The Effect of Principal Stress Rotation and Intermediate Principal Stress Changes on the Liquefaction Resistance and Undrained Cyclic Response of Ottawa Sand

Erdem O. Tastan¹, J. Antonio H. Carraro²

ABSTRACT

In laboratory testing, the liquefaction resistance of sands is typically evaluated using cyclic triaxial and simple shear tests. These tests cannot be used in a rigorous manner to systematically assess the effects of principal stress rotation and intermediate principal stress changes on the undrained cyclic response of sands. In this study, the effect of these two factors on the liquefaction resistance of Ottawa sand was investigated using a cyclic hollow cylinder apparatus. At similar initial states of fabric and mean effective stress following K_0 consolidation, the liquefaction resistance of Ottawa sand deposited underwater can: (i) decrease by 50% to 80 % as the major principal stress direction moves away from the vertical with $\sigma'_2 = \sigma'_3$, or (ii) increase by 200% to 380%, as σ'_2 increases while σ'_1 remains vertical depending on the liquefaction criterion (strain levels). When the stress state defined by the imposed boundary condition deviates from axisymmetric compression, the combined effect on the liquefaction resistance is governed by principal stress rotation.

Keywords: underwater deposition, slurry, fabric, liquefaction, principal stress rotation, intermediate principal stress, hollow cylinder test.

¹ Senior Geotechnical Engineer, CDM Smith Inc., Charlotte, NC 28209, United States <tastaneo@cdmsmith.com>, Corresponding Author

² Senior Lecturer in Experimental Geotechnical Engineering, Imperial College London, SW7 2AZ, United Kingdom <antonio.carraro@imperial.ac.uk>

INTRODUCTION

Most soils are anisotropic materials and soil anisotropy is construed of two components: inherent and induced anisotropy (Casagrande and Carillo 1944). Inherent anisotropy relates to the original deposition of soil layers *in situ*, whereas induced anisotropy results from plastic deformations experienced by the soil following deposition. Unfortunately, undisturbed sand specimens are rarely used in practice and most sand specimens are reconstituted in the laboratory. To be representative, the selected method of reconstitution should therefore replicate inherent soil anisotropy to the extent possible. Subsequent testing procedures should also be able to reproduce the actual loading conditions imparted to the soil *in situ* and the induced anisotropy that derives from actual loading scenarios.

Upstream tailings dam construction, natural sloping ground and cyclic loading induced by earthquakes represent typical scenarios encountered in practice that induce non-trivial loading conditions to soil elements in the ground/embankment. Conversely, idealized loading imposed by conventional triaxial tests, which solely apply to centerline conditions under circular footings/embankments, and/or by direct simple shear tests are crude simplifications of the actual typical scenarios cited above. In general, all three principal stresses and their directions should be taken into account in rigorous analyses of real boundary value problems encountered in practice. During an earthquake, shear and compressional waves induce cyclic changes to deviatoric ($\sigma_v - \sigma_h$) and shear (τ_{vh}) stresses on soil elements in the ground. According to Ishihara (1983), the induced cyclic shear and deviatoric stresses vary in such a way that the principal stress directions may remain approximately constant during cyclic loading for sloping ground conditions such as those existing in earth dams and embankments. Advanced numerical modelling may be

necessary in the future to further verify this as well as the true generalized stress conditions that may apply for other complex geotechnical boundary value problems involving true 3D anisotropic soil behavior associated with principal stress rotation and large deformations imparted by cyclic loading.

In practice, most liquefaction analyses involve semi-empirical model calibration relying on cyclic triaxial and cyclic simple shear tests. More recently, cyclic hollow cylinder tests have gained popularity due to their ability to simulate more generalized three-dimensional stress conditions. This study sheds new light into the three-dimensional undrained cyclic response of sands deposited under water and/or in a slurry environment (e.g. alluvial, fluvial, marine and tailings deposits). The inherent anisotropy of the specimens tested was accounted for by using a slurry deposition method during specimen reconstitution (Tastan and Carraro 2013, Carraro and Prezzi 2008). Induced anisotropy before and during cyclic loading was addressed using a cyclic hollow cylinder apparatus. As such, the effects of principal stress rotation and intermediate principal stress changes on the liquefaction resistance of Ottawa sand were systematically assessed.

LIQUEFACTION RESISTANCE UNDER CYCLIC LOADING

Available Methods and Relevant Parameters

The liquefaction susceptibility of a soil subjected to cyclic loading depends on various factors including (but not limited to) the initial state of the soil on e - p' - q space compared to the location of key bounding surfaces (e.g. undrained instability line or CSL) and the magnitude and direction of the cyclic loading experienced by the soil. The deformation behavior of a liquefied soil subjected to cyclic loading might be generally idealized as contractive or dilative. A contractive behavior would lead to large cumulative deformations

after undrained instability is mobilized, whereas a dilative response would result in incremental build up of deformations as cyclic loading progresses (Casagrande 1971). Vaid and Chern (1985) observed another deformation pattern during cyclic loading whereby a combination of undrained instability and cyclic mobility occurred, which they referred to as limited liquefaction followed by cyclic mobility.

Soil liquefaction induced by cyclic loading is typically evaluated by cyclic triaxial and cyclic simple shear tests. Cyclic hollow cylinder tests (also referred to as cyclic torsional shear tests) have also been used to evaluate soil liquefaction. Cyclic triaxial tests impose axi-symmetric states to soil, and therefore cannot rigorously simulate the true *in situ* stress conditions in sloping ground where a soil element under the sloping surface is subjected to initial shear stresses on vertical and horizontal planes before cyclic loading commences. Cyclic simple shear tests can induce an initial amount of principal stress rotation to test specimens but most commercially available devices are not commonly used in this manner, except for studies like Sivathayalan and Ha (2011) and Porcino and Caridi (2007), and suffer from additional issues related to stress and strain non-uniformities (Budhu 1988). Other limitations associated with these tests are discussed elsewhere (Arthur et al. 1980). For sloping ground, the existence of initial static shear stresses, which may be smaller or larger than the cyclic shear stresses induced by an earthquake, cause the major (σ_1) and minor (σ_3) principal stresses to rotate and not be aligned with the vertical and horizontal directions after soil deposition. In other geotechnical applications (e.g. design and analyses of onshore and offshore foundations), soil elements nearby foundation edges are loaded by combinations of shear and normal stresses prior to earthquakes. Such stress conditions

cannot be simulated in cyclic triaxial tests but may be approximated in cyclic simple shear tests, albeit these are still subject to nonuniformity issues (Budhu 1988).

Empirical corrections (Eq. 14 as provided by Idriss and Boulanger 2008) are used in practice to mitigate issues associated with a combination of triaxial and simple shear boundary conditions. Even so, such analyses are often conducted without a more comprehensive and rigorous assessment of the implications resulting from such a combination of boundary conditions. Ultimately, this reflects the inherent limitations associated with the original framework underpinning such analyses (Lambe 1967), whose limitations may have been justifiable in the past due to the prevailing experimental constraints of the last century. But as a result of major technological advances in experimental geomechanics, hollow cylinder tests have gained popularity in analyses of sand liquefaction due to cyclic loading where anisotropy is assessed in a more rigorous way (Ishihara and Yamazaki 1984, Tatsuoka et al. 1984, Ishihara et al. 1985, Tatsuoka et al. 1989, Koester 1992, Altun et al. 2005, Al-Rkaby et al. 2017, Prasanna et al. 2018, Gu et al. 2018, Chen et al. 2020, Prasanna et al. 2020). Stress conditions imposed by hollow cylinder tests (Figures 1a and 1b) allow for systematic variations in major principal stress direction (α) and intermediate principal stress (σ_2) (Figure 1c) and are more robust and consistently defined than those imparted by cyclic triaxial and cyclic simple shear tests. In cyclic hollow cylinder tests, principal stress rotations can be simulated prior to and/or during cyclic loading through the independent application of torque, vertical load and inner and outer boundary pressures (Hight et al. 1983). All related parameters for hollow cylinder testing and their definitions are provided in Appendix A.

As defined in the typical cyclic stress approach used to assess the liquefaction potential of soils, cyclic resistance can be represented in various ways. The ratio of 2D deviatoric to 3D mean stress invariants (t/p') may be applicable to allow general comparisons of the results from cyclic triaxial, simple shear and hollow cylinder tests where

$$t = \frac{\sigma_1 - \sigma_3}{2} \quad (1)$$

$$p' = \frac{\sigma'_1 + \sigma'_2 + \sigma'_3}{3} \quad (2)$$

For hollow cylinder tests, K_0 effects may be implicitly considered by defining the cyclic stress ratio (CSR) as $\Delta t/p'_c$ (Ishibashi and Sherif 1974), where Δ denotes a change in the parameter of interest (in this case, t) and p'_c denotes the mean effective stress prior to the cyclic shearing. This CSR definition is applicable to both cyclic simple shear and hollow cylinder tests and is also equivalent to that typically used for cyclic triaxial tests ($=\Delta\sigma_d/2p'_c$), where $\Delta\sigma_d$ is the change in deviator stress ($\sigma_d = \sigma_a - \sigma_r$) in a triaxial test with axial and radial stresses equal to σ_a and σ_r , respectively. However, while keeping all other factors equal, the $\Delta t/p'_c$ ratio measured at a state corresponding to a particular failure criterion may not be strictly comparable among cyclic triaxial, simple shear and hollow cylinder tests since this CSR definition does not explicitly consider σ_2 as part of t or simply assumes σ_2 to be equal to σ_3 . If the $\Delta t/p'_c$ ratio is used as CSR, isotropically consolidated cyclic triaxial ($\Delta\sigma_d/2p'_c$) and isotropically and anisotropically consolidated hollow cylinder tests ($\Delta t/p'_c$) may yield similar cyclic strengths (Ishihara and Yasuda 1975). But this may not always be true because the cyclic triaxial strengths defined as $\Delta\sigma_d/2p'_c$ were found to be different than cyclic hollow cylinder strengths defined as $\Delta t/p'$ for dense Sengenyama sand (Tatsuoka et al. 1986).

Specimen reconstitution has also been shown to induce cyclic resistance differences between cyclic triaxial and hollow cylinder test results even when $\Delta t/p_c'$ is uniquely used to define CSR (Yamashita and Toki 1993). Bhatia et al. (1985) compiled cyclic triaxial, simple shear and hollow cylinder test results and pointed that, even when $\Delta t/p_c'$ is used, sandy specimens exhibit different test-dependent liquefaction resistances. Although they observed a large scatter in liquefaction resistance, they claimed that sandy specimens exhibited the largest, lowest, and intermediate liquefaction resistance values in cyclic triaxial, simple shear and hollow cylinder testing, respectively. Therefore, $\Delta t/p_c'$ may not allow for a consistent comparison of the liquefaction resistance obtained from cyclic triaxial, simple shear or hollow cylinder tests.

In this study, CSR was defined as $\Delta q/p_c'$ where the octahedral deviatoric stress invariant q is expressed as

$$q = \sqrt{\frac{1}{2}\{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2\}} \quad (3)$$

The $\Delta q/p_c'$ ratio represents the 3D generalized stress state and is therefore more appropriate than $\Delta t/p_c'$ to systematically account for σ_2 effects. The change in the deviatoric stress invariant (Δq) is that applied during cyclic loading and p' is the mean effective stress at the start of the undrained cyclic loading. Since the earliest cyclic stress tests were cyclic triaxial tests, axial strains were the ones of original concern (Seed and Lee 1966, Lee and Seed 1967). However, a more general parameter such as the deviatoric strain invariant (ϵ_q) is more representative to assess liquefaction due to cyclic loading under generalized stress states and used in this study (Zdravkovic and Jardine 2001):

$$\varepsilon_q = \frac{2}{\sqrt{6}} \sqrt{(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_1 - \varepsilon_2)^2} \quad (4)$$

Effect of α and b on the Cyclic Behavior of Sand

Most past studies involving undrained cyclic loading in hollow cylinder tests imposed instantaneous jumps (or stress reversals) during principal stress rotation (Drnevich 1972, Ishibashi and Sherif 1974, Towhata and Ishihara 1985, Tatsuoka et al. 1989, Yamashita and Toki 1993, Chaudhary et al. 2002, Shibuya et al. 2003, Altun et al. 2005). Shibuya et al. (2003) and Gu et al. (2018) investigated the effect of intermediate principal stress coefficient b (Eq. 5) on the 3D undrained cyclic response of sands and clays, respectively,

$$b = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3} \quad (5)$$

RESEARCH METHODOLOGY

Materials

De-aired water and Ottawa sand were used in this study. Ottawa sand is a silica sand with round to sub-round particles that is manufactured by U.S. Silica (Ottawa, IL). It complies with ASTM C 778 and its basic properties are summarized in Table 1.

Hollow Cylinder Specimen Reconstitution

The specimen reconstitution method used is described in detail in Tastan and Carraro (2013); only a summary is provided here. The method is an alternative version of the slurry-deposition method of reconstitution of solid triaxial specimens of sands with or without fines (Carraro and Prezzi 2008). Thus, it also yields homogeneous specimens with high initial degree of saturation and fabric that is representative of sand deposits formed under water. Specimens had nominal outer diameter, inner diameter and height equal to 100, 60,

and 200 mm, respectively. Uniformity analysis showed that maximum local D_R variations across the specimen height was around $\pm 5\%$, which is considered acceptable (Tastan and Carraro 2013). Briefly, the method consists of (1) mixing de-aired water with dry sand, (2) transferring the saturated sand into a bespoke mixing tube, (3) placing the tube with sample into the hollow cylinder mold, and (4) carefully raising the tube to allow sample deposition inside the mold. By conducting step (4) as carefully as possible, the procedure can produce loose specimens with relative density (D_R) around 25%. Denser specimens were obtained by tapping the sides of the outer mold. Additional details about the method used are available in Tastan and Carraro (2013) and Tastan (2009).

Hollow Cylinder Testing

Equipment

The dynamic hollow cylinder apparatus used in this study was manufactured by Wykeham Farrance. Its closed-loop control is achieved by the high-speed 32-bit data acquisition system and PC-software. The system independently controls five parameters: (1) inner cell pressure, (2) outer cell pressure, (3) back pressure, (4) vertical actuator displacement, and (5) horizontal actuator displacement (for torque). In addition to these five parameters, specimen volume is also monitored/controlled by allowing drained or undrained shearing conditions during testing. Table 2 summarizes the basic characteristics of the transducers used in the system.

General Conditions

Typical hollow cylinder boundary conditions are shown in Fig. 1a. Stress-strain (Fig. 1b), and principal stress (Fig. 1c) definitions comply with those given by Hight et al. (1983) (summarized in Appendix A).

Stress non-uniformities for the stress components shown in Fig. 1b can derive from specimen geometry effects. Therefore, the specimen geometry used in this study was guided by the following geometry criteria (Sayao and Vaid 1991): (a) wall thickness between 20 and 26 mm (20 mm was used), (b) $0.65 \leq r_i/r_o \leq 0.82$ (0.60 was used), and (c) $1.8 \leq H / 2r_o < 2.2$ (2.0 was used). Avoiding α and b combinations of $\alpha = 0^\circ$ and $b = 1$; $\alpha = 90^\circ$ and $b = 0$; and $\alpha = 45^\circ$ can help keep stress nonuniformities within reasonable levels (Hight et al.1983, Wijewickreme and Vaid 1991, Naughton and O’Kelly 2007). This was observed for all stress paths employed in this study. The testing protocol followed in this study had four main stages, which are discussed next.

Saturation

Specimens were flushed with de-aired water and back pressure saturated at $p' = 20$ kPa. All specimens tested had Skempton’s pore pressure coefficient B (Skempton 1954) equal to or greater than 0.98.

Anisotropic Consolidation

An initial anisotropic consolidation stage was conducted to simulate K_0 deposition. Specimens were consolidated to $p' = 100$ kPa following a target effective stress ratio K ($=\sigma'_3/\sigma'_1$) aimed at yielding zero lateral strains (or as low as possible). A trial-and-error procedure was initially used to define the value of K to be used. This led to $K = 0.43$, which resulted in radial (ϵ_r) and circumferential (ϵ_θ) strains (Fig. 1b) ranging between 0.01 and 0.32% (average = 0.15%), and -0.06 and -0.33% (average = 0.12%), respectively. Target stress parameters ($p' = 100$ kPa and $K=0.43$) leading to these near zero lateral strain

conditions were ramped over 8 hours to allow full dissipation of pore pressure changes during this stage. Once the target anisotropic stress conditions were achieved after 8 hours, specimens were allowed to creep until the volumetric strain rate $\Delta\epsilon_p/\Delta t$ was less than 0.05%/day (Zdravkovic and Jardine 2001); where $\Delta\epsilon_p$ is the change in volumetric strain ($\epsilon_p = \epsilon_1 + \epsilon_2 + \epsilon_3 = \Delta V/V$) and ΔV , V and Δt are the specimen volume change, specimen volume and time change associated with the analysis, respectively.

Drained α and/or b Changes

After anisotropic consolidation and drained creep, α and/or b were ramped to their target values over a period of 8 hours, while p' and K were kept constant and equal to their values at the end of anisotropic consolidation. Next, specimens were allowed to creep again until $\Delta\epsilon_p/\Delta t \leq 0.05\%/day$.

Undrained Cyclic Loading

Finally, specimens were subjected to undrained cyclic loading at constant p' , α and/or b (depending on the testing conditions used). This was achieved by cycling the maximum shear stress t sinusoidally (period = 1 second). The starting point of this sinusoidal loading was the t level achieved at the end of the drained α and/or b change stages, and σ'_3/σ'_1 is not constant during shearing. As mentioned previously, 2-D seismic analyses showed that the major principal stress direction remains approximately constant during cyclic shaking (Ishihara 1983) under sloping ground conditions. This is consistent with the testing protocol used in this study and with other systematic approaches (Sivathayalan and Vaid 2002) used to investigate the effect of pre-shearing α and b on the undrained monotonic

response of Fraser River sand. Even though t is controlled during shearing, CSR is computed as $\Delta q/p'$ as discussed earlier.

Detailed information about the 32 hollow cylinder tests are summarized in Table 3.

ANALYSIS OF RESULTS

Drained (α) Principal Stress Rotation

Typical strains induced by principal stress rotation on slurry-deposited Ottawa sand specimens are shown in Fig. 2. For specimens with similar post-consolidation D_R , drained principal stress rotation to 60° induces larger shear strains $\gamma_{z\theta}$ (about 5 %) than rotation to 30° (about 1.5 %). The specimen subjected to drained 30° rotation exhibited steady contraction in the axial direction. For the specimen subjected to 60° rotation, axial strains also increased up to $\alpha = 45^\circ$ but then decreased subsequently. At $\alpha = 60^\circ$, the induced axial strain is close to zero.

Undrained Cyclic Response of Ottawa Sand

Axisymmetric Condition

Under axisymmetric loading (i.e., $\alpha = 0^\circ$ and $b = 0$) following an anisotropic state, Ottawa sand shows two distinct patterns of deformation and pore pressure changes that directly relate to CSR. For CSR = 0.20, large deformations suddenly develop within the first loading cycle (Fig. 3a) due to a temporary excursion through undrained instability (Murthy et al. 2007). Another specimen subjected to CSR = 0.14 (not shown in Fig. 3), experiences undrained instability only after the second cycle. If CSR is further reduced to 0.10 (Fig. 3b), the sand does not experience undrained instability at all but rather displays a steady build up in deformations and pore pressure (i.e., cyclic mobility).

A transient excursion through undrained instability during cyclic loading is noted when pore pressure changes deviate significantly from the common trend expected for a given CSR at a given cycle (Figs. 3a, 3c and 3e). This behavior was much quicker than the cyclic loading period used (= 1s).

When evaluating deformation and pore pressure changes induced during undrained instability, the following factors should be considered from a rigorous experimental-critical-state soil mechanics standpoint: (1) inertia forces (Uthayakumar and Vaid 1998), and (2) actuator control capability. For the Ottawa sand specimens tested, induced pore pressure changes do not exceed the initial anisotropic p' (i.e., p' does not become zero during undrained cyclic loading (Vaid and Chern 1985). But as soon as undrained instability takes place, Ottawa sand starts to exhibit dilative behavior after phase transformation. The stress states on t - p' space where the undrained instability occurred are shown in Fig. 4 for all tests where samples showed such behavior (Table 4). Also shown in Fig. 4 are the critical state lines for compression (M_c) and extension (M_e) for a critical state angle of 30° (Bolton, 1986). [Actual stress paths are not shown in Fig. 4 for clarity but can be found in Tastan \(2009\).](#) Undrained instability occurs before samples reach the critical state line. This is consistent with the definition of phase transformation point (PTP) by Shibuya et al. (2003), albeit shown for monotonic loading, where the sand samples transition from contractive to dilative behavior.

The rates of pore pressure changes and strain mobilization become much lower than those observed during undrained instability (Fig. 3a). This behavior cannot be classified as either classical undrained instability or cyclic mobility. The deviatoric strain (ϵ_q) response

of Ottawa sand with $\alpha = 0^\circ$ and $b = 0$ is similar to that described by Vaid and Chern (1985) as limited liquefaction followed by cyclic mobility.

The various CSR levels used yield the liquefaction resistance curves shown in Fig. 5 for strain-based liquefaction criteria with ε_q (single amplitude) equal to 2.5 %, 5.0 % or 8.75% (the latter being equivalent to the typical $\varepsilon_z = 5\%$ double-amplitude axial strain criterion). Results reported in Figs. 3a and 3b are consistent with those reported for anisotropically-consolidated Ottawa sands at around the same density states by Vaid and Chern (1983).

Effect of Principal Stress Rotation

The effect of α on the undrained cyclic response and liquefaction resistance of Ottawa sand was evaluated through a series of tests with $b = 0$ and $\alpha = 30^\circ$ or 60° . Target α values were achieved as described above and kept constant during undrained cyclic loading. These tests were conducted at the same pre-shearing p' ($= 100$ kPa) with CSR values ranging from 0.06 to 0.10.

The pore pressure change and deviatoric strain responses of Ottawa sand during undrained cyclic loading with CSR = 0.10 and $\alpha = 30^\circ$ and 60° are shown in Fig. 3c and 3d, respectively. Increasing α from 0° to 60° (Figs. 3b-d) (1) reduces the number of cycles required to induce pore pressure changes that lead to $\varepsilon_q = 8.75$ % (a liquefaction criterion that is never mobilized within 20 cycles for $\alpha = 0^\circ$), and (2) increases ε_q at 20 cycles. Similar behavior (i.e., limited flow liquefaction followed by cyclic mobility) is also observed for both $\alpha = 30^\circ$ and $\alpha = 60^\circ$ with CSR = 0.08 (not shown in Fig. 3, refer to Table 3).

Because the inherent slurry deposition fabric and any subsequent changes induced by anisotropic consolidation leads to similar post-consolidation fabric (created under vertical σ_1), the responses in Fig. 3b-d result from principal stress rotation and the subsequent undrained cyclic loading that follows at constant α . The contractive fabric in the axial direction prior to undrained cyclic loading for the specimen with $\alpha = 30^\circ$ (Fig. 3c) may explain its sudden contractive tendency and pore pressure changes generated during undrained cyclic loading. Conversely, the dilative axial response of the specimen subjected to principal stress rotation with $\alpha = 60^\circ$ may explain its relatively lower pore pressure change build up compared to its counterpart with $\alpha = 30^\circ$ during cyclic loading.

The effect of principal stress rotation on the liquefaction resistance of Ottawa sand is summarized in Fig. 6. Each data point represents the number of cycles required to mobilize $\epsilon_q = 2.5\%$, 5.0% , or 8.75% at a given CSR. Increasing α systematically from 0° to 60° significantly reduces the liquefaction resistance of Ottawa sand for all three liquefaction criteria (Fig. 6a, 6b, and 6c) with more pronounced reduction when α increases initially from 0° to 30° (Fig. 6).

A more comprehensive description of the cyclic resistance ratio (CRR) of Ottawa sand as a function of α and b is summarized in Fig. 7. In this plot CRR represents the CSR required to reach one of the aforementioned strain-based liquefaction criteria at 20 cycles. Each point in Fig. 6 requires the determination of a full liquefaction resistance curve (Fig. 6). All CRR values shown in Fig. 7 are normalized by the corresponding axisymmetric CRR (with $\alpha = 0^\circ$ and $b = 0$). The same overall trend is observed for all plots, where CRR significantly decreases with increasing α . The CRR decreases by as much as 80 % when α

is increased to 30° , with no significant additional changes in CRR observed as α increases further to 60° .

Undrained cyclic response and liquefaction resistance of Ottawa sand are critically affected by its anisotropic fabric that derives from underwater deposition followed by anisotropic consolidation (Figs. 5 through 7). As the major principal stress rotates from the vertical towards the horizontal direction, Ottawa sand exhibits increasingly weaker undrained cyclic behavior. This suggests that K_0 -consolidated loose to medium-dense uniform sands deposited under water (a deposition mode that is extremely common in alluvial, fluvial, offshore and tailings deposits) are critically sensitive to drained principal stress rotation. The liquefaction resistance and undrained cyclic response of such deposits may be much weaker than those determined from conventional undrained cyclic triaxial tests even under small cyclic disturbances ($CSR < 0.10$). Liquefaction events associated with such deposits may be sudden due to undrained instability and principal stress rotation. Such deposits may exhibit highly unstable behavior during undrained cyclic loading depending upon the amount of principal stress rotation (α) and shaking magnitude (CSR) induced, in addition to more typical state variables (D_R and p') commonly used in engineering practice.

Effect of Intermediate Principal Stress (σ_2) Changes

The effect of changes in σ_2 (or b) on the undrained cyclic response and liquefaction resistance of Ottawa sand was evaluated in a series of undrained cyclic tests with b kept constant at 0, 0.5 or 0.8 and $\alpha = 0^\circ$ and $p' = 100$ kPa.

The pore pressure change and deviatoric strain responses of three Ottawa sand specimens subjected to undrained cyclic loading with $b > 0$ and CSR between 0.09 and 0.52 were analyzed. Only one specimen (Fig. 3e) subjected to undrained cyclic loading with $b = 0.5$ and CSR = 0.52 exhibits temporary undrained instability (UI) behavior. All other specimens exhibit cyclic mobility. The specimen subjected to undrained cyclic loading with $b = 0.8$ and CSR = 0.55 (Fig 3f) experiences a steady increase of excess pore pressure with a progressively decreasing rate, in contrast to the quick, temporary UI behavior displayed by the specimen with $b = 0.5$ (Fig 3e). This difference in UI behavior may be due to b and/or the difference in D_r between the specimens tested (43% versus 51%). For other tests with lower CSRs (results not shown here), no discernible differences were observed between the UI behavior (cyclic mobility type behavior) of specimen sheared with $b = 0.5$ (CSR = 0.26, $D_r = 41\%$) and that of specimen sheared with $b = 0.8$ (CSR = 0.24, $D_r = 44\%$).

The liquefaction resistances for the axisymmetric case ($b = 0$ and $\alpha = 0^\circ$) lie systematically below the liquefaction resistance curves for higher b values ($= 0.5$ and 0.8) (Fig. 8).

The overall variation of the normalized CRR ratio defined previously ($CRR/CRR_{\alpha=b=0}$) as a function of b is given in Fig. 7. Increases in b from zero to 0.5 or 0.8 at $\alpha = 0^\circ$ increases the normalized CRR of Ottawa sand regardless of the liquefaction criterion adopted, although the rate of such increases depends on the liquefaction criterion used, generally decreasing with increasing ϵ_q . Increases in b from zero to 0.5 or 0.8 increases the CRR of Ottawa sand by 1.5 to 2.8 or 2.0 to 3.8 times, respectively.

Fig. 7 and 8 show that the liquefaction resistance of Ottawa sand with $\alpha = 0^\circ$ systematically increases with an increase in b . Bolton (1986) and Shibuya et al. (2003) showed that the drained monotonic plane-strain peak strength of sand is higher than its axisymmetric strength. This implies that when b is increased to 0.3 – 0.5 at $\alpha = 0^\circ$ (plane strain conditions) the resulting drained monotonic strength is larger than the strength observed for $b = 0$ and $\alpha = 0^\circ$. Ladd et al. (1977) also arrived at the same conclusion for both friction angle and stiffness. A corollary might be that relatively lower liquefaction resistance would be obtained from cyclic triaxial tests with $\alpha = 0^\circ$ and $b = 0$ compared to those related to in situ plane strain conditions with $\alpha = 0^\circ$ and $b > 0$. Therefore, liquefaction design based on cyclic triaxial test results may be more conservative considering the plane strain conditions observed in the field. The findings shown in Figs. 7 and 8 confirm this conclusion, albeit only when $\alpha = 0^\circ$. However, it is important to point out that Sayao and Vaid (1996) compiled results to illustrate the effect of b on the friction angle, and some results suggested decrease in friction angle when b was increased from 0.5 to 1. Yoshimine et al. (1998) reported that increase in b resulted in higher pore water pressure during undrained shearing of sands. Therefore, further studies may be necessary on the effect of b on undrained shearing resistance of sands.

Effect of Simultaneous Principal Stress Rotation & Intermediate Principal Stress Change

Increasing α from 0° to either 30° or 60° reduces the liquefaction resistance of Ottawa sand (Fig. 6). Conversely, increasing σ_2 (or b) increased the liquefaction resistance of Ottawa sand (Fig. 8). The combined effect of α and b changes on the liquefaction resistance of Ottawa sand was assessed through an additional series of twelve cyclic hollow cylinder

tests with both α and b maintained constant at values greater than zero during undrained cyclic loading.

Comparing at the same CSR level allows delineating the relative effects of α and b on the undrained cyclic response.

The specimen subjected to undrained cyclic loading with $\alpha = 30^\circ$, $b = 0.5$ and CSR = 0.10 experiences temporary undrained instability after five cycles (Fig. 9a). This response is in accordance with the limited flow liquefaction followed by cyclic mobility, which was also observed for the specimen subjected to undrained cyclic loading with $\alpha = 30^\circ$, $b = 0$, and CSR = 0.10 (Fig. 3c) after 3 cycles. The difference in number of cycles to reach temporary undrained instability between these two specimens might be due to their relative density difference and/or the effect of b . Thus, a direct conclusion regarding the effect of b is difficult to make.

Increasing b to 0.5 with $\alpha = 30^\circ$ and CSR = 0.10 does not prevent temporary undrained instability despite the higher D_R of the specimen with $b = 0.5$. On the other hand, a further increase in b to 0.8 at $\alpha = 30^\circ$ and CSR = 0.09 eliminates the sudden effect of temporary undrained instability from specimen response (Fig. 8b).

At $\alpha = 60^\circ$ and CSR = 0.09 or CSR = 0.10, specimens subjected to $b = 0.5$ (Fig. 9c) and $b = 0.8$ (Fig. 9d) experience temporary undrained instability with large sudden increase in deviatoric strain during the first few cycles after which the deviatoric strain steadily increases at a lower rate. Increasing b from 0 to 0.5 or 0.8 when $\alpha = 60^\circ$ does not fundamentally alter the response.

The combined effect of simultaneous α and b changes on the liquefaction resistance of Ottawa sand is summarized in Fig. 10. The $\epsilon_q = 8.75\%$ liquefaction criterion is not included

in Fig. 10, since most specimens required large number of cycles (>1000) to reach $\varepsilon_q = 8.75$. For the $\varepsilon_q = 2.5\%$ and $\varepsilon_q = 5.0\%$ liquefaction criteria, the normalized $CRR/CRR_{\alpha=0, b=0}$ curves and liquefaction resistance curves are given in Figs. 7 and 10, respectively.

When α increases from 0° to 60° , the $CRR/CRR_{\alpha=b=0}$ ratio decreases significantly regardless of b . The effect of b on the $CRR/CRR_{\alpha=b=0}$ ratio is negligible when $\alpha = 60^\circ$ and marginal when $\alpha = 30^\circ$. For all b , the reduction in $CRR/CRR_{\alpha=b=0}$ ratio is more pronounced when α increases from 0° to 30° , compared to the reduction for additional α increases between 30° and 60° . The increase in the $CRR/CRR_{\alpha=b=0}$ ratio as a function of b is highest when $\alpha = 0^\circ$ but diminishes when $\alpha > 0^\circ$. When both α and b increase, the $CRR/CRR_{\alpha=b=0}$ ratio generally becomes less than unity. The only exception to this is the $\alpha = 30^\circ$ and $b = 0.8$ case for the $\varepsilon_q = 2.5\%$ liquefaction criterion, when the $CRR/CRR_{\alpha=b=0}$ ratio increases slightly above 1 (Fig. 7a).

For $\alpha = 0^\circ$, b increases can almost quadruple the $CRR/CRR_{\alpha=b=0}$ ratio (Fig. 7a). But as soon as α is increased to 30° with simultaneous increase in b , the drastic improvement in $CRR/CRR_{\alpha=b=0}$ ratio due to b increase is suppressed, showing the dominant effect of α . At $\alpha = 30^\circ$, increasing b still increases the $CRR/CRR_{\alpha=b=0}$ ratio albeit at a much lower rate than that observed for $\alpha = 0^\circ$. At $\alpha = 60^\circ$, the effect of b on the $CRR/CRR_{\alpha=b=0}$ ratio is eliminated. These findings indicate that when α and b are increased simultaneously, the liquefaction resistance of Ottawa sand is governed by principal stress rotation.

CONCLUSIONS

Temporary excursions through undrained instability were observed in many of the undrained cyclic tests conducted in this study. Increases in CSR and α raises the likelihood of elements experiencing undrained instability. Conversely, increases in b and D_R leads to

more stable responses reducing the likelihood of undrained instability. Most specimens undergoing undrained instability displayed a deformation pattern resembling limited flow liquefaction followed by cyclic mobility (instead of classical undrained instability). But for CSR levels lower than 0.06, undrained instability did not take place and both the pore pressure change and deformation responses of Ottawa sand displayed cyclic mobility. Within the loading conditions and D_R ranges studied, no Ottawa sand specimen experienced full liquefaction ($p' = 0$).

Increasing α from 0° to 30° with $b = 0$ significantly reduced the liquefaction resistance of Ottawa sand for all three liquefaction criteria used. Further α increase from 30° to 60° induces further reduction in liquefaction resistance, but this reduction was not as pronounced as the reduction observed initially when α increased from 0° to 30° . As the major principal stress rotates from the vertical to the horizontal direction, Ottawa sand exhibits a weaker undrained cyclic response.

Increases in b from 0 to 0.5 or 0.8 with $\alpha = 0^\circ$ increased the liquefaction resistance (or CRR) of Ottawa sand regardless of the liquefaction criterion used. However, the amount of increase in CRR appears to depend on the liquefaction criterion selected. Increase in b from 0 to 0.5 or 0.8 increased the CRR of Ottawa sand by 1.5 to 2.9 or 2.0 to 3.8 times, respectively, compared to the CRR obtained with $b = 0$. Therefore, undrained cyclic triaxial tests may yield conservative liquefaction resistances for Ottawa sand in situations when the intermediate principal stress is between the major and minor principal stresses. However, this is only true when $\alpha = 0^\circ$.

The CRR of Ottawa sand decreases significantly when α is increased from 0° to 60° , regardless of b . Additionally, for all b values used, the reduction in the normalized CRR

ratio, which is defined as the ratio of the CRR for a given combination of α and b to the axisymmetric CRR (with $\alpha = 0^\circ$ and $b = 0$), is more pronounced when α increased from 0° to 30° , compared to further α increases from 30° to 60° . Increase in b significantly increased the CRR of Ottawa sand for $\alpha = 0^\circ$, but this strengthening effect due to b is significantly suppressed for $\alpha > 0^\circ$. The undrained cyclic response of Ottawa sand is primarily controlled by principal stress rotation when both α and b are greater than zero.

Analyses of soil elements subjected to stresses induced by foundations, embankments (or slopes) show that both α and b are typically greater than zero prior to earthquake loading (Zdravkovic and Jardine 2001). Similarly, upstream tailings dam construction, natural sloping ground and cyclic loading induced by earthquakes represent typical scenarios encountered in practice that induce non-trivial loading conditions to soil elements in the ground/embankment where α and b may be greater than zero. Therefore, liquefaction analyses for these applications require rigorous understanding of the effect of simultaneous α and b changes. Cyclic triaxial tests with $\alpha = 0^\circ$ and $b = 0$ may not reflect the true behavior of in situ materials, and may not always provide conservative estimates of liquefaction resistance (not considering *in situ* bi-directional loading effects). The effect of α changes is more severe and influential than that of b . When both parameters are greater than zero, the liquefaction resistance of Ottawa sand will typically decrease compared to values obtained for the axisymmetric case ($\alpha = 0^\circ$ and $b = 0$) due to the predominant effect of α . Thus, b values in the range of 0.3 – 0.5, which approximates plane strain conditions, may not ensure higher liquefaction resistance. The combined effect of α and b must be considered in any rigorous analysis of liquefaction resistance of sands to improve the design and resilience of civil engineering infrastructure

Results of this study in combination with finite element models delineating true three dimensional stress conditions can be useful to assess the soil behavior under three dimensional stress conditions.

DATA AVAILABILITY STATEMENT

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

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639 **APPENDIX A.**

640 Average vertical (or axial) stress: $\sigma_z = \frac{W}{\pi \cdot (r_o^2 - r_i^2)} + \frac{p_o r_o^2 - p_i r_i^2}{r_o^2 - r_i^2}$

641 Average radial stress: $\sigma_r = \frac{p_o r_o + p_i r_i}{r_o + r_i}$

642 Average circumferential stress: $\sigma_\theta = \frac{p_o r_o - p_i r_i}{r_o - r_i}$

643 Average shear stress: $\tau_{z\theta} = \frac{3 \cdot T}{2\pi \cdot (r_o^3 - r_i^3)}$

644 Average vertical (or axial) strain: $\varepsilon_z = \frac{\Delta H}{H}$

645 Average radial strain $\varepsilon_r = -\frac{l_o - l_i}{r_o - r_i}$

646 Average circumferential strain: $\varepsilon_\theta = -\frac{l_0 + l_i}{r_0 + r_i}$

647 Average shear strain: $\gamma_{z\theta} = \frac{2\theta \cdot (r_0^3 - r_i^3)}{3H \cdot (r_0^2 - r_i^2)}$

648 Major principal stress: $\sigma_1 = \frac{\sigma_z + \sigma_\theta}{2} + \sqrt{\left(\frac{\sigma_z - \sigma_\theta}{2}\right)^2 + \tau_{z\theta}^2}$

649 Intermediate principal stress: $\sigma_2 = \sigma_r$

650 Minor principal stress: $\sigma_3 = \frac{\sigma_z + \sigma_\theta}{2} - \sqrt{\left(\frac{\sigma_z - \sigma_\theta}{2}\right)^2 + \tau_{z\theta}^2}$

651 Major principal strain: $\varepsilon_1 = \frac{\varepsilon_z + \varepsilon_\theta}{2} + \sqrt{\left(\frac{\varepsilon_z - \varepsilon_\theta}{2}\right)^2 + \gamma_{z\theta}^2}$

652 Intermediate principal strain: $\varepsilon_2 = \varepsilon_r$

653 Minor principal strain: $\varepsilon_3 = \frac{\varepsilon_z + \varepsilon_\theta}{2} - \sqrt{\left(\frac{\varepsilon_z - \varepsilon_\theta}{2}\right)^2 + \gamma_{z\theta}^2}$

654 Major principal stress direction from vertical: $\alpha = \frac{1}{2} \tan^{-1} \left(\frac{2\tau_{z\theta}}{\sigma_z - \sigma_\theta} \right)$

655 σ_v = vertical stress

656 $\sigma_\square$ = horizontal stress

657 W = vertical load;

658 T = torque about vertical axis;

659 p_i and p_o = inner and outer cell pressures, respectively;

660 r_i and r_o = inner and outer hollow cylinder specimen radii, respectively;

661 H = specimen height, and ΔH = change in specimen height;

662 l_i and l_o = inner and outer surface displacements, respectively;

663 θ = rotation angle.

Table 1. Basic properties of tested Ottawa Sand.

Mean grain size, $D_{50}(\text{mm})$	0.40
Coefficient of uniformity, C_u	1.7
USCS Classification	SP
Specific gravity, G_s	2.65
$e_{\max}$ (ASTM D 4254-Method B)	0.495
$e_{\min}$ (ASTM D 4253-Method 1A)	0.767
SiO_2 (%)	99.7
Fe_2O_3 (%)	0.02
Al_2O_3 (%)	0.06

Table 2. Transducers used in the cyclic hollow cylinder apparatus.

Component	Capacity	Resolution	Calibration Parameters	
			Maximum Error	Calibration Range
Vertical Actuator Displacement Transducer	50 mm	0.1 mm	0.58 mm	48.26 mm
Horizontal Actuator Displacement Transducer	100 mm (± 50 mm) (90°)	0.1 mm (0.09°)	1.23 mm	42.58 mm
Vertical Load Cell	10 kN	1 N	0.084 kN	7.3 kN
Torque Load Cell	300 Nm	0.1 Nm	0.14 Nm	12.6 Nm
Pressure Transducers	1000 kPa	1 kPa	11.3 kPa	665 kPa
Volume Change Transducers	100 ml	0.1 ml	0.88 ml	98 ml

Table 3. Summary of Tests

Test #	α (deg)	b	CSR	Undrained instability (UI) mobilized?	Number of cycles at UI	q at UI (kPa)	p' at UI (kPa)	Relative density (%)		
								Initial	End of anisotropic consolidation	End of α and/or b stage
1	0	0	0.60	Yes	0.10	113	109	46	48	-
2	0	0	0.30	Yes	0.2	115	106	50	53	-
3	0	0	0.20	Yes	0.4	112	102	39	42	-
4	0	0	0.14	Yes	2.3	104	93	40	43	-
5	0	0	0.10	No				47	48	-
6	30	0	0.10	Yes	3.3	103	100	38	40	41
7	30	0	0.08	Yes	14.5	95	94	37	40	41
8	30	0	0.06	Yes	44.2	86	93	46	48	49
9	60	0	0.10	Yes	0.3	99	100	38	39	43
10	60	0	0.08	Yes	0.3	103	108	44	48	50
11	60	0	0.06	No	27.3	99	97	46	48	50
12	0	0.5	0.52	Yes	0.1	114	108	40	42	43
13	0	0.5	0.48	No				49	51	51
14	0	0.5	0.40	No				47	50	52
15	0	0.5	0.26	No				40	41	41
16	0	0.5	0.09	No				51	54	55
17	0	0.8	0.55	No				46	49	51
18	0	0.8	0.37	No				43	44	45
19	0	0.8	0.24	No				41	43	44
20	0	0.8	0.09	No				50	52	55
21	30	0.5	0.17	Yes	0.2	97	98	41	44	45
22	30	0.5	0.10	Yes	5.2	89	91	52	56	58
23	30	0.5	0.07	No				43	45	47
24	30	0.8	0.20	Yes	0.1	100	101	50	51	53
25	30	0.8	0.15	No				52	53	55
26	30	0.8	0.09	No				49	51	53
27	60	0.5	0.09	Yes	1.1	87	104	49	51	54
28	60	0.5	0.07	Yes	1.2	91	106	51	55	57
29	60	0.5	0.05	No				41	45	49
30	60	0.8	0.09	Yes	0.2	96	107	41	45	47
31	60	0.8	0.06	No				45	48	51
32	60	0.8	0.07	No				45	48	51

(a)

(b)

Figure 1. (a) Typical geometry and loading of hollow cylinder specimen, (b) stress and strain components on a soil element in the wall of a hollow cylinder specimen, (c) applied principal stresses on a soil element in the wall of a hollow cylinder specimen (after Hight et al. 1983 and Tastan and Carraro 2013).

Figure 2. Strain response of Ottawa sand specimens subjected to drained principal stress rotation to (a) $\alpha = 30^\circ$, and (b) $\alpha = 60^\circ$ (D_R at the end of anisotropic consolidation)

Figure 3. Pore pressure change, deviatoric and/or axial strain responses of Ottawa sand under undrained cyclic loading with (a) $\alpha = 0^\circ$, $b = 0$, $CSR = 0.20$, (b) $\alpha = 0^\circ$, $b = 0$, $CSR = 0.10$, (c) $\alpha = 30^\circ$, $b = 0$, $CSR = 0.10$, (d) $\alpha = 60^\circ$, $b = 0$, $CSR = 0.10$, (e) $\alpha = 0^\circ$, $b = 0.5$, $CSR = 0.52$, and (f) $\alpha = 0^\circ$, $b = 0.5$, $CSR = 0.55$ (D_R before cyclic shearing).

Figure 4. Locus of Undrained Instability (UI) points, critical state line for compression (M_c), and critical state line for extension (M_e)

Figure 5. Axisymmetric liquefaction resistance curves for anisotropically consolidated Ottawa sand. Failure defined for deviatoric strain (ϵ_q) equal to 2.5 %, 5 % and $\epsilon_q = 8.75$ % ($\epsilon_q = 8.75$ % equivalent to 5% axial strain, and D_R before cyclic shearing).

Figure 6. Effect of principal stress rotation on the liquefaction resistance curves of Ottawa sand for (a) $\varepsilon_q = 2.5\%$, (b) $\varepsilon_q = 5\%$, and (c) $\varepsilon_q = 8.75\%$ (D_R before cyclic shearing).

Figure 7. Normalized CRR as a function of α and b for (a) $\varepsilon_q = 2.5\%$, and (b) $\varepsilon_q = 5.0\%$, (c) $\varepsilon_q = 8.75\%$ (D_R before cyclic shearing).

Figure 8. Liquefaction resistance curves of Ottawa sand for (a) $\varepsilon_q = 2.5\%$, (b) $\varepsilon_q = 5\%$, and (c) $\varepsilon_q = 8.75\%$ (D_R before cyclic shearing).

Figure 9. Pore pressure change and deviatoric strain responses of Ottawa sand specimens cyclically sheared with (a) $\alpha = 30^\circ$, $b = 0.5$, $CSR = 0.10$; (b) $\alpha = 30^\circ$, $b = 0.8$, $CSR = 0.09$; (c) $\alpha = 60^\circ$, $b = 0.5$, $CSR = 0.09$; and (d) $\alpha = 60^\circ$, $b = 0.8$, $CSR = 0.09$ (D_R before cyclic shearing).

Figure 10. Liquefaction resistance curves of Ottawa sand for various α and b as (a) $\varepsilon_q = 2.5\%$, and (b) $\varepsilon_q = 5.0\%$ (D_R before cyclic shearing).