Punching shear design of RC flat slabs supported on wall corners

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Abstract

Reinforced concrete buildings are typically braced with shear walls positioned around lift shafts and stairs. Vertical transfer of load from slab to walls leads to a concentration of shear stress in the slab at wall ends and corners, which needs to be considered in punching shear design. This issue is not addressed in EN 1992 (2004) and only partially addressed in *fib* Model Code 2010 leaving engineers to resort to their own judgment. Consequently, consideration of punching shear at wall corners can be overlooked entirely or not properly addressed through lack of knowledge. The paper addresses this issue by proposing a method for calculating the design shear stress at wall corners for use in conjunction with the Critical Shear Crack Theory. The method is initially validated against test results for slabs supported on elongated columns as well as numerical simulations. Subsequently, the method is extended to the punching design of a slab supported by a wall corner. The proposed analysis of the slab-wall corner junction is validated against the predictions of nonlinear finite element analysis (NLFEA) employing 3-D solid elements as well as the joint-shell punching model (JSPM) previously developed by the authors.

KEYWORDS

design procedure, finite element methods, punching

1 | INTRODUCTION

Design methods for punching shear typically compare the nominal shear stress calculated on a specified control perimeter with the design shear resistance. In general, the shear stress around the control perimeter is nonuniform even under concentric loading, as illustrated in Figure 1a for a wall end and in Figure 1b for a wall corner. This non-uniformity of shear stress needs to be accounted for in

design to achieve safe results. Most of the pertinent experimental campaigns in the literature examine the influence of large or elongated columns^[1–5] with almost no consideration of wall supports. Available studies show that the punching capacity of slabs supported on elongated columns is less than comparable slabs supported on columns having the same perimeter length but square or circular shape. This arises because the shear stress around the control perimeter becomes nonuniform^[3] as the support cross section (maximum dimension c_{max}) becomes large relative to the slab effective depth (d).

Under concentric loading, the need to account for nonuniform shear stress around supports depends on the proximity of the adopted control perimeter to the

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FIGURE 1 One- and two-way shear forces transferred from the slab to the support at: (a) end of wall; (b) corner of wall

FIGURE 2 Control perimeter for checking local punching of slabs supported on: (a) wall end; (b) wall corner according to MC2010. Notes: d_v is the effective slab depth for punching shear

column. For example, EN 1992 (2004)^[6] positions the control perimeter at $2d$ from the column face where the shear force distribution under concentric loading is relatively uniform regardless of the support shape or size^[1]. Consequently, EN 1992 (2004) includes no special provisions for large supports. On the other hand, when the control perimeter is positioned at $0.5d$ from the column face,^[7,8] an explicit reduction factor is required to account for stress concentrations at support corners. For example, MC2010^[8] limits the length of the straight segments along each edge of the full control perimeter (b_0) to $3d$ as shown in Figure 2 for various scenarios. The reduced control perimeter is depicted $b_{0,3d}$ in MC2010 and throughout this paper. In the case of large columns, MC2010 assumes that the full design shear force is resisted by the reduced perimeter $b_{0,3d}$ with no contribution from linear shear along the neglected straight lengths of the perimeter. As shown by Sagaseta et al.,^[3] this approach works well for $\frac{c_{max}}{d} \leq 6$ but becomes increasingly conservative for longer supports^[9]. This paper proposes a procedure, based on shear field analysis,^[10] for estimating the design shear stress at the corners and ends of walls. The method is validated using available experimental data from punching tests on slabs with elongated supports as well as nonlinear finite

element analysis. Subsequently, the method is used to examine punching failure at wall corners.

2 | SHEAR FIELD ANALYSIS OF SLABS SUPPORTED ON ELONGATED COLUMN

2.1 | Background

Vaz Rodrigues et al.^[10] used shear field analysis to evaluate punching shear failure in bridge deck slabs. In this approach the design shear stress is obtained using linear elastic finite element analysis (LFEA) with shell elements. The shear field is a vector field representing the direction and magnitude of the principal shear force per unit length in a slab^[10]. Vaz Rodrigues et al. defined the effective control perimeter length as:

$$b_{0,elast} = \frac{V}{v_{el,max}} \quad (1)$$

where V is the total applied load and $v_{el,max}$ is the maximum shear force per unit length acting perpendicular to

the control perimeter. Equation 1 is conservative since it implies failure occurs when the peak elastic shear stress reaches the shear resistance whereas, in reality, some shear redistribution^[3,9] occurs around the control perimeter.

2.2 | Refined shear field analysis

This study refines the design procedure of Vaz Rodrigues et al.^[10] by accounting for the beneficial effect of shear redistribution. The refined method is intended for punching shear design at large columns as well as wall ends and corners. The key difference with the method of Vaz Rodrigues et al.^[10] is that the design shear stress is taken as the average stress around the critically loaded quadrant of the effective control perimeter $b_{0, 3d}$ (for wall corners, see Figure 2b) rather than the peak stress. Studies by Setiawan^[11] show that the design shear stress calculated with LFEA depends significantly on the choice of element out-of-plane shear stiffness. To illustrate this, results are presented in Figure 3 for slab OC15 of Teng et al.^[2] which was analyzed using LFEA with ETABS^[12]. Slab OC15 measured 2,800 mm $\times$ 2,200 mm $\times$ 150 mm with an effective depth of 114 mm and c_{max}/d of 8.77. The slab was loaded with eight equal point loads evenly positioned around its perimeter. One quarter of the specimen was modeled using a uniform square mesh of 15 mm. Seven

shear stiffness multipliers were considered: 1.0 (unreduced); 0.8; 0.6; 0.4; 0.2; 0.15, and 0.1.

Figure 3 shows the resulting contours of shear force/length (v_{xz}) along the longer side of $1/4$ of a control perimeter located at $0.5d$ from the column face. The broken-black line in Figure 3 indicates the parts of the control perimeter omitted by MC2010 (outside $b_{0, 3d}$) while the red line indicates the border between downward (yellow) and upward shear forces (green). Figure 3 shows that as the shear stiffness is reduced the border between downward and upward shear force shifts towards the slab centreline resulting in a more uniform shear force distribution. Through comparison with the results of NLFEA with solid elements using ATENA^[13], Setiawan^[11] found that using an out-of-plane shear stiffness modifier of between 0.2–4 in LFEA provides estimates that are in better agreement with distribution of shear stress at failure from the nonlinear analyses. However, further study is required to justify the generality of this proposal.

2.3 | Validation of the proposed method for long supports ($c_{max}/d > 6$)

The proposed shear field method was used in conjunction with the Critical Shear Crack Theory (CSCT)^[14] to predict the punching resistance of two-way loaded slabs with

FIGURE 3 Contours of shear forces/unit length v_{xz} along the longer side of the control perimeter of OC15 for various stiffness modifiers (LFEA)

$6.5 \leq \frac{c_{max}}{d} \leq 10$ for which very few experimental data are available^[9,11]. The analyzed slabs comprise: a) three previously tested slabs and b) four virtual slabs from the parametric study of Setiawan^[11]. Shear force demand was taken as the average shear stress around $b_{0,3d}$. This shear stress was calculated as $\frac{(V - V_{straight})}{b_{0,3d} \cdot d}$ in which V is the total column reaction and $V_{straight}$ is the shear force resisted by the straight lengths of the full control perimeter b_1 neglected in $b_{0,3d}$. The force $V_{straight}$ was determined with LFEA. Based on comparisons with NLFEA solid element modeling, the out-of-plane shear stiffness modifier used in the LFEA was taken as 0.2. The CSCT punching resistance was calculated in terms of the maximum slab rotation which was determined using MC2010 Level of Approximation (LoA) III and NLFEA (MC2010 LoA IV). The results of analyses using the proposed method (depicted “s.field”) are shown in Figure 4a,b for the tested and virtual slabs respectively. Also shown are MC2010 predictions for which the demand shear force was calculated as $\frac{V}{b_{0,3d}}$ (depicted “no one-way”). Additionally, Figure 4a shows the failure load of slab OC15 determined by Setiawan^[11] with 3-D solid element modeling using ATENA. For slabs of Series b) in Figure 4b, the ATENA predictions of Setiawan^[11] are used as the strength reference. Figure 4 shows that the proposed method (depicted “s.field”) gives reasonable yet conservative strength

predictions. The increased strength given by the proposed method (2–10%) compared with MC2010 (“no-one-way”) is due to one-way shear becoming progressively more significant as the support length increases^[9,11]. All the CSCT methods in Figure 4 include a degree of shear redistribution^[15] unlike that of Vaz Rodrigues et al.^[10] in which the peak shear stress is assumed critical. Strength predictions obtained with rotations calculated using MC2010 LoA III and NLFEA are similar, which is convenient since LoA III only requires LFEA which is commonly used in design. The next section considers punching at a wall corner.

3 | EXTENSION OF THE PROPOSED METHOD TO DESIGN SLABS SUPPORTED ON WALL CORNER

3.1 | Description of building geometry used to study wall-slab behavior

This example considers a flat slab measuring 24.5 m square in plan supported on a central core and perimeter columns. As shown in Figure 5a, the perimeter columns are 500 mm square and evenly distributed around the building perimeter

FIGURE 4 Comparison of classic (no one-way) vs. refined approach (shear field) of slabs with $\frac{c_{max}}{d} > 6.0$ from: (a) actual slab database; (b) parametric study

FIGURE 5 Structural arrangement used to study wall-slab behaviour modeled in ETABS (a) Plan view; (b) 3-D view

FIGURE 6 Illustration of typical extraction regions of shear force/length along the MC2010 effective control perimeter at wall corner

with floor-to-floor height of 3.25 m (see Figure 5b). The central core wall is idealized as a 500 mm thick square tube with outside dimensions of 8 m. The slab, which is 250 mm thick with an average effective depth of 200 mm, spans 8 m between the outside face of the wall and the centroid of the perimeter columns (see Figure 5a). The concrete class^[6] is C50/60 for wall and columns and C30/37 for the slab. The superimposed dead load (SIDL) and imposed load (LL) are 1.5 kN/m² and 3 kN/m² respectively while the external cladding load is 1 kN/m². Concrete self-weight was calculated using a specific weight of 25 kN/m³. The design ultimate

load was calculated as $1.35(SW + SIDL) + 1.5 LL$ where SW is the slab self-weight.

One quarter of the floor plate and supporting vertical structures were modeled in ETABS. Columns were modeled as line elements with their central nodes connected to the nodes of surrounding slab elements through rigid links extending to the column perimeter. The core wall and slab were modeled with shell-thick element type which considers out-of-plane transverse deformation according to Reissner-Mindlin plate theory. For ease of modeling, the shell elements

FIGURE 7 Influence of mesh size on shear force per unit length at: (a) $0.5d$ (MC2010 control perimeter); (b) $2d$ (EN1992 control perimeter) from the wall face

representing the wall were positioned at the face of the wall. This simplified the modeling since it avoided the need to connect the shell elements to the slab edge using rigid links of length equal to half the wall thickness. Both top- and bottom-ends of the wall were restrained in the out-of-plane direction (translation and rotation) but only the bottom-end was vertically restrained. Top and bottom columns were rigidly restrained in all 6 degrees of freedom (see Figure 5b).

3.2 | Sensitivity study

A sensitivity study was carried out to investigate the influence on shear stress of: 1) slab mesh size; 2) out-of-

plane moment restraint of the wall and 3) slab stiffness modifiers. Each of these parameters was varied individually. Shear forces were extracted around control perimeters located at $0.5d$ and $2d$ from the wall face. The required shear force is that perpendicular to the control perimeter which equals $\sqrt{v_{xz}^2 + v_{yz}^2}$ for rounded segments^[10] in which v_{xz} and v_{yz} are forces per unit length. Apart from concentric loading, each quadrant of $b_{0,3d}$ (see Figure 2) needs to be considered individually. The average shear force per unit length around the quadrant of $b_{0,3d}$ under consideration is depicted $v_{wall-ave}$ while the corresponding average force around the curved segment of the quadrant (see Figure 6) is depicted $v_{wall-peak}$. Shear redistribution was not considered when assessing punching using EN 1992 for which the peak shear force

FIGURE 8 Influence of out-of-plane moment restraint on shear force per unit length at MC2010 control perimeter

per unit length along a control perimeter at $2d$ from the wall face (depicted $v_{wall-peak-2d}$) is consequently critical.

3.2.1 | Sensitivity to mesh size

Four meshes with uniform element sizes of 25, 50, 100, and 200 mm were investigated. Shear forces per unit length were extracted along control perimeters positioned at $0.5d$ ($v_{wall-ave}$ and $v_{wall-peak}$) and $2d$ ($v_{wall-peak-2d}$) from the wall face. Figure 7a shows the resulting values of $v_{wall-ave}$ and $v_{wall-peak}$ for a control perimeter at $0.5d$ from the wall face. The shear force $v_{wall-peak}$ is seen to be well estimated by $\frac{v_{wall-ave}}{k_e}$ where $k_e = 0.75$ as suggested in MC2010 for corners of walls. The peak shear force per unit length at $2d$ is shown in Figure 7b. Significantly for practice, the influence of mesh refinement is seen to be significantly more pronounced for the MC2010 control perimeter at $0.5d$ than that of EN 1992 at $2d$. In all subsequent LFEA analyses, a mesh size of 50 mm was adopted.

3.2.2 | Sensitivity to out-of-plane moment restraint

Out-of-plane support moment restraint to the slab depends on the wall out-of-plane bending stiffness. This was investigated for the MC2010 control perimeter by varying the wall thickness from its reference value of 500 mm to 300, 400, 600, and 700 mm. Figure 8 shows that the effect of this on the shear force

per unit length at $0.5d$ from the support face stress is relatively minor.

3.2.3 | Sensitivity to slab stiffness modifiers

Previous studies^[3,10,15] suggest that the slab torsional stiffness should be reduced by a multiple of 0.125 in LFEA to account for stiffness reduction due to cracking. The effect of this was investigated in ETABS by adopting stiffness reduction factors of 0.125, both individually and simultaneously, for plate twisting (m_{xy}) and in-plane shear (f_{xy}). Figure 9a shows the influence of the adopted stiffness reduction factors on both the peak and average shear forces per unit length around the effective control perimeter. Reducing the f_{xy} stiffness had no effect while reducing the m_{xy} stiffness increased average and peak shear forces by an insignificant 2% and 4%, respectively. In order to evaluate the influence of modifying the out-of-plane plate shear stiffness, which is shown to be significant in Section 2.2, four out-of-plane stiffness modifiers were evaluated: 0.8, 0.6, 0.4, and 0.2. The results are presented in Figure 9b, which shows that both $v_{wall-ave}$ and $v_{wall-peak}$ decrease with decreasing slab out-of-plane shear stiffness as found for elongated columns in Section 2.2. Comparison with NLFEA in Section 4 suggests that a reduction factor of 0.4 is justifiable. However, for practical design, it is suggested that the beneficial effect of reducing the out-of-plane shear stiffness is neglected. Figure 9a,b also show that $\frac{v_{wall-ave}}{0.75}$ reasonably approximates the peak shear force ($v_{wall-peak}$) as found for elongated columns.

FIGURE 9 Influence of slab (a) torsional and (b) out-of-plane stiffness modifiers on shear force/length at MC2010 control perimeter

3.3 | Designing the required shear reinforcement around the wall corner

Once the design shear force per length v_{Ed} is known, the need for shear reinforcement can be determined. This requires the shear resistance provided by concrete alone $v_{Rd,c}$ to be determined.

For the CSCT, this is done using: (a) MC2010 LoA I, (b) MC2010 LoA III and the closed-form CSCT expression of Muttoni et al.^[16] with (c) standard and (d) refined input parameters. EN 1992 is also considered. Calculation of punching resistance using each of these approaches is described in Appendix S1. The shear resistance provided by the concrete depends on the flexural reinforcement ratio which was determined from bending moments calculated in the LFEA. Reinforcement areas were

calculated within the support strip width b_s (A.10) as required by MC2010. The resulting flexural reinforcement ratios were 0.89% at the support and 0.35% in the span. To validate the proposed procedure, comparative NLFEA was carried out using 3D solid elements with ATENA as well as the extended joint-shell punching model (JSPM) of the authors^[9,17].

4 | NONLINEAR FINITE ELEMENT ANALYSIS (NLFEA) OF THE WALL-SLAB SPECIMEN

For comparison with failure loads obtained with shear field analysis, the wall-slab specimen was analyzed with NLFEA using ATENA with 3-D solid elements as well as

FIGURE 10 Mesh and boundary conditions of wall-slab model in ATENA (3-D solid)

the joint-shell punching model (JSPM) of Setiawan et al.^[9,17]. In each case, the slab was unreinforced in shear.

4.1 | 3-D solid analysis performed in ATENA

4.1.1 | Numerical methodology and chosen boundary conditions of the 3-D solid model

The NLFEA with ATENA^[14] was carried out following the procedure described previously by Setiawan et al.^[17,18], which was used in Reference 17 to analyze seventeen slabs without shear reinforcement having either square supports (14) or elongated supports (3). All the slabs failed in punching. The resulting mean value of $P_{\text{test}}/P_{\text{predicted}}$ was 0.953 with a coefficient of variation of 7.4%. To reduce model size, the slab in Figure 5 was only modeled to just beyond the line of elastic contraflexure around which load was evenly applied through plates (see Figure 10). The wall support was modeled by clamping the top and bottom surface of the slab with vertically restrained steel plates having the same width as the wall. Based on previous studies,^[9,17,18] the slab was meshed with 10 linear brick elements (8-noded) through

its thickness. A fine mesh with uniform size of $25 \times 25 \times 25 \text{ mm}^3$ was used adjacent to the wall corner (see Figure 10). Loading and support plates were modeled using linear tetrahedral (4-noded) elements with elastic steel properties. Flexural reinforcement was modeled with embedded 1-D 2-noded linear truss elements assuming perfect bond. In ATENA, a fully rotating smeared crack approach was used in conjunction with concrete material model CC3DNonLinCementitious2, which combines constitutive models for tensile (fracture) and compressive (plastic) behavior. The Rankine tensile failure criterion was used with exponential softening. The input parameters used in the ATENA analysis are presented in Table 1. Vertical loads were applied uniformly to all loading plates in load-control using the Arc-Length method. The number of iterations was limited to 100 for each load step. The convergence tolerance was set at 1% for displacement, residual, and absolute residual error and 0.01% energy error as defined in Cervenka et al.¹⁴

4.1.2 | Results of the 3-D solid analysis

Localized punching shear failure is difficult to detect from the load-rotation response at wall ends and corners since no clear softening branch develops^[9,19]. Consequently, the

TABLE 1 Summary of material parameters and numerical input for NLFEA in ATENA

No.	Parameter	Value/reference
<i>Concrete constitutive model</i>		
A1	Concrete elastic modulus	Model Code 2010 ^[8]
A2	Fracture energy	Model Code 2010 ^[8]
A3	Concrete tensile strength	Model Code 2010 ^[8]
A4	Smearred crack model	Fully-rotating crack
A5	Critical compressive displacement	0.5 mm
A6	Limit of compressive strength reduction due to cracking	$0.8f'_c$
A7	Eccentricity (defining the shape of the failure surface)	0.52
A8	Volume dilatation plastic factor	0
<i>Reinforcement bar model</i>		
B1	Stress-strain relationship	Bilinear with minimal strain hardening
B2	Bond-slip model	Perfect bond
<i>Loading procedure and convergence criteria</i>		
C1	Loading procedure	Static (force-controlled)
C2	Iteration method	Arc-length method
C3	Convergence criteria for displacement, residual, and absolute residual error	1%
C4	Convergence criteria for energy error	0.01%
<i>Mesh properties</i>		
D1	Mesh size (finest)	$25 \times 25 \times 25$ mm (10 elements along height) at critical punching region
D2	Mesh element for concrete slab	8-noded hexahedral (linear)
D3	Mesh element for loading apparatus	4-noded tetrahedral (linear)
D4	Mesh element for reinforcement bar	2-noded truss element (embedded)

failure load was determined using the strain-based failure criterion of Setiawan^[9,11] in which failure is assumed to occur when the radial strain in the slab soffit at $0.5d$ from the wall face first drops to zero. This criterion is based on experimental observations^[20–22] that show that near failure the radial compressive strain in the slab soffit near the column face drops to zero or even goes into tension. The total wall reaction at failure was predicted to be 785.3 kN. According to previous LFEA of the full floor model, the total wall reaction for the considered ULS load combination ($1.35(\text{SW} + \text{SIDL}) + 1.5 \text{ LL}$) is 746.6 kN. Thus, the ratio of concrete capacity (mean) to the force demand, $\lambda_V = \frac{785.3}{746.6} = 1.05$. Figure 11a,b show the crack patterns (crack width > 0.1 mm) and principal tensile strain respectively at the predicted failure load. At failure, tensile strain is seen to develop locally in the slab soffit near the corner of the wall as observed experimentally^[20–22] and in the authors^[9,11] analyses of elongated columns with ATENA.

4.2 | Extended joint-shell punching model (JSPM) for long or large support scenario

4.2.1 | Brief background

The JSPM of the authors^[9,17] simulates punching failure by combining 2-D nonlinear layered-shell elements with nonlinear joint elements located around a rectangular control perimeter positioned at $0.5d$ from the column face. The joint shear resistance is calculated using the CSCT in terms of the slab rotation which is monitored throughout the analysis allowing the joint shear resistance to be updated at each load step. Individual joints fail once the joint shear force reaches the joint shear strength. Subsequently, the joint shear resistance softens in accordance with the CSCT. Shear redistribution and post-punching resistance are realistically simulated using this modeling strategy^[17]. At large supports with c_{max}/d

between 3 and 6, dummy joints with minimal resistance are provided along the straight segments of perimeter discarded by MC2010 in $b_{0,3d}$ (see Figure 2). Calibration studies^[9,11] show this approach to work well. However, for $c_{max}/d > 6$ the contribution of linear shear becomes significant along the discarded straight lengths of the reduced perimeter $b_{0,3d}$. In this case, one-way joints are inserted along the discarded straight lengths of the control perimeter that are further than $1.5d$ from the support corners (see Figure 12a). The one-way joint shear resistance is conservatively taken as that given by the model of Cavagnis et al.^[24] at first yield of flexural reinforcement. The accuracy of the JSPM has been established^[9,11,18] by modeling the punching failure of 61 slabs from the literature of which 24 had $c_{max}/d > 3$ including six with $6 < c_{max}/d < 9$. In order to extend the database, Setiawan^[11] simulated a further four virtual slabs with $6 \leq c_{max}/d \leq 10$ with ATENA. The mean failure criterion of Muttoni^[14] was used to calculate the two-way joint shear resistance in all these analyses. The resulting mean value of $P_{test}/P_{predicted}$ for slabs with $c_{max}/d < 3$ is 1.01 with a coefficient of variation of 10.3%. For slabs with elongated supports having $3 \leq c_{max}/d \leq 10$, the mean and covariance of $P_{test}/P_{predicted}$ are 0.99 and 11%. For $c_{max}/d < 6$, the JSPM gives similar strength predictions to MC2010 Level of Approximation IV provided the same CSCT resistance function is used. For longer supports MC2010 becomes progressively more conservative, due to the neglect of one-way shear, unless shear field analysis is used as described in the next section for a slab supported on a wall corner. In this paper, the joint shear resistance is determined using mean material parameters and partial material factors of 1.0. The JSPM is implemented^[9,17] in the nonlinear structural analysis program ADAPTIC^[23].

4.2.2 | Analysis of floor plate using the JSPM

A quarter of the floor plate was modeled with nonlinear shell elements^[25–26] in ADAPTIC^[23] following the procedure described by Setiawan et al.^[17]. The slab was modeled from the wall face up to mid-span where the slab was rotationally restrained. A uniform mesh of 100 mm was used as shown in Figure 12b. The wall was modeled using spring elements which for consistency with the LFEA were positioned along the slab edge at the junction with the wall face. The spring stiffness was calculated in terms of the uncracked wall stiffness neglecting reinforcement. For consistency with the LFEA, stiffnesses were calculated assuming that the wall was fully restrained at each floor level as shown in Figure 5b.

The axial component of the spring stiffness (per unit length of wall) was calculated as:

$$K_{axial} = \frac{Et}{h} \quad (2)$$

where E is the Young's modulus of concrete, h is the floor-floor wall height and t is the wall thickness.

The out-of-plane rotational stiffness (per unit length of wall) was calculated as:

$$K_{rotational} = 2 \cdot \frac{4EI}{h} = \frac{2}{3} \cdot \frac{Et^3}{h} \quad (3)$$

The out-of-plane translational stiffness (per unit length of wall) was calculated as:

$$K_{translational} = 2 \cdot \frac{6EI}{h^2} = \frac{Et^3}{h^2} \quad (4)$$

The constitutive models and material properties adopted in ADAPTIC for concrete, reinforcement bars and joints were as used previously^[9,17]. Two- and one-way joints were positioned around a control perimeter located at $0.5d$ from the wall face as shown in Figure 12a. As previously described, the $1.5d$ gap between the one- and two-way joints in Figure 12a is necessary to avoid overestimate of shear resistance due to excessive shear redistribution^[9,11]. The slab was uniformly loaded and punching failure was assumed to occur when the last two-way joint failed within the $b_{0,3d}$ region (see Figure 12a). Failure occurred at a load factor λ_V of 0.94. Figure 13a–c show the variation of shear force/length, perpendicular to the control perimeter, along the $b_{0,3d}$ (see Figure 12a) region predicted by ATENA and the JSPM at 0.20, 0.90, and 1.0 P_u where P_u is the total predicted wall reaction at failure from ATENA. Also shown is the shear force/length acquired using LFEA with out-of-plane shear stiffness modifiers of 1.0 and 0.4 (depicted “LFEA-NOMOD” and “LFEA-0.4MOD,” respectively). At 0.20 P_u the slab behaves elastically so the ATENA prediction agrees well with LFEA-NOMOD (see Figure 13a). However, with approaching failure (0.90–1.0 P_u), the JSPM and LFEA-0.4MOD shear forces compare favorably with ATENA (see Figure 13b,c). Comparison of the LFEA results in Figure 13, without and with stiffness modifier, shows that reducing shear stiffness has the effect of reducing the peak shear force and redistributing shear force away from the wall corner. This redistribution of shear force with increasing load is also evident in the ATENA results shown in Figure 13. In ATENA, the shear stiffness is dictated by the level of damage within the concrete which increases with load. Conversely, the shear stiffness is independent of load in the

FIGURE 11 (a) Crack patterns (width > 0.1 mm) and (b) plot of principal tensile strain at top and bottom surfaces of the slab. Notes: support plates not shown for clarity

FIGURE 12 Modeling of wall-slab model in ADAPTIC (a) Positioning of joint elements in the extended JSPM; (b) Mesh and boundary conditions

FIGURE 13 Variation of shear force/length along the $b_{0,3d}$ region of wall-slab model acquired using both NLFEA and LFEA at: (a) $0.20P_u$; (b) $0.90P_u$; and (c) P_u

LFEA as well as the joints of the JSPM. Prior to failure Figure 13 shows that a greater proportion of the total shear force is resisted around $b_{0,3d}$ in the JSPM than estimated with either ATENA or the LFEA. Crucially, the shear force distributions around $b_{0,3d}$ in Figure 13c are similar at failure for both the JSPM and ATENA. This is due to the JSPM adequately simulating the shear redistribution that occurred around $b_{0,3d}$ near failure when the joint shear capacity is limited by the failure criterion of the CSCT. The similarity of the shear force distributions around $b_{0,3d}$ obtained with ATENA and ADAPTIC also suggests that the difference in boundary conditions adopted in the two models had little effect on the calculated failure loads.

5 | COMPARISON OF THE PROPOSED METHOD WITH NLFEA RESULTS

The LFEA shear field based methods were evaluated by comparing their load factors, λ_V , at concrete shear failure

with those obtained with ATENA and the JSPM. For the CSCT based methods with control perimeters at $0.5d$ from the wall face, λ_V was calculated using: (a) the average shear force along $b_{0,3d}$ ($v_{wall-ave}$) and (b) the peak design shear force derived as $\frac{v_{wall-ave}}{0.75}$. The CSCT shear resistance was calculated in terms of maximum slab rotation. For EN 1992 (2004), the critical design shear force was taken as the greatest shear force/unit length around the corner of a control perimeter at $2d$ from the wall face.

Figure 14a,b show the resulting load factors λ_V for LFEA without out-of-plane shear stiffness modifier and with modifier of 0.4 (only at $0.5d$) respectively. The predictions of MC2010 LoA III and the CSCT closed-form expression^[16] (both with default and refined LFEA parameters) agree well with the predictions of JSPM and 3-D solid analysis (ATENA) when average shear forces around $b_{0,3d}$ ($v_{wall-ave}$) (depicted average forces) are used as proposed. Basing shear failure on the peak shear force ($\frac{v_{wall-ave}}{0.75}$) (depicted MC2010 expression) is overly conservative as is the lower bound of MC2010 LoA I. The EN 1992 (2004) load factor is closest to that of ATENA. Comparison of Figure 14a,b shows that the stress field methods compare best with ATENA if a stiffness modifier of 0.4 is used as implied by Figure 13c.

6 | CONCLUSION

This paper investigates punching failure of slabs supported on elongated supports with $c_{max}/d > 6$ and wall corners. The main findings, which relate to large supports and wall corners, are as follows:

- Punching failure initiates at support corners with peak support reaction developing prior to full mobilization of shear resistance around a control perimeter at $0.5d$ from the column face. MC2010 addresses this issue through the introduction of the effective control perimeter $b_{0,3d}$ shown in Figure 2. For large columns ($c_{max}/d > 3$), MC2010 assumes that the full design shear force is resisted by punching shear around $b_{0,3d}$ with no contribution from linear shear (one-way). This approach becomes increasingly conservative as c_{max}/d increases above 6 since part of the support reaction is resisted by linear shear outside $b_{0,3d}$.
- The joint shell punching model (JSPM) previously developed by the authors^[9,17] is shown to give comparable strength predictions for elongated columns and wall corners to NLFEA with 3-D solid elements carried out with ATENA. The advantage of the JSPM is that it is much more efficient computationally than solid element modeling.

FIGURE 14 Comparison of λ_V calculated using various approaches for: (a) model without out-of-plane shear stiffness modifier and (b) model with stiffness modifier = 0.4

- Comparison with NLFEA calibrated using test results shows that good estimates of support reaction at punching failure are obtained by taking the critical design shear stress to be used in the CSCT as the average elastic stress around the most highly stressed quadrant of $b_{0, 3d}$. This approach accounts for the contribution of one-way shear along the neglected straight lengths of $b_{0, 3d}$ unlike MC2010. Comparable results are obtained with EN 1992 (2004) if the design shear stress is taken as the peak shear stress on a control perimeter at $2d$ from the wall face.
- Comparison with NLFEA using solid elements suggests that the out-of-plane elastic plate shear stiffness should be reduced by a multiple of 0.4 in stress field analysis but further research is required to confirm this.

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SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of this article.

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