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A MATLAB finite element toolbox for the efficient nonlinear analysis of axisymmetric shells

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Abstract

Shells of revolution under axisymmetric conditions exhibit a circumferentially uniform pre-buckling stress state and are important fundamental systems which often serve as reference systems for those under more complex conditions. Given this status, work is continuing on a careful and complete characterization of their buckling response with the aid of the Reference Resistance Design (RRD) framework for the ultimate benefit of the EN 1993-1-6 Eurocode on the strength of stability of metal shells. The situation is greatly complicated by the fact that while modern finite element software packages offer axisymmetric shell elements in an efficient 2D modelling plane, these are not capable of detecting bifurcation buckling into non-axisymmetric modes which are often critical for slender systems. Reverting to a full 3D plane is possible, but grossly inefficient and the explicitly modelled circumferential direction is parasitic and detrimental to the overall solution quality. AQUINAS is an accessible and intuitive toolbox developed by the Authors in MATLAB for the efficient analysis of axisymmetric shell structures, aiming to reintroduce a modelling capability that was once standard in the field. Data input is entirely object-oriented and matrix assembly is parallelized with pre-compiled C++ routines, with users being able to take direct advantage of MATLAB's visualization properties. The software natively supports the LA, LBA, MNA, GMNIA etc. Eurocode analysis taxonomy. This paper demonstrates the current capabilities of the toolbox, describes the extensive programme of verification against existing established solutions that has been performed, and illustrates its ability to efficiently compute very detailed capacity curves using the EN 1993-1-6 capacity curve framework.

Keywords

axisymmetric shells, finite element analysis, bifurcation, LA, LBA, GNA, MNA, GMNA, imperfections, MATLAB

1 Introduction

The load bearing capacity of thin axisymmetric shell structures with a circumferentially uniform stress state is often controlled by their buckling resistance, characterised by a harmonic distribution of the buckling displacement field along the circumference of the shell. The circumferential uniformity of the pre-buckling stress state does not allow any obvious localisation of the buckling mode. This causes awkward numerical issues for FE software attempting to solve the axisymmetric shell eigenproblem in a full 3D modelling space.

Fourier series may be used for the description of the circumferentially varying displacements once buckling occurs, a specialised treatment that was once commonplace in the axisymmetric shell related research of a previous era. Custom-written computational tools were distributed between shell researchers, with some very notable and widely used ones being the B0S0R5 by D. Bushnell [1], the FELASH suite by J.G. Teng and J.M. Rotter [2-3] and the INCA software by A. Combescure. Unfortunately, none of the above programs appear to have been maintained through the years, do not currently appear available in a format that can be compiled and are hence no longer an option for the solution of nominally axisymmetric shell buckling problems.

While this specialised mathematical treatment is nowhere to be found in the native 2D modelling space of commercially available FE packages, a workaround for the computation of the buckling load associated with any competing circumferential mode n has been devised by shell analysts aiming to amend this shortcoming of general computational software. This method, termed as the *'panel analysis technique'*, targets the buckling load of an unsymmetrical circumferential mode $n \geq 1$ through the analysis of a shell panel of angular span $\theta = \pi / n$. As discussed in [4],

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the panel analysis technique is very inefficient and in some case of poor solution quality, mostly due to numerical issues arising from the explicit modelling of the parasitic circumferential dimension of the shell. The additional complexity that comes with its implementation makes it an uninviting approach to the bifurcation analysis of axisymmetric shell problems, especially when compared with the alternative of directly including the circumferential harmonic *n* in the shell's finite element formulation.

With this in mind, the Authors have developed AQUINAS, an open-source modern FE toolbox written in MATLAB [5] for the nonlinear analysis of axisymmetric shell problems under a uniform pre-buckling stress state. AQUINAS is capable of tracing the equilibrium path of the shell allowing for both geometrically and materially nonlinear effects to be accounted for as the shell deforms, detecting bifurcation into any circumferential mode $n \geq 0$ by treating the problem of axisymmetric shell buckling in its inherently 2D nature. The software is built using MATLAB's object-oriented programming capabilities, exposing multiple options for the definition of the objects through *name-value* pairs and permitting complete control over the definition of a shell problem.

AQUINAS fully supports all EN 1993-1-6 [6] described types of analyses, making it a very efficient tool for exhaustive parameterised analyses of axisymmetric shell problems. The generation of high-quality nominal capacity curves, needed to make the Reference Resistance Design (RRD) and LBA-MNA methods of EN 1993-1-6 viable options for the design of shells, can be considerably sped up with its use.

2 AQUINAS-FE

The AQUINAS toolbox has been tested on Windows and Linux operating systems for both single-thread and multi-threading analyses. Its source code is publicly accessible through a GitHub repository at https://github.com/AchilleasF/AQUINAS-FE, where a detailed user manual may also be found.

Apart from the class definitions and analysis scripts, an extensive collection of over 30 examples can be found in the repository, serving not only as a guide for the generation of an axisymmetric shell problem in AQUINAS but also as validation of the solution accuracy for a wide range of meridional geometries and analysis types. The FE solution of the software is compared against analytical solutions wherever possible, but also against the results of other well-established computational tools [1-3,7] that are able to perform axisymmetric shell analyses. The thoroughly documented library of example scripts may be regarded as a valuable resource on its own, as prEN 1993-1-6 [6] and prEN 1993-1-14 [8] now require the verification and validation of FE software used for the generation of thin-walled shell models. Further examples will be added with later releases of AQUINAS, extending the repository of reference solutions for axisymmetric shell problems.

2.1 Shell finite element formulation

The finite element formulation presented in [2-3] that was implemented in the FELASH suite has been adopted for AQUINAS, with its existing record of successful applications on the nonlinear analysis of axisymmetric shell structures [2-3,9-11] being a chief reason for this choice. The cubic interpolation field of the element, with an independent definition for the meridional curvature, allows for a very precise computation of the strain field along the meridian.

A key component of the formulation that has motivated the development of AQUINAS is the description of the displacement field associated with bifurcation into an n_{th} harmonic along the circumference of the shell as:

$$u = u_n \cos n\theta \quad \text{(1a)}$$

$$v = v_n \sin n\theta \quad \text{(1b)}$$

$$w = w_n \cos n\theta \quad \text{(1c)}$$

where *u*, *v* and *w* are the radial, circumferential and axial displacements of the shell respectively. The terms associated with torsional effects [2-3] are not included in the present AQUINAS distribution where the pre-buckling stress state in the software is currently limited to one of circumferential uniformity.

2.2 Software structure

AQUINAS capitalises on MATLAB's object-oriented programming capabilities to provide a clean and intuitive scripting interface for the generation of an axisymmetric shell structure. Default values have been assigned to most of the objects' properties, allowing users of varying expertise in nonlinear FE modelling different levels of control at the point of analysis submission.

Assembly of the stiffness matrices and residual force vector (if relevant to the analysis type) can be accelerated through pre-compiled C++ code, generated using MATLAB's MEX functionality. An additional boost in computational performance may also be achieved by utilising the OpenMP API for multi-threaded parallelism, subject to the number of CPUs available in the executing device. The AQUINAS-FE repository currently includes the .mexw64 and .mexa64 pre-compiled binaries, requiring independent compilation if AQUINAS is to run on a different operating system (the dependencies for such a compilation to be successful are outlined in the documentation of the software). True to its open-source nature, the .cpp and .h files necessary to produce MATLAB's MEX files, to be linked to AQUINAS during runtime, are included in the repository.

The AQUINAS_Protocol acts as the main interface function between the software and the analyst, as can be illustrated in the flowchart of Fig. 1. Error checks are performed on the assemblage of AQUINAS object arguments before any analysis script is employed, with meaningful error messages displayed in cases where ill-defined inputs have been provided.

2.3 Analysis capabilities

Generation of a multi-segment meridional geometry can be achieved through the creation of separate segment objects, with the corresponding AQUINAS class offering the abstraction of conical, elliptical and 3-point-arc geometries. The derivatives of the meridional geometric properties, necessary for the formulation of [2-3] to be employed, are computed at the nodal positions within the constructor of the object, reducing the effort required for an analysis submission. An interpolated geometry option is also provided, where an Akima spline [12] is fitted through a series of radial and axial input coordinates, produces a stable fit for geometries of potentially rapidly varying meridional curvature. This allows for the definition of meridians not composed of the basic geometry families above (cones-ellipses-arcs) so that the abstraction at the segment generation level does not lead to a loss of generality. It is also a very useful tool for the explicit inclusion of axisymmetric imperfections in the shell model.

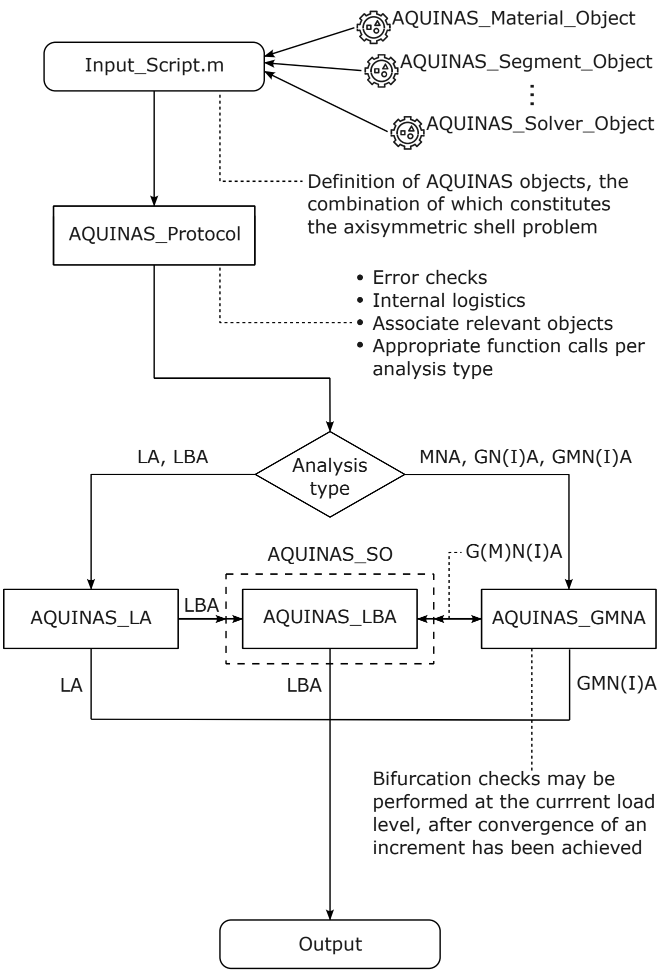

Figure 1 AQUINAS flow of control.

AQUINAS has been built around the EN 1993-1-6 classification of analysis types, as is reflected in its internal structure (Fig. 1). As AQUINAS_LBA implements the eigensolver for the software, computing the material and geometric stiffness matrices for an input of membrane stress resultants at integration stations, it is a crucial part of any solution where bifurcation checks need to be carried out.

Identifying the dependency of the bifurcation resistance of an axisymmetric shell model on the circumferential wavenumber *n*, the critical bifurcation load may be computed through a optimisation process. Application of a minimisation algorithm can lead to considerable acceleration on the computation of the shell's buckling resistance, as bifurcation checks will be performed on just a handful of harmonics instead of all competing ones. A surrogate optimisation algorithm has been included in the distribution of AQUINAS to carry out this task, coded in the AQUINAS_SO script. This acts as a wrapper to the bifurcation check operations of AQUINAS_LBA, with constant communication between the two scripts as the surrogate model is being trained.

Geometric and material nonlinearities are handled within the AQUINAS_GMNA script. While constant load incrementation is supported, the versatile arc-length method can be employed to efficiently trace the nonlinear response of any axisymmetric shell model, with two different implementations of the arc-length included [13,14]. For geometrically nonlinear analyses (GN(I)A, GMN(I)A), bifurcation checks may also be executed once convergence of a load increment has been achieved. Once any of the termination conditions provided by the user at the level of object generation has been satisfied, AQUINAS will output a struct with the FE results of the software, allowing the analyst to take advantage of MATLAB's functionality for post-processing.

3 Capacity curve generation with AQUINAS

With the novel Reference Resistance Design method introduced in EN 1993-1-6 as outlined in [15], the engineer tasked with the design of a shell structure has the option of an entirely algebraic approach to compute its resistance. However, the algebraic characterisation of each reference shell system requires the exploration of its response across different domains of mechanical behaviour, most commonly through the construct of a capacity curve. The analyst may then benefit from a conservative relationship that accounts for any nonlinear phenomena, without resorting to the strenuous execution of a GMNIA.

The generation of a nominal capacity curve for unpressurised cylinders of varying slenderness and imperfections under axial compression with AQUINAS will be shortly presented. Accurate evaluation of the cylinders' ultimate resistance constitutes a very demanding computational task not only for AQUINAS, but for any FE software capable of a GMNIA, since very different nonlinear effects trigger the failure of the appreciably varying models as they advance through the slenderness spectrum.

The dimensionless length of the cylindrical models to be considered in the present illustration is kept constant to a dimensionless Ω parameter which is defined as

$$\Omega = \frac{L}{r}\sqrt{\frac{t}{r}} \qquad (2)$$

with the slenderness of the cylinders varying through a series of r/t ratios. Keeping $\Omega = 1$ and for a unit thickness t, the radius r is manually varied which in turn causes L to change. A BC1r boundary condition is considered at the base of all cylindrical models and a BC2r at their top edge where the axially compressive edge load N is also applied.

Two distinct imperfection shapes will be examined for a range of nominal imperfection amplitudes to illustrate the capabilities of AQUINAS. The cylinders' sensitivity to a mid-height weld depression imperfection shape [10] will be investigated first, with the effect of an eigenmode affine imperfection shape obtained through an AQUINAS LBA for mode $n = 0$ to be subsequently explored.

3.1 Capacity curves for axially compressed cylinders with weld depression imperfections

A realistic imperfection shape that replicates the deviation from the perfect meridional geometry due to the anticlastic bending as the shell plates are rolled and the cooling deformations of a circumferential weld has been proposed by Rotter and Teng [10]. Direct access to the mathematical representation of a Type A weld allows for the generation of the meridional coordinates of the cylinder, with the deviation δ from the straight cylindrical geometry given as:

$$\delta = \delta_m e^{-\frac{\pi|z-z_\delta|}{\lambda}}\left(\cos\frac{\pi|z-z_\delta|}{\lambda} + \sin\frac{\pi|z-z_\delta|}{\lambda}\right) \qquad (3)$$

in which δ_m is the mathematical amplitude of the depression, z_δ is the axial position of the weld and λ is the linear bending half wavelength, found as:

$$\lambda = \frac{\pi\sqrt{rt}}{[3(1-\nu^2)]^{1/4}} \qquad (4)$$

It is crucial to distinguish between the mathematical amplitude of the imperfection δ_m (controlled by the analyst during modelling) and the code-compliant dimple tolerance amplitude δ_0 (verified on the completed physical structure) that is defined relative to an appropriate gauge length l_g. For systems dominated by meridional compression this is set to $l_g = l_{gx}$ [6] given by:

$$l_{gx} = 4\sqrt{rt} \qquad (5)$$

For a meaningful exploration of the sensitivity of cylinders to different kinds of imperfection shapes to be made, the mathematical amplitude δ_m which controls the generation of the model needs to be calibrated as to achieve a target tolerance δ_0 of a specific magnitude [16] (Fig. 2).

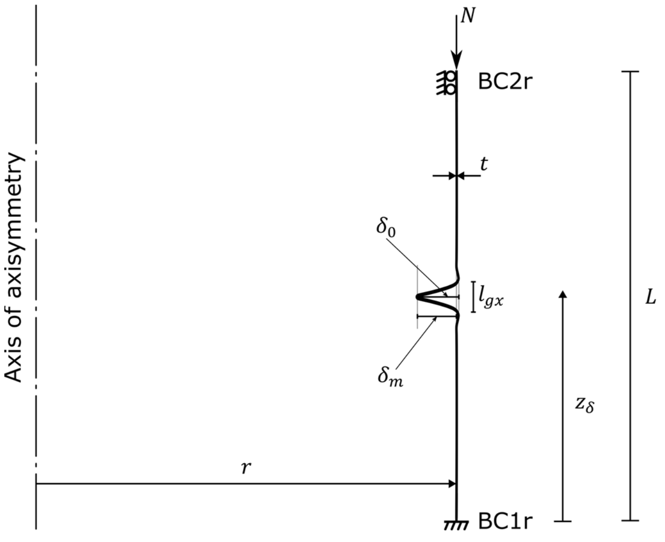

Figure 2 Exaggerated imperfect meridional geometry for a Type A weld depression imperfection.

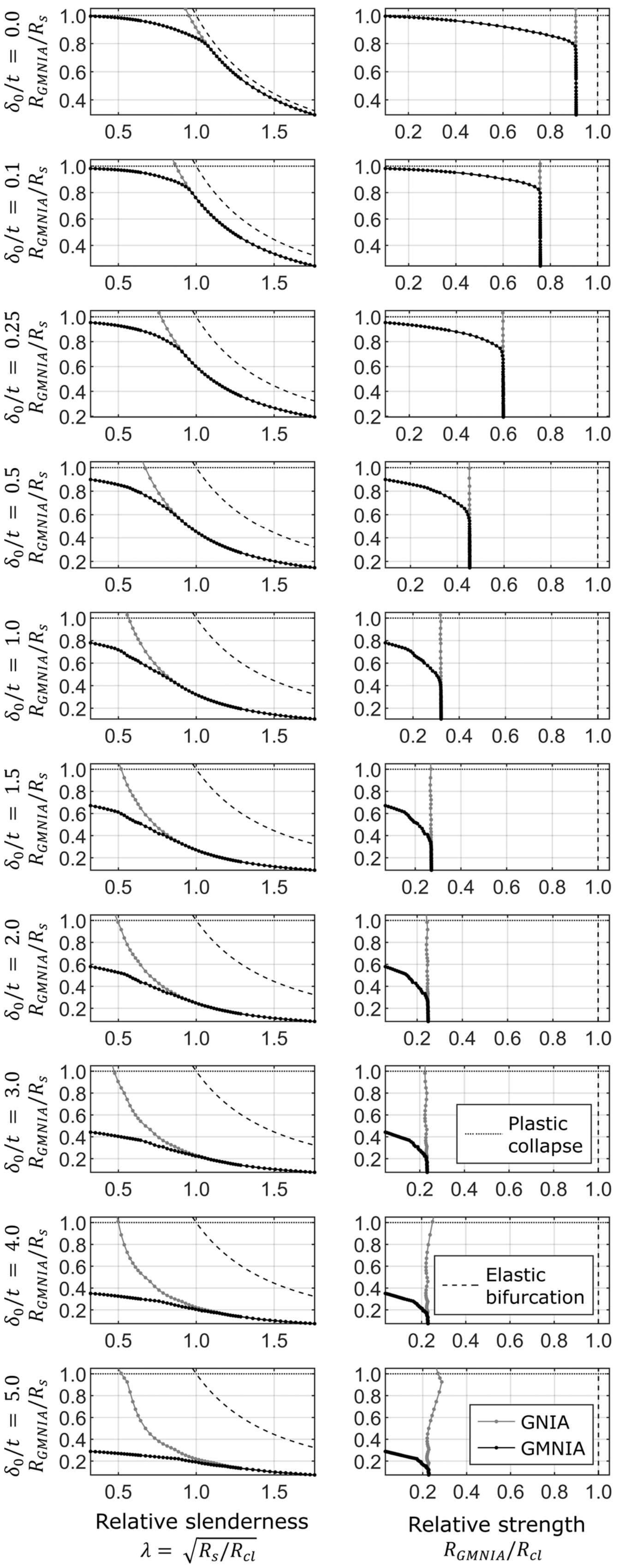

Figure 3 Traditional dimensionless (left) and modified (right) nominal capacity curves for axially compressed cylinders with a Type A weld depression imperfection shape.

Example nominal GMNIA capacity curves for the δ_0/t ratios examined here are presented in Fig. 3, where a strong imperfection sensitivity, exceptionally detrimental cylinders of high slenderness, is evident. These results are a showcase of the capabilities of AQUINAS.

The generation of a capacity curve typically requires an additional set of MNAs and LBAs for each model considered to evaluate the reference plastic R_{MNA} and critical elastic buckling R_{LBA} resistances. This step is omitted in the present exploration, as the reference system of an axially compressed cylinder has been thoroughly researched and analytical expressions for these resistances have been established, a simplification that has also been applied in a similar study [17]. The R_{MNA} reference resistance can here be replaced with the squash load R_s corresponding to the load level that would cause meridional membrane yielding, while R_{LBA} can be replaced with the R_{cl} resistance relating to the classical elastic critical stress:

$$\sigma_{cl} = \frac{E}{\sqrt{3(1-\nu^2)}}\frac{t}{r} \qquad (6)$$

3.2 Capacity curves for axially compressed cylinders with axisymmetric eigenmode affine imperfections

A second set of GMNIAs was performed to estimate the sensitivity of the axially compressed cylinders to an eigenmode affine imperfection shape. Imperfections of this type are commonly considered by analysts in their FE models as they are quite unfavourable to the resistance of the structure, convenient in terms of implementation or simply in the absence of alternative shapes rather than on the grounds of realism or proven worst conceivable deviations.

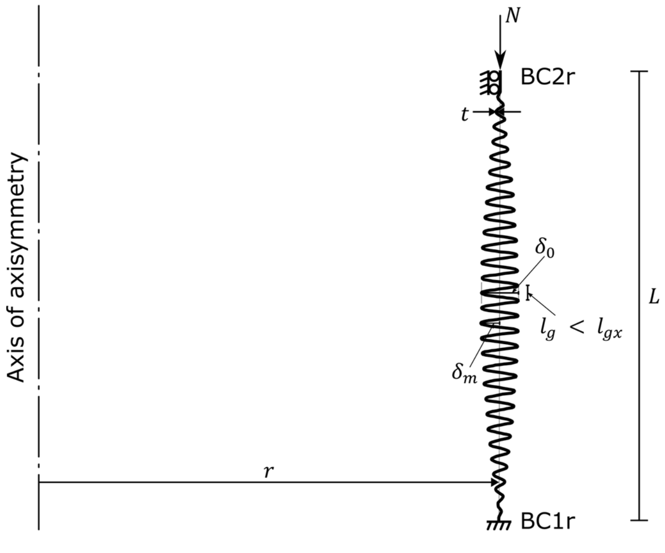

Figure 4 Exaggerated imperfect meridional geometry for an eigenmode affine imperfection shape.

For the present computational treatment, a perfect cylindrical model is submitted through an LBA to obtain the eigenmode shape corresponding to an axisymmetric mode (n = 0), normalised so that the maximum displacement degree of freedom is equal to unity. Calibration of the imperfection shape is not trivial since the mathematical amplitude δ_m (= 1 due to the normalisation and the form of the eigenmode) may not be positioned at the same meridional location as that of the dimple tolerance δ_m (Fig. 4) estimated from the outside of the cylinder with an appropriate gauge length l_{gx}. The fact that eigenmodes are sign reversible is accounted for, and the one leading to the higher scaling factors [16] is considered for the GMNIA step of each model.

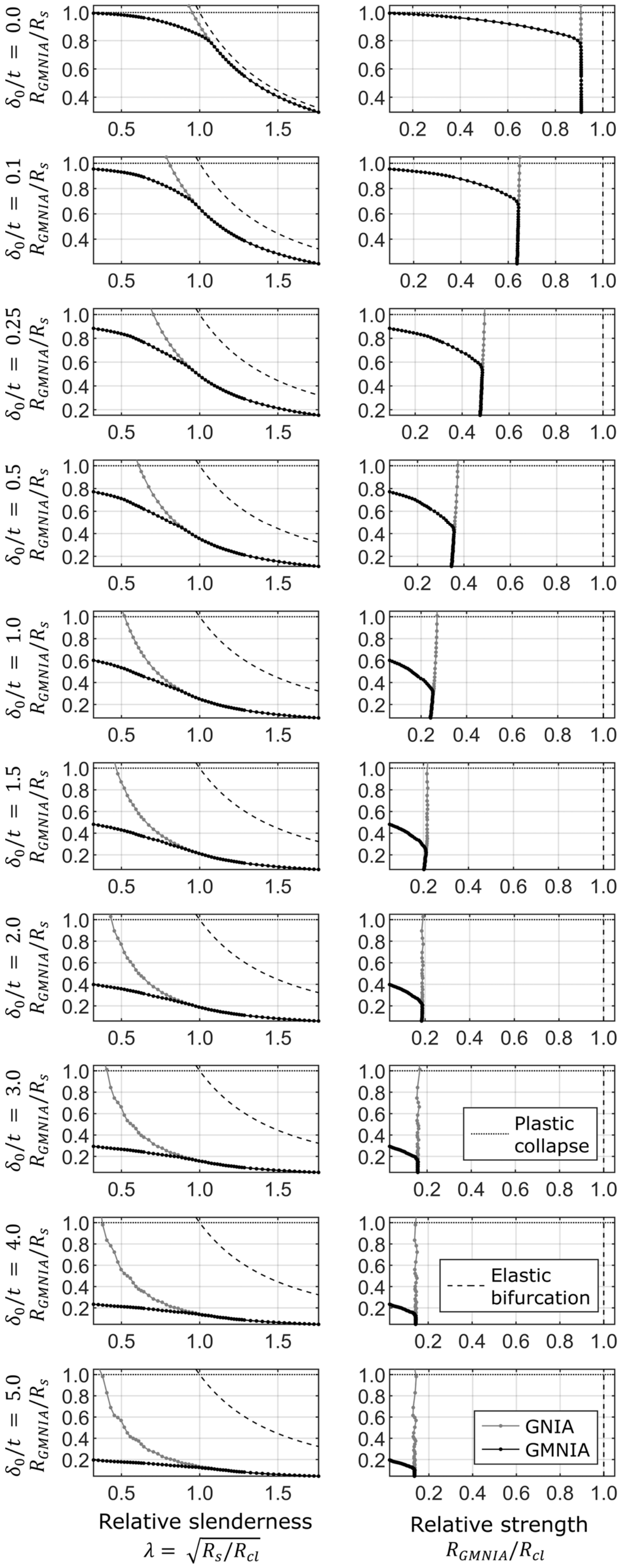

Figure 5 Traditional dimensionless (left) and modified (right) capacity curves for axially compressed cylinders with an eigenmode affine imperfection shape.

In Figure 5 the GMNIA results of AQUINAS are presented for this second set of nonlinear analyses. The algebraic R_s and R_{cl} resistances are once again used in place of the corresponding R_{MNA} and R_{LBA}. The shape of the nominal capacity curves for such deeply imperfect shells bares a strong resemblance with those computed for deeply imperfect cylinders under uniform bending by Wang et al [17] using ABAQUS [7].

4 Concluding remarks

All of the FE analyses were executed on a Red Hat Enterprise Linux 8.5 operating system over several days, running on the High Performance Computing cluster of the Imperial College. The parallelisation of the assemblies was exploited by splitting the calculations through 64 CPUs. It can be appreciated that the computations for the full algebraic characterisation of this reference shell system are exhaustive, as a series of different dimensionless length parameters Ω and boundary conditions need also be considered. A panel analysis [4] approach to perform the evaluation of these capacity curves would be excessively onerous and prohibitively time consuming. The strategy for setting up an analysis hierarchy has been outlined elsewhere [18], a process that may be fully automated with the API exposed by the AQUINAS software.

AQUINAS can be a useful FE tool for any research related to the nonlinear analysis of axisymmetric shells, not limited to analysts interested in intensive computational endeavours for the characterisation of axisymmetric shell systems. The aim is for its repository to be actively maintained and constantly updated with versions of the software that extend its capabilities and enrich the library of reference solution scripts.

5 Acknowledgements

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