Failure of Lightly Reinforced Concrete Members under Fire – Part I: Analytical Modelling

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ABSTRACT

This paper is concerned with the failure of lightly reinforced concrete members under fire conditions, with particular emphasis given to the catenary action arising from axial restraint at the supports and the ensuing rupture of the reinforcement. The relevance of this work stems from the need to make a fundamental step towards understanding the conditions that influence the failure of a steel-decked composite floor slab, which is shown to become effectively lightly reinforced at elevated temperature. A new analytical model is proposed for lightly reinforced members subject to axial restraint, which accounts for the compressive arch and tensile catenary stages, bond-slip, yielding and rupture of the steel reinforcement as well as the effect of elevated temperature. The versatility of the proposed model and the conditions which govern its validity are illustrated in this paper through comparisons with detailed computations based on nonlinear finite element analysis. The companion paper utilises the proposed analytical model to perform a parametric investigation into the factors influencing the failure of lightly reinforced members, and to highlight key implications for structural fire resistance design.

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INTRODUCTION

Over the past few years, the performance of steel framed buildings with steel-deck composite floor slabs under fire conditions has been the focus of intensive experimental, analytical and design-related research (Wang *et al.*, 1995; O'Connor & Martin, 1998; Bailey *et al.*, 1999; Bailey & Moore, 2000-b; Elghazouli & Izzuddin, 2001; Gillie *et al.*, 2001; Izzuddin & Moore, 2002). One of the main outcomes of this research has been the identification of the importance of the composite floor slab in sustaining the gravity load after the loss of strength in the supporting secondary steel beam due to elevated temperature (Fig. 1-a). While the deck in such a slab (Fig. 1-b) is itself subjected to very high temperature leading to a negligible contribution to the overall resistance, the nominal mesh of reinforcement is relatively insulated and contributes to a significant overall resistance due to tensile membrane action. The resulting structural system at elevated temperature is therefore similar to a lightly reinforced concrete (LRC) slab which supports the gravity load mainly through membrane action (Izzuddin *et al.*, 2002).

Although recently developed nonlinear analysis models and design procedures provide a realistic representation of the deflection response of composite floors, there is still a general lack of adequate failure criteria (Izzuddin & Moore, 2002) which determine the levels of deflection that may be sustained. In this context, one of the most important failure criteria is related to the rupture of reinforcement, which is addressed in current design procedures by empirical means (Bailey & Moore, 2000). Significantly, the current empirical failure criteria are very simplistic based on an allowable average reinforcement strain which ignores the influence of such crucial factors as the geometric configuration, bond characteristics, reinforcement ratio and steel stress-strain response, amongst others.

This work is aimed at making a fundamental step towards assessing the failure of LRC members under fire conditions, with particular emphasis given to the rupture of the steel reinforcement. While consideration is restricted here to one-dimensional beam systems, this is believed to be a significant step and an essential precursor to the formulation of an adequate model for the failure of composite floor slabs under fire. As shown later, not only

do one-dimensional models provide significant insight into failure under fire conditions, but they also provide an appropriate vehicle for identifying which of the parameters are of primary importance and which may otherwise be ignored. The latter consideration is relevant for achieving model simplification, this being desirable for several reasons including the ease of model application in the design process.

The paper proceeds with a background on the behaviour of LRC members, highlighting the parameters influencing the compressive arch and tensile catenary stages of the response as well as the rupture of steel reinforcement, both at ambient and elevated temperature. A one-dimensional model for an axially restrained LRC beam is then proposed, which addresses the compressive and tensile stages, bond-slip, elevated temperatures and the rupture of steel reinforcement. A solution procedure for this nonlinear model is then discussed, which is followed by a simplified version of the model that is more suitable for design-based calculations. Finally, several examples are provided which compare the proposed model in its full and simplified forms to detailed nonlinear finite element analysis, illustrating the accuracy of the model and the factors influencing the validity of its assumptions. The companion paper (Elghazouli & Izzuddin, 2002) adopts the proposed model in an extensive parametric study aimed at gaining improved understanding of the failure of LRC members under fire, and leading to the identification of key design implications.

RESPONSE OF LRC MEMBERS

The load-deflection response of a simply supported LRC beam at ambient temperature is illustrated in the schematic diagram of Fig. 2, where consideration is given to the cases with and without axial restraint. The initial response is characterised by snap-back (Bosco *et al.*, 1990) after first crack formation, the extent of which is determined largely by the difference between the ‘uncracked’ and ‘cracked’ bending moments corresponding to the cracking curvature. Subsequently, the ambient response is influenced to a great extent by the presence of axial restraint at the supports.

No axial restraint

Without axial restraint, the ambient response of the LRC beam is governed by bending behaviour, where the maximum load is determined by the ultimate bending moment capacity which is directly related to the ultimate strength of the steel reinforcement. The displacement at which the reinforcement ruptures, however, depends on the extent of cracking, which in turn is determined by the relative values of the cracking and ultimate bending moment capacities, where the cracking moment indicates the ‘uncracked’ value at the cracking curvature.

If the reinforcement is light to the extent that the ultimate moment is less than the cracking moment, only a single crack typically forms, across which high reinforcement strains are concentrated to a level dependent on the bond characteristics. For reinforcement significantly above this limit, multiple cracks can form, the spacing of which depends on the bond strength and reinforcement ratio (Park & Paulay, 1975). Such multiple cracks effectively reduce the strain concentration in the reinforcement, thus increasing the deflection failure limit at which the reinforcement rupture strain is attained.

With axial restraint: compressive arch stage

In the presence of axial restraint, the post-cracking ambient response is initially dominated by a compressive arch effect (Fig. 2) up to deflections comparable to the beam depth, which is due to the development of a resultant compressive axial force. This compressive force and the scale of the corresponding catenary effect are positively dependent on the beam cross-section depth and the support axial stiffness.

A relatively small support axial stiffness in comparison with the beam uncracked axial stiffness can render the compressive arch effect negligible, the response thus converging to that of the axially unrestrained case. Interestingly, the compressive arch effect increases the potential of developing multiple cracks in the LRC beam, since a compressive axial force increases the ultimate moment proportionally more than the cracking moment.

With axial restraint: tensile catenary stage

Assuming that the reinforcement does not rupture in the compressive arch stage, a tensile catenary stage follows which is characterised by a resultant tensile axial force sustained by the reinforcement at one or more through-depth cracks. In this stage, no further cracks are developed beyond what has occurred previously, provided the reinforcement is sufficiently light for the ultimate reinforcement axial force to be less than the cracking axial force, of course taking into account the effect of any reinforcement eccentricity.

For the case of a single crack, a strain concentration is achieved in the reinforcement across the crack, the level of concentration depending largely on the bond characteristics, the steel material response and the member length. For the case of multiple cracks, the overall response is generally less stiff than that of the single crack case, as illustrated in Fig. 2. However, with multiple cracks the level of strain concentration is relatively reduced, enabling the attainment of greater deflections and, consequently, greater resistance at the rupture point of reinforcement. In both cases of single and multiple cracks, the increase in the ultimate structural resistance due to axial restraint and the development of the tensile catenary effect can be significant, often exceeding by far the schematic depiction of Fig. 2.

The development of the tensile catenary stage is governed by a relatively high value for the support axial stiffness in comparison with the cracked beam secant axial stiffness, the latter depending on the reinforcement ratio, the bond strength as well as the ultimate strength and rupture strain of steel. Since the tensile secant axial stiffness of the beam is typically much smaller than its uncracked compressive axial stiffness, it is entirely possible to have a support axial stiffness which is sufficiently small to diminish the compressive arch effect, yet sufficiently large to allow the full development of the tensile catenary effect without inducing a significant axial support displacement. This observation, illustrated by example in the ‘Verification’ Section of the paper, is subsequently used to support the single crack assumption made in the proposed model for axially restrained LRC members.

Effect of elevated temperature

At elevated temperature, the response of a LRC member is determined by the consequential degradation of material properties – including those of steel, concrete and bond – and in many cases by the effects of thermal expansion. For an axially unrestrained member, the bending resistance degrades progressively as the temperature in the steel reinforcement increases. However, it is the axially restrained case which is primarily of interest in the context of LRC members under fire, as it allows considerable gravity loads to be sustained at relatively high reinforcement temperatures through the development of tensile catenary action. Importantly, if no limit is imposed on the reinforcement strain, unrealistically high loads for a given reinforcement temperature, or equivalently unrealistically high reinforcement temperature for a given level of loading, may be sustained through excessive deflections. This emphasises the need for a failure criterion based on the rupture of reinforcement for assessing the fire resistance of LRC members.

In the presence of significant axial restraint, the LRC beam buckles at relatively low temperatures due to restrained thermal expansion (Elghazouli & Izzuddin, 2000), overcoming in the process most of the compressive arch stage of the response. However, if the support axial stiffness is relatively low in comparison with the beam uncracked axial stiffness, the initial response to elevated temperature becomes dominated by bending rather than by buckling behaviour. In either case, a tensile catenary stage follows, provided an adequate level of support axial stiffness exists, as discussed for the ambient response.

The formation of single or multiple cracks under elevated temperature is governed principally by the reinforcement ratio, as outlined previously for the ambient response, taking into account the effect of material degradation. The presence of significant compressive axial restraint, however, can lead to significantly higher compressive axial forces than for the ambient case, thus increasing the potential for multiple crack formation.

Finally, the mechanical strain concentration in the reinforcement is determined by similar factors to the ambient case, including the member length, bond characteristics and the steel material response, but allowing for the effect of material and bond degradation due to

elevated temperature. Besides material degradation, however, thermal curvature due to a temperature gradient over the cross-section depth can lead to a significant reduction in the failure displacement and hence the fire resistance of the LRC member, as will be shown in the companion paper (Elghazouli & Izzuddin, 2002).

MODEL PURPOSE AND ASSUMPTIONS

It is evident from the previous discussion that the failure of LRC members under fire, leaving aside the original problem of composite floor slabs, is a rather complex problem, necessitating the adoption of an incremental research strategy for its treatment. As a significant first step, consideration is given in this work to one-dimensional modelling of LRC beams under fire, where answers to the following questions are sought:

- At what stage – bending, compressive arch or tensile catenary – does the rupture of reinforcement occur for common geometric configurations, material properties, bond characteristics etc.?
- What is the significance of elevated temperatures, particularly in relation to the effects of material degradation, thermal expansion, and the influence of thermal curvature?
- What is the influence of the member length and section depth?
- Is failure sensitive to small variation in the bond characteristics or the steel material law, even if in the latter case the ultimate strength and rupture strain remain unchanged?

In order to address the above questions, as undertaken in the companion paper (Elghazouli & Izzuddin, 2002), an analytical model is developed for axially restrained LRC beams subject to midspan point loading, as illustrated in Fig. 3. This model is concerned with the post-cracking ambient and elevated temperature response, and is based on the following simplifying assumptions:

1. The beam cross-section is rectangular with a single layer of reinforcement.

2. The beam is simply supported at the level of the reinforcement, where full axial restraint is provided thus enabling the development of compressive arch and tensile catenary actions.
3. Axial and shear forces are transferred fully to the concrete at the supports, hence bond-slip is neglected in this region.
4. The reinforcement is assumed to be light enough to cause only a single crack at midspan.
5. Plane sections of the concrete section remain plane, in accordance with the Euler-Bernoulli hypothesis, hence transverse shear deformation is ignored. At the midspan crack, an effective centre of rotation is considered at a prescribed distance h_c from the reinforcement (Fig. 3).
6. The concrete stresses are within its compressive strength.
7. Temperature varies linearly over the cross-section depth, but does not vary over the beam length, where different relative values of temperature may be considered for the top and bottom fibres.
8. Concrete spalling at elevated temperature is ignored, although its effect may be approximated by means of a reduced cross-section geometry.
9. Given the light reinforcement, unrestrained thermal curvature (or bowing) is dominated by the differential thermal expansion of concrete.
10. Equilibrium is based on the deformed configuration, accounting for large midspan deflections and thermal curvature of the concrete. Deformations due to mechanical curvature of the concrete are comparatively small, and hence their effects on equilibrium are ignored.
11. Steel strains and bond-slip deformations vary monotonically.

The validity of much of the above assumptions is considered later in the ‘Verification’ Section of this paper. Hereafter, the details of model formulation based on the above assumptions are presented.

MODEL FORMULATION

The model for the LRC beam of Fig. 3 is formulated on the basis of symmetry about midspan. Half of the beam is first considered in a local reference system, as illustrated in Fig. 4, where the reference axis is positioned at the level of reinforcement. Subsequently, compatibility and equilibrium conditions are employed to establish the response of the full beam in its deflected configuration.

Material modelling

The modelling of steel reinforcement, concrete and bond-slip, including the effects of elevated temperature, are discussed hereafter.

Steel reinforcement

For steel reinforcement, the commonly adopted Ramberg-Osgood model is used, which has the advantage of providing a single stress-strain constitutive function over the full range of elasto-plastic response. This model relates the mechanical strain (ϵ_{sm}) to the stress (σ_s) as follows:

$$\epsilon_{sm} = \frac{\sigma_s}{E_s} + a_s \sigma_s^{n_s} \quad (1)$$

where, E_s is the elastic Young’s modulus, and a_s and n_s are two parameters which influence the elasto-plastic response.

The influence of elevated temperature is considered through allowing E_s and a_s to vary with the steel temperature (t_s). As shown in Fig. 5-a, the variation of E_s is assumed to follow a piecewise trilinear curve, which is realistic for most practical applications (Song *et al.*, 2000). On the other hand, the variation of a_s with temperature is established in terms of the variation of a characteristic strength (σ_{sp}) measured at a given constant plastic strain (ϵ_{sp}), as follows:

$$a_s = \frac{\varepsilon_{sp}}{\sigma_{sp}^{n_s}} \quad (2)$$

where, the variation of σ_{sp} with temperature is again assumed to follow a piecewise trilinear curve (Fig. 5-a).

Finally, the total strain in the steel (ε_s) includes the thermal strain (ε_{st}), which is evaluated using a constant coefficient of thermal expansion (α_s):

$$\varepsilon_s = \varepsilon_{sm} + \varepsilon_{st} \quad (3)$$

$$\varepsilon_{st} = \alpha_s t_s \quad (4)$$

With the assumption of monotonic straining, equations (1-4) can be readily applied for a given temperature (t_s) and stress (σ_s) to evaluate the mechanical and total strains in the steel, ε_{sm} and ε_s , respectively.

Concrete

Given that the proposed model is intended for the post-cracking response following the formation of a single midspan crack, the model does not require the tensile strength of concrete. Accordingly, and considering the assumption that concrete remains within its compressive strength, the response of concrete is realistically assumed to be linear elastic. However, the influence of elevated concrete temperature (t_c) on the elastic modulus (E_c) and on the thermal strain (ε_{ct}) is accounted for:

$$\varepsilon_c = \varepsilon_{cm} + \varepsilon_{ct} \quad (5)$$

$$\varepsilon_{cm} = \frac{\sigma_c}{E_c} \quad (6)$$

$$\varepsilon_{ct} = \alpha_c t_c \quad (7)$$

where, ε_{cm} is the concrete mechanical strain, α_c is the coefficient of thermal expansion, and E_c is assumed to vary with t_c according to a piecewise trilinear curve, as shown in Fig. 5-b.

Bond-slip

In view of the light reinforcement, the influence of bond-slip on the overall member response, leading to the ultimate failure state, is reasonably approximated using a rigid-plastic relationship. Accordingly, only the bond strength (σ_b) is required in the model formulation, where its variation with temperature at the steel/concrete interface (t_s) is assumed to follow a piecewise trilinear curve, as illustrated in Fig. 5-b. It is worth noting that σ_b is defined here as the maximum bond force per unit length, thus encapsulating the effects of the maximum bond stress and the cumulative reinforcement bar perimeters.

Cross-sectional response

The contributions of concrete and steel to the overall cross-sectional response are established separately, in view of the different response characteristics over the two regions of the member length with and without bond-slip (Fig. 4), as discussed in the following Subsection.

The temperature distribution over the concrete cross-section depth is assumed to be linear:

$$t_c = t_s + \nabla t y \quad (8)$$

where,

$$\nabla t = \frac{t_t - t_b}{h} \quad (9)$$

$$t_s = t_b + \nabla t d_s \quad (10)$$

In the above, ∇t is the temperature gradient, t_s is the temperature at the level of the steel reinforcement, t_b and t_t are the bottom and top fibre temperatures, h is the cross-section depth, and d_s represents the reinforcement cover distance, as illustrated in Fig. 6.

Considering concrete first and in view of the material response assumed in the previous Subsection, the relationship between the generalised stresses (axial force/bending moment) and the generalised strains (axial strain/curvature), in the presence of thermal expansion effects, can be readily shown to have the following linear form:

$$\begin{Bmatrix} f_c \\ m_c \end{Bmatrix} = \mathbf{k}_c \begin{Bmatrix} \varepsilon_a - \alpha_c t_s \\ \kappa + \alpha_c \nabla t \end{Bmatrix} \quad (11)$$

with,

$$\mathbf{k}_c = \begin{bmatrix} EA_c & EY_c \\ EY_c & EI_c \end{bmatrix} \quad (12)$$

$$EA_c = b \int_{-d_s}^{h-d_s} E_c dy \quad (13)$$

$$EY_c = -b \int_{-d_s}^{h-d_s} E_c y dy \quad (14)$$

$$EI_c = b \int_{-d_s}^{h-d_s} E_c y^2 dy \quad (15)$$

where, (f_c, m_c) are the concrete axial force and bending moment, (ε_a, κ) are the axial strain and curvature, and b is the cross-section width (Fig. 6). Note that a positive sign convention is adopted for tensile axial force/strain and for sagging bending moment/curvature.

For a given linear variation of t_c with y according to (8), and in view of the piecewise linear variation of E_c with t_c , the determination of the concrete section constants in (13-15) involves the integration over the depth of piecewise linear, quadratic and cubic functions, respectively.

Steel, on the other hand, contributes only to the cross-sectional axial force, (considering the chosen reference axis) as follows:

$$f_s = A_s \sigma_s \quad (16)$$

In the bond-slip region of the member, the nonlinear stress-strain relationship of (1) is considered; however, in the region without bond-slip, a linearised form of (1) is required to facilitate the determination of the bond length (L_d) in the following Subsection. This linearisation is performed around a steel stress σ_{s0} corresponding to a total steel strain equal to the thermal strain of concrete at the level of reinforcement, which is obtained as the solution of the following nonlinear equation:

$$\frac{\sigma_{s0}}{E_s} + a_s \sigma_{s0}^{n_s} = (\alpha_c - \alpha_s) t_s \quad (17)$$

Once σ_{s0} is determined, the following linear form representing a tangential approximation of the steel stress-strain relationship can be employed for the no bond-slip region:

$$\sigma_{sn} = \sigma_{s0} + E_{st}(\epsilon_{an} - \alpha_c t_s) \quad (18)$$

where,

$$E_{st} = \frac{E_s}{1 + E_s n_s a_s \sigma_{s0}^{n_s-1}} \quad (19)$$

The above enables the cross-sectional response to be realistically linearised in the region without bond-slip, taking into consideration the dominance of concrete over the light reinforcement. This dominance implies that the unrestrained cross-sectional response to thermal effects will induce strains in the steel which are very close to the thermal strain of concrete at the reinforcement level, thus justifying the linearisation of the steel response in the proposed manner.

Local response

With reference to Fig. 4, the resultant forces acting at the crack are the tensile force in the steel bar (T_s), a resultant compressive force in the concrete (C_c) and an equilibrating shear force (V). The moment induced by C_c about the reference axis is denoted by M_c . For a specific temperature distribution and values of T_s , C_c , M_c and V , the steel and concrete deformations at the crack location are to be determined.

Equilibrium conditions are enforced with reference to the thermally curved configuration, determined by the thermal expansion of concrete, which is defined by (Fig. 4):

$$\kappa_t = -\alpha_c \nabla t \quad (20)$$

$$\theta_{t0} = \frac{\kappa_t L}{2} \quad (21)$$

$$\theta_t(x) = -\theta_{t0} + \kappa_t x \quad (22)$$

$$v_t(x) = -\theta_{t0} x + \frac{\kappa_t}{2} x^2 \quad (23)$$

where, κ_t is the thermal curvature, and $\theta_t(x)$ is the slope along the reference line.

No bond-slip region

The generalised stresses in the region of no bond-slip are determined from equilibrium in the thermally curved configuration, accounting for second-order geometric effects:

$$\begin{Bmatrix} f(x) \\ m_c(x) \end{Bmatrix} = \begin{Bmatrix} (T_s - C_c) \left(1 - \frac{(\theta_{t0} + \theta_t(x))^2}{2} \right) + V(\theta_{t0} + \theta_t(x)) \\ M_c \left(1 - \frac{x}{L} \right) + \left((T_s - C_c) \left(1 - \frac{\theta_{t0}^2}{2} \right) + V\theta_{t0} \right) v_t(x) \end{Bmatrix} \quad (24)$$

where,

$$V = \frac{M_c + (T_s - C_c)\theta_{t0} L}{\left(1 - \frac{\theta_{t0}^2}{2} \right) L} \quad (25)$$

Given that the above generalised stresses consist of contributions from the concrete, according to (11) and (12), and from the steel, according to (16) and (18), the generalised strains in this region (ϵ_{an}, κ_n) can be determined from:

$$\begin{Bmatrix} \epsilon_{an}(x) \\ \kappa_n(x) \end{Bmatrix} = \mathbf{k}^{-1} \begin{Bmatrix} f(x) - A_s \sigma_{s0} \\ m_c(x) \end{Bmatrix} + \begin{Bmatrix} \alpha_c t_s \\ -\alpha_c \nabla t \end{Bmatrix} \quad (26)$$

where,

$$\mathbf{k} = \mathbf{k}_c + \begin{bmatrix} E_{st} A_s & 0 \\ 0 & 0 \end{bmatrix} \quad (27)$$

with $f(x)$ and $m_c(x)$ as given by (24).

Bond-slip region

Given the assumption of a constant bond resistance (σ_b), the axial force in the reinforcement reduces linearly over the bond-slip region, leading to the following variation in the steel stress:

$$\sigma_s(x) = \frac{T_s - \sigma_b x}{A_s} \quad (28)$$

The corresponding variation of the reinforcement strain follows the steel material law given by (1) and (3):

$$\varepsilon_s(x) = \frac{\sigma_s(x)}{E_s} + a_s \sigma_s(x)^{n_s} + \alpha_s t_s \quad (29)$$

For concrete, the generalised stresses are determined from the distributed bond force imparted by the steel and from the actions at the crack face, accounting for equilibrium in the thermally curved configuration:

$$\begin{Bmatrix} f_c(x) \\ m_c(x) \end{Bmatrix} = \begin{Bmatrix} \sigma_b x - C_c - (T_s - C_c) \left(\frac{(\theta_{t0} + \theta_t(x))^2}{2} \right) + V(\theta_{t0} + \theta_t(x)) \\ M_c \left(1 - \frac{x}{L} \right) + \left((T_s - C_c) \left(1 - \frac{\theta_{t0}^2}{2} \right) + V \theta_{t0} \right) v_t(x) \end{Bmatrix} \quad (30)$$

Considering the above generalised stresses, the concrete generalised strains are obtained from (11) as:

$$\begin{Bmatrix} \varepsilon_a(x) \\ \kappa(x) \end{Bmatrix} = \mathbf{k}_c^{-1} \begin{Bmatrix} f_c(x) \\ m_c(x) \end{Bmatrix} + \begin{Bmatrix} \alpha_c t_s \\ -\alpha_c \nabla t \end{Bmatrix} \quad (31)$$

where, $\mathbf{k}_c$ is given by (12).

Local deformations

Firstly, the length of the bond-slip region (x_d) is determined as the location at which the two values of the steel stress, associated with the bond-slip and no bond-slip regions, become identical (Fig. 4):

$$\sigma_s(x_d) = \sigma_{sn}(x_d) \quad x_d \in [0, L] \quad (32)$$

$$L_d = L - x_d \quad (33)$$

Considering the expressions required for σ_{sn} , mainly (18) and (26), and the expression for σ_s in (28), the determination of x_d involves the solution of a quadratic equation, only one root of which would typically be in the valid range $[0, L]$. The upper limit (L) is assigned to

x_d if both roots are outside the valid range or if the required bond transfer at the support exceeds the bond resistance according to:

$$-A_s \frac{d\sigma_{sn}(x)}{dx} \Big|_{x=L} > \sigma_b \quad (34)$$

The extension of steel (Δ_s) and the shortening of concrete (Δ_c) along the thermally curved reference line as well as the local rotation (θ_c) can now be obtained (Fig. 7):

$$\Delta_s = \int_0^{x_d} \varepsilon_s(x) dx + \int_{x_d}^L \varepsilon_{an}(x) dx \quad (35)$$

$$\Delta_c = -\int_0^{x_d} \varepsilon_a(x) dx - \int_{x_d}^L \varepsilon_{an}(x) dx \quad (36)$$

$$\theta_c = \int_0^{x_d} \left(1 - \frac{x}{L}\right) \kappa(x) dx + \int_{x_d}^L \left(1 - \frac{x}{L}\right) \kappa_n(x) dx \quad (37)$$

Finally, the axial component of the concrete shortening (δ_c) is required (Fig. 7), which is obtained in a similar manner to Δ_c but accounting for the second-order thermal curvature effects:

$$\delta_c = \delta_{c0} - \int_0^{x_d} \left(1 - \frac{\theta_t(x)^2}{2}\right) \varepsilon_a(x) dx - \int_{x_d}^L \left(1 - \frac{\theta_t(x)^2}{2}\right) \varepsilon_{an}(x) dx \quad (38)$$

in which,

$$\delta_{c0} = \int_0^L \frac{\theta_t(x)^2}{2} dx = \frac{\theta_{t0}^2 L}{6} \quad (39)$$

where, δ_{c0} represents the unrestrained axial pull-in due to thermal bowing.

Overall compatibility and equilibrium

In enforcing compatibility between the two symmetric halves of the LRC beam (Figs. 3 & 4), it is assumed that the reinforcement has negligible bending resistance, thus stretching in a horizontal straight line across the midspan crack. With reference to Fig. 8, the vertical deflection can be obtained from the steel and concrete deformations determined previously:

$$U = \sqrt{(L - \delta_c)^2 - (L - \Delta_s - \Delta_c)^2} \quad (40)$$

Due to the discontinuity in the direction of the reinforcement at the crack, a small compressive force (C_{c0}) is imparted on the concrete at the level of the reinforcement, with the remainder of the compressive force ($C_c - C_{c0}$) taken at the contact point:

$$C_{c0} = T_s \left[1 - \left(1 - \frac{\theta_{t0}^2}{2} \right) \left(\frac{L - \Delta_s - \Delta_c}{L - \delta_c} \right) - \theta_{t0} \left(\frac{U}{L - \delta_c} \right) \right] \quad (41)$$

Considering that M_c is the resultant concrete moment about the reference axis (Fig. 9), the shear force at the crack, (V_l), and the support reactions, (V_r) and (T_r), are obtained accounting for the concrete shortening due to mechanical and thermal deformations:

$$V_l = \frac{M_c + (T_s - C_c)(L - \delta_c)\theta_{t0}}{\left(1 - \frac{\theta_{t0}^2}{2} \right)(L - \delta_c)} \quad (42)$$

$$V_r = \frac{M_c}{(L - \delta_c)} \quad (43)$$

$$T_r = (T_s - C_c) \left(1 - \frac{\theta_{t0}^2}{2} \right) + V_l \theta_{t0} \quad (44)$$

It is worth noting that V (Fig. 4) and V_l (Fig. 9) are slightly different, as the latter shear force includes the small effect of concrete shortening due to mechanical deformations.

Now P , which is half of the applied load, can be determined from the support reactions (Figs. 8 & 9):

$$P = \frac{U T_r + (L - \Delta_s - \Delta_c) V_r}{L - \delta_c} \quad (45)$$

The above expressions can be evaluated for a given temperature distribution and tensile force in the steel reinforcement (T_s), assuming that the concrete axial force (C_c) and moment (M_c) are known. Two additional conditions are required to enable the determination of C_c and M_c for a given T_s , the first of which is the following expression for M_c (Figs. 8 & 9):

$$M_c = (C_c - C_{c0})h_c \quad (46)$$

The second condition is the contact constraint at distance h_c above the reinforcement (Figs. 3, 7 & 8):

$$\left. \begin{array}{ll} \theta_c = \theta_{c0} & \text{if } C_c - C_{c0} - P_a \geq 0 \\ C_c - C_{c0} - P_a = 0 & \text{if } \theta_c \geq \theta_{c0} \end{array} \right\} \quad (47)$$

where, θ_{c0} is the lower limit on the cross-section rotation at contact (Fig. 8):

$$\theta_{c0} = \frac{h_c U - (L - \delta_c)(\Delta_s + \Delta_c)}{h_c(L - \Delta_s - \Delta_c)} \quad (48)$$

and, P_a is the axial component of P accounting for the thermal rotation:

$$P_a = P \left[\left(\frac{U}{L - \delta_c} \right) \left(1 - \frac{\theta_{t0}^2}{2} \right) - \theta_{t0} \left(\frac{L - \Delta_s - \Delta_c}{L - \delta_c} \right) \right] \quad (49)$$

In the compressive arch stage the first expression of (47) applies, thus indicating compressive contact, whereas in the tensile catenary stage the second expression applies, thus indicating separation and therefore through-depth cracking.

MODEL SOLUTION PROCEDURE

The proposed model results in a highly nonlinear system of equations requiring an incremental-iterative strategy for its solution. The following procedure is employed for this purpose:

-
- Consider values for the top and bottom fibre temperatures (t_t, t_b); for each combination:
 1. Determine ∇t from (9) and t_s from (10)
 2. Establish k_c from (12–15)

3. Obtain σ_{s0} from the solution of (17), E_{st} from (19) as well as θ_{t0} , $\theta_t(x)$ and $v_t(x)$ from (21–23)
4. Consider values of T_s up to the ultimate reinforcement force T_{su} ; for each value:
 - i. Set iterative values for C_c and M_c
 - Obtain x_d from the solution of (32)
 - Establish Δ_s , Δ_c , θ_c and δ_c from (35–38)
 - Determine U from (40) and P from (45)
 - ii. If conditions (46) and (47) are not satisfied to an acceptable tolerance, correct C_c and M_c , and repeat (i)
 - iii. Store current state (t_t, t_b, T_s, P, U)

In the above, the value of T_{su} corresponds to the rupture of reinforcement, which in the present work is based on an ultimate plastic strain ε_{pu} that may depend on temperature if necessary. Considering (1), T_{su} is determined as follows:

$$T_{su} = A_s \left(\frac{\varepsilon_{pu}}{a_s} \right)^{1/n_s} \quad (50)$$

The use of a symbolic computation tool facilitates considerably the implementation of the above procedure, where a Maple (2001) worksheet has been developed for this purpose (Izzuddin & Elghazouli, 2002). Amongst other capabilities, this tool readily performs the integrations in (13–15, 35–38) and solves (17, 32). Furthermore, it establishes the gradients of the two conditions (46, 47) with respect to (C_c, M_c) , thus enabling the iterative correction in step (4.ii) to be implemented as a Newton-Raphson type of procedure.

SIMPLIFIED MODEL

Clearly, the proposed model, described in the two previous Sections, is rather too complex for application in design-based calculations, since it involves difficult integrations, lengthy expressions and the solution of two simultaneous nonlinear equations. A simplified model

with a reduced scope is developed here, which is verified against the full model in the next Section.

The simplified model deals only with the tensile catenary stage, with the expectation that the rupture of reinforcement for most realistic cases occurs in this stage (Elghazouli & Izzuddin, 2002). The first simplification is in the assumption of linear variation in the concrete stiffness over the depth due to the temperature gradient, instead of the piecewise linear variation of the full model. This leads to the following explicit expressions for the terms of $\mathbf{k}_c$:

$$EA_c = \frac{bh}{2} [E_{cb} + E_{ct}] \quad (51)$$

$$EY_c = -\frac{bh}{6} [E_{cb}(h - 3d_s) + E_{ct}(2h - 3d_s)] \quad (52)$$

$$EI_c = \frac{bh}{12} [E_{cb}(h^2 - 4hd_s + 6d_s^2) + E_{ct}(3h^2 - 8hd_s + 6d_s^2)] \quad (53)$$

where, E_{ct} and E_{cb} are the top and bottom fibre concrete stiffness, respectively, as influenced by temperature.

The second simplification is ignoring the moment about the reference axis and the resultant compressive concrete force at the crack face, leading to the following explicit expression for the generalised strains in the no bond-slip region:

$$\begin{bmatrix} EA_c + E_{st}A_s & EY_c \\ EY_c & EI_c \end{bmatrix} \begin{Bmatrix} \varepsilon_{an} - \alpha_c t_s \\ \kappa_n + \alpha_c \nabla t \end{Bmatrix} = \begin{Bmatrix} T_s - A_s \sigma_{s0} \\ 0 \end{Bmatrix} \Rightarrow$$

$$\begin{Bmatrix} \varepsilon_{an} \\ \kappa_n \end{Bmatrix} = \begin{Bmatrix} \alpha_c t_s \\ -\alpha_c \nabla t \end{Bmatrix} + \frac{T_s - A_s \sigma_{s0}}{(EA_c + E_{st}A_s)EI_c - (EY_c)^2} \begin{Bmatrix} EI_c \\ -EY_c \end{Bmatrix} \quad (54)$$

which are now independent of x .

The solution of (32) for x_d is now simplified to:

$$x_d = \frac{T_s - A_s \sigma_{sn}}{\sigma_b} \quad (55)$$

where σ_{sn} is obtained from ε_{an} of (54) according to (18).

The extension of steel (Δ_s) is also obtained explicitly as:

$$\Delta_s = \int_0^{x_d} \varepsilon_s dx + \varepsilon_{an} L_d = \int_0^{x_d} \left(\frac{\sigma_s(x)}{E_s} + a_s \sigma_s(x)^{n_s} \right) dx + \varepsilon_{st} x_d + \varepsilon_{an} L_d \Rightarrow$$

$$\Delta_s = \left[\frac{1}{2E_s} \left(\frac{T_s + A_s \sigma_{sn}}{A_s} \right) + a_s \frac{T_s^{n_s+1} - (A_s \sigma_{sn})^{n_s+1}}{(n_s+1)(T_s - A_s \sigma_{sn}) A_s^{n_s}} + \alpha_s t_s \right] x_d + \varepsilon_{an} L_d \quad (56)$$

The third simplification is the assumption that the load is applied at the level of the reinforcement, leading to an inclined length of reinforcement across the crack, as illustrated in the modified compatibility diagram of Fig. 10. Furthermore, the effect of thermal bowing on the shortening of concrete due to mechanical deformations is ignored, leading to the following simplified expression:

$$\delta_c = \delta_{c0} + \Delta_c \quad (57)$$

where, δ_{c0} is given by (39).

Considering Fig. 10, the beam deflection and applied load are given by:

$$U = \sqrt{(L - \delta_{c0} + \Delta_s)^2 - L^2} \quad (58)$$

$$P = \frac{U}{L - \delta_{c0} + \Delta_s} T_s \quad (59)$$

Evidently, the simplified model does not require an iterative solution procedure, and is conveniently based on explicit expressions, except for σ_{s0} which is still obtained as the solution of (17). However, the simplified model is only valid in the tensile catenary stage after the formation of a through-depth crack, or mathematically:

$$\theta_c \geq \theta_{c0} \Leftrightarrow \text{valid simplified model} \quad (60)$$

where, from Fig. 10:

$$\theta_{c0} = \frac{U}{L} - \frac{\Delta_s + \Delta_c}{h_c} \quad (61)$$

The required concrete deformations are obtained by first evaluating the generalised strains in the bond-slip region:

$$\mathbf{k}_c \begin{Bmatrix} \varepsilon_a - \alpha_c t_s \\ \kappa + \alpha_c \nabla t \end{Bmatrix} = \begin{Bmatrix} \sigma_b x \\ 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} \varepsilon_a \\ \kappa \end{Bmatrix} = \begin{Bmatrix} \alpha_c t_s \\ -\alpha_c \nabla t \end{Bmatrix} + \frac{\sigma_b x}{EA_c EI_c - (EY_c)^2} \begin{Bmatrix} EI_c \\ -EY_c \end{Bmatrix} \quad (62)$$

and then performing the integration over the full length, leading to the following explicit expressions:

$$\Delta_c = - \int_0^{x_d} \varepsilon_a dx - \varepsilon_{an} L_d = -\alpha_c t_s x_d - \left[\frac{\sigma_b EI_c}{EA_c EI_c - (EY_c)^2} \right] \frac{x_d^2}{2} - \varepsilon_{an} L_d \quad (63)$$

$$\theta_c = \int_0^{x_d} \left(1 - \frac{x}{L}\right) \kappa dx + \int_{x_d}^L \left(1 - \frac{x}{L}\right) \kappa_n dx \Rightarrow$$

$$\theta_c = -\alpha_c \nabla t \frac{(L + L_d) x_d}{2L} - \left[\frac{\sigma_b EY_c}{EA_c EI_c - (EY_c)^2} \right] \frac{(L + 2L_d) x_d^2}{6L} + \kappa_n \frac{L_d^2}{2L} \quad (64)$$

As for the full model, the simplified model has been implemented as a Maple worksheet (Izzuddin & Elghazouli, 2002), thus enabling a direct comparison between the two models in terms of accuracy and efficiency.

VERIFICATION

The verification of the detailed and simplified analytical models proposed for restrained LRC members is undertaken here with reference to results obtained using the nonlinear finite element analysis program ADAPTIC (Izzuddin, 1991), which has been extensively verified for reinforced concrete (Karayannis *et al.*, 1994; Izzuddin & Lloyd Smith, 2000) and steel (Izzuddin & Elnashai, 1993-a, 1993-b; Izzuddin *et al.*, 2000) framed structures under ambient and fire conditions. The modelling of a simply supported LRC member with ADAPTIC is first outlined, and is subsequently employed in justifying the single-crack assumption of the present work. Finally, the accuracy of the proposed analytical models is demonstrated with reference to a single-crack ADAPTIC model, where consideration is given to both the ambient and elevated temperature member responses.

Modelling with ADAPTIC

One-dimensional beam-column elements are used for modelling LRC members with ADAPTIC, accounting for both geometric nonlinearity (Izzuddin & Elnashai, 1993-a) and material nonlinearity by means of the fibre approach (Izzuddin & Elnashai, 1993-b). With reference to Figs. 3, 5 and 6, the structural details of the LRC members considered are given in Table 1, whereas the material properties of steel and concrete are provided in Tables 2 and 3. Since the aim here is to provide basic verification of the proposed models, the task of justifying the choice of geometric and material characteristics as well as their variation within practical limits is left to the companion paper (Elghazouli and Izzuddin, 2002).

With ADAPTIC, the concrete and steel reinforcement are each modelled using 15mm long beam-column elements, which are connected at their coincident nodes with joint elements representing the bond-slip characteristics of the reinforcement. Accordingly, for the case ($L = 1.5\text{m}$), 100 beam-column elements are used for each of the steel reinforcement and concrete, and 99 joint elements are employed to represent the bond-slip response, where consideration is given only to half the span due to symmetry. This fine level of discretisation is found to be necessary for achieving convergence in the prediction of finite element analysis. It should also be noted that, in addition to its significant computational demand, finite element approximation is found to require considerable skill in order to overcome the numerical difficulties associated with the severe geometric and material nonlinearities of the current problem.

With reference to the steel material properties in Table 2, a bilinear approximation is required by ADAPTIC for the beam-column elements representing the reinforcement, as illustrated in Fig. 11. Furthermore, the tensile strength (f_t) and compressive strength (f_{cu}) of concrete, as given in Table 3, are required by the ADAPTIC elements representing the concrete part of the LRC member, even though such properties are not actually employed within the analytical models proposed in this work.

Single-crack assumption

In order to assess the adequacy of the single-crack assumption made in the formulation of the proposed models, the influence of multiple cracking on the response of a LRC member is investigated under ambient conditions using the nonlinear finite element models of ADAPTIC. Two alternative models are considered in representing the discrete nature of cracks: 1) a single-crack model (**S**) where only the 15mm concrete element attached to the midspan node is allowed to crack, and 2) a multi-crack model (**M**) where cracks are allowed to develop every 10 elements starting from midspan. For the **M**-model, the choice of 150mm as the minimum distance between cracks is made in view of the inability of one-dimensional elements to account for complex strain variations over the member depth, due to the use of the Euler-Bernoulli hypothesis. Nevertheless, the **M**-model should still provide a good picture of whether multiple cracks are likely to occur, although the extent of multiple cracking, both longitudinally and transversely, may not be accurately predicted.

Consideration is first given to the shorter member length ($L=1.5\text{m}$) with the lower reinforcement ratio ($\rho=0.0833\%$), for which both the ultimate bending moment and axial force capacities are less than the respective cracking capacities, where the variation of load and maximum steel strain with midspan displacement is depicted in Fig. 12. For the case of full axial restraint (**F**) at the supports, it is clear that a significant compressive arch action develops prior to the initiation of tensile catenary action. In this case, the multi-crack model (**MF**) is clearly less stiff than its single-crack counterpart (**SF**), though the level of strain in the steel reinforcement is significantly reduced by almost 50%, thus indicating the formation of more than one central crack. However, for a reduced support axial stiffness of 5000kN/m, referred to as case (**T**), the compressive arch effect disappears while the tensile catenary is maintained. This is due to the fact that the support axial stiffness is small in comparison with the compressive member stiffness but large in relation to the secant tensile member stiffness. For this case, the absence of the compressive arch action combined with the fact that ultimate bending moment and axial force capacities are less than the respective cracking capacities leads to the formation of only a single crack, as can be observed from the identical results of (**MT**) and (**ST**) for both load and strain variations. This highlights that, in evaluating the

maximum steel strain, the single-crack assumption is conservative and in fact necessary if the level of axial support stiffness is uncertain. It is also clear from the results that the absence of a limit on the maximum steel strain can lead to an unrealistically excessive prediction of the ultimate resistance due to the progressively stiffening tensile catenary effect. This is unlike the case of low axial support stiffness of 50kN/m, referred to as case (**L**), where the post-cracking ultimate resistance is almost independent of the steel strain due to the absence of a significant tensile catenary effect.

The significance of the compressive arch action and its implication on multiple crack formation reduce also with an increase in the member length, as illustrated for a longer LRC member ($\rho = 0.0833\%$, $L = 3.0\text{m}$) in Fig. 13. In addition to the evident reduction in the compressive arch action, it is clear that the single- and multi-crack models provide identical results for the most demanding case of full axial restraint at the supports, thus indicating that only a single crack occurs for this longer member at midspan.

In addition to the compressive arch effect, multiple cracking can occur if the ultimate cross-section capacity exceeds its cracking limit, taking into account any eccentricity of the reinforcement. Such multiple cracks are guaranteed to occur prior to failure if the ultimate axial capacity exceeds the cracking axial force, particularly when failure is associated with the rupture of reinforcement in the tensile catenary stage. On the other hand, multiple cracks may also form even if only the ultimate bending capacity exceeds the cracking limit, in which case additional cracks form away from the initial midspan crack during the bending stage of the response. This is illustrated by considering the shorter member ($L = 1.5\text{m}$) with the larger reinforcement ratio ($\rho = 0.25\%$), for which the ultimate bending moment capacity, but not the axial force capacity, exceeds the cracking limit. The results in Fig. 14 indicate discrepancies between the single- and multi-crack models regardless of the axial support stiffness, whether full (**F**) or partial (**T**), thus highlighting that multiple cracks form due to the ultimate bending moment exceeding the cracking limit rather than due to a significant compressive arch action. However, given the uncertainty in multiple crack prediction with one-dimensional modelling of the bending stage, and the conservative predictions of the

single-crack model in respect of both the loading and displacement for a specific rupture reinforcement strain (eg. $\varepsilon_{pu} = 15\%$), a single-crack assumption is justified even for the larger reinforcement ratios. A key requirement, however, is that the ultimate axial capacity must be less than the cracking limit, thus forming a suitable definition of what constitutes a LRC member for the purpose of the present work.

Accuracy of proposed models

Having made a case for the single-crack assumption, the accuracy of the two proposed models, detailed and simplified, is investigated here with reference to the single-crack nonlinear finite element models of ADAPTIC assuming full axial restraint at the supports. For this purpose, the last structural configuration of the previous Section ($\rho = 0.25\%$, $L = 1.5\text{m}$) is considered at ambient and elevated temperature, where in the latter case the bottom and top fibre temperatures are assumed as ($t_b = 800^\circ\text{C}$, $t_t = 0^\circ\text{C}$) leading to a steel temperature of ($t_s = 400^\circ\text{C}$).

The results for the ambient case are depicted in Fig. 15, where two alternative scenarios are considered with the detailed model in respect of the location (h_c) of the contact point at the midspan crack ($h_{c1} = h - d_s$, $h_{c2} = 0.75 h_{c1}$). These results indicate a reasonable comparison between the proposed detailed model and ADAPTIC in respect of the compressive arch action, particularly for the lower position of the contact point ($h_c = h_{c2}$). Nevertheless, the assumption of contact at the top fibre ($h_c = h_{c1}$) is more conservative with regard to the steel strain in the compressive arch stage, and is hence preferred for failure assessment purposes under general conditions. Importantly, both detailed and simplified models compare very well with ADAPTIC in respect of the tensile catenary action and the maximum steel strain, where the slight discrepancies are attributed to the use of a bilinear model for steel with the ADAPTIC elements instead of the Ramberg-Osgood model employed in the proposed analytical models (Fig. 11).

A similar quality of comparison is also observed for the elevated temperature case between the proposed analytical models and ADAPTIC, as shown in Fig. 16, noting that the material properties at elevated temperature for steel and concrete are assumed as in Tables 2 and 3. It

is worth noting that the elevated temperature case corresponds to an initial displacement at zero load of around 100mm, due to thermal buckling, thus effectively overcoming most of the compressive arch stage leading to a response that is subsequently dominated by tensile catenary action.

CONCLUSION

This paper presents two analytical models, detailed and simplified, for the nonlinear analysis of axially restrained LRC members under ambient and fire conditions, with particular emphasis on failure assessment in respect of the rupture of steel reinforcement. The relevance of this work is highlighted in the context of composite floor slabs under fire conditions, where the steel deck becomes ineffective at elevated temperatures leaving a lightly reinforced structure with a nominal steel mesh.

A brief background on the response of LRC members is provided, where the factors influencing the bending, compressive arch and tensile catenary stages are outlined. In particular, the issue of single vs. multiple crack formation is discussed in view of its crucial influence on the ductility of restrained LRC members. It is suggested that the compressive arch action and a reinforcement ratio for which the ultimate bending moment capacity exceeds the cracking limit are two factors which encourage the formation of multiple cracks in axially restrained LRC members. However, uncertainty in the axial support stiffness and the conservatism of a single-crack model in respect of failure assessment justify the assumption of a single crack, particularly for rupture of reinforcement in the tensile catenary stage, provided that the reinforcement ratio is low enough for the ultimate axial capacity to be within the cracking limit.

Two analytical models, one detailed and the other simplified, are proposed for simply-supported axially restrained LRC members subject to a midspan load, both models based on the assumption of a single crack at midspan. The detailed model addresses both the compressive arch and tensile catenary stages, whereas the simplified model only deals with failure in the tensile catenary stage. Furthermore, the detailed model accounts more accurately than the simplified model for i) local equilibrium, ii) the influence of elevated

temperature on the cross-sectional response, and iii) compatibility at large displacements. However, the detailed model requires a demanding iterative procedure, while the simplified model is applied by means of direct expressions rendering it more suitable for use in design based calculations.

Finally, the single-crack assumption and the accuracy of the two proposed analytical models are verified with reference to the results of nonlinear finite element analysis using ADAPTIC. It is shown that the single-crack assumption is accurate for LRC members where the reinforcement is sufficiently low for the ultimate bending and axial capacities to both be within their respective cracking limit, provided that the compressive arch action is not too significant either because of a relatively long span or because of a relatively low axial support stiffness. Otherwise, this assumption is shown to be conservative and hence appropriate even for the larger reinforcement ratios, provided the ultimate axial capacity is within the cracking limit. It is also shown that the detailed model captures the compressive arch action reasonably well, and importantly that both detailed and simplified models provide almost identical results in the tensile catenary stage which are very close to the predictions of ADAPTIC. Accordingly, the simplified analytical model can be readily applied in the assessment of axially restrained LRC members under ambient and fire conditions, except in the case of reinforcement rupture occurring within the compressive arch stage for which the detailed analytical model would be necessary, especially for failure load prediction.

The companion paper (Elghazouli & Izzuddin, 2002) applies the proposed analytical models to a range of axially restrained LRC members under ambient and fire conditions, where key implications on the design of such members for ambient and fire conditions are identified.

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TABLES

L (m)	b (mm)	h (mm)	d _s (mm)	A _s (mm ²)	ρ (%)
1.5–3.0	200	60	30	ρ b h	0.0833–0.25

Table 1. Geometric details of LRC members

E _s (GPa)		σ _{sp} (MPa)		ε _{sp}	n _s	α _s	σ _b (N/mm)	
0–200°C	400°C	0–300°C	400°C				0–200°C	400°C
210.0	136.5	346.6	277.3	0.01	5	14 × 10 ⁻⁶	16 × 10 ³ ρ	11.6 × 10 ³ ρ

Table 2. Material properties of steel reinforcement

E _c (GPa)			α _c	f _t (MPa)			f _{cu}
0–200°C	600°C	1200°C		0–200°C	600°C	1200°C	
40.0	20.0	0.0	8 × 10 ⁻⁶	2.0	1.0	0.0	0.001 E _c

Table 3. Material properties of concrete

NOTATION

a_s	: material parameter for Ramberg-Osgood steel model, function of temperature
A_s	: area of steel reinforcement
b	: cross-section width
C_c	: resultant compressive axial force in concrete at crack location
C_{c0}	: compressive axial force in concrete due to change of reinforcement slope at crack
d_s	: distance of steel reinforcement from bottom fibre
E_c	: elastic Young's modulus of concrete, function of temperature t_c
E_{cb}	: elastic Young's modulus of concrete at bottom fibre, function of temperature t_b
E_{ct}	: elastic Young's modulus of concrete at top fibre, function of temperature t_t
E_s	: elastic Young's modulus of steel, function of temperature t_s
E_{st}	: tangential modulus of steel, function of temperature t_s
EA_c	: axial rigidity of concrete section, function of temperature
EI_c	: bending rigidity of concrete section, function of temperature
EY_c	: axial/bending interaction rigidity of concrete section, function of temperature
f	: resultant axial force of steel and concrete
f_c	: axial force of concrete
f_{cu}	: compressive strength of concrete
f_s	: axial force of steel reinforcement
f_t	: tensile strength of concrete
h	: depth of cross-section
h_c	: assumed distance of contact point from reference line
$\mathbf{k}$	: overall section tangent stiffness matrix in no bond-slip region

$\mathbf{k}_c$	: concrete section stiffness matrix, function of temperature
L	: half span of member
L_d	: length on no bond-slip region
m_c	: bending moment of concrete
M_c	: resultant moment of C_c about reference line
n_s	: material parameter for Ramberg-Osgood steel model
P	: half of member midspan load
P_a	: axial component of P
t_b	: bottom-fibre temperature
t_c	: temperature of concrete, varying with y
t_s	: temperature of steel reinforcement
T_r	: axial force at right support
T_s	: tensile force in reinforcement at crack location
T_{su}	: ultimate tensile force in reinforcement at rupture
t_t	: top-fibre temperature
U	: midspan transverse displacement
v_t	: local transverse displacement due to thermal curvature
V	: shear force at crack location, ignoring concrete mechanical shortening
V_l	: shear force at crack location, accounting for concrete mechanical shortening
V_r	: shear force at right support, accounting for concrete mechanical shortening
x	: reference coordinate over member length
x_d	: length of bond-slip region
y	: reference coordinate over cross-section depth

α_c	: coefficient of thermal expansion for concrete
α_s	: coefficient of thermal expansion for steel
∇t	: thermal gradient over cross-section
δ_c	: axial shortening of concrete
δ_{c0}	: unrestrained axial pull-in of concrete due to thermal bowing
Δ_c	: shortening of concrete along thermally curved reference line
Δ_s	: extension of steel reinforcement along thermally curved reference line
ε_a	: axial generalised strain for concrete, typically in bond-slip region
ε_{an}	: axial generalised strain in no bond-slip region
ε_{ct}	: thermal strain of concrete
ε_{pu}	: plastic rupture strain of steel
ε_{sm}	: mechanical strain of steel
ε_{sp}	: material parameter for steel representing plastic strain
ε_{st}	: thermal strain of steel
κ	: curvature generalised strain for concrete, typically in bond-slip region
κ_n	: curvature generalised strain in no bond-slip region
κ_t	: thermal curvature of concrete
θ_c	: local rotation of concrete at crack location
θ_{c0}	: limiting local rotation used for contact constraint at crack location
θ_t	: local rotation along length due to thermal curvature
θ_{t0}	: local rotations at crack location and right support due to thermal curvature
ρ	: reinforcement ratio
σ_b	: bond strength in units of force per length, function of temperature t_s

σ_c : stress of concrete

σ_s : stress of steel, typically in bond-slip region

σ_{s0} : stress of steel for linearising σ_{sn} in no bond-slip region, function of temperature t_s

σ_{sn} : stress of steel in no bond-slip region, function of temperature t_s

σ_{sp} : steel stress corresponding to ε_{sp} , function of temperature

(a) Fire compartment

(b) Steel-decked composite slab

Figure 1. Composite floor subject to fire

Figure 2. Ambient response of simply supported LRC beam

Figure 3. Geometric and loading configuration of cracked member

Figure 4. Partial member model in local system

Figure 5. Influence of elevated temperature on material response

Figure 6. Cross-sectional configuration

Figure 7. Local deformations at crack location

Figure 8. Compatibility diagram (exaggerated depth)

Figure 9. Equilibrium diagram (exaggerated depth)

Figure 10. Compatibility diagram of simplified model (exaggerated depth)

Figure 11. Steel material models at ambient and elevated temperature

(a) Response

(b) Steel strain

**Figure 12. LRC member under various axial support conditions:
($\rho = 0.0833\%$, $L = 1.5\text{m}$)**

(a) Response

(b) Steel strain

**Figure 13. LRC member subject to full axial restraint:
($\rho = 0.0833\%$, $L = 3.0\text{m}$)**

(a) Response

(b) Steel strain

Figure 14. LRC member under various axial support conditions:
($\rho = 0.25\%$, $L = 1.5m$)

(a) Response

(b) Steel strain

Figure 15. Accuracy of proposed models under ambient conditions

(a) Response

(b) Steel strain

Figure 16. Accuracy of proposed models at elevated temperature