in which the front is perturbed) may tilt as the fluid front progresses through the model. This difference between 2D and 3D has important consequences for free fluids; where in two-dimensions vortices that are created tend to clump together to form larger vortices, in three-dimensions vortices that are formed tend to break into smaller vortices. This creates a fundamental difference in turbulence dynamics between the two systems. However, in reservoir models this is unlikely to be the case as flow at reservoir rates is in the creeping flow regime (low Reynolds numbers), and flow in real reservoirs is constrained by the porous medium.

The heterogeneity index $H_v$, as currently defined, is the inverse of the coefficient of variation of $|\Omega|$. We will investigate whether the standard definition of $H_v$ provides a suitable ranking for 3D reservoir models or whether the three individual components of vorticity provide any additional insight into reservoir behaviour.
The primary concern is that in 3D there is no realistic well pattern to approximate the line-drive boundary conditions imposed on the 2D models in our earlier simulations. The well pattern we will use are two vertical wells opposite each other, where each is completed over the reservoir interval. This is the 3D equivalent of the pseudo-Q5 spot well pattern we investigated in Chapter 5. In that case the results were not satisfactory. It is possible that 3D results may have a similar problem; being affected by the flow geometry rather than problems associated with calculating or interpreting vorticity in 3D.

We will test three permeability models in this chapter:

1. a lens permeability model, with a high permeability block embedded in a homogeneous background,

2. a set of reservoir models where we have set \( k_v = k_h \),

3. a set of reservoir models where we have set \( k_v = k_h / 10 \).

Initially, we examine models with regular Cartesian geometry, but we end this chapter by providing a method to calculate vorticity for a corner-point geometry case.

For simplicity we ignore viscosity and density differences. These will be the topic of future work. However viscosity differences were investigated in Chapters 5 & 6 for 2D models. Whilst it is generally accepted that viscous fingering behaviour is not radically different between 2D and 3D models for line-drive displacements, simulations by Riaz and Meiburg (2002) have shown that for the quarter-five spot geometry 3D models exhibit greater vorticity, and thus a more variable concentration gradient which leads to faster growing fingers and earlier breakthrough. Density differences for 2D models were investigated in Chapter 6, and it is well known that density differences in three dimensions leads to significantly more complex flow behaviour, which is outside the scope of this chapter.

### 7.1 Methodology - Regular Cartesian Geometry

In regular Cartesian geometry the vorticity was calculated from the face-centred velocity field, as described in Chapter 5. Averaging was used to construct a block-centred velocity field and then the central finite-difference method was used to estimate velocity gradients. The models were set up with an injector-producer pair diagonally across the model, as shown in Fig. 7.4. We modelled a matched-viscosity matched-density displacement where injection was at a steady rate and uniform along the vertical wells.

These boundaries are realistic in their geometry, however uniform injection over the interval is unlikely to be realistic. Permeability varies vertically and it is the permeability of the rock at the well perforations (in reality) or the grid block permeability (in the model) which governs the amount of fluid injection into that layer. The calculation of the vorticity heterogeneity index will depend on these boundary conditions,
but the nature of the permeability will govern the difference in index between the two well models. In models where there are layers with very low permeabilities fluid will move from the lower permeability to the higher permeability layers. If this happens close to the well bore then the resultant high levels of vorticity may dominate the heterogeneity index calculation, even when the model is generally homogeneous. In cases such as this it is probably better to inject at a constant rate and allow the permeability of the well grid blocks to govern that amount of flow into each layer.

The first reservoir model we have tested consisted of a high permeability lens embedded in a homogeneous background. We set $k_v = k_h$, with the lens permeability ten times greater than the background. This is an analog for the 2D lens permeability model discussed in Chapter 3 which allowed us to check whether the results from the vorticity calculation were plausible.

The geologically realistic reservoir models were extracted from SPE10 Model 2 by splitting the model into two reservoirs; the top 35 layers represented the Tarbert formation and the bottom 50 represented the fluvial upper-Ness layers. Each
The reservoir was then split areally and vertically. The Upper-Ness formation layers were used to create twelve separate models [e.g. Fig. 7.2a], and the Tarbert layers were used to create eight models [e.g. Fig. 7.2b]. Sixteen of these models were constructed with $110 \times 30 \times 15$ grid blocks, whilst the remaining four were slightly thicker with $110 \times 30 \times 20$ grid blocks. Each grid block had dimensions of $3 \times 6 \times 0.6$ m. We used the values of $k_h$ from SPE10 Model 2, with $k_v$ calculated using $k_h$ based on the level of anisotropy required. We ran two sets of simulations, one isotropic, and one anisotropic with $k_v/k_h = 0.1$. The model porosity was a constant. This provided us with twenty geologically realistic heterogeneous models.

### 7.1.2 Results and Discussion

**Lens Permeability Model** We show tracer concentrations greater than 0.5 after injecting 0.5 PV into the reservoir in Fig. 7.3. The primary features of a lens model are apparent; the water velocity is greater in the lens than the background, and there is flow between regions at the boundaries of the lens. In the 2D model we found that vorticity was large where the permeability gradient was large perpendicular to the direction of flow i.e. at the edges of the lens. In 3D we might expect that the magnitude of vorticity would be large along each face of the lens, and we would expect different components of vorticity to be large on different faces. For instance, $\Omega_z$ represents perturbations to the front in the $x-y$ plane, so this should identify those faces perpendicular to the $x-y$ plane. $\Omega_x$ and $\Omega_y$ should represent perturbations to the front in the $y-z$ and $x-z$ planes.

What we find is consistent with the 2D picture, we find the boundaries of the lens (the edges and faces) are identified clearly by $|\Omega|$ in Fig. 7.4. However, the individual components of vorticity identify only certain vertices. $\Omega_z$ does not include derivatives in the $z$ direction so does not identify the upper and lower faces, whilst $\Omega_x$ and $\Omega_y$, on the other hand, identify only the upper and lower faces of the lens. This is because both of these components involve derivatives of $\delta v_z$ which will be zero between the upper and lower faces of the lens and small at the faces where there may be some cross flow, particularly because the permeability contrast between the two regions in this model is only 10. These components also involve derivatives of $v_x$ and $v_y$ in the $z$ direction which will also be zero except at the upper and lower faces. We find the behaviour of this model, as far as sweep and recovery are concerned, is similar to the 2D model, and so we would expect $1/C_v(\Omega_z)$ to provide the best measure of the impact of heterogeneity on recovery.
7.1 Vorticity in 3D

Flow direction

Figure 7.3: The lens permeability model showing the concentration at 0.5 PVI, with the concentration clipped at 0.5.

Figure 7.4: The different components of vorticity for the 3D lens model. Note the scale in each case represents the unnormalised minimum and maximum of the plotted data.
isotropic heterogeneous models, $k_v/k_h = 1$ First we show results for the more correlated permeability models from the Upper-Ness formation. Fig. 7.5a shows the time to tracer breakthrough as a function of $H_v$ for these models. For comparison we have plotted $H_v$ as a function of breakthrough time for the 2D models in the background. It is clear that the breakthrough time for these models shows approximately the same sensitivity to $H_v$ as it did for the 2D models, despite the very different geometries between the two sets of models. The 2D models were line-drive geometries whilst these models use a pseudo-Q5-spot arrangement. We find that a change in $H_v$ of 0.2 leads to a delay in breakthrough of approximately 0.2 PVI. The relation between the two is linear with a correlation coefficient, $R^2$ of 0.88.

To investigate the usefulness of the individual components of vorticity we show $1/C_v$ of $\Omega_x$, $\Omega_y$, and $\Omega_z$, which we have labelled $H_{vx}$, $H_{vy}$, and $H_{vz}$. Based on the results of the lens permeability model we would expect $H_v$ and $H_{vz}$ to be similar. We actually find that the correlations with each component is strong, and all three give the same ranking, however the correlation coefficient is weakest for $H_{vz}$ ($R^2 = 0.75$).

Compare the performance of $H_v$ with that using the Dykstra-Parsons coefficient. Fig. 7.5e shows breakthrough time as a function of $V_{dp}$ for all the isotropic 3D models, both from the Upper-Ness formation and the Tarbert formations. $V_{dp}$ for the Upper-Ness models is approximately 0.98 despite breakthrough time varying from 0.1 to 0.3 PVI. For the Tarbert formation models $V_{dp}$ varies from 0.86 to 0.92 but shows little correlation with performance.

To demonstrate, qualitatively, that $H_v$ is capturing the difference in sweep between models we have shown concentration maps for two Upper-Ness models in Fig. 7.6, one which is relatively homogeneous ($H_v = 0.5$, breakthrough at 0.28 PVI) and one which is more heterogeneous ($H_v = 0.28$, breakthrough at 0.09 PVI). The figures are both clipped at a tracer concentration of 0.5 to given an indication of the swept volume at breakthrough. The one which appears most heterogeneous using $H_v$ has only a small region of the reservoir swept. The Dykstra-Parsons coefficient for both models is 0.98.

The results for the Tarbert formation layers are shown in Fig. 7.7. They have been overlain on those from the Upper-Ness formation. We find that when result from both formations is combined the correlation is significantly weaker, the greater scatter being introduced by the Tarbert results. This mirrors our findings from the results of §5.5 for the 2D pseudo-Q5 spot well pattern models. In that case it was found that breakthrough time for the Upper-Ness formation layers correlated well with $H_v$ but the correlation was weaker for the Tarbert formation layers.

The Tarbert formation layers, and particularly these 3D models, do not have large permeability differences between regions, and those regions with higher permeabilities are not correlated on significant length-scales. The typical correlation length in the Tarbert region is approximately 1/3 of the inter-well distance in the x direction. With the Q5-spot pattern in 3D the apparent correlation length in the direction of flow is reduced even further. This lack of correlation means that $H_v$ provides a more
Figure 7.5: (a-d): Breakthrough time against $1/C_v$ for each of the three components of vorticity and the magnitude of vorticity for 3D reservoir models from the Upper Ness formation undergoing a matched-viscosity matched-density Q5-spot flood. (e): Breakthrough time as a function of $V_{dp}$. 

(a) $H_v, \Omega, R^2 = 0.88$
(b) $H_{vx}, \Omega_x, R^2 = 0.84$
(c) $H_{vy}, \Omega_y, R^2 = 0.80$
(d) $H_{vz}, \Omega_z, R^2 = 0.75$
(e) $V_{dp}$
pessimistic view of performance, as we can see from Fig. 7.7a, where the Tarbert formation models show later breakthrough than the Upper-Ness formation layers with the same value of $H_v$.

It is possible to understand this by considering a reservoir model with a random uncorrelated distribution of permeability, or a checked permeability model, as discussed in §5.2.2. Such a model will behave homogeneously but $H_v$ may indicate a heterogeneous reservoir on account of the small-scale variations in vorticity. This discussion is also related to that on grid refinement in §5.3 where we found that for the homogeneous models vorticity was dominated by just one derivative of the velocity field. This explains why the strongest correlations we see are using $\Omega_y$ and $\Omega_z$, whilst the asymmetry we find between the two is due to the aspect ratio of the models, which are longer in $x$ than in $y$.

What we can deduce from these results is that different components of vorticity will be useful for different depositional environments. For instance, in reservoirs where there are lots of baffles to flow, fluid may trickle vertically as well as horizontally to reach the production well. In this cases we would assume that the components of vorticity that are large at the upper and lower boundaries of channels may best capture the effect of heterogeneity on recovery. Conversely, in cases where there is little vertical movement recovery will be controlled by the injection boundary conditions (the relative amounts injected into each layer). For uniform injection models we would presume that the $x$ and $y$ components of vorticity, which identify lateral channel boundaries, best capture the impact of heterogeneity on recovery.

**Anisotropic models** The anisotropic models were investigated to understand whether $H_v$ could be calculated and whether it would provide an acceptable ranking of these models in terms of their performance. We would expect these layers to behave in a similar to the 2D models; with a low $k_v/k_h$ there will be relatively small levels
of cross-flow between layers. We found that the correlations, shown in Fig. 7.8, with breakthrough time as a function of $H_v, H_{vx}, H_{vy}, H_{vz}$ for both the Tarbert and Upper-Ness formations, are weaker. However the strongest correlation is still present for the more correlated models of the Upper-Ness models, with their highly permeable fluid channels. As in the isotropic case, we found that $\Omega_z$ does provide a better correlation than $\Omega$ or $\Omega_y$, however, the strongest correlation is still $\Omega_y$. This is perhaps because these models have their fluid channels broadly in the y direction.
Figure 7.7: Isotropic: Breakthrough time against $1/C_v$ for each component of vorticity and the magnitude of vorticity for 3D reservoir models from both the Upper-Ness and the Tarbert formations.
Figure 7.8: Anisotropic: Breakthrough time against $1/C_v$ for each of the three components of vorticity and the magnitude of vorticity for 20 3D anisotropic reservoir models undergoing a matched-viscosity matched-density Q5-spot flood from Upper-Ness and Tarbert formations.
7.2 VORTICITY ON CORNER-POINT GEOMETRY

At the time of writing the vast majority of field-scale reservoir simulations are carried out on corner-point grids. Corner-point grids are structured but distorted and so are better able to represent the complex structure of subsurface reservoir geometry. Geological features like faults or the merging of two distinct horizons are difficult, if not impossible, to represent using uniform or non-uniform rectilinear grids.

To calculate vorticity a robust estimate of the velocity gradients over the grid is required. The finite-difference method normally used approximates these derivatives using a Taylor series expansion. Partial derivatives of this sort require the underlying velocity field to be sampled on an regular Cartesian grid where these derivatives are aligned to lines parallel to the coordinate axes. In the case of numerical simulation using corner-point or unstructured grids the velocity field is often not sampled on a rectangular grid making it difficult to calculate derivatives. This problem is not unique to reservoir engineering and occurs in most applications of computational fluid dynamics.

One category of solutions to this problem involves re-sampling, by interpolation, the velocity field on to a fine rectilinear grid, and then estimating the vorticity field using finite-difference methods. Another involves using methods such as Delaunay triangulation to re-grid the problem and calculate velocity gradients. Both of these are unsuitable for large reservoir models. Fine-scale geological models contain millions of grid blocks, superimposing a finer grid or generating another grid is computationally expensive, leads to errors where the velocity field may not be continuous (as may occur in Delaunay gridding) and requires the engineer to manually decide the level of re-gridding required and the interpolation technique suitable to the problem. In most cases this would introduce considerable error to the problem.

An alternative approach is to use MPFA, multi-point flux approximation [Edwards 2002], or MIMETIC [Klausen and Russell 2004] methods which reduce grid orientation effects, and which have generic implementations for non-orthogonal grids/polyhedral cells. These have discrete differential operators that will allow us to calculate the vorticity reliably.

For our calculations we have simply used a directional derivative to estimate the gradient. If we define a scalar field $\psi$ at some point $p$ with point $p_j$ in the neighbourhood, and $v_j = p_j - p$, then the directional derivative is,

$$\frac{\Delta \psi}{|v_j|} \approx \frac{v_j}{|v_j|} \cdot \nabla \psi.$$  \hspace{1cm} (7.2)

Applying Eq. (7.2) for all points $p_j$ in the neighbourhood results in a system of simultaneous equations that may be solved for $\nabla \psi$. This system of equations is over-determined and requires the relatively inexpensive inversion of a three-by-three matrix for each data point. This calculation is often less expensive than re-gridding the
model and calculating velocity gradients by interpolation, and reduces to the central-finite-difference approximation for a regular Cartesian grid.

This derivation assumes that,

1. the velocity field is continuous and differentiable over the calculation domain,

2. and that there are enough adjacent grid points which are approximately collinear.

The first assumption may not be entirely valid for highly fractured reservoirs, those with significant faults, or those with thin shales, especially if these shales have been modelled as transmissibility barriers. The second assumption requires that the corner-point grid should not be too non-orthogonal. In reality most reservoir simulators that require structured grids will not be able to use corner-point grids that are too distorted.

7.3 Conclusions

3D models present challenges that were not present in their 2D counterparts, even in the absence of gravity and density differences. We showed in chapter 5 how $H_v$ could be used to accurately rank 2D linear-displacement reservoir models in terms of their performance. This allowed the accurate quantification of the impact of geological heterogeneity on recovery. It was also found that with the pseudo-Q5 well spot pattern the correlation was much weaker, and in particular was dependent on the nature of the heterogeneity, with the more heterogeneous Upper-Ness models more easily ranked.

The 3D results shown in this chapter reflect the findings for the pseudo-Q5 well spot pattern. $H_v$ and the other components of vorticity may be used to rank these reservoir models, but the accuracy is not as good as for the 2D models. However, the results for the isotropic Upper-Ness formation layers using $H_v$ show a remarkably linear relationship that compares well with the 2D results. The Tarbert formation layers however show a weaker correlation, however, the ranking ability of $H_v$ must be compared with $V_{dp}$ which was unable to rank any of our models reliably.

The vorticity based heterogeneity index in three-dimensions needs more investigation. This study was restricted in the number of reservoir models we could use and in their size. The models were constructed by splitting SPE 10 Model 2 which meant they were less realistic than they would be otherwise, with many of the fluvial channels split along arbitrary lines.

Having introduced a 3D index we would suggest that as most real reservoirs are much larger areally than they are vertically, it may be that breakthrough time is best assessed by estimating areal sweep rather than vertical sweep. In that case, one suggestion would be to rank reservoirs by estimating $H_v$ for each horizontal cross-section rather than calculating a 3D index.
Despite the challenges with understanding the significance of $H_v$ for 3D models we have shown that $H_v$ is better able to capture the effect of heterogeneity on recovery than $V_{dp}$ which not only does not account for well arrangements but also cannot be calculated for models where the permeability field is anisotropic.
Vorticity in its two-dimensional form is a result of curvature of the flow, and of differences in flow-velocity in the direction perpendicular to flow. In this Chapter we will attempt to understand, more thoroughly, the nature of vorticity in the reservoir by using a system of natural co-ordinates which make it easier to place a physical interpretation on the link between vorticity and the flow.

Our motivation stems from the results of Chapter 5 where we demonstrated the value of using $H_v$ to rank reservoir realisations for direct line-drive models (see Fig. 5.11). The preliminary investigation into its use for the quarter-five spot well pattern and for 3D models (Chapter 7) showed it to be less suitable for these cases.

$H_v$ depends on the velocity field, as defined on a finite-difference grid. In the case of a direct line-drive the vorticity in the homogeneous case is zero, so any non-zero values are due to reservoir heterogeneity, viscosity, or density differences, and not related to the pattern geometry. However, for a different well model or reservoir geometry, there will be contributions to vorticity even in the absence of heterogeneity. This means the heterogeneity index for these cases will be different.

This is a useful property of $H_v$ as pattern geometry is a significant control on recovery (Morel-Seytoux 1966). However the results show that $H_v$, used directly, has a weaker correlation with recovery for other well patterns.

The ideal heterogeneity index should allow the engineer to compare reservoir realisations for a given well pattern with other models that differ both geologically and in the location of wells. Such an index would be valuable at the development phase of a field where the engineer may be interested in finding the well arrangement that provides the smallest range in possible breakthrough times given the uncertainty in reservoir heterogeneity.

To explore this further we propose, in this chapter, to examine how a similar or equivalent heterogeneity index may be calculated using ideas from streamline simulation. We will show how the vorticity field may be decomposed into different components which may be used to understand the effect of heterogeneity and well pattern on recovery in a way that allows us to compare different well-patterns and different reservoir realisations within the same framework.

**Notation**

In this chapter we will model other measures of heterogeneity using the same overall model used for $H_v$. They will be defined as the inverse of the coefficient of variation.
8.1 definitions

Consider the 2D Cartesian system of natural co-ordinates \((\hat{n}, \hat{s})\), shown in Fig. 8.1. \(\hat{s}\) is the unit vector in the direction of flow and \(\hat{n}\) is the vector that points perpendicular and to the left of the direction of flow. The velocity field in such a system is defined as

\[
v = v(\hat{n}, 0),
\]

\[
= v \hat{s}.
\]

We proceed by writing the curl and the divergence of a vector field in these natural co-ordinates.

The curl of the velocity-field, \(\nabla \times \mathbf{v}\), in two dimensions was presented in Eq. (3.41) as,

\[
\Omega_z = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}.
\]

The magnitude of the vorticity, as discussed previously, is a measure of the rotation of a material element of fluid that is exposed to a flow-field. Such a parcel of fluid actually has two rotational motions.

The first is due to velocity differences across its body, as shown in Fig. 8.2a. This is, effectively, due to the change in \(|\mathbf{v}|\) perpendicular to the direction of flow (streamlines). In porous media this would capture the growth of viscous fingers or the channelling of fluids in a high permeability region.

The second rotational motion is due to the curvature of streamlines. Suppose a parcel of fluid is confined to a tube, and suppose that the tube is curved. Fig. 8.2b shows such a tube filled with an inviscid fluid, i.e. the fluid flow within the pipe is uniform with no boundary-layer effects. In the trough the parcel of fluid will rotate anticlockwise (positive vorticity), in the peak it will rotate clockwise (negative vorticity), and at the point of zero curvature there will be no rotation. There is no fluid-shear in the
pipe so the only contribution to vorticity is due to the curvature of the pipe. In porous media we can envisage streamlines being curved for two reasons. Firstly due to well pattern, and secondly due to permeability or porosity heterogeneity. This will be because fluid is moving from one layer or region to another (cross-flow). These two different rotational motions are easily quantified by re-writing Eq. (8.3) in natural co-ordinates as,

\[ \Omega_z = v \frac{\partial \beta}{\partial s} - \frac{\partial v}{\partial n}, \]  

(8.4)

\[ = \frac{1}{R_s} \frac{\partial v}{\partial n}, \]  

(8.5)

where \( \beta \) is the angle between the local co-ordinate system and the direction of flow and \( R_s \) is the radius of curvature of the streamline. The first term is the contribution to the angular velocity of the particle due to curvature of the streamlines, and the second is the contribution due to shear perpendicular to flow. This allows us to move away from a representation of the fluid in terms of changes parallel to the \( x \) and \( y \) axes which are arbitrary, to a system which represents the fluid in terms of the flow direction, which does have physical meaning.

In 2D it is common for flow to have both shear and curvature. In most cases, in porous media, these contributions will differ, particularly for realistic geological models where the heterogeneity is stochastic in nature. However, it is possible that the contributions from both curvature and shear are of equal magnitudes and opposite signs that the total vorticity is zero, even though the vortex is rotating rapidly. This is often associated with tornadoes in the atmosphere.

The divergence of the velocity-field, \( \nabla \cdot \mathbf{v} \), in regular co-ordinates is,

\[ \nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}. \]  

(8.6)

In natural co-ordinates this may be written as,

\[ \nabla \cdot \mathbf{v} = \frac{\partial v}{\partial s} + v \frac{\partial \beta}{\partial n}. \]  

(8.7)

The first term is the change in \( |v| \) in the direction of flow and so represents the expansion (or contraction) of a parcel of fluid parallel to the streamlines. We identify
this term as the acceleration [of the particle]. The second term may be identified as
the diffuence or confluence term. In terms of streamlines this represents streamlines
diverging from each other (or converging). Both these terms represent the expansion
or contraction of a parcel of fluid.

In porous media with incompressible fluids the divergence of the velocity field in
the reservoir is zero which places a strict condition on the acceleration or diffuence
terms. This will mean that when characterising heterogeneity we would not expect
the divergence of the velocity field to give any information, however the diffuence
component should, in a similar way to curvature vorticity, represent the extent to
which streamlines are diverging.

Consider a reservoir with higher permeability at the top than at the bottom into
which we inject uniformly along the upstream face and produce uniformly from the
downstream face [Fig. 8.3]. By setting the vertical permeability, \(k_v\), to zero we ensure
that there is no cross-flow between layers. This gives a velocity field that only changes
perpendicular to flow. The curvature component of vorticity in this case is zero so the
only rotational motion experienced by a parcel of fluid in this stream will be due to
shear-vorticity.

In the alternative case where \(k_v\) is not zero, as is more likely in a real reservoir,
there will be flow both between as well as within layers. The streamlines in this
case, shown in the cartoon in Fig. 8.4a will curve so that even in the absence of
any velocity differences between streamlines the parcel of fluid will rotate due to
curvature vorticity. Thus, both curvature and shear vorticity are likely to arise due to
permeability differences in the model. However as rotation due to curvature vorticity
merely requires streamlines that change direction it may also be caused by different
well arrangements. For instance, in the case of Fig. 8.4b where flow is from the bottom
left to the top right, the streamlines are curved. In a real reservoir with this geometry
there will also be a higher velocity along the centre of the model and lower velocity
towards the edges.

To summarise, permeability heterogeneity may give rise to both curvature and
shear vorticity; curvature vorticity when there is cross-flow between layers, and shear
vorticity when there is channelling or viscous fingers being formed. The location of
wells and the nature of the boundary conditions, whether constant rate or constant
pressure, will contribute to curvature vorticity in the reservoir, as well as shear and
divergence. If the fluids are compressible then the divergence, diffluence and acceleration components may be non-zero and may perhaps be useful to consider when assessing the effect of heterogeneity on recovery.

8.2 Calculation of Vorticity and Divergence

The divergence and curl of a 2D vector field may be calculated using Eqs. (8.3) & (8.6) and the method discussed in §3.6 for the numerical evaluation of vorticity on a regular Cartesian grid. In Eqs. (8.4) and (8.7) we can identify $\frac{\partial v}{\partial s}$ as the flow-shear in the direction of flow and $\frac{\partial v}{\partial n}$ as the flow-shear perpendicular to flow. These two components can be calculated directly by estimating,

$$
\hat{s} \cdot (\nabla \cdot \mathbf{v}) = \frac{v_x}{|\mathbf{v}|} \frac{\partial |\mathbf{v}|}{\partial x} + \frac{v_y}{|\mathbf{v}|} \frac{\partial |\mathbf{v}|}{\partial y},
$$

$$
\hat{n} \cdot (\nabla \cdot \mathbf{v}) = \frac{v_x}{|\mathbf{v}|} \frac{\partial |\mathbf{v}|}{\partial y} - \frac{v_y}{|\mathbf{v}|} \frac{\partial |\mathbf{v}|}{\partial x}.
$$

(8.8)

The finite-volume formulation used in most simulators, including MISTRESS, defines the velocity on a face-centred grid. For the purposes of this calculation we have used Eqs. (3.43) and (3.43) and the method detailed in §3.6 to calculate $v_x$ and $v_y$.

If the velocity, $\mathbf{v}(v_x, v_y)$, is defined on a block-centred grid as shown in Fig. 8.5, then by using the central finite-difference approximation we may estimate the derivatives using,

$$
\frac{\partial v}{\partial s} = \frac{v_x(i,j)}{v(i,j)} \frac{v(i+1,j) - v(i-1,j)}{2\Delta x} + \frac{v_y(i,j)}{v(i,j)} \frac{v(i,j+1) - v(i,j-1)}{2\Delta y},
$$

$$
\frac{\partial v}{\partial n} = \frac{v_x(i,j)}{v(i,j)} \frac{v(i,j+1) - v(i,j-1)}{2\Delta y} - \frac{v_y(i,j)}{v(i,j)} \frac{v(i+1,j) - v(i-1,j)}{2\Delta x}.
$$

(8.9)

The curvature vorticity and diffluence components of vorticity and the divergence may now be calculated using Eqs. (8.4) and (8.7).
We have now split the curl and the divergence of the velocity field into four components each. We now use the same form as $H_v$, the inverse of the coefficient of variation of a scalar field, to write down the following additional measures of heterogeneity:

1. $H_{cv}$, based on the curvature vorticity component of vorticity,
2. $H_{sv}$, based on the shear vorticity component of vorticity.

We propose to test $H_{cv}$ and $H_{sv}$ as we believe these will capture the effect of the two processes that may occur in a reservoir that will contribute to recovery. The divergence should generally be zero in the reservoir unless the fluids are compressible. The nature of the grid will mean that the divergence will be non-zero in those grid blocks that have an injection or a production well.

8.3 Models examined

One of the goals in this section is to show how the combined effect of well pattern and heterogeneity on recovery may be estimated using a single heterogeneity index. We found in previous chapters that $H_v$ was less reliable for non-line drive reservoir models. For that reason the reservoir models studied are the 2D layers Model A as described in §4.2.1 where we have varied the locations of the injection and production wells.

We begin by using a simple lens permeability model to demonstrate the relationship between the different components of vorticity. We then verify the ability of $H_{cv}$ and $H_{sv}$ to rank direct line-drive reservoir models. Finally we use these results to understand how we may compare different well-patterns and different reservoir realisations in the same framework. Fig. 8.6 shows the different well patterns used.

The line-drive and pseudo-Q5 spot simulations were carried out using all 85 layers from Model A, however simulations using other well-patterns only used the 50
Figure 8.6: The different well arrangements considered in this chapter. A solid line represents completions in an injection well, a dashed line represents completions in a production well. The 0.5 and 1.0 lines of isoconcentration are displayed after 1 PVI of FCM fluid to show the dependence of breakthrough time on pattern geometry.

Upper-Ness formation layers and 10 representative layers from the Tarbert formation, forming 60 different reservoir realizations. The reduction in the number of Tarbert formation layers was primarily to reduce the time taken for these simulations, however as the Tarbert formation layers are less heterogeneous, a sample of these models were sufficient to demonstrate the concept.

In this chapter we do not consider viscosity and density differences, and dispersive processes. We showed in §5.4 that for adverse mobility displacements the sensitivity of breakthrough time to $H_v$ was changed however the ranking was maintained. Both density differences and dispersive effects are ignored as they are dealt with in Chapter 6.

8.4 results

Fig. 8.7 demonstrates using a reservoir model the link between the components of vorticity and streamlines. The reservoir model is a rectangular homogeneous model with a higher permeability lens embedded in the centre. The peak vorticity in the model is in two locations, the upper and lower boundaries of the higher permeability block and at the four corners. The boundaries have high vorticity due to shear, the velocity in the higher permeability layer is much greater than that in the lower permeability region. This is also where there is a large permeability gradient perpendicular to flow. The four corners are those where there is the greatest turn in the streamlines;
these are regions where streamlines curve taking fluid into the higher permeability layer.

Figure 8.7: Maps showing vorticity, curvature, shear terms overlain on streamlines for a lens permeability map. The streamlines were launched at fixed intervals from the left face.
8.4.1 Line Drive

We showed the strong correlation between breakthrough time and \( H_v \) for line-drive models in §5.2. Fig. 8.8 shows the equivalent graphs for \( H_{cv} \) and \( H_{sv} \), the indices calculated from the curvature and shear components of the vorticity field. For these models \( H_{cv} (R^2 = 0.9) \) correlates as well as \( H_v (R^2 = 0.9) \) with breakthrough time and both can be used in this case to rank the models in terms of sweep.

Compare this strong correlation with that between \( H_{sv} \) and breakthrough time in Fig. 8.8b. The correlation is weaker with a \( R^2 \) correlation coefficient of 0.86. The correlation exhibits both greater scatter and greater sensitivity over the breadth of heterogeneous models that were tested. For instance, a change in \( H_{sv} \) from 0.2 to 0.5 results in a delay in breakthrough from 0.1 to 0.6 PVI. The greater scatter is mainly due to layers from the Tarbert formation, which unlike the heterogeneous Upper-Ness formation layers, do not have well defined and highly permeable fluid channels in the direction of flow.

We calculated the mean difference between the value of \( H_{cv} \) and \( H_{sv} \) for each layer (Table 8.1). For the upper-Ness formation layers the difference in value between the two indices was negligible at 0.05. The Tarbert formation layers have an average difference of 0.27 between the two indices.

To understand this consider a layer from each formation. Figs. 8.9a and 8.9b show the permeability map for layer 21 from the Tarbert formation and layer 62 from the Upper-Ness formation. Layer 21 represents a prograding near-shore environment with a smaller variation in \( k_x (V_{dp} = 0.87) \) than layer 62 (\( V_{dp} = 0.98 \)), which is from a fluvial channel environment with a high permeability channel embedded in a very

<table>
<thead>
<tr>
<th>Table 8.1: Comparison between ( H_{cv} ) and ( H_{sv} )</th>
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<td>Tarbert Layers</td>
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low permeability background. As well as very large differences in permeability in layer 62, the correlation length of permeability is larger with a few narrow channels linking the injector to the producer. Layer 21, by comparison, has no well defined channel and is generally much more homogeneous.

The breakthrough time in layer 21 was 0.45 PVI whilst in layer 62 it was 0.23 PVI. The corresponding values for \( H_v \) and \( H_{cv} \) are, appropriately, different, however \( H_{sv} \) is 0.36 for both layers and does not account for the difference in their performance. The map of curvature vorticity and shear vorticity, in Fig. 8.9, shows the range of both to be between \(-30\) and \(+30\) for layer 62 which suggests that there is a similar contribution from both shear and curvature to the overall vorticity. The tortuous channel gives rise to large values of curvature as well as differences in fluid velocity perpendicular to flow. Contrast this with layer 21 which is relatively homogeneous. The differences in fluid velocity between streamlines is greater than their curvature, which means that whilst the model appears quite heterogeneous if we calculate \( H_{sv} \), it is in fact the curvature of the streamlines that best represents its performance.

From these results it seems that both curvature vorticity and vorticity appear to better capture, at least for these direct line-drive models, the variation in breakthrough time than shear vorticity, which can be dominated by large changes in velocity in small regions of the reservoir.
Figure 8.9: Maps showing the vorticity and its two components, curvature and shear vorticity for Layer 21 and Layer 62 from SPE10 Model 2 for a line-drive from left to right.
8.4.2 Non Line-Drive Models

**Fig. 8.10** shows breakthrough time as a function of the various indices for the pseudo-Q5 spot well pattern that was discussed in §5. The red dot on each graph shows the heterogeneity index and breakthrough time for the homogeneous model with the same geometry and well arrangement.

The correlation with \( H_v \) for this well pattern was not as good as it was for the line-drive models, however the comparison with \( H_{cv} \) is significantly improved, with a correlation coefficient of 0.75. The scatter seen for the Tarbert formation layers when using \( H_v \) is considerably reduced, such that \( H_{cv} \) may be used to better rank these models in terms of performance.

**Fig. 8.10b** still shows a few heterogeneous realisations (and the equivalent homogeneous case marked by a red dot) which have a later breakthrough and so a better sweep than their heterogeneity indices would suggest. One such realisation is layer 36 from the Upper-Ness formation. The permeability map shown in **Fig. 8.11** suggests that the permeability variation in this layer is restricted to a narrow high permeability streak which is disconnected from the wells. Much of the rest of the reservoir is relatively homogeneous. Presumably it is this feature that contributes to the variation in vorticity that results in a small heterogeneity index, despite its relatively low impact on performance. A coefficient of variation is clearly unable to account for correlations in the vorticity field (correlations in the permeability field are reflected in the velocity field and hence in the vorticity) so we would expect the heterogeneity index to provide a more pessimistic view of behaviour in those cases where the vorticity may be large at random locations in the reservoirs but not correlated sufficiently to have much of an impact on behaviour.

Despite that, it seems from these results that \( H_{cv} \) may be used to rank heterogeneous reservoirs in terms of performance, with indications that predictions for some models may be unduly pessimistic.

The suspect values of the heterogeneity indices for the homogeneous cases are expected because of the method by which we estimate vorticity and calculate the coefficient of variation. The coefficient of variation is \( \sigma/\bar{x} \). For homogeneous reservoirs the mean of the vorticity field, \( \bar{x} \), will be close to zero. The standard deviation will be non-zero but very small, on account of the relatively small differences in velocity. As the coefficient of variation is one divided by the other small numerical errors in the calculation of the vorticity field may result in spurious values for the coefficient of variation, and hence the heterogeneity index.

Numerical simulations were also carried out with other well patterns. In total we ran simulations with 420 reservoir models, each with different heterogeneities and different well patterns. We have attempted to show results from all the well patterns tested, including the line-drive and the pseudo-Q5 spot pattern, in **Fig. 8.12** where we have compared \( H_v, H_{cv}, H_{sv} \) against breakthrough time for each model.
Table 8.2 summarises the strength of the correlation for the line-drive, the pseudo-Q5 spot models, and all the models together. The best correlation is seen using curvature vorticity, with a correlation coefficient for all models of 0.67. The vorticity based heterogeneity index performs relatively well ($R^2 = 0.62$) whilst shear vorticity shows the weakest correlation (0.54). It is interesting to note that whilst the degree of scatter increases for $H_{cv}$ and $H_v$ as more models are added, particularly models with different well patterns, the opposite is true for $H_{sv}$, which has a significantly poor correlation ($R^2 = 0.44$) for the Q5-spot well pattern compared with the overall correlation (0.54).

There are two features of note here. Firstly, the reservoir realisations which appeared more heterogeneous than they really were (Layer 36, for example) appear so even with other well patterns. Secondly much of the scatter seen in these results is due to those well patterns where the mean direction of flow was not from left to
Figure 8.11: Permeability map for Layer 36 from the Upper-Ness formation

Figure 8.12: All models - The red dots and labels indicate the breakthrough time and the heterogeneity index for the homogeneous cases with the same geometry and the well pattern indicated by the label. The labels for each homogeneous model refer to Fig. 8.6
right but where it was from the lower face to the upper face. In those cases $H_v$ and its associated indices were unable to accurately rank the realisations in terms of performance, particularly for the Tarbert formation layers which had fluid channels that were much smaller than the inter-well distance and which were, on average, more homogeneous.

However, despite the greater scatter seen for the Tarbert formation models, we have shown that $H_{cv}$ may be used to rank different reservoir realisations and different well patterns quickly and reliably.

### 8.5 Summary

We have discussed how the curl of the velocity field may be decomposed into curvature and shear vorticity components in two dimensions.

Curvature vorticity is greatest in those regions where there is cross-flow between layers. We assert that the variability in cross-flow is best correlated with the effects of heterogeneity on recovery. This is most likely because cross-flow allows more fluid to enter preferential flow-paths within the reservoir, which leads to a greater difference between fluid velocity in different parts of the reservoir. This is what leads to early breakthrough of the injected fluid and a poor areal sweep.

Shear vorticity describes differences in fluid velocity between layers, so it is best able to capture the growth rate of initial perturbations when viscous fingers are being formed. This means that it may be less sensitive to heterogeneity as it does not capture the transfer of fluids between different channels.

We have shown that $H_{cv}$, the heterogeneity index calculated from curvature vorticity, is a more accurate indicator of performance than $H_v$ and $H_{sv}$ for all well patterns, but particularly for those that differ most from a line-drive. It may be used to rank reservoirs with arbitrary well-arrangements in terms of performance without running a fine-scale numerical simulation for each combination of well pattern and realisation.

Further work to extend this to 3D will need to focus either on,

1. understanding how vorticity may be decomposed into curvature and shear vorticity components in 3D,
2. or on calculating curvature and shear vorticity using streamline methods which allow an easy conversion from Cartesian co-ordinates to a more natural co-ordinate system.
CONCLUSIONS & FUTURE WORK

To close, we now summarise the results of this thesis, and demonstrate how they may be used in practical applications. We also discuss some of the questions that have arisen during our work, and the scope for further work.

9.1 KEY FINDINGS

The primary purpose of this thesis was to demonstrate a measure of permeability heterogeneity that could be combined with flow-regime dimensionless numbers to understand fluid displacement processes in porous media. For this purpose, the central theme has been the use of the vorticity of the displacement velocity (Heller 1966) to understand the balance of forces and mechanisms that affect such a displacement. We have shown that three of the four terms in Eq. 3.13 may be related to dimensionless numbers already in the literature, and we have used the dimensionless vorticity due to heterogeneity to define a new measure of reservoir heterogeneity calculated from the single-phase pressure/velocity field.

Using $H_v$, the new heterogeneity index, we were able to rank realistic 2D reservoir models accurately, reliably, and rapidly in terms of their performance. Further we found for adverse mobility displacements that whilst instabilities grow in the form of fingers of faster moving fluid, the effect of these fingers in very heterogeneous media on recovery did not change the rankings calculated using $H_v$. This was seen for first contact miscible and immiscible displacement processes.

A refinement of the technique introduced in Chapter 8 showed that by decomposing the vorticity field into curvature and shear-vorticity components ($H_{cv}$) we could extend this to those reservoirs with non line-drive well arrangements.

In addition to using $H_v$ as a tool to rank models, we also analysed the effect of heterogeneity on flows which are influenced by gravity, viscous, and dispersive effects. In Chapter 6 we have focused particularly on adverse mobility miscible floods as these are most likely to be influenced by both gravity and diffusion/dispersion. We found the competing influences of gravity and viscosity ratio could be captured through a modification to the gravity number originally defined by Craig et al. (1957), and the influence of dispersion using a number derived from that of Lake and Hirasaki (1981) and Tungdumrongsub and Muggeridge (2010).

We found that using $H_v$ with these existing dimensionless numbers we could determine the dominant flow-regime for heterogeneous systems. This demonstrates that the dimensionless numbers determined by the perturbation approach shown in Chapter 3, which is strictly only valid for linear systems, nonetheless can provide meaning-
ful results for real reservoir flows (which can be extremely non-linear, particularly in the presence of heterogeneity). In particular the growth rate of perturbations to a solvent front due to heterogeneity may be modified as the front moves through the reservoir. Our analysis suggests that these further perturbations, for the models tested, are (in most cases) adequately captured by the new measure of heterogeneity.

Our calculations of the Dykstra-Parsons coefficient and the dynamic Lorenz coefficient supported the findings of earlier authors which suggest that $V_{dp}$ is not an appropriate measure of the impact of heterogeneity on reservoir performance for non-layered reservoirs. We also found that the dynamic $L_c$ could not rank the most heterogenous models in a reliable way [unlike the results of Shook and Mitchell (2009)].

This work has gone some way to addressing the two areas that we identified in Chapter 2 as requiring further investigation - the use of dimensionless numbers for the design of oil recovery processes in heterogeneous reservoirs and the use of heterogeneity indices to estimate the uncertainty in reservoir behaviour due to permeability heterogeneity. The vorticity-based framework provides a foundation for comparing the effects of mobility, gravity, diffusion, and heterogeneity on recovery.

9.2 Modelling challenges and future work

There were a number of simplifying assumptions that we made in this study. Some assumptions were required so that our results were easy to interpret, and some were imposed by the experimental tools available.

We restricted the validation of our analysis to 2D flows and the geological models we constructed, mainly, from SPE10 Model 2. These models are generally correlated, in permeability, over the inter-well length scale. Whilst it is the inter-well length-scale heterogeneities that tend to govern the behaviour of reservoirs in terms of breakthrough time there are reservoirs with small-correlation length heterogeneities which causes dispersion (Warren and Cosgrove 1964; Kelkar and Gupta 1988). In these reservoirs the mobility ratio, density differences, and longitudinal and transverse dispersion coefficients may be of more interest as viscous/gravity instabilities will be a stronger control on recovery.

A more complete assessment of this method would require numerical simulation of models which are based on different depositional environments, and different permeability correlation lengths. The dependence of $H_v$ on the correlation structure of the permeability field would be interesting, and particularly useful in fields where sharp changes in reservoir permeability (such as turbidites) mean that flow may be viscous dominated within the channels but the field itself may be extremely heterogeneous in terms of the distribution and size of these channels.

The extension to 3D which we showed in Chapter 7 suggests that a careful interpretation of the heterogeneity indices is required for models where vorticity varies
over the reservoir in both direction and magnitude. This will be complicated by the presence of complex geometries, fractures, and faults. The 2D work on curvature vorticity provides some interesting insight into 3D behaviour, and curvature vorticity calculated in 3D, may provide a better way of assessing heterogeneity for these models. This is particularly important as a result of our investigation of models with quarter-five-spot geometries in 2D, which showed that the correlation for these was weaker than for the line-drive models. Using streamline methods may allow a geometry-independent method of calculating these indices.

Our work on dimensionless numbers has not considered capillary pressure effects; for immiscible displacements capillary pressure effects can accentuate the growth of viscous fingers (or in some cases help reduce their impact). An extension of Heller’s theory is required to introduce a capillary-viscous dimensionless number, however various such numbers already exist in the literature. Recent work by [Schmid and Geiger (2012)] has shown the power of a new method to rigorously derive a universal scaling group for spontaneous imbibition in water-wet systems. This new scaling group has the particularly useful feature that previous scaling groups are included as special cases. This approach should be explored for any future work on flow-regime dimensionless numbers.

Our modifications of $N_{TD}$ and the viscous-gravity number $G$ has used the effective permeability of the models in the direction of flow. Whilst this has been useful further investigations are required to determine whether $k_{eff}$ is the most suitable number to calculate these ratios.

### 9.3 Recommendations

The heterogeneity indices and flow-regime dimensionless numbers we have used from the literature may be used by reservoir engineers both for laboratory studies and the development of fields.

To fully explore the uncertainty space of probable geological models of the subsurface and to gauge the impact of heterogeneity on recovery, the most robust approach is to construct a statistically significant number of such models and numerically simulate multi-phase flow. As this will be impractical for the foreseeable future, we suggest using a combination of the new heterogeneity index (and the more traditional measures) to rank these models. This data can be used to decide which models would be most useful for a detailed multi-phase flow study.

More importantly this method could be used at the field development stage to perform a screening study on well locations. We envisage the engineer constructing a large number of reservoir models, encompassing the full range of geological uncertainty, and a range of possible well locations and using the heterogeneity index to inform the decision on well placement. For instance, well locations could be optimised to reduce the uncertainty in performance, or optimised to maximise the
likelihood of later breakthrough. This would not be a replacement for detailed flow simulation however it would provide an objective indication of the range of parameters to explore.

Any evaluation of heterogeneity should be performed in light of the balance of forces and processes in the reservoir, which will require the calculation of the appropriate dimensionless flow-regime numbers. For development plans which may be represented by a miscible displacement model we would recommend the use of $M$, $G$, $N_{TD}$, and $H_{cv}$. In combination with the phase diagram shown in Fig. 6.8, this will identify those fields which are being driven by a viscous dominated stable process ($M << 1$), or a gravity dominated process, where heterogeneity may have little impact on performance. In these cases the reservoir models may be considerably simplified.
The purpose of calculating the effective permeability of the reservoir models in this study was primarily to estimate a value of $k$ that could be used in the gravity-viscous number in Eq. (6.3). The effective permeability of any heterogeneous porous medium (including laboratory permeability measurements of core samples) is the permeability for an equivalent homogeneous model that will give the same flow rate, given the same boundary conditions. To calculate this we use the pressure-solver method of \cite{Begg1985, Begg1989}.

The porous medium is described using the permeability distribution on a regular Cartesian grid. The left and right faces of the reservoir model are held at pressure $p_1$ and $p_2$, whilst the top and bottom faces have no-flow boundary conditions imposed (Fig. A.1). The pressure equation for a single-phase incompressible fluid,

$$\nabla \cdot k(x, y, z) \nabla P = 0,$$

(A.1)

is solved to estimate the pressure in each individual grid block. Darcy’s law is used to estimate the total flux through the model. The total flux for the heterogeneous model is used in Darcy’s law to estimate the permeability required for the same flux in a homogeneous model.

Fig. A.2 shows the effective permeability, in the $x$–direction, and breakthrough time for each layer from Model A (SPE10 Model 2).
Figure A.1: Schematic of the pressure-solver upscaling method of Begg and King (1985).

Figure A.2: The effective permeability, $k_{\text{eff}}$, in the $x$–direction, and the breakthrough time for a $M = 1$ FCM flow simulation without gravity or dispersion effects. Continuous lines are used for $b_t$ merely to guide the eye.
The streamline simulations of the 2D horizontal cross-sections that were performed for the calculation of the dynamic Lorenz coefficient in Chapter 5 yielded distributions of time-of-flight for each model. This data was used to calculate two measures:

- $C_v(\tau)$: The coefficient of variation of the time-of-flight distribution, Fig. B.1
- $\frac{\max(\tau) - \min(\tau)}{\bar{\tau}}$: The normalised range in time-of-flights, Fig. B.2

The logic for both of these measures was that they reflect, at least for 1-phase flow, differences in breakthrough time due to heterogeneity. For instance, the larger the range (or the variation) in time-of-flights of the launched streamlines, the more likely it was that breakthrough would be premature.

Whilst the results do show some ranking ability, it would be difficult to suggest that they may be used to reliably rank realisations.

Please refer to Chapter 5 for the models, fluid properties, and methods used to generate these results.
Figure B.1: Breakthrough time for all 85 models as a function of the coefficient of variation of the 1-phase time-of-flight distribution for $M = 1$ and $M = 10$ and immiscible displacements.
Figure B.2: Breakthrough time for all 85 models as a function of the normalised range of the 1-phase time-of-flight distribution for $M = 1$ and $M = 10$ and immiscible displacements.
THE HETEROGENEITY INDEX, $H_s$, FOR OTHER RESERVOIR MODELS

In this thesis we have shown results using reservoir models extracted from SPE 10 Model 2. These were used because they are notoriously difficult to upscale and because they are geologically realistic.

We also ran some simulations using 2D layers extracted from the Stanford V dataset and a set of models populated randomly using a rather crude method to generate a permeability field with different correlation lengths.

C.1 STANFORD V

The static model properties are described in §4.2.2. This was a simulation of a line-drive model where injection was uniformly along the upstream face and production along the downstream face.

The results in Fig. C.1 show the shear based heterogeneity index, $H_s$, as a function of breakthrough time for a $M = 1$ displacement. The grey markers are the results for the 30 layers extracted from this model. The black markers are shown for comparison and are the results for the SPE10 Model 2 layers.

Both data sets show a linear relationship, and it is useful to note that $H_s$ may be used to successfully rank these reservoir models as well.

C.2 RANDOMLY POPULATED MODELS

These models had the same fluid properties and well arrangements as the Stanford V model, however it had $100 \times 50$ grid blocks with a grid block aspect ratio of $1 : 1$. One hundred such models were generated, the permeability was populated randomly from a normal distribution. The standard deviation of the permeability field and the correlation length of the permeability was varied. The permeability was correlated scaled by the aspect ratio and varied from correlations over 5 grid blocks to correlations up to half the inter-well distance (50 grid blocks).

These models have not been included in the main study as they were not generated very rigorously. Fig. C.2 shows the results for these models overlain on the previous results for SPE 10 Model 2 and the Stanford V datasets. These are considerably more homogeneous and, it appears they are difficult to rank using $H_s$. 

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Figure C.1: Breakthrough time as a function of $H_s$ for models from the Stanford V dataset overlain on the results for the SPE 10 Model 2 layers

Figure C.2: Breakthrough time as a function of $H_s$ for some randomly populated models overlain on results from the Stanford V dataset and the SPE 10 Model 2 layers
This is a sample MISTRESS input deck for a homogeneous reservoir model.

TITLE Example data set for MISTRESS
NGRID 100 50
GSIZE 1.0 0.5
SOLVER ICCGS

* Fluid Properties
  VISCW 1.0
  VISCO 1.0
  VISCS 1.0
  DENSITYS 1.0 ! Density of Water is = 1
  DENSITYO 1.0
  THETA 1.0 ! Implicitness parameter = 1.0 for Pc and Diff.
  SINIT 0.00 ! Initial water saturation
  CINIT 0.00 ! Initial solvent concentration
  SWCRIT 0.00
  SORSDL 0.00

* Relative Permeability
  KROSWC 1.0
  KRWSOR 1.0
  NW NO
  RELPERM 2.0 2.0 ! Corey parameters
  *PCMAX 10.0

* Diffusion, Dispersion, Viscous Fingering
  DIFF 0.000014 0.000014
  ALPHA 0.0018 0.000036

* Modelling diffusion and Dispersion
  MODERANX 1 5427896
  MODERANY 1 5427896
  MODRANSX 1 5427896
  MODRANSY 1 5427896

* Activate these keywords to initiate viscous fingering

* Specify Permeability
  READTRAN
  READPERM
  MODSINIT 1 10 1 1 0.8
* Gravity - Model tilted reservoirs by varying GX/GY
* -------------------------------------------------------------------
* GX   GY  
GRAV  0.0  0.0  
* -------------------------------------------------------------------
* Output Time Steps (PVI) for visualisations - Sim ends at last TOUT
* -------------------------------------------------------------------
* TOUT  0.5
TOUT  1.0
TOUT  1.5
* QINJ  1.0
* VAL   TIME
*FWINJ  0  2.0
*FRQOUT  1
FRQDBG  10000
FRQRST  5000
* Well specification, BLX = x co-ordinate of bottom left corner  
* TRX = x co-ordinate of top right corner  
* CINJ = fraction of solvent injected.  
* -------------------------------------------------------------------
* BLX   BLY  TRX   TRY   TYPE   BHP   PI   CINJ
  WELL   1   1   1  NY  INJN   1.0
* BLX   BLY  TRX   TRY   TYPE   BHP   PI
  WELL  NX  1   NX   NY  PROD   0.0  100000.0
* COUR  0.4
* CHANGEVT  0.05
* Activate to enable flux-corrected transport  
FCTS
FCTC
* Ask for pseudos for flow in x-direction.
* -------------------------------------------------------------------
* Direction No of. Output frequency
* grid blocks (ie every 50 timesteps)
* -------------------------------------------------------------------
*XPSUEUDO  1   50
*XPSUEUDO  2   50
*XPSUEUDO  4   50
* Ask for output so we can generate effective relative permeabilities  
at a later date.
* -------------------------------------------------------------------
* Direction Output frequency Fluid (WATER or SOLV)
* -------------------------------------------------------------------
*EFFREPX  50   WATER
*READREG
OUTLEVEL  1
FULLSIZE
*DTMOVIE  1.0E-03 WATER
END


