Super-Resolution Broad Null Beamforming for Cochannel Interference Cancellation in Mobile Radio Networks

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Abstract—In this paper, an innovative beamforming-type approach for suppressing cochannel interference in mobile radio networks is introduced. This approach combines a super-resolution direction and power profile finding algorithm, together with a signal subspace-type broad null beamformer which can steer both sharp and controlled broad nulls in the appropriate directions. The latter property makes the beamformer ideal for mobile communication systems where the angular spread of the incoming rays from a cochannel interfering mobile station is often too large to be eliminated by a sharp null, resulting in a significant degradation in the performance of a conventional system.

Index Terms—Beamformers, broad nulls, diffused sources, interference cancellation.

NOMENCLATURE

A, a	Scalar.
$\underline{A}, \underline{a}$	Vector.
$\mathbb{A}, \underline{\underline{a}}$	Matrix.
$(\cdot)^T$	Transpose.
$(\cdot)^*$	Complex conjugate.
$(\cdot)^H$	Conjugate transpose.
$ a $	Absolute value of scalar.
$[a]$	Integer part of a .
$\ \underline{a}\ $	Euclidian norm of vector.
$\exp(\underline{a})$	Element by element exponential.
$\underline{a}^b$	Element by element power.
$\odot$	Hadamard product (element by element product).
$\text{eig}_i(\mathbb{A})$	i th eigenvalue of $\mathbb{A}$.
$\text{eig}_{\min}(\mathbb{A})$	Minimum eigenvalue of $\mathbb{A}$.

I. INTRODUCTION

THE EXPLOSIVE increase in mobile communication products and services requires systems which can accommodate a large number of users by utilizing the available frequency

spectrum in the most efficient way. Undoubtedly, cochannel interference, caused by channel reuse, is one of the most restrictive factors in the design of mobile radio systems. Therefore, considerable research effort is being devoted to developing systems that either tolerate a higher level of interference (i.e., by using advanced modulation and coding schemes) or try to reduce the interference by efficient cell planning, dynamic channel assignment, or transmission power control. Besides these methods, the use of beamforming and array signal processing at the base station has also been proposed for interference suppression [1], [2].

Most of the beamforming techniques which have been proposed for mobile communications perform spatial filtering by forming comparatively sharp nulls in the direction of the interfering mobile stations. However, there are cases where the angular spread of the rays impinging on the array from an interfering mobile is so large that a sharp null cannot provide efficient interference suppression. Such cases, which arise when the mobile station is located relatively close to the base station or when the direction-of-arrival (DOA) changes quickly due to the motion of the mobile, can significantly degrade the performance of the system. They can be handled, however, if a broad null is formed toward the direction of the interference.

There has been considerable interest in recent years in beamformers which are able to synthesize controlled broad nulls [3]–[8]. However, there is nothing in the literature concerning the use of controlled broad nulls in mobile communications, regardless of the considerable advantages they have in cases of cochannel interferences with large angular spread or when motion is involved.

In this paper, an innovative approach employing beamforming with controlled broad nulls in mobile radio networks is proposed. Section II contains a detailed modeling of the array output as well as a classification of the incoming signals as point or diffuse sources. This classification is based on the angular spread of the cluster of rays impinging on the array from a mobile unit in conjunction with the resolution capabilities of the array configuration. In Section III, a broad null beamformer is proposed which is based on a new super-resolution direction finding algorithm which also provides estimates of the received power levels of the incoming signals as well as of the angular spreads of the clusters of rays arriving from each mobile unit. Section IV provides some computer simulation results which depict the performance of the proposed approach. Finally, the paper is concluded in Section V.

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II. THE PROPAGATION MODEL OF THE ARRAY ENVIRONMENT

Consider a base station with an array of N antennas receiving signals from M mobile stations. The array output $\underline{x}(t)$ can be expressed as

$$\underline{x}(t) = [\mathbb{S}_1(t), \mathbb{S}_2(t), \dots, \mathbb{S}_M(t)] \cdot \begin{bmatrix} \underline{m}_1(t) \\ \underline{m}_2(t) \\ \vdots \\ \underline{m}_M(t) \end{bmatrix} + \underline{n}(t) \quad (1)$$

where

$$\underline{m}_i(t) = [m_{i1}(t), m_{i2}(t), \dots, m_{iK_i}(t)]^T, \quad K_i\text{-dimensional column vector of the base-band signals arriving at the array reference point from the } K_i \text{ reflectors/scatterers in the vicinity of the } i\text{th mobile station;}$$

$$\underline{n}(t) \quad N\text{-dimensional column vector of the additive white Gaussian noise at the array antennas;}$$

$$\mathbb{S}_i(t), i = 1, 2, \dots, M \quad \text{the following matrix:}$$

$$\mathbb{S}_i(t) = [\underline{S}(\theta_i + \Delta\theta_{i1}(t), \phi_i + \Delta\phi_{i1}(t)), \underline{S}(\theta_i + \Delta\theta_{i2}(t), \phi_i + \Delta\phi_{i2}(t)), \dots, \underline{S}(\theta_i + \Delta\theta_{iK_i}(t), \phi_i + \Delta\phi_{iK_i}(t))] \quad (2)$$

with columns the manifold vectors corresponding to the K_i rays originating from the i th mobile. The parameters θ_i, ϕ_i denote the azimuth and elevation bearings of the i th mobile station, and $\Delta\theta_{ik}, \Delta\phi_{ik}$ are the differences in azimuth and elevation between the direction of the i th mobile station and the direction of the k th reflector.

The N -dimensional manifold vector $\underline{S}(\theta, \phi) = \underline{g}(\theta, \phi) \odot \exp\{-j \cdot \underline{r} \cdot \underline{k}(\theta, \phi)\}$ represents the complex array response to a single far-field source of unity power with direction parameters θ and ϕ , where $\underline{g}(\theta, \phi)$ is the complex vector of the gain and phase response of the antennas in the direction (θ, ϕ) , $\underline{r} = [r_x, r_y, r_z]$ denotes the locations of the array elements, and $\underline{k}(\theta, \phi) = ((2\pi/\lambda) \cdot [\cos(\theta) \cdot \cos(\phi), \sin(\theta) \cdot \cos(\phi), \sin(\phi)]^T$ is the wavenumber vector.¹ The vector $\underline{m}_i(t)$, $i = 1, 2, \dots, M$ contains the base-band waveforms of all the rays arriving at the array reference point from the i th mobile station at instant t . The k th component of this vector can be modeled as

$$m_{ik}(t) = \sqrt{G_{p_{ik}} \cdot G_{s_{ik}}} \cdot G_{f_{ik}} \cdot \exp\left(j \cdot 2 \cdot \pi \cdot \left(F_{di} \cdot \cos(\psi_{ik})t - \frac{\Delta d_{ik}}{c} \cdot F_c\right)\right) \cdot u_i\left(t - \frac{\Delta d_{ik}}{c}\right) \quad (3)$$

¹Without any loss of generality in this paper, the elevation angle (ϕ) is assumed to be constant and equal to zero.

where

$G_{p_{ik}}$	path-loss factor;
$G_{s_{ik}}$	slow fading factor;
$G_{f_{ik}}$	amplitude component of the fast fading factor;
F_c	propagation frequency;
Δd_{ik}	path difference between the direct and the k th paths;
$F_{di} = (v_i \cdot F_c / c)$	maximum Doppler frequency shift for the i th mobile station;
v_i	speed of the i th mobile station;
c	speed of light in the propagation medium;
ψ_{ik}	direction of the k th reflector/scatterer with respect to the vector of the mobile station velocity (i.e., ψ_{ik} is uniformly distributed in $[0, 2\pi)$);
$u_i(t)$	message signal transmitted by the i th mobile station.

The reflectors/scatterers in the vicinity of the i th mobile are located relatively close to the mobile station. Therefore, the K_i rays experience similar path loss, slow fading, and path difference components, i.e., $\forall k, G_{p_{ik}} \simeq G_{p_i}, G_{s_{ik}} \simeq G_{s_i}$, and $\Delta d_{ik} \simeq \Delta d_i$. Any remote scatterers, out of the vicinity of the mobile, generate either individual rays with different DOA, amplitude, etc., or another cluster of rays with the above parameters different from the original one and can be handled as extra “effective” mobiles using the above model.

It is important to point out that the energy of the signals impinging on the array from the i th mobile station, say, is in general distributed around a central direction. If the mobile is an intracell one, it is more likely to correspond to a diffuse source. The maximum angular ray spread $\Delta\theta_i$ can be expressed as a function of the radius r of the scattering area and the distance d_i between the base station and the i th mobile as follows:

$$\Delta\theta_i = 2 \cdot \arcsin\left(\frac{r}{d_i}\right). \quad (4)$$

Theoretically, over an observation interval of L snapshots, the maximum number of resolvable rays in this case is equal to $\lfloor \Delta\theta_i / \Delta\theta_{\text{res}} \rfloor$, where $\Delta\theta_{\text{res}}$ is the *resolution threshold* of the array configuration which provides the minimum azimuth-angular separation for resolving two sources close together at bearings θ_1 and $\theta_2 = \theta_1 + \Delta\theta$ and is given [9], [10] by the following:

$$\Delta\theta_{\text{res}} = \frac{\sqrt[4]{2} \cdot \left(1 + \sqrt[4]{\frac{|g(\theta_1)|^2 \cdot P_1}{|g(\theta_2)|^2 \cdot P_2}}\right)}{\dot{s}(\theta_1) \cdot \sqrt[4]{\text{SNR}_{\theta_1} \cdot L \cdot \left(\hat{\mathcal{K}}_1^2(\theta_m) - \frac{1}{N}\right)}} \quad (5)$$

where $\hat{\mathcal{K}}_1(\theta_m) = \sqrt[4]{\mathcal{K}_1^2(\theta_m) - \text{sum}^2(\tilde{R}_\theta^3)}$, $\mathcal{K}_1(\theta_m)$ is the first curvature of the array manifold,² $\tilde{R}_\theta = (R_\theta / \|R_\theta\|)$, $R_\theta = (\partial R / \partial \theta)$, and $R = r_x \cdot \cos(\theta) + r_y \cdot \sin(\theta)$.

In (5), P_1 and P_2 represent the powers of the received signals at the array reference point, SNR_{θ_1} is the signal-to-noise power

² θ_m is the azimuth angle which corresponds to the arc length $(s_1 + s_2)/2$ on the array manifold. It is assumed that $\theta_m \simeq ((\theta_1 + \theta_2)/2) = \theta_1 + (\Delta\theta/2)$.

ratio of the emitter with bearing θ_1 at the array reference point, and $\dot{s}(\theta_1)$ is the rate of change of the arc length of the manifold curve at θ_1 .

III. A PROPOSED COCHANNEL INTERFERENCE CANCELLATION APPROACH

This section is concerned with the estimation of a weight vector $\underline{w}$ which suppresses cochannel interference by steering sharp nulls toward the directions of the mobile stations which are seen by the beamformer as point sources (i.e., $\Delta\theta_i < \Delta\theta_{\text{res}}$) and applying broad nulls to eliminate the interference from mobile units whose energy spread around the central directions makes the use of sharp nulls ineffective.

The estimation of the DOA's as well as of the maximum angular spread $\Delta\theta_i$ for all the mobile stations is based on a one-parameter minimization of a specially designed cost function $\xi(\alpha, \theta)$, and it can be expressed as

$$\forall \theta \text{ find } \alpha_o = \arg \min_{\alpha} (\xi(\alpha, \theta)). \quad (6)$$

$\xi(\alpha, \theta)$ is a function of the eigenvalues of the matrix $\mathbb{R}(\alpha, \theta)$ which is formed from the data covariance matrix $\mathbb{R}_{xx}$ by removing the effects of the noise and of any signal component impinging on the array from a direction θ , i.e.,

$$\mathbb{R}(\alpha, \theta) = \mathbb{R}_{xx} - \text{eig}_{\min}(\mathbb{R}_{xx}) \cdot \mathbb{I}_N - \alpha \cdot \underline{S}(\theta) \cdot \underline{S}(\theta)^H \quad (7)$$

where α is the power of the signal component arriving from direction θ and $\text{eig}_{\min}(\mathbb{R}_{xx})$ is the minimum eigenvalue³ of the matrix $\mathbb{R}_{xx}$ which theoretically (i.e., for infinite observation interval $L \rightarrow \infty$) is equal to the power of the noise P_n .

The cost function $\xi(\alpha, \theta)$ can be written as

$$\begin{aligned} \xi(\alpha, \theta) = & \sum_{\substack{i=1 \\ \text{eig}_i > 0}}^N (1 + \text{eig}_i(\mathbb{R}(\alpha, \theta))) \\ & + 10 \cdot \log_{10} \left(\sum_{\substack{i=1 \\ \text{eig}_i < 0}}^N |\text{eig}_i(\mathbb{R}(\alpha, \theta))| \right) \end{aligned} \quad (8)$$

and its minimization over α provides the point α_o where the rank of the matrix $\mathbb{R}$ is reduced by one. If no signal component is arriving from the direction θ , the estimated value of α_o is zero.

For M signals, $\underline{m}_i(t)$, $i = 1, 2, \dots, M$, it can be proved that the value of α_o [provided by (6)] for the k th path of i th signal, is given by

$$\alpha_{o_{ik}} = \det(\mathbb{R}_{mm}) / R_{mm_{ik}} \quad (9)$$

³Theoretically the N minus the multiplicity of the minimum eigenvalue of $\mathbb{R}_{xx}$ is used to specify the number M of incident signals. For a finite observation interval of L snapshots, however, M is estimated by using *Akaike information criterion* (AIC), *minimum description length* (MDL), etc., and the mean of the $(N - M)$ smaller eigenvalues, rather than $\text{eig}_{\min}(\mathbb{R}_{xx})$, is used in (7).

where $\mathbb{R}_{mm}$ is the $(\sum_{i=1}^M K_i) \times (\sum_{i=1}^M K_i)$ covariance matrix of the received signals

$$\underline{m}(t) = \begin{bmatrix} \underline{m}_1(t) \\ \vdots \\ \underline{m}_M(t) \end{bmatrix}$$

i.e., $\mathbb{R}_{mm} = \mathcal{E}(\underline{m}(t) \cdot \underline{m}^H(t))$, and $R_{mm_{ik}}$ is the cofactor of the matrix $\mathbb{R}_{mm}$ corresponding to the

$$\left(\left(\sum_{j=1}^{i-1} K_j + k \right), \left(\sum_{j=1}^{i-1} K_j + k \right) \right) \text{th element.}$$

If the signal $m_i(t)$ is a point signal source (i.e., $K_i = 1$), uncorrelated with the other signals impinging on the array, the estimated value $\alpha_{o_{i1}}$ is equal to the power P_i , of this signal i.e.,

$$\alpha_{o_{i1}} = \frac{\det(\mathbb{R}_{mm})}{R_{mm_{i1}}} = \frac{P_i \cdot R_{mm_{i1}}}{R_{mm_{i1}}} = P_i. \quad (10)$$

It is important to point out that as long as $\alpha < \alpha_o$, the matrix $\mathbb{R}$ is a semipositive definite matrix since it can be written as a summation of semipositive definite matrices. That is,

$$\begin{aligned} \mathbb{R}(\alpha, \theta) = & \mathbb{R}_1 + \mathbb{R}_2 = P_{\theta_1} \cdot \underline{S}(\theta_1) \cdot \underline{S}(\theta_1)^H \\ & + (P_{\theta_2} - \alpha) \cdot \underline{S}(\theta_2) \cdot \underline{S}(\theta_2)^H \end{aligned}$$

for two uncorrelated point sources. As a result, the matrix $\mathbb{R}$ has nonnegative eigenvalues and the second part of the cost function $\xi(\alpha, \theta)$ is equal to zero. As the value of α increases, toward α_o , a portion of the signal arriving from direction θ is removed and therefore the eigenvalues of $\mathbb{R}$ are decreasing, leading to a monotonic decrease of the cost function. When $\alpha = \alpha_o$, all the effects of the signal in the direction θ have been removed. This causes a dramatic drop in the cost function which creates the global minimum. For $\alpha > \alpha_o$, the matrix $\mathbb{R}$ is not semipositive definite any more (i.e., there are some negative eigenvalues). These negative eigenvalues are employed constructively by the second part of the cost function to produce a monotonic increase, as can be seen in Fig. 1. Note that the $10 \log_{10}(\cdot)$ factor on the second part of the cost function is used to enhance the depth of the minimum point and so to accelerate the convergence of the optimization algorithm.

Based on the above approach, the directional power profile (i.e., α_o as a function of θ) can be estimated and the directions of point sources as well as the mean θ and the angular distribution of diffuse sources can be found. An illustrative example is shown in Fig. 2(a) for two point sources with directions 30 and 140° and one diffuse source $(\theta, \Delta\theta) = (90, 60^\circ)$.

Having identified a diffuse interference source with estimated angular width $\Delta\theta_I$ at direction θ_I based on the above directional power profile approach, a beamformer can now be designed to steer a controlled broad null to suppress this interference. The approach is to synthesize an array pattern whose power response over the region of interest [i.e., $\theta_I - (\Delta\theta_I/2) \leq \theta \leq \theta_I + (\Delta\theta_I/2)$] is smaller than or equal to a threshold γ .

Fig. 1. The cost function $\xi(\alpha, \theta)$ as a function of α for the desired signal direction (90°). A uniform linear array ($N = 13$) is assumed receiving four uncorrelated signals.

This threshold determines the degree of cancellation which is performed by the beamformer for all the rays impinging on the array from the above DOA's.

Thus, if we define the $N \times N$ Hermitian matrix

$$\mathbf{Q}_I = \int_{\theta_I - (\Delta\theta_I/2)}^{\theta_I + (\Delta\theta_I/2)} \underline{\mathbf{S}}(\theta) \cdot \underline{\mathbf{S}}^H(\theta) \cdot d\theta$$

with eigenvectors $\underline{\mathbf{E}}_{Ik}$, $k = 1, \dots, N$, then if $\mathbf{P}_{\underline{\mathbf{E}}_{Ib}}^\perp$ is the complement projector operator on the subspace spanned by the columns of $\underline{\mathbf{E}}_{Ib} = [\underline{\mathbf{E}}_{I1}, \underline{\mathbf{E}}_{I2}, \dots, \underline{\mathbf{E}}_{I\tau_I}]$ and $\underline{\mathbf{S}}_d$ is the manifold vector of the desired signal, it can be proved that the weight vector $\underline{\mathbf{w}} = \mathbf{P}_{\underline{\mathbf{E}}_{Ib}}^\perp \cdot \underline{\mathbf{S}}_d$ provides a broad null in the array pattern over the angular sector $\theta_I \pm (\Delta\theta_I/2)$ and satisfies the following:

$$\sum_{k=\tau_I+1}^N (\lambda_{Ik} \cdot (\underline{\mathbf{w}}^H \cdot \underline{\mathbf{E}}_{Ik})^2) \leq \gamma \quad (11)$$

where λ_{Ik} , $\underline{\mathbf{E}}_{Ik}$ are the $N - \tau_I$ smallest eigenvalues of matrix $\mathbf{Q}_I$ and the associated eigenvectors and τ_I is an unknown integer.

By applying the Cauchy-Schwartz inequality, the left side of (11) can be expressed as

$$\begin{aligned} & \sum_{k=\tau_I+1}^N (\lambda_{Ik} \cdot (\underline{\mathbf{w}}^H \cdot \underline{\mathbf{E}}_{Ik})^2) \\ & \leq \sum_{k=\tau_I+1}^N (\lambda_{Ik} \cdot \|\underline{\mathbf{w}}\|^2 \cdot \|\underline{\mathbf{E}}_{Ik}\|^2) \\ & = \sum_{k=\tau_I+1}^N (\lambda_{Ik} \cdot \|\underline{\mathbf{w}}\|^2) \end{aligned} \quad (12)$$

since $\|\underline{\mathbf{E}}_{Ik}\| = 1 \forall k$. By normalizing the weight vector $\underline{\mathbf{w}}$ to have $\|\underline{\mathbf{w}}\| = 1$, i.e.,

$$\underline{\mathbf{w}} = \frac{\mathbf{P}_{\underline{\mathbf{E}}_{Ib}}^\perp \cdot \underline{\mathbf{S}}_d}{\sqrt{\underline{\mathbf{S}}_d^H \cdot \mathbf{P}_{\underline{\mathbf{E}}_{Ib}}^\perp \cdot \underline{\mathbf{S}}_d}} \quad (13)$$

the last part of (12) is simplified to

$$\sum_{k=\tau_I+1}^N \lambda_{Ik}.$$

The number τ_I can now be estimated as the minimum number of the eigenvalues of the matrix $\mathbf{Q}_I$ satisfying the condition

$$\sum_{k=\tau_I+1}^N \lambda_{Ik} \leq \gamma. \quad (14)$$

At this point, it should be noted that $N - \tau_I - 1$ determines the remaining degrees of freedom which may be used to steer sharp or broad nulls toward other directions.

The weight vector of (13) can be generalized to the case where more than one broad null is needed, as well as when a combination of sharp and broad nulls are required to suppress the signals coming from both point and diffuse-like types of interfering mobile units as follows:

$$\underline{\mathbf{w}} = \frac{\mathbf{P}_{\underline{\mathbf{S}}_I}^\perp \cdot \underline{\mathbf{S}}_d}{\sqrt{\underline{\mathbf{S}}_d^H \cdot \mathbf{P}_{\underline{\mathbf{S}}_I}^\perp \cdot \underline{\mathbf{S}}_d}} \quad (15)$$

where

$$\underline{\mathbf{S}}_I = [\underline{\mathbf{E}}_{Ib_1}, \underline{\mathbf{E}}_{Ib_2}, \dots, \underline{\mathbf{E}}_{Ib_{M_b}}, \underline{\mathbf{S}}_{I1}, \underline{\mathbf{S}}_{I2}, \dots, \underline{\mathbf{S}}_{IM_p}], \quad \underline{\mathbf{E}}_{Ib_i}$$

is the matrix of the eigenvectors corresponding to the τ_{oi} largest eigenvalues of the i th broad null, $\underline{\mathbf{S}}_{Ii}$ is the manifold vector corresponding to the DOA of the i th point source, and M_p and M_b are the numbers of point and diffuse (broad) sources, respectively. An illustrative example is presented in Fig. 2(b) which shows the array pattern of the proposed beamformer for the environment associated with Fig. 2(a) (one diffuse and two point sources where the desired source is the one located at 30°).

It is important to point out that the proposed weight vector $\underline{\mathbf{w}}$, which provides a super-resolution beamformer, cancels completely (asymptotically) the cochannel interference associated with point sources and rejects diffuse interfering sources according to the prespecified gain thresholds γ_i , $i = 1, 2, \dots, M_b$. Thus, overall the output SIR is maximized although the power of the desired signal is multiplied by the $\cos^2$ of the angle between the desired signal subspace and interference subspace. This $\cos^2$ term will reduce the desired signal power when an interference source is located very close to the direction of the desired signal, and in this case, an *intrabase-station channel assignment* scheme should be used to prevent codirectional (overlapping in space) or nearly codirectional desired and interfering signals. If there are overlapping interferences, these are handled as a combined interference. Note also that the number of array elements N must be greater than the degrees of freedom which are required to form the broad and the sharp nulls $\tau_1 + \tau_2 + \dots + \tau_{M_b} + M_p < N$.

Fig. 2. (a) Directional power profile and (b) beamformer's array pattern provided by the proposed approach for a signal environment having two point sources with DOA's 30° and 140° and one diffused source with mean DOA 90° and angular spread 60°.

If this condition is violated, then the concept of the “effective array” (which has more than N effective sensors) may be used in conjunction with the fourth-order (cumulant) matrices [11], [12] instead of covariance matrices. However, the number of antennas can be kept adequately small by choosing a reasonable intracell channel reuse. This is because the mobile units belonging to the cell of interest are the most likely to be classified as diffuse sources due to the small distance from the base station.

The overall algorithm for employing beamforming with sharp and controlled broad nulls in a mobile radio network can be presented in a brief step form as follows.

- Step 1) Over an observation interval of L snapshots, estimate the data covariance matrix $\mathbb{R}_{xx}$.
- Step 2) Minimize the cost function $\xi(\alpha, \theta)$ of (8) for the angular sector of interest (i.e., say 120°) and estimate the DOA's and the angular spread of the incoming signals from the values of $\alpha_o = \arg \min_{\alpha} (\xi(\alpha, \theta))$. Note

this step involves a one-dimensional (1-D) exhaustive search over the angular sector of interest with a simple minimization performed for every θ or, otherwise, a two-dimensional (2-D) exhaustive search over both the angular sector of interest and the range of α .

Step 3) Based on the results of Step 2), classify the mobile stations as point or diffuse-type sources. For a desired mobile station form the manifold vector $\underline{S}_d$.

Step 4) For each point interfering source form the manifold vector $\underline{S}_I$. For each of the diffuse-type interfering mobile stations, estimate the matrix $\mathbf{Q}_I$ and using (14) determine the τ_I most significant eigenvectors $\mathbf{E}_{I_b}$ of $\mathbf{Q}_I$. Form the matrix

$$\mathbf{S}_I = [\mathbf{E}_{I_{b_1}}, \mathbf{E}_{I_{b_2}}, \dots, \mathbf{E}_{I_{b_{M_b}}}, \underline{S}_{I1}, \underline{S}_{I2}, \dots, \underline{S}_{I_{M_p}}].$$

Step 5) Form the weight vector

$$\underline{w} = \frac{\mathbf{P}_{\mathbf{S}_I}^\perp \cdot \underline{S}_d}{\sqrt{\underline{S}_d^H \cdot \mathbf{P}_{\mathbf{S}_I}^\perp \cdot \underline{S}_d}}$$

to receive the desired signal and suppress the rest as unwanted interferences, where $\mathbf{P}_{\mathbf{S}_I}^\perp$ is the complement projector operator on the subspace spanned by the columns of $\mathbf{S}_I$.

Note that if the desired source is a diffuse-type source, then the desired manifold vector $\underline{S}_d$ can be determined as the manifold vector corresponding to the maximum $\cos^2$ of the angle between $\underline{S}_d$ and the interference subspace of Step 3). This approach is especially appropriate if the angle of spread of the desired signal is relatively large compared to the beamwidth of the array (i.e., it will result in a smaller loss of the desired signal's energy). Furthermore, if there is more than one cluster associated with the desired signal, then the one with the highest power profile is chosen as the desired and the rest are suppressed as additional "interferences." The identification of the "desired" signal can be performed by a "signature" checking procedure based on a user (or time slot) specific training sequence. This identification can be carried out in Step 3) (if there is *a priori* information to map "signatures" to clusters of the directional power profile) or at the end of Step 5) by using the weight vector $\underline{w}$ for every cluster/source to map "signatures" to clusters. An indication of the computational requirements of the proposed approach can be given by the number of floating point operations (flops) which can be estimated using the MATLAB flops built in facility. Table I provides the number of flops required by MATLAB as a function of the number of array antennas (N)

TABLE I
COMPLEXITY OF THE PROPOSED
ALGORITHM

Number of Antenna Array Elements (N)	Eigendecomposition of $\mathbf{R}_{xx}$ in Mflops	Total in Mflops (including Directional Power Profile & Beamforming)
2	0.0005	0.08
3	0.0025	0.4
4	0.005	0.8
5	0.011	1.8
6	0.017	2.7
7	0.030	4.8
8	0.042	7.5
9	0.063	11.1
10	0.090	15.5
11	0.120	21.2
12	0.156	28.6
13	0.196	35
14	0.251	43
15	0.312	53

NB : 1) The Directional Power Profile is two orders of magnitude (in flops) more complex than MUSIC.

2) The number of sources used consume $N - 1$ degrees of freedom.

operating in the presence of sources using all available degrees of freedom.

IV. COMPUTER SIMULATION STUDIES

In this section, some examples are considered, using a uniform linear array of $N = 13$ antennas, in order to investigate the performance and the main features of the proposed algorithms. The first example investigates the directional power profile capabilities of the proposed algorithm operating in the presence of seven closely located point sources with directions (85° , 87° , 90° , 93° , 95° , 97° , and 99°) and a signal covariance matrix $\mathbf{R}_{mm}$ as shown in (16) at the bottom of the page. Fig. 3 depicts the results indicating that the algorithm manages to find accurately the DOA's of all the incoming signals, despite their small angular separation. However, the algorithm fails to estimate the last two signals which are coherent (fully correlated). For these two signals, no peaks appear since the value of α which is estimated by the algorithm is zero. The algorithm can also estimate the power level of the third signal which is uncorrelated with the others while, for the rest of the signals (i.e., the partially correlated ones) the estimates of the algorithm are equal to the result given in (9). Furthermore, it is clear that the higher the correlation between the signals, the smaller the value of the parameter α which is estimated by the algorithm. Note that if there

$$\mathbf{R}_{mm} = \begin{bmatrix} 0.75 & \sqrt{0.75 \cdot 1} \cdot 0.3 \cdot e^{-j \cdot (\pi/6)} & 0 & 0 & 0 & 0 & 0 \\ \sqrt{0.75 \cdot 1} \cdot 0.3 \cdot e^{j \cdot (\pi/6)} & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.75 & \sqrt{0.75 \cdot 1} \cdot 0.7 \cdot e^{-j \cdot (\pi/3)} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{0.75 \cdot 1} \cdot 0.7 \cdot e^{j \cdot (\pi/3)} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & \sqrt{0.5 \cdot 0.75} \\ 0 & 0 & 0 & 0 & 0 & \sqrt{0.5 \cdot 0.75} & 0.75 \end{bmatrix} \quad (16)$$

Fig. 3. The directional power profile (α_o as a function of θ) for a uniform linear array ($N = 13$) operating in the presence of closely located point sources with covariance matrix $\mathbf{R}_{mm}$ given by (16).

are coherent sources, then (9) becomes $0 \div 0$ and therefore the L'Hôpital's rule should be employed.

In order to examine a realistic scenario, applying the proposed techniques in a mobile radio network, the above antenna array is employed with 120° sectorization at the base station ($30^\circ \leq \theta \leq 150^\circ$) to receive signals from seven cochannel mobile stations. The directions of the mobile stations, with respect to the base station of interest, are (40° , 60° , 75° , 90° , 110° , 116° , and 130°), respectively. Ten rays $K = 10$ impinge on the array from each mobile station. The second, fourth, and seventh are assumed to be diffuse sources (intracell) with angular spread (5° , 5° , and 7°), respectively, with the fourth source to be the desired (intracell) mobile station and the remaining four unwanted point interferences (intercell). Without any loss of generality, it is assumed that the path loss and the slow fading factors are normalized to one (i.e., $G_{p_i} = G_{s_i} = 1 \forall i$). The received power of all the mobile stations is assumed to be 1 W and the noise power is $P_n = 10^{-3}$ W. This scenario corresponds to an array input SIR_{in} of -7.78 dB i.e.,

$$\begin{aligned} \text{SIR}_{\text{in}} &= 10 \cdot \log_{10} \left(\frac{P_4}{\sum_{j=1, j \neq 4}^7 P_j} \right) \\ &= 10 \cdot \log_{10} \left(\frac{1}{6} \right) \simeq -7.78 \text{ dB} \end{aligned}$$

and an input SNR_{in} of

$$\begin{aligned} \text{SNR}_{\text{in}} &= 10 \cdot \log_{10} \left(\frac{P_4}{P_n} \right) \\ &= 10 \cdot \log_{10} \left(\frac{1}{10^{-3}} \right) \simeq 30 \text{ dB} \end{aligned}$$

(both after averaging the fast fading). An observation interval of $L = 100$ snapshots is taken which, for instance, in the case of a GSM system corresponds to 0.613 ms assuming a sampling frequency equal to twice the Gaussian filter bandwidth 81.3 kHz. The results of the proposed algorithm are shown in Fig. 4(a) and (b). For example, in Fig. 4(a), the algorithm estimates with negligible errors the DOA's of all point mobile stations while, in the case of diffuse sources, it provides the mean DOA as well as the angular spread of the incident rays. It is clear also from Fig. 4(b) that the proposed beamformer steers a beam toward the direction of the desired mobile station and nulls toward the directions of the cochannel interferences. In this example, the degrees of freedom required to achieve the predefined thresholds of $\gamma_1 = -50$ dB and $\gamma_2 = -50$ dB for the two diffused interfering sources (second and seventh) are $\tau_1 = 3$ and $\tau_2 = 5$, respectively.

In the final example, the performance of the proposed algorithm is compared with the performance of a single element antenna system for the uplink of a large mobile radio network. The network consists of 19 clusters of hexagonal cells. The base stations are located in the centers of the cells using 120° sectorization and they employ a uniform linear array of $N = 13$ elements in each sector to communicate with the corresponding mobile units. The mobiles, having omnidirectional antennas, are uniformly distributed in the sector areas. Each array receives the signals from a desired mobile station and eight interfering ones. One of them is assumed to be an intracell cochannel interference which is considered as a diffuse source. The rest are intercell interferences which are point sources (three tiers of intercell cochannel interferences are taken into account in each sector). Ten rays (i.e., $K = 10$) impinge on the array from each mobile station with an angular spread $\Delta\theta_{\text{intra}} = 5^\circ$ and

Fig. 4. The results provided by the super-resolution proposed approach for the mobile radio network scenario described in Section IV. (a) Directional power profile and (b) array pattern.

$\Delta\theta_{\text{inter}} = 0^\circ$ with the depth of the broad null (threshold) $\gamma = -50$ dB. An intelligent intrabase-station channel assignment scheme has been assumed to prevent codirectional or nearly codirectional cochannel mobiles. The evaluation of the outage probabilities $\Pr(\text{SNR}_o < \text{SNR}_{\text{pr}})$ and the noise probabilities $\Pr(\text{SNR}_o < \text{SNR}_{\text{pr}})$ at the output of the beamformer were performed using Monte Carlo simulations of 500 independent trials using MATLAB. Fig. 5 depicts the outage probability and the noise probability for this network using the proposed algorithm as compared to the outage probability of a base station with a single element sectorized antenna. According to the results,

there is a significant improvement in the SNR_o due to the successful cancellation of the cochannel interference both from the intracell and the intercell mobile stations. It can also be seen that there is only a small difference between the two curves which means that the algorithm achieves almost complete interference cancellation. This difference is caused by a small portion of interference which sometimes remains at the output of the beamformer due to errors in the DOA estimates. It is caused by the noise or by the finite observation interval and is common to all signal subspace-type beamformers which are dominated by the product $\text{SNR}_{\text{in}} \times L$. Note that all the simulations in this section

Fig. 5. The outage and the noise probability curves for the proposed algorithm using a uniform linear array ($N = 13$) and the outage probability curve for a single element antenna.

have been carried out using a step size of 0.1° for θ and the range for the parameter α is $[0, \text{trace}(\mathbf{R}_{xx} - \text{eig}_{\min}(\mathbf{R}_{xx}) \cdot \mathbf{I}_N)/N]$.

V. CONCLUSIONS

In this paper, a novel approach employing beamforming-type techniques in order to eliminate the unwanted cochannel interference in mobile radio networks is introduced. The proposed approach is based on a powerful direction and power finding algorithm and provides a super-resolution beamformer capable of handling both point and diffuse-type interfering sources.

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