The determination and enhancement of compliant modes for high-amplitude actuation in lattices

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Abstract

This paper details the nonlinear design of adaptive lattices by determination and enhancement of compliant modes and optimizing the designed structure for delivering high amplitude actuation. The particular focus is the kagome lattice geometry — a pattern with some unique and useful actuation properties. Developing a novel design tool, the stiffness matrix of the beam assembly is calculated using a developed second-order geometrically nonlinear beam finite element formulation allowing large rotations. Based on this formulation in conjunction with singular value decomposition of the stiffness matrix, the modal optimization technique reduces the continuous structure with many degrees of freedom to a small number of low energy modes, which form the basis of designing the adaptive structure. For delivering high-amplitude actuation, the designed structure needs to be re-optimized due to changes in the nonlinear stiffness matrix under large deformation. This is performed via Bayesian optimization and by removing some internal members of the lattice. The integrity and feasibility of the optimum design is guaranteed via defining some constraints on removed members.

Keywords: Adaptive structures, Large deformation, Nonlinear finite element, Bayesian optimization, Kagome lattice

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1. Introduction

Lattice structures can be analysed in two main categories, depending on whether the lattices are formed with members having pin-joined or rigid-joined connections. The adaptive behaviour of a pin-joined framework is fully understood in terms of its kinematic and static indeterminacies, which are defined as the modes of inextensional deformation and the states of self-stress, respectively. In turn, infinitesimal inextensional mechanisms tighten up with the higher orders of displacement which have a detrimental effect on the adaptivity of the assembly [1]. McCallen and Romstad [2] developed an equivalent continuum model for the geometrically nonlinear analysis of beam-like lattice structures, by requiring the strain energy of the continuum equal to the strain energy of the lattice for a finite set of assumed deformation modes. Cao et al. [3] extended the continuum model to nonlinear beam-like lattice truss structures by introducing a co-rotational coordinate system to decompose the motion into rigid body motion and pure deformations. Generalized quasicontinuum modeling of metallic lattices with geometrical and material nonlinearity and variability is studied in Ref [4]. Jamshidian et al. [5] investigated the large-deformation nonlinear elastic behaviour of beam-lattice metamaterials by micromechanical nonlinear buckling analysis, where the effective constitutive behaviour of representative unit cells is derived by the uniaxial standard test. In this paper we consider the design and optimization of lattice structure subject to high amplitude actuation. Based on an FE discretization method and using the modal reduction technique, the design of compliant lattice is performed via determination and enhancement of its compliant modes. Under high amplitude actuation, the effectiveness of the designed lattice reduces significantly due to the induced internal stresses. This implies the necessity of re-optimizing the designed lattice based on the functionality of the structure for delivering the desired actuation response.

The developed formulation is general, but the present work focuses on the so-called kagome lattice, with a representative hexagonal unit cell shown in Fig. 1(a). The kagome lattice has high potential for use in lightweight structures

due to its high relative stiffness and strengths. Specifically, compared to the triangular lattice, in which the members are intersecting at the center of the hexagonal unit cell (Fig. 1(b)), the kagome lattice with the same relative density has four-times greater buckling strength [6].

Figure 1: Infinite repetitive (a) kagome and (b) triangular lattice structures with a representative hexagonal unit cell

35 We denote each row as an actuation ‘corridor’. Kagome lattices possess some unique and attractive actuation properties in the laterally-confined and far-reaching deformation along each corridor via the actuation of a single member [7]. These characteristic make kagome lattices suitable for the production of in-plane surface waves of variable wave length. The principle is demonstrated
40 in Fig. 2 where adjacent corridors are driven with different amplitudes to create two waves of different lengths. A quasi-stiffness matrix is presented by Bird et al.[8] for determination of the compliant modes of a kagome lattice and the subsequent optimization of the lattice by manipulating these modes.

Figure 2: Using kagome lattices for producing in-plane waves of variable wave length [9]

In an experimental study[9], a novel compliant adaptive surface was created and measured, via attaching the kagome lattice to a membrane skin, to
45 be capable of generating controllable in-plane travelling waves in a structurally

supported continuum. The innovative surface has been shown experimentally to be capable of producing waveforms of variable length and speed and recording a significant reduction in skin friction[10]. However in previous works designs
50 were predominately based on a linear formulation which is valid only for the design of lattices under small amplitude actuation. The present work which employs a novel nonlinear formulation in conjunction with Bayesian optimization, presents a bespoke design tool for the optimum design of lattices with a significant reduction in the actuation force. A large deformation theory for
55 in-plane beam problems, analogous to Budiansky’s nonlinear shell theory, is formulated by Epstine and Murrey [11]. The geometrically nonlinear finite element formulation of elastic beams in this work is derived for the general 3D load condition. The formulation stems from a second-order geometric approximation for finite rotations presented by Argyris [12] and was used for developing a non-
60 linear layer-wise shell element formulation in the study of compliant composite morphing wings [13], [14]. The developed formulation is applied in an in-house code and compliance modes of the lattice are extracted using the singular value decomposition as a model reduction technique due to the cost benefits of computing and complexity under high-amplitude actuation. The routine is used as
65 a design tool to refine the kagome lattice by determination and enhancement of some key compliant modes for delivering desired actuation modes. However, it is shown that compliance modes change significantly under high-amplitude actuation and subsequently reduce the functionality of the designed structure. This implies the necessity of the re-optimization of the adaptive structure under
70 large deformation. This is performed using a machine-learning-based optimization method. The process illustrates how by removing some internal members subject to feasibility constraints, a modified integrated feasible design of the lattice can be obtained which is effective for high-amplitude actuation.

2. Nonlinear second order beam finite element formulation

75 In this section, a second-order kinematic and geometrically nonlinear finite element formulation of elastic beams is derived under large displacements and rotations, with the assumption of small strain values.

2.1. Second-order kinematics of elastic beams under large rotations

Figure 3 shows the motion of a beam in a stationary Cartesian coordinate system, where it can experience large displacements and rotations with small strains. The convected orthogonal coordinate system at each point, ${}^t x_i$ ($i = 1, 2, 3$), is defined along the principle axes of beam at time t , where ${}^t \hat{e}_3$ denotes the unit tangential vector along the beam axis. In the following, a right subscript $i = 1, 2, 3$ indicates that the variables apply to the convected, rather than
85 the global coordinate system. Also, the left superscript is used to denote the time that the variable value is measured.

Figure 3: Motion of beam axis in Cartesian coordinate frame

The main assumptions in this study are as follows:

1. The kinematic conditions do not include warping deformation (i.e., plane sections remain plane);

- 90 2. The strains normal to the beam axis are negligible (i.e., the cross section areas remain constant).

The kinematics of the beam are studied based on the Updated Lagrangian (UL) incremental approach, which has more numerical effectiveness in comparison with the Total Lagrangian (TL) formulation[15]. In the UL framework,
 95 referring to Fig. 3, the position vector of a generic point $\mathbf{P}$ along the beam axis at a known configuration of the beam at time t can be described as

$${}^t\mathbf{r} = x_3 {}^t\hat{\mathbf{e}}_3 \quad (1)$$

Then, the position vector of a point in the corresponding cross section is expressed as

$${}^t\mathbf{R} = {}^t\mathbf{r} + x_1 {}^t\hat{\mathbf{e}}_1 + x_2 {}^t\hat{\mathbf{e}}_2 \quad (2)$$

During motion of the beam to an unknown configuration at time $t + \Delta t$,
 100 making use of assumptions 1 and 2, this same point deforms to

$${}^{t+\Delta t}\mathbf{R} = {}^{t+\Delta t}\mathbf{r} + x_1 {}^{t+\Delta t}\hat{\mathbf{e}}_1 + x_2 {}^{t+\Delta t}\hat{\mathbf{e}}_2 \quad (3)$$

where ${}^{t+\Delta t}\mathbf{r} = {}^t\mathbf{r} + \mathbf{W}$, in which $\mathbf{W}(x_3) = \begin{Bmatrix} w_1 & w_2 & w_3 \end{Bmatrix}^T$ expresses the incremental displacement vector of the beam axis and is measured in the convected coordinate system at time t . If the incremental finite rotation components around the principle axes of beam ${}^t\hat{\mathbf{e}}_i$ at the generic point $\mathbf{P}$ in Fig. 3 are
 105 denoted with $\boldsymbol{\Theta}(x_3) = \begin{Bmatrix} \theta_1 & \theta_2 & \theta_3 \end{Bmatrix}^T$, the updated director vectors after some geometric manipulations can be expressed as

$${}^{t+\Delta t}\hat{\mathbf{e}}_i = \boldsymbol{\Theta} \times {}^t\hat{\mathbf{e}}_i + \frac{1}{2}\boldsymbol{\Theta} \times (\boldsymbol{\Theta} \times {}^t\hat{\mathbf{e}}_i) \quad (4)$$

Further details concerning this relation can be found in Ref [12]. Using Eq (4), the new transverse director vectors are obtained as

$${}^{t+\Delta t}\hat{\mathbf{e}}_1 = \left\{ 1 - \frac{1}{2}(\theta_2^2 + \theta_3^2) \quad \theta_3 + \frac{1}{2}\theta_1\theta_2 \quad -\theta_2 + \frac{1}{2}\theta_1\theta_3 \right\}^T \quad (4a)$$

$${}^{t+\Delta t}\hat{\mathbf{e}}_2 = \left\{ -\theta_3 + \frac{1}{2}\theta_1\theta_2 \quad 1 - \frac{1}{2}(\theta_1^2 + \theta_3^2) \quad \theta_1 + \frac{1}{2}\theta_2\theta_3 \right\}^T \quad (4b)$$

Substituting Eqs (4a)-(4b) in Eq (3) and subtracting Eq (1) from the result, yields the following second-order approximations for increments in the displacement field with finite rotations from time t to time $t + \Delta t$.

$$u_1(x_1, x_2, x_3) = w_1(x_3) - x_2\theta_3(x_3) + \frac{x_2}{2}\theta_1\theta_2 - \frac{x_1}{2}(\theta_2^2 + \theta_3^2) \quad (5a)$$

$$u_2(x_1, x_2, x_3) = w_2(x_3) + x_1\theta_3(x_3) + \frac{x_1}{2}\theta_1\theta_2 - \frac{x_2}{2}(\theta_1^2 + \theta_3^2) \quad (5b)$$

$$u_3(x_1, x_2, x_3) = w_3(x_3) - x_1\theta_2(x_3) + x_2\theta_1(x_3) + \frac{x_1}{2}\theta_1\theta_3 + \frac{x_2}{2}\theta_2\theta_3 \quad (5c)$$

It is worth noting that the first linear terms in Eqs(5) are in accordance with
 110 the well-known first order displacement field for an Euler-Bernoulli beam [16].

Large deformation with small strain values during bending deformation of the beam structure, calls for the Green-Lagrange (GL) strain tensor that is invariant under rigid body rotations. The incremental components of GL strain in the convected coordinate system are given by

$$\epsilon_{ij} = \frac{1}{2} \left(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j} \right) \quad (6)$$

115 where a comma denotes differentiation with respect to the following coordinates. Introducing Eqs (5) into the above equation and ignoring the third and higher-order terms, the non-zero components of incremental GL strains are obtained by

$$\begin{aligned} \epsilon_{33} = & w'_3 - x_1\theta'_2 + x_2\theta'_1 + \frac{\theta_3}{2}(x_1\theta'_1 + x_2\theta'_2) + \frac{\theta'_3}{2}(x_1\theta_1 + x_2\theta_2) \\ & + \frac{1}{2} \left[(w'_1 - x_2\theta'_3)^2 + (w'_2 + x_1\theta'_3)^2 + (w'_3 - x_1\theta'_2 + x_2\theta'_1)^2 \right] \end{aligned} \quad (7a)$$

$$\begin{aligned} 2\epsilon_{13} = & w'_1 - x_2\theta'_3 - \theta_2 - x_1(\theta_2\theta'_2 + \theta_3\theta'_3) + \frac{x_2}{2}(\theta_1\theta'_2 + \theta'_1\theta_2) + \frac{1}{2}\theta_1\theta_3 \\ & + \theta_3(w'_2 + x_1\theta'_3) - \theta'_2(w'_3 - x_1\theta'_2 + x_2\theta'_1) \end{aligned} \quad (7b)$$

$$\begin{aligned} 2\epsilon_{23} = & w'_2 + x_1\theta'_3 + \theta_1 + \frac{x_1}{2}(\theta_1\theta'_2 + \theta'_1\theta_2) - x_2(\theta_1\theta'_1 + \theta_3\theta'_3) + \frac{1}{2}\theta_2\theta_3 \\ & - \theta_3(w'_1 - x_2\theta'_3) + \theta'_1(w'_3 - x_1\theta'_2 + x_2\theta'_1) \end{aligned} \quad (7c)$$

where the prime symbol is used to denote x_3 -derivatives.

2.2. Geometrically nonlinear FE formulation

Figure 4 shows a beam element with two nodes at two consecutive time steps which undergoes a general three dimensional displacement field.

Figure 4: Beam element under large deformation

Each node has six degrees of freedom, consisting of three displacements and three rotations. The incremental displacement vector at node k in the convected local coordinate system is expressed as $\mathbf{U}^k = \{w_1^k \ w_2^k \ w_3^k \ \theta_1^k \ \theta_2^k \ \theta_3^k\}^T$. Transformation between the local and global Cartesian coordinate systems can be performed using the updated director vectors at time t as follows.

$$\mathbf{q}^k = \begin{bmatrix} {}^t\mathbf{T}^K & \mathbf{0} \\ \mathbf{0} & \mathbf{T}^k \end{bmatrix} \mathbf{U}^k \quad (8)$$

where

$${}^t\mathbf{T}^k = \begin{bmatrix} {}^t\hat{\mathbf{e}}_1 & {}^t\hat{\mathbf{e}}_2 & {}^t\hat{\mathbf{e}}_2 \end{bmatrix} \quad (9)$$

and $\mathbf{q}^k$ is the incremental displacement vector at node k in the global Cartesian coordinate system.

The finite element formulation is implemented using linear interpolations to describe axial displacement and torsional rotation, and Hermitian functions to interpolate the transverse bending displacements and rotations. Then, with respect to the right-handed convected coordinate system defined in Fig. 4, the interpolations are as follows:

$$w_1 = h_3 w_1^1 + h_4 \theta_2^1 + h_5 w_1^2 + h_6 \theta_2^2 \quad (10a)$$

$$w_2 = h_3 w_2^1 - h_4 \theta_1^1 + h_5 w_2^2 - h_6 \theta_1^2 \quad (10b)$$

$$w_3 = h_1 w_3^1 + h_2 w_3^2 \quad (10c)$$

$$\theta_1 = -h'_3 w_2^1 + h'_4 \theta_1^1 - h'_5 w_2^2 + h'_6 \theta_1^2 \quad (10d)$$

$$\theta_2 = h'_3 w_1^1 + h'_4 \theta_2^1 + h'_5 w_1^2 + h'_6 \theta_2^2 \quad (10e)$$

$$\theta_3 = h_1 \theta_3^1 + h_2 \theta_3^2 \quad (10f)$$

130 where the relations for linear and Hermitian shape functions h_i are given in the Appendix.

In the development of the geometrically nonlinear finite element formulation in the UL framework, the equilibrium of the body at time $t + \Delta t$ is expressed by employing the principle of virtual displacements as follows

$$\int_{tV} {}^{t+\Delta t}_t S_{ij} \delta {}^{t+\Delta t}_t \epsilon_{ij} d^t V = {}^{t+\Delta t} \mathfrak{R} \quad (11)$$

135 where ${}^{t+\Delta t} \mathfrak{R}$ is the external virtual work. Also, ${}^{t+\Delta t}_t S$ denotes the 2nd-Piola Kirchhoff (PK2) stress tensor, which is the work-conjugate with the GL strain

tensor [17] and is related to the Cauchy stress tensor by

$${}^{t+\Delta t}_t S_{ij} = \frac{{}^t \rho}{{}^{t+\Delta t}_t \rho} {}^{t+\Delta t}_t x_{i,m} {}^{t+\Delta t}_t \sigma_{mn} {}^{t+\Delta t}_t x_{j,n} \quad (12)$$

where ${}^{t+\Delta t}_t x_{i,j} = \partial^{t+\Delta t} x_i / \partial^t x_j$ is the deformation gradient tensor, which describes the stretches and rotations that material fibers undergo from time t to

time $t + \Delta t$.

By analogy to the notation used for coordinates in Section 2.1, the left superscript indicates in which configuration the quantity occurs. In addition, the left subscript indicates the configuration with respect to which the quantity is measured. Also, a quantity without a left superscript and subscript, refers to the incremental value, similar to the notation used for displacement and strain fields in Eqs (5) and (7), respectively.

Satisfying the equilibrium equation (11) through the complete history of load application, the large deformation of compliant structures is determined via a step-by step solution and using incremental decompositions of PK2 stress and GL strain tensors in UL framework, i.e. ${}^{t+\Delta t}_t S_{ij} = {}^t \sigma_{ij} + {}_t S_{ij}$ and ${}^{t+\Delta t}_t \epsilon_{ij} = {}_t \epsilon_{ij}$. In each increment, since the internal force and stiffness matrix depend on the nodal displacements, an iterative solution scheme should be applied. In this fashion, the incremental finite element form of equilibrium Eq (11) using the Newton-Raphson method can be expressed as follows

$$\begin{aligned} \left({}^{t+\Delta t}_t \mathbf{K}_L^{(i-1)} + {}^{t+\Delta t}_t \mathbf{K}_{NL}^{(i-1)} \right) \mathbf{q}^{(i)} &= {}^{t+\Delta t}_t \mathbf{R} - {}^{t+\Delta t}_t \mathbf{F}^{(i-1)} \\ {}^{t+\Delta t}_t \mathbf{q}^{(i)} &= {}^{t+\Delta t}_t \mathbf{q}^{(i-1)} + \mathbf{q}^{(i)} \end{aligned} \quad (13)$$

where

$${}^{t+\Delta t}_t \mathbf{K}_L^{(i-1)} = {}^{t+\Delta t}_t \mathbf{T}^{(i-1)} {}^{t+\Delta t}_t \mathbf{K}_L^e {}^{t+\Delta t}_t \mathbf{T}^{(i-1)T} \quad (14a)$$

$${}^{t+\Delta t}_t \mathbf{K}_{NL}^{(i-1)} = {}^{t+\Delta t}_t \mathbf{T}^{(i-1)} {}^{t+\Delta t}_t \mathbf{K}_{NL}^{e(i-1)} {}^{t+\Delta t}_t \mathbf{T}^{(i-1)} \quad (14b)$$

$${}^{t+\Delta t}_t \mathbf{F}^{(i-1)} = {}^{t+\Delta t}_t \mathbf{T}^{(i-1)} {}^{t+\Delta t}_t \mathbf{F}^{e(i-1)} \quad (14c)$$

in which ${}^{t+\Delta t}\mathbf{T}^{(i-1)}$ is the transformation matrix from the local element coordinate systems to global one which is formed from individual sub-matrices Eq(9), in which the director vectors at each iteration are updated using Eqs(4). Also, the element linear and nonlinear stiffness matrices and internal force vector in
160 local coordinate system are obtained as follows

$${}^{t+\Delta t}_t \mathbf{K}_L^e = \int_V {}^{t+\Delta t}_t \mathbf{B}_L^{eT} {}^t \mathbf{C}^e {}^{t+\Delta t}_t \mathbf{B}_L^e dV \quad (15a)$$

$${}^{t+\Delta t}_t \mathbf{K}_{NL}^e = \int_V {}^{t+\Delta t}_t \mathbf{B}_{NL}^{eT} {}^t \hat{\boldsymbol{\sigma}}^e {}^{t+\Delta t}_t \mathbf{B}_{NL}^e dV \quad (15b)$$

$${}^{t+\Delta t}_t \mathbf{F}^e = \int_V {}^{t+\Delta t}_t \mathbf{B}_L^{eT} {}^t \hat{\boldsymbol{\sigma}}^e dV \quad (15c)$$

where $\mathbf{B}_L^e$ and $\mathbf{B}_{NL}^e$ are linear and nonlinear strain-displacement matrices in element coordinate system, which are calculated by substituting the second-order incremental displacement field Eqs (5) and the interpolations Eqs (10) in linear, $e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i})$, and nonlinear, $\eta_{ij} = \frac{1}{2} u_{k,i} u_{k,j}$, parts of GL strain
165 Eq (6). Denoting ${}^{t+\Delta t}_t \mathbf{B}^e = \left[{}^{t+\Delta t}_t \mathbf{B}^{e^1} \mid {}^{t+\Delta t}_t \mathbf{B}^{e^2} \right]$ for both linear and nonlinear matrices, they can be expressed after some mathematical manipulation as

$${}_{t+\Delta t}^t \mathbf{B}_L^{e^i} = \begin{bmatrix} -x_1 h''_{(2i+1)} + x_2 g'_{(2i+1)} & -x_2 h''_{(2i+1)} + x_1 g'_{(2i+1)} & 0 & \dots \\ g(2i+1) + \frac{[1,1]}{[-2,2]} \Gamma_{(2i+1):[1,2]}^{[2,4]} & \frac{[2,2,1]}{[-2,1,-2]} \Gamma_{(2i+1):[1,2,2]}^{[2,4,5]} & 0 & \dots \\ \frac{[1,2,2,1]}{[-2,-2,2,-1]} \Gamma_{(2i+1):[1,1,2,2]}^{[1,2,4,5]} & -g(2i+1) + \frac{[2]}{[-2]} \Gamma_{(2i+1):[2]}^{[5]} & 0 & \dots \\ \dots x_2 h''_{(2i+2)} + x_1 g'_{(2i+2)} & -x_1 h''_{(2i+2)} + x_2 g'_{(2i+2)} & 0 & \\ \dots \frac{[2,1]}{[-2,2]} \Gamma_{(2i+2):[2,2]}^{[4,5]} & g(2i+2) & x_1 h'_i + \frac{[2]}{[-2]} \Gamma_{i:[-k]}^{[6]} & \\ \dots g(2i+2) + \frac{[2]}{[2]} \Gamma_{(2i+2):[2]}^{[5]} & \frac{[1]}{[-2]} \Gamma_{(2i+2):[2]}^{[5]} & -x_2 h'_i + \frac{[1]}{[-2]} \Gamma_{i:[-k]}^{[6]} & \end{bmatrix} \quad (16)$$

$${}_{t+\Delta t}^t \mathbf{B}_{NL}^{e^i} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -h_i \\ h'_{2i+1} & 0 & 0 & 0 & h'_{2i} & -x_2 h'_i \\ 0 & 0 & 0 & 0 & 0 & h_i \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & h'_{2i+1} & 0 & -h'_{2i} & 0 & x_1 h'_i \\ -h'_{2i+1} & 0 & 0 & 0 & -h'_{2i} & 0 \\ 0 & -h'_{2i+1} & 0 & h'_{2i} & 0 & 0 \\ -x_1 h''_{2i+1} & -x_2 h''_{2i+1} & h'_i & x_2 h''_{2i} & -x_1 h''_{2i} & 0 \end{bmatrix} \quad (17)$$

where

$$\frac{[a_1 \dots a_n]}{[b_1 \dots b_n]} \Gamma_{m:[d_1 \dots d_n]}^{[c_1 \dots c_n]} = \sum_{j=1}^n \sum_{k=1}^{\#Node} \frac{x_{a_j}}{b_j} \mathbf{U}_{(6(k-1)+c_j)}^e (h'_m h''_{2k+d_j} + h'_{2k+d_j} h''_m) \quad (18)$$

and

$$g_m = \frac{1}{2} h'_m (h_1 \theta_3^1 + h_2 \theta_3^2) \quad (19)$$

In addition, terms ${}^t\mathbf{C}^e$ and ${}^t\hat{\boldsymbol{\sigma}}$ in Eqs (15a) and (15c) are the incremental stress-strain tensor and the Cauchy stress vector in the element coordinate system at time t , respectively. Also, ${}^t\bar{\boldsymbol{\sigma}}^e$ in Eq (15b) denotes the Cauchy stress matrix in the following form.

$${}^t\bar{\boldsymbol{\sigma}}^e = \begin{bmatrix} {}^t\boldsymbol{\sigma}^{*e} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & {}^t\boldsymbol{\sigma}^{*e} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & {}^t\boldsymbol{\sigma}^{*e} \end{bmatrix} \quad (20)$$

where

$${}^t\boldsymbol{\sigma}^{*e} = \begin{bmatrix} {}^t\sigma_{11}^e & {}^t\sigma_{12}^e & {}^t\sigma_{13}^e \\ {}^t\sigma_{21}^e & {}^t\sigma_{22}^e & {}^t\sigma_{23}^e \\ {}^t\sigma_{31}^e & {}^t\sigma_{32}^e & {}^t\sigma_{33}^e \end{bmatrix} \quad (21)$$

For reasons of simplicity, the superscript $(i-1)$ is dropped from Eq(15) in subsequent formulation.

3. Determination and enhancement of compliant modes

The singular value decomposition (SVD) decomposes the stiffness matrix with n degrees of freedom into its singular values according to

$$\begin{aligned} \underbrace{\mathbf{K}}_{(n \times n)} &= \underbrace{\mathbf{K}_L}_{(n \times n)} + \underbrace{\mathbf{K}_{NL}}_{(n \times n)} = \underbrace{\boldsymbol{\Psi}}_{(n \times n)} \underbrace{\boldsymbol{\Omega}}_{(n \times n)} \underbrace{\boldsymbol{\Phi}^T}_{(n \times n)} \\ &= \begin{bmatrix} | & | & & | \\ \psi_1 & \psi_2 & \dots & \psi_n \\ | & | & & | \end{bmatrix}_{(n \times n)} \begin{bmatrix} \omega_1 & & 0 \\ & \omega_2 & \\ & & \ddots \\ 0 & & & \omega_n \end{bmatrix}_{(n \times n)} \begin{bmatrix} - & \phi_1^T & - \\ - & \phi_2^T & - \\ & \vdots & \\ - & \phi_n^T & - \end{bmatrix}_{(n \times n)} \end{aligned} \quad (22)$$

where the matrix $\boldsymbol{\Omega}$ is the diagonal matrix of singular values sorted in decreas-

ing order, $\omega_1 \geq \omega_2 \geq \dots \geq \omega_n \geq 0$. The matrices $\mathbf{\Psi}$ and $\mathbf{\Phi}$ are orthogonal matrices containing the left and right singular vectors. Then, from Eq (13), the
180 incremental displacement of structure can be obtained by summation of inverted singular values and their compliant modes at time t , as

$$\mathbf{q} = \mathbf{\Psi} \mathbf{\Omega}^{-1} \mathbf{\Phi}^T ({}^{t+\Delta t} \mathbf{R} - {}^t \mathbf{F}) = \sum_{i=1}^n \frac{1}{\omega_i} \left\{ \boldsymbol{\psi}_i \boldsymbol{\psi}_i^T ({}^{t+\Delta t} \mathbf{R} - {}^t \mathbf{F}) \right\} \quad (23)$$

where for the square, symmetric and positive definite stiffness matrices $\mathbf{\Psi} = \mathbf{\Phi}$, contains the compliant modes $\boldsymbol{\psi}_i$. As it can be observed from Eq (23), the SVD provides a model reduction technique in which the smallest singular values of
185 $\mathbf{K}$ and their compliant modes have the most contribution to the incremental deformation.

In the following section, the generally applicable formulation detailed above is applied for the determination and enhancement of compliant modes of kagome lattice geometry.

190 4. Kagome lattice design

The work presented in this paper details the design of the compliant lattice for delivering large amplitude actuation based on the kagome geometry. Figure 5 shows a finite kagome lattice with six corridors ($r_1 - r_6$) and nineteen columns ($c_1 - c_{19}$). The internal geometry of the lattice is fully parameterized by the
195 values of height, d_1 , thickness d_2 and length, d_3 , of the member.

Figure 5: Finite kagome lattice structure defined with geometrical parameters: width d_1 , thickness d_2 and length d_3

The kagome lattice is loaded at three actuation points along each corridor, shown using blue triangular symbols in Fig. 5, where the in-plane load direction changes alternately between two adjacent corridors. Also, boundary conditions are applied between actuating corridors, shown with filled-in red circles, to suppress out-of-plane compliant modes. To get large values of deformation, standing leaf springs are used between corridors as supporting flexible boundary conditions. More details on the compliant boundary conditions are discussed in Section 4.2. Also, the kagome lattice is given material properties corresponding to fully hardened AISI 301 graded stainless steel with Young's modulus $E = 186$ GPa and Poisson ratio $\nu = 0.3$.

4.1. Geometric design

The internal geometry of the kagome lattice is determined based on the requirement of minimum actuation effort. Using non-dimensional parameters, the slenderness ratio of the kagome lattice can be defined as the quotient between the length of the hexagonal unit cell, shown in Fig. 5 and its height, i.e. $\lambda = \frac{\sqrt{3}d_3}{d_1}$. Also, the parameter $\beta = \frac{d_1}{d_2}$ is used for measuring the aspect ratio of an individual member.

Figure 6 shows the variation of actuation force versus β for different values of λ . It can be observed that for a kagome lattice with a defined slenderness ratio λ , the actuation effort decreases significantly with reducing member's thickness. On the other hand, increasing the member's length for a defined ratio of height to thickness reduces the actuation effort for in-plane bending deformation.

Figure 6: Variation of actuation force versus aspect ratio, $\beta = \frac{d_1}{d_2}$, for kagome lattices with different values of slenderness ratio, $\lambda = \frac{\sqrt{3}d_3}{d_1}$.

In many practical cases, the dimension d_3 is determined based on the design requirements for the distance between adjacent corridors, i.e., $\sqrt{3}d_3$. This work aims to make a lattice with a distance of 20 mm between corridors. From Fig. 6 it can be observed that for an aspect ratio greater than around 80, the asymptotic actuation force reaches its minimum value. On the other hand, from the previous work [8], the propensity to generate exclusively in-plane displacement modes can be ensured for a kagome lattice with an aspect ratio of ≈ 120 and a slenderness ratio of ≈ 5.5 , in which out-of-plane compliant modes have higher levels of activation energy. This will be clarified further in Figs 10 and 11, where

the confined corridor and alternating shapes are obtained by the summation of low-energy in-plane compliant modes. Ultimately, Table 1 shows the value of parameter d_3 and two other geometrical parameters, d_{1-2} to ensure a lattice with the minimum required in-plane actuation force.

Table 1: Dimensions of lattice structure, where dimensions d_{1-3} are in accordance with Fig. 5

d_1	d_2	d_3
3.62 mm	30 μm	11.55 mm

4.2. Compliant boundary condition

The boundary conditions are designed to have minimum effect on the in-plane movement of the kagome lattice under large displacement values. This is accomplished by using leaf springs at boundary points as compliant boundary conditions, see Fig. 7. The leaf springs are designed from the same grade of steel as the structure.

Compliant boundary conditions are modelled as three longitudinal springs K_i ($i = 1, 2, 3$) and one torsional spring K_6 . Denoting geometrical parameters of the leaf spring with S_{1-3} , as shown in Fig. 7, the equivalent stiffness in span (K_1) and chord (K_2) directions is calculated based on the deflection of a cantilever beam with zero slope at the tip point, where its movement is confined within the plan of the kagome lattice. The equivalent stiffness in normal direction (K_3) is determined as the stiffness of bar element with the cross-sectional area of $S_1 S_2$. The torsional stiffness K_6 is estimated based on the torsion constant J given in Ref [18] for bars with rectangular cross section.

Figure 7: Modeling of compliant boundary condition with three longitudinal springs in three direction and one rotation spring normal to the plane.

From a practical point of view, the load is applied to the kagome lattice through some load leaf springs at the actuation points. More details on the specific actuation mechanism can be found in Ref. [9]

Tables 2 and 3 give the dimension and equivalent stiffness values of standing and load leaf springs which are used in boundary conditions and load points, respectively. Standing leaf springs with a small thickness in the chord direction, S_2 , provide a compliant boundary condition which allows the structure to move normal to the direction of actuation. Also, assigning a large value of s_1 , and consequently a large stiffness K_1 in the span direction, results in a higher independence of actuating corridors. In addition, a high value of normal stiffness K_3 suppresses undesired out-of-plane deformation modes. It should be mentioned that the span-wise stiffness K_1 is neglected for load leaf springs which transfer span-wise movement to the lattice.

Table 2: Mechanical properties of leaf springs at boundary and load points, with notations presented in Fig. 7

	S_1	S_2	S_3
Standing leaf spring	6.35 mm	50 μm	12 mm
Load leaf spring	6.35 mm	50 μm	20 mm

Table 3: Equivalent stiffness of leaf springs at boundary and load points in FE model, calculated using the relations given in Fig. 7

	K_1	K_2	K_3	K_6
Standing leaf spring	$1.4 \times 10^6 \text{ N.m}^{-1}$	85 N.m^{-1}	$4.9 \times 10^6 \text{ N.m}^{-1}$	1.6 N.mm
Load leaf spring	0	18 N.m^{-1}	$2.9 \times 10^6 \text{ N.m}^{-1}$	1 N.mm

With the geometry and constraints given above, Fig. 8 illustrates the performance of the designed compliant boundary conditions in comparison with built-in boundary conditions in reducing the actuation forces under large deformation. This figure also compares the actuation forces obtained by the linear and geometrically nonlinear analyses. The significant difference between the two solutions, which is observed from the early stages of displacement, illustrates

the necessity of employing the nonlinear formulation even for small-amplitude actuation.

Figure 8: Comparison of force-displacement responses of kagome lattice under alternating load condition obtained via linear and geometrically nonlinear FE analysis with built in and compliant boundary conditions

4.3. Modal optimization

270 In the case of the kagome lattice structure, we aim to produce a continuous structure capable of producing in-plane confined corridor modes. As per the model reduction technique, the desired deformation is invoked by summation of the compliance modes with low energy. Based on the initial stiffness matrix for the undeformed configuration, the compliance modes of the finite kagome lattice
275 are obtained from Eq 22 and the six lowest energy modes are shown in Fig. 9. It should be mentioned that the low energy out-of-plane modes are avoided by applying the boundary conditions presented in Section 4.2.

Figure 9: First mode shapes of kagome lattice calculated based on the initial configuration

280 In this manner, the modes that should be activated, and to what degree so that a desired displacement q can be delivered, can be found as follows

$$\overbrace{\left[\Phi_r \right]}^{(n \times c)} \overbrace{\left\{ r \right\}}^{(c \times 1)} = \overbrace{\left[q \right]}^{(n \times 1)} \quad (24)$$

In this equation, Φ_r is a subset of the compliance modal matrix in Eq 22 that contains the c lowest required energy modes and r shows their corresponding weighting coefficients.

285 The required compliance mode shapes and the corresponding weights for producing confined corridor mode shapes are given in Table 4. Figure 10 shows the resultant summation of the weighted compliance modes. As can be expected, the symmetry in the geometry of the lattice is reflected in the repeated modes and weight coefficients in Table 4.

290 Also, the alternating deformed shape, according what is desired in Fig. 5, would be obtained by summation of mode shape 5 with weight 0.824 and mode shape 10 with weight 0.182, shown in Fig. 11.

At this point, it can be observed that the presented kagome lattice with aspect ratio of $\beta \approx 120$ and slenderness ratio of $\lambda \approx 5.5$ under the defined compliant boundary conditions provides a suitable design for producing the

295

Table 4: Weighting coefficients to produce the corridor modes

	Mode 5	Mode 6	Mode 10	Mode 12	Mode 17	Mode 18
Corridor mode 1	0.611	-0.721	0.788	0.513	-0.558	0.000
Corridor mode 2	-0.736	0.521	0.000	0.514	-0.948	0.682
Corridor mode 3	0.714	0.000	-0.591	-0.516	0.000	0.913
Corridor mode 4	-0.714	0.000	0.591	-0.516	0.000	0.913
Corridor mode 5	0.736	0.521	0.000	0.514	0.948	0.682
Corridor mode 6	-0.611	-0.721	-0.788	0.513	0.558	0.000

Figure 10: Confined corridor shapes obtained by summation of compliant mode shapes weighted according to Table 4

actuation corridor as well as alternating mode shapes with summation of compliance modes with low energy. However, under large deformation the stiffness matrix and consequently the compliance modes change. The change of compliant modes during deformation can be quantified using the Modal Assurance Criterion (MAC) [19] defined as

$$\text{MAC}({}^0\boldsymbol{\psi}^i, {}^t\boldsymbol{\psi}^i) = \frac{({}^0\boldsymbol{\psi}^{iT} {}^t\boldsymbol{\psi}^i)^2}{({}^0\boldsymbol{\psi}^{iT} {}^0\boldsymbol{\psi}^i)({}^t\boldsymbol{\psi}^{iT} {}^t\boldsymbol{\psi}^i)} \quad (25)$$

where ${}^0\boldsymbol{\psi}^i$ and ${}^t\boldsymbol{\psi}^i$ are the i -th compliant mode of the compliant structure at the initial configuration and time t , respectively. The parameter is calculated for the first six mode shapes under the applied displacement $u/d3 = 10\%$. The compliance modes are illustrated in Fig. 12, and the corresponding MAC values

Figure 11: Two compliant mode shapes 5 and 10 and the resultant alternating mode shape, obtained by component weights 0.824 and weight 0.182, respectively.

305 are given in Table 5.

Figure 12: First mode shapes of kagome lattice under applied displacement $u/d_3 = 10\%$

Table 5: Modal Assurance Criterion for six first mode shapes under applied displacement $u/d_3 = 10\%$

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
MAC%	7.8	2.2	1.8	5.4	0.3	0.3

The significant change in compliant mode shapes is due to tightening of internal members under large deformation. Figure 13 compares the deformed shapes

of the kagome lattice under displacement value of $u/d_3 = 10\%$ obtained from nonlinear and linear analyses. Importantly, the tightening of internal members
 310 causes a significantly large increase in actuation force, as shown in Fig. 8, in comparison with the linear solution that ignores the additional stiffness induced by the stress state. The tightening of internal members also causes a lateral shrinkage deformation toward the center of the kagome lattice structure in Fig. 13(b). To mitigate the tightening problem, some internal members can be re-
 315 moved. This implies the necessity of optimizing the kagome lattice design to find an optimum continuous lattice with the aim of reducing actuation force under large deformation.

Figure 13: Kagome lattice under applied displacement $u/d_3 = 10\%$ obtained via linear and geometrically nonlinear analysis

As can be observed in Eqs (15b) and (15c), the nonlinear stiffness matrix and the internal force vector are determined based on the instantaneous stress field in each increment. By removing each member of the lattice, the stress distribution
 320 changes in the new structure which in turn will change the incremental deformation path of the lattice under large amplitude actuation. As per the large numbers of configurations that can be obtained by removing different members of the lattice as well as the nonlinear deformation path which each candidate configuration will undergo, the optimum design is obtained using Bayesian opti-
 325 mization [20], which is a class of machine-learning-based optimization methods. Bayesian optimization, as a black-box derivative-free global optimization, provides a powerful tool in the development of structurally efficient lattice design under large deformation.

330 5. Optimization of lattices under large deformation using Bayesian optimization

In this current work the complete kagome lattice geometry corresponds to the full extent of the design space, which defines the possible location of the nodes and the connectivity of members. The optimization problem assumes only the
335 removal of members, in conjunction with feasibility constraints to ensure connectivity. Also, the objective function to be minimized is defined the required actuation force to achieve a displacement value of $u/d_3 = 50\%$.

As illustrated in the flowchart shown in Fig 14, at the first step, an initial data set is constructed using the developed nonlinear FE code and based on the
340 definition of the problem, including the geometry, material properties, boundary and load conditions. The initial lattice topologies are generated randomly, so that each topology is represented as a subset of the complete kagome lattice with regards to the constraints which defines design domain. Then a surrogate model for the objective is built using the ‘Regression Learner’ app in MATLAB [21]
345 with optimized hyper-parameters, which provides a Bayesian posterior probability distribution using all available data points. The obtained distribution describes potential values for actuation force at each candidate topology. Next, the ‘bayesopt’ function is used to find an optimum topology that minimizes the actuation force, where the ‘expected-improvement-plus’ acquisition function is
350 computed using the current posterior distribution. The actuation force in the candidate topology is calculated using the FE code and the new data point is added to the current data set. The loop will continue running until it reaches the convergence criterion.

5.1. Definition of design variables

355 Each subtopology of the kagome lattice is uniquely specified using a binary vector $\mathbf{t} = \{t_1, \dots, t_m\}$, called the topology vector. The vector has a length equal to the total number of lattice members, m , and the existence and absence of each member is represented by 0 or 1 as

Figure 14: Flowchart of the proposed optimization algorithm

$$t_i = \begin{cases} 1 & \text{if member } i \text{ exists in the lattice} \\ 0 & \text{otherwise} \end{cases}$$

360 Note that the topology vector defines only the existence of members in the lattice. The size of members in the topology vector are the same and are defined based on the design criteria as discussed in Section 4.1.

5.2. Definition of constraints

365 In order to guarantee feasibility of the optimal lattice as an integrated actuating structure, some constraints are defined on the topology of lattice within the design domain.

First, a candidate topology should be symmetric with respect to the middle column of the lattice. i.e. column c_{10} in Fig. 5. Indeed, we are interested to find the optimal design under the alternating load condition, in which load direction
370 changes from right to left (or from left to right) between two neighbouring corridors. In other words, the obtained solution should be the optimum design

under the applied load condition with left direction on the rows $r_{1,3,5}$ and right direction on rows $r_{2,4,6}$, as shown in Fig. 5, as well as the opposite applied load with left direction on the rows $r_{2,4,6}$ and right direction on rows $r_{1,3,5}$. With
375 regard to the symmetric location of boundary and load points, a symmetric geometry ensures that the solution will remain optimum with reversing the load direction at neighbouring corridors.

The remaining constraints are categorized in two classes — the constraints (1) along the corridors and (2) at joints between corridors defined as follows:

- 380 1. Constraints along each corridor: These constraints are applied to transfer loads along each corridor as active members.
 - 1.1. A member placed at a load point can not be removed (i.e., members on columns c_4 , c_{10} and c_{16} along each corridor at Fig. 5).
 - 1.2. Between two line loads only one member can be removed.
- 385 2. Constraints at joints: This constraint aims to keep the structural integrity of the lattice at joints between two neighbouring corridors. If four crossing members at a typical joint are denoted by t_a, t_b, t_c and t_d as shown in Fig. 15(a), then the joint constraint is defined as

$$(t_a + t_b) \cdot (t_c + t_d) \neq 0 \quad (26)$$

Three typical feasible joint topologies are illustrated in Fig. 15(b)-(d).
390 Note that the grey members indicate removed members with $t_i = 0$ in the topology vector.

As an illustration, Fig. 16 presents constraints 1 and 2 for feasible topologies on a small part of the studied kagome lattice in Fig. 5.

5.3. Optimum kagome lattice for producing large amplitude in-plane deformation

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Figure 17 shows the obtained optimum design for the finite lattice and the total actuation force versus displacement for the optimum lattice obtained via geo-

Figure 15: (a) A sample joint with four members between two corridors, and three typical feasible joint topologies related to (b): $t_a = 0$, (b): $t_a = t_d = 0$ and (b): $t_a = t_c = 0$ in Eq (26)

Figure 16: Constraints for feasible topologies: (a) a sample part of studied kagome lattice in Fig. 5 containing columns $c_4 - c_9$ of corridors $r_3 - r_4$ and, (b) a typical feasible topology, infeasible topologies: (c) violating constraint 1.1 (removing member under load) (d) violating constraint 1.2 (removing two members between two line loads, which makes the member on row r_4 and column c_7 ineffective) and (d) violating constraint 2 (loosing the connection between two corridors)

metrically nonlinear analysis. It can be observed that by the effective release of the internal stress field in the structure, the optimum lattice gives the significant
400 reduction of two orders of magnitude in the actuation force from 1240 N to 80 N at the displacement of $u/d_3 = 50\%$.

Figure 18 shows the deformed shapes of the optimum kagome lattice obtained by nonlinear FE analysis under alternating applied displacements at four different values, with equal intervals of $u/d_3 = 10\%$. In comparison with Fig 13(b) at
405 $u/d_3 = 10\%$, the vertical shrinkage has been eliminated in the deformed shape of optimum kagome lattice at Fig 18(a). Due to the boundary condition discontinuities at the top and bottom of the kagome lattice, the boundary areas shrink in the vertical direction with increasing actuation amplitude. However, the inner corridors are still capable of generating the large amplitude alternating
410 deformed shape, as shown in Fig 18(b)-(d).

Figure 17: Nonlinear force-displacement diagrams under alternating applied load for the optimum lattice in comparison with the complete kagome lattice. The grey members in the optimal design of kagome lattice indicate the removed members.

Figure 18: Deformed shape of the inner part of the optimum kagome lattice under large amplitude alternating actuation. With increasing the actuation amplitude, the top and bottom boundary areas shrink in vertical direction due to boundary condition discontinuities.

6. Conclusions

This paper has demonstrated the determination and enhancement of compliant modes for adaptive beam assemblies – with a particular focus on the kagome

lattice geometry. As a continuous system with many degrees of freedom, deter-
415 mination of compliance modes are carried out based on finite element discretiza-
tion method and then singular value decomposition of stiffness matrix. Finite
element analysis is performed using an in-house FE code developed based on the
elaborated second-order nonlinear beam finite element formulation presented for
beam assemblies under large rotations. Adaptivity of the structure under large
420 deformation has been also improved by introducing compliant boundary condi-
tions. Using a reduced modal model as the basis for the design of adaptive struc-
ture, the desired actuation mode is constructed by weighted summation of small
number of key modes with low energy. Conversely, the locations of boundary
points are specified to suppress undesired out-of-plane compliant mode shapes.

425 With increasing deformation, the induced internal stress state causes change
of the compliant mode shapes which are substantial and reduce the efficiency
of designed structure for delivering the high amplitude actuation. This implies
the necessity of re-optimization of the designed adaptive structure for large
actuating values. This work employs the Bayesian optimization algorithm, as
430 a black-box derivative-free global optimization tool for the design of effective
lattice. The optimization problem is defined with the complete kagome lattice
as the largest design space and assumes only the removal of members. Also,
some constraints are defined on removed members to guarantee feasibility of
the optimal lattice as an integrated actuating structure. The process detailed
435 yields an optimum design, in which the internal stress is released effectively,
with the significant reduction of two orders of magnitude in actuation force.

In continuing work a full-scale actuated module is being fabricated and will
be used to validate the nonlinear dynamic response of the optimized lattice. This
adaptive surface will form the basis of a large scale flow control experiment in
440 Imperial College’s 10×5 wind tunnel.

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445 8. Appendix

The FE formulation of the nonlinear beam element is implemented using the linear interpolations (h_1 and h_2 in Eqs(27)) to describe axial displacement and torsional rotation, and Hermitian functions ($h_i; i = 3..6$ in Eqs(27)) to interpolate the transverse bending displacements and rotations.

$$h_1 = 1 - \frac{x_3}{L} \quad (27a)$$

$$h_2 = \frac{x_3}{L} \quad (27b)$$

$$h_3 = 1 - 3\frac{x_3^2}{L^2} + 2\frac{x_3^3}{l^3} \quad (27c)$$

$$h_4 = x_3(1 - 2\frac{x_3}{L} + \frac{x_3^2}{l^2}) \quad (27d)$$

$$h_5 = \frac{x_3^2}{L^2}(3 - 2\frac{x_3}{l}) \quad (27e)$$

$$h_6 = \frac{x_3^2}{L}(-1 + \frac{x_3}{l}) \quad (27f)$$

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