Modelling Traffic Action in High Speed Railway Bridges

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Abstract

There are a significant variety of approaches taken by researchers when considering the response due to traffic in high speed railway (HSR) bridges. This paper will focus on the comparison between methods employed by different authors for the structural models developed for the bridge, the load models for the high-speed trains, and the interaction between the train vehicles and the bridge. The structural bridge models range from simple beam models to complex 3D solid models, with important implications in terms of the ability of the models to predict realistic responses, as well as in terms of the appropriateness for design purposes considering the computational requirements. The load models vary from simple moving loads to a full vehicle moving system (with its own masses, stiffness, dampers, and contacts). For the latter approach, the interaction between the vehicle and the bridge becomes important. The major models in literature are identified and compared herein. The aim of this paper is to compile these different approaches and compare the different available methods, in order to enable a clearer judgement when setting the corresponding models for design purposes, as well as to provide a deeper understanding about some of the key definitions within the HSR bridge models.

Keywords: Bridges, Dynamics, Railway systems

1 Introduction

High-speed-railway (HSR) lines are present in many countries around the world. The development of these lines has stemmed from the first HSR line to be built, known as the Tokaido Shinkansen, linking the 515 *km* between Tokyo and Osaka in Japan in 1964, allowing train speeds of up to 270 *km/h* (Smith, 2003).

This paper will compile the research for modelling HSR bridges under traffic loading. The paper focuses on the bridges, the vehicles, the track, the track-vehicle interaction and the irregularities. Finally, focus will be made to the HSR bridge design constraints.

1.1 Current High-Speed-Railways

Although the definition of a HSR line depends on many factors, the widely accepted definition, used by the International Union of Railways (UIC), is based on two speeds. For railway lines built specifically for HSR, 250 *km/h* is the minimum main operating speed of vehicles to be classified as high speed. Conversely, a line upgraded from conventional speeds is classified as a HSR line when operating speeds exceed 200 *km/h* (Leboeuf, 2018). Although faster, with speeds up to 430 *km/h*, magnetic levitation (maglev) trains are not considered in this paper, as the vehicle load is spread over a much larger region than the concentrated HSR wheel loads.

The United Kingdom (UK) currently has one dedicated HSR line, with an operating speed of 300 *km/h* (Bennett, 2007), built between the Channel Tunnel and London St Pancras, and known as High Speed 1 (HS1) (formerly the Channel Tunnel Rail Link (CTRL)). Through upgrading of infrastructure, the UK also has a few lines that are high speed (operating at 200 *km/h*). In addition, construction has begun on High Speed 2 (HS2) between London and the West Midlands (Phase 1) with future phases continuing up to Manchester and Leeds from the West Midlands (Phase 2) (Durrant, 2015). With operating speeds of 360 *km/h*, it will be among the fastest HSR lines in the world (UIC, 2019).

China is the highest user of HSR, both in terms of number of passengers, with 800 million per year in 2015 (Leboeuf, 2018), as well as total track length, as seen in Table 1 (compiled from UIC (2019)). In 2014, there was a total worldwide HSR network length of 22954 *km* (Leboeuf, 2018), which has more than doubled to 46403 *km* by 2019, and further substantial growth is expected, with Asia and Europe being the major markets.

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Table 1: Table of the current and planned line lengths and maximum operating speeds of countries around the world with dedicated HSR lines around the world as of 2019 (*link to Bahrain; **link to Singapore)

Country	Operational		Planned	
	Length	Max Speed	Length	Max Speed
Qatar*	-	-	180	350
China	31043	350	8535	350
India	-	-	508	320
Indonesia	-	-	712	300
Iran	-	-	3104	250
Israel	-	-	85	250
Japan	3041	320	596	320
Kazakhstan	-	-	1011	250
Malaysia**	-	-	350	320
Saudi Arabia	453	300	-	-
South Korea	887	305	49	305
Taiwan-China	354	300	-	-
Thailand	-	-	2878	250
Turkey	594	250	6242	300
Vietnam	-	-	1600	350
ASIA	36372	350	29976	350
Austria	263	250	352	250
Baltic	-	-	870	240
Belgium	209	300	-	-
Czechia	-	-	810	350
Denmark	-	-	60	250
France	2734	320	1725	320
Germany	1571	300	438	300
Italy	896	300	205	300
Netherlands	90	300	-	-
Norway	-	-	333	250
Poland	224	200	1082	350
Portugal	-	-	596	350
Russia	-	-	2970	400
Spain	2852	300	1965	300
Sweden	-	-	750	320
Switzerland	144	250	15	250
UK	113	300	550	360
EUROPE	9096	320	12721	400
Australia	-	-	1749	350
Brazil	-	-	511	300
Canada	-	-	290	250
Egypt	-	-	1210	?
Mexico	-	-	210	300
Morocco	200	320	1114	320
South Africa	-	-	2390	300
USA	735	240	2351	350
OTHER	935	320	9825	350
WORLD	46403	350	52522	400

2 HSR Bridges

Bridges are an important component of HSR lines, due to strict geometrical requirements for the longitudinal alignment. For example in Taiwan, the HSR project utilised 251 *km* of bridges, out of a total line length of 345 *km* (Tai *et al.*, 2010). In comparison with a highway bridge, the loading is much heavier and sudden, although less continuous over the time, leading to stockier span to depth ratios in HSR bridges. The repeating distance of axles on a train can also lead to resonance problems in HSR bridges (see Section 7.5.1). This paper focuses on small to medium HSR bridges (spans up to around 80 *m*). When larger spans are required, bridges may use alternative structural forms, such as arch, cable stayed or suspension bridges (He *et al.*, 2017). These present different challenges (for example rail expansion, deck flexibility and external loading), to those discussed in this paper.

2.1 Materials

Medium span HSR bridge decks are generally constructed with prestressed concrete, whilst steel and concrete composites are used for longer spans and reinforced concrete for shorter spans (Hseih and Wu, 2014). Concrete bridges tend to be less prone to vibrations than composite bridges due to their larger mass. They are also found to be better than a comparable composite bridge, as the critical speed (defined as the speed at which resonance vibration may occur (Frýba, 2001)), of concrete bridges tends to be higher. Nevertheless, the critical speed can still be within the running speeds of HSR bridges.

2.2 Construction Methods

The main construction methods considering on site construction are full span casting, balanced cantilever and incremental launching methods. The use of prefabricated construction typically involves full span placement, segmental cantilevered or segmental span-by-span construction. Cast in-situ concrete methods are more traditionally used and are more financially viable for small scale production. However, the use of these construction methods are time intensive, requiring more on site space to cast, resulting in longer construction times. In cases where standardisation can occur, the cast in-situ methods become uneconomical and with a sufficient number of bridges, prefabricated methods become achievable by a reduction in construction costs through economies of scale, with savings covering the initial set-up costs.

Choosing precast construction over cast in situ allows higher quality control, reduced on site construction times, better control of prestressing loads, lower maintenance costs and better riding comfort levels. Generally the durability of construction may be increased, although joints between precast elements need to be carefully designed. This is particularly the case for segmental construction, where epoxy resins and post tensioning are used to join segments. Cast in-situ design allows greater design flexibility and does not require the specialist heavy lifting and installation equipment (Hseih and Wu, 2014).

2.3 Cross sections

A database of published short and medium spanned HSR bridges from around the world have been compiled by the authors and their properties compared. The bridges are split into categories based on their cross sectional properties, as described in Table 2. For these bridge types Figure 1 shows a comparison between the span and the depth of the bridges. It can clearly be seen that there is an upper span length of around 45 *m* for precast construction due to construction constraints (mainly transportation). It can also be seen that in-situ construction can generally achieve more slender profiles.

Precast HSR beam bridges will generally conform to approximate span to depth ratios. According to Montagut (2010) spans of up to 30 to 35 *m* can be achieved in simply supported bridges, with depth to span ratios (slenderness) between 1/10 and 1/12. Through post-tensioning for continuity, up to 40 *m* spans can be achieved, with more slender profiles such that the depth to span ratio reaches between 1/14 and 1/15.

The majority of these HSR solutions use a box girder, matching the claims of Hseih and Wu (2014). According to Montagut (2010), the use of box girder bridges in general can lead to depth to span ratios of 1/12 to 1/14 with some cases of up to 1/17. However, these figures are based on cast-in-place solutions, and examples of precast construction indicated stockier solutions (Tai *et al.*, 2010). Some cases in Spain have utilised a haunched profile during in-situ construction to half the midspan depth (Llombart Jaques *et al.*, 2014). In the case of wide decks, or a bridge that requires a higher level of stiffness and shear strength, a double cell box girder can be used (cross section c of Table 2) (Marí and Montaner, 2000).

Figure 1: Bridge span against cross section depth. The shape of the point refer to the cross section in Table 2. Solid markers refer to continuous bridges and hollow are simply supported. Grey colours refer to precast construction and black is cast in-situ.

Table 2: Different types of cross section in the bridge database

cross section	name	key
	Box girder	a
	Dual U-Beam	b
	Straight sided box girder	c
	Triple webbed box girder	d
	Voided slab	e

The use of U-beams is the most popular type of precast beam used in HSR bridges, due to its torsional resistance and attractive design. It is combined with an often cast in-situ slab to complete the deck.

Although the aforementioned cross sections are the most popular, other sections are possible. Trough bridges, where the train runs in the trough of a beam with a wide U shape without an additional deck slab being constructed, has had some limited use in HSR. By running in the trough of the beam there is a reduction in the noise pollution without a need for sound barriers (Rosignoli, 2014). An example is the Italian Modena HSR viaducts, which have a total length of 24.8 km. For shorter spans less than 30 m, when shallow depths of the bridge are required, then steel I beams have been encased in concrete, to form a solution called filler beam. This is particularly used for spans less than 12 m. This form of composite construction is relatively simple and widely used (Hoorpah, 2008), however this results in a lack of published constructed examples, so it is not included as a solution in Figure 1.

2.4 Longitudinal continuity and articulation

Continuous bridges offer a lower vibration response and therefore a better dynamic performance than simply supported bridges (Goicolea, 2008). Integral bridges are found to be suitable for spans of up to 60 m for HSR (Rodriguez *et al.*, 2011). The continuity of the rail over joints is often ignored for dynamic analysis due to the relatively low rail stiffness compared to the bridge (Majka and Hartnett, 2009). However, rail-structure interaction is studied for longitudinal actions such as braking, acceleration, thermal expansion and long-term bridge deformations. These actions introduce stresses into the rail, which generally to avoid expansion joints in the rail on the bridge, limits the distance from a fixed support to a bridge expansion joint to 90 m for concrete bridges (60 m for composite) (Cagaró *et al.*, 2010).

Through vehicle braking and accelerating, large horizontal forces are required to be transported from the deck into the foundations. This is achieved through the piers and abutments. The horizontal load is generally transferred through a stocky pier, an abutment or V shaped piers, all of which become stiff enough to transfer the horizontal load.

When piers are slender they may need modelling for the dynamic effects of the bridge. This is normally the case for tall viaducts. The flexibility of the pier can lead to larger lateral displacements of the bridge (Antolín *et al.*, 2010; Antolín *et al.*, 2012). The stiffness of the piers, abutments, bearings and foundations are usually represented by springs attached to the support conditions, as modelled by Cuadrado *et al.* (2008) and Ramondenc *et al.* (2008).

2.5 Bridge Modelling

The modelling of the bridge structure itself depends largely on the focus of the investigation or the design analyses. For research focusing on the passenger comfort or the track-structure interaction, it often is the case that the bridge is modelled by simple beam elements, ranging from two dimensional Euler-Bernoulli (Gabaldon *et al.*, 2008) and Timoshenko methods (Lou *et al.*, 2012), to three dimensional beams element models (Montens and Huyard, 2008).

For focus on the bridge design, the need to capture the sectional deck deformations requires shell element models. These elements, used by Song *et al.* (2003) among many others, are often preferred over the solid elements (used by Xia *et al.* (2003) in conjunction with a simple beam element model), as they have the ability to model the bridge more accurately for the same computational time. Theoretically, 3D solid elements could be used, but the usage is limited with current computing capacity, as once these models are appropriately calibrated through sensitivity analysis, an extremely large number of elements is required, so the computational time becomes unrealistic for complex dynamic modelling. Modelling with shell elements requires significantly fewer elements for accurate analysis, leading to a smaller computational time for comparable results. In addition, the more realistic representation gained from using 3D solid elements, rather than shells, has very little implication for the dynamic analysis problems raised during the design of these bridges.

3 Train-Vehicle Models

There are many types of train vehicle, with different weights and wheel spacings. Table 3 (compiled from UIC (2019)), shows a few examples of the high speed rolling stock in use in the world, their primary countries of use and the type of train that they are. This is a small selection of the trains used around the world, with the maximum train speed (V_{max}) in some cases limited by the line speed.

Train vehicles can broadly be placed into three layout categories: regular, conventional or articulated. Articulated vehicles are characterised by the bogies, which each hold two wheelsets, and are shared between carriages. In contrast conventional train carriages do not share bogies and therefore have two dedicated bogies of two wheelsets

Table 3: Table of a selection of trains, country of operation, layout type and maximum operating velocity. A=Articulated, C=Conventional, R=Regular

Train	Country	Type	$V_{\max}$ [km/h]
Thalys PBKA	France, Belgium	A	320
AVE S100	Spain	A	300
Eurostar	France, Belgium, UK	A	300
ICE2	Germany	C	280
ICE3	Germany	C	300
ETR-500	Italy	C	300
Virgin	UK	C	200
S102	Spain	R	300
TGV Duplex	France	A	320
CR400AF	China	C	350
JRE-E6	Japan	C	320

Figure 2: Train dynamic motions

each for every carriage of the train vehicle. Regular trains do not feature bogies, with the wheelsets directly connected to the carriages.

The motion of the vehicles is important in the study of the vehicle dynamics. A set of three perpendicular axes (u, v, w), can be used to define the six degrees of freedom (DOF) of the vehicle motion, with three displacements (forth and back, lateral swing and floating) and three rotations (rolling, pitching and yawing). Axis u is defined in the direction of travel, v perpendicular to u and horizontal, and w perpendicular to u and v .

3.1 Point Load Methods

The simplest method of modelling is through a set of moving point loads (Figure 3a). This is widely accepted to be an appropriate method for determining the dynamics in bridge design, especially for cases where spans exceed 30 m. Yang *et al.* (2004), stated that although the moving load model is widely used in literature, it ignores the interaction between the bridge and the moving vehicle. Hence, it is only suitable for situations where the vehicle mass is small compared to the mass of the bridge and the vehicle response is an undesired output.

Moving point loads were found to be inappropriate for short span bridges (with literature differing between spans of $L < 40$ m in Martin (2008) and $L < 30$ m in BS EN 1991-2:2003 (2010)), as the interaction of the mass of the bridge with the vehicle mass is important. Goicolea *et al.* (2008) compared a model of moving loads with a sprung mass model. It was found that the model of point loads consistently and significantly overestimated both

Figure 3: Loading models, moving at velocity v

the displacements and accelerations of the bridge, particularly for simply supported spans less than 30 m. To take into account of this effect, **BS EN 1991-2:2003 (2010)** suggests either additional structural damping depending on the span length or performing a vehicle-structure dynamic analysis.

3.2 Moving Mass Models

For short spans, less than 30 m, the study of the bridge dynamics may require incorporation of the effects of the vehicular mass, achieved by a moving mass model (Figure 3b). It is also useful for situations where vehicle mass is large in proportion with the bridge mass, for which differences between moving load and moving mass models can be up to 80 % (**Akin and Mofid, 1989**). The lack of a vehicle in this model means that the vehicle motion cannot be determined, which is important for HSR in the presence of track irregularities (**Yang et al., 2004**).

3.3 Sprung Mass Model

A sprung mass model (Figure 3c), incorporates a mass suspended by either a spring or a spring and damper, which moves along the bridge with the velocity of the train. For the bridge design, it allows incorporation of the mass effects that are needed for proportionally heavy loading (with respect to the bridge mass) or short spans (**Goicolea and Gabaldón, 2008**), but it also is the simplest possible study of vehicular dynamics (passenger comfort). There are several levels of complexity from a mass hung from a spring (**Yang et al., 2004**), to a moving unsprung mass attached to a sprung mass and multiple layers of sprung masses. The unsprung mass refers to the wheelset (**Vale and Calçada, 2014**). The carriage mass is either incorporated as an additional sprung layer, combined with the bogie mass or as a moving load on top of the wheelset mass (**Vale and Calçada, 2014**).

Using a sprung mass model, **Yang et al. (2004)** conducted analysis comparing different values of vehicle suspension damping and its effect on the bridge and vehicles. It was concluded that with higher damping, the vertical acceleration of the sprung mass was higher, and the bridge had a reduced response. The sprung mass suspension stiffness was found to have negligible impacts on the bridge response, but was important to the sprung mass response. A drawback of using the sprung mass model, is that the vehicle motion does not include pitching motion which is important especially during the presence of rail track irregularities (**Yang et al., 2004**).

Figure 4: 2D vehicle model, with mass (M_i), stiffness (I_i) and modulus of elasticity (E_i), where i represents the particular body for the wheelsets (w), bogies (b) and carriage (c)

3.4 Two dimensional Vehicle Model

When modelling in two dimensions (Yang *et al.*, 2004; Lou, 2005), the sprung mass model can be modified to connect the components of the vehicle with rigid beam connecting the bogie sprung masses, such that they are linked like a real carriage (with the stiffness of members modelled as rigid in the beam elements in Figure 4). Studies by Lou (2005, 2007), showed that there was no significant difference in the bridge response between the sprung mass model and the two dimensional vehicle model, for velocities up to 360 km/h. However, it was found that the vehicle accelerations were increased by the connection between bogies and wheelsets (Lou, 2005). This method is more commonly used for the analysis of the passenger comfort, due to the two dimensional bridge model being insufficient for accurate representation of the bridge dynamics.

Some researchers have opted to replace the rigid links between bogies and wheelsets with flexible links (as represented in Figure 4 by I_i and E_i) to represent the vehicle deformability (Chen *et al.*, 2015). These 2D models are all limited to one plane, with the lateral actions ignored.

3.5 Complete Vehicle Models

Yang *et al.* (2004), Xu *et al.* (2004), Zhang *et al.* (2008) and Antolín *et al.* (2013) are among many to extend the train vehicles to three dimensions (as seen in Figure 5). This model is important for the full study of passenger comfort. Some of the common assumptions of the three dimensional models are expressed as:

1. Car bodies and bogies have five DOFs each, with the movement in the longitudinal direction restrained by the velocity of the train which runs at a constant speed.
2. Each vehicle consists of a rigid carriage body, two rigid bogies and generally four rigid wheelsets, each component joined by viscoelastic elements (with linear properties).
3. The connection between a bogie and its wheelsets (the primary suspension system) consists of both vertical, longitudinal and lateral springs and dampers, which are located on both sides of the bogie.
4. The secondary suspension system acts between the car body and the bogies.

In general the carriages are modelled independently, except for certain articulated trains (Zhang *et al.*, 2008). It is common that the mass and the inertial moments of the bodies are lumped at their respective centre of masses (Kwark *et al.*, 2004; Majka and Hartnett, 2008; Dinh *et al.*, 2009; Ju, 2012). Due to the constant velocity modelling, the horizontal acceleration and braking forces are usually studied in combination with vertical loads in a static analysis (BS EN 1991-2:2003, 2010).

According to Zhang *et al.* (2008) the main difference between models in literature is the number of degrees of freedom (DOF) allocated to each of the wheelsets. Xu *et al.* (2004) and Yang *et al.* (2004) allocate three DOF per wheelset, restraining in the yaw, pitching and longitudinal directions, resulting in 27 DOF per vehicle (carriage: 1×5 , bogies: 2×5 , wheelsets: 4×4). Alternatively, Dinh *et al.* (2009) and Antolín *et al.* (2010, 2013) allocate 4 DOF per

Figure 5: An example of a three dimensional model, showing wheelsets (W), bogies (B), the carriage, the suspension links between them and the coordinate system (x, y, z and θ for rotations)

wheelset, with pitching and longitudinal restrained, resulting in a vehicle model with 31 DOF in total (carriage: 1×5 , bogies: 2×5 , wheelsets: 4×4).

Most two and three dimensional vehicle models seen in literature use a conventional train layout (Lou, 2007; Majka and Hartnett, 2008; Zhang *et al.*, 2008; Wen *et al.*, 2009; Antolín *et al.*, 2010). However, some assumptions previously made are not acceptable if an articulated train is used (Xia *et al.*, 2003; Kwarck *et al.*, 2004; Lee and Kim, 2010). In this case, additional visco-elastic elements are required in the lateral and longitudinal directions between carriages.

This model is increasingly used for bridge dynamics design, but the complex vehicle-structure interaction often limits the ability to use with shell element bridge models. Most often, these models are used in combination with simple beam models for bridges (Antolín *et al.*, 2010, 2013; Kwarck *et al.*, 2004; Lee and Kim, 2010). In other models, a more computationally conservative approach is taken (Lee and Kim, 2010), using a hybrid model of the full moving vehicle for the front and rear powered carriages and moving forces for the rest of the carriages, resulting in underestimates of the response compared to the full vehicle of up to 3 % (for speeds less than 150 km/h, with higher speeds showing convergence of results).

3.6 Eurocode Dynamic Load Models

In order to satisfy the dynamic loading of all possible train variations, load models were developed to take into account current and future train vehicles possible for operation within Europe. Static load models with dynamic load factors are used for design (BS EN 1991-2:2003, 2010), however under certain conditions (such as the bridge natural frequency exceeding a variable limit, vehicle velocities above 200 km/h, or specific structural forms, for example those with significant skew effects), dynamic analyses are also required. This uses one of two High Speed Load Models (HSLM) that have been developed (HSLM-A and HSLM-B), with the specific model used determined by bridge characteristics (BS EN 1991-2:2003, 2010). Most commonly the HSLM-A model is used, which comprises of ten different universal train load arrangements. Analysing with all ten provides a range of dynamic excitations, with an envelope of loading frequencies and corresponding amplitudes studied, covering the possible loading signature of real trains, as seen in Figure 6 (Dominguez, 2001; Goicolea *et al.*, 2006).

Figure 6: Comparison of the dynamic signatures of high speed trains, with the dynamic envelope of the HSLM-A models

4 Track Model

The track is an important subsystem of the global vehicle track bridge interaction. By track we refer to the components between the vehicle and the bridge. This includes the rail, rail pads, fasteners, sleepers and either ballast (ballasted track) or concrete slab (slab track, also known as non-ballasted track). Ballasted track is currently more widely implemented when modelling bridges (Dinh *et al.*, 2009). The track life cycle is maximum 50 years. Therefore the usual bridge design life of 150 years would determine that the track-bridge system must be designed to allow track replacement.

4.1 Ballasted Track vs Slab-Track

Traditionally railway track has included a ballasted bed, for the placement of the sleepers and rails above them. Some of the benefits of ballasted track over slab track are lower initial costs, a more flexible structural system, good noise absorption, speed of construction, improved water drainage and wider load distribution. On the contrary, some of the advantages that slab track has over ballasted track are smaller long term track movements, higher resistance to lateral movements, a lower train running height, a lower lifetime cost due to reduced maintenance costs (particularly important on HSR bridges), smaller weight, and a higher capacity to resist axle loads (Fédération internationale du béton. Task Group 6.5., 2006; Bastin, 2006; Bezin *et al.*, 2010). In addition, the lack of ballast removes the long-term mass gain from material in the voids, and removes the potential for dislodged ballast material to cause damage to the wheels and rails.

Many countries with HSR, use a combination of slab-track and ballasted track, with different segments of the lines utilising different track structures (Fagan, 2016). France, Italy, Turkey and the UK are amongst countries to predominantly use ballasted track. On the other hand, Germany, China, South Korea and Japan are a few countries where usage of slab track dominates. The recent trend is towards using slab track (Fagan, 2016). However, in the past there has been a tendency towards using ballasted track for its advantages outlined above and also due to uncertainties over the long term performance of slab track especially with regards to cracking, vibration and drainage issues.

Figure 7: Side view of a full track model

4.2 Modelling the Track

Modelling of the track is found to be important to the vehicular dynamic response, with Yang *et al.* (2004), finding it sensitive to ballast stiffness changes, particularly at lower vehicle speeds. The modelling of the track structure is less commonly performed for the study of the bridge dynamics.

Some authors ignore all of the track components, and assume that the track and the bridge deck deflect and accelerate in the same manner (Xia *et al.*, 2003; Antolín *et al.*, 2013). This method can be used for both slab track and ballasted track, using the least amount of elements, so it is the least computationally expensive. However, the distribution of vehicular load and the elastic effects of the track system are ignored.

When considering the longitudinal stresses induced into the rail and substructure by horizontal actions or temperature, the rail may be modelled and connected to the bridge by elastic linear and non linear longitudinal springs (Ramondenc *et al.*, 2008; BS EN 1991-2:2003, 2010). This is commonly used for the track design using static loading. For vertical loading, a series of vertical springs can support the track modelled by a beam (Winkler foundation model). These models require additional parameters to represent the longitudinal stiffness (Zhaohua and Cook, 1983; Song *et al.*, 2003).

A four layer track model is required to represent the rail, rail pad, sleeper and ballast (Wu and Thompson, 2002; Kargarnovin *et al.*, 2005). A linear approach is accurate enough to describe track impacts on the vehicle comfort levels (Kargarnovin *et al.*, 2005). The sleepers are modelled as a lumped mass in these models. Longitudinal and lateral springs between the ballast can also be implemented (Figure 7)(Wen *et al.*, 2009). Variants of this model are more commonly used for the determination of passenger comfort, rather than the bridge design.

In addition, track models may include flexible sleeper models (Auersch, 2005), or modelling of the ballast in a continuous or half space manner (Vale and Calçada, 2014). However, these complex models have yet to be used for dynamic bridge modelling, instead only for complex track structure analysis.

Slab tracks are less commonly found to be modelled in literature. De Man (2002) and Proença *et al.* (2011) model the track system in two dimensions, incorporating the continuous rail beam, elastic railpads, a series of smaller beams for the slab, an elastic membrane layer and then the underlying substructure.

5 Wheel-Rail Interaction

The contact between the wheel and the rail, allows the coupling of the bridge and vehicle models (Yang *et al.*, 2004). This is important for the study of passenger comfort, although bridge dynamics, particularly in the lateral direction, are also impacted by this interaction. However, the inclusion of complex interaction models normally comes at the expense of using simpler bridge models. The simplest interaction sets the wheels of the vehicle on a perfectly guided predefined path (Song *et al.*, 2003; Xia *et al.*, 2003). Alternatively, a linear model (Xu *et al.*, 2004; Yang *et al.*, 2004), can allow a relative velocity and displacement between the wheel and rail, linearising the shape of the wheel and rail profiles. A more complex approach considers the actual non-linear wheel and rail profiles (Nguyen *et al.*, 2009; Antolín *et al.*, 2010; Antolin *et al.*, 2012), allowing contact between the wheel flange and rail,

keeping the wheelset within the rails.

In order to solve the wheel-rail interaction problem, it is necessary to solve in order: the contact position, the contact area, the normal force (perpendicular to the contact) and the tangential forces (within the contact surface) (Antolín *et al.*, 2012).

5.1 Contact Position

The point of contact may be found by the location at which the two bodies come into contact, or by the largest penetration. It is possible for multiple points of contact to occur simultaneously as found in Piotrowski and Chollet (2005). Methods used to find the point of contact can vary from searching at discrete points and interpolating between points, to using equations to solve the contact (Shabana *et al.*, 2005).

5.2 Contact Area

There are two different approaches for solving the contact area, depending on the assumed shape of the contact area which is either elliptical or non-elliptical (Antolín, 2013). Due to the changing curvatures of the profile of the wheel and rail, non-elliptical contact areas are found. However, the area is often assumed to be an ellipse or series of ellipses (Piotrowski and Chollet, 2005). Formulations to calculate the shape and size of the elliptical contact patch have been given in multiple papers (Antolín *et al.*, 2012; Shabana *et al.*, 2004). This depends on the normal load, Poisson's ratio and Young's modulus, and uses Hertz's Contact theory (as mentioned in Antolín *et al.* (2012)).

The alternative is to assume that the contact area is non-elliptical. This is less commonly used in literature. To model this, the area may be approximated as multiple elliptical contact regions (Pascal and Sauvage, 1993; Piotrowski and Chollet, 2005), or be split into strips (Ayasse and Chollet, 2005). Moreover, multiple points of contact may occur as the rail and wheel do not have constant curvature.

5.3 Normal Force

After finding the contact between the wheel and rail, the next step to solving the wheel-rail interaction (WRI), is to define the normal contact. The normal force is split into methods that consider an elastic contact, normally using elliptical Hertz contact and those that use a simpler rigid contact.

5.3.1 Elastic Normal Contact

The use of elastic theory for the determination of the contact requires an iteration process to solve the equations (Xia *et al.*, 2012). The majority of elastic solutions use the theory of Hertz (1882), of which the assumptions for its validity are expressed by Antolín *et al.* (2012).

The contact force between the two bodies is determined from the apparent penetration between these two bodies. Although this penetration does not really exist, it is a representation of the deformation of the bodies. The magnitude of the normal force (F_{WR}) is normally defined through a non-linear Hertzian contact equation (Shabana *et al.*, 2004; Antolín *et al.*, 2012; Xia *et al.*, 2012) expressed as:

$$F_{WR} = K\delta^{3/2} \quad (1)$$

where K the Hertzian contact stiffness and δ the representative penetration between the two bodies. Alternatively, the expression can be linearised (Antolín *et al.*, 2012), instead relying on a stiffness, K_s :

$$F_{WR} = K_s\delta \quad (2)$$

In some cases, it is mentioned that the non-linear Hertzian model can allow for separation between the rail and the wheel. When this occurs the normal contact force becomes zero (Sun and Dhanasekar, 2002; Kargarnovin *et al.*, 2005; Dinh *et al.*, 2009). The definition of the parameter K , for the Hertzian contact stiffness, is not uniformly defined across the different literature (Antolín *et al.*, 2012). Kargarnovin *et al.* (2005) predefines the value, whereas Sun and Dhanasekar (2002) defines the relationship as in Equation 3, assuming identical wheel and rail materials.

$$K = \frac{2E}{3(1-\nu^2)}(a_r a_w)^{1/4} \quad (3)$$

where: E is the Young's modulus of the wheel and the rail; ν the Poisson's ratio; a_r and a_w are values defining the wheel radius and the rail head radius.

Table 4: Selection of Wheel-Rail Contact models being used in literature for bridges

Source	Contact type	
	Normal	Tangential
Li <i>et al.</i> (2005)	Rigid	
Vale and Calçada (2014)	Linear Hertz	n/a (2D)
Kargarnovin <i>et al.</i> (2005)	Non-linear Hertz	n/a (2D)
Dinh <i>et al.</i> (2009)	Non-linear Hertz	Linear theory
Antolin <i>et al.</i> (2012)	Non-linear Hertz	FASTSIM
Goicolea and Antolín (2011)	Non-linear Hertz	USETAB

Torstensson and Nielsen (2011) propose the addition of contact damping by coupling the spring with a non-linear damper as well. Although using a different formulation, according to Shabana *et al.* (2004, 2005) the damping force is proportional to the rate of penetration of the two bodies.

An alternative method proposed to calculate the normal force is the Adaptive Wheel Rail Contact Model. It is argued that the non-linear Hertzian normal force model overestimates the forces during impact loadings and underestimates them at low speeds (Zhu *et al.*, 2007, 2009).

5.3.2 Rigid Normal Contact

The alternative method to the elastic method, assumes rigid contact. The rigid contact (or constraint) approach, is more simplistic and it does not allow for the derailment (Xia *et al.*, 2012; Shabana *et al.*, 2004). It takes as input the displacements of the bridge and the rail, including the displacements due to irregularities and the hunting motion. From this equation based approach to the positioning of the wheels, the forces can be determined by a set of lagrangian multipliers (Zaazaa *et al.*, 2009). The formulation is widely used for bridges in literature due to its simplicity, as shown by Table 4.

According to Xia *et al.* (2012), this method has been widely verified by many in literature. Due to the inability to naturally develop hunting motion with this method, hunting motion is a prescribed displacement (Antolín *et al.*, 2012). The number of DOF in the constraint approach is reduced as the wheels only have 5 DOF with respect to the rail, whereas in the elastic approach there is an additional DOF for the wheel penetration (Shabana *et al.*, 2004). A similar rigid approach has also been employed by Song *et al.* (2003), with the force acting on the bridge based on the forces acting on the wheels, taking into account the sprung and unsprung masses.

5.4 Tangential Force

The tangential force (sometimes known as the creep force), is essentially the friction force acting in the plane of contact between the wheel and rail. Its consideration is important for modelling the lateral hunting motion, derailment issues and the ride quality (Shen *et al.*, 1983). The tangential force cannot exceed the value given by Coulomb’s law, which is the normal force multiplied by the friction coefficient (Antolín *et al.*, 2012, 2010).

The relative difference in velocity between the rotation of the wheel at contact and the forward motion of the wheelset are nondimensionalised and known as creepages. The creepages are used in the calculation of the tangential forces and are calculated in the lateral, longitudinal and the rotational planes between the wheel and rail (Antolín *et al.*, 2012). Different methods to calculate this tangential force are detailed herein.

5.4.1 Kalker Linear theory

The most simple tangential contact is the linear theory, of which Kalker Linear theory is the most prominently used (as described in Antolín *et al.* (2012) who references the original text of Kalker (1967)). This method has very small computation times, but it is unrealistic for anything but small creepages, as it is for these larger creepages that the linear theory violates Coulombs law.

5.4.2 Heuristic approach

Some manipulation can be made of the linear theory, correcting the forces to satisfy the Coulombs law (Shen *et al.*, 1983; Shabana *et al.*, 2007). This theory is used by Zhang *et al.* (2008) and Wen *et al.* (2009), but according to

(Polach, 2005) is less accurate than Polach’s approach despite similar times for computation. However, these methods are not widely used in literature.

5.4.3 Polach Method

An advanced method is the Polach method (Polach, 1999). It has advantages over Kalker’s Simplified theory through shorter computational time (up to 17 times shorter (Polach, 1999)), making it more suitable for complicated multi-body simulations (Six *et al.*, 2015). It differs from the Linear Kalker method by including slip, such that the maximum force is limited to Coulomb’s theory. Antolín (2013) found that the Polach method is not accurate for high values of rotational (spin) creepage.

5.4.4 Kalker Simplified Theory

A more complicated theory is Kalker’s simplified theory. The accuracy of the simplified method is much closer to that of the exact theory compared to the linear theory. Implementation of this theory takes place via an algorithm as proposed by Kalker (1982), named a fast algorithm for the simplified theory of rolling contact (FASTSIM) (Garg and Dukkipati, 1984; Antolín *et al.*, 2010; Torstensson and Nielsen, 2011). This method is one of the most widely used algorithms due to its high speed of computation and accuracy. It requires multiple calculations per contact as the contacting area is discretized into strips and cells for calculation. It is found to be a factor of 100 times less computationally expensive than the exact theory of rolling contact (Garg and Dukkipati, 1984), but Kalker (1982) finds FASTSIM to only be 15-25 times faster. Errors are found to be less than 15% compared to Exact Rolling contact theory (Antolín *et al.*, 2012). Some of the underlying assumptions (Antolín *et al.*, 2010), behind FASTSIM are as follows:

- the normal contact is independent of the tangential contact
- the use of Hertz contact theory is applied correctly
- the coefficient of friction remains constant throughout the contact patch and is independent of the local slip
- the contact occurs over a single area between the wheel and rail

5.4.5 Kalker’s Exact Theory

The most exact approach that has been created is Kalker’s exact three-dimensional rolling contact theory (also known as variational theory). Through its program (CONTACT (Vollebregt, 2016)), it is often used as the benchmark of comparisons for accuracy with other models (Kalker, 1990; Antolín, 2013). In this theory the contact problem is solved with a theory of virtual work, and as a result the method is computationally very expensive. Hence, it is not used for solving dynamic problems. The increased accuracy comes from a lack of limiting assumptions of the spin creepage and contact areas (Garg and Dukkipati, 1984).

5.4.6 USETAB

As the use of Kalker’s exact theory is demanding computationally, this method uses precalculated lookup tables based on the creepages and elliptical semi-axes and normal forces, such that for a given ellipse and creepage the values for the forces can be found. However, interpolation is often required to find the forces for the exact set of input variables (Antolín, 2013). For small tables this reduces the potential accuracy, but in larger tables, despite greater accuracy, the greater quantity of values increases computational demand, so a trade-off is required (Polach, 1999; Shabana *et al.*, 2004; Antolín *et al.*, 2013).

5.5 Wheel Hunting Behaviour

Wheel hunting movement (also known as nosing motion), is a sinusoidal lateral motion of the vehicle as it moves along the rails. It can be triggered by irregularities decentralising the wheelset. This causes differential wheel running radii, leading to oscillation about the centre of the rails, with yawing and lateral motions of the wheelset, to reach equilibrium. At critical instability limit speeds (Antolín *et al.*, 2012), this motion may become uncontrollable and resonate, causing a derailment risk and large lateral impact forces (Zaazaa *et al.*, 2009), but this is more of a train manufacturing problem, rather than a bridge design issue. On straight track this hunting motion is one of the major factors in causing lateral movements of a bridge (Antolín *et al.*, 2010). Overall, the hunting motion can

cause lateral resonant problems in the bridge if the vehicle hunting and lateral bridge frequencies align (Zhang *et al.*, 2012). This motion can be developed naturally in elastic non-linear contact models, but in linear models and rigid contact models, forces or motions need to be imposed (Goicolea and Antolín, 2011).

6 Irregularities

Irregularities exist in the track system and wheels, which are imperfections from the design occurring through processes of manufacturing (short wavelength irregularities), construction (long wavelength) or long term wear (intermediate wavelength) (Yang *et al.*, 2004; Hamid *et al.*, 1983). Irregularities may arise as isolated irregularities or more continuous track variations, including random track irregularities and corrugations. Irregularities are found to be essential in determining both the bridge and vehicular accelerations (Pring, 2019; Cantero *et al.*, 2016). However, existing literature finds contrasting conclusions, with irregularities found to have an impact on the vertical bridge accelerations in Cantero *et al.* (2016), but Yang *et al.* (2004) finds the vertical effect negligible, with the lateral bridge response significantly affected. Poorer quality track is found to worsen only the vehicle and lateral bridge responses (Au *et al.*, 2002; Yang *et al.*, 2004). Lateral irregularities are found to be the predominant cause of the dynamic lateral bridge response (Majka and Hartnett, 2009), but the lateral displacement is unaffected in Xia *et al.* (2000), with the opposite found by Yang *et al.* (2004). A comprehensive investigation of this area has been done by the first author (Pring, 2019), and supervised by the second.

6.1 Definition of Irregularities

There are four types of track geometry irregularity (Garg and Dukkipati, 1984; Li *et al.*, 2005; Xia *et al.*, 2012), related to:

The vertical profile: It is the deviation of the average height of the rail from the intended height

The horizontal alignment: It is the deviation of the centre of the two rails from the intended centre

The cross level: It is the unintended differences in elevation between the two rails

The gauge: It is the deviation from the intended gauge of the horizontal distance between the two rails measured on a plane 1.5 cm below the railhead top

These irregularities are shown in Figure 8 (Garg and Dukkipati, 1984). The gauge irregularity is commonly ignored, due to negligible impacts on analysis (Song *et al.*, 2003; Xu *et al.*, 2004; Yang *et al.*, 2004; Li *et al.*, 2005). The cross level irregularity differs from the track cant, as the cant is an intended difference between the vertical heights of the two tracks for steering around curves.

6.2 Random Track Geometry Variations

The definition of random track geometry irregularities is commonly made either by real data (Xia *et al.*, 2000; Xu *et al.*, 2004; Xia and Zhang, 2005; Zhang *et al.*, 2008), or a random function, such as a power spectral density function (Song *et al.*, 2003; Li *et al.*, 2005; Dinh *et al.*, 2009; Au *et al.*, 2002). Real data is limited, site specific and dependent on the track operating speeds.

Power Spectral Density (PSD) functions, give rise to a series of amplitudes for different wavelengths. For irregularities, these amplitudes and wavelengths can be used in sinusoidal functions, superimposing different wavelengths to generate random profiles. Each PSD is calculated for a range of wavelengths, based on the highest and lowest frequencies considered to contribute to the irregularities of the track (Yang *et al.*, 2004).

The irregularities produced by the PSD functions depend on the characteristics of the track of that particular country, which are often proposed by organisations from within that country such as the Société nationale des chemins de fer français (SNCF) in France or the Federal Railroad Administration (FRA) in the USA. As highlighted within these models, the speed of the line is an important consideration, as tracks of higher operating speeds will tend to be better quality. Five of the common PSD functions are known as the FRA, German, Chinese, SNCF and Braun functions (Berawi, 2013). The SNCF and Braun functions only have capabilities for defining the vertical irregularities, whereas the FRA, German and Chinese functions can account for all four irregularities.

Figure 8: Definitions of the track irregularities

6.2.1 Federal Railroad Administration PSD function

This function is the most commonly used in literature, utilising the class of tracks proposed by the Federal Railroad Administration (FRA) (Zhang *et al.*, 2001; Song *et al.*, 2003; Majka and Hartnett, 2009; Dinh *et al.*, 2009). The FRA define the different track qualities in classes ranging from 1-9 (class 1 is poorest quality, class 9 the best). PSD functions have been defined up to class 6, a quality corresponding to a non-HSR line speed of 177 km/h (Berawi, 2013). Equations have yet to be proposed for classes 7-9, due to the age of this PSD definition predating the widespread need for these HSR track qualities. Hence, despite use in literature, even the highest quality PSD defined, is unlikely to be suitable for HSR.

6.2.2 German Rail PSD function

The second most common PSD function is the German one (Li *et al.*, 2005; Berawi, 2013). There are two types: the German Rail Spectrum of Low Irregularity (GRSLI) and German Rail Spectrum of High Irregularity (GRSHI). These can simulate the irregularities of modelling a bridge that runs HSR vehicles with design velocities up to 350 km/h. Some variation of the model exists with different coefficients given for the generation of the PSD (Claus and Schiehlen, 1998; Cuadrado *et al.*, 2008). The Claus and Schiehlen model is found to match well with measured track data from a Deutsche Bahn Train travelling at 250 km/h, although Liu *et al.* (2009) questions the suitability instead fitting the variables to match a specific track profile.

6.2.3 Track irregularity generation from the PSD functions

Conversion of the PSD function into the track irregularities can be performed with the Monte-Carlo method (Podworna, 2014), which is essentially an inverse Fourier transform (Zhang *et al.*, 2001). The expression used can be seen in Equation 4.

$$r(x) = \sum_{n=1}^N \sqrt{4S(\omega_n)\Delta\omega} \cos(\omega_n x - \phi_n) \quad (4)$$

$$\Delta\omega = (\omega_{max} - \omega_{min})/N \quad (5)$$

$$\omega_n = \omega_{min} + \delta\omega(n-1) \quad (6)$$

where $r(x)$ is the longitudinal spatial variation of the irregularity, ω_n is a circular frequency found in the frequency range of the PSD being converted, $S(\omega_n)$ is the PSD value for the particular frequency, x the longitudinal coordinate, N is the number of frequency points between the upper and lower limits of the frequency range (ω_{max} and ω_{min} respectively). The random phase angle (ϕ_n) is a uniformly distributed between 0 and 2π , with each irregularity type having independent angles (Zhang *et al.*, 2001).

Alternatively, the irregularities can be obtained from the PSD by the spectral representation method (Claus and Schiehlen, 1998; Yang *et al.*, 2004; Antolín *et al.*, 2012). This alternative method gives different results to the other model. It is shown in equation 7.

$$r(x) = \sqrt{2} \sum_{n=0}^{N-1} A_n \cos(\Omega_n x + \phi_n) \quad (7)$$

where $\Omega_n = n \frac{\Omega_{max}}{N}$, for $n = 0, 1, 2, \dots, N-1$, Ω_{max} is the upper circular frequency considered for the PSD spectrum and:

$$A_n = \begin{cases} 0 & n = 0 \\ \sqrt{\frac{1}{\pi}(S(\Delta\Omega) + \frac{4}{6}S(0))\Delta\Omega} & n = 1 \\ \sqrt{\frac{1}{\pi}(S(2\Delta\Omega) + \frac{1}{6}S(0))\Delta\Omega} & n = 2 \\ A_n = \sqrt{\frac{1}{\pi}S(\Omega_n)} & n > 2 \end{cases} \quad (8)$$

According to Yang *et al.* (2004), the final results of the generation of the irregularity profile may not be satisfactory, due to the superposition of random components. Hence, they may require some form of normalisation to make sure they are representative.

6.3 Periodic Irregularities

Due to the repeating axle distance of the vehicle, some irregularities, although isolated, create a periodic loading. In addition, these periodic irregularities may be found at fixed regular intervals, for example wheel flats and rail welds.

6.3.1 Isolated Irregularities

Whilst the typical track variations can be described by PSD functions, this random process is not able to pick up the local irregularities, such as Cusp, Bump, Jog, Plateau, Trough, Sinusoid and Damped Sinusoid profiles (Hamid *et al.*, 1983; Garg and Dukkipati, 1984). Similarly sleepers with voids beneath can displace under load (Vale and Calçada, 2014), causing large impact forces.

Due to the restriction of rail joints and other rail features that can cause periodic irregularities on a bridge, they are rarely modelled in proximity of a bridge. However, in long viaducts expansion rail joints may be necessary, which then have to be specially designed for safety and maintenance issues (Hsieh and Wu, 2014).

6.3.2 Corrugations

Rail corrugations is another periodic type of irregularity, caused by track degradation after initial imperfections. Corrugations take the form of particularly prominent sinusoidal variation (with one dominant period, as opposed to superposition of multiple periods, as found in random irregularity generation). They can lead to damage of the track and the vehicle as well as to cause vibration which translates to noise (Torstensson and Nielsen, 2011). Lou (2005) found that this single repeating irregularity could cause significant effect on the vertical accelerations of the bridge but previous studies on irregularities, suggest that changes to the vertical bridge dynamic effects are negligible (Yang *et al.*, 2004).

6.3.3 Wheel Flats

Wheelsets can also contain defects that could cause a dynamic response to the bridge and train, with its regular occurrence due to the rotation of the wheel causing a harmonic effect. These defects, commonly called wheel flats, can be created during braking, which causes the locking and subsequent sliding of the wheels. Both Sun and Dhanasekar (2002) and Wu and Thompson (2002) use a cosine function to model the wheel flat and convert it to an equivalent irregularity on the track, instead of modelling the flat on the wheel.

6.4 Track Variation Limits

Owing to maintenance, the irregularities on a track will be smoothed and reduced when they progress beyond certain threshold limits, expressed in regional codes, such as **BS EN 13848-5:2008 +A1:2010** (2010). There are three limit types: the alert limit (maintenance is planned as part of the regular maintenance schedule if exceeded), intervention limit (corrective maintenance is required before the next inspection) and immediate action limit (immediate remediation if exceeded due to safety reasons). The limits are made both from the peak to mean values of the irregularities, and the standard deviation defined over a set length. Values are given for gauge, alignment and longitudinal profiles, with the cross level not given directly, but linked to the rate of twist and cant limits.

7 Specific Design Constraints

7.1 Load Combinations for Persistent Situations

For design to eurocode, the vehicles are modelled by moving point loads. Different combinations of load models are expressed in Table 6.11 of **BS EN 1991-2:2003** (2010), taking into account the direction of the loading and number of tracks to be loaded. The combinations apply to static models, with dynamic models only requiring the loading of one track at a time per bridge. The positioning of the tracks should be considered to be in the least favourable possible location on the bridge.

7.2 Derailment

Determining whether a vehicle may derail is possible when using a non-linear vehicle bridge numerical interaction model (Antolín *et al.*, 2013). Yang *et al.* (2004) states five limits that determine the derailment and the safety assessment for trains. They are found as the axle load decrement ratio (ratio of vertical static to dynamic contact force), the lateral track force (the static vertical force determines the upper bound to the lateral force), the single wheel lateral to vertical force ratio, the wheelset lateral to vertical force ratio and the bogie-side lateral to vertical force ratio. Many of these limits are used by other authors (Dinh *et al.*, 2009; Ju, 2012; Six *et al.*, 2015; Zhang *et al.*, 2008). With the exception of the load decrement ratio, these ratios are found to be dependent on the track quality and train velocity.

When not considering a full vehicle model, the bridge accelerations are the only indicator for the derailment safety, as explained in Section 7.3.1. In the unfortunate case when derailment occurs, the bridges are designed such that the extraordinary loads during derailment are accounted for, as per **BS EN 1991-2:2003** (2010).

7.3 Serviceability Limit State

The bridge accelerations are an important factor in determining the running safety of the vehicles. However, the suspension in the vehicle reduces the accelerations experienced by the vehicle. The riding comfort is directly related to the vehicle accelerations. Much of the research in literature focuses on the riding comfort and running safety of the bridge under various loading conditions.

The static bridge design uses the results of the dynamic analyses through a dynamic enhancement factor (also known as a dynamic amplification factor). This is used to consider the impact of the dynamic actions and the resonant behaviour on the static stresses and displacements (**BS EN 1991-2:2003**, 2010).

7.3.1 Bridge Accelerations

Accelerations of HSR bridges are often the critical design factor (Calgaro *et al.*, 2010). **BS EN 1991-2:2003** (2010) specifies that the bridge deck must not have accelerations above a certain threshold. These values for the vertical deck acceleration are $0.35g$ (or 3.5 m/s^2) for ballasted track, and $0.5g$ (or 5 m/s^2) for non-ballasted track (**BS EN 1991-2:2003**, 2010). Ballasted track has a lower limit to ensure the ballast stability, as destabilisation can cause long term track and vehicle damage, due to ballast thrown or settling. Both accelerations limits include a factor of safety (FOS) of 2 (Salcher *et al.*, 2014). The slab track limit of $0.5g$ ($1.0g$ without FOS) is related to the acceleration causing derailment. These conditions ensure that the HSR line continues to function effectively. In addition, codes from China limit the lateral HSR bridge acceleration to 1.4 m/s^2 (Xia and Zhang, 2005; Zhang *et al.*, 2008), a limit not found in **BS EN 1991-2:2003** (2010).

According to BS EN 1991-2:2003 (2010), the limits to acceleration should take into account a low pass filter, removing high frequency acceleration components. This filter is as follows:

$$f_f = \max(30, 1.5f_1, f_3) \quad [\text{Hz}] \quad (9)$$

where f_f is the cutoff frequency, f_1 is the frequency of the fundamental bending mode and f_3 is the frequency of the third bending mode.

7.3.2 Vehicle Accelerations

The vehicle accelerations (and hence riding comfort), are often calculated from within the carriage. Very good comfort is considered as vertical accelerations below 1m/s^2 , with accelerations below 2m/s^2 acceptable (BS EN 1992-2:2005, 2005). Despite the lateral accelerations being sensitive to the presence of irregularities (Majka and Hartnett, 2009), they are not limited in BS EN 1992-2:2005 (2005). According to Zhang *et al.* (2008), the Chinese codes limit the vertical carriage acceleration to 1m/s^2 , and laterally to 1.3m/s^2 . Only numerical models implementing the vehicles can provide the values of accelerations in the vehicles. Otherwise, these values need to be inferred from those in the bridge.

7.3.3 Deformations

Excessive bridge deflections can cause changes to the track profile increasing rail stress and potentially causing vehicle running problems. The maximum vertical deflection of the bridge varies between $L/600$ (BS EN 1990:2002 +A1:2005, 2010), and $L/1800$ (Dinh *et al.*, 2009), where L is the span of the bridge. BS EN 1991-2:2003 (2010) also limits the relative longitudinal displacement between the bridge and the abutment to 2 mm for a line speeds over 160 km/h. For HSR speeds of over 200 km/h, BS EN 1990:2002 +A1:2005 (2010) requires that the twist of the deck over a width equal to the gauge of the rails and a 3 m length should be no more than 1.5 mm.

7.4 Fatigue

The rails can undergo fatigue due to their heavy dynamic loading (Sun and Dhanasekar, 2002; Wu and Thompson, 2002). However, the bridge is more of a concern, due to both its longer design life, and the relative ease of track replacement. Steel and composite HSR bridges are particularly susceptible to fatigue damage, but it is also important for reinforced and prestressed concrete bridges. In prestressed concrete bridges, ensuring that under the service loads the section remains fully prestressed, helps to avoid fatigue problems. Fatigue damage characteristics depend on the construction material, and is determined using load model 71 and a series of fatigue trains in BS EN 1991-2:2003 (2010). The fatigue design should also consider excessive dynamic effects, which increase the fatigue damage, through using appropriate dynamic enhancement factors.

7.5 Dynamic Behaviour

Both the resonance and cancellation phenomenon relate to the behaviour of the bridge under the forced vibration stage, but can influence the behaviour in the free vibration stage. If the free vibrations from multiple loads are in phase, then resonance can occur, resulting in an amplified response. Whereas if they are out of phase by half a period, the free vibration response will cancel out, leaving a cancellation effect. The formulas proposed are based on a simply supported bridge.

7.5.1 Amplification effects

Resonant speeds are speeds at which, due to an excitation frequency matching one of the bridge's natural frequencies, causing amplification of the bridge (resonance) (Podworna, 2014). Equation 10 shows the first method used to calculate the resonant speeds ($v_{r1,in}$), where n is the bridge vibration mode under consideration ($n = 1, 2, 3, \dots$) and i stands for the particular harmonic of that mode of vibration ($i = 1, 2, 3, \dots, 1/2, 1/3, 1/4, \dots$) (Xia *et al.*, 2014; Goicolea *et al.*, 2008). In addition, d is the regular repeating distance between the loads and f_n is the n^{th} modal frequency of the bridge.

$$v_{r1,in} = \frac{f_n d}{i} \quad [\text{m/s}] \quad (10)$$

A second resonant set of speeds $v_{r2,n}$ found at higher speeds, hence often ignored, and is found in Equation 11 relating the span of the bridge L to the natural frequencies of the bridge (Frýba, 2001).

$$v_{r2,n} = \frac{2Lf_n}{n} \quad [m/s] \quad (11)$$

Resonance may also occur in the lateral direction due to the action of hunting motion and irregularities. This utilises a formula very similar to Equation 10, but instead of the train length it uses the dominant wavelength of the irregularities or hunting motion with the lateral frequencies of the bridge (Xia *et al.*, 2006). Cuadrado *et al.* (2008) suggests that the lateral response of the bridge does not become resonant under normal conditions, as generally the lateral frequencies of vehicles were found to be less than 1 Hz, which is smaller than the minimum frequency of the first lateral mode which is required to be at least 1.2 Hz (BS EN 1991-2:2003, 2010).

7.5.2 Cancellation Effects

Under cancellation phenomenon (also known as the resonance disappearance effect (Xia *et al.*, 2014)), the residual response of the wheel loads on the bridge is minimal. Where a vehicle matches or is similar to a cancellation speed, the dynamic response is significantly reduced (Lou, 2007). There are multiple forms of cancellation. The commonly quoted cancellation speed is defined in Equation 12 (closely related to Equation 11), with the frequency of the mode f_n and length of bridge, L , contributing to the cancellation velocity of $v_{c1,in}$ [m/s], for different harmonics of $i = 1, 2, 3, \dots$ (Xia *et al.*, 2014).

$$v_{c1,in} = \begin{cases} \frac{2Lf_n}{2i-1} & \text{for } n = 1, 3, 5 \text{ but } n \neq 2i - 1 \\ \frac{2Lf_n}{2i} & \text{for } n = 2, 4, 6; \text{ but } n \neq 2i \end{cases} \quad [m/s] \quad (12)$$

The second cancellation velocity in Equation 13 (closely related to Equation 10) (Xia *et al.*, 2014), is based on any regular repeating train distance d_c , such as the distance between wheels, bogies or vehicles.

$$v_{c2,in} = \frac{2d_cf_n}{2i-1} \quad \text{for } i = 1, 2, 3... \quad [m/s] \quad (13)$$

7.6 Simultaneous Actions

HSR bridges may be subject to simultaneous actions that affect the traffic action such as earthquakes, wind, waves and ship or vehicle impact. Under such situations the running safety of the vehicles are the primary concern, due to imposed deformations on the rail or due to extreme accelerations of the vehicle or bridge. Strong cross winds can have particular affect on the vehicle, leading to derailment concerns (Li *et al.*, 2005). Long span bridges which inherently have greater flexibility require more extensive structural wind design than short and medium spans.

Some study has been made to vehicle safety under earthquakes in Ju (2012), using non-linear vehicle bridge interaction models. The best method is to bring the vehicle to a stop on first detection of the earthquake. Conventionally, structures are designed to be ductile and flexible in earthquake regions, however this contrasts the rigid structures needed to conform with the strict displacement requirements of HSR (Cao *et al.*, 2019). Design for earthquakes and other external loads needs particular care for differential deformations between spans, as this could lead to derailment issues.

Other actions such as long term deformations of the bridge will potentially impact the running safety of the vehicles. These deformations need to be compensated for within track maintenance, else it can lead to larger vehicle accelerations, and consequently impact the dynamic traffic load the vehicle transfers into the bridge.

8 Conclusions

This paper outlines the most current literature regarding the modelling of high speed rail bridges. It shows feasible methods to model the dynamic loading of the bridge, including full vehicles with wheel-rail interaction, irregularities and track structure models. This enables the full determination of the passenger comfort and bridge accelerations.

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References

- Akin JE and Mofid M (1989) Numerical Solution for Response of Beams with Moving Mass. *Journal of Structural Engineering* **115**(1): 120–131, [10.1061/\(ASCE\)0733-9445\(1989\)115:1\(120\)](https://doi.org/10.1061/(ASCE)0733-9445(1989)115:1(120)).
- Antolín P (2013) *Efectos Dinámicos Laterales en Vehículos y Puentes Ferroviarios sometidos a la Acción de Vientos Transversales*. PhD thesis, Universidad Politécnica de Madrid.
- Antolin P, Goicolea J, Oliva J and Astiz M (2012) Nonlinear Train-Bridge Lateral Interaction Using a Simplified Wheel-Rail Contact Method Within a Finite Element Framework. *Journal of Computational and Nonlinear Dynamics* **7**(4): 41014, [10.1115/1.4006736](https://doi.org/10.1115/1.4006736).
- Antolín P, Goicolea JM and Astiz MÁ (2012) Strategies for Modelling Train-Bridge Lateral Dynamic Interaction. In *Bridge Vibration and Controls* (Xia H, De Roeck G and Goicolea JM, eds), chap. 6, Nova Science Publishers, pp. 195–235.
- Antolín P, Goicolea JM, Astiz MA and Alonso A (2010) A methodology for analysing lateral coupled behavior of high speed railway vehicles and structures. *IOP Conference Series: Materials Science and Engineering* **10**: 012001, [10.1088/1757-899X/10/1/012001](https://doi.org/10.1088/1757-899X/10/1/012001).
- Antolín P, Zhang N, Goicolea JM, Xia H, Astiz MÁ and Oliva J (2013) Consideration of nonlinear wheelrail contact forces for dynamic vehiclebridge interaction in high-speed railways. *Journal of Sound and Vibration* **332**(5): 1231–1251, [10.1016/j.jsv.2012.10.022](https://doi.org/10.1016/j.jsv.2012.10.022).
- Au F, Wang J and Cheung Y (2002) Impact study of cable-stayed railway bridges with random rail irregularities. *Engineering Structures* **24**(5): 529–541, [10.1016/S0141-0296\(01\)00119-5](https://doi.org/10.1016/S0141-0296(01)00119-5).
- Auersch L (2005) The excitation of ground vibration by rail traffic: theory of vehicle-track-soil interaction and measurements on high-speed lines. *Journal of Sound and Vibration* **284**(1-2): 103–132, [10.1016/j.jsv.2004.06.017](https://doi.org/10.1016/j.jsv.2004.06.017).
- Ayasse J and Chollet H (2005) Determination of the wheel rail contact patch in semi-Hertzian conditions. *Vehicle System Dynamics* **43**(3): 161–172, [10.1080/00423110412331327193](https://doi.org/10.1080/00423110412331327193).
- Bastin R (2006) Development of German non-ballasted track forms. *Proceedings of the Institute of Civil Engineers: Transport* **159**(1): 25–39.
- Bennett D (2007) Channel Tunnel Rail Link section 2: railway systems. *Proceedings of the Institution of Civil Engineers - Civil Engineering* **160**(6): 49–53, [10.1680/cien.2007.160.6.49](https://doi.org/10.1680/cien.2007.160.6.49).
- Berawi ARB (2013) *Improving Railway Track Maintenance Using Power Spectral Density (PSD)*. PhD thesis, Universidade do Porto.
- Bezin Y, Farrington D, Penny C, Temple B and Iwnicki S (2010) The dynamic response of slab track constructions and their benefit with respect to conventional ballasted track. *Vehicle System Dynamics* **48**(sup1): 175–193, [10.1080/00423111003693201](https://doi.org/10.1080/00423111003693201).
- BS EN 13848-5:2008 +A1:2010 (2010) Railway applications - Track - Track geometry quality - Part 5: Geometric quality levels - Plain line.
- BS EN 1990:2002 +A1:2005 (2010) Eurocode - Basis of structural design. [10.1680/cien.144.6.8.40609](https://doi.org/10.1680/cien.144.6.8.40609).
- BS EN 1991-2:2003 (2010) *Eurocode 1: Actions on structures - Part 2: Traffic loads on bridges*. 3 edn., BSI.
- BS EN 1992-2:2005 (2005) Eurocode 2 : Design of concrete structures. Concrete bridges. Design and detailing rules.
- Calgaro J, Tschumi M and Gulvanessian H (2010) Traffic loads on railway bridges. In *Designers' Guide to Eurocode 1: Actions on Bridges* (Gulvanessian H, ed.), chap. 6, Thomas Telford Limited, London, pp. 145–190, [10.1680/dgeab.31586.0006](https://doi.org/10.1680/dgeab.31586.0006).
- Cantero D, Arvidsson T, Obrien E and Karoumi R (2016) Train track bridge modelling and review of parameters. *Structure and Infrastructure Engineering* **12**(9): 1051–1064, [10.1080/15732479.2015.1076854](https://doi.org/10.1080/15732479.2015.1076854).

- Cao Y, Ude T, Getter D and Gill B (2019) Design of High-Speed Railway Bridges with Earthquakes in the United States. *Structural Engineering International* **29**(1): 18–25, [10.1080/10168664.2018.1515582](https://doi.org/10.1080/10168664.2018.1515582).
- Chen Y, Zhang B, Zhang N and Zheng M (2015) A Condensation Method for the Dynamic Analysis of Vertical VehicleTrack Interaction Considering Vehicle Flexibility. *Journal of Vibration and Acoustics* **137**(4): 041010, [10.1115/1.4029947](https://doi.org/10.1115/1.4029947).
- Claus H and Schiehlen W (1998) Modeling and Simulation of Railway Bogie Structural Vibrations. *Vehicle System Dynamics* **29**(sup1): 538–552, [10.1080/00423119808969585](https://doi.org/10.1080/00423119808969585).
- Cuadrado M, Gonzalez P, Goicolea JM, Nasarre J and Dias R (2008) Analysis of lateral displacements in large railway viaducts under traffic loads. Impact on ride safety and passenger comfort. *Proceedings of the 8th World Congress on Railway Research* : 1–12.
- De Man A (2002) *Dynatrack: A survey of dynamic railway track properties and their quality*. Doctoral thesis, Delft University.
- Dinh VN, Kim KD and Warnitchai P (2009) Dynamic analysis of three-dimensional bridge-high-speed train interactions using a wheel-rail contact model. *Engineering Structures* **31**(12): 3090–3106, [10.1016/j.engstruct.2009.08.015](https://doi.org/10.1016/j.engstruct.2009.08.015).
- Domingu J (2001) *Dinmica de puentes de ferrocarril para alta velocidad: mtodos de clculo y estudio de resonancia*. PhD thesis, Universidad Politcnica De Madrid.
- Durrant DW (2015) The controversial discourse on speed in the case of HS2. *Proceedings of the Institution of Civil Engineers - Urban Design and Planning* **168**(5): 241–250, [10.1680/udap.14.00050](https://doi.org/10.1680/udap.14.00050).
- Fagan N (2016) Choosing the Right Trackform for a High Speed Railway. In *Fixed Track Forms for High Speed Lines*, Permanent Way Institute, Manchester, UK, pp. 4–8.
- Fdration internationale du bton. Task Group 6.5. (2006) *Precast concrete railway track systems : state-of-art report*. International Federation for Structural Concrete.
- Frba L (2001) A rough assessment of railway bridges for high speed trains. *Engineering Structures* **23**(5): 548–556, [10.1016/S0141-0296\(00\)00057-2](https://doi.org/10.1016/S0141-0296(00)00057-2).
- Gabaldon F, Riquelme F, Goicolea JM and Arribas J (2008) Dynamic analysis of structures under high speed train loads: case studies in Spain. In *Dynamics of High-Speed Railway Bridges: Selected and revised papers from the Advanced Course on Dynamics of High-Speed Railway Bridges, Porto, Portugal, 20-23 September 2005* (Delgado R, Calada R, Goicolea JM and Gabaldn F, eds), CRC Press, London, UK, pp. 113–132.
- Garg VK and Dukkipati RV (1984) *Dynamics of railway vehicle systems*. Academic Press Canada, Toronto.
- Goicolea J, Gabaldn F and Riquelme F (2006) Design issues for dynamics of high speed railway bridges. In *Bridge Maintenance, Safety, Management, Life-Cycle Performance and Cost*, CRC Press, Taylor & Francis Group, 6000 Broken Sound Parkway NW, Suite 300, Boca Raton, FL 33487-2742, pp. 163–164, [10.1201/b18175-50](https://doi.org/10.1201/b18175-50).
- Goicolea JM and Antoln P (2011) Dynamic Effects of Railway Traffic due to Lateral Motion in Long Viaducts with High Piers. In *COMPDYN 2011: 3rd ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering* (Papadrakakis M, Fragiadakis M and Plevris V, eds), Corfu, Greece, pp. 1–16.
- Goicolea JM and Gabaldn F (2008) Design issues related to dynamic effects for high speed railway bridges in Spain. In *Dynamics of High-Speed Railway Bridges: Selected and revised papers from the Advanced Course on Dynamics of High-Speed Railway Bridges, Porto, Portugal, 20-23 September 2005* (Delgado R, Calada R, Goicolea JM and Gabaldn F, eds), CRC Press, London, UK, pp. 9–18.
- Goicolea JM, Gabaldn F, Doming J and Navarro JA (2008) Dynamic loads in new engineering codes for railway bridges in Europe and Spain. In *Bridges for High-Speed Railways: Revised Papers from the Workshop, Porto, Portugal, 3-4 June 2004*, CRC Press, p. 31.
- Goicolea JM (2008) Service Limit States for Railway Bridges in new Design Codes IAPF and Eurocodes. In *Track-Bridge Interaction on High-Speed Railways: Selected and revised papers from the Workshop on Track-Bridge Interaction on High-Speed Railways, Porto, Portugal, 15-16 October, 2007* (Calada R, Delgado R, Campos e Matos A, Goicolea JM and Galbadon F, eds), CRC Press, Boca Raton, pp. 7–18.

- Hamid A, Rasmussen K, Baluja M and Yang TL (1983) *Analytical Descriptions of track geometry variations. Technical report*, Federal Railroad Administration.
- He X, Wu T, Zou Y, Chen YF, Guo H and Yu Z (2017) Recent developments of high-speed railway bridges in China. *Structure and Infrastructure Engineering* **13**(12): 1584–1595, [10.1080/15732479.2017.1304429](https://doi.org/10.1080/15732479.2017.1304429).
- Hertz H (1882) Ueber die Berührung fester elastischer Körper. *J. Fur Reine und Angewandte Mathematik* **92**: 156–171.
- Hoorpah W (2008) Dynamic calculations of high-speed railway bridges in France some case studies. In *Dynamics of High-Speed Railway Bridges: Selected and revised papers from the Advanced Course on Dynamics of High-Speed Railway Bridges, Porto, Portugal, 20-23 September 2005* (Delgado R, Calcada R, Goicolea J and Gabaldon F, eds), CRC Press, London, UK, pp. 133–146.
- Hsieh J and Wu FH (2014) High-Speed Railway Bridges. In *Bridge Engineering Handbook, Second Edition: Fundamentals* (Chen WF and Duan L, eds), 2 edn., chap. 8, CRC Press, Boca Raton, pp. 159–183.
- Ju SH (2012) Nonlinear analysis of high-speed trains moving on bridges during earthquakes. *Nonlinear Dynamics* **69**(1-2): 173–183, [10.1007/s11071-011-0254-5](https://doi.org/10.1007/s11071-011-0254-5).
- Kalker J (1967) *On the rolling contact of two elastic bodies in the presence of dry friction*. Doctoral thesis, Delft University of Technology, Delft, Netherlands.
- Kalker J (1982) A Fast Algorithm for the Simplified Theory of Rolling Contact. *Vehicle System Dynamics* **11**(1): 1–12, [10.1080/00423118208968684](https://doi.org/10.1080/00423118208968684).
- Kalker JJ (1990) *Three-Dimensional Elastic Bodies in Rolling Contact, Solid Mechanics and Its Applications*, vol. 2. Kluwer Academic Publishers, Dordrecht, [10.1007/978-94-015-7889-9](https://doi.org/10.1007/978-94-015-7889-9).
- Kargarnovin M, Younesian D, Thompson D and Jones C (2005) Ride comfort of high-speed trains travelling over railway bridges. *Vehicle System Dynamics* **43**(3): 173–197, [10.1080/00423110512331335111](https://doi.org/10.1080/00423110512331335111).
- Kwark JW, Choi ES, Kim YJ, Kim BS and Kim SI (2004) Dynamic behavior of two-span continuous concrete bridges under moving high-speed train. *Computers and Structures* **82**(4-5): 463–474, [10.1016/S0045-7949\(03\)00054-3](https://doi.org/10.1016/S0045-7949(03)00054-3).
- Leboeuf M (2018) *High Speed Rail: Fast Track to Sustainable Mobility*. UIC Passenger Department.
- Lee YS and Kim SH (2010) Structural analysis of 3D high-speed train-bridge interactions for simple train load models. *Vehicle System Dynamics* **48**(2): 263–281, [10.1080/00423110902751912](https://doi.org/10.1080/00423110902751912).
- Li Y, Qiang S, Liao H and Xu Y (2005) Dynamics of wind-rail vehicle-bridge systems. *Journal of Wind Engineering and Industrial Aerodynamics* **93**(6): 483–507, [10.1016/j.jweia.2005.04.001](https://doi.org/10.1016/j.jweia.2005.04.001).
- Liu Z, Luo S, Ma W and Song R (2009) Application Research of Track Irregularity PSD in the High-Speed Train Dynamic Simulation. In *International Conference on Transportation Engineering 2009* (Peng Q, Wang KCP, Qiu Y, Pu Y, Luo X and Shuai B, eds), American Society of Civil Engineers, Reston, VA, pp. 2845–2850, [10.1061/41039\(345\)469](https://doi.org/10.1061/41039(345)469).
- Llombart Jaques JA, Revoltós Fort J, Fernández Vega J and Catalán Martín I (2014) Viaducto Sobre El Rio Oria. In *VI Congreso de ACHE*, pp. 1–5.
- Lou P (2005) A vehicle-track-bridge interaction element considering vehicle's pitching effect. *Finite Elements in Analysis and Design* **41**(4): 397–427, [10.1016/j.finel.2004.07.004](https://doi.org/10.1016/j.finel.2004.07.004).
- Lou P (2007) Finite element analysis for train-track-bridge interaction system. *Archive of Applied Mechanics* **77**(10): 707–728, [10.1007/s00419-007-0122-4](https://doi.org/10.1007/s00419-007-0122-4).
- Lou P, Yu Z and Au F (2012) Rail-bridge coupling element of unequal lengths for analysing train-track-bridge interaction systems. *Applied Mathematical Modelling* **36**(4): 1395–1414, [10.1016/j.apm.2011.08.041](https://doi.org/10.1016/j.apm.2011.08.041).
- Majka M and Hartnett M (2008) Effects of speed, load and damping on the dynamic response of railway bridges and vehicles. *Computers and Structures* **86**(6): 556–572, [10.1016/j.compstruc.2007.05.002](https://doi.org/10.1016/j.compstruc.2007.05.002).

- Majka M and Hartnett M (2009) Dynamic response of bridges to moving trains: A study on effects of random track irregularities and bridge skewness. *Computers and Structures* **87**(19-20): 1233–1252, [10.1016/j.compstruc.2008.12.004](https://doi.org/10.1016/j.compstruc.2008.12.004).
- Marí A and Montaner J (2000) Continuous precast concrete girder and slab bridge decks. *Proceedings of the Institution of Civil Engineers - Structures and Buildings* **140**(3): 195–206, [10.1680/stbu.2000.140.3.195](https://doi.org/10.1680/stbu.2000.140.3.195).
- Martin D (2008) Railway bridges for high speed lines and Eurocodes. In *Bridges for High-Speed Railways: Revised Papers from the Workshop, Porto, Portugal, 3-4 June 2004* (Calcada R, Delgado R and Campos e Matos A, eds), CRC Press, pp. 23–30.
- Montagut IB (2010) *Tipologia de Viaductos en las Lineas de Alta Velocidad en Espana*. PhD thesis, Universitat Politecnica de Catalunya, Barcelona, Spain.
- Montens S and Huyard J (2008) Last developments of dynamic analysis of bridges under moving train loads. In *Dynamics of High-Speed Railway Bridges: Selected and revised papers from the Advanced Course on Dynamics of High-Speed Railway Bridges, Porto, Portugal, 20-23 September 2005* (Delgado R, Calcada R, Goicolea JM and Gabaldon F, eds), CRC Press, London, UK, pp. 57–72.
- Nguyen DV, Kim KD and Warnitchai P (2009) Simulation procedure for vehicle-substructure dynamic interactions and wheel movements using linearized wheel-rail interfaces. *Finite Elements in Analysis and Design* **45**(5): 341–356, [10.1016/j.finel.2008.11.001](https://doi.org/10.1016/j.finel.2008.11.001).
- Pascal J and Sauvage G (1993) The Available Methods to Calculate the Wheel/Rail Forces in Non Hertzian Contact Patches and Rail Damaging. *Vehicle System Dynamics* **22**(3-4): 263–275, [10.1080/00423119308969028](https://doi.org/10.1080/00423119308969028).
- Piotrowski J and Chollet H (2005) Wheel-rail contact models for vehicle system dynamics including multi-point contact. *Vehicle System Dynamics* **43**(6): 455–483, [10.1080/00423110500141144](https://doi.org/10.1080/00423110500141144).
- Podworna M (2014) Vibrations of Bridge / Track Structure / High-speed Train System with Vertical Irregularities of the Railway Track. *Procedia Engineering* **91**: 148–153, [10.1016/j.proeng.2014.12.037](https://doi.org/10.1016/j.proeng.2014.12.037).
- Polach O (1999) A Fast Wheel-Rail Forces Calculation Computer Code. In *16th International Association for Vehicle System Dynamics*, vol. 33, pp. 728–739.
- Polach O (2005) Creep forces in simulations of traction vehicles running on adhesion limit. *Wear* **258**(7-8): 992–1000, [10.1016/j.wear.2004.03.046](https://doi.org/10.1016/j.wear.2004.03.046).
- Pring B (2019) *Structural Performance of Prestressed Precast High Speed Railway Bridges using High Performance Concrete*. Doctoral thesis, Imperial College London.
- Proença JM, Casal H and Neves M (2011) Effect of the Type of Track on the Dynamic Behaviour of High Speed Railway Bridges. In *3rd ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering* (Papadrakakis M, Fragiadakis M and Plevris V, eds), pp. 1–18.
- Ramondenc P, Martin D and Schmitt P (2008) Track-Bridge interaction - the SNCF experience. In *Track-Bridge Interaction on High-Speed Railways* (Calcada R, Delgado R, Campos e Matos A, Goicolea JM and Gabaldon F, eds), CRC Press, London, UK, pp. 63–76.
- Rodriguez J, Martinez F and Marti J (2011) Integral Bridge for High-Speed Railway. *Structural Engineering International* **21**(3): 297–303, [10.2749/101686611X13049248219881](https://doi.org/10.2749/101686611X13049248219881).
- Rosignoli M (2014) Full-Span Precasting of Railway Bridges. *Aspire* **8**(2): 16–18.
- Salcher P, Pradlwarter H and Adam C (2014) Reliability of high-speed railway bridges with respect to uncertain characteristics. In *Proceedings of the 9th International Conference on Structural Dynamics, EUROLYN 2014* (Cunha A, Caetano E, Ribeiro P and Müller G, eds), pp. 2729–2736.
- Shabana AA, Tobaa M, Sugiyama H and Zaazaa KE (2005) On the Computer Formulations of the Wheel/Rail Contact Problem. *Nonlinear Dynamics* **40**(2): 169–193, [10.1007/s11071-005-5200-y](https://doi.org/10.1007/s11071-005-5200-y).
- Shabana AA, Zaazaa KE, Escalona JL and Sany JR (2004) Development of elastic force model for wheel/rail contact problems. *Journal of Sound and Vibration* **269**(1-2): 295–325, [10.1016/S0022-460X\(03\)00074-9](https://doi.org/10.1016/S0022-460X(03)00074-9).

- Shabana AA, Zaazaa KE and Sugiyama H (2007) *Railroad Vehicle Dynamics: A Computational Approach*. CRC Press, Boca Raton, [10.1201/9781420045857](https://doi.org/10.1201/9781420045857).
- Shen ZY, Hedrick JK and Elkins JA (1983) A Comparison of Alternative Creep Force Models for Rail Vehicle Dynamic Analysis. *Vehicle System Dynamics* **12**(1-3): 79–83, [10.1080/00423118308968725](https://doi.org/10.1080/00423118308968725).
- Six K, Meierhofer A, Müller G and Dietmaier P (2015) Physical processes in wheel-rail contact and its implications on vehicle-track interaction. *Vehicle System Dynamics* **53**(5): 635–650, [10.1080/00423114.2014.983675](https://doi.org/10.1080/00423114.2014.983675).
- Smith RA (2003) The Japanese Shinkansen: Catalyst for the Renaissance of Rail. *The Journal of Transport History* **24**(2): 222–347, <https://doi.org/10.7227/TJTH.24.2.6>.
- Song MK, Noh HC and Choi CK (2003) A new three-dimensional finite element analysis model of high-speed train-bridge interactions. *Engineering Structures* **25**: 1611–1626, [10.1016/S0141-0296\(03\)00133-0](https://doi.org/10.1016/S0141-0296(03)00133-0).
- Sun Y and Dhanasekar M (2002) A dynamic model for the vertical interaction of the rail track and wagon system. *International Journal of Solids and Structures* **39**(5): 1337–1359, [10.1016/S0020-7683\(01\)00224-4](https://doi.org/10.1016/S0020-7683(01)00224-4).
- Tai JC, Jang D, Gaffney H and Flint J (2010) Full Span Precast and Launching Construction in Taiwan’s High Speed Rail Project. In *2010 Joint Rail Conference, Volume 1*, ASME, pp. 167–173, [10.1115/JRC2010-36096](https://doi.org/10.1115/JRC2010-36096).
- Torstensson PT and Nielsen JC (2011) Simulation of dynamic vehicle-track interaction on small radius curves. *Vehicle System Dynamics* **49**(11): 1711–1732, [10.1080/00423114.2010.499468](https://doi.org/10.1080/00423114.2010.499468).
- UIC (2019) HS Lines. <https://uic.org/high-speed-database-maps>.
- Vale C and Calçada R (2014) A Dynamic Vehicle-Track Interaction Model for Predicting the Track Degradation Process. *Journal of Infrastructure Systems* **20**(3): 04014016, [10.1061/\(ASCE\)IS.1943-555X.0000190](https://doi.org/10.1061/(ASCE)IS.1943-555X.0000190).
- Vollebregt E (2016) *User guide for CONTACT, Rolling and sliding contact with friction*. Technical report, Vortech, Delft, Netherlands.
- Wen ZF, Mao GW, Mao XB, Jin XS and Zhu MH (2009) Dynamic vehicle-track interaction and plastic deformation of rail at rail welds. *Engineering Failure Analysis* **16**(4): 1221–1237, [10.1016/j.engfailanal.2008.08.001](https://doi.org/10.1016/j.engfailanal.2008.08.001).
- Wu TX and Thompson DJ (2002) A Hybrid Model for the Noise Generation due to Railway Wheel Flats. *Journal of Sound and Vibration* **251**(1): 115–139, [http://dx.doi.org/10.1006/jsvi.2001.3980](https://doi.org/10.1006/jsvi.2001.3980).
- Xia H, Li HL, Guo WW and De Roeck G (2014) Vibration Resonance and Cancellation of Simply Supported Bridges under Moving Train Loads. *Journal of Engineering Mechanics* **140**(5): 04014015, [10.1061/\(ASCE\)EM.1943-7889.0000714](https://doi.org/10.1061/(ASCE)EM.1943-7889.0000714).
- Xia H, Wang K and Yu Z (2012) Dynamic Interaction Analysis of Coupled Train-Bridge System. In *Bridge Vibration and Controls* (Xia H, De Roeck G and Goicolea JM, eds), chap. 1, Nova Science Publishers, pp. 1–65.
- Xia H, Xu Y and Chan T (2000) Dynamic Interaction of Long Suspension Bridges with Running Trains. *Journal of Sound and Vibration* **237**(2): 263–280, [10.1006/jsvi.2000.3027](https://doi.org/10.1006/jsvi.2000.3027).
- Xia H and Zhang N (2005) Dynamic analysis of railway bridge under high-speed trains. *Computers & Structures* **83**(23-24): 1891–1901, [10.1016/j.compstruc.2005.02.014](https://doi.org/10.1016/j.compstruc.2005.02.014).
- Xia H, Zhang N and De Roeck G (2003) Dynamic analysis of high speed railway bridge under articulated trains. *Computers & Structures* **81**(26-27): 2467–2478, [10.1016/S0045-7949\(03\)00309-2](https://doi.org/10.1016/S0045-7949(03)00309-2).
- Xia H, Zhang N and Guo WW (2006) Analysis of resonance mechanism and conditions of train-bridge system. *Journal of Sound and Vibration* **297**(3-5): 810–822, [10.1016/j.jsv.2006.04.022](https://doi.org/10.1016/j.jsv.2006.04.022).
- Xu Y, Zhang N and Xia H (2004) Vibration of coupled train and cable-stayed bridge systems in cross winds. *Engineering Structures* **26**(10): 1389–1406, [10.1016/j.engstruct.2004.05.005](https://doi.org/10.1016/j.engstruct.2004.05.005).
- Yang J, Wu Y and Yau Y (2004) *Vehicle-Bridge interaction Dynamics: With Applications to High-Speed Railways*. World Scientific Publishing Co., Singapore.

- Zaazaa KE, Martin TP, Whitten B, Marquis B, Curtis E, El-Sibaie M and Tajaddini A (2009) A Nonlinear Rail Vehicle Dynamics Computer Program Sams/Rail Part 3: Applications to Predict Railroad Vehicle-Track Interaction Performance. In *Proceedings of the ASME/IEEE Joint Rail Conference*, pp. 165–173, [10.1115/JRC2009-63046](https://doi.org/10.1115/JRC2009-63046).
- Zhang N, Xia H and Guo WW (2008) Vehicle-bridge interaction analysis under high-speed trains. *Journal of Sound and Vibration* **309**(3-5): 407–425, [10.1016/j.jsv.2007.07.064](https://doi.org/10.1016/j.jsv.2007.07.064).
- Zhang N, Xu M, Chen S and Yu Z (2012) Dynamic Responses of a Simply-Supported beam Subjected to Various Moving Loads. In *Bridge Vibration and Controls* (Xia H, De Roeck G and Goicolea JM, eds), Nova Science Publishers, pp. 113–150.
- Zhang QL, Vrouwenvelder A and Wardenier J (2001) Numerical simulation of train-bridge interactive dynamics. *Computers & Structures* **79**(10): 1059–1075, [10.1016/S0045-7949\(00\)00181-4](https://doi.org/10.1016/S0045-7949(00)00181-4).
- Zhaohua F and Cook RD (1983) Beam Elements on Two-Parameter Elastic Foundations. *Journal of Engineering Mechanics* **109**(6): 1390–1402, [10.1061/\(ASCE\)0733-9399\(1983\)109:6\(1390\)](https://doi.org/10.1061/(ASCE)0733-9399(1983)109:6(1390)).
- Zhu JJ, Ahmed AKW and Rakheja S (2007) An Adaptive Contact Model for Simulation of Wheel-rail Impact Load due to a Wheel Flat. In *13th National Conference on Mechanisms and Machines (NaCoMM07)*, Bangalore, India, pp. 157–164.
- Zhu JJ, Ahmed AKW, Rakheja S and Hu YS (2009) Impact load due to railway wheels with multiple flats predicted using an adaptive contact model. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* **223**(4): 391–403, [10.1243/09544097JRRT258](https://doi.org/10.1243/09544097JRRT258).