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Model based adaptive control for a soft robotic manipulator

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Abstract— The application of model based adaptive control to an underactuated system representative of a class of soft continuum manipulators is investigated. To this end, a rigid-link model with elastic joints is employed and an energy shaping controller is designed. Additionally, model uncertainties and external disturbances, both matched and unmatched, are compensated with an adaptive algorithm. This results in a control law that only depends on the orientation and on the angular velocity of the distal link and it is therefore independent of the number of links. Finally, stability conditions are discussed and the effectiveness of the controller is verified via simulations.

I. INTRODUCTION

Soft robots are characterized by high dexterity, light weight, and low stiffness, which are desirable for safe interactions with operators and the environment. Notable applications include minimally invasive surgery [1], handling of fragile objects, and collaborative tasks with human operators without barriers [2]. Among the different actuation strategies, pneumatics is one of the most popular due to its high power-to-weight ratio, fast response, affordable cost, and ease of miniaturization [3]. While there have been important advances in design and manufacturing of soft robots [4], the accurate position control of these systems remains a challenging engineering problem [5]. This is mostly due to the compliance of soft robots, which makes them particularly susceptible to external disturbances.

Numerical model-free controllers are beneficial when a dynamical model of the system is not available [6], but they can be computationally expensive and difficult to exploit in a general scenario. Model-based controllers are therefore preferable when fast control and high accuracy are required. For this reason, rigid-link models have been increasingly employed for control purposes due to their computational efficiency [7]–[10]. Notable implementations of model-based controllers for soft robots include feedback linearization [11], optimal control [12], and sliding mode control [13]. The main drawback of model-based controllers is the need for an accurate model of the system. When this is not available,

robustification or adaptation is necessary to ensure the required performance. Since soft robots are typically underactuated, this last aspect is particularly challenging and consequently it remains an active area of research.

In this work the application of adaptive model-based control for a rigid-link mechanism with elastic joints, representative of a soft robotic manipulator [14], is investigated. The manipulator is fixed at the base and is actuated by three controlled pressure inputs. The distal end of the manipulator interacts with external loads and has 6 degree-of-freedom (DOF) in 3D space, thus the system is underactuated. The control aim is the regulation of the tip orientation. The main contributions of this work are a partial state feedback control law constructed employing an energy shaping approach and an adaptive algorithm to compensate for model uncertainties and external disturbances, both matched and unmatched. The proposed algorithm relies only on measurements of the rotation and of the angular velocity of the distal link, which corresponds to the tip of the manipulator. Finally, differently from other works, prior knowledge of the disturbances is not required. Stability conditions are discussed and simulations results are presented to demonstrate the effectiveness of the control.

The rest of the paper is organized as follows. Section II introduces the system model and provides the problem statement; Section III details the controller design and discusses the stability conditions; Section IV presents the simulation results, while Section V contains concluding remarks.

II. SYSTEM MODEL AND PROBLEM STATEMENT

A. System Overview

The soft manipulator with pneumatic actuation presented in [14] serves as motivating example for this work. Similarly to the flexible micro actuator (FMA) [3], the system consists of a tubular structure made of a hyper-elastic material with a constant cross section defining three equal internal chambers spaced at $2\pi/3$ radians, as shown in Figure 1. Additionally, an inextensible nylon fiber is embedded at the center of the cross section along the entire device to prevent elongation of its neutral axis. A further inextensible fiber is wound around the device in order to prevent radial expansion of the outer wall while allowing longitudinal strain. The result is a device that can bend in any direction by controlling the pressures P_0, P_1 and P_2 in the internal chambers, whereas variations in length and external diameter are negligible. In the absence of external disturbances, the orientation of the bending plane φ and the rotation of the tip θ on the plane of bending are defined as in [3]:

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$$\tan(\varphi) = \frac{\sqrt{3}(P_1 - P_2)}{P_1 + P_2 - 2P_0} \quad (1)$$

$$\theta = \frac{1}{k} \sqrt{P_0^2 + P_1^2 + P_2^2 - P_1P_2 - P_1P_0 - P_2P_0}$$

where the term k is the bending stiffness of the manipulator expressed in Nm/rad, assumed uniform over the whole length. It follows from (1) that the three internal pressures result in two independent control inputs corresponding to the magnitude and the direction of a resultant pressure vector:

$$u_1 = \sqrt{P_0^2 + P_1^2 + P_2^2 - P_1P_2 - P_1P_0 - P_2P_0} \quad (2)$$

$$u_2 = \text{atan}\left(\frac{\sqrt{3}(P_1 - P_2)}{P_1 + P_2 - 2P_0}\right)$$

Figure 1. Prototype of manipulator (a); schematic of the model (b).

B. System Model

Assuming negligible twist, the soft robotic manipulator is modelled for control purposes with a rigid-link system consisting of $2n + 1$ virtual elastic pin joints (see Figure 1). The rigid-link model approximates the dynamics of a continuum soft robot with increasing accuracy as larger values of n are employed [7]. Nevertheless, the effects of disturbances should be accounted for. In particular, in the presence of disturbances acting outside the plane of bending the neutral axis of the manipulator does not lie on a plane. The system dynamics is expressed in port-controlled Hamiltonian form as¹:

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & I_{2n+1} \\ -I_{2n+1} & -R \end{bmatrix} \begin{bmatrix} \nabla_q H \\ \nabla_p H \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} u - \begin{bmatrix} 0 \\ \delta \end{bmatrix}. \quad (3)$$

The system states are the position $q(t) \in \mathbb{R}^{2n+1}$ and the corresponding momenta $p = M\dot{q} \in \mathbb{R}^{2n+1}$, where $M(q)$ is the positive-definite inertia matrix, defined as in [7]. In particular, the positions q_1 to q_n are the joint angles on the plane of bending, the positions q_{n+1} to q_{2n} are the corresponding out-of-plane angles, while the position

$q_{2n+1} = \varphi$ defines the orientation of the plane of bending. This results in a vector of $(2n + 1)$ virtual positions that relate to the tip angles $\theta = \sum_{i=1}^n q_i$ and $\gamma = \sum_{i=n+1}^{2n+1} q_i$. The open-loop Hamiltonian is $H = T(q, p) + V(q)$, with $T(q, p) = \frac{1}{2} p^T M^{-1} p$ representing the kinetic energy, and $V(q)$ representing the potential energy. The control input is $u \in \mathbb{R}^2$, while the input matrix is $G^T = \begin{bmatrix} 1_n & 0_n & 0 \\ 0_n & 0_n & 1 \end{bmatrix}$, with $\text{rank}(G) = 2$. The constant damping matrix is $R > 0$, while the lumped disturbance $\delta \in \mathbb{R}^{2n+1}$ represents the effect of model uncertainties and external forces on the virtual joints. The system is further qualified by the following assumptions.

Assumption 1: Only the tip angles θ, γ and their time derivatives are known at any instant $t \geq 0$.

Assumption 2: The potential energy is defined as $V(q) = \frac{k}{2} \sum_{i=1}^{2n} q_i^2$, where the constant k , defined in (1), is identical for all joints. Additionally, the position q is bounded, thus $|q| \leq Q$, for some known $Q > 0$. Finally, the damping matrix is defined as $R = R_0 I_{2n+1}$ with constant $R_0 > 0$.

Assumption 3: In the presence of constant disturbances, which do not include twisting moments, the attainable equilibria q^* satisfy the condition

$$G^\perp (\nabla_q V(q^*) + \delta) = 0, \quad (4)$$

where $G^\perp$ is a full-rank left annihilator of G , that is $G^\perp G = 0$.

Assumption 4: The lumped disturbance δ is bounded and constant, thus $|\delta| \leq \Delta$ for some unknown $\Delta > 0$, while $|p| \leq c_1 m_T |\dot{q}|$ for some $c_1 > 0$ and the mass of the manipulator is $m_T \ll 1$.

Assumption 1 implies that the manipulator is instrumented with sensors only at its tip which provide readings of the angles θ, γ . *Assumption 2* states that the potential energy resulting from the mass of the manipulator is negligible compared to its elastic energy, which is reasonable due to the small mass of the system (see also Section IV). Notably, no elastic energy is associated to the position q_{2n+1} since the latter depends only on the direction of the resultant pressure vector, which is defined by the control input. Finally, the constant section of the manipulator motivates the assumption of uniform damping and elasticity, while the maximum strain of the material (i.e. 700% maximum elongation for a soft robotic manipulator made of Elastosil M 4601) defines the bound Q on the position. *Assumption 3* quantifies the effect of the disturbances on the unactuated states (i.e. unmatched disturbances) employing the same notation as [15]. *Assumption 4* limits the study to constant disturbances which do not include twisting moments. This is a limiting but recurring assumption in similar soft manipulators [7].

III. MAIN RESULT

The control aim consists in achieving the desired values θ_d, γ_d of the tip rotations in spite of the external disturbances (which can include out-of-plane components) while only relying on the measurements of θ, γ and of their time derivatives. Thus the control law should be independent of the virtual positions q_i .

¹ The symbols $\nabla_q(\cdot)$ and $\nabla_p(\cdot)$ represent the vectors of partial derivatives with respect to q and p respectively.

A. Energy Shaping Design

The control approach employed in this work is based on the so-called Interconnection-and-damping-assignment passivity-based-control (IDA-PBC), see e.g. [16]. Thus, the desired closed-loop dynamics is defined as:

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & M^{-1}M_d \\ -M_d M^{-1} & J_2 - Gk_v G^T - RM^{-1}M_d \end{bmatrix} \begin{bmatrix} \nabla_q H_d + \Lambda \\ \nabla_p H_d \end{bmatrix} \quad (5)$$

where the closed-loop Hamiltonian is $H_d = \frac{1}{2} p^T M_d^{-1} p + V_d$, while the term Λ accounts for the effect of the disturbances δ [15], [17]. The closed-loop inertia matrix is M_d while the potential energy V_d has a strict minimizer at the desired equilibrium, i.e. $q_d = \text{argmin}(V_d)$. The term $k_v = k_v^T > 0$ is a tuning parameter governing the closed-loop damping, while $J_2 = -J_2^T$ is a free matrix depending on p . The open-loop damping R is typically left unchanged by the control. The control law that applied to (3) achieves the dynamics (5) is:

$$u = G^\dagger (\nabla_q H - M_d M^{-1} \nabla_q H_d) + (G^\dagger J_2 - k_v G^T) \nabla_p H_d + u^* \quad (6)$$

$$u^* = G^\dagger (\delta - M_d M^{-1} \Lambda).$$

where $G^\dagger = (G^T G)^{-1} G^T$. Since the matrix G is rank deficient, the following conditions termed kinetic-energy PDE and potential-energy PDE should be satisfied by V_d, M_d and J_2 for all q, p :

$$G^\perp (\nabla_q (p^T M^{-1} p) - M_d M^{-1} \nabla_q (p^T M_d^{-1} p) + 2J_2 M_d^{-1} p) = 0 \quad (7.a)$$

$$G^\perp (\nabla_q V - M_d M^{-1} \nabla_q V_d) = 0 \quad (7.b)$$

In addition to (7) the condition

$$G^\perp (\delta - M_d M^{-1} \Lambda) = 0 \quad (8)$$

should be verified by Λ , which can be interpreted as a vector of closed-loop non-conservative forces, to account for the effect of the unmatched disturbances [15]. Finally, the existence of a strict minimizer at the equilibrium $(q^*, 0)$ translates in the conditions:

$$\nabla_q V_d(q^*) + \Lambda(q^*) = 0, \quad (9.a)$$

$$\nabla_q V_d^2(q^*) + \nabla_q \Lambda(q^*) > 0. \quad (9.b)$$

It follows from *Assumption 3* that the attainable equilibria q^* in the presence of constant disturbances are such that:

$$q_i = \begin{cases} q_n + \frac{\delta_n - \delta_i}{k} & 1 \leq i \leq n \\ -\frac{\delta_i}{k} & n+1 \leq i \leq 2n. \end{cases} \quad (10)$$

Since q_n, q_{2n+1} are not uniquely defined by (10), they can be chosen as:

$$q_n = \frac{1}{n} \left(\theta_d - (n-1) \frac{\delta_n}{k} + \sum_{i=1}^{n-1} \frac{\delta_i}{k} \right) \quad (11.a)$$

$$q_{2n+1} = \gamma_d + \frac{\delta_0}{k} + \sum_{i=n+1}^{2n+1} \frac{\delta_i}{k} \quad (11.b)$$

to obtain the regulation goal $(\theta, \gamma) = (\theta_d, \gamma_d)$. In order to render the control (6) independent of the individual positions q_i of the virtual joints which appear in M , we define $M_d = k_m M$, with $k_m > 0$ a constant tuning parameter. This choice of M_d solves the kinetic-energy PDE (7.a) with $J_2 = 0$.

Additionally, the closed-loop damping becomes $(Gk_v G^T + Rk_m) > 0$, thus ensuring closed-loop dissipation also for $k_v = 0$ [18]. Substituting the expressions of $V, G, G^\perp$ in the potential-energy PDE (7.b) leads to:

$$V_d = \frac{k}{k_m} \frac{n-1}{2n} \left(\sum_{i=1}^n q_i \right)^2 + \frac{k}{2k_m} \sum_{i=1}^{2n} q_i^2 - \frac{k}{2k_m} \left(\sum_{i=1}^n q_i \right)^2 + \frac{k_p}{2} \left(\sum_{i=1}^n q_i - \theta_d \right)^2 + \frac{k_p}{2} (q_{2n+1} - \gamma_d)^2, \quad (12)$$

where $k_p > 0$ is a tuning parameter. Solving (8) yields:

$$\Lambda_i = \begin{cases} \frac{1}{n} \left(\frac{(n-1)\delta_i}{k_m} - \sum_{j=1}^n \frac{\delta_{j \neq i}}{k_m} \right) & 1 \leq i \leq n \\ \frac{\delta_i}{k_m} & n+1 \leq i \leq 2n, \end{cases} \quad (13)$$

which is constant and satisfies (9.a). Finally, computing the minimizer condition (9.b) gives:

$$\nabla_q V_d^2(q^*) + \nabla_q \Lambda(q^*) = n(k/k_m)^{n+2} k_p^2 > 0 \quad (14)$$

Substituting (12), (13) into (6) yields the feedback control:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{k}{n} \theta - k_p (\theta - \theta_d) - \frac{k_v}{k_m} \dot{\theta} \\ -k_p (\varphi - \gamma_d) - \frac{k_v}{k_m} \dot{\varphi} \end{bmatrix} + \begin{bmatrix} u_1^* \\ u_2^* \end{bmatrix} \quad (15)$$

where

$$\begin{bmatrix} u_1^* \\ u_2^* \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^n \frac{\delta_i}{n} \\ \delta_{2n+1} \end{bmatrix} \quad (16)$$

Finally, substituting (13) into (6) gives $G^\dagger M_d M^{-1} \Lambda = 0$, thus (16) reduces to $u^* = G^\dagger \delta$, which only depends on the matched disturbances. Since the disturbances δ_i are typically unknown, they need to be estimated adaptively, as discussed in the next section.

For the time being, the following preliminary result is presented to streamline the stability analysis of the next section.

Proposition 1: Consider system (3) with *Assumptions 1-4* in closed loop with the control (15) and (16), and assume additionally that the disturbances are exactly known. Then the equilibrium $(q^*, 0)$ defined in (10) is asymptotically stable for all $k_m > 0$ and q^* is a strict minimizer of $V_d' = V_d + \Lambda^T (q - q^*)$ for all $k_p > 0$.

Proof: Define the Lyapunov function candidate $W_1 = H_d + \Lambda^T (q - q^*) + \mathcal{C}$, where $\mathcal{C} > 0$ is a constant that ensures positive definiteness of W_1 . Computing its time derivative along the trajectories of the closed-loop system, and substituting (5) with $M_d = k_m M, J_2 = 0$ and Λ as defined in (13) gives:

$$\dot{W}_1 = -\nabla_p H_d^T (Gk_v G^T + Rk_m) \nabla_p H_d \quad (17)$$

It follows from (17) that $p \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, while computing $\dot{p}$ from (5) yields that $\dot{p} \in \mathcal{L}_\infty$, thus p is bounded and converges

to zero asymptotically according to Barbalat's lemma. Substituting $p = \dot{p} = 0$ in (5) gives $\nabla_q V_d + \Lambda = 0$ which corresponds to (9.a), hence the equilibrium $(q^*, 0)$ is asymptotically stable. The inequality (14) shows that q^* is a strict minimizer of V'_d ■

B. Adaptive Disturbance Rejection

The combined disturbance $G^\dagger \delta$ appearing in (16) is estimated adaptively employing a modification of the Immersion & Invariance method [19]. In particular, $u^* \in \mathbb{R}^2$ is defined as

$$\begin{bmatrix} u_1^* \\ u_2^* \end{bmatrix} = \alpha \begin{bmatrix} \int_0^t \left(-\frac{k}{n} \theta - \frac{R_0}{n} \dot{\theta} + u_1 - \frac{1}{n} \sum_{i=1}^n \delta_i \right) d\tau \\ \int_0^t (u_2 - \hat{\delta}_{2n+1}) d\tau \end{bmatrix}, \quad (18)$$

where $\alpha > 0$ is a constant tuning parameter. Note that the integral action (18) can be implemented by a dynamic controller as illustrated in the next statement.

Proposition 2: Consider the system (3) with *Assumptions 1-4* in closed loop with the control (15) and (18). Define the adaptation law

$$\dot{\hat{\delta}} = -\alpha(\nabla_q V + R\dot{q} - Gu + \hat{\delta}) \quad (19)$$

where the vector of the disturbance estimation errors is defined according to [19] as $z = \hat{\delta} - \alpha p - \delta$. Then there exist $\alpha > 0, k_m > 0$, and $k_v > 0$ such that the equilibrium $(q, p, z) = (q^*, 0, 0)$ is locally stable. Additionally q^* is a strict minimizer of V'_d for all $k_p > 0$, and (θ, φ) converge to (θ_d, γ_d) asymptotically. Finally, the control component u^* in (18) is implemented with the adaptation law (19).

Proof: Computing the time derivative of z gives:

$$\dot{z} = \dot{\hat{\delta}} + \alpha(\nabla_q H + R\nabla_p H - Gu + \hat{\delta} - \alpha p - z). \quad (20)$$

Substituting in (20) the adaptation law (19) gives²:

$$\dot{z} = \alpha(\nabla_q(p^T M^{-1}p)/2 - \alpha p - z) \quad (21)$$

Since equation (21) contains apart from z also terms depending on p , the error dynamics is studied in conjunction with the dynamics of the closed-loop system (3),(15). Define the Lyapunov function candidate $W_2 = W_1 + \frac{1}{2} \log(1 + z^T z)$. Its time derivative along the trajectories of the closed-loop system using (5),(21) gives:

$$\begin{aligned} \dot{W}_2 = & -\nabla_p H_d^T \left((Gk_v G^T + Rk_m) \nabla_p H_d - \alpha p - z \right) \\ & + \frac{z^T}{1 + z^T z} (\alpha \nabla_q(p^T M^{-1}p)/2 - \alpha^2 p - \alpha z) \end{aligned} \quad (22)$$

Isolating common factors in (22), recalling that $M_d = k_m M$ and exploiting the inequalities $|p| \leq c_1 m_T |\dot{q}|$, and $\nabla_q(p^T M^{-1}p)/2 \leq c_2 m_t |\dot{q}|^2$, for some $c_1 > 0$ and $c_2 > 0$, gives:

² In classical Immersion & Invariance the update law $\dot{\hat{\delta}} = -\alpha(\nabla_q H + R\nabla_p H - Gu + \hat{\delta} - \alpha p)$ would be used, which substituted in (20) for a constant disturbance δ would yield the error dynamics $\dot{z} = -\alpha z$. This adaptation law is however not implementable in the present scenario, since it depends on the positions q_i of the virtual joints.

$$\begin{aligned} \dot{W}_2 \leq & -\frac{|\dot{q}|^2 (Gk_v G^T + R_0 k_m)}{k_m^2} + \frac{|\dot{q}|(\alpha c_1 m_T |\dot{q}| + |z|)}{k_m} \\ & - \frac{\alpha |z|^2}{(1 + |z|^2)} + \frac{|z| m_t (\alpha c_2 |\dot{q}|^2 + \alpha^2 c_1 |\dot{q}|)}{(1 + |z|^2)}. \end{aligned} \quad (23)$$

Observing that $\frac{1}{1+|z|^2} \leq 1$ and $\frac{|z|}{1+|z|^2} \leq \frac{1}{2}$ and regrouping common factors, inequality (23) becomes:

$$\begin{aligned} \dot{W}_2 \leq & -\frac{k_v}{k_m^2} |G^T \dot{q}|^2 - |\dot{q}|^2 \left(\frac{R_0}{k_m} - \frac{\alpha c_1 m_T}{k_m} - \frac{\alpha c_2 m_T}{2} \right) \\ & |\dot{q}| |z| \left(\frac{1}{k_m} + m_T \alpha^2 c_1 \right) - \frac{\alpha |z|^2}{1 + |z|^2}. \end{aligned} \quad (24)$$

Considering that $m_T \ll 1$ (see *Assumption 4*), there exist $c_3 < 1, c_4 > 1$ such that $(R_0 - \alpha m_T (c_1 + c_2 k_m)) \geq R_0 c_3$ and $\left(\frac{1}{k_m} + m_T \alpha^2 c_1 \right) \leq \frac{c_4}{k_m}$, thus inequality (24) can be rewritten as:

$$\dot{W}_2 \leq -\frac{k_v}{k_m^2} |G^T \dot{q}|^2 - [|\dot{q}| \quad |z|] \begin{bmatrix} \frac{R_0 c_3}{k_m} & \frac{-c_4}{2k_m} \\ \frac{-c_4}{2k_m} & \frac{\alpha}{1 + |z|^2} \end{bmatrix} \begin{bmatrix} |\dot{q}| \\ |z| \end{bmatrix} \quad (25)$$

Employing a Schur complement argument in (25), we conclude that $\dot{W}_2 \leq 0$ locally if:

$$4k_m R_0 \alpha c_3 / c_4^2 > 1 + |z|^2, \quad (26)$$

which shows that the equilibrium is locally stable. Note that for all trajectories close to the equilibrium one has $|\dot{q}| \in \mathcal{L}_2 \cap \mathcal{L}_\infty, |z| \in \mathcal{L}_\infty$, while computing $\dot{p}$ from (5) gives:

$$\dot{p} = \left(-k_m (\nabla_q H_d + \Lambda) - \frac{Gk_v G^T \dot{q}}{k_m} - R\dot{q} + z + \alpha p \right) \quad (27)$$

Since $q \in \mathcal{L}_\infty, \Lambda \in \mathcal{L}_\infty$ (see *Assumptions 2, 4*) then $\dot{p} \in \mathcal{L}_\infty$, thus $|\dot{q}|$ converges to zero asymptotically. Substituting $p = \dot{p} = 0$ in (27) gives $(\nabla_q V_d + \Lambda)k_m - z = 0$. From (22) and (26), z converges to zero asymptotically. As a result $G^\dagger ((\nabla_q V_d + \Lambda)k_m - z) = k_p \begin{pmatrix} \theta - \theta_d \\ \varphi - \gamma_d \end{pmatrix} = 0$ and the equilibrium is locally attractive hence asymptotically stable. The minimizer claim follows directly from (14). Finally, integrating (19) and multiplying it by $G^\dagger$ yields (18) ■

The adaptation law (18),(19) can be extended to discrete-time systems similarly to [20], [21]. This point is part of our future work.

While the component u_1 of the control input (15) depends on θ and $\dot{\theta}$ as initially required, the control u_2 contains φ and $\dot{\varphi}$, which are not directly measurable. If the disturbances are only acting in the plane of bending, then at the equilibrium $\gamma = \varphi$. Otherwise, computing γ at the equilibrium and substituting (10) gives:

$$\varphi = \gamma + \sum_{i=n+1}^{2n} \frac{\delta_i}{k}. \quad (28)$$

The control (15) regulates φ to γ_d and consequently $\gamma \neq \gamma_d$. Instead, regulating γ to γ_d requires replacing φ with γ in the control (15). The resulting control law is obtained from (6) using:

$$V_d = \frac{k}{k_m} \frac{n-1}{2n} \left(\sum_{i=1}^n q_i \right)^2 + \frac{k}{2k_m} \sum_{i=1}^{2n} q_i^2 - \frac{k}{2k_m} \left(\sum_{i=1}^n q_i \right)^2 + \frac{k_p}{2} \left(\sum_{i=1}^n q_i - \theta_d \right)^2 + \frac{k_p}{2} (\gamma - \gamma_d)^2 \quad (29)$$

Substituting (29) in the potential-energy PDE (7.b) gives:

$$G^\perp(\nabla_q V - M_d M^{-1} \nabla_q V_d) = \begin{bmatrix} 0_{n-1} \\ 1_n \end{bmatrix} k_p k_m (\gamma - \gamma_d), \quad (30)$$

which holds if $\gamma = \gamma_d$, while the minimizer condition (14) remains unchanged. The stability conditions for this case are summarized in the following result.

Proposition 3: Consider the system (3) with *Assumptions 1-4* in closed loop with the control law:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{k}{n} \theta - k_p(\theta - \theta_d) - \frac{k_v}{k_m} \dot{\theta} \\ -k_p(\gamma - \gamma_d) - \frac{k_v}{k_m} \dot{\gamma} \end{bmatrix} + \begin{bmatrix} u_1^* \\ u_2^* \end{bmatrix}, \quad (31)$$

where the control component u^* is given by (18) and (19). Then there exists $\alpha > 0, k_m > 0$ and $k_v > 0$ such that the equilibrium $(q, p, z) = (q^*, 0, 0)$ is locally stable. Additionally q^* is a strict minimizer of V_d' for all $k_p > 0$, and (θ, γ) converge to (θ_d, γ_d) asymptotically.

Proof: Consider the closed-loop system with $z = 0$ and define the Lyapunov function candidate $W_1 = H_d + \Lambda^T(q - q^*) + C$, with V_d as given in (29). Computing (17) for the control (31) while defining $\dot{\omega} = \sum_{i=n+1}^{2n} \dot{q}_i = \dot{\gamma} - \dot{\phi}$ gives:

$$\dot{W}_1 = -\frac{R_0}{k_m} \left(\sum_{i=1}^{2n+1} \dot{q}_i^2 \right) - \frac{k_v}{k_m^2} (\dot{\theta}^2 + \dot{\phi}^2 + \dot{\omega}^2). \quad (32)$$

Substituting in (32) the Cauchy-Schwarz inequality $\sum_{i=1}^{2n+1} \dot{q}_i^2 \geq \frac{\dot{\theta}^2}{n} + \frac{\dot{\omega}^2}{n}$ and the Young's inequality $|\dot{\phi}| |\dot{\omega}| \leq \varepsilon |\dot{\phi}|^2 + |\dot{\omega}|^2 / 4\varepsilon$, verified for all $\varepsilon > 0$, yields

$$\begin{aligned} \dot{W}_1 \leq & -|\dot{\theta}|^2 \left(\frac{R_0}{nk_m} + \frac{k_v}{k_m^2} \right) - |\dot{\omega}|^2 \left(\frac{R_0}{nk_m} - \frac{k_v}{4\varepsilon k_m^2} \right) \\ & - |\dot{\phi}|^2 \frac{(R_0 k_m + k_v(1 - \varepsilon))}{k_m^2} \end{aligned} \quad (33)$$

Consequently, $\dot{W}_1 \leq 0$, if $R_0 k_m > n k_v / 4\varepsilon, \forall \varepsilon \leq 1$ thus the equilibrium $(\theta, \phi) = (\theta_d, \gamma_d)$ is locally asymptotically stable. Considering the complete system and introducing the constant $0 < c_5 < 1$ while computing $\dot{W}_2$ as in (25) gives

$$\begin{aligned} \dot{W}_2 \leq & -|\dot{\theta}|^2 \left(\frac{R_0 c_5}{nk_m} + \frac{k_v}{k_m^2} \right) - |\dot{\omega}|^2 \left(\frac{R_0 c_5}{nk_m} - \frac{k_v}{4\varepsilon k_m^2} \right) \\ & - |\dot{\phi}|^2 \frac{(R_0 k_m c_5 + k_v(1 - \varepsilon))}{k_m^2} \\ & - \begin{bmatrix} |\dot{q}| & |z| \end{bmatrix} \begin{bmatrix} \frac{R_0 c_3(1 - c_5)}{k_m} & \frac{-c_4}{2k_m} \\ \frac{-c_4}{2k_m} & \frac{\alpha}{1 + |z|^2} \end{bmatrix} \begin{bmatrix} |\dot{q}| \\ |z| \end{bmatrix} \end{aligned} \quad (34)$$

The rest of the proof follows a similar structure to (25)-(27) in *Proposition 2* ■

Figure 2. Time histories of the tip angles (θ, γ) with the control (18),(31) and $k_m = 5, \alpha = 0.5$ (a); control input (b).

Figure 3. Time histories of the control component u^* (a) and of the controlled pressures P_0, P_1, P_2 (b).

Figure 4. Time histories of the tip angles (θ, γ) with $k_m = 10$ and $\alpha = 1$ (a), and of the corresponding control input (b).

IV. SIMULATION RESULTS

In this section the effectiveness of the proposed control is demonstrated via simulations employing a rigid-link model representative of a soft robotic prototype [14] which has a total length l_T and a total mass m_T . Simulations have been conducted in Matlab with the parameters: $n = 3, k = 5 \text{ Nmm/rad}, l_T = 30 \text{ mm}, m_T = 1.5 \text{ g}, R_0 = 0.1, k_v = 0.1, k_p = 0.2, k_m = 5$, or $k_m = 10$, and $\alpha = 0.5$ or $\alpha = 1$. The desired equilibrium has been defined as $(\theta_d, \gamma_d) = (\frac{\pi}{2}, \frac{\pi}{6})$. The constant disturbances have been defined as $\delta = \delta_0 [0.5 \ 0.5 \ 0.5 \ 0.1 \ 0.1 \ 0.1 \ 1]$ with $\delta_0 = -0.1$, where all terms apart from the last correspond to constant moment loads. Since no elastic constant is associated to the angle ϕ ,

the last component of δ can be attributed to model uncertainties. The results of the simulations showing the time histories of the tip angles (θ, γ) and of the control input computed from (18) and (31) are reported in Figure 2. These indicate that the regulation goal is achieved in spite of the disturbances. The initial overshoot on the tip angles is due to the fact that the constant disturbance has a negative sign and therefore acts in the same direction as the control input. The time histories of the pressures P_0, P_1 and P_2 computed from (1) and the disturbance compensation (18) are plotted in Figure 3. For illustrative purposes we set $P_0 = 0$, while the values P_1 and P_2 are relative to the atmospheric pressure. The effect of k_m and of α are illustrated in Figure 4, indicating that a larger k_m and a larger α result in a more responsive control than in Figure 2, although for different reasons. While increasing k_m effectively scales down the parameter k_v , which affects the closed-loop damping, increasing α results in a faster convergence of the disturbance estimates to the correct value which is then compensated earlier on in the time history. Notably, setting $\alpha = 0$, which corresponds to setting $u_1^* = u_2^* = 0$ in (31), the position error settles to the value $(\theta - \theta_d, \gamma - \gamma_d) = (0.25, 0.09)$. Finally, the system response with a variable disturbance ($\delta_0 = 0.05$ between 2.5 seconds and 2.6 seconds), is showed in Figure 5. Although the study of this scenario is beyond the scope of this work, the results indicate that the proposed controller can effectively reject the variable disturbance.

Figure 5. Time histories of the tip angles (θ, γ) with $k_m = 10$ and $\alpha = 1$ and variable disturbance (a), and of the corresponding control input (b).

V. CONCLUSION

A new model based adaptive control for an underactuated mechanical system with flexible joints representative of a soft robotic manipulator with pneumatic actuation has been proposed. The control law only depends on the orientation of the distal end of the manipulator and compensates the effect of matched and unmatched disturbances resulting from external moments and model uncertainties. Stability conditions have been discussed and simulation results have been presented to illustrate the effectiveness of the proposed approach. Future work will extend the result to time varying disturbances and will investigate the regulation and tracking problem for more complex manipulators consisting of multiple segments connected in series and actuated independently. Finally, the results will be validated with experiments on a soft robotic manipulator prototype.

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