Laser-Profiled Continuum Robot with Integrated Tension-Sensing for Simultaneous Shape and Tip Force Estimation*

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Abstract

The development of miniaturised continuum robots has a wide range of applications in minimally invasive endoluminal interventions. In order to navigate inside tortuous lumens without impinging on the vessel wall and causing tissue damage or the risk of perforation, it is necessary to have simultaneous shape sensing of the continuum robot and its tip contact force sensing with the surrounding environment. Miniaturisation and size constraint of the device have precluded the use of conventional sensing hardware and embodiment schemes. In this paper, we propose the use of optical fibres for both actuation and tension/shape/force-sensing. It uses a model-based method with structural compensation, allowing direct measurement of the cable tension near the base of the manipulator without increasing the dimensions. It further structurally filters out disturbances from the flexible shaft. In addition, a model is built by considering segment differences, cable interactions/cross talks and external forces. The proposed model-based method can simultaneously estimate the shape of the manipulator and external force applied onto the robot tip. Detailed modelling and validation results demonstrate the accuracy and reliability of the proposed method for the miniaturised continuum robot for endoluminal intervention.

Keywords: Endoluminal intervention, continuum robot, tension-sensing cable, modelling, shape and force sensing

Running title: SHAPE/TIP FORCE ESTIMATION OF CONTINUUM ROBOTS

Introduction

Lung cancer is one of the most common causes of cancer death worldwide. Endobronchial optical biopsy with probe-based confocal laser endomicroscopy (pCLE) is an emerging diagnostic technique for endoluminal diagnosis and intervention. During intervention, an imaging probe is inserted into the airways to take microscopic images for analysis, thus allowing optical biopsy and providing instant feedback and oncological margin assessment. However, the imaging probe does not have the capability to steer itself to different airway branches, and standard bronchoscopes are too big to access small peripheral airways. To enable tissue scanning at distal airways, a cable-driven continuum robot with multiple lumens has been developed.¹ The robot consists of a bendable continuum manipulator and a long flexible shaft. The manipulator is composed of a series of interlocked rigid segments and an elastic imaging probe inside. Multiple disks are used to guide actuation cables and provide channels for the imaging probe or other small endoscopic instruments. The probe can be steered by the actuation cables to achieve active positioning and trajectory tracking. The nonlinear couplings among rigid segments, elastic probe and actuation cables, however, pose a significant challenge to estimate the manipulator's deflection characteristics under free or loaded bending configurations.

A schematic diagram of endobronchial intervention for tissue scanning with pCLE is shown in Fig. 1. After inserting the bronchoscope to the segmental bronchus near the target region, the robot is first passed through the bronchoscope's instrumental channel and then steered to the target airway location where the bronchoscope tip cannot reach. To ensure safe and steady tissue scanning, it is vital to maintain an appropriate probe pose and tip contact force with the tissue, requiring accurate shape and force sensing in situ. As it is difficult to attach additional force sensors to the probe tip due to size constraints, it is desirable to consider integrated actuation, shape and force sensing by using the same cable.

Literature review

Continuum robots have been used extensively for minimally invasive surgery.²⁻⁸ A range of actuation mechanisms and sensing methods have been utilised for control and shape estimation. In this paper, we focus on cable-driven continuum robots, which are more commonly used in flexible surgical instruments.

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A variety of technologies have been reported for shape sensing of continuum robots. Sensor-based approaches including using fibre Bragg grating (FBG) sensors, electromagnetic (EM) sensors and piezoelectric Polyvinylidene fluoride (PVDF) sensors have been investigated to measure the shape directly. Ryu *et al.*⁹ used a braided polymer tube as the substrate for three optical fibres with FBG sensor arrays to measure 3D curvature of a continuum robot. Liu *et al.*¹⁰ developed a sliding shape sensor assembled using an optical fibre and two nitinol wires. Xu *et al.*¹¹ used three helically wrapped FBG sensors to calculate the shape of continuum manipulators. FBG based shape sensors¹²⁻¹⁴ were developed by using multiple fibres with multiple FBG sensors along each fibre to estimate the shape of a continuum manipulator. EM-based shape reconstruction approaches¹⁵⁻¹⁷ usually rely on the assumption of constant curvature along the manipulator. However, the assumption does not always hold for continuum manipulators with non-constant curvature.¹⁸⁻²⁰ Therefore, Song *et al.*²¹ proposed a shape sensing algorithm based on multiple EM sensors, but their method is only applicable to long continuum manipulators with a diameter large enough to house multiple EM sensors. Hence, shape sensing techniques based on multiple EM sensors are not applicable to continuum manipulators with compact sizes. In practice, it is a trade-off between the number of sensors and the remaining space in the continuum manipulator. PVDF sensors have been used as the sensing elements because of its piezoelectric property. Several PVDF film sensors have been successfully integrated with a hyper-flexible beam to reconstruct its shape by using linear interpolation.²² A fast and simple shape/kinematic model has been built with PVDF films as the curvature sensors to calculate the shape of a pneumatic hyperflexible actuator with a good positional accuracy.²³ Techniques based on computer vision have also been developed for shape estimation. Hannan *et al.*²⁴ proposed a vision-based real-time shape estimation method for continuum manipulators. Croom *et al.*²⁵ presented a shape sensing algorithm by using stereo vision with an average error as low as 1.53 mm. For endobronchial intervention, although these algorithms could be implemented using the images acquired with the bronchoscope camera, the inevitable occlusion of the manipulator's tip when it is inside an airway branch can lead to detection failure (Fig. 1). For error correction, X-ray fluoroscopy was used at intervals to obtain 2D projections of the continuum manipulator during the procedure,²⁶ but it carries the risk associated with the excessive exposure of ionising radiation to both patients and surgeons. For the force sensing of continuum robots, several force sensors based on FBG sensors have been developed by our previous work,²⁷⁻²⁹ however, all of them require additional sensing elements, and the inner lumens of continuum manipulators are always occupied. They are not suitable for our application of optical biopsy with the demand of an inner lumen inside continuum manipulators.

Model-based methods represent a promising technique for simultaneously estimating the shape and tip force of cable-driven continuum manipulators. The methods only require the mathematical model, cable tension and tip/base pose to estimate the shape and tip force of the manipulator. It helps reduce the number of sensors, therefore occupying less space compared with other sensor-based techniques. The majority of model-based methods for cable-driven continuum robots are thus far based on the kinematic or mathematical model as illustrated in the review list of Table 1. However, the cable interaction is vital because its effect on the propagation of cable tension along the flexible shaft and manipulator has not been fully investigated in the model. In addition, most of them require several sensors and additional space for them to achieve shape and tip force estimation. Moreover, the long flexible shaft connected to the proximal end of the continuum manipulator for remote motion transmission can cause additional problems. The actuation force or the cable length measured at the proximal end of the flexible shaft is not exactly equal to that at the base of the continuum manipulator due to friction, even with the commonly used Bowden cable mechanisms. Especially for the miniaturised continuum manipulator with a large inner lumen, the coil springs are constrained in a confined wall where the mechanism cannot fully exert its performance. Furthermore, the hysteresis from the friction, cable elongation during tension and the gap between the actuation cable and the outer sheath has been reported when using Bowden cables. These effects can be compensated by modelling the hysteresis, but a certain amount of deviation still exists due to model inaccuracy. A better alternative is to put sensors at the base of continuum manipulator to measure the cable length or tension directly, but it is challenging due to the constrained space, sensor dimension and configuration. Lai *et al.*⁴⁰ proposed a method to measure the compression force on the nitinol tube by using a FBG sensor, which can derive the cable tension at the base of continuum manipulator. This method requires an extra tube as the substrate for each cable, which occupies more channel space if it is to be integrated into the continuum manipulator. Pattanshetti *et al.*⁴¹ and Karthikeyan⁴² integrated a PM-FBG fibre⁴³ with a pre-tensioned Kevlar fibre to accomplish the sensorised tendons with a steerable needle for force sensing and the preliminary results successfully validated the effectiveness. It requires the bonding of the PM-FBG fibre and Kevlar fibre, which leaves smaller space inside the needle's inner lumen. Hence, the challenge is to acquire the tension at the base of the continuum manipulator directly without sacrificing its inner space.

Objectives and contributions

This paper presents a model-based method with structural compensation to estimate the shape and tip force of a laser-profiled continuum manipulator. First, the integration of actuation and tension-sensing is introduced to drive the continuum manipulator while measuring the actuation force directly by filtering out the disturbances introduced by the connected flexible shaft. Second, the accurate model with due consideration of cable interactions and segment differences is built to describe its deflection characteristics. Third, a model-based method is developed to estimate the shape and tip force of the continuum manipulator

simultaneously under free or loaded bending. Finally, detailed experiments are conducted to validate the proposed design and method.

To the best of our knowledge, this is the first study to integrate the tension-sensing cables with our developed laser-profiled continuum manipulator to filter out unintended adverse effects from the connecting long flexible shaft. Moreover, this paper proposes an accurate mathematical model considering cable interactions and segment differences, as well as a model-based estimation method to simultaneously predict the shape and tip force for the laser-profiled continuum manipulator.

Robot Design and Estimation Principle

In this section, we introduce the robot design consisting of a continuum manipulator at the distal end and a long-thin flexible shaft. Also, a framework based on the accurate mathematical model and sensor feedbacks for shape and tip force estimation is illustrated.

Robot design, fabrication and sensor configuration

Fig. 2 shows the robot design for endobronchial intervention. It is composed of:

- 1) Laser-profiled continuum manipulator: The manipulator consists of two components, namely:
 - i) A series of discrete interlocked segments fabricated by laser profiling a thin-walled stainless-steel tube.
 - ii) Disks with multiple holes, laser cut from a thin stainless-steel plate. The holes are used to guide the imaging probe, actuation cables, or small interventional instruments.Both (i) and (ii) are laser cut using a 4-axis laser machine. Fig. 2D illustrates the CAD renderings of the laser cut joints and disks.
- 2) Flexible shaft: The flexible polymer shaft with multiple lumens is fabricated via a thermal drawing process. The thermal drawing process starts from a macroscopic preform containing the desired cross-section, which is consequently drawn into a long and thin flexible shaft. The preform was 3D printed with polycarbonate filaments. Polycarbonate is amorphous and drawable, which guarantees the desired stiffness and hardness of the shaft. The preform was then thermally drawn into tens of metres of the shaft in a custom-built draw tower.
- 3) Elastic imaging probe: The probe serves as the elastic backbone defining the continuum characteristics. The imaging fibre is made of silica glass and an outer elastic sleeve.

The joints and disks (green) are assembled as shown in Figs. 2B and 2C. The probe is then inserted through the lumen (V) and tightly fit at the tip. The continuum manipulator is then connected to the long flexible shaft to make a steerable robot as shown in Fig. 2A. Several sensors are placed inside those lumens of the continuum manipulator to provide different feedbacks. An optical fibre with one FBG sensor is used as its actuation cable, which also acts as a force sensor to directly measure the cable tension. The location of the FBG sensor is near the proximal end of the continuum manipulator, which enables direct tension measurement at this location (see Fig. 2B). The optical fibre is also made using the thermal drawing process, which simultaneously writes the grating and coats the fibre during the fibre draw, ensuring a high strength – about 50 N tension load at break.⁴⁴ The fibre is then inserted through one lumen of each disk and flexible shaft and terminated and fixed at the tip of continuum manipulator (see Fig. 2C). The other lumens can be used for EM sensors configured at the proximal and distal ends of continuum manipulator to derive the global pose. They can also be used for passing a Chip-on-Tip camera to provide the tip's pose relative to the anatomy using the method developed in our previous work.⁴⁵

When the cable (the optical fibre) is actuated, the tension is transmitted through the flexible shaft to deflect the continuum manipulator. The FBG sensor inscribed in the optical fibre is located at the base of the continuum manipulator to directly measure the actuation tension for the manipulator, while EM sensors embedded in the continuum manipulator can be used to derive the global pose of the tip and base. The tension and global poses can be combined to accomplish the sensing work. The principle can be found in the next subsection.

The merits of the proposed robot design include four aspects. First, it ensures a thin-walled structure, therefore allowing a larger space for its inner lumens. Second, the manipulator segments are interlocked right after the laser profiling without the need of being assembled manually. Third, the disk can be customised easily to enable a variety of configurations. Finally, the manufacturing cost is relatively low, which makes it ideal for cheap, disposable surgical instruments. In comparison with the traditional techniques to measure the actuation tension at the proximal end of the flexible shaft, the introduction of tension-sensing cables can help filter out the disturbances caused by the unknown deflection of the long flexible shaft and obtain the accurate tension input to the manipulator. Moreover, the integration of actuation and tension-sensing can save more space and make the robot more compact.

Principle of estimation

During endoluminal intervention, it is important to allow surgeons to visualise the shape of the continuum manipulator, in order to avoid excessive contact forces with the surrounding tissue. However, given the confined space within the miniaturised continuum manipulator, it is challenging to add additional sensing capabilities (e.g. shape sensor, tip force sensor).

In this section, we propose a model-based estimation method to predict the shape and/or tip force of the continuum manipulator using the tension-sensing cables (i.e. optical fibres). The deflection of the continuum manipulator can be categorised into two cases: free bending configuration (Case I) and loaded bending configuration (Case II), which have different inputs and outputs. For Case I, the aim is to estimate the shape under the free bending using the current proximal cable tension and its loading history to the manipulator; for Case II, the aim is to estimate the shape and tip force under the loaded bending using the current proximal cable tension, tip and base poses and their histories.

A general estimation principle for the continuum manipulator under free or loaded bending configuration is shown in Fig. 3. First, the pose sensors (e.g. EM sensors) are used to get the tip and base poses of the continuum manipulator in the global frame. Second, a model-based method using the tension-sensing cables and local poses is used to estimate the shape and tip force in the local frame. Finally, the global shape and tip force are obtained by registering the local estimates to the global frame. Both cases solve the developed mathematical model using the estimation method (Fig. 3A), but with different solving algorithms.

In this paper, we define the shape of the continuum manipulator as a series of discrete points at its joints' centres. Their absolute positions defined at the global frame - $\{C_G\}$ in Fig. 3B can be calculated as

$$\mathbf{P}_{C_i}^{C_G} = \mathbf{T}_{C_r}^{C_G} \cdot \mathbf{P}_{C_i}^{C_r} \quad (1)$$

where $\mathbf{P}_{C_i}^{C_r}$ is the i -th discrete point at $\{C_i\}$ defined at the local frame - $\{C_r\}$; $\mathbf{T}_{C_r}^{C_G}$ is the transformation matrix from the local frame - $\{C_r\}$ to the global frame - $\{C_G\}$. The absolute tip force in Fig. 3B can be calculated as

$$\mathbf{F}_T^{C_G} = \mathbf{T}_{C_r}^{C_G} \cdot \mathbf{F}_T^{C_r} \quad (2)$$

where $\mathbf{F}_T^{C_r}$ and $\mathbf{F}_T^{C_G}$ are the tip forces defined at $\{C_r\}$ and $\{C_G\}$.

Here, $\mathbf{P}_{C_i}^{C_r}$ and $\mathbf{F}_T^{C_r}$ can be calculated using the developed model-based estimation method. Details for the mathematical model and estimation method can be found in the following sections.

Model Considering Cable Interactions and Segment Differences

In this section, a mathematical model is developed to derive the deflection characteristics of the continuum manipulator. The model consists of several sub-models: a) static model of each segment; b) constitutive model of each segment; c) constitutive model of the tension-sensing cable; and d) friction model and tension propagation. Then, the system identification method for each segment is illustrated.

Method and assumptions

In general, the continuum manipulator can be deflected by the cable tension, as well as the tip force. To estimate its deflection characteristics, a mathematical model considering the cable tension and the tip force is developed. The cable length-based estimation method cannot measure the cable length applied to the continuum manipulator directly and accurately due to the elongation within the flexible long-thin shaft under unknown disturbances, while the tension-based estimation method can directly measure the actuation force using the optical fibre with the FBG sensor at the proximal end of the continuum manipulator.

The free body diagram of the continuum manipulator under cable tension and applied tip force is shown in Fig. 4. Here, the central imaging probe can be treated as an elastic rod, and the protruded probe tip outside the continuum manipulator is treated as a rigid rod due to its metal sheath. Because of the interlocked structure, we can assume that each segment can only bend at the interlocked point, as shown in Fig. 4. Based on this, the continuum manipulator is simplified to a skeleton with several serial sections, each of which has an articulated joint. The simplified structure can be found in Fig. 4. For the free bending configuration, the local shape can be estimated only given the cable tension at the proximal end; while for the loaded bending configuration, the additional tip force is required. Hence, in general, the inputs should include the cable tension and the tip force.

To build the mathematical model, we make several assumptions. First, the deflection of each segment is based on the constant curvature assumption, which means that the shape of the whole manipulator can be described as piecewise constant curvatures. Second, the friction at each interlocked joint is ignored or not explicitly modelled. Third, the actuation cable

interacts with each disk, where the normal force applied to the disk by the cable lies in the middle of two adjacent cable tensions, and the friction force is perpendicular to the normal force. Finally, the stiffness of each segment and the coefficient of static friction at each interaction point are different segment by segment.

Static model of each segment

A static model is developed for each segment by assuming that each segment deflection is with constant curvature, then the whole deflection of continuum manipulator is described by piecewise constant curvatures. Considering a general case of the i -th segment with external force and cable actuation in Fig. 4, it may bend toward the right side under the free bending (Status I) or may bend toward the left under the loaded bending (Status II or III). Here, we define the bending angle to be positive when it bends to the right side. Status I shows a positive bending angle for the i -th segment; Status II shows a negative bending angle for the i -th segment, but with a positive bending angle for the $i-1$ -th segment; Status III shows negative bending angles for both the i -th segment and the $i-1$ -th segment. We can observe that their main differences are the cable routine and the interaction points between the actuation cable and the body of continuum manipulator. The locations of $\mathbf{P}_{2i}$, $\mathbf{P}_{2i-1}$ and $\mathbf{P}_{2i-2}$ are different between Status I and Status II/III due to the change of cable routine.

Based on the assumptions above, the simplified manipulator model is shown in Fig. 4, where the red lines represent the elastic backbone and the blue lines represent the actuation cable. Take the i -th segment of the continuum manipulator for example. The tip of the i -th segment is only subject to the cable tension $-T_{2i}$, which generates a bending moment to the i -th segment. The statics equations of the i -th segment are expressed as

$$\sum M_x^{\{C_i\}} = 0 \rightarrow M_i - M_{F_r}^{\{C_i\}} - r_i \cdot T_{2i} \cdot \cos(\theta_i/2) = 0 \quad (3)$$

where T_{2i} is the cable tension at the i -th segment; θ_i is the bending angle of the i -th segment; M_i is the resulting moment applied to the i -th segment; $M_{F_r}^{\{C_i\}}$ is the moment applied to the i -th segment by the tip force $-\mathbf{F}_r^{C_i}$; r_i is the distance from the interaction point at the disk to the centre of disk (see Fig. 4A).

Constitutive model of each segment

The constitutive equation of the i -th segment is

$$M_i = k_i(\theta_i) \cdot \kappa_i = k_i(\theta_i) \cdot \theta_i / l_i \quad (4)$$

where κ_i is the curvature of the i -th segment; l_i is the arc length of the imaging probe in the i -th segment; $k_i(\theta_i)$ is the equivalent stiffness of the i -th segment. Here, considering the change of stiffness during the deflection due to the extension/contraction of probe, friction between articulated joints and other fabrication effects, we assume that the equivalent stiffness of the i -th segment has a linear relationship with the bending angle as

$$k_i(\theta_i) = EI_i \cdot (1 + \zeta_i \cdot \theta_i) \quad (5)$$

where EI_i is the stiffness of the i -th segment at the straight status, ζ_i is the linear coefficient to indicate the change of the stiffness of the i -th segment. They can be calibrated using the proposed identification method for each segment. This is useful to define the segment differences and will be validated by experiments.

The arc length of the imaging probe inside the i -th segment of the continuum manipulator is considered as an arc between two adjacent disks, which is calculated as

$$l_i = \frac{d}{\tan(\theta_i/2)} \cdot \theta_i \quad (6)$$

where d is the distance from the disk centre to the joint centre (see Fig. 4A).

Constitutive model of the tension-sensing cable

The tension-sensing cable we use is an optical fibre with one FBG sensor inscribed on it (DTG, 1550-125, FBGS, Belgium). The principle of FBG sensor is to use Bragg reflector in the optical fibre to reflect a specific wavelength of light, which can be measured by the FBG interrogator. The change of wavelength has a relationship with the change in strain and temperature of the FBG sensor as

$$\frac{\Delta\lambda_B}{\lambda_B} = (1 - p_e)\varepsilon + (\alpha_\lambda + \alpha_n)\Delta t \quad (7)$$

where $\Delta\lambda_B$ is the wavelength change; λ_B is the centre wavelength of Bragg grating under strain-free conditions; p_e is the photoelastic constant; α_λ is the thermal expansion coefficient; α_n is the thermos-optic coefficient.

When the optical fibre actuates the continuum manipulator, the fibre is tensioned and bent with the deflection of continuum manipulator. Due to the sliding status of the optical fibre, the deflection of the optical fibre along its own neutral axis does not introduce the strain change. Then, the strain at the Bragg grating is only generated from the tension, which is calculated as

$$\varepsilon = T_1 / (E_f \cdot A_f) \quad (8)$$

where T_1 is the tension at the Bragg grating, E_f is the Young's modulus of the optical fibre, A_f is the cross-section area of the optical fibre. We substitute (8) into (7) and ignore the temperature effect, then the tension force shows a linear relationship with respect to the change of wavelength as

$$T_1 = \frac{\Delta\lambda_B}{\lambda_B \cdot (1 - p_e)} \cdot (E_f \cdot A_f) = k_f \cdot \Delta\lambda_B \quad (9)$$

where k_f is a linear coefficient, which can be calibrated using the axial loading experiment. Hence, once the change of wavelength is measured by the interrogator, the tension of the optical fibre at the Bragg grating can be calculated using (9).

Friction model and tension propagation

In this model, in order to propagate the cable tension of the i -th segment to the upward or backward segment, the statics equations of cable at $\mathbf{P}_{2i}$ are expressed as

$$T_{2i+1} \cdot \cos \alpha_{2i} + f_{2i} = T_{2i} \cdot \cos \alpha_{2i} \quad (10)$$

$$T_{2i+1} \cdot \sin \alpha_{2i} + T_{2i} \cdot \sin \alpha_{2i} = N_{2i} \quad (11)$$

where T_{2i+1} and T_{2i} are the cable tensions shown in Fig. 4; f_{2i} and N_{2i} are the friction and normal force at $\mathbf{P}_{2i}$; α_{2i} is the relative angle between the cable tension - T_{2i} and the friction force - f_{2i} at $\mathbf{P}_{2i}$. Here, the friction can be static or kinetic, where the static friction depends on the resulting force, and the kinetic friction is proportional to the normal force. A general friction model can be expressed as

$$f_{2i} = N_{2i} \cdot \mu_{2i} \quad (12)$$

where μ_{2i} is the friction coefficient with its direction at $\mathbf{P}_{2i}$. It has a relationship with an equivalent static/kinetic friction coefficient as

$$\mu_{2i} = \eta_{2i} \cdot \mu_{2i}^{k,s} \quad (13)$$

where η_{2i} is the adjustable coefficient; $\mu_{2i}^{k,s}$ is the equivalent friction coefficient, which will be calibrated using the proposed identification method in the next subsection. The adjustable coefficient is described as

$$\eta_{2i} = \begin{cases} 1 & \text{Slide backward} \\ \in (-1, 1) & \text{Stick} \\ -1 & \text{Slide forward} \end{cases} \quad (14)$$

For the free bending configuration, the sliding status can be determined using the calibration method^{20, 46} by comparing the current cable profile along the whole continuum manipulator with its previous one; while for the loaded bending configuration, the sliding status can be determined by comparing the current cable location with the previous cable location, where the sticking status is ignored. Details can be found in the algorithms of appendixes.

By substituting (12) to (10~11), the cable tension is propagated as

$$T_{2i+1} = T_{2i} \cdot \frac{\cos \alpha_{2i} - \mu_{2i} \cdot \sin \alpha_{2i}}{\cos \alpha_{2i} + \mu_{2i} \cdot \sin \alpha_{2i}} \quad (15)$$

where α_{2i} is determined by

$$\alpha_{2i} = \begin{cases} \frac{\theta_{i+1} + \theta_i}{4} & \theta_i < 0 \text{ \& } \theta_{i+1} < 0 (\text{Status III}) \\ \frac{\theta_i}{4} & \text{others} (\text{Status I \& II}) \end{cases} \quad (16)$$

where α_{2i} is not the same for different bending situations (see Fig. 4). Similarly, we can also solve the free body diagram of cable at $\mathbf{P}_{2i-1}$ to get the above equations. Given (3) to (16) for $\mathbf{P}_{2i}$ and $\mathbf{P}_{2i-1}$, the shape of the continuum manipulator can be derived. But the solutions for the identification and shape estimation are different. Algorithms of identifying the model and estimating the shape under the free and loaded bending configurations can be found in the following sections.

System identification

This section identifies the segment parameters for the laser-profiled continuum manipulator. The equivalent stiffness, linear coefficient and friction coefficient may be different segment by segment, so we need to calibrate one by one carefully using the free bending experiment. Our previous work has proposed an identification method to calibrate the continuum manipulator,^{20, 46} but it does not consider the differences of segments from the laser-profiled continuum manipulator. In other words, the equivalent stiffness, linear coefficient and friction coefficient may not be same for all segments of the proposed laser-profiled continuum manipulator due to its deflection, fabrication and assembly process. In this subsection, we introduce an identification method using the free bending experiment to implement the identification for each segment's parameters.

The inputs for the identification process are the deflections of the continuum manipulator under the free bending and the corresponding actuation tensions at the proximal end of the continuum manipulator - $T_1(1:m)$ (m is the total number of loading cases); while the outputs are the identified parameters including EI_i , ζ_i , $\mu_{2i-2}^{k,s}$ and $\mu_{2i-1}^{k,s}$ for each segment. The method is illustrated in Algorithm I of Appendix A, which identifies each segment from the base segment to the tip segment as the cable tension is propagated from the base to the tip. Here, the deflection of the continuum manipulator is denoted using a series of points at the centres of joints and disks as the input.

The identification of each segment is an optimisation problem and the objective is to minimize the distance errors between the real tip positions of each segment and the values from the model, defined as

$$\begin{aligned} \text{argmin } f(EI_i, \zeta_i, \mu_{2i-2}^{k,s}, \mu_{2i-1}^{k,s}) &= \frac{1}{m} \sum_{k=1}^m \|\hat{\mathbf{P}}_{C_i}^{C_{i-1}}(k) - \mathbf{P}_{C_i}^{C_{i-1}}(k)\| \\ \text{subject to } 0 &\leq EI_i \leq \xi_{EI} \\ 0 &\leq \zeta_i \leq \xi_{\zeta} \\ 0 &\leq \mu_{2i-2} \leq \omega \\ 0 &\leq \mu_{2i-1} \leq \omega \end{aligned} \quad (17)$$

where $\hat{\mathbf{P}}_{C_i}^{C_{i-1}}(k)$ and $\mathbf{P}_{C_i}^{C_{i-1}}(k)$ are the tip positions of the i -th segment at $\{C_{i-1}\}$ under the k -th load from the model and the measurement of the absolute sensor, respectively.

The optimisation is implemented from the base to the tip to derive the equivalent stiffness, linear coefficient and equivalent friction coefficient of each segment. For every segment, the actuation cable interacts with the continuum manipulator at two points - $\mathbf{P}_{2i-2}$ & $\mathbf{P}_{2i-1}$. The initial guesses of stiffness - EI_i , linear coefficient - ζ_i and equivalent friction coefficients - $\mu_{2i-2}^{k,s}$ and $\mu_{2i-1}^{k,s}$ for each segment are given first, then the optimisation in (17) is implemented to find the optimised values using the Matlab function - `fmincon` (MATLAB2017b, The MathWorks, Natick, USA). The solving results of each segment is denoted as *segRes_i*, including the profiles of cable tension, curvature, normal force, friction force and friction coefficient, which are used as the inputs for the solution of next segment. For the first segment, EI_i , ζ_i and $\mu_{2i-1}^{k,s}$ ($i=1$) are derived after the optimisation given the input cable tensions - $T_1(1:m)$. For the other segments ($i \geq 2$), the inputs are the last segment's solution - *OneSegRes_{i-1}*($1:m$) including the cable tensions for the i -th segment - $T_{2i-2}(1:m)$, then EI_i , ζ_i and two friction coefficients - $\mu_{2i-2}^{k,s}$ and $\mu_{2i-1}^{k,s}$ are derived. During the solution for each segment, the sliding and sticking status are determined by finding the transition point using the similar method introduced in our previous work.⁴⁶ The direction of tension change determines an initial direction for friction, then the loading history is combined to find if there is a transition point and where it is, therefore updating the segment's solution - *OneSegRes_i*. The procedure is shown in the Algorithm I of Appendix A.

Estimation of Shape and Tip Force

In this section, we first introduce the method for simultaneous shape and tip force estimation, then the details for the shape estimation under the free and loaded bending configurations are illustrated.

Method

In the previous sections, we have proposed the estimation principle. Here, the detailed procedure including the identifications is shown in Fig. 5, which includes two steps.

The first step is the identifications for the tension-sensing cable and the continuum manipulator, where the former is to find the relationship between the change of wavelength reading of the FBG sensor and the actual tension; while the latter is to find the stiffness, linear coefficient and friction coefficients for each segment.

The second step is to execute the estimation of shape, tip force or their combination. The implementation of identification has been introduced in the previous section. In this section, we focus on the shape and tip force estimation.

The local estimations for the continuum manipulator under the free bending (denoted as Case I) and the loaded bending (denoted as Case II) are different due to the different solving procedures considering the cable and tip force interactions. Two estimations are implemented first to get the local shape and tip force, then the global shape and tip force of the continuum manipulator can be obtained by registering the local shape and tip force with the absolute pose from the sensor feedback together using (1) and (2) (see Figs. 3 & 5).

Case I - Shape estimation under the free bending

To solve the deflection of continuum manipulator under the free bending, the algorithm is developed given the inputs including the identified stiffness, linear coefficient and friction coefficient of each segment, and the loading history of the proximal cable tension - $T_1(k) (k \in [1, m])$. The details of the algorithm can be found in Appendix B.

For the continuum manipulator with n segments starting from the straight status, the actuation cable slides toward the base first, where the friction directions along all interaction points are positive. Then, if the tension is increased, the directions will be positive, or else the directions will change to be negative segment by segment from the base to the tip. If the tension - $T_1(k)$ changes its direction, the profile of friction direction is updated again. The method is similar as that presented in our previous work.⁴⁶ Take 3 times loading histories as an example with the adjustable coefficients as

$$\begin{aligned}\eta(1) &= [1 \quad 1 \quad \dots \quad 1]; \\ \eta(2) &= [-1 \quad -1 \quad -1 \quad \eta_4 \quad 1 \quad \dots \quad 1]; \\ \eta(3) &= [1 \quad \eta_2 \quad -1 \quad \eta_4 \quad 1 \quad \dots \quad 1]; \\ &\dots\end{aligned}\tag{18}$$

Here, the directions of friction along all interaction points remain positive when $k=1$ with the profile of the adjustable coefficient - $\eta(1)$, then the deflection of the continuum manipulator is solved segment by segment from the base to the tip. Then, if the cable is released to a specific tension, all directions of friction are set to be negative first, then the deflection of the continuum manipulator is solved from the base to the tip. Finally, the transition point can be found by comparing the current tension profile with the previous profile, and the solution - $AllSegRes(k)$ is updated by combining the current one with the previous one. The procedure keeps running until the final loading case is completed. The solving procedure is to solve the deflection of the continuum manipulator considering all segments together when a new loading case is provided, while the solving procedure of identification in last section is to use all the loading cases to solve the deflection of each segment one by one. The same characteristic is that they both solve the deflection from the base segment to the tip segment.

Case II - Shape and force estimation under the loaded bending

For the deflection of the continuum manipulator under the loaded bending configuration, we need the tip/base pose and proximal cable tension - $T_1(k) (k \in [1, m])$ as inputs to predict the shape and tip force. The solving procedure starts from the tip segment to the base segment because the tip force changes the deflection of each segment from the tip to the base. The solution is treated as a root-finding problem defined as

$$\operatorname{argmin} f(\hat{\sigma}_k) = \|\hat{\beta}(\hat{\sigma}_k) - \beta\|\tag{19}$$

where $\hat{\beta}(\hat{\sigma}_k)$ and β are the estimated state and real state, $\hat{\sigma}_k$ is the estimated root of the root-finding algorithm, they are denoted as

$$\hat{\beta}(\hat{\sigma}_k) = \begin{bmatrix} \hat{P}_{C_i}^{C_g}(k) \\ \hat{\theta}_{all}(k) \\ \hat{T}_1(k) \end{bmatrix}, \quad \beta = \begin{bmatrix} P_{C_i}^{C_g}(k) \\ \theta_{all}(k) \\ T_1(k) \end{bmatrix}, \quad \hat{\sigma}_k = \begin{bmatrix} F_T(k) \\ \theta_{F_T}(k) \\ T_{2n}(k) \end{bmatrix} \quad (20)$$

where $F_T(k)$ is the tip force, $\theta_{F_T}(k)$ is the orientation of tip force defined at $\{C_T\}$, $T_{2n}(k)$ is the cable tension at the distal segment, $\hat{P}_{C_i}^{C_g}(k)$ and $P_{C_i}^{C_g}(k)$ are tip positions of the model and the measurement from the absolute sensor, $\hat{\theta}_{all}(k)$ and $\theta_{all}(k)$ are deflection angles of the continuum manipulator from the model and measurement from the absolute sensor, $\hat{T}_1(k)$ and $T_1(k)$ are the cable tension at the proximal end from the model and the measurement from FBG sensor in the tension-sensing cable.

The procedure is shown in Appendix C. In each iteration, the calculation of each segment is implemented from the tip to the base, because the tip force will affect the deflection of all segments. An initial guess - $\hat{\sigma}_k$ is given first for the optimisation in (19), then the segment deflection is solved from the tip to the base. If $f(\hat{\sigma}_k)$ is not the minimum to meet with the convergence criteria, a new guess of $\hat{\sigma}_k$ will be made to continue the iteration to solve each segment's deflection until the minimum of $f(\hat{\sigma}_k)$ is found. The optimisation is also implemented using the Matlab function - `fmincon`. During the iteration, the cable location is calculated for each segment to determine the direction of friction coefficient at the interaction points of current segment using Eq. (14). The status of each segment is determined by the bending angle of each segment. If $\theta_i(k) < 0$ & $\theta_{i+1}(k) < 0$, then it changes to Status III, or else it keeps Status I and Status II by considering Eq. (16). It is also worth mentioning that if both of two adjacent segments belong to Status III, the tension profile needs to be updated by deleting one element as shown in Fig. 4G.

Experiments and Validation

Experimental setup for validations

To validate the model, an experimental platform was developed as shown in Fig. 6A. The robot consists of a continuum manipulator at the distal end, a flexible polymer shaft, an operation handle, optical fibres and an imaging probe. The prototype of the assembly is shown in Fig. 6B. The continuum manipulator has the interlocked joints and several multi-lumen disks, which were cut by a laser machine (Rofin LFS Cutting System with MPS, UK). The flexible polymer shaft was made of polycarbonate, which was drawn using our custom-built draw tower. The optical fibres with a cladding diameter of 125 μm and Ormocer coating (DTG, 1550-125, FBGS, Belgium) were chosen, which could stand for up to 50 N tensile load⁴⁴. An optical fibre (Cellvizio, Mauna Kea Technologies) was chosen as the imaging probe. Details can be found in Table 2.

The motor (MX-28, Dynamixel, ROBOTIS, Korea) at the proximal end was used to control the actuation cable directly. A camera was placed over the head of the continuum manipulator to record the process of deflection. The tip of the continuum manipulator was fixed with a thread, which went through a series of pulleys to a weight on a scale, then the tip force could be measured indirectly by the change of the scale reading when the continuum manipulator was deflected with a tensioned thread at its tip.

After capturing the images of the continuum manipulator, the detection of joint centres was executed, which consisted of two steps: detection of the circular joint region using deep convolutional neural networks (CNN) and estimation of joint centres using Hough transform. CNN has shown its advantages over conventional computer vision.^{47, 48} To achieve robust and accurate detection results, we adopted the U-Net⁴⁹ architecture for segmenting the circular U-shape region between each joint. The training data consisted of 120 manually labelled images. A variety of probe shapes and illumination conditions are included in the training dataset. We use Adam solver⁵⁰ with a batch size of 1. The Pytorch framework (<https://github.com/pytorch/pytorch>) is adopted to implement the deep convolution neural networks. We used a workstation with Xeon E5-2630 and NVIDIA GeForce Titan Xp. After generating the binary mask of the joint regions, the centre of each joint region is computed using Hough transformation.⁵¹ The centre points are sorted along the axial direction of the continuum manipulator to represent the shape of the laser-profiled continuum manipulator.

Based on this, the following experiments were conducted for the identification and validation processes:

1) Identification of the tension-sensing cable was implemented to figure out the relationship between the cable tension and the wavelength change of FBG sensor.

- 2) Identification of the continuum manipulator was implemented to derive the stiffness, linear coefficient and equivalent friction coefficient for each segment.
- 3) The shape and tip force estimation method was validated by the experiments under free and loaded bending configurations.

Identification of the tension-sensing cable

To implement the identification process of the tension-sensing cable, a group of weights were hanged to one side of FBG sensor, while the optical fibre was configured vertically and fixed at the other end. Hence, the weights are the forces directly applied to the FBG sensor with values 0 g, 200 g, 400 g, 600 g, 800 g, 1000 g. In the same time, the raw reading of wavelength change for each weight was recorded simultaneously. The whole weights were hanged for five times, then five groups of data were derived. The Levenberg–Marquardt algorithm was implemented to fit the derived data to Eq. 9 and get the coefficient - k_f as 0.78 N/nm for the FBG sensor, which will be used to transform the change of wavelength to the cable tension.

Identification and validation of mathematical model

As introduced in the identification section, we actuated the continuum manipulator under the free bending and recovering to identify its intrinsic parameters. The measured tension from the tension-sensing cable - T_1 was collected as the actuation force, the tension measured at the proximal handle end was collected in the same time for comparison with T_1 . The shapes with respect to T_1 were captured by the overhead camera. Here, the proximal tension was applied from 0 N to 10 N in 1 N increments, and then released to 0 N in 1 N decrements. The identification procedure based on the optimisation in Eq. (17) was implemented using the Algorithm I in Appendix A. Results are displayed in Fig. 7, while the identified intrinsic parameters are shown in Table 3.

Figs. 7A and 7B indicate that the proposed mathematical model is able to estimate the manipulator shape with errors in an acceptable range. The average errors of the tip position and orientation angle are 0.70 mm (2.65% of the manipulator's length) and 0.003 rad. Fig. 7C shows an obvious hysteresis for the tension at the proximal end of the flexible shaft (the handle end), as well as a large deviation from the tension of the tension-sensing cable measured at the proximal end of the continuum manipulator. It means that the tension cannot be fully propagated from the handle end to the proximal end of the continuum manipulator due to the friction within the long-thin flexible shaft, where the proposed sensing method using the FBG sensor located at the proximal end of the continuum manipulator can effectively avoid this negative effect. Figs. 7D and 7E show the profile of cable tension along the whole length of the continuum manipulator, where the discontinuous change and propagation of tension can be observed. Figs. 7F ~ 7N show the deflection of each segment, where the obvious segment differences can be observed and captured by the proposed mathematical model. It is consistent with the segment characteristics shown in Figs. 7D and 7E.

Validation of simultaneous shape and force estimation

The interaction with the tissue could be active or passive. The former indicates that the continuum manipulator is steered to interact with the tissue actively to achieve the necessary tissue contacts to acquire high-quality cellular images; while the latter indicates that the respiratory motion of tissue applies the external force to the continuum manipulator, thereby affecting its shape. Based on this, two experimental cases were implemented to validate the proposed estimation method.

For the active contact, the continuum manipulator was actuated freely without tip force first, then the tip force was added when the actuation cable was pulled again and the thread at the tip started to be tensioned gradually. The thread was connected to a 50 g weight, which was put on the electronic scale. The change of scale reading indicated the force added to the tip of the continuum manipulator, which was sent to the computer via an RS-232 connector. Also, the shapes of the continuum manipulator were collected simultaneously during the procedure by the overhead camera. This experiment can emulate the active contact of the continuum manipulator with the surrounding soft tissue. Results are shown in Figs. 8A ~ 8E, where the predicted shapes can capture the experimental shapes accurately. The average errors of the tip position and orientation are 0.14 mm and 0.01 rad, where some minimal deviations can be observed. The average errors of the tip force and tip force orientation are -3.07 mN and 0.22 rad. Hence, the average error of the tip position is 0.53% of the continuum manipulator's total length (26.45 mm), and the average error of the tip force is -1.73% of the range of tip force change in the active contact case (177.07 mN). Detailed error measurements are listed in Table 4.

For the passive contact, the continuum manipulator was first actuated to a specific deflection angle with a tensioned thread at the tip, and then the thread was loaded or released randomly by manual operations for several times when the actuation cable was fixed at the specific position. Similarly, the change of scale reading and the shape of the continuum manipulator were collected at the same time. This experiment can emulate the passive interaction from the anatomical environment due to its respiratory motion. Results are shown in Figs. 8F ~ 8J, where the shapes have some overlap because the tip was loaded or released for several times, and the loading history of tip force can be observed and estimated by the proposed estimation method. The average errors of tip position and orientation are 1.51 mm and -0.04 rad, where some small deviations can be observed.

The average errors of tip force and tip force orientation are -5.77 mN and 0.003 rad. Hence, the average error of the tip position is 5.70% of the continuum manipulator's total length (26.45 mm), and the average error of the tip force is -2.91% of the range of tip force change in the passive contact case (198.36 mN). Detailed error measurements are listed in Table 5.

Demonstration using the overlay of estimated and real shapes/forces

Given the estimated local shape and tip force, we can register the estimated tip pose of the continuum manipulator to the measured one from the absolute sensor using (1) and (2), then the estimated local shape and tip force can be shown in the global frame. In this section, we implement this method by registering them to the real manipulator captured by the camera. Results of Cases I and II can be found in Fig. 9, where A, B and C show the overlay results of the free bending configuration, the loaded bending configuration with the active contact, the loaded bending configuration with the passive contact, respectively. The estimated shapes are displayed by using 3D transparent models of real interlocked segments, disks, the imaging probe's tip and the estimated forces are displayed at the tip of the probe by using the red arrow, where a longer arrow stands for a larger force amplitude. Results show that the estimated tip poses can accurately match the tip poses of the real continuum manipulator after the registration because the errors from the estimation method occur at the base segment. That means that the estimated tip pose can be trusted during the intervention, and the other part of shape can be treated as a reference to the surgeons during the intervention. In sum, results indicate that the proposed estimation method can effectively predict the shape and tip force of the laser-profiled continuum manipulator.

Discussion

The integrated optical tension-sensing cable has been successfully integrated into the continuum robot to accomplish the simultaneous actuation and tension-sensing without increasing dimensions of the robot. The developed mathematical model has successfully described the segment differences and cable interactions to improve its accuracy. We also demonstrate that the proposed model-based estimation method has successfully predicted the shape and tip force simultaneously. It is worth noting that several potential improvements can be made to the proposed method, including the structure, model and accuracy of estimation as outlined below.

Structure and Model

The proposed robot design enables a more compact design with only tension-sensing cables and absolute sensors configured with the thin-walled continuum manipulator, when compared to other approaches using multiple FBG-based shape sensors to measure the shape and additional sensors to measure the tip force.⁵² The tension-sensing cables can also help structurally avoid the unintended effects from the long-thin flexible shaft under the unknown disturbances. However, the manufacturing and assembly process of the continuum manipulator can be improved to ensure consistency of the model and estimation. A consistent and high manufacturing quality is important especially for such a small dimension of the continuum manipulator.

The interlocked joints of the developed continuum manipulator were cut by the CO₂ laser machine (Rofin LFS Cutting System with MPS, UK) on a stainless tube, so the edges of cutting area can be rough with some unavoidable heat-affected zones. These can introduce more friction, and the large heat-affected zones may have some pre-deformation and can affect the joint performance, as the performance of each joint may be inconsistent. In practice, the number of cycles does affect the performance, because it can cause the wear to the joints of the continuum manipulator with articulated joints or interlocked joints. Moreover, the disks in Fig. 2D were inserted into the slots manually. Any small misplacement can affect the free body diagram in Fig. 4, therefore affecting the propagation of cable tension.

By taking these factors into consideration, we have in this paper introduced a strong assumption of the linear relationship between each segment's equivalent stiffness and its bending angle in (5) and an identification approach in (17) to capture the phenomena segment by segment as much as possible. The results in Figs. 7F ~ 7N have shown that the deflections of all segments are not the same, most of which have been successfully captured by the proposed model with this assumption. Figs. 7A & B also show that the shapes of the whole manipulator reconstructed from each segment's deflection can successfully describe the experimental results, although with some small noticeable deviation. The reasons for this include two aspects: a) The linear assumption is not perfect, and it does not always capture the whole loading and unloading procedure of each segment, such as Fig. 7G with a noticeable error during the loading procedure; b) The identification and shape reconstruction are implemented from the base to the tip, where slight angular errors at the base segments (e.g. Fig. 7G) can introduce a large accumulated error to the tip pose.

To mitigate these problems, the manufacturing quality can be improved by using a femtosecond laser to reduce the dimension of micro cutting slot, generate a smaller heat-affected zone and enhance the smoothness of cutting surfaces. Moreover, the post-processing treatments (e.g. electropolishing, surface treatment) could be carried out to improve the surface

quality, consistency of the joint properties and avoidance of wear. Moreover, the assembly accuracy could be improved by designing a dedicated assembly jig to help assemble the disks (Fig. 2D) to the notched tube precisely, then the slots at each disk could be aligned properly along the axial direction. It could help the actuation cables as well as the image probe to go through the slots of the disks smoothly and be configured in the proper locations as required.

Accuracy of Estimation

In this paper, the validation results show the good accuracy with some deviations from the experimental results, which we attribute to the following reasons. The developed mathematical model is based on some assumptions that may not always hold, which has to adapt the equivalent stiffnesses and equivalent friction coefficients to handle all manufacturing errors and assembly tolerance. The measurements of the cable tension and tip pose also have some errors, introducing further uncertainty to the estimation method. Thus, all uncertainties are accumulated to the estimated shape and tip force. The error and accuracy are discussed in the following three aspects:

a) The average error of the tip position for the active contact is 0.14 mm (0.53%); and that for the passive contact is 1.51 mm (5.70%). According to our experience, it is better to guarantee the mean position error within 10%. The accuracy of our method is better than this, and it still has the scope for improvement in terms of structure and assembly as discussed above. As a reference, the current commercial image navigation software such as Archimedes™ System⁵³ can provide a virtual image navigation for bronchoscopy with a positional error at about 2.2 mm. Therefore, a 2 mm error derived in this paper is acceptable for navigation and guidance.

b) The estimation shapes in Figs. 3 and Fig. 9 represent the global shape reconstructed from the local shape calculated based on the proposed model and the absolute tip pose from the pose sensor (e.g. EM sensors). Hence, in real world applications, the tip pose in the global frame is always the real value from the pose sensor, but the shape in the global frame is the estimated value based on the method illustrated in Fig. 3 and Fig. 5, which renders the accumulated error after the registration of their tip poses. Although the estimated shape has inevitable errors, it is acceptable to provide the guidance assistance during the intervention based on our experience. The absolute poses were derived from an external camera for the validation in this paper. To achieve this in practical applications, we could consider using one EM sensor at the tip and another one at the base of the manipulator to derive the poses, and then combine the localisation method⁴⁵ using a Chip-on-Tip camera to recover the relative relationship between the tip pose and the anatomy. Finally, the situational awareness of continuum manipulator in the anatomy could be achieved to ensure safe and efficient interventions.

c) The environment can also introduce certain errors. In our application, the continuum manipulator is expected to work in the airways and therefore it can be affected by the local environment. It is true that temperature changes may affect the wavelength of the FBG sensor in (7). However, a 10°C temperature change only results in 0.1 nm wavelength change, which is around 0.78% of the range of wavelength change (12.82 nm with respect to 10 N cable tension according to the calibrated coefficient of tension-sensing cable - k_f). Therefore, errors from less than 10°C temperature change could be ignored, but it can also be compensated in future if another FBG sensor is put in parallel to compensate such errors. For endovascular applications, a protection rubber sheath could be assembled onto the manipulator's outer surface to achieve waterproofness.

Conclusion

In conclusion, this paper has presented an effective compensation method based on tension-sensing cables to actuate a miniaturised laser-profiled continuum manipulator, which can structurally filter out the disturbance from the flexible shaft without requiring additional space. An accurate mathematical model has been proposed by considering segment differences, cable interactions and tip force, and a model-based estimation method has been developed to simultaneously estimate the shape and tip force of the laser-profiled continuum manipulator accurately. The benchtop experiments have been conducted to validate the proposed model and the estimation method. The results derived have demonstrated the effectiveness of the proposed method for endoluminal intervention. It is worth noting that the proposed estimation method using the tension-sensing cable and absolute sensors could effectively minimise structural diameters, thereby enabling a more compact configuration. Moreover, the manipulator and optical fibres are low-cost, therefore allowing it to be used for disposable applications.

Further improvements to the structure and assembly are required to increase the accuracy of model and the proposed estimation method. A general model could be built to describe the continuum manipulator with the contact-aided compliant mechanism.⁵⁴ Also, EM sensors and Chip-on-Tip cameras can be incorporated to achieve the real-time shape and tip force estimation. Furthermore, augmented see-through displays using an augmented reality glass could be used to provide a more intuitive navigation for the operator.⁵⁵

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FIG. 1. Endobronchial intervention using the miniaturised continuum robot to enable the steering of an imaging probe to perform the optical biopsy.¹ Here, the robot with a continuum manipulator and a flexible shaft is inserted through the instrument lumen of bronchoscope to access the distal airways, where the bronchoscope's camera cannot capture its shape as its sight is limited and often occluded.

FIG. 2. The diagram of the proposed handheld robot with a continuum manipulator actuated by the tension-sensing cables. The optical fibre is used to actuate the continuum manipulator, as well as to measure the cable tension at the proximal end of the continuum manipulator. EM sensors could be integrated to derive the global tip and base poses of manipulator.

FIG. 3. The framework of shape and tip force estimation for the laser-profiled continuum manipulator.

FIG. 4. A diagram of the continuum manipulator: where **A ~ C**, **D ~ F**, **G ~ I** represent Status I, II and III, respectively. Here, **B**, **E** and **H** represent the free body diagrams of i -th segment; **C**, **F** and **I** represent the free body diagrams of actuation cable at P_{2i} & P_{2i-1} . Status I shows a positive bending angle for i -th segment; Status II shows a negative bending angle for i -th segment, but with a positive bending angle for $i-1$ -th segment; Status III shows the negative bending angles for both i -th segment and $i-1$ -th segment. The locations of P_{2i} & P_{2i-1} changes from the interaction points between the cable and the body of continuum manipulator in Status I to the those between the cable and the disks in Status II & III. And in Status III, P_{2i-2} and T_{2i-1} disappear due to the change of cable routine.

FIG. 5. The procedure of identifications and the shape/tip force estimation. The identifications of the tension-sensing cable and the continuum manipulator are implemented first, then the identified parameters are used in the estimation method to get the shape and tip force under the free and loaded bending configurations.

FIG. 6. Experimental setup for the validations: **A** shows the whole setup with an overhead camera used to capture the deflection of the continuum manipulator during the bending and recovering procedure; **B** shows the enlarged figure of the continuum manipulator under the tip force from the tensioned thread; **C** shows the measurement of scale reading's change to reflect the tip force.

FIG. 7. Comparison of results from model and experiments: **A** & **B** show the results comparison for the bending and recovering shapes; **C** shows the tension comparison at the proximal end of flexible shaft (red lines) and the manipulator (blue lines) of the continuum manipulator; **D** and **E** show the profile of cable tension from the model during the bending and recovering procedure; **F** ~ **N** show the change of each segment's deflection angle with respect to the cable tension.

FIG. 8. Comparison of results from the experiments and model-based estimations for actively loaded case and passively loaded case: **A ~ E** show the results of active contact case; **F ~ J** show the results of passive contact case. Here, **A** and **F** show the shape comparison; **B** and **G** show the comparison of tip position; **C** and **H** show the comparison of tip orientation; **D** and **I** show the comparison of estimated tip load; **E** and **J** show the comparison of estimated tip load orientation.

FIG. 9. The overlay of the estimated shape and tip force to the real laser-profiled continuum manipulator: **A** shows the free bending case; **B** shows the active contact case; **C** shows the passive contact case. In **B** and **C**, the estimated tip force and the real tip force (shown in parenthesis) are displayed on each sub-figure.

TABLE 1. Literature review of model-based estimation methods for cable-driven continuum robots*

Authors	OD (mm)	Inner Lumen	Long Shaft	Shape Sensing	Force Sensing	Friction	Principle
Back <i>et al.</i> ³⁰	3	N	N	N	Y	N	Cosserat rod model-based method
Back <i>et al.</i> ³¹	3	N	N	Y	Y	Y	Static model-based method
Xu <i>et al.</i> ³²	7.5	N	N	Y	Y	N	Virtual work principle
Bajo <i>et al.</i> ^{33, 34}	4.2	N	N	Y	Y	N	Kinematics and screw motion deviation
Rucker <i>et al.</i> ³⁵	20	N	N	Y	Y	N	Extended Kalman filter approach
Khoshnam <i>et al.</i> ³⁶	2.33	N	Y	Y	Y	N	Kinematic model and curvature change
Rone <i>et al.</i> ³⁷	30	N	N	Y	N	Y	Mechanics based method and passive cable length
Roesthuis <i>et al.</i> ³⁸	>15	N	N	Y	Y	N	Rigid-link kinematics
Hasanzadeh <i>et al.</i> ³⁹	2.33	N	Y	Y	Y	N	Virtual work principle

* Y: yes, N: no

TABLE 2. Components and dimensions

Part	Dimension	Description
Optical fibre	Φ 0.19 mm	Actuate the continuum manipulator and sense the actuation tension
Image probe	Φ 0.8 mm	Glass fibre based probe with a rigid tip
Disk	Lumen I/IV Φ 0.35 mm	For the tension-sensing cables
	Lumen II/III Φ 0.45 mm	For the absolute sensors like EM sensors
	Lumen V Φ 0.85 mm	For the imaging probe
Continuum manipulator	OD = Φ 2.20 mm ID = Φ 1.90 mm $d = 1.15$ mm $r_t = 0.78$ mm	Assembled by the laser-profiled stainless-steel tube and disks
Flexible shaft	OD = Φ 2.20 mm	Made by the multi-lumen polymer tube drawn by a draw tower

TABLE 3. The identified parameters for each segment of the continuum manipulator

<i>Segment No.- i</i>	μ_{2i-2}	μ_{2i-1}	$EI_i (Nmm^2)$	ζ_i
1 (base)	0	1.50	25.82	4.97
2	0.58	1.20	152.75	-3.08
3	1.49	1.49	43.69	4.98
4	0.06	0.06	119.51	-2.74
5	0.02	0.02	135.93	-3.08
6	0.70	0.52	113.02	-3.10
7	0.01	0.01	117.43	-2.96
8	0.01	0.02	186.90	-4.88
9 (tip)	1.49	1.50	44.83	1.18

TABLE 4. Estimation errors of the continuum manipulator for the active contact case

Sample No.	1	2	3	4	5	6	7	8
<i>Measured tip position (mm)</i>	[3.25,26.53]	[5.79,26.01]	[7.54,25.47]	[8.95,24.89]	[9.89,24.45]	[10.57,24.06]	[11.10,23.73]	[11.55,23.37]
<i>Estimated tip position (mm)</i>	[3.26,26.64]	[5.81,26.12]	[7.57,25.57]	[8.99,24.99]	[9.93,24.54]	[10.62,24.17]	[11.17,23.84]	[11.65,23.53]
<i>Tip position error (%)*</i>	0.40	0.41	0.39	0.42	0.38	0.45	0.51	0.71
<i>Measured tip force (mN)</i>	0.10	5.00	12.75	28.45	47.58	69.06	91.23	120.1725
<i>Estimated tip force (mN)</i>	8.03	15.99	23.85	34.56	48.07	73.41	89.23	106.74
<i>Tip force error (%)*</i>	4.48	6.20	6.27	3.45	0.28	2.46	-1.13	-7.59
Sample No.	9	10	Mean	STD	Max of Abs			
<i>Measured tip position (mm)</i>	[11.93,23.11]	[12.22,22.85]						
<i>Estimated tip position (mm)</i>	[12.03,23.27]	[12.35,23.04]						
<i>Tip position error (%)*</i>	0.71	0.86	0.53	0.17	0.86			
<i>Measured tip force (mN)</i>	149.70	177.17						
<i>Estimated tip force (mN)</i>	123.55	147.09						
<i>Tip force error (%)*</i>	-14.77	-16.99	-1.73	8.51	16.99			

* The percentage of tip position error is the tip position error to the manipulator's length; while the percentage of tip force error is the tip force error to the range of measured tip force.

TABLE 5. Estimation errors of the continuum manipulator for the passive contact case

Sample No.	1	2	3	4	5	6	7	8
<i>Measured tip position (mm)</i>	[11.65,23.32]	[10.64,23.78]	[6.51,25.35]	[7.32,25.08]	[11.18,23.56]	[12.23,22.96]	[12.12,22.99]	[10.31,23.89]
<i>Estimated tip position (mm)</i>	[11.75,23.47]	[11.24,23.73]	[6.53,25.58]	[10.82,23.92]	[11.48,23.55]	[13.10,22.51]	[15.04,20.85]	[10.85,23.69]
<i>Tip position error (%)[*]</i>	0.70	2.28	0.86	13.95	1.14	3.69	13.68	2.15
<i>Measured tip force (mN)</i>	123.80	203.26	245.45	194.83	187.08	111.34	115.17	211.21
<i>Estimated tip force (mN)</i>	113.01	174.96	252.07	224.83	216.98	141.34	122.07	200.61
<i>Tip force error (%)[*]</i>	-5.44	-14.27	3.34	15.12	15.07	15.12	3.48	-5.34
Sample No.	9	10	11	12	13	14	15	16
<i>Measured tip position (mm)</i>	[8.22,24.75]	[7.15,25.09]	[5.19,25.65]	[5.17,25.66]	[10.66,23.80]	[12.29,22.90]	[12.22,22.90]	[9.32,24.31]
<i>Estimated tip position (mm)</i>	[10.60,23.82]	[4.70,26.12]	[2.88,26.44]	[2.77,26.46]	[10.78,23.97]	[12.40,23.03]	[10.51,24.06]	[6.36,25.70]
<i>Tip position error (%)[*]</i>	9.65	10.05	9.23	9.58	0.78	0.62	7.81	12.36
<i>Measured tip force (mN)</i>	233.18	231.03	258.40	219.35	152.45	116.05	113.99	220.14
<i>Estimated tip force (mN)</i>	204.02	203.96	228.40	228.09	148.60	95.15	94.99	190.14
<i>Tip force error (%)[*]</i>	-14.70	-13.64	-15.12	4.41	-1.94	-10.54	-9.58	-15.12
Sample No.	17	18	19	20	21	Mean	STD	Max of Abs
<i>Measured tip position (mm)</i>	[5.25,25.58]	[1.79,26.18]	[4.11,25.91]	[8.84,24.61]	[11.57,23.33]			
<i>Estimated tip position (mm)</i>	[3.25,26.39]	[-0.52,26.69]	[4.38,26.22]	[8.58,25.01]	[11.68,23.48]			
<i>Tip position error (%)[*]</i>	8.17	8.94	1.54	1.80	0.71	5.70	4.82	13.95
<i>Measured tip force (mN)</i>	251.82	309.70	204.83	176.68	127.53			
<i>Estimated tip force (mN)</i>	221.82	279.70	228.33	184.70	132.28			
<i>Tip force error (%)[*]</i>	-15.12	-15.12	11.85	4.04	2.39	-2.91	11.17	15.12

* The percentage of tip position error is the tip position error to the manipulator's length; while the percentage of tip force error is the tip force error to the range of measured tip force.

Algorithm 1: Identification of each segment of the continuum manipulator

Inputs:

m -times loading history of the proximal tendon force - $T_1(1:m)$.

Outputs:

All variables' profiles - $OneSegRes(1:m)$, $calParas_{1:n}$, etc.

function $FreeCostFun(calParas_i, OneSegRes_{i-1}(1:m), i, expData)$

for $k = 1 : m$ **do**

if $i == 1$ **then**

$\mu_{2i-2}^{k,s} = []$; $\mu_{2i-1}^{k,s} = calParas_i(1)$; $EL_i = calParas_i(2)$; $\zeta_i = calParas_i(3)$;

else

$\mu_{2i-2}^{k,s} = calParas_i(1)$; $\mu_{2i-1}^{k,s} = calParas_i(2)$; $EL_i = calParas_i(2)$; $\zeta_i = calParas_i(4)$;

end if

$calShape(\mu_{2i-2}^{k,s}, \mu_{2i-1}^{k,s}, EL_i, \zeta_i, OneSegRes_{i-1}((k-1):k), i, k)$;

end for

return $f(EL_i, \zeta_i, \mu_{2i-2}^{k,s}, \mu_{2i-1}^{k,s})$ in (17);

end function

function $calShape(\mu_{2i-2}^{k,s}, \mu_{2i-1}^{k,s}, EL_i, \zeta_i, OneSegRes_{i-1}((k-1):k), i, k)$

if $i == 1$ **then**

$dir = sign(T_{2i-1}(k) - T_{2i-1}(k-1))$;

$\mu_{2i-1} = dir \cdot |\mu_{2i-1}^{k,s}|$;

$calSeg(T_{2i-1}(k), \mu_{2i-1}, EL_i, \zeta_i)$;

derive $T_{(2i-1):(2i)}(k), \theta_i(k), \kappa_i(k), N_{2i-1}(k), f_{2i-1}(k), \mu_{2i-1}(k)$;

if $dir \cdot (T_{2i}(k) > T_{2i}(k-1)) < 0$ **then**

$OneSegRes_i(k) \leftarrow updated T_{(2i-1):(2i)}(k), \theta_i(k), \kappa_i(k), N_{2i-1}(k), f_{2i-1}(k), \mu_{2i-1}(k)$;

$break$;

end if

else

$dir = sign(T_{2i-2}(k) - T_{2i-2}(k-1))$;

$[\mu_{2i-2}, \mu_{2i-1}] = dir \cdot [\mu_{2i-2}^{k,s}, \mu_{2i-1}^{k,s}]$;

$T_{2i-1}(k) = T_{2i-2}(k) \cdot \frac{\cos(\theta_{i-1}(k)/4) - \sin(\theta_{i-1}(k)/4) \cdot \mu_{2i-2}}{\cos(\theta_{i-1}(k)/4) + \sin(\theta_{i-1}(k)/4) \cdot \mu_{2i-2}}$;

$calSeg(T_{2i-1}(k), \mu_{2i-1}, EL_i, \zeta_i)$;

derive $T_{(2i-1):(2i)}(k), \theta_i(k), \kappa_i(k), N_{(2i-2):(2i-1)}(k), f_{(2i-2):(2i-1)}(k), \mu_{(2i-2):(2i-1)}(k)$;

if $dir \cdot (T_{2i-1}(k) > T_{2i-1}(k-1)) < 0$ **then**

$OneSegRes_i(k) \leftarrow updated T_{(2i-1):(2i)}(k), \theta_i(k), \kappa_i(k), N_{(2i-2):(2i-1)}(k), f_{(2i-2):(2i-1)}(k), \mu_{(2i-2):(2i-1)}(k)$;

$break$;

end if

if $dir \cdot (T_{2i}(k) > T_{2i}(k-1)) < 0$ **then**

$OneSegRes_i(k) \leftarrow updated T_{(2i-1):(2i)}(k), \theta_i(k), \kappa_i(k), N_{(2i-2):(2i-1)}(k), f_{(2i-2):(2i-1)}(k), \mu_{(2i-2):(2i-1)}(k)$;

$break$;

end if

end if

end function

Algorithm 2: Solve the deflection of the continuum manipulator under the free bending**Inputs:**

m -times loading history of the proximal tendon force - $T_1(1:m)$, the equivalent friction coefficient - $\mu_{1:2n-1}^{k,s}$, the stiffness - $EI_{1:n}$, the linear coefficient - $\zeta_{1:n}$.

Outputs:

All variables' profiles - $AllSegRes(1:m)$, etc.

function $FreeDeflecFun(T_1(k), \mu_{1:2n-1}^{k,s}, EI_{1:n}, \zeta_{1:n})$

for $k = 1 : m$

$dir(k) = sign(T_1(k) - T_1(k-1));$

$\mu_{1:2n-1}(k) = dir \cdot |\mu_{1:2n-1}^{k,s}(k-1)|;$

$AllSegRes(k) = segBaseTip(T_1(k), \mu_{1:2n-1}(k), dir(k), EI_{1:n}, \zeta_{1:n}, ResFree(k-1));$

end for

return $AllSegRes(1:m)$;

end function

function $AllSegRes(k) = segBaseTip(T_1(k), \mu_{1:2n-1}(k), dir(k), EI_{1:n}, \zeta_{1:n}, ResFree(k-1))$

for $i = 1 : n$ **do**

if $i=1$ **then**

$calSeg(T_{2i-1}(k), \mu_{2i-1}(k), EI_i, \zeta_i);$

derive $T_{(2i-1):(2i)}(k), \theta_i(k), \kappa_i(k), N_{2i-1}(k), f_{2i-1}(k), \mu_{2i-1}(k);$

if $dir(k) \cdot (T_{2i}(k) > T_{2i}(k-1)) < 0$ **then**

$AllSegRes(k) \leftarrow updated T_{1:2i}(k), \theta_{1:i}(k), \kappa_{1:i}(k), N_{1:2i-1}(k), f_{1:2i-1}(k), \mu_{1:2i-1}(k);$

$break;$

end if

else

$T_{2i-1}(k) = T_{2i-2}(k) \cdot \frac{\cos(\theta_{i-1}(k)/4) - \sin(\theta_{i-1}(k)/4) \cdot \eta_{2i-2}(k)}{\cos(\theta_{i-1}(k)/4) + \sin(\theta_{i-1}(k)/4) \cdot \eta_{2i-2}(k)};$

$calSeg(T_{2i-1}(k), \mu_{2i-1}(k), EI_i, \zeta_i);$

derive $T_{(2i-1):(2i)}(k), \theta_i(k), \kappa_i(k), N_{(2i-2):(2i-1)}(k), f_{(2i-2):(2i-1)}(k), \mu_{(2i-2):(2i-1)}(k);$

if $dir(k) \cdot (T_{2i-1}(k) > T_{2i-1}(k-1)) < 0$ **then**

$AllSegRes(k) \leftarrow updated T_{1:2n}(k), \theta_{1:n}(k), \kappa_{1:n}(k), N_{1:2n-1}(k), f_{1:2n-1}(k), \mu_{1:2n-1}(k);$

$break;$

end if

if $dir(k) \cdot (T_{2i}(k) > T_{2i}(k-1)) < 0$ **then**

$AllSegRes(k) \leftarrow updated T_{1:2n}(k), \theta_{1:n}(k), \kappa_{1:n}(k), N_{1:2n-1}(k), f_{1:2n-1}(k), \eta_{1:2n-1}(k);$

$break;$

end if

end if

end for

return $AllSegRes(k);$

end function

Algorithm 3: Solve the deflection of the continuum manipulator under the loaded bending

Inputs:

m -times loading history of the proximal tendon force - $T_1(1:m)$, the equivalent friction coefficient - $\mu_{1:2n-1}^{k,s}$, the stiffness - $EI_{1:n}$, the linear coefficient - $\zeta_{1:n}$, the tip pose - $P_{C_T}^{C_o}(k)$ and $\theta_{all}(k)$.

Outputs:

All variables' profiles - $AllSegRes(1:m)$, etc.

```

function LoadedCostFun( $\hat{\sigma}_k, \mu_{1:2n-1}^{k,s}, EI_{1:n}, \zeta_{1:n}, AllSegRes(k-1)$ )
  for  $i = n : -1 : 1$  do
    if  $i == n$  do
      calSegStatusI & II( $T_{2i}(k), \mu_{2i-1}^{k,s}(k), EI_i, \zeta_i, M_{F_T}^{[C_i]}, AllSegRes(k-1), OneSegRes_i(k-1)$ );
      derive  $T_{(2i-1):(2i)}(k), \theta_i(k), \kappa_i(k), N_{2i-1}(k), f_{2i-1}(k), \mu_{2i-1}(k), l_{(2i-1):(2i)}(k)$ ;
       $OneSegRes_i(k) \leftarrow updated T_{1:2n}(k), \theta_{1:n}(k), \kappa_{1:n}(k), N_{1:2n-1}(k), f_{1:2n-1}(k), \mu_{1:2n-1}(k), l_{1:2n-1}(k)$ ;
    else
      calSegStatusI & II( $T_{2i+1}(k), \mu_{2i-1}^{k,s}(k), \mu_{2i}^{k,s}(k), EI_i, \zeta_i, M_{F_T}^{[C_i]}, AllSegRes(k-1), OneSegRes_i(k-1)$ );
      derive  $T_{(2i-1):(2i)}(k), \theta_i(k), \kappa_i(k), N_{(2i-1):(2i)}(k), f_{(2i-1):(2i)}(k), \mu_{(2i-1):(2i)}(k), l_{(2i-1):(2i)}(k)$ ;
      if ( $\theta_i(k) < 0$ ) & ( $\theta_{i+1}(k) < 0$ ) do
        calSegStatusIII( $T_{2i-1}(k), \mu_{2i-1}^{k,s}(k), EI_i, \zeta_i, M_{F_T}^{[C_i]}, AllSegRes(k-1), OneSegRes_i(k-1)$ );
        derive  $T_{(2i-1):(2i)}(k), \theta_i(k), \kappa_i(k), N_{(2i-1):(2i)}(k), f_{(2i-1):(2i)}(k), \mu_{(2i-1):(2i)}(k), l_{(2i-1):(2i)}(k)$ ;
      end if
       $OneSegRes_i(k) \leftarrow updated T_{1:2n}(k), \theta_{1:n}(k), \kappa_{1:n}(k), N_{1:2n-1}(k), f_{1:2n-1}(k), \mu_{1:2n-1}(k), l_{1:2n-1}(k)$ ;
    end if
  end for
  derive  $AllSegRes(k)$ ;
  return  $f(\sigma_k)$  in (19);
end function

```