System Stability Improvement through Optimal Control Allocation in VSC HVDC Links

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<td>Pipelzadeh, Yousef; Imperial College London, Electrical and Electronic Engineering Chaudhuri, Nilanjan Ray; Imperial College London, Electrical and Electronic Engineering; Chaudhuri, Balarko; Imperial College London, Electrical and Electronic Engineering Green, T.; Imperial College London, Department of Electrical Engineering</td>
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System Stability Improvement through Optimal Control Allocation in VSC HVDC Links

Yousef Pipelzadeh, Nilanjan Ray Chaudhuri, Balarko Chaudhuri, Tim C Green *
Abstract

Control of both active and reactive power in voltage source converter (VSC) based High Voltage Direct Current (HVDC) links could be very effective for system stability improvement. The challenge, however, is to properly allocate the overall control duty among the available control variables in order to minimize the total control effort and hence allow use of less expensive converters (actuators). Here relative gain array (RGA) and residue analysis are used to identify the most appropriate control loops avoiding possible interactions. Optimal allocation of the secondary control duty between the two ends of the VSC HVDC link is demonstrated. Active and reactive power modulation at the rectifier end, in a certain proportion, turns out to be the most effective. Two scenarios, with normal and heavy loading conditions, are considered to justify the generality of the conclusions. Subspace-based multi-input-multi-output (MIMO) system identification is used to estimate and validate linearized state-space models through pseudo random binary sequence (PRBS) probing. Linear analysis is substantiated with non-linear simulations in DlgsILENT PowerFactory with detailed representation of HVDC links.

1 Introduction

Modulation of the active power order of an High Voltage Direct Current (HVDC) link [1, 2] could be extremely effective for damping low frequency power oscillations, thereby increasing the transfer capacity of an AC transmission system. Network operators like WECC have considered this in 1970s to improve their system dynamic performance [3]. Over the past few decades a lot of research attention was focused on identifying ways to maximize the impact of HVDC modulation control on AC system stability [4, 5, 6, 7]. In the recent past, with an increasing number of HVDC installations around the world, especially in countries with long transmission corridors like Brazil, China and India, there has been a renewed interest in this area [8, 9, 10, 11]. Even in smaller countries, such as the U.K. the HVDC links would increase the stability limits in addition to direct expansion of transmission capacity.

Most of the work on HVDC control to damp oscillations, including those mentioned...
above, have primarily focused HVDC systems based on Line Commutated Converter (LCC) based HVDC technology. This, of course, is a proven and mature technology and constitutes the bulk of the HVDC installations around the world especially, at higher power levels. Since the first commercial installation of the Voltage Source Converter (VSC) based HVDC system in the late nineties, its size and use has grown due to its advantages over its LCC counterpart [1, 2, 12, 13]. Some of the larger VSC installations that are either already commissioned or about to be in near future include the 400 MW Transbay project in the US [14], 400 MW BorWin 1 offshore link north of Germany and 600 MW Caprivi link (with both poles operational) in Namibia [15].

A VSC HVDC link allows independent modulation of both the active and reactive power (or voltage) at both ends and hence offers more flexibility than LCC systems whereby only the active power can be modulated. There is tremendous potential for VSC HVDC systems to contribute towards improvement in AC system dynamic performance as discussed in [16, 17, 18]. For power oscillation damping through supplementary modulation of active and/or reactive power in a VSC-HVDC link, the converters dynamic ratings (MVA) are set not only according to their steady-state values, but also their expected range of modulation ('headroom') during dynamic conditions. There exist multiple control options within a VSC HVDC in damping inter-area oscillations, which is not well reported in the literature. The primary objective of this paper is to highlight the importance of allocating the secondary control duty appropriately among the available options in terms of reducing the overall control effort (energy). The motivation behind control energy minimization is to invoke lesser excursion on the control variables around their steady-state values by ensuring that the control effort demanded from the actuators is optimal. This translates into lower converter dynamic rating/headroom required and hence, lower associated converter costs since it would be very expensive to increase a converter station rating to incorporate secondary controls.

The main research question addressed is: How to optimally allocate the control duty among the multiple control options that exist within a VSC HVDC for supplementary damping control, in terms of reducing the overall control energy, and hence, minimizing the dynamic ratings of the expensive converters?
The paper begins by presenting a case study (see Section 2) with a VSC HVDC link added in parallel with the existing AC tie-lines of a multi-machine test system [19] modelled in DlgSILENT PowerFactory [20]. In Section 3, linearized state-space model of the system was estimated by measuring system responses to injecting pseudo random binary sequence (PRBS) probing signal at the VSC HVDC control inputs, since DlgSILENT does not provide a linear model directly. This has been a common practice for model validation in field at WECC and other systems [21]. Numerical algorithm for subspace state-space system identification (N4SID) [22] which is a proven technique for obtaining linearized state-space models using input/output data was used. N4SID is particularly useful tool for high-order multi-variable systems [23]. This technique has been shown to provide accurate linear models for the SE Australian test system [24]. VSC HVDC offers multiple control-loop options through modulation of active power (at the rectifier end) and reactive power (at both ends). In Section 4, residue analysis and relative gain array (RGA) and were used to identify the most appropriate control-loops avoiding possible interactions [25].

In Section 5, optimal allocation of the secondary control duty between the two ends of the VSC HVDC link was formulated as a constrained optimization problem. Since it is not straightforward to solve such a problem analytically [26], a simulation-based design of experiment (DoE) [27] approach which is widely used in industries was adopted to identify an optimal combination of control options in VSC HVDC. The overall control energy requirement was expressed as a function of allocation factors in order to determine the appropriate combination. Dynamic simulations are presented in Section 6 for two scenarios considering normal and heavy loading conditions to justify the generality of the conclusions.

2 Test Cases in DlgSILENT PowerFactory

The AC part of the test system considered here is the well known 4-machine, 2-area test system in [19]. There are four generators (G1, G2 and G3, G4), two in each area (marked West and East) as shown in Fig. 3. The generators were represented by sub-transient models and equipped with IEEE DC1A excitation systems [28]. The active component of the loads at buses 7 and 9 have constant current characteristics, while the reactive components have
constant impedance characteristics. The power-flow and dynamic data for the system are given in [19].

A point-to-point VSC-based HVDC link was added in parallel to the AC corridor between buses 7 and 9. A single-line diagram for both the LCC system (see Fig. 1) and VSC HVDC (see Fig. 2) systems with their associated parameters are provided in the Appendix. The converter and the line ratings of the both DC links were chosen in line with those of the LCC system in [19]. The AC-DC system was modelled in DIgSILENT PowerFactory after verifying the AC part against the results in [19].

Out of several operating scenarios considered, two different loading conditions are presented here. Under normal loading, the power transfer through the 230 kV AC corridor from West to East is approximately 400 MW. The loads at buses 7 and 9 were adjusted to simulate a heavy loading scenario with 600 MW tie-line flow. Under both scenarios, the steady-state active power order for the rectifier was fixed at 200 MW. The reactive power order was set to maintain close to unity power factor at the terminal AC buses, 7 and 9.

Standard current control strategy in the $d-q$ reference frame was used for the primary control loops because of the obvious advantages over voltage control strategy [29]. The limits on direct and quadrature axes current for both the converters were set at 7.5 and 3.75 kA, respectively with standard current limiting logic (from DIgSILENT) in place. Secondary modulation (control) of $P_r$, $Q_r$, $V_{dc1}$ and $Q_i$ were studied in terms of their effectiveness to damp inter-area oscillations. All nodes were considered as potential sites for Phasor Measurement Unit (PMU) feedback, providing time-synchronized phase angle measurements. The most effective signals were identified and made available at the control centers at either end of the VSC HVDC link. Appropriate feedback signals for modulating each of the secondary control variables were chosen systematically as described later in the paper.

3 Linear Model Estimation and Validation

It is not possible to obtain the linearized system models directly from DIgSILENT. Therefore, a system identification technique was used to estimate the linear model from the
simulated outputs in response to appropriate probing signals at the inputs. Here, the linear model has 4 control inputs; $P_r$, $Q_r$, $V_{dc1}$ and $Q_i$, for the VSC HVDC and 11 possible phase angle measurements available from the PMUs. Identification of such MIMO systems is quite challenging and gets further complicated with increase in number of output signals [30].

### 3.1 Probing Signal

Selection of an appropriate probing signal plays an important role in system identification. Different probing signals, such as repeated pulses, band-limited gaussian white noise, PRBS, Fourier sequence, have been reported in the literature [30], [31]. Here PRBS was chosen due to its richer spectrum [30].

At every time step, one of the two pre-specified binary values was generated randomly by rounding the magnitudes of a complex combination of transcendental functions. The amplitude of the PRBS was chosen to be high enough to sufficiently excite the critical modes without pushing the responses into nonlinear behaviour. Persistent excitation of at least the model order of interest was provided. Moreover, the probing sequence for different inputs were ensured to be uncorrelated [31]. Typical PRBS injection signals used for probing the test system are shown in Fig. 4.

### 3.2 MIMO Subspace Identification

The discrete state space model of the MIMO system with $m$ inputs and $l$ outputs can be expressed as:

$$
\begin{align*}
x(k+1) &= A_d x(k) + B_d u(k) + w(k) \\
y(k) &= C_d x(k) + D_d u(k) + v(k)
\end{align*}
$$

where, $A_d \in \mathbb{R}^{n \times n}$, $B_d \in \mathbb{R}^{n \times m}$, $C_d \in \mathbb{R}^{l \times n}$ are the matrices to be estimated and $w(k) \in \mathbb{R}^{n \times 1}$ and $v(k) \in \mathbb{R}^{l \times 1}$ are non-measurable observation and process noise vector sequences. Here, the input vector $u(k) \in \mathbb{R}^{4 \times 1}$ is the set of PRBS injection signals at the 4 control inputs of VSC HVDC and the simulated output responses $y(k) \in \mathbb{R}^{11 \times 1}$ are the phase angles of voltage measured from the 11 PMUs.

Using the input probing signal \{u_i(0), u_i(1), ...u_i(N), i = 1, 2, ...4\} and output responses \{y_i(0), y_i(1), ...y_i(N), i = 1, 2, ...11\} the matrices $A_d$, $B_d$, $C_d$ and $D_d$ were calculated such
that the simulated (actual) data resembled the responses from the estimated (identified) linear model. The estimated model in discrete domain was converted to continuous domain for linear analysis and control design. N4SID [22] was used to estimate the above matrices. A model order of 45 was found to be appropriate for both load scenario.

3.3 Model Validation

To validate the estimated linear model of the MIMO system, pulses of 0.5 s duration were applied at the 4 control inputs separately and in all possible combinations. The blue (black in grey-scale) traces in Fig. 5 show the simulated responses from DIgSILENT against the red (grey in grey-scale) ones which were obtained from the identified linear model. Very close correlation between the red and blue traces in all cases (only a few representative cases are shown here) confirms the accuracy of the estimated linear models. These models were used for eigen value analysis, control loop selection and control allocation described later.

4 Control Loop Selection

4.1 Residue Analysis

Residues provide a combined measure of controllability and observability of the modes of interest of a linearized system model which can be expressed in state-space form as:

\[
\begin{bmatrix}
\Delta \dot{x} \\
\Delta y
\end{bmatrix}
= \begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
\Delta u
\end{bmatrix};
G(s) \triangleq \begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\]

(2)

Applying an appropriate transformation, (2) can be transformed into a modal (normal or decoupled) form [32] as:

\[
\begin{bmatrix}
\dot{z} \\
\Delta y
\end{bmatrix} = \begin{bmatrix}
\Lambda & \Phi^{-1}B \\
C\Phi & D
\end{bmatrix}
\begin{bmatrix}
z \\
\Delta u
\end{bmatrix}
\]

(3)

where \(\Lambda = \Phi^{-1}A\Phi\) is a diagonal matrix and \(\Phi = \begin{bmatrix}
\phi_1 & \phi_2 & \ldots & \phi_n
\end{bmatrix}\) is the right eigen vector (or modal) matrix comprising the right eigen vectors \((\phi_1, \phi_2, \ldots)\) corresponding to
each mode.

Modal observability $Ob_{ij}$ of the $i^{th}$ mode in the $j^{th}$ output is obtained by multiplying the $j^{th}$ row vector of $C$ with the $i^{th}$ column of $\Phi$. Similarly, modal controllability $Co_{ik}$ of the $i^{th}$ mode in the $k^{th}$ control input is the product of the $j^{th}$ row vector of $\Phi^{-1}$ and the $k^{th}$ column of $B$.

$$Co_{ik} = [\Phi^{-1}]_i \times [B]_k$$

$$Ob_{ij} = [C]_j \times [\Phi]_i$$

The product of modal controllability and observability gives the residue $Res_{i-kj} = Co_{ik} \times Ob_{ij}$ which indicates the extent to which mode $i$ can be observed and controlled through input $k$ and output $j$. The elements of an eigen vector are complex numbers, in general, and as such modal controllability $Co_{ij}$, modal observability $Ob_{ik}$ and residue $Res_{i-kj}$ are all complex numbers with a magnitude and a phase angle component.

Once the residues corresponding to all possible input-output combinations are calculated and sorted in descending order of magnitude, the appropriate ones are chosen from the top (with highest residues) to ensure minimum control effort. For single-input-single-output (SISO) systems the phase angle of the residue is not important. However, for single-input-multiple-output (SIMO) or MIMO systems, the phase angle could be critical when choosing appropriate inputs and outputs [33]. RGA can be an addition to the residue analysis for a systematic application in large power system studies. It measures two-way interactions between a determined input and output [34].

### 4.2 Relative Gain Array (RGA)

RGA provides a measure of interaction between several loops in the case of decentralized control [35]. For a non-singular square system $G(s)$, the RGA at a particular frequency $\omega$ is a square matrix defined as:

$$RGA(G(j\omega)) \triangleq \Upsilon(G(j\omega)) = G(j\omega) \times (G(j\omega)^{-1})^T$$
where \( \times \) denotes element-by-element multiplication (the Hadamard or Schur product) [23]. For non-square systems, the concept can be generalized using the pseudo-inverse.

Suppose that the \( j \)th input \( u_j \) of a MIMO system \( G(s) \) is used to control the \( i \)th output \( y_i \). This leads to two extreme cases: i) all the other control loops are open, i.e., \( u_k = 0, \forall k \neq j \) and ii) all other control loops are closed, i.e., with perfect control under steady state (reasonable approximation for frequencies within the bandwidth) \( y_k = 0, \forall k \neq i \). It can be shown that [23]:

\[
    g_{ij} \triangleq \left( \frac{\partial y_i}{\partial u_j} \right)_{u_k=0, k \neq j} = [G(j\omega)]_{ij} 
\]

\[
    \hat{g}_{ij} \triangleq \left( \frac{\partial y_i}{\partial u_j} \right)_{y_k=0, k \neq i} = [G(j\omega)^{-1}]_{ji} 
\]

The \( ij \)th element of the RGA \( [\Upsilon(G)]_{ij} \) captures the ratio \( \upsilon_{ij} \) between the gains \( g_{ij} \) and \( \hat{g}_{ij} \), corresponding to the two extremes, and thus provide a useful measure of interaction.

\[
    \upsilon_{ij} \triangleq \frac{g_{ij}}{\hat{g}_{ij}} = [G(j\omega)]_{ij}[G(j\omega)^{-1}]_{ji} = [\Upsilon(G(j\omega))]_{ij} 
\]

For an \( m \times m \) MIMO system with \( m \) inputs and \( m \) outputs there are \( m! \) ways in which the control loops can be arranged. Looking at the RGA and following the rules stated below, appropriate control loops can be selected in a systematic way avoiding the ones which are likely to cause adverse interaction.

1. Choose to pair those inputs and outputs for which the RGA element in the crossover frequency range is close to 1.

2. In order to minimize adverse interactions, avoid input-output pairing where the RGA elements calculated at low frequency (steady-state) are negative.

In this paper, the second criterion is used to select appropriate input-output pairs for the two terminals of a VSC HVDC system.
5 Control Allocation

For a MIMO system $G(s)$ with $m$-inputs and $m$-outputs, the overall control duty can be allocated (distributed) amongst individual input-output pairs resulting in a diagonal $m$-input, $m$-output controller $K(s)$. The control allocation is said to be ‘optimal’ when the overall control effort (control energy) is minimum. Such an optimal control allocation problem can be expressed mathematically as follows:

$$G(s) \triangleq \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}; A \in \mathbb{R}^{n \times n}, \ B \in \mathbb{R}^{n \times m}, \ C \in \mathbb{R}^{m \times n}$$ \hspace{1cm} (10)

The objective is to determine the control allocation factors $\alpha_i$ and a stable diagonal controller $K(s)$,

$$K(s) \triangleq \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} = \begin{bmatrix} A_{k1} & 0 & 0 & B_{k1} & 0 & 0 \\ 0 & A_{k2} & 0 & 0 & B_{k2} & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & A_{km} & 0 & 0 & B_{km} \\ C_{k1} & 0 & 0 & 0 & 0 & 0 \\ 0 & C_{k2} & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & C_{km} & 0 & 0 & 0 \end{bmatrix}$$ \hspace{1cm} (11)

$$A_{kj} \in \mathbb{R}^{n_k \times n_k}, B_{kj} \in \mathbb{R}^{n_k \times 1}, C_{kj} \in \mathbb{R}^{1 \times n_k}, j = 1, 2, \cdots, m$$

which minimizes the cost function $J$:

$$\min_{K(s), \alpha_i} J = \int_0^{\infty} \sum_{i=1}^m \Delta u_i^2 \, dt$$ \hspace{1cm} (12)
such that the following constraints are satisfied:

\[
\sum_{i=1}^{m} \alpha_i = 1 \tag{13}
\]

\[
K(s) \in S \tag{14}
\]

\[
\Delta \lambda_c \left( \begin{bmatrix} A & B_i C_{ki} \\ B_{ki} C_i & A_k \end{bmatrix} \right) \in (\lambda_d - \lambda_o) \alpha_i \ \forall \ i = 1, \ldots, m \tag{15}
\]

\[
\lambda_c \left( \begin{bmatrix} A & BC_k \\ B_k C & A_k \end{bmatrix} \right) \in \lambda_d \tag{16}
\]

\[
n_k \ll n \tag{17}
\]

where, \( \Delta u_i \): dynamic variation of \( i^{th} \) control input, \( \lambda_c(\cdot) \): critical eigen values, \( \Delta \lambda_c(\cdot) \): shift of critical eigen values towards left of the complex plane, \( \lambda_o = \lambda_c[G(s)] \): critical eigen values of the open loop system \( G(s) \), \( S \): set of stable controllers and \( \lambda_d \): desired region for the critical eigen values. \( B_i : i^{th} \) column of the plant input matrix B, \( C_i : i^{th} \) row of the plant output matrix C, \( B_{ki} : i^{th} \) column of the controller input matrix \( B_k \), \( C_{ki} : i^{th} \) row of the controller output matrix \( C_k \).

The objective in (12) represents minimization of overall control energy. Constraint (13) indicates that the sum of all the control allocation factors \( \alpha_i \) should be 1. Each control input \( i \) would contribute to the overall shift of the critical eigen values (\( \lambda_d - \lambda_o \)) in proportion to the control duty \( \alpha_i \) allocated to it, which is captured in constraint (15). Constraint (16) represents the placement of the critical closed-loop eigen values (with all control loops closed) at desired location (\( \lambda_d \)) with a diagonal (decentralized) controller.

It is not straightforward to analytically solve the constrained optimization problem described above to determine \( \alpha_i \) and \( K(s) \). Here the solution was sought through a simulation based design of experiments (DoE) approach [27]. A number of scenarios with random combinations of control allocation factors \( \alpha_i \in \{0 : 1, \Sigma \alpha_i = 1\} \) were generated. A decentralized controller \( K(s) \) was designed for each combination to satisfy (14). The cost function \( J \) was evaluated from simulation results for each case. Using the data from several trials, \( J \) was expressed as a function of \( \alpha_i \) from which the optimum control allocation factors and the
corresponding controller $K(s)$ could be synthesized. Moreover, pareto of each control input $i$ on the overall control energy (indicated by cost function $J$) could also be evaluated.

6 Results and Discussions

6.1 Eigen value Analysis

The linear model of the system was identified using a subspace identification technique (N4SID) described in Section 3.2. Table 1 shows the damping ratios and frequencies of the critical modes of the system under normal loading (400 MW tie-flow) and heavy loading (600 MW tie-flow) conditions.

Table 1: Damping ratios and frequencies of critical modes. VSC HVDC operates in $P-Q$ control on the rectifier end, and $V_{dc}-Q$ control on the inverter end. LCC HVDC operates in $P$ control on the rectifier end, and $V_{dc}$ control on the inverter end. See Appendix.

<table>
<thead>
<tr>
<th>Loading Condition</th>
<th>AC system only</th>
<th>with VSC HVDC</th>
<th>with LCC HVDC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>$\zeta, %$</td>
<td>$f, Hz$</td>
<td>$\zeta, %$</td>
</tr>
<tr>
<td>0.6</td>
<td>0.519</td>
<td>1.5</td>
<td>0.568</td>
</tr>
<tr>
<td>8.5</td>
<td>1.058</td>
<td>8.4</td>
<td>1.062</td>
</tr>
<tr>
<td>8.3</td>
<td>1.089</td>
<td>7.9</td>
<td>1.097</td>
</tr>
<tr>
<td>Heavy</td>
<td>-7.8</td>
<td>0.401</td>
<td>0.7</td>
</tr>
<tr>
<td>-8.4</td>
<td>1.076</td>
<td>8.6</td>
<td>1.06</td>
</tr>
<tr>
<td>-8.9</td>
<td>1.05</td>
<td>7.7</td>
<td>1.092</td>
</tr>
</tbody>
</table>

Under the normal condition and with the DC link removed, one poorly-damped inter-area mode at 0.52 Hz and two local modes at 1.06 and 1.09 Hz are present. With the introduction of the VSC HVDC link, the frequency of the inter-area mode increases to 0.568 Hz while the damping ratio improves from 0.6% to 1.5%. As the DC link takes a 200 MW share of the total tie-flow, the power angle across the corridor decreases, thereby increasing the synchronizing torque coefficient and in turn, the frequency [19].

Under the heavy loading condition, the AC system alone is unstable but the inclusion of VSC HVDC stabilizes the system (damping ratio increases from -7.8% to 0.7%) by taking 200 MW of the tie-flow and giving reactive power support at each end. As expected, an increase in the frequency of the inter-area mode from 0.4 Hz to 0.54 Hz is also observed.

A comparison with the inclusion of a LCC HVDC link with same rating as in [19] shows that the frequency under normal and heavy loading scenarios increases to 0.578 Hz and
0.548 Hz, respectively, see Table 1. In all cases the system is better damped with HVDC.

6.2 Control Loop Selection

In this work a decentralized control framework is used at the rectifier and inverter end control centers, see Fig. 3. There are 4 possible control (modulation) inputs $P_{rmod}, Q_{rmod}, V_{dcimod}$ and $Q_{imod}$, and 11 possible outputs (the phase angles measured by the PMUs installed at 11 buses). Residues were calculated for all possible input-output combinations. The magnitudes and phase angles of the residues under normal loading condition are shown in Table 2.

Table 2: Residues with measured phase angles of voltages at 11 buses and 4 control inputs of the VSC HVDC under normal loading condition

<table>
<thead>
<tr>
<th>bus no.</th>
<th>$P_{rmod}$</th>
<th>$Q_{rmod}$</th>
<th>$V_{dcimod}$</th>
<th>$Q_{imod}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mag</td>
<td>ang</td>
<td>mag</td>
<td>ang</td>
</tr>
<tr>
<td>1</td>
<td>1.0</td>
<td>-87</td>
<td>0.64</td>
<td>88</td>
</tr>
<tr>
<td>2</td>
<td>0.91</td>
<td>-88</td>
<td>0.59</td>
<td>88</td>
</tr>
<tr>
<td>4</td>
<td>0.17</td>
<td>-106</td>
<td>0.11</td>
<td>70</td>
</tr>
<tr>
<td>5</td>
<td>0.95</td>
<td>-88</td>
<td>0.61</td>
<td>88</td>
</tr>
<tr>
<td>6</td>
<td>0.86</td>
<td>-88</td>
<td>0.55</td>
<td>88</td>
</tr>
<tr>
<td>7</td>
<td>0.77</td>
<td>-88</td>
<td>0.50</td>
<td>87</td>
</tr>
<tr>
<td>8</td>
<td>0.47</td>
<td>-91</td>
<td>0.30</td>
<td>84</td>
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<tr>
<td>9</td>
<td>0.21</td>
<td>-102</td>
<td>0.13</td>
<td>74</td>
</tr>
<tr>
<td>10</td>
<td>0.18</td>
<td>-104</td>
<td>0.12</td>
<td>72</td>
</tr>
<tr>
<td>11</td>
<td>0.15</td>
<td>-108</td>
<td>0.10</td>
<td>67</td>
</tr>
</tbody>
</table>

Since $V_{dcimod}$ has very small residue, only $P_{rmod}, Q_{rmod}$, and $Q_{imod}$ were considered for secondary control. Three measured outputs - phase angles at buses 1, 2 and 5 (marked in bold) - were chosen for reliability in order of descending residue magnitudes.

RGA at steady-state (see Section 4.2) was used to form appropriate input-output pair among $P_{rmod}, Q_{rmod}$, and $Q_{imod}$ and phase angles at buses 1, 2 and 5. Any input-output pair with negative RGA element were discarded to avoid adverse interactions. Based on the RGA elements in Table 3 for normal loading condition, bus angle 5 was chosen for $P_{rmod}$, bus angle 2 for $Q_{imod}$ and 1 for $Q_{rmod}$ to form a 3-input, 3-output decentralized control structure. Similarly, for heavy loading condition, bus angles 5, 1 and 2 were paired to $Q_{rmod}, P_{rmod}$ and $Q_{imod}$, respectively. The RGA elements corresponding to the chosen input-output pairs are marked in bold in Table 3.
### Table 3: RGA for possible input-output combinations

<table>
<thead>
<tr>
<th>bus no.</th>
<th>Normal Loading</th>
<th>Heavy Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_{rmod}$</td>
<td>$Q_{rmod}$</td>
</tr>
<tr>
<td>1</td>
<td>-10.93</td>
<td>12.48</td>
</tr>
<tr>
<td>2</td>
<td>-24.84</td>
<td>11.76</td>
</tr>
<tr>
<td>5</td>
<td>36.78</td>
<td>-23.24</td>
</tr>
</tbody>
</table>

### 6.3 Control Allocation

As previously discussed in Section 5, the objective here is to optimally allocate/distribute the overall control duty between the rectifier and inverter end variables such that the overall control energy required is minimum. Only $P_{rmod}$, $Q_{rmod}$ and $Q_{imod}$ are considered leaving out $V_{dcimod}$ due to low residues, see Table 2.

Parameters $\alpha_1$, $\alpha_2$ represents the fraction of the control duty allocated to $P_{rmod}$ and $Q_{rmod}$, respectively while $\alpha_3 = 1 - (\alpha_1 + \alpha_2)$ is the share of $Q_{imod}$. A design of experiment (DoE) with several random (but permissible) combinations of $\alpha_i \in \{0,1; \Sigma \alpha_i = 1\}$ was conducted. A diagonal 3-input, 3-output controller was designed in each case with the appropriate control loops described in Section 6.2. The specification for control design was to ensure a minimum settling time $T_s = 10.0$ s for the critical inter-area mode. Non-linear simulations in DiGILENT PowerFactory were carried out to calculate the overall control energy $J$ from time variations of $P_{rmod}$, $Q_{rmod}$ and $Q_{imod}$, see Fig. 13, in each case:

$$
J = \int_0^\infty [P_{rmod}(t)^2 + Q_{rmod}(t)^2 + Q_{imod}(t)^2] dt
$$

A three phase self-clearing fault near bus 9 (the inverter bus) was considered for the heavy loading condition, whereas under the normal loading scenario fault near bus 8 was simulated.

Using the simulation based DoE results, a function relating $J$ to the two independent variables $\alpha_1, \alpha_2$ was constructed as:

$$
\mathcal{J} = f(\alpha_1, \alpha_2) \ \forall \alpha_1, \alpha_2 \in \{0,1; (\alpha_1 + \alpha_2) \leq 1\}
$$

A representative result is shown in Fig 6 for the heavy loading condition (see Section 6). More control energy is required towards either extreme $\{0 : 1\}$ of $\alpha_2$ axis indicating the fact
that an appropriate combination of control variables at both ends could be more effective than exercising only the rectifier or inverter end options. For both normal and heavy loading similar trends were observed. However, the control energy required for the heavy loading is much more than for the normal loading condition because of the larger power transfer and the fault near the inverter bus (bus 9).

The bar graphs in Figs 7 and 8 show the minimum control energies for the following options:

- \( P_r \): \( P \) control at rectifier end only, \( \alpha_1 = 1, \alpha_2 = 0 \);
- \( Q_r \): \( Q \) control and rectifier only, \( \alpha_1 = 0, \alpha_2 = 1 \);
- \( Q_i \): \( Q \) control at inverter only, \( \alpha_1 = 0, \alpha_2 = 0 \);
- \( P_r - Q_r \): \( P \) and \( Q \) control at rectifier only, \( \alpha_1 = \alpha_{1\text{opt}}, \alpha_2 = \alpha_{2\text{opt}} \forall \alpha_1, \alpha_2 \in \{0, 1; (\alpha_1 + \alpha_2) = 1\} \);
- \( P_r - Q_i \): \( P \) at rectifier and \( Q \) control at inverter, \( \alpha_1 = \alpha_{1\text{opt}}, \alpha_2 = 0 \);
- \( Q_r - Q_i \): \( Q \) control at both rectifier and inverter, \( \alpha_1 = 0, \alpha_2 = \alpha_{2\text{opt}} \);
- \( P_r - Q_r - Q_i \): \( P, Q \) at rectifier and \( Q \) control at inverter, \( \alpha_1 = \alpha_{1\text{opt}}, \alpha_2 = \alpha_{2\text{opt}} \).

7 Simulation Results

This Section shows a representative set of time-domain simulation results in DIgSILENT PowerFactory for the cases considered in Figs 7 and 8.

7.1 Normal Loading Condition

The dynamic performance of the system for different secondary control loop combinations is shown in Figs 9 and 10.

A three-phase fault at bus 8 that self-clears after 5 cycles is considered. Note that the legends marked on the top right corner of the figures indicate the control loop combinations while the plotted variables are described below each subplot.

In all cases the desired settling time of around 10.0 s is achieved. With \( P_r \) control, see subplots 9(a), 9(b), 9(c), 9(d), \( I_{dr} \) varies while \( I_{qr} \) is nearly zero to maintain unity power factor. For \( Q_r \) and \( Q_i \) control \( I_{qr} \) and \( I_{qi} \) are seen to vary in subplots 9(f), 9(h). The high
frequency components are due to successive hitting of the current limits immediately after
the large disturbance. Fig. 10 shows the dynamic performance with more than one control
loop. For $P_r - Q_r$ pairing, see subplots 10(a), 10(b), both $I_{dr}$ and $I_{qr}$ changes. In all the
cases $V_{dc1}$ is kept constant - a representative plot is shown for the case with $P_r - Q_i$ pairing,
see subplot 10(d).

### 7.2 Heavy Loading Condition

Figs 11 and 12 show the dynamic behavior of the system following a three-phase self-clearing
fault for 5 cycles near the inverter bus (bus 9). As expected a settling time of about 10.0 s
is achieved under this heavy loading condition with a 600 MW tie flow, see subplot 11(a).
During the fault, the dc bus voltages at both the ends increase sharply due to reduction of ac
side power transfer. This is followed by oscillations in $V_{dcr}$ due to the dc link dynamics while
the $V_{dci}$ is regulated to a constant value, see subplots 11(c), 11(d). Dynamic performance of
the system with a combination of control loops is shown in Fig. 12. Because of the severity
of the fault close to inverter bus $I_{dr}$ and $I_{qr}$ are seen to violate their respective limits, see
subplots 12(a), 12(b).

Fig. 13 shows the outputs of the controllers for different combinations of secondary con-
trol loops. These plots confirm that the control effort needed with modulation of $P_r$ is much
less compared to $Q_r$ modulation. Similarly control effort required for $Q_r - Q_i$ combination
is larger than other alternatives except $Q_i$. Interestingly, $P_r - Q_r - Q_i$ combination demands
higher control energy as compared to the $P_r - Q_r$ and $P_r - Q_i$ cases. The range of variation
of the system variables is higher in Figs 11 and 12 compared to Figs 9 and 10 because of
heavy loading condition and fault close to the inverter bus. To summarize, similar dynamic
performance (10.0 s settling time in this case) can be obtained through different combina-
tions of rectifier and inverter end control parameters ($P_r$, $Q_r$ and $Q_i$). However, less control
energy is required when control duty is allocated/distributed in a proper way.
8 Conclusions

In this paper, the importance of properly allocating the secondary control duty at both ends of a VSC HVDC link is highlighted. For the case study presented, active and reactive power modulation at the rectifier end (in a certain proportion) is shown to be the most effective in terms of minimum overall control energy (effort) required. This was true for both normal as well as heavy loading conditions with short-circuit faults at various locations. However, this need not be a general conclusion for other systems and would depend on the type and distribution of loads with respect to the HVDC terminals and also the power flow patterns. Nonetheless, the significance of considering the control allocation aspect in order to reduce the overall control energy and hence the headroom required from expensive actuators (terminal converters in this case) is demonstrated.

Although the test system used for this study is relatively small, it is widely reported in the literature for inter-area oscillations studies. It is used as a first step for developing a basic understanding of the proposed concept. The simulation-based design of experiment (DoE) approach is popular in industries and is used here to demonstrate that there exists an optimal combination of control options in VSC HVDC. This paper thus should set the motivation for future work on systematic and elegant control design techniques to address this problem on larger networks with many controllable devices. With increasing penetration of controllable power electronics in the form of FACTS, HVDC, wind generators etc. appropriate control allocation is going to be crucial in future.

Acknowledgment

The authors would like to acknowledge the suggestions received from Dr Rajat Majumder at Siemens Energy, USA and from the DlgsILENT support team.
Appendix

LCC HVDC Model

A single-line diagram representing the LCC HVDC, as modelled in PowerFactory DIgSILENT is shown in Fig. 1. The system parameters are provided in Table 4. The converter and the line ratings of the LCC HVDC link were chosen in line with those of the LCC system in [19].

![Single-line diagram of the bipolar LCC HVDC system](image)

Figure 1: Single-line diagram of the bipolar LCC HVDC system

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectifier control mode</td>
<td>( R_{\text{cc}, \text{control}} )</td>
<td>( P )</td>
</tr>
<tr>
<td>Inverter control mode</td>
<td>( I_{\text{inv}, \text{control}} )</td>
<td>( V_{\text{dc}} )</td>
</tr>
<tr>
<td>Converter rating</td>
<td>( S_{\text{rating}} )</td>
<td>224 MVA</td>
</tr>
<tr>
<td>DC line voltage</td>
<td>( V_{\text{dc}} )</td>
<td>56 kV</td>
</tr>
<tr>
<td>DC line current</td>
<td>( I_{\text{dc}} )</td>
<td>3.6 kA</td>
</tr>
<tr>
<td>DC line resistance</td>
<td>( R_{L} )</td>
<td>0.15 Ω</td>
</tr>
<tr>
<td>DC line inductance</td>
<td>( L_{L} )</td>
<td>100 mH</td>
</tr>
<tr>
<td>Commutating reactance</td>
<td>( X_{c} )</td>
<td>0.57 Ω</td>
</tr>
<tr>
<td>Smoothing reactor</td>
<td>( L )</td>
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<tr>
<td>Rated reactive power</td>
<td>( Q_{\text{rated}} )</td>
<td>125 MVAr</td>
</tr>
<tr>
<td>Terminal ac voltage</td>
<td>( V_{t} )</td>
<td>30 kV</td>
</tr>
<tr>
<td>Terminal PCC voltage (grid)</td>
<td>( V_{\text{PCC}} )</td>
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<tr>
<td>Firing angle set-point</td>
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<tr>
<td>Extinction angle (gamma) set-point</td>
<td>( \gamma )</td>
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</table>

Table 4: LCC HVDC parameters
VSC HVDC Model

A single-line diagram representing the VSC HVDC system, as modelled in PowerFactory DlgsILENT is shown in Fig. 2. The system parameters are provided in Table 5. The converter and the line ratings of the VSC HVDC link were chosen to match those of the LCC system.

![Single line diagram of the VSC HVDC system](image)

Figure 2: Single line diagram of the VSC HVDC system

Table 5: VSC HVDC parameters

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Rectifier control mode</td>
<td>Rec, control</td>
<td>P – Q</td>
</tr>
<tr>
<td>Inverter control mode</td>
<td>Inv, control</td>
<td>V&lt;sub&gt;dc&lt;/sub&gt; – Q</td>
</tr>
<tr>
<td>Converter rating</td>
<td>S&lt;sub&gt;rating&lt;/sub&gt;</td>
<td>224 MVA</td>
</tr>
<tr>
<td>DC line voltage</td>
<td>V&lt;sub&gt;dc&lt;/sub&gt;</td>
<td>56 kV</td>
</tr>
<tr>
<td>DC line current</td>
<td>I&lt;sub&gt;dc&lt;/sub&gt;</td>
<td>3.6 kA</td>
</tr>
<tr>
<td>Capacitor</td>
<td>C</td>
<td>1 mF</td>
</tr>
<tr>
<td>DC line resistance</td>
<td>R&lt;sub&gt;L&lt;/sub&gt;</td>
<td>0.15 Ω</td>
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<tr>
<td>DC line inductance</td>
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<tr>
<td>Terminal PCC voltage (grid)</td>
<td>V&lt;sub&gt;PCC&lt;/sub&gt;</td>
<td>230 kV</td>
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</table>
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**Figure 3** 4-machine, 2-area test system with a VSC HVDC link. Secondary control loops with PMU signals are shown.

**Figure 4** Typical PRBS injection used for system identification

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**Figure 6** Overall control energy required for different control allocation levels under heavy loading

**Figure 7** Minimum control energy required for each control loop pairing under normal loading

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**Figure 9** Dynamic performance of the system under normal loading. Legends on top right corner show the secondary control loops used. Plotted variables described below each subplot

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**Figure 11** Dynamic performance of the system under heavy loading. Legends on top right corner show the secondary control loops used. Plotted variables described below each subplot.
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