The transpose null-space matrix N^T becomes

$$N^T = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}.$$

Since columns 1, 2, and 4 include only zeroes, the state variables θ_1 , θ_2 , and θ_4 are observable.

Finally, if active power injection measurements in buses 1 and 5 and active power flow measurements 1–2, 1–3, and 2–4 are considered, matrix H becomes

$$H = \begin{bmatrix} 2 & -1 & -1 & & \\ & -1 & & -1 & \\ 1 & -1 & & & \\ 1 & & -1 & & \\ & 1 & & -1 & \end{bmatrix}.$$

The transpose null-space matrix N^T becomes

$$N^T = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}.$$

Since all columns include only zeroes, all state variables are observable.

Numerical tests carried out using the IEEE Reliability Test System [8], including voltage, active and reactive power injection, and active and reactive power flow measurements, show the effective behavior of the proposed algorithm.

IV. CONCLUSION

This letter proposes and illustrates a novel technique to state estimation observability analysis. This technique relies on computing the null space of the Jacobian measurement matrix. The computational burden involved is, therefore, that pertaining to the null space calculation, which is similar to the burden of solving a linear homogeneous system ($O(N^3)$). A simple example is used to demonstrate the efficacious functioning of the technique. Extensive computational analysis involving voltage and active and reactive power measurements as well as current measurements is the subject of a future paper.

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Coherency Identification in Power Systems Through Principal Component Analysis

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Abstract—In this letter, a new technique to identify coherent generators in large interconnected power system using measurements of generator speed and bus angle data has been presented. This is based on the application of principal component analysis (PCA) to measurements obtained from simulation studies that represent examples of interarea events. The results of application of PCA separately to data sets of generator speeds and bus angles, respectively, are presented. The approach of PCA was able to highlight clusters of generators showing common features when compared with the conventional modal analysis technique.

Index Terms—Clusters, coherency, modal analysis, principal component analysis (PCA).

I. INTRODUCTION

Increasing interconnection of power plants in modern large electric power systems has made power system dynamic studies much more complex. Under the deregulated business environment, the interconnections are increasingly being used for trading between utilities. This stresses the interconnections through large power transfer. As a result, the system is prone to low-frequency interarea oscillations. In such an operating scenario, the system behaves coherently with groups of coherent generators separated from other groups of coherent generators linked through weak interconnections.

The analysis of an interconnected power system is generally for a specified portion, called the study system, while the rest is the external portion of the system approximated to its equivalent of lower dimension. Since the impacts of major disturbances propagate through tie lines to neighboring systems, it is important to represent not only the power system in question but also the neighboring utilities or the external system in terms of its dynamic equivalent.

Coherency-based approaches to dynamic equivalents have been adopted in reducing the size of the power system model [1]–[3]. However, coherency was identified with the help of the linearized model of the power systems. More often than not, the access to system dynamic data involving a number of utilities is incredibly difficult. The accuracy of the system model influences the accuracy of the results. Also, owing to the dimension of the interconnected systems, it is neither easy nor desirable to represent the entire system model in detail.

In this letter, coherency is obtained from measured data, obviating the need for detailed modeling information. The method is based on principal component analysis (PCA), which is computationally simple and fast. An industrial application of this method will take the wide area measurements from GPS-based time-stamped phasor measurement units (PMUs). For the purposes of demonstration in this letter, the data measurements are generated from the Simulink model of a 16-machine 68-bus test system for generator and bus coherency identification

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of the system. The proposed method has been able to correctly identify the coherent groups, as would be evident through visual inspection later.

II. PCA TECHNIQUE

PCA is a technique that transforms a set of p correlated variables to a new set of uncorrelated variables called *principal components*. These new variables are a linear combination of the original variables and are derived in decreasing order of importance so that, for example, the first principal component accounts for as much as possible of the variation in the original data. The transformation is, in fact, an orthogonal rotation in p -space. The usual objective of the analysis is to see if the first few components account for most of the variation in the original data. A description of PCA can be found in [4] and [5]. Thornhill *et al.* [6] had used spectral PCA of data in a chemical process to highlight clusters of process measurements showing common features.

The data matrix $X_{m \times n}$, consisting of m process variables, is obtained from the online measurements of the considered plant variables measured over time t for n ($m \ll n$) samples. A full PCA decomposition reconstructs the X matrix as a sum over m orthonormal basis functions w'_1 to w'_m , which are arranged as row vectors

$$X = \begin{pmatrix} t_{1,1} \\ \vdots \\ t_{m,1} \end{pmatrix} w'_1 + \begin{pmatrix} t_{1,2} \\ \vdots \\ t_{m,2} \end{pmatrix} w'_2 + \cdots + \begin{pmatrix} t_{1,m} \\ \vdots \\ t_{m,m} \end{pmatrix} w'_m. \quad (1)$$

The w'_i -vectors are the normalized right eigenvectors of the $n \times n$ matrix $X'X$. The ratio between the eigenvalue and the sum of all the eigenvalues gives a measure of the total variation captured by that eigenvector.

The expression (1) may be written compactly as

$$X = TW' \quad (2)$$

where the i th column of T is $(t_{1,i}, \dots, t_{m,i})'$, and the rows of W' are w'_1 to w'_m . The orthonormality of the rows of W' means that $T = XW$. Singular value decomposition $X = UDV'$ provides a means for computation of the requisite vectors with $T = UD$ and $W' = V'$. The elements of D are singular values of X .

The weightings of each principal component in each data variable of X may be represented graphically. When three w' -vectors are in use, the i th component maps to a point having the coordinates $t_{i,1}$, $t_{i,2}$, and $t_{i,3}$ in a three-dimensional space called a scores plot. Similar components have similar t -coordinates. Therefore, such groups form clusters. This formation of the clusters can be used in identifying the coherent group of generators in a vast interconnected power system. Each time trend in the raw data is preprocessed by subtracting its mean and scaling it to unit standard deviation. This is the only preprocessing required.

III. RESULTS AND ANALYSIS

A reduced-order model of the New England/New York 16-machine 68-bus interconnected system is considered as the test system. The single-line diagram, bus data, line data, and the dynamic characteristics for the machine, exciters, and loads can be found in [7]. For the case of a disturbance of 10% increase in mechanical input torque to each of the generators, nonlinear simulations were performed in Simulink, and the simulation data consisting of 2001 samples of generator speed, and bus angle measurements were recorded with a sampling time of 0.01 s.

PCA was applied to the data matrix using the singular value decomposition to compute the requisite vectors T and W' . It was found that variation of X was captured by the first three principal components

Fig. 1. Identification of coherent groups from the generator speed data.

Fig. 2. Bus coherency identification of 68-bus test system.

and, hence, was truncated to the first three terms. Fig. 1 represents the scores plot of the data.

It can be observed that the first nine machines (G1 to G9) are along the t_1 - t_2 plane, machines G10 to G13 are along the t_2 - t_3 plane, whilst the last three machines G14 to G16 are in different planes far away from the machines in other areas. One can conclude that the machines in different planes form separate coherent groups. This was compared with the 16-machine test system and was found out that those respective machines, in fact, fall into different areas.

The proposed method of PCA was also applied to another case of data matrix defined by phase angle measurements of bus voltages. Fig. 2 depicts the bus coherency captured by the PCA technique. Bus pairs #14 and #41 ($\square$ s) of Areas #3, #15, and #42 (∇ s) of Area #4 and #16 and #18 ($\circ$ s) of Area #5 are separated and are far apart from the buses that belong to Areas #1 ($\triangle$ s) and #2 ($*$ s). It can be observed that some buses of Area #2 drift toward Areas #3 and #5 (indicated by the arrows). This is because they are electrically close to each other connected through tie lines. The coherent machine groups and bus areas obtained through our PCA approach match exactly with that obtained through direction cosine approach detailed in [7] and [8].

IV. CONCLUSION

This letter proposes a new technique of coherency identification of generators and buses through PCA. Unlike conventional modal analysis technique, this method relies only on measurements of generator speeds and bus angles from the test system. The technique of PCA requires simple calculation involving little time and, hence, can be thought of as a possible tool for frequent identification of the coherent generators/buses and the corresponding weak tie lines. The results of generator and bus coherency calculation in a study system has been presented.

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Performance Curves of Voltage Relays for Islanding Detection of Distributed Generators

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Abstract—This letter proposes a method for systematically evaluating the voltage-based relay's capability to detect islanding of distributed generators connected to distribution systems. The method is based on a set of detection time versus reactive power imbalance curves, which can be obtained through repeated dynamic simulations. With the curves, one can determine the characteristics of voltage-based anti-islanding protection systems.

Index Terms—Distributed generator, islanding detection, synchronous generator, voltage relay.

I. INTRODUCTION

Islanding occurs when a part of the distribution system is electrically isolated from the main source of supply yet continues to be energized by distributed generators [1]. This is an undesirable phenomenon since damage may occur to the loads and generators connected to the islanded system. The current industry practice is to disconnect all distributed generators immediately after the occurrence of islands [1], [2]. Voltage-based relays have been used by the industry to detect islanding of synchronous generators [2]. Thus, this letter proposes a new approach that permits to evaluate the capability of these relays systematically. The approach is based on a set of detection time versus reactive power imbalance curves, which are obtained through repeated

Fig. 1. Single-line diagram of the system.

Fig. 2. Detection time versus active power imbalance curves.

dynamic simulations considering different reactive power imbalance levels in the islanded system. As the simplest voltage-based device is the under/over voltage relay, this is the chosen relay to explain and exemplify the proposed method.

II. NETWORK COMPONENT MODELS

Islanding is a dynamic phenomenon. Thus, time-domain dynamic simulation is the tool used in this letter to analyze the performance of voltage relays. The system employed is presented in Fig. 1. Such network consists of a 132-kV, 60-Hz substation with short-circuit level of 1500 MVA, feeding a 33-kV distribution system. In this system, there is one synchronous generator with a capacity of 30 MVA. All network components are represented by three-phase models. Distribution feeders are modeled as series RL impedances. Transformers are modeled using a T circuit. The synchronous generator is represented by a sixth-order model in the dq rotor reference frame and equipped with an automatic excitation system, which is controlled to keep operation with constant reactive power, as is usual in distributed generation [1]. The islanding situation is simulated by opening the circuit breaker CB at bus 2; consequently, the values of reactive and active power imbalances in the islanded system are equal to the reactive and active power supplied by the substation just before the islanding instant.

III. PERFORMANCE CURVES OF VOLTAGE RELAYS

Recently, a method based on a set of detection time versus active power imbalance curves was proposed to evaluate the performance of frequency-based anti-islanding relays [3]. Such curves are denominated performance curves. However, this approach has shown to be inappropriate to evaluate voltage-based anti-islanding relays. Fig. 2 presents the detection time versus active power imbalance curves of the under/over voltage relay installed at bus 5 considering different settings. These curves were obtained gradually varying the active power

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