Adaptive Formation Tracking Control of Directed Networked Vehicles in a Time-Varying Flowfield

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Nomenclature

$\mathbb{R}$	=	the set of real numbers
$\mathbb{R}^{n \times m}$	=	the set of $n \times m$ matrices with real entries
I	=	the identity matrix
0	=	the zero matrix of appropriate dimension
$\mathbb{T}$	=	the one-torus
$\ \cdot\ $	=	the Euclidean norm
$ M $	=	the determinant of matrix M
$\otimes$	=	the Kronecker product
p_i	=	the position of vehicle i
p_{x_i}	=	the position abscissa of vehicle i
p_{y_i}	=	the position ordinate of vehicle i
u_i	=	the control input of vehicle i
u_{x_i}	=	the control input of vehicle i along the abscissa
u_{y_i}	=	the control input of vehicle i along the ordinate
f_i	=	the flow velocity at position p_i and time t
λ_i	=	the path following error of vehicle i
Ω_i	=	an open set associated with vehicle i
f_{α_i}	=	the flow amplitude at λ_i
f_{β_i}	=	the basis flow function evaluated at position p_i and time t

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θ_i	=	the set of unknown time-varying parameter for vehicle i
δ_*	=	the unknown constant bound of θ_i
s_i	=	the arc-length of vehicle i moving along the orbit
ϕ_i^*	=	the responding parameter of the starting point used to evaluate the arc-length of vehicle i
ξ_i	=	the generalized arc-length of vehicle i
N_i	=	the normal vector at position p_i
v_{N_i}	=	the control input of vehicle i projected on N_i
T_i	=	the tangent vector at position p_i
v_{T_i}	=	the control input of vehicle i projected on T_i
ς_i	=	the formation error associated with vehicle i
l	=	the constant vector selected to be the center of the set of time-varying parameter θ_i
ρ_M	=	the maximum eigenvalue of the matrix $L_1^T L_1$
ρ_0	=	the minimum eigenvalue of the matrix $(L_1^T + L_1)/2$
$\hat{\theta}_i$	=	the estimate of l as implemented by vehicle i
$\hat{\rho}_{\alpha_i}$	=	the estimate of $\rho_\alpha = \rho_M^2 \delta_*/\rho_0$ as implemented by vehicle i
$\hat{\rho}_{\beta_i}$	=	the estimate of $\rho_\beta = \rho_M^2 \delta_*^2/\rho_0$ as implemented by vehicle i
$\hat{\delta}_i$	=	the estimate of δ_* as implemented by vehicle i
$\hat{\delta}_{2i}$	=	the estimate of $\delta_{2*} = \delta_*^2$ as implemented by vehicle i
γ, γ_m	=	the adaptation gains
k_m	=	the control gains
Subscripts		
i, j	=	the vehicle indices, $1, \dots, n$

I. Introduction

IN the past decade there has been growing interest in the use of formation tracking control technology for oceanic and planetary explorations [1–3]. Formation tracking control, which requires multiple vehicles to simultaneously follow a set of given orbits with a desired formation, can achieve better performance than a single monolithic vehicle in seeking measurements of biological variables across a range of spatial and temporal scales. Typical approaches in this line of research deal with each vehicle’s motion without the effect of external flowfields; see for example in [4–7]. However, the motion of each vehicle is inevitably affected by external flowfields. More importantly, the flowfields may force the vehicles to deviate from the desired orbit and even disrupt the formation [8], which yields the urgency of designing formation tracking methods in the presence of flowfields.

Many studies of flow-based formation tracking control exploit bidirectional communication topologies. In

[9], two proposed circular formation control laws in the presence of a time-invariant flowfield were presented in the case of all-to-all topology and undirected, connected, topology, respectively. In [10], the circular formation control methods introduced in [9] have been modified by using a state observer, which permits an unknown time-invariant flowfield and a flowfield uniformly rotating at an unknown constant angular velocity. In [11], formation tracking is achieved in the presence of a parameterized flowfield with unknown constant parameters by using a consensus-based filter. The formation tracking control laws are designed based on an adaptive neural network (ANN) in [12] and on an adaptive method in [13]. In addition to the aforementioned methods, consensus in the presence of disturbances has been investigated and can help in the solution of the flow-based formation tracking control problem. In [14], a consensus problem is solved with an adaptive estimation algorithm in the case of jointly connected topologies. In [15], a disturbance observer solves the leader-following consensus problem, under the assumption that the time derivative of the disturbance converges to zero and the communication topology of the followers is bidirectional. Finally, [16] presents an alternative filter for the consensus problem, assuming the unknown nonlinearities satisfy a Lipschitz-like condition.

In some complex environments, especially underwater, considering a directed communication topology is more realistic [17]. In the cases of sensor networks (e.g., navigation beacons [18]) some sensors/agents may have transceivers, while others may only have receivers. Even if bidirectional communication is achievable, such a method is not preferable because of the limited compatibility and power requirement of the communication equipment. It is therefore essential to design formation tracking methods for directed communication topologies, a problem which has not yet been extensively studied.

An adaptive formation tracking control problem with unknown constant parameters is considered in the case of directed and ring topologies in [19] and in the case of strongly connected topologies in [20]. In [21], ANNs are exploited for formation tracking in the case of a spanning tree, yet using ANNs only guarantees uniform boundedness of the tracking error. Moreover, observer-based cooperative output regulation algorithms for exosystem-like disturbances are designed for leader-following digraphs in [22] and for time-varying digraphs in [23]. The dynamics of the flowfield are often spatio-temporal and contain unknown time-varying parameters [24]. The aforementioned methods cannot be used for solving the formation tracking control problem of vehicles in the case of a directed communication topology, especially in a spatio-temporal flowfield without restrictions on the rate of parameter variations. To cope with unknown time-varying parameters, a novel adaptive method, the *congelation of variables* method, has been proposed in [25–28]. This method has been exploited to solve the formation tracking control problem in the presence of an unknown spatio-temporal flowfield and bidirectional topologies in [29].

This paper intends to solve the robust formation tracking control problem for vehicles with first order dynamics connected by a directed topology. The flowfield under consideration is a spatio-temporal flow with

unknown time-varying parameters; the only restriction is that the parameters are bounded. Different from our previous result for bidirectional topologies [29], in this paper, the topology associated to the virtual leader and the followers is directed. This topology is similar to what discussed in [1], for deep-space formation flying of leader and follower spacecrafts, in [3], for AUVs adaptive sampling in Monterey Bay, and in [4], for spacecraft formation flying based on a virtual structure. By integrating estimation algorithms for the eigenvalues of the Laplacian matrix and an unknown bound on the time-varying parameters into the *congelation of variables* method, a novel distributed formation tracking control law to achieve asymptotic converge with a stationary virtual leader is designed. A novel *formation adjustment unit* for the virtual leader provides formation movement with a desired speed.

The paper has the following outline. Section II provides some preliminaries and formulates the adaptive formation tracking control problem in the presence of an unknown spatio-temporal flowfield. Section III gives a solution for the problem for the case in which the virtual leader is stationary and in the case in which the virtual leader moves along an orbit. Simulation results are presented in Section IV. Conclusions are given in Section V.

II. Problem Formulation

Consider a formation composed of a virtual leader, labeled as the 0th vehicle, and n followers, labeled as the 1st to the n th vehicles. Each follower is position in a fixed inertial reference frame evolves according to the dynamic equation

$$\dot{p}_i = v_i + f_i(p_i, t), \quad (1)$$

where $p_i(t) = [p_{x_i}(t), p_{y_i}(t)]^T \in \mathbb{R}^2$ denotes the position, $v_i(t) = [v_{x_i}(t), v_{y_i}(t)]^T \in \mathbb{R}^2$ denotes the control input, and $f_i : \mathbb{R}^2 \times \mathbb{R} \rightarrow \mathbb{R}^2$ denotes the action of the spatio-temporal flowfield. The dynamics of the virtual leader, that is the 0th vehicle, are

$$\dot{p}_0 = v_0, \quad (2)$$

where $p_0(t) = [p_{x_0}(t), p_{y_0}(t)]^T \in \mathbb{R}^2$ and $v_0(t) = [v_{x_0}(t), v_{y_0}(t)]^T \in \mathbb{R}^2$ are its position and control input, respectively.

The network topology of the formation can be described by a digraph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ in which the vertices/nodes $\mathcal{V} = \{\mathcal{V}_0, \dots, \mathcal{V}_n\}$ are associated to the virtual leader and the n vehicles, respectively, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of network links. A directed path from node $\mathcal{V}_j$ to node $\mathcal{V}_i$ is a sequence of edges $(\mathcal{V}_i, \mathcal{V}_{i_1}), (\mathcal{V}_{i_1}, \mathcal{V}_{i_2}), \dots, (\mathcal{V}_{i_{l-1}}, \mathcal{V}_{i_l}), (\mathcal{V}_{i_l}, \mathcal{V}_j)$ in the network topology with distinct nodes $\mathcal{V}_{i_k}, k = 0, \dots, m$. A digraph is called a directed tree if there exists a node, called the root, that has directed paths to all other nodes of

the digraph. Let, for $i, j = 0, \dots, n$, $a_{ij} = 1$ if $(\mathcal{V}_j, \mathcal{V}_i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. In addition, define the Laplacian matrix $L \triangleq [l_{ij}]_{i,j=0}^n$ with $l_{ii} = \sum_{j=1}^n a_{ij}$ and $l_{ij} = -a_{ij}$, for any $i \neq j$, $i, j = 0, \dots, n$. For the considered formation the Laplacian matrix L can be written as

$$L = \begin{bmatrix} l_{00} & \beta_0 \\ \alpha_0 & L_1 \end{bmatrix},$$

where $\alpha_0 = [l_{10}, \dots, l_{n0}]^T$, $\beta_0 = [l_{01}, \dots, l_{0n}]^T$, and $L_1 \in \mathbb{R}^{n \times n}$. If there is no path from the followers to the leader, then $l_{00} = 0$ and $\beta_0 = 0$. This observation motivates the following assumption.

Assumption 1. The digraph consisting of the virtual leader and the n vehicles contains a directed spanning tree with root $\mathcal{V}_0$.

Lemma 1 [30]. Consider the digraph $\mathcal{G}$. Suppose that Assumption 1 holds. Then L_1 is a nonsingular M -matrix and all eigenvalues of L_1 have positive real parts.

The formation tracking task consists of a path-following task and a formation-motion task. In the former, the desired orbit associated to each vehicle is a simple, closed, and regular curve with nonzero curvature. As discussed in detail in [7] the path-following error associated with the i th vehicle can be defined by a smooth function $\lambda_i : \Omega_i \rightarrow (-\varepsilon_i, \varepsilon_i)$, where $\Omega_i \subset \mathbb{R}^2$ is an open set and ε_i is a positive constant. As shown in Figure 1a, the path-following task is achieved if, along the trajectories of the system,

$$\lim_{t \rightarrow \infty} \lambda_i(p_i(t)) = 0, \quad (3)$$

and $p_i(t) \in \Omega_i$, for all $t \geq 0$, where

$$\Omega_i = \{p_i \in \mathbb{R}^2 \mid |\lambda_i(p_i(t))| < \varepsilon_i\}. \quad (4)$$

Note that the formation preserves the relative positions along the paths and the arc-length can be regarded as the positions on the path. It is natural to use generalized arc-lengths to define the formation [6, 7, 13, 20, 29]. The arc-length is

$$s_i(\lambda_i, \phi_i) \triangleq \int_{\phi_i^*}^{\phi_i} \frac{\partial s_i(\lambda_i, \tau)}{\partial \tau} d\tau, \quad (5)$$

where ϕ_i^* is the parameter associated with the starting point of the arc. Then, consistent with [7], the generalized arc-lengths $\xi_i(s_i)$ are C^1 functions of s_i and $|\partial \xi_i / \partial s_i| \geq \varepsilon > 0$. The generalized arc-lengths are selected in such a way that if the formation is achieved then

$$\lim_{t \rightarrow \infty} (\xi_i(t) - \xi_0(t)) = 0. \quad (6)$$

Fig. 1 Sketches of path followed and of formation. The meshed area denotes the set Ω_i

Similarly to [29], assume that the flowfield is a unknown spatio-temporal flow parameterized as

$$f_i(p_i, t) = f_{\alpha_i}(\lambda_i) f_{\beta_i}(p_i, t) \theta_i(t), \quad (7)$$

where $f_{\alpha_i} : \mathbb{R} \rightarrow \mathbb{R}$ denotes the known amplitude of the flowfield; $f_{\beta_i} : \mathbb{R}^2 \times \mathbb{R} \rightarrow \mathbb{R}^{2 \times m}$ is the $2 \times m$ matrix of known bounded basis functions of the flowfield; and $\theta_i(t) \in \mathbb{R}^m$ is the set of unknown time-varying parameters. Eq.(7) can model, for example, the time-varying spatial flowfield in [10], the parameterizable flowfield in [11] and the Eulerian flowfield in [13].

Remark 1. The C^1 invertible mappings $\xi_i : \mathbb{R} \rightarrow \mathbb{R}$ define a change of coordinates that allows formulating the formation tracking problem with state variables s_i into the consensus problem described by Eq. (6), with state variables $\xi_i(s_i)$. To form the desired formation, the desired arc-length s_i^* of the i th follower vehicle is determined by the arc-length of the leader s_0 via $s_i^* = g_{s_i s_0}(s_0)$, where $g_{s_i s_0} : \mathbb{R} \rightarrow \mathbb{R}$ is an invertible mapping explicitly defined by the desired formation. By Eq. (6), ξ_0 and ξ_i are such that

$$\xi_i(s_i^*) = \xi_0(s_0).$$

Noting $s_0 = g_{s_i s_0}^{-1}(s_i^*)$ yields $\xi_i = \xi_0 \circ g_{s_i s_0}^{-1}$, which properly defines ξ_i for any given ξ_0 , since $g_{s_i s_0}$ is known. To clarify, consider the tracking problem depicted by Figure 1b, in which the vehicles move along concentric circles with radii R_i and the desired formation is the radially in-line formation; that is, all vehicles stay on the same radius when moving along the circles. A natural selection of ξ_0 is the azimuth associated with s_0 , namely $\xi_0(s_0) = \frac{1}{R_0} s_0$; note that being in the desired formation implies $g_{s_i s_0}(s_0) = \frac{R_i}{R_0} s_0$. The corresponding

ξ_i is given by

$$\xi_i(s_i) = \xi_0 \circ g_{s_i s_0}^{-1} \circ s_i = \frac{1}{R_i} s_i.$$

Assumption 2 [29]. The functions f_{α_i} are bounded and satisfy $\nabla \Psi_i f_{\alpha_i} \geq f_{\alpha_i}^2$, for all $\lambda_i \in (-\varepsilon_i, \varepsilon_i)$, and $f_{\alpha_i}(0) = 0$, where $\nabla \Psi_i$ is the derivative of the barrier function $\Psi_i : (-\varepsilon_i, \varepsilon_i) \rightarrow \mathbb{R}$ with barrier $2\varepsilon_i$.

Assumption 3. There exists a constant $c_1 > 0$ such that $\|f_{\beta_i}\| \leq c_1$, for all $p_i \in \Omega_i$ and $t \geq 0$.

Assumption 4 [28]. The vectors of unknown time-varying parameters θ_i is piecewise continuous and $\theta_i(t) \in \Theta_i$, for all $t \geq 0$, where Θ_i is a compact set possibly unknown. Moreover the radius of Θ_i satisfies

$$|\theta_i(t) - l_i| \leq \delta_*, \quad (8)$$

where a constant vector $l_i \in \mathbb{R}^m$ is selected to be the center of the parameter set θ_i and δ_* is a unknown constant.

Remark 2. The open interval $(-\varepsilon_i, \varepsilon_i)$ is the range of the smooth function λ_i . The orbit associated to $\mathcal{C}_{i0}$ is such that $\lambda_i(p_i) = 0$, which implies that the positions of the agents are on the given orbits. We do not require the condition $\lambda_i(p_i(0)) = 0$.

Remark 3. The path-following task dictates that each vehicle satisfies Eq. (3). The formation task dictates that each vehicle follows the virtual leader and maintains the desired relative positions with its neighbors along given orbits, that is, Eq. (6) holds. When both tasks are achieved, the agents move along the closed orbit at the desired speed and their trajectories can be naturally described by periodic solutions in the Euclidean space.

Remark 4. Eq. (7) provides a linearly parameterizable expression of a time-varying flowfield such that the effect of the flowfield disappears on the desired orbit associated to each vehicle. For example, consider a rotating flowfield

$$f_i(t) = \tanh\left(1 - \sqrt{p_{x_i}^2 + p_{y_i}^2}\right) e^{j\sigma(t)} \vartheta_i, \quad (9)$$

where j is the imaginary unit, $\vartheta_i \in \mathbb{R}$ is the magnitude of the flow, and $\sigma(t) \in \mathbb{R}$ is the rotation rate. Due to the fact that $\lambda_i(p_i) = 1 - \sqrt{p_{x_i}^2 + p_{y_i}^2}$ when the given orbit is a unit circle, we have that $\tanh\left(1 - \sqrt{p_{x_i}^2 + p_{y_i}^2}\right) = \tanh \lambda_i$. Let $f_{\alpha_i} = \tanh \lambda_i$, $f_{\beta_i} = I$, that is the 2×2 identity matrix, and $\theta_i = e^{j\sigma(t)} \vartheta_i$. The above rotating flowfield can be rewritten as in Eq. (7).

Remark 5. Assumption 2 requires that the signs of f_{α_i} and $\nabla \Psi_i$ are consistent and $|f_{\alpha_i}| \leq |\nabla \Psi_i|$. The condition $f_{\alpha_i}(0) = 0$ means that the effect of the flowfield disappears on the desired orbit associated to each vehicle.

As a result the corresponding problem studied in the paper can be formulated as follows.

Formation tracking problem with a virtual leader. Let $i \in [1, n]$. Consider the system (1) and the initial position $p_i(0) \in \Omega_i$. Suppose Assumptions 1 to 4 hold. Design a formation tracking control u_i , with an adaptive estimate for the unknown time-varying flow parameter θ_i , such that the closed-loop system satisfies the conditions (3), (4) and (6).

III. Main Results

A. Open-Loop System

Let $\theta(t) = [\theta_1^T(t), \dots, \theta_n^T(t)]^T \in \mathbb{R}^{nm}$ and $\bar{f}_{\beta_i}(p_i, t) = [0_{2 \times m(i-1)}, f_{\beta_i}(p_i, t), 0_{2 \times m(n-i)}] \in \mathbb{R}^{2 \times nm}$ and rewrite (7) as *

$$f_i(p_i, t, \theta_i(t)) = f_{\alpha_i}(\lambda_i) \bar{f}_{\beta_i}(p_i, t) \theta(t). \quad (10)$$

To obtain the path-following dynamics for the i th follower it is sufficient to differentiate λ_i , yielding

$$\dot{\lambda}_i = \|\nabla \lambda_i\| v_{N_i} + g_{\lambda_i} f_{\beta_{N_i}} \theta, \quad (11)$$

where $v_{N_i} = N_i^T v_i$ denotes the control input projected on the vector N_i normal to the level orbit of the current position of the vehicle, that is, $N_i = \frac{\nabla \lambda_i}{\|\nabla \lambda_i\|}$, $f_{\beta_{N_i}} = N_i^T \bar{f}_{\beta_i}$, and $g_{\lambda_i} = \|\nabla \lambda_i\| f_{\alpha_i}$. For the i th follower, differentiating both sides of Eq.(5) yields

$$\dot{s}_i = v_{T_i} + \frac{\partial s_i}{\partial \lambda_i} \|\nabla \lambda_i\| v_{N_i} + f_{\alpha_i} f_{\theta_i} \theta, \quad (12)$$

where $v_{T_i} = T_i^T v_i$ denotes the control input projected on the vector T_i tangent to the level orbit of the current position of the vehicle, that is, $T_i = [R_1, R_2]^T N_i$, with $R_1 = [0, 1]^T$ and $R_2 = [-1, 0]^T$. Note that $f_{\theta_i} = f_{\beta_{T_i}} + \frac{\partial s_i}{\partial \lambda_i} \|\nabla \lambda_i\| f_{\beta_{N_i}}$ and $f_{\beta_{T_i}} = T_i^T \bar{f}_{\beta_i}$. As a result, the dynamics of ξ_i are

$$\dot{\xi}_i = \frac{\partial \xi_i}{\partial s_i} v_{T_i} + g_{\xi_{\alpha_i}} + g_{\xi_{\beta_i}} f_{\theta_i} \theta, \quad (13)$$

where $g_{\xi_{\alpha_i}} = \frac{\partial \xi_i}{\partial s_i} \frac{\partial s_i}{\partial \lambda_i} \|\nabla \lambda_i\| v_{N_i}$ and $g_{\xi_{\beta_i}} = \frac{\partial \xi_i}{\partial s_i} f_{\alpha_i}$.

Similarly, the path-following dynamics λ_0 and the dynamics of the generalized arc-length ξ_0 for the virtual leader are given as (11) and (13). Without loss of generality, assume the position $p_0(t)$ of the virtual leader is on a virtual orbit for all $t \geq 0$, which implies that $\lambda_0(p_0(0)) = 0$, $v_{N_0} = 0$, $g_{\lambda_0} f_{\beta_{N_0}} = 0$, $g_{\xi_{\alpha_i}} = 0$, and $g_{\xi_{\beta_i}} f_{\theta_i} = 0$. The design for v_{T_0} is as follows.

*In what follows, to simplify notation, we may use θ to denote $\theta(t)$.

For $i = 1, \dots, n$, let

$$\varsigma_i = \sum_{j=0}^n a_{ij} (\xi_i - \xi_j) = a_{i0} (\xi_i - \xi_0) + \sum_{j=1}^n a_{ij} (\xi_i - \xi_j) \quad (14)$$

denote the formation errors. The dynamics of ς_i are

$$\dot{\varsigma}_i = \sum_{j=0}^n a_{ij} \left(\frac{\partial \xi_i}{\partial s_i} v_{T_i} + g_{\xi_{\alpha i}} + g_{\xi_{\beta i}} f_{\theta_i} \theta - \frac{\partial \xi_j}{\partial s_j} v_{T_j} - g_{\xi_{\alpha j}} - g_{\xi_{\beta j}} f_{\theta_j} \theta \right). \quad (15)$$

The equations of the formation-tracking control system consisting of the virtual leader and the followers are given by Eqs. (11), (13) and (15).

B. Controller Design

We first analyze the simple case of stationary virtual leader and then the case of moving virtual leader. For the virtual leader, the controller v_{T_0} is

$$v_{T_0} = 0, \quad (16)$$

which yields $\dot{\xi}_0 = 0$. The point on the virtual orbit associated with the motion of the virtual leader is determined therefore by its initial position.

Consider the candidate Lyapunov function

$$\begin{aligned} V_I = & \sum_{i=1}^n \Psi(\lambda_i) + \frac{1}{2} \sum_{i=1}^n \varsigma_i^2 + \frac{1}{2\gamma} \sum_{i=1}^n \left(l - \hat{\theta}_i \right)^T \left(l - \hat{\theta}_i \right) + \frac{\rho_0}{2\gamma_1} \sum_{i=1}^n (\rho_\alpha - \hat{\rho}_{\alpha i})^2 \\ & + \frac{\rho_0}{2\gamma_2} \sum_{i=1}^n (\rho_\beta - \hat{\rho}_{\beta i})^2 + \frac{1}{2\gamma_3} \sum_{i=1}^n \left(\delta_* - \hat{\delta}_i \right)^2 + \frac{1}{2\gamma_4} \sum_{i=1}^n \left(\delta_{2*} - \hat{\delta}_{2i} \right)^2, \end{aligned} \quad (17)$$

where $l = [l_1^T, \dots, l_n^T]^T \in \mathbb{R}^{nm}$. $\hat{\rho}_{\alpha i}$, $\hat{\rho}_{\beta i}$, $\hat{\delta}_i$ and $\hat{\delta}_{2i}$ are the estimates of l , ρ_α , ρ_β , δ_* and $\delta_{2*} = \delta_* |l|$, respectively. Moreover $\rho_\alpha = \rho_M^2 \delta_*/\rho_0$, $\rho_\beta = \rho_M^2 \delta_{2*}/\rho_0$, where ρ_M is the maximum eigenvalue of the matrix $L_1^T L_1$, and ρ_0 is the minimum eigenvalue of the matrix $(L_1^T + L_1)/2$. The constants γ , γ_1 , γ_2 , γ_3 and γ_4 are positive adaptation gains.

In the candidate Lyapunov function (17), the first term contributes to achieving the path-following objective, i.e., Eqs. (3) and (4). It vanishes when $\lambda_i = 0$. The second term contributes to achieving the formation, i.e., the objective expressed by Eq. (12); it vanishes when $\varsigma_i = 0$. The third term is used to design the parameter update law $\dot{\hat{\theta}}_i$ using the *congelation of variables* method, in which the parameter update law $\dot{\hat{\theta}}_i$ is used to estimate an unknown constant vector l instead of $\theta(t)$. The next four terms contribute to achieving the estimation of the eigenvalue-based parameters associated with L_1 and the estimation of the unknown

parameters associated with the bound on the time-varying parameters. Differentiating both sides of Eq.(17) along the trajectories of Eqs. (11), (13) and (15) yields

$$\begin{aligned}
\dot{V}_I &= \sum_{i=1}^n \nabla \Psi_i (\|\nabla \lambda_i\| v_{N_i} + g_{\lambda_i} f_{\beta_{N_i}} \theta) + \sum_{i=1}^n \varsigma_i \sum_{j=0}^n a_{ij} \left(\frac{\partial \xi_i}{\partial s_i} v_{T_i} + g_{\xi_{\alpha_i}} + (g_{\xi_{\beta_i}} f_{\theta_i} - g_{\xi_{\beta_j}} f_{\theta_j}) \theta \right. \\
&\quad \left. - \frac{\partial \xi_j}{\partial s_j} v_{T_j} - g_{\xi_{\alpha_j}} \right) - \frac{1}{\gamma} \sum_{i=1}^n (l - \hat{\theta}_i)^T \dot{\hat{\theta}}_i - \frac{\rho_0}{\gamma_1} \sum_{i=1}^n (\rho_{\alpha} - \hat{\rho}_{\alpha_i}) \dot{\hat{\rho}}_{\alpha_i} - \frac{\rho_0}{\gamma_2} \sum_{i=1}^n (\rho_{\beta} - \hat{\rho}_{\beta_i}) \dot{\hat{\rho}}_{\beta_i} \\
&\quad - \frac{1}{\gamma_3} \sum_{i=1}^n (\delta_* - \hat{\delta}_i) \dot{\hat{\delta}}_i - \frac{1}{\gamma_4} \sum_{i=1}^n (\delta_{2*} - \hat{\delta}_{2i}) \dot{\hat{\delta}}_{2i} \\
&= \sum_{i=1}^n \nabla \Psi_i (\|\nabla \lambda_i\| v_{N_i} + g_{\lambda_i} f_{\beta_{N_i}} (\theta - l + \hat{\theta}_i)) \\
&\quad + \sum_{i=1}^n \varsigma_i \sum_{j=0}^n a_{ij} \left(\frac{\partial \xi_i}{\partial s_i} v_{T_i} + g_{\xi_{\alpha_i}} + (g_{\xi_{\beta_i}} f_{\theta_i} - g_{\xi_{\beta_j}} f_{\theta_j}) (\theta - l + \hat{\theta}_i) - \frac{\partial \xi_j}{\partial s_j} v_{T_j} - g_{\xi_{\alpha_j}} \right) \\
&\quad + \sum_{i=1}^n (l - \hat{\theta}_i)^T \left(-\frac{1}{\gamma} \dot{\hat{\theta}}_i + \nabla \Psi_i g_{\lambda_i} f_{\beta_{N_i}}^T + \varsigma_i \sum_{j=0}^n a_{ij} (g_{\xi_{\beta_i}} f_{\theta_i}^T - g_{\xi_{\beta_j}} f_{\theta_j}^T) \right) \\
&\quad - \frac{\rho_0}{\gamma_1} \sum_{i=1}^n (\rho_{\alpha} - \hat{\rho}_{\alpha_i}) \dot{\hat{\rho}}_{\alpha_i} - \frac{\rho_0}{\gamma_2} \sum_{i=1}^n (\rho_{\beta} - \hat{\rho}_{\beta_i}) \dot{\hat{\rho}}_{\beta_i} - \frac{1}{\gamma_3} \sum_{i=1}^n (\delta_* - \hat{\delta}_i) \dot{\hat{\delta}}_i - \frac{1}{\gamma_4} \sum_{i=1}^n (\delta_{2*} - \hat{\delta}_{2i}) \dot{\hat{\delta}}_{2i} \quad (18)
\end{aligned}$$

consider

$$v_{N_i} = -\frac{1}{\|\nabla \lambda_i\|} (g_{\lambda_i} f_{\beta_{N_i}} \hat{\theta}_i + k_1 f_{\alpha_i}), \quad (19a)$$

$$v_{T_i} = \left(\frac{\partial \xi_i}{\partial s_i} \right)^{-1} \left(-g_{\xi_{\alpha_i}} - \sum_{j=0}^n a_{ij} (g_{\xi_{\beta_i}} f_{\theta_i} - g_{\xi_{\beta_j}} f_{\theta_j}) \hat{\theta}_i - k_2 \varsigma_i \right), \quad (19b)$$

$$\dot{\hat{\theta}}_i = \gamma \left(\nabla \Psi_i g_{\lambda_i} f_{\beta_{N_i}}^T + \varsigma_i \sum_{j=0}^n a_{ij} (g_{\xi_{\beta_i}} f_{\theta_i}^T - g_{\xi_{\beta_j}} f_{\theta_j}^T) \right), \quad (19c)$$

in which the control parameters k_1 and k_2 are be selected. Let $\varsigma_0 = 0$ and $\hat{\theta}_0 = 0$, then v_{T_i} in Eq. (19b) is the same as v_{T_0} in Eq. (16). Substituting Eq. (19) into Eq. (18) yields

$$\begin{aligned}
\dot{V}_I &= \sum_{i=1}^n \nabla \Psi_i (-k_1 f_{\alpha_i} + g_{\lambda_i} f_{\beta_{N_i}} (\theta - l)) - k_2 \varsigma^T \frac{L_1^T + L_1}{2} \varsigma + \Xi_1 + \Xi_2 - \frac{\rho_0}{\gamma_1} \sum_{i=1}^n (\rho_{\alpha} - \hat{\rho}_{\alpha_i}) \dot{\hat{\rho}}_{\alpha_i} \\
&\quad - \frac{\rho_0}{\gamma_2} \sum_{i=1}^n (\rho_{\beta} - \hat{\rho}_{\beta_i}) \dot{\hat{\rho}}_{\beta_i} - \frac{1}{\gamma_3} \sum_{i=1}^n (\delta_* - \hat{\delta}_i) \dot{\hat{\delta}}_i - \frac{1}{\gamma_4} \sum_{i=1}^n (\delta_{2*} - \hat{\delta}_{2i}) \dot{\hat{\delta}}_{2i}, \quad (20)
\end{aligned}$$

where $\varsigma = [\varsigma_1, \dots, \varsigma_n]^T$, $\Xi_1 = \sum_{i=1}^n \varsigma_i \sum_{j=0}^n a_{ij} (g_{\xi_{\beta_i}} f_{\theta_i} - g_{\xi_{\beta_j}} f_{\theta_j}) (\theta - l)$ and $\Xi_2 = \sum_{i=1}^n \varsigma_i \sum_{j=0}^n a_{ij} \sum_{i=0}^n a_{ji} (g_{\xi_{\beta_j}} f_{\theta_j}$

$-g_{\xi_{\beta i}} f_{\theta_i}) \hat{\theta}_j$. By definition of λ_i in [7] and by Assumptions 3 and 4 the inequalities

$$\begin{aligned} \|f_{\beta_{N_i}}\| &\leq c_1, \quad \|f_{\beta_{T_i}}\| \leq c_1, \quad \|\nabla \lambda_i\| \leq c_2, \quad \left| \frac{\partial \xi_i}{\partial s_i} \right| \leq c_3, \quad \left| \frac{\partial s_i}{\partial \lambda_i} \right| \leq \int_0^{2\pi} \sqrt{(\dot{p}_{x_i}(\tau))^2 + (\dot{p}_{y_i}(\tau))^2} d\tau \leq c_4, \\ \left\| \frac{\partial \xi_i}{\partial s_i} f_{\theta_i} \right\| &\leq c_1 c_3 (1 + c_2 c_4) = c_5, \quad |g_{\xi_{\beta i}} f_{\theta_i}| \leq c_3 c_5 |f_{\alpha_i}|, \quad |\theta - l| \leq \delta_* \end{aligned} \quad (21)$$

hold on the set Ω_i , for some positive constants c_k , $k = 1, \dots, 5$.

Let $\varsigma_\theta = [\varsigma_1(\theta - l), \dots, \varsigma_n(\theta - l)]^T$, $g_{\xi_\beta} = [g_{\xi_{\beta 1}} f_{\theta_1}, \dots, g_{\xi_{\beta n}} f_{\theta_n}]^T$ and $f_\alpha = [f_{\alpha_1}, \dots, f_{\alpha_n}]^T$ and note that

$$\Xi_1 = \Xi_1^T = \sum_{i=1}^n \varsigma_i(\theta - l)^T \sum_{j=0}^n a_{ij} (g_{\xi_{\beta i}} f_{\theta_i}^T - g_{\xi_{\beta j}} f_{\theta_j}^T). \quad (22)$$

Due to the fact that $g_{\xi_{\beta 0}} f_{\theta_0}^T = 0$ we have

$$\sum_{j=0}^n a_{ij} (g_{\xi_{\beta i}} f_{\theta_i}^T - g_{\xi_{\beta j}} f_{\theta_j}^T) = a_{i0} g_{\xi_{\beta i}} f_{\theta_i}^T + \sum_{j=1}^n a_{ij} (g_{\xi_{\beta i}} f_{\theta_i}^T - g_{\xi_{\beta j}} f_{\theta_j}^T) = L_1 \otimes I g_{\xi_\beta}, \quad (23)$$

which yields

$$\Xi_1 = \varsigma_\theta^T L_1 \otimes I g_{\xi_\beta}. \quad (24)$$

From Eqs. (21) one has

$$\varsigma_\theta^T L_1 \otimes I g_{\xi_\beta} \leq \|\varsigma_\theta^T\| \|L_1 \otimes I\| \|g_{\xi_\beta}\| \leq \rho_M \|\varsigma_\theta^T\| \|g_{\xi_\beta}\| \leq c_3 c_5 \rho_M \delta_* \|\varsigma^T\| \|f_\alpha\|. \quad (25)$$

Finally let $\hat{\theta}_{a_i} = a_{ji} \hat{\theta}_i^T$, $\hat{\theta}_a = [\hat{\theta}_{a_1}^T, \dots, \hat{\theta}_{a_n}^T]^T$ and $\hat{\theta} = [\hat{\theta}_1^T, \dots, \hat{\theta}_n^T]^T$. Then

$$\Xi_2 = \sum_{i=0}^n a_{ji} \hat{\theta}_i^T \sum_{j=0}^n a_{ij} (g_{\xi_{\beta i}} f_{\theta_i}^T - g_{\xi_{\beta j}} f_{\theta_j}^T) = \sum_{i=1}^n a_{ji} \hat{\theta}_i^T \sum_{j=0}^n a_{ij} (g_{\xi_{\beta i}} f_{\theta_i}^T - g_{\xi_{\beta j}} f_{\theta_j}^T) = \hat{\theta}_a^T L_1 \otimes I g_{\xi_\beta}. \quad (26)$$

From (21) one has

$$\hat{\theta}_a^T L_1 \otimes I g_{\xi_\beta} \leq \|\hat{\theta}_a^T\| \|L_1 \otimes I\| \|g_{\xi_\beta}\| \leq c_3 c_5 \rho_M \|\hat{\theta}^T\| \|f_\alpha\|. \quad (27)$$

According to Lemma 2 in [31],

$$n^{1/2} \|\varsigma\| = n^{1/2} \left(\sum_{i=1}^n \varsigma_i^2 \right)^{1/2} \geq \sum_{i=1}^n |\varsigma_i|. \quad (28)$$

Exploiting Eqs. (21), (25), (27) and (28) we conclude that

$$\begin{aligned}
\dot{V}_I \leq & - \sum_{i=1}^n k_1 \nabla \Psi_i f_{\alpha_i} + \sum_{i=1}^n c_1 c_2 \delta_* |\nabla \Psi_i f_{\alpha_i}| - k_2 \varsigma^T \frac{L_1^T + L_1}{2} \varsigma + c_3 c_5 \rho_M \delta_* \|\varsigma^T\| \|f_\alpha\| \\
& + n^{1/2} c_3 c_5 \rho_M \delta_* \|\hat{\theta}^T\| \|f_\alpha\| \|\varsigma\| - \frac{\rho_0}{\gamma_1} \sum_{i=1}^n (\rho_\alpha - \hat{\rho}_{\alpha_i}) \dot{\rho}_{\alpha_i} - \frac{\rho_0}{\gamma_2} \sum_{i=1}^n (\rho_\beta - \hat{\rho}_{\beta_i}) \dot{\rho}_{\beta_i} - \frac{1}{\gamma_3} \sum_{i=1}^n (\delta_* - \hat{\delta}_i) \dot{\delta}_i \\
& - \frac{1}{\gamma_4} \sum_{i=1}^n (\delta_{2*} - \hat{\delta}_{2i}) \dot{\delta}_{2i}.
\end{aligned} \tag{29}$$

On the set $\Phi_I = \left\{ \left(\lambda_i, \varsigma_i, l - \hat{\theta}_i, \rho_\alpha - \hat{\rho}_{\alpha_i}, \rho_\beta - \hat{\rho}_{\beta_i}, \delta_* - \hat{\delta}_i, \delta_{2*} - \hat{\delta}_{2i} \right) \mid V_I \leq c_6 \right\}$, for some $c_6 > 0$, one has

$$|\hat{\theta}_i| \leq |l - \hat{\theta}_i| + |l| \leq \sqrt{2\gamma c_6} + |l| = c_7 + |l|. \tag{30}$$

By applying Young's inequality

$$\|\varsigma\| \|f_\alpha\| = \left(\sum_{i=1}^n \varsigma_i^2 \right)^{1/2} \left(\sum_{i=1}^n f_{\alpha_i}^2 \right)^{1/2} \leq \frac{\rho_M}{2} \sum_{i=1}^n \varsigma_i^2 + \frac{1}{2\rho_M} \sum_{i=1}^n f_{\alpha_i}^2 \tag{31}$$

and Eq. (30) to Eq. (29) yields

$$\begin{aligned}
\dot{V}_I \leq & - \left(k_1 - \left(c_1 c_2 - \frac{1}{2} c_3 c_5 \left(1 + n^{1/2} c_7 \right) \right) \hat{\delta}_* - \frac{1}{2} c_3 c_5 n^{1/2} \hat{\delta}_{2*} \right) \sum_{i=1}^n \nabla \Psi_i f_{\alpha_i} - \frac{1}{2} c_3 c_5 \delta_* \left(1 + n^{1/2} (c_7 + |l|) \right) \\
& \times \sum_{i=1}^n (\nabla \Psi_i f_{\alpha_i} - f_{\alpha_i}^2) - \rho_0 \left(k_2 - \frac{1}{2} c_3 c_5 \left(1 + n^{1/2} c_7 \right) \hat{\rho}_{\alpha_i} - \frac{1}{2} c_3 c_5 n^{1/2} \hat{\rho}_{\beta_i} \right) \sum_{i=1}^n \varsigma_i^2 \\
& + \rho_0 \sum_{i=1}^n (\rho_\alpha - \hat{\rho}_{\alpha_i}) \left(-\frac{1}{\gamma_1} \dot{\rho}_{\alpha_i} + \frac{1}{2} c_3 c_5 \left(1 + n^{1/2} c_7 \right) \varsigma_i^2 \right) + \rho_0 \sum_{i=1}^n (\rho_\beta - \hat{\rho}_{\beta_i}) \left(-\frac{1}{\gamma_2} \dot{\rho}_{\beta_i} + \frac{1}{2} c_3 c_5 n^{1/2} \varsigma_i^2 \right) \\
& + \sum_{i=1}^n (\delta_* - \hat{\delta}_i) \left(-\frac{1}{\gamma_3} \dot{\delta}_i + \left(c_1 c_2 - \frac{1}{2} c_3 c_5 \left(1 + n^{1/2} c_7 \right) \right) \nabla \Psi_i f_{\alpha_i} \right) \\
& + \sum_{i=1}^n (\delta_{2*} - \hat{\delta}_{2i}) \left(-\frac{1}{\gamma_4} \dot{\delta}_{2i} + \frac{1}{2} c_3 c_5 n^{1/2} \nabla \Psi_i f_{\alpha_i} \right).
\end{aligned} \tag{32}$$

Let

$$\begin{aligned}
k_1 &= \left(c_1 c_2 - \frac{1}{2} c_3 c_5 \left(1 + n^{1/2} c_7 \right) \right) \hat{\delta}_* + \frac{1}{2} c_3 c_5 n^{1/2} \hat{\delta}_{2*} + \epsilon_1, \\
k_2 &= \frac{1}{2} c_3 c_5 \left(1 + n^{1/2} c_7 \right) \hat{\rho}_{\alpha_i} + \frac{1}{2} c_3 c_5 n^{1/2} \hat{\rho}_{\beta_i} + \epsilon_2, \\
\dot{\rho}_{\alpha_i} &= \frac{1}{2} \gamma_1 c_3 c_5 \left(1 + n^{1/2} c_7 \right) \varsigma_i^2, \\
\dot{\rho}_{\beta_i} &= \frac{1}{2} \gamma_2 c_3 c_5 n^{1/2} \varsigma_i^2,
\end{aligned}$$

$$\begin{aligned}\dot{\delta}_i &= \gamma_3 \left(c_1 c_2 - \frac{1}{2} c_3 c_5 \left(1 + n^{1/2} c_7 \right) \right) \nabla \Psi_i f_{\alpha_i}, \\ \dot{\delta}_{2i} &= \frac{1}{2} \gamma_4 c_3 c_5 n^{1/2} \nabla \psi_i f_{\alpha_i},\end{aligned}\tag{33}$$

with ϵ_1 and ϵ_2 positive constants. Substituting Eqs. (33) into Eq. (32) yields

$$\dot{W}_I \leq W_I,\tag{34}$$

where

$$W_I = -\epsilon_1 \sum_{i=1}^n \nabla \Psi_i f_{\alpha_i} - \rho_0 \epsilon_2 \sum_{i=1}^n \varsigma_i^2.$$

Recalling Assumptions 1 and 2, Lemma 1 and Eq. (33), we conclude that $W_I \leq 0$ hence $\dot{W}_I \leq 0$. Note now that, by Eq. (19), the closed-loop system associated to the formation tracking control system for the i th follower is

$$\begin{aligned}\dot{\lambda}_i &= -k_1 f_{\alpha_i} + g_{\lambda_i} f_{\beta_{Ni}} \left(\theta - \hat{\theta}_i \right), \\ \dot{\varsigma}_i &= \sum_{j=0}^n a_{ij} \left(-k_2 (\varsigma_i - \varsigma_j) + (g_{\xi_{\beta_i}} f_{\theta_i} - g_{\xi_{\beta_j}} f_{\theta_j}) \left(\theta - \hat{\theta}_i \right) - \sum_{i=0}^n a_{ji} (g_{\xi_{\beta_j}} f_{\theta_j} - g_{\xi_{\beta_i}} f_{\theta_i}) \hat{\theta}_j \right), \\ \dot{\hat{\theta}}_i &= \gamma \left(\nabla \Psi_i g_{\lambda_i} f_{\beta_{Ni}}^T + \varsigma_i \sum_{j=0}^n a_{ij} \left(g_{\xi_{\beta_i}} f_{\hat{\theta}_i}^T - g_{\xi_{\beta_j}} f_{\hat{\theta}_j}^T \right) \right),\end{aligned}\tag{35}$$

with the control gains k_1 and k_2 satisfying conditions (33).

In the following we will consider the case of the moving virtual leader. The control v_{T_0} for the moving virtual leader is set as

$$\begin{aligned}v_{T_0} &= - \left(\frac{\partial \xi_0}{\partial s_0} \right)^{-1} k_2 \varsigma_0, \\ \dot{\varsigma}_0 &= -k_2 \sum_{i=1}^n a_{i0} \varsigma_i + \eta_*(t),\end{aligned}\tag{36}$$

where ς_0 is used to adjust the speed of the virtual leader to maintain the formation performance. $\eta_*(t) \in \mathbb{R}$ is a bounded signal, which forces the virtual leader to move along its virtual orbit at a specific orbital speed. The control term $k_2 \varsigma_0$ in the first of equations of (36) is a *formation adjustment unit* to keep the formation accuracy of the whole formation. If we let $\eta_*(t) = 0$, $\varsigma_0(0) = 0$ and $k_2 = 0$, the control law (36) for the moving virtual leader is the same as the one in Eq. (16) for the stationary virtual leader.

Remark 6. A similar idea has been proposed in [32], in which a function of the error between the state

of the i th reference node and the state of the i th agent is added into the instantiation dynamics of the i th reference node in the virtual structure. It is indicated therein that such additional function can improve the consensus accuracy but a specific form was not given.

Remark 7. When the formation objective is achieved, one finds that $\varsigma_i = 0$ and then $\dot{\varsigma}_0 = \eta_*$ as $t \rightarrow \infty$. Note that $\dot{\xi}_0 = -k_2\varsigma_0$. As a result, the virtual leader and all the followers move along their desired orbits with the given speed.

Consider now the candidate Lyapunov function

$$V_{II} = V_I + \frac{1}{2} \left(\varsigma_0 - \int_0^t \eta_*(\tau) d\tau \right)^2. \quad (37)$$

Differentiating both sides of Eq. (37) along v_{N_i} , v_{T_i} and $\dot{\theta}_i$ as in Eqs. (19) yields

$$\begin{aligned} \dot{V}_{II} &= \sum_{i=1}^n \nabla \Psi_i \left(-k_1 f_{\alpha_i} + g_{\lambda_i} f_{\beta_{N_i}} (\theta - l) \right) + \left(\varsigma_0 - \int_0^t \eta_*(\tau) d\tau \right) (\dot{\varsigma}_0 - \eta_*) \\ &\quad + \sum_{i=1}^n \varsigma_i \sum_{j=0}^n a_{ij} \left(-k_2 (\varsigma_i - \varsigma_j) + (g_{\xi_{\beta_i}} f_{\theta_i} - g_{\xi_{\beta_j}} f_{\theta_j}) (\theta - l) - \sum_{i=0}^n a_{ji} (g_{\xi_{\beta_j}} f_{\theta_j} - g_{\xi_{\beta_i}} f_{\theta_i}) \hat{\theta}_j \right) \\ &\quad - \frac{\rho_0}{\gamma_1} \sum_{i=1}^n (\rho_\alpha - \hat{\rho}_{\alpha_i}) \dot{\hat{\rho}}_{\alpha_i} - \frac{\rho_0}{\gamma_2} \sum_{i=1}^n (\rho_\beta - \hat{\rho}_{\beta_i}) \dot{\hat{\rho}}_{\beta_i} - \frac{1}{\gamma_3} \sum_{i=1}^n (\delta_* - \hat{\delta}_i) \dot{\hat{\delta}}_i - \frac{1}{\gamma_4} \sum_{i=1}^n (\delta_{2*} - \hat{\delta}_{2i}) \dot{\hat{\delta}}_{2i} \\ &= \sum_{i=1}^n \nabla \Psi_i \left(-k_1 f_{\alpha_i} + g_{\lambda_i} f_{\beta_{N_i}} (\theta - l) \right) + \left(\varsigma_0 - \int_0^t \eta_*(\tau) d\tau \right) (\dot{\varsigma}_0 - \eta_* + k_2 \sum_{i=1}^n a_{i0} \varsigma_i) \\ &\quad - k_2 \varsigma^T \frac{L_1^T + L_1}{2} \varsigma + \sum_{i=1}^n \varsigma_i \sum_{j=0}^n a_{ij} \left((g_{\xi_{\beta_i}} f_{\theta_i} - g_{\xi_{\beta_j}} f_{\theta_j}) (\theta - l) - \sum_{i=0}^n a_{ji} (g_{\xi_{\beta_j}} f_{\theta_j} - g_{\xi_{\beta_i}} f_{\theta_i}) \hat{\theta}_j \right) \\ &\quad - \frac{\rho_0}{\gamma_1} \sum_{i=1}^n (\rho_\alpha - \hat{\rho}_{\alpha_i}) \dot{\hat{\rho}}_{\alpha_i} - \frac{\rho_0}{\gamma_2} \sum_{i=1}^n (\rho_\beta - \hat{\rho}_{\beta_i}) \dot{\hat{\rho}}_{\beta_i} - \frac{1}{\gamma_3} \sum_{i=1}^n (\delta_* - \hat{\delta}_i) \dot{\hat{\delta}}_i - \frac{1}{\gamma_4} \sum_{i=1}^n (\delta_{2*} - \hat{\delta}_{2i}) \dot{\hat{\delta}}_{2i}. \quad (38) \end{aligned}$$

Substituting Eqs. (36) into Eq. (38) and applying a similar analysis to that of the case of the stationary virtual leader yield $\dot{V}_{II} \leq W_I \leq 0$, from which we obtain the following result.

Theorem 1. Suppose that the initial position of each vehicle is such that $p_i(0) \in \Omega_i$. Assume moreover that Assumptions 1 to 4 hold. Then, for $l = 0, \dots, n$, the formation-tracking problem with a virtual leader is solved by the formation tracking control

$$u_l = \begin{bmatrix} N_l^T \\ T_l^T \end{bmatrix}^{-1} \begin{bmatrix} v_{N_l} \\ v_{T_l} \end{bmatrix}, \quad (39)$$

where $v_{N_0} = 0$, v_{T_0} is given in (36) and v_{N_i} , v_{T_i} and $\dot{\theta}_i$ are as given in Eqs. (19).

Proof. Let $p_i(0) \in \Omega_i$ and note that the set $\Phi_{II} = \left\{ \left(\lambda_i, \varsigma_i, l - \hat{\theta}_i, \rho_\alpha - \hat{\rho}_{\alpha_i}, \rho_\beta - \hat{\rho}_{\beta_i}, \delta_* - \hat{\delta}_i, \delta_{2*} - \hat{\delta}_{2i}, \varsigma_0 - \right. \right.$

$\int_0^t \eta_*(\tau) d\tau \Big| V_{II} \leq c_6 \Big\}$ is closed by continuity. Note now that $|\lambda_i| < \varepsilon_i$, $|\varsigma_i| \leq \sqrt{2c_6}$, $|l - \hat{\theta}_i| \leq \sqrt{2c_6\gamma}$, $|\rho_\alpha - \hat{\rho}_{\alpha_i}| \leq \sqrt{2c_6\gamma_1}$, $|\rho_\beta - \hat{\rho}_{\beta_i}| \leq \sqrt{2c_6\gamma_2}$, $|\delta_* - \hat{\delta}_i| \leq \sqrt{2c_6\gamma_3}$, $|\delta_{2*} - \hat{\delta}_{2i}| \leq \sqrt{2c_6\gamma_4}$ and $\left| \varsigma_0 - \int_0^t \eta_*(\tau) d\tau \right| \leq \sqrt{2c_6}$. Thus the set Φ_{II} is compact. As a result the closed-loop system (35) is Lipschitz continuous on the set Φ_{II} and also piecewise continuous in t , which implies that solutions exist and are unique.

Note now that the value of V_{II} is non-increasing along the trajectories of the system. When the initial value of V_{II} is finite, the solution stays in Φ_{II} , which implies that $|\lambda_i(p_i(t))| < \varepsilon_i$ by the definition of the barrier function Ψ_i in [29]. By the LaSalle-Yoshizawa Theorem cite, we have that

$$\lim_{t \rightarrow +\infty} W_I(t) = 0,$$

that is

$$\lim_{t \rightarrow +\infty} \nabla \Psi_i(t) f_{\alpha_i}(\lambda_i(t)) = 0 \quad (40)$$

and

$$\lim_{t \rightarrow +\infty} \varsigma_i(t) = \lim_{t \rightarrow \infty} \sum_{j=0}^n a_{ij}(\xi_i - \xi_j) = 0. \quad (41)$$

From Eq.(40) and Assumption 2 we therefore conclude that $\lim_{t \rightarrow +\infty} \lambda_i(t) = 0$. Moreover condition (41) and Assumption 1 imply that condition (6) is satisfied. $\square$

Remark 8. The proof of Lemma 1 in [7] yields $\|\nabla \lambda_i\| = \left\| \frac{\partial \lambda_i}{\partial p_i} \right\| \neq 0$, for all $p_i \in \Omega_i$. Reviewing the proof of Theorem 1 we conclude that $|\lambda_i(p_i(t))| < \varepsilon_i$, which implies $p_i(t) \in \Omega_i$, for all $t \geq 0$. On the set Φ_{II} the terms $\nabla \Psi_i$, g_{λ_i} , $f_{\beta_{N_i}}$, $g_{\xi_{\beta_i}}$ and f_{θ_i} are bounded, so is $\hat{\theta}_i$ in (19c). Due to the fact that $|l - \hat{\theta}_i| \leq \sqrt{2c_6\gamma}$ in the set Φ_{II} , and l is a constant vector, we conclude that $\hat{\theta}_i$ is also bounded. Note that the definition of the generalized arc-lengths yields $|\partial \xi_i / \partial s_i| \geq \varepsilon > 0$. Therefore, the control law (19) is well defined.

Remark 9. Reviewing the proof of Theorem 1 we observe that the closed-loop solutions stay in Φ_{II} . Due to the fact that $|\rho_\alpha - \hat{\rho}_{\alpha_i}| \leq \sqrt{2c_6\gamma}$ in the set Φ_{II} and $\rho_\alpha = \rho_M^2 \delta_* / \rho_0$ is a constant we can conclude that $\hat{\rho}_{\alpha_i}$ is bounded. The same applies to $\hat{\rho}_{\beta_i}$, $\hat{\delta}_i$ and $\hat{\delta}_{2i}$. As a result, the control gains k_1 and k_2 are bounded.

Remark 10. From Eqs. (35) we conclude that the convergence rates of the states of the resulting path following subsystem and of the resulting formation subsystem depends on the control gains k_1 and k_2 . The convergence rates of the systems with states $\hat{\theta}_i$, $\hat{\rho}_{\alpha_i}$, $\hat{\rho}_{\beta_i}$, $\hat{\delta}_i$ and $\hat{\delta}_{2i}$ depend on the adaptation gains γ , γ_1 , γ_2 , γ_3 and γ_4 , respectively. To speed up the convergence rates one can increase the gains. However, the increase in gains results in increased control energy and potential lack of noise attenuation.

Remark 11. From Eqs. (19) we conclude that the control law contains only ς_i , but not $\dot{\varsigma}_i$. $\dot{\varsigma}_i$ is used during the analysis. Note that ς_i in Eq. (14) only depends on the ξ_i , the state of the agent itself, and on ξ_j , the neighboring states.

Remark 12. The states and the estimate of the i th vehicle and its neighboring states ξ_j and $g_{\xi_{\beta_j}} f_{\theta_j}$ are

Fig. 2 Digraph illustrating the connections between the vehicles.

used to design the control law (19). Hence our proposed control law is distributed.

IV. Simulation Results

The directed network among vehicles is shown in Figure 1. The selected trajectories for the vehicles are concentric ellipses with different semi-major axis and semi-minor axis, that is,

$$C_{l0} : \left(\frac{p_{x_l}}{e_l a} \right)^2 + \left(\frac{p_{y_l}}{e_l b} \right)^2 = 1,$$

where $e_l = 1 + 0.5(l - 1)$, $a = 3$, $b = 2$, $l = 0, 1, 2, 3, 4$. For $i = 1, \dots, 4$ the effect of the time-varying flowfield is described by

$$\begin{aligned} f_i(p_i, t) &= f_{\alpha_i}(\lambda_i) f_{\beta_i}(p_i, t) \theta_i(t) \\ &= \lambda_i \begin{bmatrix} (\cos(0.1 + 0.01(i-1))t) & 0 \\ 0 & \sin((0.1 + 0.01(i-1))t) \end{bmatrix} \begin{bmatrix} \sin((0.1 + 0.01(i-1))t) \\ \cos((0.2 + 0.01(i-1))t) \end{bmatrix}. \end{aligned}$$

The parameters in this case have been selected as $k_1 = 50 + 10\hat{\delta}_* + 10\hat{\delta}_{2*}$, $k_2 = 50 + 10\hat{\rho}_{\alpha_i} + 10\hat{\rho}_{\beta_i}$ and $\gamma = \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 1$. In these simulations the desired formation is given by an in-line pattern and thus the generalized arc-lengths ξ_l are selected as $\xi_l = s_l/e_l$.

Case 1: The bounded signal for the virtual leader is $\eta_* = 0.1$. The motion of the vehicles is illustrated in Figure 3a, where o , $\square$, $+$ and $\star$ denote the vehicles' positions at $t = 0$, $t = 5$, $t = 15$ and $t = 20$, respectively. From this figure one can see that the four vehicles converge to the given orbits, form the desired formation and moves along the given orbits. The time histories of the path following errors λ_i and of the formation errors $\xi_i - \xi_0$ are plotted in Figures 3b and 3c, respectively. The time histories of the control inputs v_{N_i} and v_{T_i} are plotted in Figures 3d. Consistently with Theorem 1, path following and formation tracking are achieved.

Fig. 3 Formation tracking motion with a moving virtual leader.

Case 2: The performance in the case of input saturation $sat(v_i(t)) = [sat(v_{N_i}(t)), sat(v_{T_i}(t))]^T$, where $sat(v_{N_i}(t)) = sign(v_{N_i}(t)) \min\{|v_{N_i}|, 100\}$ and $sat(v_{T_i}(t)) = sign(v_{T_i}(t)) \min\{|v_{N_i}|, 100\}$, is now discussed. The motion of the vehicles, the path following errors λ_i , the formation errors $\xi_i - \xi_0$ and the control inputs v_{N_i} and v_{T_i} are displayed in Figures 4a–4d, where o , $\square$, $+$ and $\star$ denote the vehicles' positions at $t = 0$, $t = 5$, $t = 15$ and $t = 20$, respectively. Compared to Case 1 the saturated inputs continue to achieve the control task and its performance remains of a good level.

V. Conclusion

An adaptive method to solve the formation tracking control problem in the presence of a flowfield with time-varying flow parameters has been presented. First, the case of a stationary virtual leader with unidirectional connected followers is considered and the formation tracking control law is designed by using the *congelation of variables* method. In order to avoid the use of the eigenvalues of the Laplacian matrix and a bound on the time-varying flow parameters in the control law design, adaptive estimations are designed. By designing the *formation adjustment unit* for the virtual leader the proposed control can be extended to the case of moving virtual leader. In future work the leaderless formation tracking problem for directed networked vehicles will be considered.

Fig. 4 Formation tracking motion with saturation inputs.

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