Crossed-Dipole Arrays for Asynchronous DS-CDMA Systems

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Abstract—In this paper, the use of a crossed-dipole array is proposed in joint space-time channel estimation for asynchronous multipath direct sequence code division multiple access (DS-CDMA) systems. The polarization diversity offered by such an array, unlike linearly polarized arrays, is able to detect and estimate any arbitrary completely polarized signal path. By utilizing the polarization information inherent in the received signal to construct the polar-spatio-temporal array (polar-STAR) manifold vector, the accuracy and resolution of the polar-STAR parameters' estimation are significantly improved, and its signal detection capability is enhanced. To alleviate the need for a multidimensional search in the polarization space, a computationally efficient joint polarization-angle-delay channel parameter estimation algorithm is proposed for a "desired user" that operates in an asynchronous multiuser and multipath environment. The proposed algorithm, which can be seen as an application of MUSIC-type techniques, is based on combining a two-dimensional STAR-Subspace type technique with a set of analytical equations and is supported by representative examples and computer simulation studies.

Index Terms—Antenna arrays, code division multiple access (CDMA), crossed-dipole, diversity methods, MUSIC, polarization, space-time.

NOMENCLATURE

A	Scalar.
$\underline{A}$	Column Vector.
$\mathbf{A}$	Matrix.
$(\cdot)^T$	Transpose.
$(\cdot)^H$	Hermitian transpose.
$\lceil \cdot \rceil$	Round up to integer.
$\odot$	Hadamard (Schur) product.
$\otimes$	Kronecker product.
$\text{diag}(\underline{A})$	Diagonalization of vector $\underline{A}$.
$\exp(\underline{A})$	Element by element exponential of vector $\underline{A}$.
$\underline{A}^b$	Element by element power.
$\underline{0}_N$	Zero vector of N elements.
$\mathbb{I}_N$	Identity matrix $N \times N$ dimension.

I. INTRODUCTION

IN WIRELESS communications, the signal emitted by a mobile terminal normally suffers multiple reflections and scattering along the transmission path, hence creating several

replicas with different arriving angle, path delay, polarization and fading. However, among these four channel parameters, the polarization factor of the signal, which describe the orientation of its electric field, often receives disproportionately little attention in traditional model of array processing. Most array processing techniques assume the employment of polarization-insensitive sensors, which therefore presume that the polarization of the received signal is perfectly aligned with respect to the orientation of the sensors, thus obviating any polarization mismatches. But in typical mobile environments, the received signal rarely takes on its transmitted polarization due to the depolarization mechanism [1] intrinsic in the propagation channel (especially in an urban environment). This is further aggravated by the frequent random angular orientation of most portable handheld devices. Such diversity in signal's polarization, which is normally treated as part of signal fading (i.e., polarization fading), can be exploited to provide an extra degree of signal discrimination, and as such improve the receiver's detection/estimation capabilities. Note, that the polarization state of the signal can be either completely or partially polarized, but as with many studies, the main focus of this work will assume complete polarization. Investigation on partially polarized scenario can be found in [2] and [3].

Polarization diversity has been studied in a number of direction finding algorithms to ameliorate its angle-of-arrival estimation [4]–[10]. This is achieved by means of diversely polarized arrays which are sensitive to the polarization of the received signal. Ferrara and Parks [4] have shown that by employing an array of diversely polarized sensors, the angle-of-arrival estimation is significantly improved as multiple signals can now be resolved on the basis of polarizations in addition to their arriving angles. Schmidt [5], on the other hand, demonstrated the ability of distinguishing two highly correlated signals by incorporating their signal polarizations. Since then, various diversely polarized arrays have been proposed in a correlated signal environment. For instance, in [6] a diversely polarized array consisting of circularly-polarized sensors is used with the Cramer–Rao bound to evaluate the angle estimation accuracy of correlated signals. In [7], a linear array of crossed dipoles, which measures its horizontal and vertical responses separately, is used with the ESPRIT algorithm to estimate the angle and polarization of coherent signals. The performance of the angle and polarization estimation is then further improved in [8] by using a co-centered orthogonal loop and dipole (COLD) array. Other diversely polarized arrays include, for instance, the use of dipole triad and/or loop triad(s) for multisource angle and polarization estimation [11]. Note that a dipole, or loop, triad consists of

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three identical and orthogonally colocated electrically short dipoles, or magnetically small loops, respectively. In [12], the concept of “electromagnetic vector sensor” is employed for self-initiating MUSIC-based direction finding and polarization estimation algorithm in the spatio-polarizational beamspace. An electromagnetic vector sensor is comprised of six spatially colocated nonidentical nonisotropic antennas where the signal’s three electric-field components and three magnetic-field components are each measured separately. These electromagnetic vector sensors were also employed in [13] for ESPRIT-based blind beamforming or geolocation of wideband fast frequency-hopping signals. However, little work [14]–[16] has been done to include polarization in the area of joint space-time channel estimation [17], especially in a correlated signal environment. Hence, in this study a novel joint direction of arrival and time delay estimation approach is proposed for asynchronous direct sequence code division multiple access (DS-CDMA) systems, in conjunction with analytical expressions providing the estimate of the polarization parameters. The proposed approach is a subspace-type method having superresolution capabilities. However, it requires knowledge of the array manifold, which implies that the array should be properly calibrated.

To realistically model the multipath channel using polarization-sensitive sensors, instead of using the traditional isotropic sensors, a generalized signal model is formulated as detailed in Section II. The well-known spatial array manifold vector commonly-used for polarization-insensitive array is first extended to include the polarization element to form its corresponding manifold vector. The temporal dimension is then incorporated to construct the polar-spatio-temporal array (polar-STAR) manifold vector for the asynchronous DS-CDMA systems. Based on the formulation, a subspace-based polarization-angle-delay estimation (PADE) algorithm is then proposed in Section III, together with a novel temporal smoothing technique which restores the desired subspace dimensionality. Following that, Section IV provides several simulation studies which depict the performance of the proposed algorithm. The paper is finally concluded in Section V.

II. SIGNAL MODEL

Consider an antenna array of N polarization-insensitive sensors, its corresponding spatial array manifold vector due to a signal path arriving from the direction (θ, ϕ) , with θ and ϕ representing the azimuth and elevation directions respectively, can be expressed as

$$\underline{S}(\theta, \phi) = \exp(-j [\underline{r}_x, \underline{r}_y, \underline{r}_z] \underline{k}(\theta, \phi)) \quad (1)$$

where $[\underline{r}_x, \underline{r}_y, \underline{r}_z]^T$ is a $3 \times N$ matrix denoting the Cartesian coordinates of the sensors, and $\underline{k}(\theta, \phi) = (2\pi/\lambda) \underline{u}(\theta, \phi)$ is the wavenumber vector with the unit vector $\underline{u}(\theta, \phi) = [\cos(\theta) \cdot \cos(\phi), \sin(\theta) \cdot \cos(\phi), \sin(\phi)]^T$.

This can be easily extended to the polarization-sensitive array manifold vector [7] by specifying the polarization of the signal path. Thus consider the case of an array of N tripole [18] sensors (i.e., sets of three co-centered orthogonal dipoles with the signal from each dipole being processed separately in the array). Given a completely polarized transverse electromag-

Fig. 1. Linear crossed-dipole array.

netic (TEM) signal impinging onto the array, its polarization ellipse [19] generated by the electric field can be uniquely described by the parameters γ and η , where $0 \leq \gamma \leq \pi/2$ and $-\pi \leq \eta < \pi$. Hence for a given path of arbitrary elliptical polarization propagating into the array, its associated manifold vector can be derived as

$$\underline{A}(\underline{\Theta}) = \underline{S}(\theta, \phi) \otimes \underline{q}(\underline{\Theta}) \quad (2)$$

where $\underline{\Theta} \triangleq [\theta, \phi, \gamma, \eta]^T$ is the path’s parameter vector, $\otimes$ is the Kronecker product, and $\underline{q}(\underline{\Theta}) = [E_x(\underline{\Theta}), E_y(\underline{\Theta}), E_z(\underline{\Theta})]^T$ is a 3-dimensional (3-D) vector containing the electric field components induced on each dipole, that is given by

$$\underline{q}(\underline{\Theta}) = \text{diag}(\underline{\mathcal{V}}) \cdot \mathbb{T}(\theta, \phi) \cdot \underline{p}(\gamma, \eta) \quad (3)$$

where $\underline{\mathcal{V}} = [V_x, V_y, V_z]^T$ is the sensor’s sensitivities to unit electric fields solely polarized in the x , y , and z directions respectively, $\mathbb{T}(\theta, \phi) = [\{1/\cos(\phi)\} \partial \underline{u}/\partial \theta, \partial \underline{u}/\partial \phi]^T$ is the Spherical-to-Cartesian transformation matrix, and $\underline{p}(\gamma, \eta) = [\cos(\gamma), \sin(\gamma) e^{j\eta}]^T$ is the signal’s state of polarization which can be envisioned by making use of the Poincaré sphere concept [20].

However, for a crossed-dipole array as illustrated in Fig. 1, (3) is reduced to $\underline{q}(\underline{\Theta}) = [E_x(\underline{\Theta}), E_z(\underline{\Theta})]^T$ consisting of only the field components induced on the horizontally and vertically polarized dipoles respectively. By assuming that the sensors and the sources are coplanar (i.e., $\phi = 0$; $\underline{\Theta} \triangleq [\theta, \gamma, \eta]^T$), the manifold vector from (2) thus becomes

$$\underline{A}(\underline{\Theta}) = \mathbb{M}(\theta) \underline{p}(\gamma, \eta) \in C^{2N \times 1} \quad (4)$$

where

$$\begin{aligned} \mathbb{M} &\triangleq \mathbb{M}(\theta) = [\underline{S}_h(\theta), \underline{S}_v(\theta)] \in C^{2N \times 2}; \\ \underline{p} &\triangleq \underline{p}(\underline{\Theta}) = [\sin(\theta) \cos(\gamma), \sin(\gamma) e^{j\eta}]^T \in C^{2 \times 1}. \end{aligned}$$

Note, that in this case the two columns of $\mathbb{M}$ are defined as: $\underline{S}_h(\theta) \triangleq S(\theta) \otimes [-V_x, 0]^T$ and $\underline{S}_v(\theta) \triangleq S(\theta) \otimes [0, V_z]^T$, and may be viewed, respectively, as the spatial array manifold vectors for horizontally and vertically polarized signals arriving from the azimuth direction θ .

Equation (4) provides a general framework where the manifold vector $\underline{A}(\underline{\Theta})$ is expressed as a two-columns matrix $\mathbb{M}$ multiplied by a two-dimensional vector $\underline{p}$. This framework can also be used for some arrays that do not measure and process each polar-

ization component separately. For instance, for a circularly-polarized sensors array [6], it can be proven that its corresponding manifold vectors are given as

$$\underline{A}(\Theta) = \underline{S}(\theta, \phi) \odot \underline{q}(\Theta) \in C^{N \times 1} \quad (5)$$

where $\odot$ is the Hadamard product, and $\underline{q}(\Theta)$ is given by

$$\underline{q}(\Theta) = \mathbb{V}^T \cdot \mathbb{T}(\theta, \phi) \underline{p}(\gamma, \eta) \in C^{N \times 1} \quad (6)$$

where $\mathbb{V} = [\underline{V}_x, \underline{V}_y, \underline{V}_z]^T$ is a $3 \times N$ matrix with its n th column $[\underline{V}_x^{(n)}, \underline{V}_y^{(n)}, \underline{V}_z^{(n)}]^T$ representing the complex voltages induced at the n th sensor output in response to incoming signals with unit electric fields polarized solely in the x , y , and z directions respectively. By assuming that the sensors and the sources are coplanar, (5) can be cast into the framework of (4) where $\underline{p}$ remains the same, as above, while $\mathbb{M}$ now becomes

$$\mathbb{M} \triangleq \mathbb{M}(\theta) = \begin{bmatrix} -S(\theta) \odot \underline{V}_x & S(\theta) \odot \underline{V}_z \\ \equiv \underline{S}_h(\theta) & \equiv \underline{S}_v(\theta) \end{bmatrix} \in C^{N \times 2}.$$

Note, that in order to account for the mutual coupling in the array, the manifold vector described in (2) and (5) may be modified to include the mutual coupling matrix $\mathbb{C}$ [22](i.e., $\mathbb{C}\underline{A}(\Theta)$), which is a square complex matrix modeling the coupling effects amongst the sensor elements. Without loss of generality, and in order to simplify the notation, the mutual coupling matrix will be taken as an identity matrix (i.e., it is assumed that there are no mutual coupling effects between the antenna elements).

Now let us consider an M -user asynchronous DS-CDMA system where the i th user's transmitted baseband signal is given by

$$m_i(t) = \sum_{n=-\infty}^{+\infty} a_i[n] c_{PN,i}(t - nT_{cs}) \quad (7)$$

$$nT_{cs} \leq t < (n+1)T_{cs}$$

where $\{a_i[n] \in \{-1, +1\}, \forall n \in \mathcal{N}\}$ is the i th user's data symbol, and T_{cs} is the channel symbol period. The pseudo-noise spreading waveform associated with the i th user, $c_{PN,i}(t)$, is modeled as

$$c_{PN,i}(t) = \sum_{m=0}^{N_c-1} \alpha_i[m] c(t - mT_c), \quad mT_c \leq t < (m+1)T_c \quad (8)$$

where $\{\alpha_i[m] \in \{-1, +1\}, m = 0, 1, \dots, N_c-1\}$ corresponds to the i th user's PN-code sequence of period $N_c = T_{cs}/T_c$, and $c(t)$ denotes the unit amplitude chip pulse-shaping waveform of duration T_c which is zero outside of $0 \leq t < T_c$.

Suppose the transmitted signal due to the i th user arrives at the receiver via K_i multipaths. To refer to the parameters Θ of the j th path of the i th user we will use two subscripts, i.e., $\Theta_{ij} = [\theta_{ij}, \gamma_{ij}, \eta_{ij}]^T$. Using the polarization-sensitive array manifold vector $\underline{A}_{ij} \triangleq \underline{A}(\theta_{ij}, \gamma_{ij}, \eta_{ij})$ of (4) and letting β_{ij} , τ_{ij} be the complex path coefficient and path delay for the j th path of the i th user, the net baseband vector representation of the received signal can therefore be written as

$$\underline{x}(t) = \sum_{i=1}^M \underline{A}_i \text{diag}(\underline{\beta}_i) \underline{m}_i(t) + \underline{n}(t) \quad (9)$$

where $\underline{n}(t) \in C^{2N \times 1}$ is a complex additive white Gaussian noise vector and

$$\underline{A}_i = [\underline{A}_{i1}, \underline{A}_{i2}, \dots, \underline{A}_{iK_i}] \in C^{2N \times K_i}$$

$$\underline{\beta}_i = [\beta_{i1}, \beta_{i2}, \dots, \beta_{iK_i}]^T \in C^{K_i \times 1}$$

$$\underline{m}_i(t) = [m_i(t - \tau_{i1}), m_i(t - \tau_{i2}), \dots, m_i(t - \tau_{iK_i})]^T \in \mathcal{R}^{K_i \times 1}.$$

The $2N$ -dimensional signal vector $\underline{x}(t)$, received from the crossed-dipole array, is then sampled at chip rate and passed through a bank of $2N$ tapped-delay lines (TDL), each of length $2N_c$. Upon concatenating its outputs, a $4N\mathcal{N}_c$ -dimensional discretised signal vector is thus formed and read for every T_{cs} with the n th observation interval represented as

$$\underline{x}[n] = [\underline{x}_1[n]^T, \underline{x}_2[n]^T, \dots, \underline{x}_{2N}[n]^T]^T \quad (10)$$

where $\underline{x}_k[n]$ is the $2N_c$ -dimensional output frame from the k th TDL. Note, however, that due to the lack of synchronization and with a multipath delay spread comparable to T_{cs} , the content of each TDL contains contributions from not only the current but also the previous and next symbols. To model such contributions, the manifold vector due to the j th path of the i th user is modified from (4) to form the polar-STAR manifold vector given as

$$\underline{h}_{ij} \triangleq \underline{A}_{ij} \otimes \mathbb{J}^{l_{ij}} \underline{c}_i \in C^{4N\mathcal{N}_c \times 1} \quad (11)$$

where $l_{ij} = \lceil \tau_{ij}/T_c \rceil \bmod \mathcal{N}_c$ is the discretised path delay; $\underline{c}_i$ is the i th user's PN-code sequence padded with $\mathcal{N}_c$ zeros at the end, i.e.

$$\underline{c}_i = \underbrace{[\alpha_i[0], \alpha_i[1], \dots, \alpha_i[\mathcal{N}_c - 1], \mathbf{0}_{\mathcal{N}_c}^T]^T}_{i\text{th user's PN-code}} \quad (12)$$

and the matrix $\mathbb{J}$ (or $\mathbb{J}^T$) is a $2N_c \times 2N_c$ time down-shift (or up-shift) matrix given as follows:

$$\mathbb{J} = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{2N_c-1}^T & 0 \\ \mathbb{I}_{2N_c-1} & \mathbf{0}_{2N_c-1} \end{bmatrix}. \quad (13)$$

By taking into account the contributions of the current, previous and next symbols, the polar-spatio-temporal discretised signal vector $\underline{x}[n]$, hence, becomes

$$\underline{x}[n] = \sum_{i=1}^M \left[\mathbb{H}_{i,\text{prev}} \underline{\beta}_i, \mathbb{H}_i \underline{\beta}_i, \mathbb{H}_{i,\text{next}} \underline{\beta}_i \right] \begin{bmatrix} a_i[n-1] \\ a_i[n] \\ a_i[n+1] \end{bmatrix} + \underline{n}[n] \quad (14)$$

where $\mathbb{H}_i = [\underline{h}_{i1}, \underline{h}_{i2}, \dots, \underline{h}_{iK_i}]$ and $\underline{n}[n] \in C^{4N\mathcal{N}_c \times 1}$ is the sampled noise vector and

$$\mathbb{H}_{i,\text{prev}} = (\mathbb{I}_{2N} \otimes (\mathbb{J}^T)^{\mathcal{N}_c}) \mathbb{H}_i \in C^{4N\mathcal{N}_c \times K_i}$$

$$\mathbb{H}_{i,\text{next}} = (\mathbb{I}_{2N} \otimes \mathbb{J}^{\mathcal{N}_c}) \mathbb{H}_i \in C^{4N\mathcal{N}_c \times K_i}.$$

Notice, that by rearranging the terms in (14), $\underline{x}[n]$ can be decoupled into four constituents, namely the desired, inter-symbol-interference (ISI), multiple-access interference (MAI) and noise

Fig. 2. Temporal smoothing procedure.

components. Taking $\mathcal{D}$ th user as the desired user of interest, (14) may thus be rewritten as follows:

$$\underline{x}[n] = \mathbb{H}_{\mathcal{D}} \underline{\beta}_{\mathcal{D}} a_{\mathcal{D}}[n] + \underline{I}_{\text{ISI}}[n] + \underline{I}_{\text{MAI}}[n] + \underline{n}[n] \quad (15)$$

where $\mathbb{H}_{\mathcal{D}} \underline{\beta}_{\mathcal{D}} a_{\mathcal{D}}[n]$ is the desired signal component

$$\begin{aligned} \underline{I}_{\text{ISI}}[n] &= [\mathbb{H}_{\mathcal{D}, \text{prev}} \underline{\beta}_{\mathcal{D}}, \mathbb{H}_{\mathcal{D}, \text{next}} \underline{\beta}_{\mathcal{D}}] \begin{bmatrix} a_{\mathcal{D}}[n-1] \\ a_{\mathcal{D}}[n+1] \end{bmatrix} \\ \underline{I}_{\text{MAI}}[n] &= \sum_{\substack{i=1 \\ i \neq \mathcal{D}}}^M [\mathbb{H}_{i, \text{prev}} \underline{\beta}_i, \mathbb{H}_{i, \text{next}} \underline{\beta}_i] \begin{bmatrix} a_i[n-1] \\ a_i[n] \\ a_i[n+1] \end{bmatrix}. \end{aligned}$$

III. POLARIZATION, ANGLE, AND DELAY ESTIMATION (PADE)

In order to isolate the desired signal component as shown in (15), we propose that the discretised signal vector $\underline{x}[n]$ be preprocessed by the $\mathcal{D}$ th user's preprocessor which is given by

$$\mathbb{Z}_{\mathcal{D}} = \mathbb{I}_{2N} \otimes (\text{diag}(\mathbb{F} \underline{\mathbf{c}}_{\mathcal{D}})^{-1} \mathbb{F}) \quad (16)$$

where $\mathbb{F}$ is a $2N_c \times 2N_c$ Fourier transformation matrix, i.e.

$$\begin{aligned} \mathbb{F} &= [\underline{\Phi}^0, \underline{\Phi}^1, \underline{\Phi}^2, \dots, \underline{\Phi}^{(2N_c-1)}] \\ \text{where } \underline{\Phi} &= [1, \Phi^1, \Phi^2, \dots, \Phi^{(2N_c-1)}]^T \\ \text{with } \Phi &= \exp\left(-j \frac{2\pi}{2N_c}\right). \end{aligned} \quad (17)$$

To see this, let us apply the preprocessor to (11) as follows:

$$\begin{aligned} \mathbb{Z}_{\mathcal{D}} \underline{h}_{ij} &= \{\mathbb{I}_{2N} \otimes (\text{diag}(\mathbb{F} \underline{\mathbf{c}}_{\mathcal{D}})^{-1} \mathbb{F})\} \cdot (\underline{A}_{ij} \otimes \mathbb{J}^{l_{ij}} \underline{\mathbf{c}}_i) \\ &= \underline{A}_{ij} \otimes \{(\text{diag}(\mathbb{F} \underline{\mathbf{c}}_{\mathcal{D}})^{-1} \mathbb{F}) \cdot (\mathbb{J}^{l_{ij}} \underline{\mathbf{c}}_i)\} \\ &= \underline{A}_{ij} \otimes \{\text{diag}(\mathbb{F} \underline{\mathbf{c}}_{\mathcal{D}})^{-1} \text{diag}(\mathbb{F} \underline{\mathbf{c}}_i) \underline{\Phi}^{l_{ij}}\}. \end{aligned} \quad (18)$$

Notice, that the above expression can be reduced to simply $\underline{A}_{\mathcal{D}j} \otimes \underline{\Phi}^{l_{\mathcal{D}j}}$ if and only if $i = \mathcal{D}$ which corresponds to the desired user of interest. Hence, by applying the same operation to (15), the discretised signal vector is thus transformed to

$$\begin{aligned} \underline{y}[n] &= \mathbb{Z}_{\mathcal{D}} \underline{x}[n] \\ &= \mathbb{H}_{\mathcal{D}} \underline{\beta}_{\mathcal{D}} a_{\mathcal{D}}[n] + \mathbb{Z}_{\mathcal{D}} \underline{I}_{\text{ISI}}[n] + \mathbb{Z}_{\mathcal{D}} \underline{I}_{\text{MAI}}[n] + \mathbb{Z}_{\mathcal{D}} \underline{n}[n] \end{aligned} \quad (19)$$

where

$$\mathbb{H}_{\mathcal{D}} = [\underline{A}_{\mathcal{D}1} \otimes \underline{\Phi}^{l_{\mathcal{D}1}}, \underline{A}_{\mathcal{D}2} \otimes \underline{\Phi}^{l_{\mathcal{D}2}}, \dots, \underline{A}_{\mathcal{D}K_{\mathcal{D}}} \otimes \underline{\Phi}^{l_{\mathcal{D}K_{\mathcal{D}}}}].$$

However, the second order statistics of the vector $\underline{y}[n]$, instead of providing a basis for the desired signal subspace, would result in a rank deficiency with the desired signal subspace dimension being reduced to one. This is not due to the signal coherence problem (signal clustering or diffused signals) but because, as shown in (19), the columns of $\mathbb{H}_{\mathcal{D}}$ are linearly combined by the path coefficient vector $\underline{\beta}_{\mathcal{D}}$, hence its contribution to the observation space of $\underline{y}[n]$ will consequently lead to a subspace of only one dimension. To restore the dimensionality of this subspace back to $K_{\mathcal{D}}$, we will make use of the Vandermonde structure of the submatrices of $\mathbb{H}_{\mathcal{D}}$ provided by the preprocessing operation. This can be achieved by performing a technique referred to as temporal smoothing, a concept similar to that of spatial smoothing described in [21].

For clarity, let us reshape the $\underline{y}[n] \in \mathcal{C}^{4N_c \times 1}$ vector to $\mathbb{Y}[n] \in \mathcal{C}^{2N \times 2N_c}$ matrix so that each successive two rows reflect the preprocessed output for each crossed-dipole sensor, as illustrated in Fig. 2. By extracting a set of Q (where $Q = 2N_c - d + 1$) overlapping submatrices of length d (where $d < 2N_c$), and concatenating each of the submatrices via vectorization, Q concatenated subvectors each of length $2Nd$ are thus formed, i.e.

$$\underline{y}_q[n] = \text{vec}(\mathbb{Y}_q[n]), \quad \forall q = 1, 2, \dots, Q \quad (20)$$

where $\text{vec}(\bullet)$ is the row-wise vectorization operator. With that, the $2Nd \times 2Nd$ temporal-smoothed covariance matrix $\mathbb{R}_{\text{Tsmooth}}$ can, therefore, be obtained as follows:

$$\mathbb{R}_{\text{Tsmooth}} = \frac{1}{Q} \sum_{q=1}^Q \mathbb{R}_{\underline{y}_q} \quad (21)$$

where $\mathbb{R}_{\underline{y}_q}$ is the covariance matrix obtained from the subvector $\underline{y}_q[n]$. This technique also applies for paths arriving from the same direction (co-directional). However, for paths arriving at the same time (co-delay), singularity in $\mathbb{R}_{\text{Tsmooth}}$ will occur. This special case cannot be resolved for a general array geometry but for a uniform linear array where spatial smoothing can be performed on top of (21) to form the spatial-temporal-smoothed covariance matrix $\mathbb{R}_{\text{STsmooth}}$.

From the above discussion, it can be shown that by utilizing the polar-STAR manifold vector, the MUSIC-type cost function is based on the following criterion:

$$\begin{aligned}\xi_1(\theta, l, \gamma, \eta) &= (\mathbf{A}(\Theta) \otimes \mathbf{\Phi}_d^l)^H \mathbf{E}_n \mathbf{E}_n^H (\mathbf{A}(\Theta) \otimes \mathbf{\Phi}_d^l) \\ &= \mathbf{p}(\Theta)^H (\mathbf{M}(\theta) \otimes \mathbf{\Phi}_d^l)^H \mathbf{E}_n \mathbf{E}_n^H (\mathbf{M}(\theta) \otimes \mathbf{\Phi}_d^l) \mathbf{p}(\Theta)\end{aligned}\quad (22)$$

where $\mathbf{\Phi}_d$ is a subvector of $\mathbf{\Phi}$ with length d , and $\mathbf{E}_n$ is a matrix whose columns are the generalized noise eigenvectors of $(\mathbb{R}_{\text{STsmooth}}, \mathbb{D})$ due to the transformed noise in (19), with $\mathbb{D}$ representing the spatial-temporal smoothed diagonal matrix $\mathbb{Z}_D \mathbb{Z}_D^H$. However, such operation involves a multidimensional search over θ, l, γ and η for its minima. A more efficient minimization method can be obtained by letting the 2×2 matrix $\mathbb{B} \triangleq \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ be

$$\mathbb{B} \triangleq \mathbb{B}(\theta, l) = (\mathbf{M}(\theta) \otimes \mathbf{\Phi}_d^l)^H \mathbf{E}_n \mathbf{E}_n^H (\mathbf{M}(\theta) \otimes \mathbf{\Phi}_d^l). \quad (23)$$

Note, that minimization over the polarization space of $\mathbf{p}(\Theta)$ is equivalent to finding the eigenvectors corresponding to the minimum eigenvalues of $\mathbb{B}$. Hence by dropping the 2-D vector $\mathbf{p}(\Theta)$ and applying the quadratic formula, it can be established that (22) can be simplified to the following ‘‘STAR-Subspace’’ cost function:

$$\xi_2(\theta, l) = \text{trace}(\mathbb{B}(\theta, l)) - \sqrt{\text{trace}(\mathbb{B}(\theta, l))^2 - 4 \det(\mathbb{B}(\theta, l))} \quad (24)$$

where $\text{trace}(\bullet)$ denotes the trace operation and $\det(\bullet)$ represents the determinant of the 2×2 matrix. Now let $\xi_2^{\min}$ be the minima obtained from the spectrum constructed using the cost function $\xi_2(\theta, l)$ in (24). The location of the minima, as such, provides the joint estimate of its direction of arrival (DOA) and time of arrival (TOA). Its corresponding polarization parameters, on the other hand, are estimated as follows:

$$\begin{aligned}\hat{\gamma} &= \tan^{-1}(|\rho|) \\ \hat{\eta} &= \arg\left(\frac{\rho}{|\rho|}\right)\end{aligned}$$

where

$$\rho = \begin{cases} \frac{(\xi_2^{\min} - 2b_{11}) \sin(\hat{\theta})}{2b_{12}} & \text{or} \\ \frac{2b_{21} \sin(\hat{\theta})}{(\xi_2^{\min} - 2b_{22})} \end{cases} \quad (25)$$

It is worthwhile noting that the above expression ρ in (25), if necessary, can be further simplified in most circumstances since the value $\xi_2^{\min}$ is usually close to zero.

In general, when $\xi_2(\theta, l)$ is minimized, the supplementary (second) eigenvalue of the matrix $\mathbb{B}$ remains large. However this is not the case in an ill-conditioned situation in which two or more multipaths of the desired user with different polarization are closely located in space and time (i.e. have almost identical (θ, l) parameters), and thus remain unresolvable in the spatial and temporal domain. An ill-conditioned situation can be detected by using the following supplementary ‘‘STAR-Subspace’’ cost function:

$$\xi_3(\theta, l) = \text{trace}(\mathbb{B}(\theta, l)) + \sqrt{\text{trace}(\mathbb{B}(\theta, l))^2 - 4 \det(\mathbb{B}(\theta, l))} \quad (26)$$

which exploits the property that at an ill-conditioned point, not only one, but both eigenvalues of the matrix $\mathbb{B}$ are minimized simultaneously (i.e., both are very small- almost zero). In other words, when

$$\arg \min_{\theta, l} \xi_2(\theta, l) = \arg \min_{\theta, l} \xi_3(\theta, l)$$

then an ill-conditioned situation has been detected.

The complete procedure outlining the major steps of the proposed polarization-angle-delay channel estimation algorithm is summarized as follows.

- 1) Sample the crossed-dipole array output at chip rate and collect the data by concatenating the tapped-delay lines (TDL) contents to form the discretised signal vector of (10);
- 2) Apply the desired user’s preprocessor (16) onto the discretised signal vector and perform the temporal smoothing technique to restore the dimensionality of the desired signal subspace. Form the matrix $\mathbb{R}_{\text{smooth}}$. Note, that for paths arriving at the same time (co-delay), spatial smoothing (only for linear arrays) can be performed on top of the temporal smoothing technique to form the spatial-temporal-smoothed covariance matrix;
- 3) Apply the cost function in (24), using the generalized eigenvector decomposition of $\mathbb{R}_{\text{smooth}}$, to obtain the joint estimate of the direction of arrival (DOA) and time of arrival (TOA) of the desired user’s multipaths. The corresponding polarization parameters are estimated using the set of analytical equations in (25);
- 4) In an ill-conditioned situation in which two or more multipaths of differing polarization are closely located in space and time, the supplementary cost function in (26) can be employed to detect such occurrences.

It is important to point out that, due to the large number of degrees of freedom provided by the proposed approach, the number of identifiable rays is no longer limited by the number of antenna elements. In particular, the manifold surface associated with the desired signal is embedded in a $2Nd$ -dimensional complex space and so long as the total number of identifiable paths of the desired plus ISI and MAI signals is less than $2Nd - 1$, i.e.

$$(\text{identifiable paths of the desired user}) \leq 2Nd - 1 - (\text{number of ISI + MAI signals})$$

the proposed approach will operate properly. This is an important performance advantage of the proposed algorithm. Note, that the larger the d , the better the estimate. However, d and Q must fulfil the condition that $d \leq 2N_c - Q + 1$.

IV. SIMULATION STUDIES

In this section, several illustrative examples are presented to demonstrate the key features of the proposed PADE algorithm. Consider a uniform linear array, of half-wavelength spacing, consisting of $N = 5$ crossed-dipoles (assuming unity sensors sensitivities, i.e., $V_x = V_z = 1$) and operating in the presence of $M = 3$ co-channel CDMA users, where each user is being assigned a unique Gold sequence of length $N_c = 31$. The observation interval is assumed to be equal to 200 data symbols and, without loss of generality, the array is considered to be ‘‘fully

Fig. 3. MUSIC-type spectrum of desired user.

TABLE I
USER'S PARAMETERS

Path	User 1				User 2				User 3			
	θ_{1j}	l_{1j}	γ_{1j}	η_{1j}	θ_{2j}	l_{2j}	γ_{2j}	η_{2j}	θ_{3j}	l_{3j}	γ_{3j}	η_{3j}
$j=1$	40°	8 T_c	0°	-	50°	18 T_c	80°	30°	30°	3 T_c	10°	0°
$j=2$	50°	20 T_c	90°	-	70°	10 T_c	10°	-70°	45°	24 T_c	60°	100°
$j=3$	60°	15 T_c	45°	90°	80°	5 T_c	40°	120°	60°	5 T_c	20°	50°
$j=4$	85°	10 T_c	45°	-90°	95°	20 T_c	50°	90°	80°	20 T_c	70°	-90°
$j=5$	100°	5 T_c	30°	-10°	95.5°	20 T_c	45°	0°	105°	11 T_c	35°	-20°
$j=6$	100°	21 T_c	5°	-180°	110°	6 T_c	70°	100°	120°	28 T_c	45°	45°
$j=7$	120°	15 T_c	20°	80°	130°	25 T_c	30°	-160°	150°	20 T_c	50°	170°
$j=8$	130°	25 T_c	60°	120°	140°	15 T_c	20°	5°	160°	12 T_c	80°	-140°

TABLE II
ESTIMATED POLARIZATION PARAMETERS (DESIRED USER)

Table II: Estimated polarisation parameters (desired user)								
Path	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$	$j=8$
$\hat{\gamma}_{1j}$	0.06°	89.89°	45.03°	44.99°	29.91°	5.06°	19.91°	59.80°
$\hat{\eta}_{1j}$	-	-	89.91°	-90.09°	-10.08°	-179.25°	79.90°	120.02°

calibrated" and without mutual coupling effects between the antenna elements.

Assume that user 1 is the desired user, with an input SNR of 20 dB, together with two other interferers each constituting an SIR of -20 dB (i.e., near-far problem). All three users are assumed to have eight multipath rays each, with their parameters as listed in Table I. By partitioning the array into two overlapping subarrays (each having four crossed-dipoles) for spatial smoothing and setting $d = 50$ for temporal smoothing, it is seen from Fig. 3 that all the eight multipaths, as well as the co-delay and co-directional paths, can be identified/estimated successfully using the proposed algorithm. Their polarization parameters can also be estimated as shown in Table II by using the set of analytical equations given in (25). Notice that the number of resolvable paths is now no longer limited by the number of sensors available in the array.

Unlike linearly polarized array, it is also clear that the crossed-dipole array is capable of handling any arbitrary polarized paths, including signal path which is horizontally polarized (path 1), vertically polarized (path 2), left-hand circularly polarized (path 3) or right-hand circularly polarized (path 4). The performance of the estimation is shown to be relatively consistent irregardless of the received signal's polarization [16].

Fig. 4 depicts the standard deviations of the polarization, DOA and TOA estimates of the desired user for a normal-incidence desired signal path delayed over $15T_c$ with a SNR of -10 dB. It is assumed that the desired user operates in the presence of four other interferers (with each interferer having eight multipaths) individually constituting a desired signal to interference ratio of 40 dB. The results are averaged over 200 Monte Carlo simulations. Although different, the three

Fig. 4. Standard deviation of estimates versus (a) γ with $\eta = 0$ and (b) η with $\gamma = \pi/4$.

Fig. 5. Standard deviation of (a) time of arrival (TOA) and (b) direction of arrival (DOA) estimates for the first multipath of the desired user, in the presence of a second multipath, versus the SNR.

parameters (deviations) of the desired signal are shown for convenience on the same graph and are plotted against the desired signal's polarization parameters γ (with $\eta = 0$) and η (with $\gamma = \pi/4$), to demonstrate the performance in response to linear (ranging between horizontal to vertical polarizations) and elliptical (ranging between right-hand circular to left-hand circular polarizations) polarizations respectively. Note, that the standard deviation of the polarization estimate is defined as the angular distance [7] on the Poincaré sphere δ , where $0 \leq \delta \leq \pi$, obtained using

$$\cos(\delta) = \cos(2\gamma) \cos(2\hat{\gamma}) + \sin(2\gamma) \sin(2\hat{\gamma}) \cos(\eta - \hat{\eta}). \quad (27)$$

From Fig. 4, it is therefore apparent that in a poor SNR environment, with strong interferers constituting a near-far situation, the proposed algorithm is still able to provide relatively consistent and accurate estimates irrespective of the polarizations of the received signal.

Next, let us compare the performance of a crossed-dipole array with that of an equivalent polarization-insensitive array, with the latter assumed to be omitting any polarization mismatches. For simplicity, we will assume user 1 to have only two linearly polarized multipaths: $(\theta_{11}, l_{11}, \gamma_{11}, \eta_{11}) = (80^\circ, 10T_c, 20^\circ, 0^\circ)$ and $(\theta_{12}, l_{12}, \gamma_{12}, \eta_{12}) = (82^\circ, 11T_c, \gamma, 0^\circ)$ with the polarization difference defined as $\Delta\gamma = (\gamma - 20^\circ)$. Using the same scenario as that in Table I, the standard deviation of the estimates associated with the first multipath is plotted as shown in Fig. 5.

As expected, paths that are well separated in polarization exhibit a better standard deviation, with those due to the crossed-dipole array having in general a lower deviation (even when the polarization difference $\Delta\gamma = 0$) than that of the polarization-insensitive array. Such deviation is also observed to be deteriorating with decreasing angular/temporal separation. Take the case whereby user 1 has the following two identically polarized ($\gamma = 20^\circ$ and $\eta = 0^\circ$) signal paths: $(\theta_{11}, l_{11}) = (80^\circ, 10T_c)$ and $(\theta_{12}, l_{12}) = (80^\circ + \Delta\theta, 11T_c)$ with a SNR = 5 dB.

Fig. 6. Standard deviation of time of arrival (TOA) and direction of arrival (DOA) estimates for one of the two multipaths of the desired user, versus (a) DOA separation and (b) TOA separation.

The standard deviation of the estimates due to the first signal path is plotted versus their angular separation $\Delta\theta$ as illustrated in Fig. 6(a). Similarly, Fig. 6(b) depicts its response toward their temporal separation Δl , with the second path given by $(\theta_{12}, l_{12}) = (81^\circ, 10T_c + \Delta l)$. Again the crossed-dipole array provides better estimation as compared with that of the polarization-insensitive array where both estimates degenerate, as anticipated, with decreasing angular and temporal separations.

Hence, it is apparent that by incorporating the polarization dependent processing, the crossed-dipole array can improve the accuracy and resolution of the estimates considerably, especially when the paths are well separated in polarization. This improvement is even more significant for paths in a poor SNR environment and paths that are very closely located. To get a clearer picture, let us take a closer look at the effect of polarization on the spectrum due to the crossed-dipole array as compared with that of the polarization-insensitive array. Now consider the case of a vertically polarized signal path of $(\theta_{11}, l_{11}, \gamma_{11}, \eta_{11}) = (50^\circ, 5T_c, 90^\circ, 0^\circ)$ together with a closely-located linearly polarized multipath given by $(\theta_{12}, l_{12}, \gamma_{12}, \eta_{12}) = (51^\circ, 6T_c, 30^\circ, 0^\circ)$. Assuming an $\text{SNR} = -3 \text{ dB}$ and $\text{SIR} = -30 \text{ dB}$, it is seen from Fig. 7(a)

that the crossed-dipole array is able to clearly resolve the two signal paths; whereas that of the polarization-insensitive array has their peaks collapsed together as depicted in Fig. 7(b).

Finally, consider an ill-conditioned case whereby 2 paths are so closely located that even the crossed-dipole array fails to distinguish/resolve their spatial and temporal locations. Let us take user 2 as the desired user, with its 4th and 5th paths being so closely located such that the resolution of their peaks collapse and merge into one as illustrated in Fig. 8(a). By making use of the supplementary cost function in (26), such an occurrence can be detected as depicted in Fig. 8(b). The capability and proximity limit of such detection is found to be even better if the polarization difference between the paths is higher or the strength of the paths increases.

V. CONCLUSION

In this paper an efficient near-far resistant channel estimation algorithm is presented using a crossed-dipole array in an asynchronous completely polarized multipath DS-CDMA system. The array is assumed to be “fully calibrated” and that there are no mutual coupling effects between the antenna elements. The

Fig. 7. Spectrum of two closely-located paths using (a) crossed-dipole array, and (b) polarization-insensitive array, over an observation interval of 200 data snapshots.

Fig. 8. (a) Spectrum of user 2 with point a indicating the ill-conditioned event due to the unresolvable 4th and 5th paths and (b) supplementary spectrum of user 2 with point b indicating the detected ill-conditioned occurrence.

proposed approach is based on the polar-STAR manifold vector parameterization and the transformation of the data to another domain. However, in this parameterization the polar-STAR manifold vectors of the various paths of the desired user are

linearly combined by the fading coefficient. This leads to the collapse of the desired signal subspace (which is a subset of the overall signal subspace) to a rank-1 subspace. To overcome this problem and to restore the rank of the desired signal subspace

a temporal smoothing technique is employed operating on the transformed domain. This leads to the "STAR-Subspace" cost function of (24) which is used in conjunction with the analytical expressions of (25) to provide a joint estimation of the polarization-angle-delay parameters associated with the desired user.

In contrast to the linearly polarized arrays, the crossed-dipole arrays employed in this paper are able to capture the polarization diversity in the signal, hence providing a better detection and estimation performance regardless of its received polarization. By exploiting the inherent polarization information in the signal, the joint space-time channel estimator is also able to resolve closely-located paths that otherwise cannot be resolved using its equivalent polarization-insensitive array. In the event when the paths are so closely-located, such that the proposed algorithm fails, its supplementary "STAR-Subspace" cost function of (26) can be employed to detect such an occurrence. Finally it is important to point out the number of multipaths that can be resolved by the proposed algorithm is not constrained by the number of antennas available in the array. This is due to the way of collecting the received data [see (14)] and the extended dimensionality of the polar-STAR array manifold vector of (11).

REFERENCES

- [1] P. Beckmann, *The Depolarization of Electromagnetic Waves*. Boulder, CO: Golden Press, 1968.
- [2] J. Li and P. Stoica, "Efficient parameter estimation of partially polarized electromagnetic waves," *IEEE Trans. Signal Processing*, vol. 42, pp. 3114–3125, Nov. 1994.
- [3] K. C. Ho, K. C. Tan, and A. Nehorai, "Estimating directions of arrival of completely and incompletely polarized signals with electromagnetic vector sensors," *IEEE Trans. Signal Processing*, vol. 47, pp. 2845–2852, Oct. 1999.
- [4] E. Ferrara Jr and T. Parks, "Direction finding with an array of antennas having diverse polarizations," *IEEE Trans. Antennas Propagat.*, vol. 31, pp. 231–236, Mar. 1983.
- [5] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propagation*, vol. 34, no. 3, pp. 276–280, Mar. 1986.
- [6] A. J. Weiss and B. Friedlander, "Performance of diversely polarized antenna arrays for correlated signals," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 28, pp. 869–879, July 1992.
- [7] J. Li and R. T. C. Jr, "Angle and polarization estimation in a coherent signal environment," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 29, pp. 706–716, July 1993.
- [8] J. Li, P. Stoica, and D. Zheng, "Efficient direction and polarization estimation with a COLD array," *IEEE Trans. Antennas Propagat.*, vol. 44, pp. 539–547, Apr. 1996.
- [9] M. D. Zoltowski and K. T. Wong, "ESPRIT-based 2-D direction finding with a sparse uniform array of electromagnetic vector-sensors," *IEEE Trans. Signal Processing*, vol. 48, pp. 2195–2204, Aug. 2000.
- [10] K. T. Wong and M. D. Zoltowski, "Uni-vector-sensor ESPRIT for multi-source azimuth, elevation, and polarization estimation," *IEEE Trans. Antennas Propagat.*, vol. 45, pp. 1467–1474, Oct. 1997.
- [11] K. T. Wong, "Direction finding/polarization estimation- dipole and/or loop triad(s)," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 37, pp. 679–684, Apr. 2001.
- [12] K. T. Wong and M. D. Zoltowski, "Self-initiating MUSIC-based direction finding & polarization estimation in spatio-polarizational beamspace," *IEEE Trans. Antennas Propagat.*, vol. 48, pp. 1235–1245, Aug. 2000.
- [13] K. T. Wong, "Blind beamforming/geolocation for wideband-FFH with unknown hop-sequences," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 37, pp. 65–76, Jan. 2001.
- [14] P. H. Chua, C. M. S. See, and A. Nehorai, "Vector-sensor array processing for estimating angles and times of arrival of multipath communication signals," in *Proc. Int. Conf. Acoustics, Speech, Signal Processing*, vol. 6, May 1998, pp. 3325–3328.
- [15] J. W. P. Ng and A. Manikas, "Diversely polarized arrays in DS-CDMA: A space-time channel estimation approach," in *Proc. Int. Conf. Acoustics, Speech, Signal Processing*, vol. 3, May 2002, pp. 2617–2620.
- [16] —, "Polarization-angle-delay estimation using crossed-dipole array for DS-CDMA systems," in *Proc. 14th Int. Conf. Digital Signal Processing*, July 2002, pp. 259–262.
- [17] L. K. Huang and A. Manikas, "Blind adaptive single-user array receiver for MAI cancellation in multipath fading DS-CDMA channels," in *EU-SIPCO Proc.*, vol. 2, Sept. 2000, pp. 647–650.
- [18] R. T. Compton, Jr., "The tripole antenna: An adaptive array with full polarization flexibility," *IEEE Trans. Antennas Propagat.*, vol. 29, pp. 944–952, Nov. 1981.
- [19] C. A. Balanis, *Antenna Theory: Analysis and Design*, 2nd ed. New York: Wiley, 1997.
- [20] G. A. Deschamps, "Geometrical representation of the polarization of a plane electromagnetic wave," *Proc. IRE*, vol. 39, pp. 540–544, May 1951.
- [21] T. Shan, M. Wax, and T. Kailath, "On spatial smoothing for direction-of-arrival estimation of coherent signals," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 33, pp. 806–811, Aug. 1985.
- [22] K. Stavropoulos and A. Manikas, "Array calibration in the presence of unknown sensor characteristics and mutual coupling," in *EUSIPCO Proc.*, vol. 3, Sept. 2000, pp. 1417–1420.

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