Determination of the mode I crack tip opening rate and the rate dependent cohesive properties for structural adhesive joints using digital image correlation

F. Sun *, R. Zhang, B. R. K. Blackman *

*Department of Mechanical Engineering, Imperial College London, Exhibition Road, London
SW7 2AZ, UK*

* Correspondence to

f.sun@imperial.ac.uk (F.S.)

b.blackman@imperial.ac.uk (B.R.K.B.)

Abstract

The present work addresses two key issues relating to the study of rate effects in adhesively bonded joints. Firstly, the accurate determination of the crack tip strain rate and secondly the accurate determination of cohesive zone length. The rate-dependent fracture behaviour of adhesive joints bonded with either a toughened epoxy or a ductile polyurethane adhesive was investigated under mode I loading rates ranging from 0.1 mm/min to 1.0 m/s with digital image correlation (DIC) analysis. The traction-separation laws (TSLs) were determined by measuring the J -integral values and the crack tip opening displacements simultaneously. An analytical method is proposed to correlate the crack tip opening velocity with the external loading rate. The lengths of the cohesive zones were measured, and the values were examined by thickness-independent and -dependent models. It was observed that the cohesive properties of the two adhesives exhibited very different rate dependences. An analytical tool is developed for the determination of the strain rates for TSLs using a direct method such as DIC.

Keywords: Crack tip opening velocity, Length of cohesive zone, Rate dependent traction-separation law, Fracture of adhesive joint, Digital image correlation

Nomenclature

English alphabet

a	crack length
b	width of a DCB specimen
c	elastic wave speed in a solid

E	Young's modulus
G_c	fracture toughness measured using LEFM methods
$G_{c,b}$	fracture toughness measured using simple beam theory
h	thickness of a beam
h_a	thickness of the adhesive layer
J	J -integral value
J_c	fracture toughness measured using J -integral
J_c^*	fracture toughness at a reference loading rate
J_{ext}	J -integral along the contour of the external boundary
J_{tip}	J -integral along the contour around the crack tip
k	initial stiffness of the cohesive law
ℓ_{cz}	length of the cohesive zone
M	scale factor for cohesive zone length
n_j	components of the unit vector normal to Γ
P	external load
t	time
t_f	total time for crack initiation
T	kinetic energy density
u_i	displacement vector components
U_k	kinetic energy

V_{ext}	external loading rate
V_{ext}^*	the reference loading rate
v_{tip}	crack tip opening velocity
$v_{\text{tip}}^{\text{n}}$	nominal crack tip opening velocity
$v_{\text{tip}}^{\text{n},*}$	crack tip opening velocity at a reference loading rate
w	strain energy density

Greek alphabet

Γ	integral contour
δ	displacement of the load point
δ_{c}	critical δ for the onset of crack propagation
Δ	initial crack tip opening displacement
Δ_{f}	critical crack tip opening displacement to failure
$\bar{\ell}$	characteristic length of deformation process
ε_{ij}	strain tensor components
ε_{yy}	strain in the loading direction
$\dot{\varepsilon}_{\text{tip}}$	crack tip opening strain rate
$\dot{\varepsilon}_{\text{tip}}^{\text{n}}$	nominal crack tip opening strain rate
$\dot{\varepsilon}_{\text{tip}}^{\text{n},*}$	crack tip opening strain rate at a reference loading rate
λ_1, λ_2	shape factors

ρ	density of a solid
σ_0	cohesive strength
σ_0^*	cohesive strength at a reference loading rate
σ_{ij}	stress tensor components
σ	cohesive stress
Ψ	dimensionless parameter
Ψ^*	dimensionless parameter at a reference loading rate
ω_L, ω_U	rotation angle of the lower/upper beam

Acronyms

CZM	Cohesive Zone Model
DCB	double-cantilever beam
DIC	digital image correlation
LEFM	Linear Elastic Fracture Mechanics
TDCB	tapered double-cantilever beam
TSL	traction-separation law

1. Introduction

Structural adhesives are being extensively adopted by the automotive industry to assemble various body structures due to their excellent load-bearing capability and their energy absorption capacity. During a crash, vehicle structures can be subjected to strain rates of 1-200 /s (Lee et al., 2000), and therefore it is important to understand the effect of the external loading rate, V_{ext} , on the fracture behaviour of the adhesive joints in the structure. Under quasi-static loading, stable cohesive failure usually prevails, whilst under dynamic loading, unstable failure characterized by stick-slip crack growth may occur (Blackman et al., 2009; Kanninen, 1974). In terms of energy absorption, the fracture toughness, G_c , may decrease with increasing V_{ext} (Blackman et al., 2009; Sun et al., 2008), however, some workers have also reported that increasing V_{ext} resulted in an increase in G_c (Carlberger et al., 2009; Schmandt and Marzi, 2018; Zhu et al., 2009). Cohesive Zone Models (CZMs), using traction-separation laws (TSLs) to describe the mechanical properties across the failure interfaces, have been widely employed to analyse the fracture of adhesive joints and composite laminates. Many studies have therefore been conducted to identify the rate sensitivities of the cohesive properties, including G_c , the cohesive strength (σ_0) and the shape of the TSL (Carlberger et al., 2009; Karac et al., 2011; Marzi et al., 2009; Rajan et al., 2018). However, there are still two key issues which remain poorly understood and should therefore be addressed.

The first issue relates to the determination of the crack tip separation strain rate, $\dot{\epsilon}_{\text{tip}}$, or the equivalent separation velocity, v_{tip} . For mode I fracture, one method to determine the rate dependent TSL is through uniaxial tensile testing combined with double-cantilever beam (DCB) or tapered double-cantilever beam (TDCB) fracture testing under varying values of V_{ext} , which give the values of σ_0 and G_c respectively. In this approach the shape of the TSL must be assumed, and a major drawback of this approach is that G_c and σ_0 are not obtained

simultaneously. In order to determine the G_c and σ_0 values for different loading rates, the values of the crack tip strain rate $\dot{\epsilon}_{tip}$ in the fracture test must be matched with the strain rate in the gauge length of the tensile specimen (the tensile velocity divided by the gauge length). Various approaches have been developed to determine the value of $\dot{\epsilon}_{tip}$ in DCB or TDCB specimens, e.g., the analytical approaches (Freund and Lee, 1990; Karac et al., 2011; Smiley and Pipes, 1987) and numerical approaches (Karac et al., 2011; Marzi et al., 2009). However, they are either not suitable for adhesive joints, or they require the measurement of the crack velocity, which can be challenging for high-rate tests, or may require many numerical trial calculations. Recently, Yang et al. (2019) derived a new expression for the crack tip opening velocity based on elastic foundation theory. However, this expression requires the value of the foundation stiffness, the definition of which usually is not straight-forward (Saseendran et al., 2018). Moreover, it is very difficult to determine the value of $\dot{\epsilon}_{tip}$ as it implicitly depends on the cohesive property. An alternative way to determine the rate dependent TSL is to use digital image correlation (DIC). This method is being extensively used to directly extract the accurate TSLs for mode I, mode II and mixed mode loading (Gorman and Thouless, 2019; Högberg et al., 2007; Rajan et al., 2018), as it can acquire the values of G_c , σ_0 and the shape of the TSL simultaneously. However, the remaining problem is the determination of $\dot{\epsilon}_{tip}$ for the TSLs. To date, the relationship between $\dot{\epsilon}_{tip}$ and V_{ext} remains unclear and thus a further exploration is timely.

The second issue relates to the determination of the length of the cohesive zone, ℓ_{cz} , which has been studied by many researchers (Bao and Suo, 1992; Harper and Hallett, 2008; Hillerborg et al., 1976; Hui et al., 2003; Rice, 1980; Soto et al., 2016; Toolabi and Blackman, 2018; Yang and Cox, 2005). Hillerborg et al. (1976), Rice (1980), Hui et al. (2003) developed thickness independent models to estimate the value of ℓ_{cz} . These models assume that ℓ_{cz} is a material property. On the other hand, Bao and Suo (1992), Yang and Cox (2005),

amongst others developed thickness dependent models to estimate ℓ_{cz} for slender structures, which suggest that ℓ_{cz} is not a material property but also depends on the thickness of the structure as well as on the shape of the TSL. These two modelling approaches have been widely followed although their validity is not proven. Harper and Hallett (2008) pointed out that the thickness independent models gave more accurate predictions for slender bodies associated with small cohesive zones whilst the thickness dependent models can better predict ℓ_{cz} in the cases with relatively large cohesive zones. The value of ℓ_{cz} is sensitive to the loading rate as some experimental results have shown decreasing values of ℓ_{cz} with increasing V_{ext} (Rajan et al., 2018), which raises a question concerning the accuracy of the thickness dependent and independent models for determining rate dependent ℓ_{cz} . Hence, an experimental evaluation of the reliability of these models in slender bodies subjected to varying V_{ext} is of interest and will benefit the model selection for analytical and FE solutions.

The main objective of the present work is therefore to evaluate the effects of loading rate on the mode I cohesive properties (G_c , σ_0 , ℓ_{cz}) of adhesive joints bonded with either a toughened epoxy or a ductile polyurethane adhesive, whilst addressing the two issues described above. DCB specimens were tested under loading rates ranging from 0.1 mm/min to 1 m/s in conjunction with DIC analysis, which represents a range of strain rates from 3×10^{-4} /s to 60 /s. Accurate TSLs were deduced by measuring the J -integral values and the crack tip opening displacements simultaneously. The evolution of J_c and σ_0 with increasing V_{ext} have been analysed. The values of ℓ_{cz} were experimentally measured and were compared with theoretical predictions to evaluate the applicability of the existing models. An analytical solution has been derived to correlate v_{tip} with V_{ext} , and the accuracy of this solution has been evaluated by comparison with the experimental results.

2. Theoretical

2.1. J -integral and TSL

The generalized J -integral along a vanishingly small contour Γ around the crack tip including inertia effects under dynamic loading is given by (Anderson, 2005)

$$J = \lim_{\Gamma \rightarrow 0} \int_{\Gamma} \left[(w + T) dy - \sigma_{ij} n_j \frac{\partial u_i}{\partial x} ds \right] \quad (1)$$

in which, w and T are the strain energy density and the kinetic energy density defined respectively as:

$$w = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij}$$

$$T = \frac{1}{2} \rho \frac{\partial u_i}{\partial t} \frac{\partial u_i}{\partial t}$$

where σ_{ij} and ε_{ij} are the components of stress and strain tensors, n_j are the components of the unit vector normal to Γ , u_i are the displacement vector components, ds is the length increment along Γ , ρ is the density of the solid and t is time. For the mode I fracture, prior to crack initiation, the total contribution of the kinetic energy in the DCB specimen, U_k , is (Blackman et al., 1996)

$$\frac{1}{b} \frac{dU_k}{da} = \frac{33Eh}{140} \left(\frac{V_{\text{ext}}/2}{c} \right)^2 \quad (2a)$$

where b is the width of the specimen, a is the crack length, $c = (E/\rho)^{1/2}$ is the elastic wave speed in the solid, E is the Young's modulus and h is the thickness of the beam respectively. For steady-state crack propagation, the kinetic energy contribution becomes (Blackman et al., 1996)

$$\frac{1}{b} \frac{dU_k}{da} = \frac{111Eh}{280} \left(\frac{V_{\text{ext}}/2}{c} \right)^2 \quad (2b)$$

Fig. 1 shows the contribution of the kinetic energy to the fracture of one adhesively bonded aluminium joint, as used in this study and described later. It shows that below $V_{\text{ext}} = 1.0$ m/s, the kinetic energy is negligible compared with the fracture energy for either adhesive used in this study, which typically ranges from a few hundred to a few thousand kJ/m² (Lopes et al., 2016; Sun and Blackman, 2020). According to our previous study (Blackman et al., 2012), when $V_{\text{ext}} \geq 10$ m/s, the load signal obtained by piezoelectric load cells (which are commonly used to measure rapidly changing loads) are associated with strong oscillations due to dynamic effects, which therefore become unreliable. Therefore, the maximum loading rate in this work has been limited to 1.0 m/s in order to ensure the measured load values are accurate.

Fig. 1 The kinetic energy generated in a DCB adhesive joint as a function of the loading rate.

If the kinetic term is negligible, the J -integral in Eq. (1) can be estimated along an exterior boundary of the DCB geometry via:

$$J_{\text{ext}} = \frac{P(\omega_L - \omega_U)}{b} \quad (3a)$$

where P/b is the applied load per unit width, and $\omega_L - \omega_U$ is the relative rotation between the lower and upper beams, with the anti-clockwise direction being positive.

The J -integral along the contour around the crack tip in the DCB specimen is,

$$J_{\text{tip}} = \int_0^{\Delta_f} \sigma(\Delta) d\Delta \quad (3b)$$

where $\sigma(\Delta)$ is the cohesive stress, Δ is the crack tip opening displacement, and Δ_f is the crack tip opening displacement to failure. As the J -integral is a path independent parameter, the relation $J = J_{\text{tip}} = J_{\text{ext}}$ exists if the region outside of the cohesive zone remains elastic during fracture. The cohesive stress in Eq. (3b) can thus be obtained as,

$$\sigma(\Delta) = \frac{dJ_{\text{ext}}}{d\Delta} \quad (4)$$

provided that the J_{ext} and Δ values can be correlated, for instance, via global fitting with a 6th order polynomial equation in this study. It is noted that Eq. (4) relies upon the assumption that the TSL is identical for all points along the interface. However, for rate dependent cohesive zone models, the TSL at the initial crack tip is different from that in the steady-state, i.e. the TSL varies along the locations ahead of the initial crack tip (Yang et al., 2019). Nevertheless, as an experimental approach, Eq. (4) can give a general estimation of the rate dependence of the TSL.

2.2. Length of cohesive zone

The length of the cohesive zone, ℓ_{cz} , is defined as the distance from the crack tip to the point where purely elastic tension begins to appear in front of the crack tip. For the mode I fracture in an infinite, isotropic body, the thickness independent models for ℓ_{cz} have the form:

$$\ell_{\text{cz}} = M \frac{EG_c}{\sigma_0^2} \quad (5a)$$

where M is a scale factor that depends on the specific TSL, and its proposed values can be found in Turon et al. (2007).

For mode I fracture in slender bodies, as illustrated in Fig. 2(a), the value of ℓ_{cz} is also related to the size of the body. The thickness dependent models estimate ℓ_{cz} as:

$$\ell_{cz} = M \left(\frac{EG_c}{\sigma_0^2} \right)^{1/4} h^{3/4} \quad (5b)$$

The value of M ranges from 0.5 to 1, and the proposed values in literature can be found in Soto et al. (2016). The above two models of ℓ_{cz} are based on rate independent TSLs. As mentioned earlier, the TSL may vary along the interface for rate dependent cohesive zone models, and so the value of ℓ_{cz} may vary accordingly. Eqs. (5a) and (5b) are therefore considered as indirect methods to estimate the rate dependence of the cohesive zone lengths.

2.3. Crack tip opening velocity, v_{tip}

The value of v_{tip} (or strain rate $\dot{\epsilon}_{tip}$) of a stationary crack varies considerably during constant loading due to the softening of the adhesive (see Appendix A), and it is not obvious at which time a representative v_{tip} can be chosen (Carlberger et al., 2009). As for the nominal strain rate in a tensile test, the nominal crack tip opening velocity can be defined as:

$$v_{tip}^n = \frac{\Delta_f}{t_f} \quad (6)$$

where t_f is the total time to when the initial crack tip fails. Accordingly, the nominal crack tip opening strain rate is defined as

$$\dot{\epsilon}_{tip}^n = \frac{v_{tip}^n}{h_a} \quad (7)$$

where h_a is the thickness of the adhesive layer in an adhesive joint. It is noted that Eq. (6) is an averaged measure of the crack tip opening velocity. However, in a DCB test the local opening velocity is not constant during the failure process, as indicated in Fig. A1 in Appendix A. This difference needs to be considered in the finite element implementation.

Bilinear and trapezoidal TSLs are widely used to model the fracture of interfaces. Fig. (2b) illustrates a bilinear TSL. Prior to the onset of damage, there is a linear relationship between the traction σ and the crack tip opening displacement, Δ . Once material is damaged, σ is reduced with Δ . Fig. (2c) illustrates a trapezoidal TSL with two shape parameters λ_1 and λ_2 . The area below the TSL is equal to G_c .

Fig. 2 Schematics of (a) a DCB specimen, (b) a bilinear TSL, and (c) a trapezoidal TSL.

Williams and Hadavinia (2002) proposed a series of analytical solutions for TSLs with various shapes using a beam on an elastic foundation. They found if the existence of a cohesive zone was taken into account, G_c deduced using simple beam theory should be corrected by:

$$G_c = G_{c,b} \left(1 + \frac{\bar{\ell}}{a} \right)^2 \quad (8)$$

where $G_{c,b} = 12(Pa)^2 / Eb^2h^3$ is the toughness estimated using simple beam theory, $\bar{\ell} = [2/3(EG_c/\sigma_0^2)]^{1/4}h^{3/4}$ is called the characteristic length of the deformation process, which

can be considered as a special case of Eq. (5b), with $M = 0.9$. Hereafter, $\bar{\ell}$ in Eq. (8) is replaced by ℓ_{cz} .

The external load P can be estimated from the load-point displacement, δ , using simple beam theory:

$$P = \frac{Ebh^3\delta}{8(a+\ell_{cz})^3} \quad (9)$$

When the cohesive zone is fully developed, $G = G_c$, and thus the crack tip starts to fail. From Eqs. (8) and (9), the critical δ corresponding to the fully developed fracture process zone (the onset of crack propagation) is obtained,

$$\delta_c = \frac{4}{\sqrt{3}} \left(\frac{a+\ell_{cz}}{h} \right)^2 \left(\frac{hG_c}{E} \right)^{1/2} \quad (10)$$

The total time taken to reach δ_c is

$$t_f = \frac{\delta_c}{V_{ext}} \quad (11)$$

For linear elastic, linear damage or bilinear TSLs,

$$\Delta_f = \frac{2G_c}{\sigma_0} \quad (12)$$

According to Eq. (6), the nominal crack tip opening velocity therefore is

$$\text{Bilinear TSL: } v_{tip,b}^n = \frac{\sqrt{3}V_{ext}}{2} \left(\frac{h}{a+\ell_{cz}} \right)^2 \left(\frac{1}{h} \frac{EG_c}{\sigma_0^2} \right)^{1/2} \quad (13)$$

For TSLs having trapezoidal shapes,

$$\Delta_f = \frac{2G_c}{(1+\lambda_2-\lambda_1)\sigma_0} \quad (14)$$

leading to,

$$\text{Trapezoidal TSL: } v_{tip,t}^n = \frac{\sqrt{3}V_{ext}}{2(1+\lambda_2-\lambda_1)} \left(\frac{h}{a+\ell_{cz}} \right)^2 \left(\frac{1}{h} \frac{EG_c}{\sigma_0^2} \right)^{1/2} \quad (15)$$

As a special case, when $\lambda_1 = 0$ and $\lambda_2 = 1$, the trapezoidal TSL becomes the constant-stress

TSL, and the value of the crack tip opening velocity using a constant-stress TSL, $v_{tip,cs}^n$, is

one half of the value determined using a bilinear TSL. In various studies, it has been widely accepted that the shape of the TSL plays a less significant role compared to G_c and σ_0 (Tvergaard and Hutchinson, 1992; Williams and Hadavinia, 2002). However, the above analysis indicates that the shape of the TSL significantly affects the value of v_{tip}^n . Table 1 summarises the experimental and analytical formulas to determine v_{tip}^n in this study.

For any two bilinear TSLs or two trapezoidal TSLs with a constant value of $\lambda_2 - \lambda_1$, a relative rate relationship is obtained from Eqs. (13) and (15),

$$\frac{v_{\text{tip}}^n}{v_{\text{tip}}^{n,*}} = \frac{\dot{\epsilon}_{\text{tip}}^n}{\dot{\epsilon}_{\text{tip}}^{n,*}} = \left(\frac{V_{\text{ext}}}{V_{\text{ext}}^*} \right) \left(\frac{\psi}{\psi^*} \right) \quad (16)$$

with

$$\psi = \left(\frac{h}{a + \ell_{cz}} \right)^2 \left(\frac{1}{h} \frac{E G_c}{\sigma_0^2} \right)^{1/2}$$

being a dimensionless parameter reflecting the effects of material properties and structural geometry. In Eq. (16), the symbols superscripted with $*$ are the values at a reference loading rate. One advantage of defining a relative tip opening velocity in this way is that the result becomes independent of the shape of the TSL if the shapes are consistent under varying loading rates.

Table 1. Methods to determine the crack tip opening velocity in a DCB test.

Methods	Equation	
DIC	$v_{\text{tip}}^n = \frac{\Delta_f}{t_f}$	Eq. (6)
via bilinear TSL	$v_{\text{tip,b}}^n = \frac{\sqrt{3}V_{\text{ext}}}{2} \left(\frac{h}{a + \ell_{cz}} \right)^2 \left(\frac{1}{h} \frac{E G_c}{\sigma_0^2} \right)^{1/2}$	Eq. (13)
via trapezoidal TSL	$v_{\text{tip,t}}^n = \frac{\sqrt{3}V_{\text{ext}}}{2(1 + \lambda_2 - \lambda_1)} \left(\frac{h}{a + \ell_{cz}} \right)^2 \left(\frac{1}{h} \frac{E G_c}{\sigma_0^2} \right)^{1/2}$	Eq. (15)

3. Experimental

3.1. Adhesive joints

3.1.1. Substrate and adhesives

Aluminium alloy grade AW6082-T6 beams with a width $b = 20$ mm and a height $h = 10$ mm were chosen as the substrates for the adhesive joints. A single-part epoxy based structural adhesive, SikaPower-497, and a two-part polyurethane adhesive, Araldite-2028, were chosen to bond the substrates.

3.1.2. Joint manufacturing

Surface pre-treatment was applied to the aluminium substrates. The substrates were firstly abraded with grit blasting and then were degreased in a Vapourwash 701-12 solvent. After degreasing, the substrates were immersed into a chromic acid bath preheated to 68 °C and held for 30 minutes. Immediately following etching, the substrates were immersed into cold running water for 20 minutes. Finally, the substrates were rinsed with distilled water and were dried in a fan-circulating oven at 60 °C. The bond-line thickness of the joints was 0.4 mm, which was controlled by inserting 0.4 mm steel wires at both ends of the joint. A PTFE film (thickness of 12.5 μ m) was inserted in the joint to act as an initial crack, and the initial crack length measured from the load line was 50 mm. The adhesives were cured according to the manufacturer's instructions. After curing, the excess adhesive on the sides of the joints was removed using a grinding wheel. Aluminium end-blocks (20 mm $\times$ 20 mm $\times$ 13 mm) were attached to the joints for loading. One lateral side of each joint was painted with a fine dark and white spray paint to form the speckle patterns required for the DIC analysis.

3.2. DCB testing

The values of V_{ext} employed in this study ranged from 1.7×10^{-3} mm/s to 1.0×10^3 mm/s, and the DCB experiments were classified as low-speed or high-speed tests shown in Table 2. The

low-speed tests were performed on a screw-driven tensile test machine (Instron 5584) installed with a 5 kN load cell. The DCB specimen was mounted to the tensile machine by connecting its end-blocks to the fixtures with stiff pins that were lubricated with a grease oil. A Nikon D7500 camera was used to take images during loading. For each test, the tensile machine and camera started simultaneously. The high-speed tests were performed on an Instron servo-hydraulic VHS machine, which utilises a piezo-electric sensor (PCB 222B, 10 kN) to measure the load. A motion lost device as illustrated in Fig. 3 was installed on the VHS machine to allow the ram to accelerate to the desired speed before the specimens were loaded. A Phantom Miro M310 high-speed camera was employed to record the images, which was triggered automatically when the VHS ram reached a set position. The image recording rates for DIC analysis are given in Table 2. In accordance with the ISO 25217 adhesive testing standard (ISO, 2009), four repeat tests were conducted at each loading rate and all tests were performed in standard laboratory conditions.

Table 2. DCB loading rate and image recording rate.

No.	Category of loading rate	V_{ext} , mm/s	Imaging rate, frames/s
1	Low-speed tests	1.7×10^{-3} (0.1 mm/min)	0.5
2		1.7×10^{-2} (1 mm/min)	5
3		1.7×10^{-1} (10 mm/min)	30
4	High-speed tests	1.0×10^2	16,000
5		1.0×10^3	19,028

Fig. 3 Experimental setup for the high rate DCB tests.

The acquired images were imported into GOM Correlate to extract the beam rotation angles, ω , and the initial crack tip opening distance, Δ . The values of ω were determined by calculating the values of dy/dx at a series of horizontal points created at the end of each beam and the values of Δ were determined by calculating the relative vertical displacement values of $dy_1 - dy_2$ at a series of vertical points created beside the initial crack tip. More details about the determinations of these two parameters can be found in Sun and Blackman (2020). Examples of the measured data including the load-point displacement, the load, the rotation of the load-point and the crack tip opening displacement for the adhesive joints tested at varying loading rates are shown in the supplementary material.

4. Results and discussions

4.1. J - Δ curves

All the joints bonded with the SikaPower-497 adhesive were found to fracture cohesively within the adhesive layer. Under low-speed testing conditions, the fracture surfaces were rough and uniform, indicating the crack growth was stable. Under high-speed testing conditions, periodic stick-slip crack growth occurred, indicating an unstable crack growth

behaviour. The main features of the fracture surfaces at varying values of V_{ext} are summarized in Table 3.

Fig. 4 shows the measured J curves for the joints bonded with the SikaPower-497 adhesive. The low-speed results in Fig. 4(a-c) suggest that the value of J continuously increased to a maximum with increasing Δ and then remained approximately constant, i.e. attaining the value of J_c . The high-speed results in Fig. 4(d, e) exhibit a similar behaviour, with the major difference being the small oscillations in the curves along the J_c plateau region. The peak points in the high-speed J - Δ curves correspond to the points of crack initiation while the valley points correspond to the points of crack arrest. The oscillations were mainly caused by the stick-slip behaviour of the crack growth in the adhesive layer. Fig. 4(f) shows the average values of J_c determined across the range of V_{ext} , indicating that the value of J_c increased from 3.8 kJ/m² to 4.4 kJ/m² as the value of V_{ext} was increased from 1.7×10^{-3} mm/s to 1.0×10^3 mm/s.

Fig. 4 The J - Δ curves for the joints bonded with the SikaPower-497 adhesive under varying loading rates.

The fracture surfaces of the joints bonded with the Araldite-2028 adhesive are shown in Table 3. Under slow-speed testing conditions, the joints failed predominantly by an interfacial failure mechanism but with a certain amount of cohesive failure. However, under high-speed testing conditions, purely interfacial failure was observed in all joints.

Fig. 5 shows the J - Δ curves measured for the Araldite-2028 joints. As before, for the low-speed tests, the values of J increased to a maximum value then remained nearly constant with Δ , as indicated in Fig. 5(a-c). The results for the high-speed tests are shown in Fig. 5(d, e) where the J - Δ curves were associated with pronounced oscillations due to the unstable stick-slip crack growth observed. Fig. 5(f) summarizes the values of J_c as a function of V_{ext} . In this figure, only the crack initiation points (the peak values) were taken for high-speed loading.

These results show that the value of J_c increased from 1.5 kJ/m^2 to 2.8 kJ/m^2 and then decreased to 0.8 kJ/m^2 . This indicates that increasing V_{ext} led to a transition in the fracture toughness in these adhesive joints from a higher energy mode to a lower energy mode.

Fig. 5 The J - Δ curves for the joints bonded with the Araldite-2028 adhesive under varying loading rates.

Table 3. The observed failure mechanisms of adhesive joint.

V_{ext} (mm/s)	SikaPower-497	Araldite-2028
1.7×10^{-3}	Cohesive failure, stable	Interfacial (predominant) - cohesive failure, stable
1.7×10^{-2}	Cohesive failure, stable	Interfacial (predominant) - cohesive failure, stable
1.7×10^{-1}	Cohesive failure, stable	Interfacial (predominant) - cohesive failure, stable
1.0×10^2	Cohesive failure, feeble stick-slip patterns	Interfacial failure, stick-slip patterns
1.0×10^3	Cohesive failure, feeble stick-slip patterns	Interfacial failure, stick-slip patterns

4.2. TSL curves

The TSLs determined for the joints bonded with the SikaPower-497 adhesive from each loading rate are shown in Fig. 6, and the cohesive parameters are summarised in Table 4. The effects of loading rate are now summarised: (1) the TSLs more closely resemble the trapezoidal shape as indicated in Fig. 6(a-e); (2) the value of σ_0 increases with increasing V_{ext} , while the value of Δ_f decreases (see Fig. 6f); (3) the softening slopes of the high-speed TSLs are steeper than those of the low-speed TSLs, implying that high-speed loading accelerated the damage process in the adhesive.

The TSLs determined for the joints bonded with the Araldite-2028 adhesive and the cohesive parameters deduced at varying values of V_{ext} are shown in Fig. 7 and Table 4 respectively.

For the high-speed tests, as unstable stick-slip crack propagation occurred, only the values at

the crack initiation points (the peak points in Fig. 5 d, e) were used to determine the TSLs. Except for $V_{\text{ext}} = 1.0 \times 10^3$ mm/s, the value of σ_0 shows an increasing trend with V_{ext} , whilst the value of Δ_f generally decreases. The shape of the TSL for the joints bonded with the polyurethane adhesive resembles more closely the bilinear form across the range of the loading rates.

The TSLs obtained were then simplified into bilinear and trapezoidal shapes for the determination of v_{tip}^n , which are also given in Figs. 6 and 7. For the simplified TSL, the cohesive strength took the average value of the measured σ_0 and the area below the curve was forced to equal to the average value of J_c . There are many ways to define the initial stiffness k in the TSL curve (Kanninen, 1973; Li et al., 2004; Turon et al., 2007), and in this work a value of k that closely matches the initial slope of TSL is given in the figure. For the trapezoidal TSL, the softening slope taken was the average value of the measured softening slopes. It was found that the simplified trapezoidal TSLs can better capture the basic shape of the curves determined from experiments, whilst the bilinear TSLs can better capture the values of Δ_f .

Fig. 6 Measured and simplified TSLs for the joints bonded with the SikaPower-497 adhesive at varying loading rates. The dark lines in (a-e) represent the average values of the measured cohesive stress, and the shaded areas in the plots represents the associated standard deviation. The coloured lines represent the simplified TSL curves. The plots in (f) are the average cohesive stress values determined at each loading rate.

Fig. 7 Measured and simplified TSLs for the joints bonded with the Araldite-2028 adhesive at varying loading rates. The dark lines in (a-e) represent the average values of the measured cohesive stress, and the shaded areas in the plots represent the associated standard deviation. The coloured lines represent the simplified TSL curves. The plots in (f) are the average cohesive stress values determined at each loading rate.

Table 4. Cohesive properties for the joints bonded with the SikaPower-497 and Araldite-2028 adhesives.

V_{ext} (mm/s)	SikaPower-497			Araldite-2028		
	J_c (kJ/m ²)	σ_0 (MPa)	Δ_f (mm)	J_c (kJ/m ²)	σ_0 (MPa)	Δ_f (mm)
1.7×10^{-3}	3.82	46.1	0.18	1.52	14.6	0.26

1.7×10^{-2}	3.88	57.9	0.13	2.54	20.5	0.26
1.7×10^{-1}	4.25	75.6	0.14	2.84	27.6	0.28
1.0×10^2	4.17	84.5	0.12	0.96	46.8	0.03
1.0×10^3	4.39	92.7	0.08	0.71	30.4	0.09

4.3. Length of cohesive zone

Fig. 8 shows the contour of fully developed strain in the loading direction, ε_{yy} , measured from DIC in the joints bonded with the SikaPower-497 adhesive. At each loading rate, a slender region with an intensive ε_{yy} developed in front of the crack tip. For this adhesive, according to the TSLs in Fig. 6, the opening displacement for the onset of damage was approximately 0.015 mm, corresponding to ε_{yy} of about 3.75%. Therefore, the region in front of the crack tip with $\varepsilon_{yy} \geq 3.75\%$ has been taken as the damaged region and its length taken as ℓ_{cz} . In joints bonded with the Araldite-2028 adhesive, according to Fig. 7, the cohesive zones for the low-speed tests showed $\varepsilon_{yy} \geq 6\%$ and the high-speed tests showed that $\varepsilon_{yy} \geq 3\%$, see Fig. 9. The measurements of ℓ_{cz} based on a threshold strain level (corresponding to the peak cohesive stress) does not account for the contribution from the elastic portion of a TSL. This definition has been widely adopted in many studies (e.g. Harper and Hallett, 2008; Turon et al., 2007), and such a definition leads to a physically reasonable crack length, i.e. the length from the load-point to the crack tip plus the length of the damaged region ahead of the crack tip. In addition, the length of the elastic region in structural adhesive joints is usually very small compared to the length of damaged region. As shown by the DIC strain contours in Figs. 8 and 9, the difference in the ℓ_{cz} determined with and without accounting for the elastic region is modest, about 10%. The values of ℓ_{cz} for the two adhesives measured by DIC are given in Tables 5 and 6. For joints bonded with the SikaPower-497 adhesive, ℓ_{cz} decreased as V_{ext} was

increased. For joints bonded with the Araldite-2028 adhesive, a decreasing trend in the values of ℓ_{cz} can be noticed as V_{ext} increased.

Fig. 8 The strain contour of ε_{yy} ahead of the crack tip for joints bonded with the SikaPower-497 adhesive. The region with $\varepsilon_{yy} \geq 3.75\%$ was taken as the cohesive zone.

Fig. 9 The strain contour of ε_{yy} ahead of the crack tip for joints bonded with the Araldite-2028 adhesive. For (a-c) the regions with $\varepsilon_{yy} \geq 6\%$ and for (d, e) $\varepsilon_{yy} \geq 3\%$ were taken as the cohesive zones.

From Tables 5 and 6, it is found that the values of ℓ_{cz} calculated by the thickness independent model using Eq. (5a) with $M = 1$, i.e. Hillerborg's model (Hillerborg et al., 1976), which corresponds to the upper bound value, overestimates the cohesive zone length by a factor of up to a few tens compared to the DIC measured values. The lower bound value calculated using Eq. (5a) with $M = 0.21$, i.e. Hui's model (Hui et al., 2003), still overestimates the value of ℓ_{cz} , especially for the lower rates of $V_{ext} = 1.7 \times 10^{-3}$ mm/s and $V_{ext} = 1.7 \times 10^{-2}$ mm/s. This overestimation is observed in joints bonded with either adhesive. The thickness dependent model Eq. (5b) with $M = 1$ (Yang and Cox, 2005) or $M = 0.9$ (Williams and Hadavinia, 2002) also overestimates the values of ℓ_{cz} significantly for both adhesives. However, with $M = 0.5$ (Harper and Hallett, 2008), the results from Eq. (5b) agree with the DIC results quite closely for both adhesives. Therefore, it is concluded that in the range of loading rates employed in this study, Eq. (5b) with a scale factor $M = 0.5$ can provide reliable estimations of ℓ_{cz} for tough and ductile adhesive joints.

It is also evident that in joints bonded with the SikaPower-497 adhesive as $V_{ext} \geq 1.7 \times 10^{-1}$ mm/s and for Araldite-2028 joints as $V_{ext} \geq 1.0 \times 10^2$ mm/s, the prediction accuracy of Hui's model begins to be improved as the size of the cohesive zone became smaller. This is consistent with the finding in Harper and Hallett (2008), confirming that thickness independent models are more suitable for structures associated with relatively small cohesive zones. In this range of loading rates, no accuracy shift between the thickness dependent and independent models occurred.

Table 5. The measured cohesive zone lengths obtained from DIC and the calculated lengths, for joints bonded with the SikaPower-497 adhesive.

V_{ext} , mm/s	ℓ_{cz} by DIC, mm	ℓ_{cz} by Eq. (5a), mm		ℓ_{cz} by Eq. (5b), mm		
		$M = 1$	$M = 0.21$	$M = 1$	$M = 0.9$	$M = 0.5$
1.7×10^{-3}	10.2	126.1	26.5	18.8	16.9	9.4
1.7×10^{-2}	9.9	80.9	17.0	16.9	15.2	8.4
1.7×10^{-1}	8.1	52.1	10.9	15.1	13.6	7.6
1.0×10^2	6.6	40.8	8.6	14.2	12.8	7.1
1.0×10^3	6.0	35.8	7.5	13.8	12.4	6.9

Table 6. The measured cohesive zone lengths obtained from DIC and the calculated lengths, for joints bonded with the Araldite-2028 adhesive.

V_{ext} , mm/s	ℓ_{cz} by DIC, mm	ℓ_{cz} by Eq. (5a), mm		ℓ_{cz} by Eq. (5b), mm		
		$M = 1$	$M = 0.21$	$M = 1$	$M = 0.9$	$M = 0.5$
1.7×10^{-3}	13.9	499.7	104.9	26.6	23.9	13.3
1.7×10^{-2}	10.1	421.4	88.5	25.5	23.0	12.7
1.7×10^{-1}	12.2	261.9	55.0	22.6	20.3	11.3
1.0×10^2	8.1	30.6	6.4	13.2	11.9	6.6
1.0×10^3	7.4	54.2	11.4	15.3	13.8	7.6

4.3. Crack tip opening velocity

The results in Section 4.2 indicate that the use of the equation $\ell_{\text{cz}} = 0.5(EG_c/\sigma_0^2)^{1/4}h^{3/4}$ led to a reliable estimation of the value of ℓ_{cz} . Our previous work also shows that if the crack

length is extended to its boundary condition where the deflection angle of the beam is equal to zero, which is close to the value of the ℓ_{cz} , Eq. (8) gives the similar toughness results as the J -integral approach (Sun and Blackman, 2020). Therefore, in this section the crack tip opening velocity is determined via Eqs. (13) and (15) with $\ell_{cz} = 0.5(EG_c/\sigma_0^2)^{1/4}h^{3/4}$. Table 7 shows the measured and predicted crack tip opening velocity, v_{tip}^n , for the joints bonded with the SikaPower-497 adhesive, in which $v_{tip,b}^n$, $v_{tip,t}^n$ and $v_{tip,cs}^n$ denote the values estimated using the bilinear, the trapezoidal and the constant-stress TSLs respectively. It is found that for $V_{ext} \leq 1.0 \times 10^2$ mm/s, the value of v_{tip}^n determined using the trapezoidal TSL is very close to the velocities measured by DIC. The prediction using the bilinear TSL overestimates v_{tip}^n by between 20-60 %, while using the constant-stress TSL it underestimates the value by between 20-40 %. For $V_{ext} = 1.0 \times 10^3$ mm/s, the constant-stress TSL gives the best prediction, while the trapezoidal and bilinear TSLs overestimate the velocities about 60% and 120% respectively. However, the reliability of the result at $V_{ext} = 1.0 \times 10^3$ mm/s needs further confirmation as the resolution of the images at high rates is much lower than those at lower rates, which may impose errors in the data processing. However, it can generally be concluded that the use of the trapezoidal TSL is more accurate for the prediction of v_{tip}^n .

The results in Table 7 show that the use of the trapezoidal model predicts the value of v_{tip}^n more closely than when the bilinear model is used although the later captures more closely the value of Δ_f . Theoretically, the trapezoidal and bilinear TSLs are associated with different values of ℓ_{cz} , and their values should be reflected in Eqs. (8) and (9). However, the thickness dependent ℓ_{cz} in this study is based on the experimental correction, i.e., with $M = 0.5$, which has intrinsically included the shape effect. Hence, the reasons for the discrepancy in the prediction using the bilinear and the trapezoidal TSLs are not further discussed.

For studies using tensile tests combined with DCB/TDCB tests to obtain trapezoidal TSLs for tough adhesives, the value of v_{tip}^n also needs to be estimated to match the two strain rates in the two testing conditions. If the value of $\lambda_2 - \lambda_1$ is preselected or is determined using numerical methods, v_{tip}^n can be estimated using Eq. (15) directly. For the cases with no knowledge of the value of $\lambda_2 - \lambda_1$, this study suggests a value of 0.35 is appropriate, which is also consistent with the value given in Tvergaard and Hutchinson (1992) and Yang et al. (1999).

Table 7. Measured and calculated crack tip opening velocity for joints bonded with the Sikapower-497 adhesive.

V_{ext} (mm/s)	v_{tip}^n by DIC (mm/s)	$\dot{\epsilon}_{\text{tip}}^n$ by DIC (/s)	$v_{\text{tip,b}}^n$ (mm/s)	$\lambda_2 - \lambda_1$	$v_{\text{tip,t}}^n$ (mm/s)	$v_{\text{tip,cs}}^n$ (mm/s)
1.7×10^{-3}	1.2×10^{-4}	3×10^{-4}	1.5×10^{-4}	0.39	1.0×10^{-4}	7.3×10^{-5}
1.7×10^{-2}	8.9×10^{-4}	2.2×10^{-3}	1.2×10^{-3}	0.33	9.0×10^{-4}	6.0×10^{-4}
1.7×10^{-1}	8.4×10^{-3}	2.1×10^{-2}	1.0×10^{-2}	0.39	7.2×10^{-3}	5.0×10^{-3}
1.0×10^2	3.3	8.3	5.4	0.43	3.8	2.7
1.0×10^3	24	60	51	0.37	37	25

Tables 8 shows the experimental and estimated v_{tip}^n for joints bonded with the Araldite-2028 adhesive. For the low-speed tests, the values of v_{tip}^n determined using trapezoidal TSLs match the experimental results more closely than when using the bilinear TSLs. The value of $\lambda_2 - \lambda_1$ is around 0.3. The bilinear TSL overestimates the velocity by between 30-60 %, while the constant-stress form underestimates the velocity by between 20-40 %. For the test at $V_{\text{ext}} = 1.0 \times 10^2$ mm/s, all the predictions overestimate v_{tip}^n markedly. For $V_{\text{ext}} = 1.0 \times 10^3$ mm/s, the

velocity determined using bilinear TSL, which is consistent with its measured shape, agrees closely with the DIC result.

The results for the two adhesives indicate that the shape of the TSL plays an important role in the prediction of v_{tip}^n . The prediction using a TSL with a more accurate shape always leads to a more accurate result.

Table 8. Measured and calculated crack tip opening velocity for joints bonded with the Araldite-2028 adhesive.

V_{ext} (mm/s)	v_{tip}^n by DIC (mm/s)	$\dot{\epsilon}_{\text{tip}}^n$ by DIC (/s)	$v_{\text{tip,b}}^n$ (mm/s)	$\lambda_2 - \lambda_1$	$v_{\text{tip,t}}^n$ (mm/s)	$v_{\text{tip,cs}}^n$ (mm/s)
1.7×10^{-3}	1.6×10^{-4}	4.0×10^{-4}	2.6×10^{-4}	0.37	1.9×10^{-4}	1.3×10^{-4}
1.7×10^{-2}	1.7×10^{-3}	4.3×10^{-3}	2.4×10^{-3}	0.26	1.9×10^{-3}	1.2×10^{-3}
1.7×10^{-1}	1.5×10^{-2}	3.8×10^{-2}	2.0×10^{-2}	0.3	1.5×10^{-2}	9.8×10^{-3}
1.0×10^2	1.4	3.5	4.7	0.44	3.3	2.4
1.0×10^3	73	183	61	0	61	30

Fig. 10 plots the relative crack tip opening velocity with the Ψ parameter defined in Eq. (16), with the reference loading rate $V_{\text{ext}} = 1.7 \times 10^{-3}$ mm/s, in which the dashed line indicates equality. It is shown that within the range of rates tested in this study, the relative crack tip opening velocity from the DIC measurement (the y axis) agrees very closely with the theoretical predictions (the x axis) for both adhesives. Discrepancies in the experimental and calculated results may occur due to errors in the DIC measurements, particularly as the resolution of images obtained at high rates were lower than those at slow rates. In addition, the dynamic effects encountered at high rates affected the magnitude of the load values, which would also cause some errors in the calculation.

Fig. 10 Comparison between experimental relative crack tip opening velocity and theoretical estimations using the external loading rate and cohesive parameters, in which the dashed line represents equality.

4.4. Correlating J_c and σ_0 with strain rate

Using the Johnson-Cook stress model (Johnson and Cook, 1983), the values of σ_0 and the relative strain rate $\dot{\epsilon}_{\text{tip}}^n / \dot{\epsilon}_{\text{tip}}^{n*}$ (or $v_{\text{tip}}^n / v_{\text{tip}}^{n*}$) may be related by a logarithmic function,

$$\sigma_0 = \sigma_0^* \left[1 + C_s \ln \left(\frac{\dot{\epsilon}_{\text{tip}}^n}{\dot{\epsilon}_{\text{tip}}^{n*}} \right) \right] \quad (17)$$

Similarly, the J_c may be related to the relative rate using the same form of relationship (Lißner et al., 2019)

$$J_c = J_c^* \left[1 + C_t \ln \left(\frac{\dot{\epsilon}_{\text{tip}}^n}{\dot{\epsilon}_{\text{tip}}^{n*}} \right) \right] \quad (18)$$

where C_s and C_t are constants, and the variables superscripted with $*$ are the values at the reference loading rate. Figs. 11 and 12 show the cohesive parameters J_c and σ_0 as a function of $\dot{\epsilon}_{\text{tip}}^n$ together with the fitted plots. It is observed that the experimental results obtained for both adhesives can be fitted well with the above relationship.

For joints bonded with the SikaPower-497 adhesive, both values of σ_0 and J_c increase with increasing $\dot{\epsilon}_{\text{tip}}^n$, confirming that this adhesive has been developed to possess good high-speed

crash performance. In contrast, the joints bonded with the Araldite-2028 adhesive exhibits a more complex rate dependence. For $V_{\text{ext}} \leq 1.0 \times 10^2$ mm/s, σ_0 increases with strain rate, while above this velocity, the strength reduces. A toughness transition appears at the strain rate of about 3.8×10^{-2} /s, corresponding to $V_{\text{ext}} = 1.7 \times 10^{-1}$ mm/s. Under this rate, J_c increases with increasing rate, while above this rate, J_c decreases. This is probably due to the transition of failure mechanism in the joints. As noted earlier, purely interfacial failure was observed for the high-speed tests, which differs from the mixed interfacial-cohesive failure observed in the low-speed tests.

Fig. 11 Cohesive properties of SikaPower-497 adhesive as a function of crack tip opening strain rate, (a) cohesive strength and (b) fracture toughness.

Fig. 12 Cohesive properties of Araldite-2028 adhesive as a function of crack tip opening strain rate, (a) cohesive strength and (b) fracture toughness.

5. Conclusions

The rate effects encountered during the mode I fracture of joints bonded with a toughened epoxy adhesive (SikaPower-497) or a ductile polyurethane adhesive (Araldite-2028) were experimentally investigated in the range of strain rates from 3×10^{-4} /s to 60 /s using the digital image correlation technique.

An analytical method was proposed to describe the relationship between the crack tip opening velocity (or opening strain rate) and the external loading velocity, which indicates that these two quantities can be linked through Linear Elastic Fracture Mechanics and Cohesive Zone Model theories. The crack tip opening velocities determined using this method agreed with the experiments closely although there was some discrepancy at high loading rates. It was shown that the shape of the TSL plays an important role in the crack tip opening velocity; a more accurate shape led to a more reliable prediction of the opening velocity.

The cohesive zone lengths for these adhesive joints over the range of test rates employed were measured experimentally by DIC. The experiments indicated that the length of cohesive zone can be accurately predicted via a thickness-dependent model, i.e. $\ell_{cz} = 0.5(EG_{Ic}/\sigma_0^2)^{1/4}h^{3/4}$. For both adhesives employed, the length of the cohesive zone ℓ_{cz} was shown to decrease with increasing loading rate.

The values of J_c and σ_0 for the toughened epoxy adhesive were shown to increase with increasing loading rate. The shape of the TSL for the adhesive joint most closely resembled the trapezoidal form within the range of the loading rates investigated. For the ductile polyurethane adhesive, below a critical loading rate, the values of J_c and σ_0 increased with increasing loading rate, whilst above the critical rate, both J_c and σ_0 decreased. The transition

was due to the change of the failure locus, changing from a mixed interfacial/cohesive failure path at low-speeds to a purely interfacial failure path at high-speeds. The shape of the TSL for the joints bonded with the polyurethane adhesive most closely resembled the bilinear form across the range of loading rates investigated.

Acknowledgements

The research in this paper was funded by Innovate UK's APC7 "RACEForm" grant (No. 113153). Sika AG (Switzerland) is acknowledged for the supplement of adhesive. We would like to acknowledge Bryan Lee from Imperial College London for assisting the experiments.

Appendix A.

Fig. A1 displays the evolution of the crack tip opening displacement and opening velocity (the tangent of the opening displacement) in one joint bonded with the SikaPower-497 adhesive under the loading rate of 1.7×10^{-2} mm/s. The values of the opening velocity were calculated as $v_{\text{tip}} = (\Delta_{t+1} - \Delta_t) / [(t + 1) - t]$. It is observed that the opening displacement continued to increase with time during the fracture process. Prior to the onset of damage, the opening velocity nearly remained at a constant value approximately about 1×10^{-3} mm/s, very close to the predicted nominal value of 8.9×10^{-4} mm/s. Once damage had been developed, the opening velocity increased with time due to the loss of the stiffness, attaining about 4×10^{-3} mm/s at the failure of the crack tip.

Fig. A1 Evolution of the crack tip opening displacement and opening velocity in the adhesive joint bonded with the SikaPower-497 adhesive under an external loading rate of 1.7×10^{-2} mm/s.

Appendix B. Supplementary material

References

- Anderson, T.L., 2005. Fracture Mechanics: Fundamentals and Applications, 3rd ed. CPC Press, Boca Raton.
- Bao, G., Suo, Z., 1992. Remarks on crack-bridging concepts. Appl. Mech. Rev. 45, 355–366.
- Blackman, B.R.K., Kinloch, A.J., Rodriguez-Sanchez, F.S., Teo, W.S., 2012. The fracture behaviour of adhesively-bonded composite joints: Effects of rate of test and mode of loading. Int. J. Solids Struct. 49, 1434–1452.
<https://doi.org/10.1016/j.ijsolstr.2012.02.022>
- Blackman, B.R.K., Kinloch, A.J., Rodriguez Sanchez, F.S., Teo, W.S., Williams, J.G., 2009.

- The fracture behaviour of structural adhesives under high rates of testing. *Eng. Fract. Mech.* 76, 2868–2889. <https://doi.org/10.1016/j.engfracmech.2009.07.013>
- Blackman, B.R.K., Kinloch, A.J., Wang, Y., Williams, J.G., 1996. The failure of fibre composites and adhesively bonded fibre composites under high rates of test-Part II: Mode I loading - dynamic effects. *J. Mater. Sci.* 31, 4451–4466. <https://doi.org/10.1007/BF00366341>
- Carlberger, T., Biel, A., Stigh, U., 2009. Influence of temperature and strain rate on cohesive properties of a structural epoxy adhesive. *Int. J. Fract.* 155, 155–166. <https://doi.org/10.1007/s10704-009-9337-4>
- Freund, L.B., Lee, Y.J., 1990. Observations on high strain rate crack growth based on a strip yield model. *Int. J. Fract.* 42, 261–276.
- Gorman, J.M., Thouless, M.D., 2019. The use of digital-image correlation to investigate the cohesive zone in a double-cantilever beam, with comparisons to numerical and analytical models. *J. Mech. Phys. Solids* 123, 315–331. <https://doi.org/10.1016/j.jmps.2018.08.013>
- Harper, P.W., Hallett, S.R., 2008. Cohesive zone length in numerical simulations of composite delamination. *Eng. Fract. Mech.* 75, 4774–4792. <https://doi.org/10.1016/j.engfracmech.2008.06.004>
- Hillerborg, A., Modéer, M., Petersson, P.-E., 1976. Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. *Cem. Concr Res* 6, 773–782.
- Högberg, J.L., Sørensen, B.F., Stigh, U., 2007. Constitutive behavior of mixed mode loaded adhesive layer. *Int. J. Solids Struct.* 44, 8335–8354.

- Hui, C.-Y., Jagota, A., Bennison, S.J., Londono, J.D., 2003. Crack blunting and the strength of soft elastic solids. *Proc. R. Soc. Lond. A* 459, 1489–1516.
- ISO 25217:2009. Adhesives-Determination of the mode I adhesive fracture energy of structural adhesive joints using double cantilever beam and tapered double cantilever beam specimens.
- Johnson, G.R., Cook, W.H., 1983. A constitutive model and data for metals subjected to large strains, high strain rates, and high temperature, in: *Proceedings 7th International Symposium on Ballistics*. The Hague, pp. 541–547.
- Kanninen, M.F., 1974. A dynamic analysis of unstable crack propagation and arrest in the DCB test specimen. *Int. J. Fract.* <https://doi.org/10.1007/BF00035502>
- Kanninen, M.F., 1973. An augmented double cantilever beam model for studying crack propagation and arrest. *Int. J. Fract.* 9, 83–92.
- Karac, A., Blackman, B.R.K., Cooper, V., Kinloch, A.J., Sanchez, S.R., Teo, W.S., Ivankovic, A., 2011. Modelling the fracture behaviour of adhesively-bonded joints as a function of test rate. *Eng. Fract. Mech.* 78, 973–989. <https://doi.org/10.1016/j.engfracmech.2010.11.014>
- Lee, D.G., Lim, T.S., Cheon, S.S., 2000. Impact energy absorption characteristics of composite structure. *Compos. Struct.* 50, 381–390.
- Li, S., Wang, J., Thouless, M.D., 2004. The effects of shear on delamination in layered materials 52, 193–214. [https://doi.org/10.1016/S0022-5096\(03\)00070-X](https://doi.org/10.1016/S0022-5096(03)00070-X)
- Lißner, M., Alabort, E., Cui, H., Rito, R., Blackman, B.R.K., Petrinic, N., 2019. Experimental characterisation and numerical modelling of the influence of bondline thickness, loading rate, and deformation mode on the response of ductile adhesive interfaces. *J. Mech.*

- Phys. Solids 130, 349–369. <https://doi.org/10.1016/j.jmps.2019.06.011>
- Lopes, R.M., Campilho, R.D.S.G., da Silva, F.L.G., Faneco, T.M.S., 2016. Comparative evaluation of the Double-Cantilever Beam and Tapered Double-Cantilever Beam tests for estimation of the tensile fracture toughness of adhesive joints. *Int. J. Adhes. Adhes.* 67, 103–111. <https://doi.org/10.1016/j.ijadhadh.2015.12.032>
- Marzi, S., Hesebeck, O., Brede, M., Kleiner, F., 2009. A rate-dependent cohesive zone model for adhesively bonded joints loaded in Mode I. *J. Adhes. Sci. Technol.* 23, 881–898. <https://doi.org/10.1163/156856109X411238>
- Rajan, S., Sutton, M.A., Fuerte, R., Kidane, A., 2018. Traction-separation relationship for polymer-modified bitumen under Mode I loading: Double cantilever beam experiment with stereo digital image correlation. *Eng. Fract. Mech.* 187, 404–421. <https://doi.org/10.1016/j.engfracmech.2017.12.031>
- Rice, J.R., 1980. The mechanics of earthquake rupture, in: Dziewonski, A.M., Boschhi, E. (Eds.), *Physics of the Earth's Interior. Proceedings of the International School of Physics "Enrico Fermi"*, Course 78, 1979. Italian Physical Society/North-Holland, Amsterdam, pp. 555–649.
- Saseendran, V., Carlsson, L.A., Berggreen, C., 2018. Shear and foundation effects on crack root rotation and mode- mixity in moment- and force-loaded single cantilever beam sandwich specimen. *J. Compos. Mater.* 52, 2537–2547.
- Schmandt, C., Marzi, S., 2018. Effect of crack opening velocity and adhesive layer thickness on the fracture behaviour of hyperelastic adhesive joints subjected to mode I loading. *Int. J. Adhes. Adhes.* 83, 9–14. <https://doi.org/10.1016/j.ijadhadh.2018.02.028>
- Smiley, A.J., Pipes, R.B., 1987. Rate effects on Mode-I interlaminar fracture toughness in

- composite materials. *J. Compos. Mater.* 21, 670–687.
- Soto, A., González, E.V., Maimí, P., Turon, A., Sainz de Aja, J.R., de la Escalera, F.M., 2016. Cohesive zone length of orthotropic materials undergoing delamination. *Eng. Fract. Mech.* 159, 174–188. <https://doi.org/10.1016/j.engfracmech.2016.03.033>
- Sun, C., Thouless, M.D., Waas, A.M., Schroeder, J.A., Zavattieri, P.D., 2008. Ductile – brittle transitions in the fracture of plastically-deforming, adhesively-bonded structures. Part I : Experimental studies. *Int. J. Solids Struct.* 45, 3059–3073. <https://doi.org/10.1016/j.ijsolstr.2008.01.011>
- Sun, F., Blackman, B.R.K., 2020. A DIC method to determine the Mode I energy release rate G , the J-integral and the traction-separation law simultaneously for adhesive joints. *Eng. Fract. Mech.* 234, 107097. <https://doi.org/10.1016/j.engfracmech.2020.107097>
- Toolabi, M., Blackman, B.R.K., 2018. Guidelines for selecting the dimensions of adhesively bonded end- loaded split joints : An approach based on numerical cohesive zone length. *Eng. Fract. Mech.* 203, 250–265. <https://doi.org/10.1016/j.engfracmech.2018.05.019>
- Turon, A., Dávila, C.G., Camanho, P.P., Costa, J., 2007. An engineering solution for mesh size effects in the simulation of delamination using cohesive zone models. *Eng. Fract. Mech.* 74, 1665–1682. <https://doi.org/10.1016/j.engfracmech.2006.08.025>
- Tvergaard, V., Hutchinson, J.W., 1992. The relation between crack growth resistance and fracture process parameters in elastic-plastic solids. *J. Mech. Phys. Solids* 40, 1377–1397. [https://doi.org/10.1016/0022-5096\(92\)90020-3](https://doi.org/10.1016/0022-5096(92)90020-3)
- Williams, J.G., Hadavinia, H., 2002. Analytical solutions for cohesive zone models. *J. Mech. Phys. Solids* 50, 809–825. [https://doi.org/10.1016/S0022-5096\(01\)00095-3](https://doi.org/10.1016/S0022-5096(01)00095-3)
- Yang, Q., Cox, B., 2005. Cohesive models for damage evolution in laminated composites.

- Int. J. Fract. 133, 107–137. <https://doi.org/10.1007/s10704-005-4729-6>
- Yang, Q.D., Thouless, M.D., Ward, S.M., 1999. Numerical simulations of adhesively-bonded beams failing with extensive plastic deformation. *J. Mech. Phys. Solids* 47, 1337–1353.
- Yang, T., Yang, X., Huang, R., Liechti, K.M., 2019. Rate-dependent traction-separation relations for a silicon/epoxy interface informed by experiments and bond rupture kinetics. *J. Mech. Phys. Solids* 131, 1–19. <https://doi.org/10.1016/j.jmps.2019.06.013>
- Zhu, Y., Liechti, K.M., Ravi-Chandar, K., 2009. Direct extraction of rate-dependent traction-separation laws for polyurea/steel interfaces. *Int. J. Solids Struct.* 46, 31–51. <https://doi.org/10.1016/j.ijsolstr.2008.08.019>