Determination of fracture mechanics parameters J and C^* by finite element and reference stress methods for a semi-elliptical flaw in a plate

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Abstract

The fracture mechanics parameters J and C^* used, respectively, to describe ductile fracture and creep crack growth can be determined either by finite element methods or reference stress techniques. In this paper solutions for a partially penetrating semi-elliptical flaw in a plate subjected to tension and bending loading are considered. Estimates of J and C^* are obtained from finite element calculations for a range of work-hardening plasticity and power law creep behaviours and from reference stresses derived from ‘global’ collapse of the entire cracked cross-section. Comparisons are made with solutions taken from the literature for a range of loading conditions, plate geometries and crack sizes and shapes. Generally it is found that although there are significant variations between the different finite element solutions, satisfactory estimates of J and C^* that are mostly conservative are obtained when the reference stress procedure is adopted.

1. Introduction

Assessments of the structural integrity of components that contain defects can be made using the elastic-plastic fracture mechanics parameter J [1] and, at elevated temperatures in the presence of creep, by the corresponding creep fracture mechanics parameter C^* [2]. The term J is relevant to characterising fast fracture whereas C^* is appropriate for describing creep crack growth. Both can be calculated numerically by finite element methods provided the relevant elastic, plastic and creep properties of the materials of construction are known [3,4]. When finite element solutions are not available, approximate reference stress procedures can be employed [4-8].

Provided enough calculations have been made, estimates of reference stress can be obtained from finite element methods. However, normally limit analysis is used. In this case, it is necessary to identify a collapse mechanism for the plane containing a crack. For through thickness cracks, failure corresponding to ‘global’ collapse of the entire cross-section containing the crack is postulated. For partially penetrating defects it is possible, in addition, to base failure on ‘local’ collapse of the uncracked ligament ahead of the crack. This latter approach results in a higher value of reference stress and therefore more conservative determinations of J and C^* . Because of this, defect assessment procedures [5-8] often recommend that reference stress estimates are based on a ‘local’ collapse approach. However, there is evidence in the literature [9] that use of a ‘local’ reference stress for partially penetrating defects in plates subjected to combined axial and bending loading can significantly over-estimate creep crack growth rates when C^* is calculated from this stress. As a

consequence, Goodall and Webster [10] have proposed the adoption of a ‘global’ reference stress for this crack geometry. In this paper, this definition of reference stress is used to estimate J and C^* . Initially expressions for J and C^* are derived in terms of reference stress. Comparisons are then made with numerical solutions taken from the literature [11-15] and with additional calculations made by ABAQUS [16].

2. Formulae for J and C^*

In general, the elastic-plastic fracture mechanics parameter J for flawed components can be written in the form

$$J = h\sigma\varepsilon a \quad (1)$$

where h is a non-dimensional factor, σ a representative stress which describes the loading applied to the component, ε the total strain at this stress and a is a measure of the defect dimensions. Typically, the value of h is sensitive to the relative crack size to component dimensions, the loading conditions and the material stress-strain properties and is determined by finite element analysis. However, it has been found [4, 17] that when σ is defined as the reference stress, σ_{ref} , of the cracked component, h becomes relatively insensitive to material properties and can be obtained from its elastic value using the relation

$$G = h\sigma_{ref} \frac{\sigma_{ref}}{E'} a = \frac{K^2}{E'} \quad (2)$$

where G is the elastic strain energy release rate and K is the stress intensity factor. To allow for stress state effects, E' is the elastic modulus E for plane stress conditions and $E/(1 - \nu^2)$ for plane strain where ν is Poisson’s ratio. Substitution of eq. (2) into eq (1) gives

$$J = \sigma_{ref} \varepsilon_{ref} \left(\frac{K}{\sigma_{ref}} \right)^2 \quad (3)$$

where ε_{ref} is the total strain obtained from the material stress-strain properties at the reference stress. Consequently, when finite element solutions for J are not available, eq (3) can be employed for determining approximate estimates. The form of this equation ensures that $J = G$ for purely elastic loading. It is particularly attractive because solutions for K and σ_{ref} are available for a wide range of crack and component geometries and loading conditions.

Before eq (3) can be evaluated, σ_{ref} must be established. It can be determined from limit analysis or numerical methods [4, 18]. When limit analysis is employed, for a component subjected to a load P , it is given by

$$\sigma_{ref} = \sigma_Y \frac{P}{P_{LC}} \quad (4)$$

where σ_Y is the material yield stress and P_{LC} the corresponding collapse load of the cracked component. For partially penetrating defects both ‘local’ and ‘global’ collapse mechanisms can be adopted. For a semi-elliptical surface defect (as shown in Fig 1) a ‘global’ collapse mechanism corresponding to collapse of the entire cracked cross-section, as proposed by Goodall and Webster [10], is employed here. For this case

$$\sigma_{ref} = \frac{(\sigma_b + 3\gamma\sigma_m) + \left\{ (\sigma_b + 3\gamma\sigma_m)^2 + 9\sigma_m^2 \left[(1-\gamma)^2 + 2\gamma(\alpha - \gamma) \right] \right\}^{0.5}}{3 \left\{ (1-\gamma)^2 + 2\gamma(\alpha - \gamma) \right\}} \quad (5)$$

where $\alpha = a/t$, $\gamma = ac/Wt$ and σ_m and σ_b are the remote axial and elastic bending stresses given, respectively, in terms of the axial load N and bending moment M in Fig 1 by

$$\sigma_m = N / 2Wt \quad (6)$$

$$\sigma_b = 3M / 2Wt^2 \quad (7)$$

In the analysis of Goodall and Webster [10], it is assumed that the semi-elliptical defect is represented by a circumscribing rectangle and the entire crack remains in the tensile stress field so that no crack closure occurs. Insertion of eq (5) into eq (3) enables J to be evaluated.

A similar procedure can be used for calculating C^* . Like J it can be expressed in the generalized form

$$C^* = h\sigma\dot{\epsilon}^c a \quad (8)$$

where $\dot{\epsilon}^c$ is creep strain rate at stress σ . The other terms are as defined previously except that h is sensitive to the creep properties of a material instead of its stress-strain behaviour. Following the approach for J , when σ is replaced by σ_{ref} , h becomes relatively insensitive to material creep properties and C^* can be determined approximately from

$$C^* = \sigma_{ref} \dot{\epsilon}_{ref}^c \left(\frac{K}{\sigma_{ref}} \right)^2 \quad (9)$$

where $\dot{\epsilon}_{ref}^c$ is the creep strain rate at stress σ_{ref} . Consequently, therefore, when finite element solutions for C^* are not available, approximate estimates can be obtained from eq (9) in the same way as J can be determined from eq (3). In both cases, the same formulae are employed for evaluating K and σ_{ref} . For a semi-elliptical flaw in a plate subjected to combined axial and bending loading K is given by [19],

$$K = F[\sigma_m + H\sigma_b] \sqrt{\frac{\pi a}{Q}} \quad (10)$$

where Q is a function of crack shape, and F and H are dependent on crack shape, relative crack depth and angular position ϕ around the crack (see Fig 1). Their values have been tabulated in several sources [5-7, 19-21].

Comparisons will now be made between estimates of J and C^* determined from finite element (FE) and reference stress methods.

3. Calculations of J and C^*

Calculations have been made for a square plate with $L = W$ and $t = W/10$ containing a semi-elliptical defect of dimensions $c = W/4$, $a/c = 0.2$ and $a/t = 0.5$ as shown in Fig 1. In making the finite element calculations only one quarter of the plate was modelled due to symmetry as shown in Fig 2. The mesh consisted of 974 elements and 7241 nodes. Solutions for J were obtained at 17 angular positions ϕ around the crack front. At each position, the value of J (termed J_{FE}) was taken as the average obtained from 11 contours around the crack tip. The deviation between individual values was less than 5%. Strain was assumed to obey the work-hardening elastic-plastic stress-strain law,

$$\varepsilon = \frac{\sigma}{E} + \alpha_o \frac{\sigma_Y}{E} \left(\frac{\sigma}{\sigma_Y} \right)^n = \frac{\sigma}{E} + A \sigma^n \quad (11)$$

where α_o , n and A are parameters which describe the plastic behaviour of the material. This same stress-strain relation was used for evaluating J (called J_{ref}) from eq (3) by the reference stress procedure; that is the total strain ε was used to evaluate ε_{ref} . Calculations were made for the separate cases of tension and bending loading for increasing values of load for $\alpha_o = 0.1$, $\sigma_Y = 170$ MPa, $E = 155$ GPa and $n = 5$ and 10 for each angular position.

The normalized results of J_{FE}/J_{ref} for the surface and deepest points of the crack are shown in Figs 3-6. Figures 3 and 4 illustrate the trends obtained for tensile loading for $n = 5$ and 10, respectively. The corresponding results for bending alone are shown in Figs 5 and 6. A ratio of $J_{FE}/J_{ref} < 1$ implies that use of eq (5) to calculate σ_{ref} results in conservative estimates of J . At low loads, by definition from eqs (2) and (3) the ratio must tend to unity as $J = G$ for purely elastic loading as is observed. Also in all cases as load is increased and plastic strains dominate, the ratio tends to a constant value. It is evident, except for high bending loads and $n = 10$ shown in Fig 6, that estimates of J based on J_{ref} are within about 15%, over most of the loading range considered, of those determined from J_{FE} . This demonstrates the general validity of the reference stress approach for calculating J . For the case shown in Fig 6, the use of J_{ref} is conservative by a factor of about 2. This degree of conservatism is less than that obtained from use of a reference stress based on a 'local' collapse mechanism [5-7].

From eq (11), for the plastic strain ε_{ref}^p at the reference stress to dominate the elastic strain ε_{ref}^e at this stress

$$\frac{\sigma_{ref}}{\sigma_Y} > \left(\frac{1}{\alpha_o} \right)^{\frac{1}{n-1}} \quad (12)$$

Also from eq (5) for the crack geometry examined, tension loading gives $\sigma_{ref} = 1.23 \sigma_m$ and bending loading $\sigma_{ref} = 0.78 \sigma_b$. Combination of these relations with eq (12) gives the applied loadings, listed in Table 1, above which the plastic term in eq (11) dominates. This corresponds in Figs 3-6 with the region where the ratio J_{FE}/J_{ref} begins to approach a constant value and where J tends to J^p , the plastic component of J which can be expressed from eq (11) as

$$J^p = \sigma_{ref} \cdot \epsilon_{ref}^p \left(\frac{K}{\sigma_{ref}} \right)^2 \quad (13)$$

$$= \frac{\alpha_o}{E} \left(\frac{\sigma_{ref}}{\sigma_Y} \right)^{n-1} K^2 = A \sigma_{ref}^{n-1} K^2 \quad (14)$$

In making the calculations of J^p in Figs 3-6 the approximation has been made that $J^p \approx J - G$. Consequently when plastic strains dominate, J_{FE} tends to J^p and the ratio J_{FE}/J_{ref} will tend to a constant value as is observed in the figures. This constant value is safely achieved at ratios of stress in Table 1 that correspond with $2\sigma_{ref}/\sigma_Y$ for $n \leq 10$. This ratio can be regarded as a useful guide for convergence but it is sensitive to the value chosen for α_o and is expected to decrease as n increases.

For the circumstance when $J_{FE} \approx J^p$, eq (14) can be employed to determine C^* using eq (9). Typically creep strain rate can be described by a power-law relation of the form

$$\dot{\epsilon}^c = \dot{\epsilon}_o \left(\frac{\sigma}{\sigma_o} \right)^m = C \sigma^m \quad (15)$$

where $\dot{\epsilon}_o$, σ_o , C and m are parameters which describe the creep properties of a material. Substitution of this equation into eq (9) gives

$$C^* = \frac{\dot{\epsilon}_o}{\sigma_o} \left(\frac{\sigma_{ref}}{\sigma_o} \right)^{m-1} K^2 = C \sigma_{ref}^{m-1} K^2 \quad (16)$$

This equation is of a very similar form to eq (14). It is apparent, by combining eqs (14) and (16), that C^* can be evaluated from solutions of J_{FE} obtained in region where $J_{FE} \approx J^p$ from

$$C^* = \frac{\dot{\epsilon}_o E}{\alpha_o \sigma_o} \left(\frac{\sigma_Y}{\sigma_o} \right)^{m-1} J_{FE} = \frac{C}{A} J_{FE} \quad (17)$$

for the case when J_{FE} is determined for n having the same value as m . Consequently finite element solutions for J_{FE} obtained when plastic strains dominate can be used for estimating C^* from eq (17). Equation (17) is particularly valuable because there are more solutions available in the literature for J than for C^* (see, for example [11-15])

The dependence of normalized J^p on angle around the crack front for each loading case when plastic strains dominate is shown in Figs 7 – 10. For tension, the normalization has been carried out by dividing J^p by

$$J_{normal}^p = \alpha_o \frac{\sigma_Y^2}{E} \left(\frac{\sigma_m}{\sigma_Y} \right)^{n+1} t \quad (18)$$

and for bending by dividing by

$$J_{normal}^p = \alpha_o \frac{\sigma_Y^2}{E} \left(\frac{2\sigma_b}{3\sigma_Y} \right)^{n+1} t \quad (19)$$

The normalising stress of $2\sigma_b/3\sigma_Y$ has been chosen for bending because it corresponds with the reference stress for an uncracked plate in bending. Included in the figures are estimates of normalized J^p determined from the reference stress procedure outlined, the current finite element calculations and additional finite element results taken from the literature [11, 13, 15]. It is evident that there are significant differences in some instances between the individual finite element results. This may be attributed to use of different plate dimensions and materials properties coefficients in eq (11), mesh distributions and possibly boundary conditions. It is apparent in most cases that the differences between the individual finite element solutions are comparable to their difference from the reference stress estimates. For tension all the calculations indicate that J^p increases from the surface to the deepest point of the crack and that values obtained from the reference stress procedure either span or exceed the maximum FE estimates. For bending, the maximum normalized J^p is neither at the surface nor the deepest point. Again the reference stress predictions either span or exceed the maximum FE determinations. In view of the previous discussion, Figs 7-10 can also be employed to obtain C^* as a function of angle around the crack front by using eq (17).

4. Discussion

Finite element and reference stress calculations have been presented for a plate under tension and under bending loads for one crack. They have shown that J and C^* can be estimated with reasonable accuracy by reference stress methods to a variation that is comparable to that between different FE calculations. For tension and one bending case ($n = 5$) it has been found that agreement to within about 15%, corresponding to an accuracy of better than 5% in σ_{ref} , is usually achieved with the most conservative FE solutions. For the remaining bending case ($n = 10$), the reference stress approach

overestimates FE predictions by a factor of about 2 corresponding to an overestimate of σ_{ref} of less than 10%.

Other calculations have been presented in the literature [11-15] for a wider range of loading conditions including combined tension and bending. They have also been made for plate geometries with $W/c = 4-2$ and crack sizes, shapes and depths covering $a/t = 0.2-0.8$, $a/c = 0.2-1.0$ and $n = 5, 10$ and 15 . These have all shown similar trends to those described earlier. Generally it has been found that reasonable agreement is obtained between reference stress and FE estimates of J although conservatism cannot be guaranteed when using σ_{ref} derived from limit analysis. There is a tendency for lack of conservatism to be associated with increasing W/c , a/c and a/t ratios and proposals have been made by Kim et al [14] and Lei [15] for obtaining improved estimates based on FE calculations. For predominantly tensile loading an elevation in σ_{ref} , determined from limit analysis based on ‘global’ collapse, of about 5% can usually ensure conservative predictions. In all cases it has been found that reference stress solutions give conservative predictions at the surface $\phi = 0$.

Although the finite element calculations taken from the literature have been made for J , they are relevant to C^* provided that they have been made in the region where plastic strains dominate and the value of n in the plasticity law has been chosen to equal m in the creep law.

5. Conclusions

In this paper solutions for J and C^* for partially penetrating semi-elliptical flaws in a plate subjected to tension and to bending loads have been considered. Comparisons have been made between estimates obtained from finite element calculations, for a range of work-hardening plasticity and power law creep behaviours, and those produced using reference stresses derived from ‘global’ collapse of the entire cracked cross-section. It has been found that variations exist between the different FE solutions for values of J and C^* for all angles around the crack front. These differences are attributed to choice of FE mesh, boundary conditions, the material properties laws used and the FE package employed. Nevertheless it has been established that satisfactory estimates of J and C^* , that are mostly conservative when compared against their maximum FE determinations, are obtained when the reference stress procedure is adopted. Also it has been demonstrated how values of C^* can be calculated from FE estimates of J .

Table 1. Ratio of applied loading above which plastic strain at the reference stress exceeds the elastic strain using eq (11) with $\alpha_o = 0.1$

n	σ_m/σ_Y	σ_b/σ_Y
5	1.44	2.29
10	1.05	1.66

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Figure Captions

- Figure 1 Plate containing partially penetrating defect subjected to axial load N and bending moment M
- Figure 2 Finite element mesh of (a) one quarter of cracked plate and (b) magnified crack region
- Figure 3 Dependence of J_{FE}/J_{ref} at deepest point and surface on normalized load for pure tension and $n = 5$
- Figure 4 Dependence of J_{FE}/J_{ref} at deepest point and surface on normalized load for pure tension and $n = 10$
- Figure 5 Dependence of J_{FE}/J_{ref} at deepest point and surface on normalized load for pure bending and $n = 5$
- Figure 6 Dependence of J_{FE}/J_{ref} at deepest point and surface on normalized load for pure bending and $n = 10$
- Figure 7 Dependence of normalized J^p on angle around crack front for $a/c = 0.2$, $a/t = 0.5$ for pure tension and $n = 5$
- Figure 8 Dependence of normalized J^p on angle around crack front for $a/c = 0.2$, $a/t = 0.5$ for pure tension and $n = 10$
- Figure 9 Dependence of normalized J^p on angle around crack front for $a/c = 0.2$, $a/t = 0.5$ for pure bending and $n = 5$
- Figure10 Dependence of normalized J^p on angle around crack front for $a/c = 0.2$, $a/t = 0.5$ for pure bending and $n = 10$