Adaptive MPC for Fire Incidents in Water Distribution Networks

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Abstract

Water utilities in UK aim in network pressure reduction as a means to reduce leakage, in order to meet regulatory targets and avoid financial penalties. At the same time, however, they are not legally bound to guaranty specific fire-flow rates at fire hydrants, thus posing a potential a risk to fire fighting. In the exiting literature, fire-flows in water distribution networks have been considered primarily with respect to design, vulnerability and capacity analysis, while control of pumps and valves has focused mainly on minimizing energy costs (and to a lesser extent pressure) under normal operating conditions. This study presents a (non-linear) adaptive model predictive control methodology, for water distribution networks, combining two separate modes of control: 1) Normal Control, when the network operates under normal conditions and the objective is to minimize energy costs and average zonal pressure. 2) Fire Control, when a fire incident occurs and fire-flows need to be delivered at hydrant nodes while, to the furthest possible extent, resume delivery of customer demand without over-pressuring the network. The proposed methodology is applied on an operational network from UK.

1 Introduction

In England and Wales, privately owned water utilities have to meet various operational targets, set by the regulatory service (Ofwat), or otherwise face potential financial penalties. These operational targets include the reduction of background leakage to acceptable levels. As a means to reduce leakage, water utilities aim in network pressure reduction, while ensuring uninterrupted customer supply. At the same time, apart from non-legally binding guidelines ([2]), they are not required to guaranty specific fire-flow rates at fire hydrants, which will be negatively affected by the pressure reduction. This financial motivation for water utilities to reduce network pressure, without any legal obligations for fire-flows, constitutes a potential risk to urban fire fighting.

In the existing literature, fire-flows in water distribution networks (WDNs) have been considered mainly with respect to design, vulnerability and capacity analysis (for example [9, 17, 12, 13, 4, 40]). Placement and control of gate-valves (open/close) for better fire-fighting, in a sectorized network, has been considered in [6].

On the other hand, control of pumps and valves in WDNs (see [20] for a review) has focused mainly on minimizing energy costs and to a lesser extent pressure, under normal operating conditions (an exception is [18] where control during limited electrical energy availability is considered). Many approaches have been proposed in order to solve this MINLP problem, either globally (for example [31]) or with various approximate formulations (for example [11, 21, 10]). The main drawback of such methods is with respect to scalability, since operational networks are usually large and include several pumping stations.

A number of approaches, for the control of pumps and valves in WDNs, have been proposed under the framework of Model Predictive Control (MPC) also known as Receding Horizon Control (RHC) (see for example [29, 23, 16, 14, 28, 15, 36, 37, 27, 35]). Although not guaranteeing global optimality with respect to the MINLP problem, the MPC methodology is computationally faster (thus more scalable) and better suited for dealing with uncertainties compared to approaches mentioned in the previous paragraph (such as [11, 31, 21, 10]), since the control can be adjusted at each timestep. Some specific examples from the MPC literature include: Grosso et al. [14], where a chance-constraint MPC approach for the modelling of demand uncertainty is presented. Wang et al. [37], where a two-layer non-linear MPC approach is proposed. In [27], Sampathirao et al. develop a GPU-accelerated stochastic MPC method, considering mass-balance hydraulics. In [35], Wang et al. apply MPC with a geometric programming formulation along with a binary search heuristic.

Motivated by the above, this paper presents a (non-linear) adaptive model predictive control methodology, for WDNs, combining two separate modes of control: 1) **Normal Control**, when the network operates under normal conditions and the objective is to minimize energy costs and average zonal pressure (for leakage minimization [19]). 2) **Fire Control**, when a fire incident occurs and fire-flows need to be delivered at hydrant nodes while, to the furthest possible extent, resume delivery of customer demand without over-pressuring the network. To the best of the authors knowledge, this is the first time that the problem of real-time fire control of WDNs is considered. The proposed methodology could potentially serve water utilities both as a tool for control and as a tool for testing, planning and prevention.

The paper proceeds as follows: In Section 2, we outline the proposed MPC scheme. Next, in Section 3, we explain our modelling choice for pumps and valves as well as the heuristic which we use for the mapping of the (continuous) control of

pumps to the (integer) on/off state. In Sections 4 and 5, we present the optimization formulations for the normal and fire control respectively. In Section 6, we present the results from the application of the proposed methodology to an operational WDN from UK. Finally, we draw our conclusions in Section 7.

2 Adaptive MPC Scheme

In this section we outline the proposed scheme for the normal and fire control of a water supply network. As mentioned in the introduction, we follow a model predictive / receding horizon control approach. At each timestep, we solve a non-linear optimization problem for the control of pumps and valves over a certain horizon (i.e. multiple timesteps). Next, only the control of the first timestep is applied/simulated. The resulting tank levels are used as the new initial state of the system and the whole process repeats (with the horizon shifting one step forward).

As shown in Figure 2, two different optimization problems (NLPs) denoted by, **NC** for the Normal Control and **FC** for the Fire Control, are considered for each mode of operation respectively, with each mode having different hydraulic model, objectives and horizon length.

Figure 1: Adaptive MPC scheme

During normal control, we consider the minimization of energy costs, due to pumping operations, and average zonal pressure (AZP) as a way for reducing background leakage. Under normal operating conditions we expect that the minimum pressures required to satisfy the customer demand can be achieved. Therefore, as is the case with the existing literature, we use a demand-driven model for the formulation of **NC**.

During a fire incident, the control switches to the solution of **FC** and the goal is to provide a certain amount of total flow, through a number of specified fire hydrants. At the same time, we want to continue delivery of customer demand, to the extent that is possible, but without over-pressurising the network. Nevertheless, the use/opening of the fire hydrants can potentially cause significant pressure reduction at multiple nodes in the network. Consequently, the pressure drop may result in reduced demand supply. A demand-driven model would not “capture” this effect, since the demand supply is assumed to be independent of pressure. Thus, for the formulation of **FC**, we use a pressure-driven model.

For the normal control, the horizon length, H_{NC} , would normally be constant and ideally (although not necessarily) span a 24-hour period, considering the “periodicity” of the demand and electricity tariff from one day to the next. For the fire control, the choice of the horizon length, H_{FC} , would depend on the predicted duration of the fire incident (normally expected to be much shorter than H_{NC}). Moreover, during fire control, we might vary the length of the horizon period at each subsequent iteration (for example, reduce the horizon, as we approach the end of the incident), especially if the initial horizon was relatively long. However, such a case is not considered in this work.

Finally, we should note that, the economic control of WDNs makes sense in the long term. Therefore, it is sensible to neglect the energy costs for the duration of a fire incident, which is an infrequent event with a duration of a several hours, and

focus on the delivery of the required fire-flows. The same is true for background leakage reduction (and thus minimization of AZP), however we would still need to consider nodal pressure bounds in order to avoid pipe bursts during fire control.

3 Pumps, valves and heuristic

In this section we describe the modelling of pumps and valves (used in the formulation of problems **NC** and **FC**) and the heuristic we use in order to map the continuous control of (fixed-speed) pumps to on/off status. We consider, only fixed-speed pumps. Nevertheless, the problem formulations can be expanded in order to include variable speed pumps. Furthermore, the valves considered are pressure reducing valves (PRVs). The pump modelling and the heuristic differ from the previous literature (for example [37, 35]) and thus we present them in this section for more clarity. The rest of the modelling techniques, for steady-state hydraulics, are well known (see for example [7]) and thus, we will only go into further details in Sections 4 and 5, where the formulations for the normal and fire control are presented respectively.

3.1 Pumps and Valves

For a link k (pump or valve), from node i to node j , the energy balance constraint is expressed as,

$$h_j - h_i = f(q_k) - u_k \quad (1)$$

$$u_k \geq 0, \quad (2)$$

$$q_k \geq 0, \quad (3)$$

where q_k is the flow through link k and h_i, h_j are the heads at nodes i, j respectively.

When link k is a PRV, $f(q_k) = -(a_k|q_k|q_k + b_kq_k)$, where the coefficients a_k and b_k are obtained by a quadratic approximation to the Hazen-Williams head-loss formula (see [3, 24]) and u_k represents the amount of (outlet) head reduced by the PRV. Thus, for a PRV, u_k is a control variable. The flow through a PRV is assumed to be non-negative.

When link k is a pump, $f(q_k) = a_kq_k^2 + b_kq_k + c_k$ is the corresponding pump (characteristic) curve. The coefficients a_k, b_k and c_k are obtained by fitting the curve to available operational data of the pump. In this case, u_k is a slack variable, used for the continuous relaxation of the, otherwise, integer-variable formulation required for the modelling of the on/off status of a pump. As shown in Figure 2, the use of u_k allows for operation points below the pump curve $f(q)$ (so that the continuous optimization formulation can set the pump on/off).

The reason behind the equality constraint formulation (1) (instead of using the convex constraint $h_j - h_i \leq f(q_k)$) is so that we can include the slack variable u_k in a bilinear penalty term, $w_k u_k q_k$ (w_k being a weight value), in the objective, with the aim of forcing the operation of pump to be closer to either the characteristic curve ($u_k = 0$) or to zero flow ($q_k = 0$), resulting in less discrepancy between the optimization-obtained operation points and those realised during simulation, based on the heuristic. Moreover, in the formulations of **NC** and **FC**, the quadratic form of the pump curve is replaced with a linear approximation (and thus constraint (1) becomes convex in practise).

3.2 Heuristic

The mapping to the on/off status of an operation point, $(q, \Delta h)$ obtained from (1)-(3), takes place with the use of the following simple heuristic: The pump is set to “off” when the operation point lies on or below the boundary curve,

$$B(q) = \frac{a}{\rho^2}q^2 + \frac{b}{\rho}q + c. \quad (4)$$

otherwise it is set to “on”. The parameters a, b and c , in (4), are the corresponding coefficients of the pump curve and $\rho \in (0, 1]$ a parameter that shifts the boundary curve to the right as its value increases.

Figure 2: **Fixed speed pump:** $\Delta h = f(q) - u$. The slack $u \geq 0$ allows for operation points below the pump curve $f(q)$. Based on the horizontal distance of the point from $f(q)$, the pump is decided to be on or off, during simulation. The boundary curve, $B(q)$ for $\rho = 0.5$, is depicted with the dashed line.

The heuristic “chooses” between the on/off status by comparing the horizontal distances of the optimization-obtained operation point, $(q, \Delta h)$, from the head gain axis (pump “off”) and the pump curve (pump “on”). An example is shown in Figure 2.

The reasoning behind this choice is as follow: If for the corresponding simulated point, $(q_{sim}, \Delta h_{sim})$, we have $\Delta h_{sim} \approx \Delta h$, then the simulated point is approximated by shifting the point $(q, \Delta h)$ to the left (if the pump was set off) or right (if the pump is set on) until it “falls” on the head axis or the pump curve respectively. Thus, we choose the status which corresponds to the smallest horizontal distance (i.e. smallest difference between optimization and simulation). Note that, when water is pumped from a source (of known head) directly to a tank, we can expect that $\Delta h \approx \Delta h_{sim}$.

For $\rho = 0.5$, as shown in Figure 2, the boundary curve divides equally the area under the pump curve between “on” and “off” status. Depending on the case, it might be preferable to adjust/set $\rho < 0.5$ or $\rho > 0.5$. Nevertheless, in our results we have used $\rho = 0.5$ for all iterations and pumps.

4 Normal control

In this section we present the formulation for the Normal Control (**NC**) and introduce the necessary notation. As mentioned earlier, for **NC**, we make use of a demand-driven, steady-state and discrete-time hydraulic model.

A water supply network can be represented as a directed graph $G(\mathbb{V}, \mathbb{E})$ where the link directions are with respect to the assigned positive flow direction. For a given network we consider, n_p links, n_n junction nodes (of unknown head), n_0 sources (of known head) and n_r tanks. A link might be pipe, a valve, or a pump. The set of vertices, $\mathbb{V}$, represents the nodes of the network with $|\mathbb{V}| = n_n + n_0 + n_r$ being the total number of nodes. The set of edges, $\mathbb{E}$, represents the set of links connecting the nodes with $|\mathbb{E}| = n_p$.

The problem variables consist of the vectors of flows $q_t \in \mathbb{R}^{n_p}$, junction heads $h_t \in \mathbb{R}^{n_n}$, tank heads, $r_t \in \mathbb{R}^{n_r}$ and the vector of pump-slack/PRV-control variables, $u_t \in \mathbb{R}^{n_u}$, where n_u is equal to the total number of pumps and PRVs. In addition, we consider the vector of known source heads, $h_{0,t} \in \mathbb{R}^{n_0}$. The (predicted) demand vector at timestep t is denoted by the vector $d_t \in \mathbb{R}^{n_n}$.

Moreover, we use the following incident matrices in order to form the mass/energy balance constraints and the tank head constraints: link-source $A_0 \in \{-1, 0\}^{n_p \times n_0}$, link-node $A_1 \in \{-1, 0, 1\}^{n_p \times n_n}$, link-tank $A_2 \in \{-1, 0, 1\}^{n_p \times n_t}$ and a link-slack matrix, $A_3 \in \{0, 1\}^{n_p \times n_u}$. Matrix A_1 is defined as,

$$A_1(k, j) = \begin{cases} 1 & \text{if link } k \text{ enters node } j. \\ 0 & \text{if link } k \text{ is not connected to node } j. \\ -1 & \text{if link } k \text{ leaves node } j. \end{cases} \quad (5)$$

Matrices A_0 and A_2 are defined similarly to A_1 . Matrix A_3 is defined as,

$$A_3(k, j) = \begin{cases} 1 & \text{if link } k \text{ is an FSP/PRV with} \\ & \text{assigned slack/control variable } u_j. \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

We also define the diagonal matrix $D_{tanks} \in \mathbb{R}^{n_r \times n_r}$, $D_{tanks} = \text{diag}(\Delta t / \text{Area}_1, \dots, \Delta t / \text{Area}_{n_t})$ where Δt is the duration of the timestep in seconds and Area_i is the watered cross-sectional area of tank i (tanks are assumed to be of cylindrical shape).

The main objectives of **NC** are the minimization of energy costs and average zonal pressure (AZP). The energy cost of an operating (fixed-speed) pump is calculated as,

$$\text{Cost}(q) = \frac{\gamma q \Delta h(q)}{\eta(q)} \Delta t c_t (\mathcal{L}) \quad (7)$$

where $\gamma = 9.81$ (kN/m^3), $\eta(q)$ is the efficiency of the pump (when operating at q), Δt is the duration of the timestep in hours and c_t ($\mathcal{L}/kWh$) is the electricity tariff during timestep t . For the optimization formulation we use a linear approximation of (7), and after assuming that the pump operates at a constant efficiency. Nevertheless, for the calculation of the actual energy costs, after simulation, equation (7) was used.

Average zonal pressure (AZP) is a weighted average of junction pressures. The weighting is with respect to the total length of the pipes connected to each junction. Minimization of AZP is used as a means of reducing background leakage, pipe damage and bursts ([19]). It is calculated as,

$$\text{AZP}(h) = \sum_{i=1}^{n_n} \frac{\sum_{k \in J(i)} L_k}{2 \sum_{k=1}^{n_p} L_k} (h_i - e_i) \quad (8)$$

where L_k is the length of link k , $J(i)$ is the set of the link indices of links that are connected to junction i and e_i is the elevation at junction i .

In addition to energy costs and AZP, we consider the inclusion of bilinear penalty terms, $w_{it}u_{it}q_{it}$, for the operation of pump i at timestep t , with w_{it} being a weight. These penalty terms aim in avoiding “unrealistic” operation of the pumps, as a consequence of the continuous relaxation introduced by constraints (1)-(2) in Section 3. If necessary, further objectives can be considered such as, smooth operation of pumps (such as in [37]), pressure variation and others. Based on the above, the formulation of **NC** is as follows:

Normal Control (**NC**):

$$\min \quad w_{cost} \sum_{i \in I_p, t} \text{Cost}(q_{it}) + \frac{w_{azp}}{H_{NC}} \sum_t \text{AZP}(h_t) + \sum_{i \in I_p, t} w_{it}u_{it}q_{it} \quad (9)$$

$$\text{s.t.} \quad A_0 h_{0t} + A_1 h_t + A_2 r_{t-1} = F(q_t) - A_3 u_t \quad (10)$$

$$A_1^T q_t = d_t \quad (11)$$

$$r_t - r_{t-1} = D_{tanks} A_2^T q_t \quad (12)$$

$$q_{min} \leq q_t \leq q_{max} \quad (13)$$

$$h_{min} \leq h_t \leq h_{max} \quad (14)$$

$$r_{min} \leq r_t \leq r_{max} \quad (15)$$

$$u_{min} \leq u_t \leq u_{max} \quad (16)$$

$$t = 1, \dots, H_{NC}. \quad (17)$$

Constraints (10)-(11) describe the energy and mass balance respectively. Constraints (12) describe the change of tank head levels during each timestep. Variable bounds are set by constraints (13) - (16). The tank level bounds, r_{min} and r_{max} , correspond not to the actual limits of the tanks, but rather on narrower, safety limits, which we aim for during network operation. If needed, slack variables to the tank levels constraints can be considered, allowing small deviations from the safety limits, but with a corresponding penalty. The set I_p , appearing as a subscript in the summation terms of the objective, is the set of the pump link indices. Finally, the components of the vector function $F(q_t)$, in constraints (10), are as defined by equation (1) in Section 3 (the energy balance for a simple pipe link k , i.e. not a pump or PRV, is given by equation (1) with $f(q_k) = -(a_k|q_k|q_k + b_k q_k)$, $u_k = 0$ and $q_{k_{min}} \leq q_k \leq q_{k_{max}}$).

5 Fire control

As explained in Section 2, during a fire incident we adapt the control to the new conditions by “switching” to a pressure-driven hydraulic model, with the main objective being the delivery of the required amount of fire-flow while maintaining, as much as possible, customer delivery. Now, the delivered demand, $\phi(h)$, depends on the nodal pressure/head and is modelled as proposed in [33]:

$$\phi(h) = \begin{cases} 0, & h \leq h_m \\ d\sqrt{\frac{h-h_m}{h_s-h_m}}, & h_m < h \leq h_s \\ d, & h > h_s \end{cases} \quad (18)$$

where h_m is the minimum service head, h_s the service head and d the customer demand. In our experiments we set, $h_m = e + 10$ and $h_s = h_m + 10$, where e is the elevation of the node. However, the formulation of $\phi(h)$ is non-differentiable and thus, for the optimization formulation of **FC**, we use a logistic approximation (see also [8] for more modelling choices):

$$\psi(h) = \frac{d}{1 + e^{\alpha - \beta h}} \quad (19)$$

where α and β are parameters determined by fitting $\psi(h)$ to $\phi(h)$. An example of the graph of $\phi(h)$ and its approximation $\psi(h)$, is shown in Figure 3 (for the pressure-driven simulations we make use of (18)).

Figure 3: Pressure-depended demand supply.

The flow, μ (m^3/s), through a fire hydrant is modelled with the orifice equation as

$$\mu = K_v \sqrt{h - e} \quad (20)$$

where K_v is the discharge coefficient and with h and e being the head and elevation, respectively, at the fire hydrant. In our case study we assume $K_v = 0.006 \left(\frac{m^3}{s\sqrt{m}} \right)$ for all fire-hydrants.

In addition to the notation introduced in the previous section, we now expand the notation as follows: With $\Psi(h_t) = [\psi_1(h_{1t}), \dots, \psi_{n_n}(h_{n_nt})]^T$ we denote the vector of pressure-depended demand supply, during timestep t . With μ_t the vector of fire-hydrant flows. For convenience we take $\mu_t \in \mathbb{R}^{n_n}$ and $K_v \in \mathbb{R}^{n_n}$, with $K_v(i) = 0$ if junction node i is not a fire hydrant. By d_{fire} we denote the total target fire-flow, where by “total” we refer to the sum of flows from all hydrant in use. We assume d_{fire} to be constant for the duration of the fire. Finally, we use the slack variable s_{f_t} and the slack variable vector $s_{d_t} \in \mathbb{R}^{n_n}$, which correspond to fire-flow deficit (how much below the target fire-flow, d_{fire} , was the achieved total fire-flow $\mathbf{1}^T \mu_t$) and demand supply deficit (how much below the demand, $d_t(i)$, was the achieved supply, $\psi_i(h_{it})$, for each node i) respectively. The proposed formulation of **FC** is as follows:

Fire Control (**FC**):

$$\min \quad w_f 10^3 \sum_t s_{f_t} + w_d^T 10^3 \sum_t s_{d_t} + \sum_{i \in I_p, t} w_{it} u_{it} q_{it} \quad (21)$$

$$\text{s.t.} \quad A_1^T q_t = \mu_t + \Psi(h_t) \quad (22)$$

$$\Psi(h_t) = d_t - s_{d_t} \quad (23)$$

$$\mu_t^2 = D_c(h_t - e) \quad (24)$$

$$\mathbf{1}^T \mu_t \geq d_{fire} - s_{f_t} \quad (25)$$

$$r_t - r_{t-1} = D_{tanks} A_2^T q_t \quad (26)$$

$$A_0 h_{0_t} + A_1 h_t + A_2 r_{t-1} = F(q_t) - A_3 u_t \quad (27)$$

$$h_t \leq h_{max} \quad (28)$$

$$q_{min} \leq q_t \leq q_{max} \quad (29)$$

$$r_{min} \leq r_t \leq r_{max} \quad (30)$$

$$u_{min} \leq u_t \leq u_{max} \quad (31)$$

$$0 \leq \mu_t \quad (32)$$

$$0 \leq s_{d_t} \leq d_t \quad (33)$$

$$0 \leq s_{f_t} \leq d_{fire} \quad (34)$$

$$t = 1, \dots, H_{FC}. \quad (35)$$

For the objective of **FC**, $w_f 10^3 \sum_t s_{f_t}$ is the sum of the fire-flow deficits (in l/s) over the horizon H_{FC} , with w_f a scalar weight. $w_d^T 10^3 \sum_t s_{d_t}$ is the sum of demand supply deficits (in l/s) for all nodes and over the horizon H_{FC} , with w_d a vector weight (in case we want to assign higher importance in certain demand nodes). $\sum_{i \in I_p, t} w_{it} u_{it} q_{it}$ are the bilinear penalty terms for the pump operation, the same as in the formulation of **NC**. Constraints (22) describe the pressure-depended mass balance. Constraints (23) “aim” in supplying the customer demand. Constraint (24) is the orifice equation (eq. (20)) squared, where D_c is the diagonal matrix with the corresponding squared discharge coefficients values in the diagonal. Note that constraint (24) implies the assumption that the pressure at a fire hydrant, in use, will be non-negative. Constraint (25) “aims” in delivering (or exceeding) the target fire-flow d_{fire} . The tank constraints (26) and the energy balance constraints (27) are the same as with **NC**. Variable bounds are set by constraints (28)-(34).

In constraint (28), h_{max} should be such that, if reached at any junction node, it would not (with high confidence) cause any pipe bursts. In our study case we have used $h_{max}(i) = h_{max,normal}(i) + 10$, where $h_{max,normal}(i)$ is the maximum head reached during normal operation for node i .

Finally, we should note that the objectives of fire-flow deficit, $w_f 10^3 \sum_t s_{f_t}$, and demand supply deficit, $w_d^T 10^3 \sum_t s_{d_t}$, may or may not be in a trade-off. Examples of both cases are presented in the results section. Nevertheless, this does not mean that one of the objectives can be neglected when there is no trade-off. If the demand supply deficit is neglected, then this could result in unnecessary demand deficit in nodes away from the fire hydrants. If the fire-flow deficit is neglected, we could have a case where the demand is fully met but we do not achieve the target flow, even though we could, by increasing the pressure to allowed levels.

6 Case study

In this section we present the results from the application of the proposed methodology. All numerical experiments were conducted on an Intel(R) Core(TM) CPU i9-9980HK @2.40 GHz, 8 Core, 32GB RAM system. For the solutions of **NC** and **FC** we used the IPOPT solver [32] with MATLAB R2020a. Both demand-driven and pressure-driven simulations were performed based on the methodology of previous work in the literature (see [5, 38, 30, 26]), where the non-linear system of hydraulic equations is formulated as a strictly convex program. The IPOPT solver was also used for the simulations. For the presented study case, the computational time, in most iterations (**NC** or **FC**), was less than a minute.

6.1 The BWFL Network

We have applied the proposed methodology to the Bristol Water Field Lab (BWFL) network (modified for the purposes of this study). The BWFL network is an operational water supply network, which is part of the water distribution network of the city of Bristol in the United Kingdom, and it serves approximately 8,000 customers. It has been used as a case study in multiple published works [39, 38, 25, 1, 22, 34].

Figure 4: The BWFL network

The BWFL network, shown in Figure 4, is comprised of 2371 links, 2310 nodes, 2 sources of fixed head, 2 (fixed-speed) pumps, two tanks, three pressure reducing valves (PRVs) and two boundary control valves (BVs). In addition, we consider the location of four fire hydrants (FHs). The two pumps are of the same characteristics and are considered to be directly connected from each corresponding source to the corresponding tank. The boundary control valves open and close according to a pre-determined time schedule. On the other hand, the PRVs can be remotely controlled in real-time and thus, in this work, their control is subject to optimization. In all presented results, the duration of each timestep is one hour.

6.2 Normal Operation

First, we present the results from the 24-hour operation of the BWFL network, under normal conditions (i.e. no fire incident takes place). The prediction horizon length for the **NC** problem, throughout the case study section, was set to $H_{NC} = 12$. The achieved electricity cost, for the 24-hour normal operation, was $\approx 34\text{£}$. There was no significant increase in electricity costs compared to trials with $H_{NC} = 24$. At the same, iterations with $H_{NC} = 12$ were faster and more robust. However, there was a considerable increase in electricity costs when $H_{NC} = 6$. Therefore, we settled with the choice of $H_{NC} = 12$.

Figure 5: Pump operations and tank levels during normal 24-hour operation. (a) Pump-1 flows, (b) Tank-1 levels, (c) Pump-2 flows and (d) Tank-2 levels. The dashed horizontal lines in (b) and (c) show the aimed operating tank limits.

The operation of the two pumps and the corresponding tank levels are shown in Figure 5. For both pumps there is good agreement between optimization and simulation. In most cases the optimization-obtained pump operating points are very close to either zero flow (pump “off”) or to the pump characteristic curve (pump “on”). This can also be seen, more clearly, from graphs (c) and (d) in Figure 6. Notice also from graph (c) of Figure 5, how the optimization points at timesteps 16 and 20 for Pump-2, which have larger deviations from the operating characteristics of the pump, are mapped by the heuristic to “on” and “off” respectively. Furthermore, the control avoids any pump operation during the timesteps with the highest tariff (i.e. $t=17,18,19$).

Figure 6: Effect of the bilinear pump penalty. (a) and (b): operation points with no penalty. (c) and (d): operation points when first-timestep-only penalty is included, with $w_{11} = w_{21} = 0.5$. The pump characteristic curve is shown as the dashed curve in each graph.

The bilinear pump-penalty weights were set to $w_{1t} = w_{2t} = 0.5$ for $t = 1$ and $w_{1t} = w_{2t} = 0$ for $t > 1$. Thus, we considered

a penalty for both pumps, but only for the first timestep of the prediction horizon. The effect of the bilinear penalty, with the chosen weights, can be seen in Figure 6 where, some of the previously “unrealistic” operating points in graphs (a) and (b), are “forced” towards either zero flow or the characteristic curve in graphs (c) and (d) respectively.

Figure 7: (a) Demand-node pressures and (b) PRV u-values under normal 24-hour operation.

The demand-node pressures and the operation of the PRVs are shown in Figure 7. Notice that, all the demand-node pressures are above or equal to the supply pressure threshold (in this case 20 meters), as would be required under normal operating conditions. Finally, the AZP during normal operation is plotted in the graph of Figure 8.

Figure 8: AZP during normal operation.

6.3 Fire Scenario 1

For the first scenario we assume that a fire takes place during timesteps 6 and 7, with a target fire-flow of 30 l/s and with fire hydrant FH-1 (see Figure 4) in use. The horizon length for the fire control problem was set to $H_{FC} = 2$ for both timesteps. All objective weights for **FC** were set equal to 1 (the weights for the bilinear penalty terms were set to 1 for both pumps, but only for the first timestep of the horizon).

Figure 9: (a) Demand-node pressures, (b) PRV u-values and (c) fire hydrant pressures during scenario 1.

In Figure 9, we show the pressures at the demand-nodes, the PRV u-values and FH-1 pressures as obtained with the proposed MPC scheme, for the 24-hour operation of the BWFL network. The "response" of the control scheme to the fire incident is evident in the graph values at timesteps 6 and 7. The lower u-values at timesteps 6 and 7 correspond to "opening" of the PRVs (which results in increase of nodal pressures). Furthermore, in this case, the network maintains full customer demand supply during the fire since all the demand-node pressures remain above the 20 meters threshold.

Figure 10: Adaptive MPC vs "No fire control" for scenario 1. (a) achieved fire-flows and (b) corresponding supply deficits.

Next, in Figure 10 we compare the fire-flows (graph (a)) and demand supply deficits (graph (b)) between the adaptive MPC scheme (both optimization and corresponding simulation values are shown) and the case where no fire control takes place. In the later case, we assume that the network operates with a pre-determined schedule for pumps and valves throughout the 24-hour period and thus we use the corresponding controls from the normal operation in Subsection 6.2. The fire control achieves (and exceeds) the required fire-flows for the duration of the fire while, at the same time, maintains full customer supply. On the other hand, without the adaptive control, we would only achieve 20 l/s of fire-flow and with (approximately) 2 l/s and 4 l/s of demand deficit during timesteps 6 and 7 respectively.

Figure 11: Pump operations during scenario 1. (a) Pump-1 flows and (b) Pump-2 flows.

Finally, in Figure 11, we plot the operation of the two pumps over the 24-hour period, as obtained from the application of the adaptive MPC scheme. Pump-2 remains off during both timesteps 6 and 7, while Pump-1 is off only on timestep 6. This is acceptable since we meet the target fire-flow and provide full customer demand (and at the same time we do not want unnecessarily increase nodal pressures). The operation of Pump-1 at timestep 7 can be explained by considering that $H_{FC} = 2$, PRV-1 and PRV-3 are already fully opened, the fire-flow is only slightly above the target at timestep 7 and the significant increase in customer demand from timestep 7 to timestep 8.

6.4 Fire Scenario 2

In this scenario we consider that a fire takes place during timesteps 17 and 18, with a target fire-flow of 25 l/s and with fire hydrants FH-2 and FH-3 in use. The horizon length and objective weights for **FC** were the same as with the previous scenario.

Figure 12: (a) Demand-node pressures, (b) PRV u-values and (c) fire hydrant pressures during scenario 2.

Similarly with scenario 1, there is no trade-off between fire-flow and demand supply. However, in contrast with scenario 1, we cannot maintain full demand supply. This can be seen from graph (a) in Figure 12, where for a number of demand nodes the pressure drops below the 20 meters threshold.

Figure 13: Adaptive MPC vs “No fire control” for scenario 2. (a) total fire-flows achieved and (b) corresponding supply deficits.

From Figure 13 we see that the adaptive MPC achieves the target fire-flow of 25 l/s for the duration of the the fire incident while, without the control, the fire-flow is approximately 20 l/s. In addition, the adaptive MPC results in less demand supply deficit compared to the case of no control.

Figure 14: % Demand supply during timestep 17 (i.e. during 16:00-17:00) of scenario 2. Nodes with no demand are plotted in smaller size.

The affected demand nodes during timestep 17 can be seen in Figure 14 (the picture is similar for timestep 18). Figure 14 is plotted based on the pressure-driven simulation results and is in good agreement with the corresponding optimization results.

Figure 15: Absolute differences of heads and flows between optimization and simulation for scenario 2. PD = Pressure-Driven, DD = Demand-Driven

The good agreement between optimization and simulation (both pressure driven and demand-driven) is evident from the histograms of the absolute differences of junction heads and pipe flows in Figure 15. The values correspond to the 24-hour period for scenario 2, with 22 timesteps of demand-driven and 2 timesteps of pressure-driven simulations.

Figure 16: Pump operations during scenario 2. (a) Pump-1 flows and (b) Pump-2 flows.

Finally, in Figure 16, we plot the operation of the pumps, as obtained from the application of the adaptive MPC scheme. In this case, Pump-1 operates throughout the duration of the fire (note that PRV-1 is fully open). Pump-2 operates only at timestep 18, due to the fact that otherwise the level of Tank-2 is predicted to fall below the safety limit by the end of timestep 20 (given that $H_{FC} = 2$).

6.5 Fire Scenario 3

In the previous two scenarios there was no trade-off between fire-flow and demand supply. However, this is not always the case. We demonstrate such a situation in this scenario. We assume that a fire takes place during timestep 10, with a target fire-flow of 25 l/s and FH-4 in use. Notice that FH-4 is located before (upstream of) PRV-3.

In order to explore a range of PRV control options as a response to the fire incident, we remove the constraints on nodal pressures (constraints (28)) and instead include AZP as a third objective in the formulation of **FC** (along with fire-flow deficit

and demand supply deficit). For simplicity, we set both pumps “on” and $H_{FC} = 1$. Next, we run multiple weighted-sum iterations of the **FC** problem by varying the objective weights with $w_f + w_d + w_{azp} = 1$ (w_d taken as a scalar weight and the supply deficit objective being $w_d 10^3 \mathbf{1}^T \sum_t s_{d_t}$).

Figure 17: A range of possible PRV control options during the fire incident of scenario 3. Each vertically aligned group of u-values corresponds to a solution of **FC** with different objective function weights. The implemented control is highlighted, in both graphs, inside the rectangle.

A selected number of the obtained solutions (PRV u-values) are shown in the two graphs of Figure 17. In graph (a) the u-values are colored based on their fire-flow deficit, while on graph (b) based on their demand supply deficit. Each vertically aligned group of u-values corresponds to a solution of **FC** with different objective function weights and with the corresponding AZP value shown on the x-axis. Some of the solutions towards the right side of the two graphs have both low fire-flow deficit and demand supply deficit, however they also have very high AZP and therefore are prohibitive in practise.

We focus on the solutions with acceptable values of AZP and particularly on the solution highlighted, in both graphs, inside the rectangle. In this solution, PRV-3 has a very high u-value (i.e. fully closed) “sacrificing” some demand supply in order to increase pressure and thus the fire-flow at FH-4.

Figure 18: a) Demand-node pressures, (b) PRV u-values and (c) FH-4 pressures during scenario 3.

The demand-node pressures, PRV controls and FH-4 pressures, for the total 24-hour period, with the aforementioned solution applied during the fire incident of scenario 3, are shown in Figure 18. Notice, in graph (b), the increase in PRV-3’s u-value (i.e closing of PRV-3) at timestep 10.

Figure 19: Adaptive MPC vs “No fire control” for scenario 3. (a) achieved fire-flows and (b) corresponding supply deficits.

From graph (a) in Figure 19 we observe that the applied control solution achieves $\approx 3 \text{ l/s}$ (or 16%) more fire-flow compared to the “No fire control” case, however with $\approx 1.3 \text{ l/s}$ more demand supply deficit as shown in graph (b). In either case, we cannot meet the target fire-flow of 25 l/s under an acceptable pressure increase. This however depends on the characteristics of the network rather than the control, as is also the case with the amount of extra fire-flow we can achieve by closing PRV-3. Nevertheless, the example can be considered more in a qualitative rather than quantitative way, in order to highlight the fact that, in certain cases, there can be a trade-off between fire-flow and demand supply.

Figure 20: Scenario 3: % Demand supply during timestep 10 (i.e. during 09:00-10:00) of scenario 3. Nodes with no demand are plotted in smaller size.

Finally, in Figure 20, we show the affected demand nodes during the fire incident, as obtained from the corresponding pressure-driven simulation. As expected, the affected area is located (mainly) downstream of PRV-3.

7 Conclusions

We have presented an adaptive model predictive control approach for water supply networks, which combines two separate modes of control: 1) Normal control, when the network operates under normal conditions and the aim is the minimization of energy costs and average zonal pressure. 2) Fire control, when a fire incident occurs and the aim is to deliver the required fire-flow from specific fire-hydrants, while maintaining, to the furthest possible extent, customer supply without over-pressurizing

the network. Each mode of control uses a different hydraulic model. For normal control, where we expect that the required pressures at demand nodes can be met, we use a demand-driven model. For fire control, where potential pressure drops can have a significant effect on customer supply, we use a pressure-driven model.

The proposed methodology was applied on an operational water distribution network from the city of Bristol in UK. We have demonstrated the potential benefits of the adaptive MPC scheme, when compared to a predefined (non-adaptive) control, which considers only energy costs and leakage reduction (current practise by UK water utilities). More specifically, we have presented three different fire scenarios. In the first two scenarios, we achieve higher fire-flows with less or no impact in the supply of customer demand. In the third scenario we show that with the proposed method, in certain situations, we can choose to sacrifice some customer demand in order to achieve higher fire-flows. Moreover, the computational time of the proposed methodology was well within practical applicability. For the presented case study and settings, most iterations (**NC** or **FC**) took less than a minute. The proposed methodology could potentially serve water utilities both as a tool for control and as a tool for testing, planning and prevention.

Several aspects of the presented work are open for improvement as future work: We have assumed that, during a fire incident, the location of the fire hydrants in use are given by the corresponding fire services. However, the choice of which and how many fire hydrants to use, from a group of available candidates, is in itself an optimization problem, with the optimal solution having the potential of improving significantly the resulting fire flows. This could be added as an extra component to the overall MPC scheme (executed only once at the beginning of the fire incident), prior to the execution of the **FC** problem. Furthermore, it would be interesting to assess the quality of the local solutions provided by the proposed adaptive MPC scheme, both for normal and fire control, with respect to the corresponding globally optimal solutions. Of course this would require testing on several networks. Finally, demand uncertainty, which has not been considered in this work.

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