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Highlight

- Superplastic behavior of near-alpha Titanium alloy (TA15) with equiaxed fine grain structure have been studied.
- A wide range of thermal-mechanical tensile test performed at temperature range of 880 – 940 °C and strain rates range of 5×10^{-4} – 10^{-2} s^{-1} .
- A mechanism based unified viscoplastic constitutive model is established.
- The proposed constitute model is implemented in FE software to check its validity against the superplastic forming experimental test for the multi-box die.

Constitutive Modelling for the Simulation of the Superplastic Forming of TA15 Titanium Alloy

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Abstract

Titanium alloy, TA15, has a high strength-to-weight ratio, high weldability, and superior creep resistance at high temperatures up to 550 °C. TA15 is difficult to deform, especially for forming complex-shaped large-scale web-rib components, due to its low plasticity, large inhomogeneous deformation and narrow processing window. The objective of this research is to model the superplastic mechanisms in TA15 alloy with equiaxed, fine grain structure, and applying the proposed constitutive model to investigate the maximum grid aspect ratio, that can be achieved in superplastic forming (SPF), for a TA15 sheet with an initial thickness of 1.2 mm. Thermo-mechanical tensile tests are conducted first to characterize the superplastic behaviour of the material in the temperature range of 880–940 °C and the strain-rate of 0.0005 – 0.01 s⁻¹. A set of mechanism-based unified visco-plastic constitutive equations has been proposed and calibrated based on the results of stress-strain data. A gradient-based optimization method is applied for the calibration of constitutive equations. The constitutive model is incorporated into FEA code through creep subroutine to check the validity of the proposed material model against the experimental SPF test of a multi-box die. Predicted sheet thickness and thinning in a die entry radius region at the end of forming are examined in detail. Preliminary results show a good agreement between the computational and experimental results.

Key words: TA15, Superplastic forming, Multi-box die, Thermomechanical test, Constitutive equations, Abaqus.

1. Introduction

Superplasticity is the capacity of a certain polycrystalline material which exhibits exceptionally large elongations without necking prior to failure. To exploit the superplastic phenomenon for the manufacturing of engineering components with complex structure in the aerospace industry, near- α titanium alloys are generally used for high-temperature owing to its superior strength and creep resistance [1].

TA15 (Ti-6Al-2Zr-1Mo-1V) is a type of near- α titanium alloy and is famous for its application in aerospace industry because of their high strength to mass ratio, high weldability, superior creep, and their ability to retain its mechanical properties at high temperatures up to 550 °C [2]. However, the low plasticity and narrow processing window (i.e. high flow stress at elevated temperatures) of TA15 makes it difficult to deform and process. Owing to this, it is difficult to form the complex shape of the large-scale web-rib component due to its hard-working property and large inhomogeneous hot deformation in a work piece. Moreover, usually undesired thickness variation of ribs, which leads to poor structural integrity, has been observed during the manufacturing of TA15 multilayer sandwich structure by superplastic forming coupled with diffusion bonding (SPF/DB) process. Thus, there is a need for reliable constitutive equations which have the ability to numerically optimize the SPF process for TA15 alloy with the microstructure evolution of fully equiaxed near α structure.

In recent years, numerical modelling has been widely used in the superplastic forming industry to provide effective guidance for optimizing processes. At present, the three main approaches to establish the constitutive modelling of materials' deformation are phenomenological, physically-based and advanced statistical [3]. Physically-based constitutive models coupled with microstructural evolution have attracted increasing attention in the field of metal forming, because it accurately predict the stress-strain relationship and microstructure distribution and evolution during the process, which provides valuable mechanical properties information for post-formed parts [4].

Fan et al. [5] studied the microstructure based constitutive behavior of TA15 alloy. The loss of Hall-Petch strengthening with deformation was modeled to predict the flow softening. Gao et al. [6] investigated TA15 alloy having a lamellar starting microstructure and proposed physically based constitutive relation to forecast the globularization and flow softening behavior of the material. A set of mechanistically based constitutive equations for predicting the evolution of grain size, dislocation density and recrystallization was developed by Sun et al. [7]. The relation was applied for predicting the microstructure evolution in TA15 alloy with the starting microstructure comprising of transformed

β matrix with α plate subjected to isothermal hot-compression tests. Yang et al. [8] worked on unified viscoplastic constitutive relation for TA15 and Ti-17 alloys both with initial lamellar microstructure. All of the above mentioned efforts are based on a SPF modeling framework which predicts grain growth and strain rate hardening. However, to widen the applications of TA15 alloy in SPF process, more fundamental research is necessary. First, the mechanism based unified constitutive model for predicting superplastic mechanism at low strain-rate in the TA15 alloy with fine microstructure are still unavailable, which demonstrates substantial softening under large strains. Second, finite element analysis (FEA) is being considered as one of the favorable technologies for extending the use of superplastic forming for producing multi-layer hollow structures. FEA has increasingly become an important technique to predict the optimal pressure profile for the material selection and minimum formation time. The accuracy of the simulation code to forecast the deformation mechanism of the sheet material is a major factor in optimizing gas pressure cycles that can produce the work piece with a uniform thickness in the shortest possible time. Much of this capacity comes from a precise constitutive model that describes the behavior of the material. The constitutive model is established as an input to the finite element program for accurate prediction of material deformation behavior under certain processing conditions.

The aim of this research is to model the superplastic mechanisms in TA15 alloy with equiaxed, fine grain structure, and develop the physically-based constitutive equations to simulate the superplastic forming (SPF), for a TA15 sheet with an initial thickness of 1.2 mm. Thermo-mechanical tensile tests are conducted first to characterize the superplastic behaviour of the material in the temperature range of 880–940 °C and the strain-rate of 0.0005 – 0.01s⁻¹. A set of mechanism-based unified visco-plastic constitutive equations will be proposed and calibrated based on the results of stress-strain data. A gradient-based optimization method will be applied for the calibration of constitutive equations. The constitutive model is incorporated into FEA code through creep subroutine to check the validity of the proposed material model against the experimental SPF test of a multi-box die.

2. Experimental procedure

The current study is using the rolled TA15 alloy sheet with a thickness of 1.2 mm. This thickness corresponds to the practical application of SPF in industries, which is obtained by large deformation forging and then cladding pack rolling. The colony microstructure is changed into equiaxed microstructure via necessary primary hot working [9]. The β -transus temperature of material is 985 °C and Table 1 describes the chemical composition of the studied TA15 alloy. TA15 sheet having a near

alpha starting microstructure is used. Fig.1 demonstrates the initial microstructure which contain small amount of ductile β phase grain dispersed in the intergranular region of a near α -phase structure. The equiaxed microstructure is one of the requirements for the SPF applications due its ability to enable the alloy to acquire a good proportion of strength and ductility.

Table1. Chemical composition of used TA15 alloy

Element	Al	Zr	Mo	V	Ti		
Mass%	5.0-7.0	1.5-2.5	0.5-2.0	0.8-2.5	Bal.		
Impurity	Fe	Si	C	N	H	O	others
Mass%	0.25	0.15	0.1	0.05	0.015	0.15	0.3

Fig. 1. Initial microstructure of the TA15 titanium alloy captured using SEM with a back scatter detector. The black regions correspond to the β phase grains and white regions are alpha grains.

2.1. Uniaxial thermal-mechanical test

It is generally agreed that the temperature must be in excess of about $0.5T_m$, where T_m is the absolute melting point [10]. The temperature at which superplasticity is found in titanium alloys is usually between about $0.47 T_m$ (about 800 °C) to $0.61 T_m$ (about 1030 °C) [11]. As the β -transus temperature of TA15 alloy is 985 °C, so superplastic deformation of TA15 was investigated in the temperature of 880 – 940 °C. The specimens for hot tensile tests (Fig. 2) were cut using wire-electrode along two directions i.e. the rolling and transverse direction, to check their anisotropy response. The 800# Silicon Carbides grinding paper is used to polish the specimens to minimize the influence of rough cutting. The isothermal tensile tests were carried out in the temperature range of 880 – 940 °C and strain rate range of $0.0005 - 0.01s^{-1}$ using a computer controlled universal testing machine (CSS-8800). Prior to tests, the tensile samples were coated with the layer of antioxidants for the prevention of oxidation at a high temperature. During these tests each specimen was held for 10 minutes at corresponding temperature for uniform temperature distribution and then deformed under constant strain rate. Finally, the specimen was air cooled immediately after deformation.

Fig. 2. Geometry of the tensile specimen (unit: mm)

2.2. SPF test of TA15

To test the SPF process, a Ni-Cr-Fe steel die is designed and manufactured. The die is designed with the aim to find the geometry of maximum grid aspect ratio without localized necking during the superplastic forming process. The experimental die (Fig. 3a) comprises of three rows of seven boxes each. The plane dimension of each box is 60 x 60 mm, with grid aspect ratio of 1: 2, 1: 1.5, 1: 1.2, 1: 1, 1: 0.75, 1: 0.6 and 1: 0.5, respectively. The different grid aspect ratios imply to different amount of deformation for the same sheet. An experience-based gas pressure profile (with the aim of strain rate of $10^{-3}s^{-1}$) at the temperature of 920 °C) shown in Fig. 3a is applied to sheet of initial thickness 1.2mm to form the seven boxes.

Fig 3. Schematic of the experimental three-row Ni-Cr-Fe steel boxes die (a) and the applied Gas pressure (b). Depths of the rows vary from 15mm to 60mm. The square boxes have the same length and width: 60x60 mm. The boxes in the row 1 have the same geometrical configuration as row 2 and 3. Row 2 of the Die (a) is used to produce an SPF part using the Gas pressure (b) for the deformation temperature of 920°C and strain rate of 10^{-3} s^{-1}

2.3. Finite element model of a superplastic forming process

An FE model is developed in ABAQUS using the proposed material model via a CREEP subroutine in which the defined strain rate potential is written as function of von Mises equivalent deviatoric stress and a series of solution-dependent variables or evolutionary equations, solved using the forward Euler method. This subroutine is called for each material integration point at each iteration of every increment. A pressure profile is generated for the simulation of superplastic forming of a multi-box experimental die (Fig. 3a) to validate the efficiency of the proposed material model. The model geometry is shown in Fig. 4 where the boundary conditions are: Edges 1 and 2 are clamped, Edge 2 is restrained in U_z direction and Edge 4 has got symmetry boundary conditions about z axis. The die is defined as discrete rigid surfaces meshed with R3D4 elements. The blank dimension is 70x700 mm with thickness of 1.2mm and is meshed with a four-node general purpose shell elements (S4R element in ABAQUS). The contact was modelled with a “surface-to-surface” algorithm with a finite sliding approach.

The accuracy of numerical integration is controlled by setting the ABAQUS parameter CETOL equal to 0.01%. This parameter is defined by choosing an acceptable stress error tolerance and dividing it by the elastic modulus. This way, the accuracy is checked by comparing the equivalent creep strain rate at the beginning and the end of each increment. The difference should be less than CETOL divided by the time increment. Otherwise, the increment is reattempted with a smaller time increment. The size of the time increment is set automatically by ABAQUS during the simulation but it is constrained by a user-

specified accuracy tolerance parameter that limits the maximum inelastic strain rate change allowed over an increment.

During the blow forming process, a gas pressure is imposed over a sheet to make it flow into a die of the desired shape. To produce maximum ductility in the SPF process, it is desirable to adjust the forming pressure to keep the strain rate within an optimum range throughout the whole forming process. Due to the geometry of the die cavity, sheet undergoes to different strain conditions according to the considered area and to the forming phase. Thus, significantly different strain rates can be found along the sheet and the location of critical zones changes during the process.

Fig. 4. Model geometry

3. Results and Discussion

3.1. Hot tensile test

Fig. 5 shows the deformed samples, correlations of yield and tensile strength and comparing the percentage elongations between rolling direction (RD) and transverse direction (TD) of the TA15 sheet. Consistent mechanical properties are found along RD and TD. It can be observed in the Fig. 5 (b) that the strength in the Rolling Direction (RD) increases with corresponding strength in Transverse Direction (TD) and dispersion of the data points along the line with a slope of 1 is negligible. In addition, Fig. 5(c) shows that the distribution trend between the RD and TD elongation is similar with that exhibited in Fig. 5(b).

The percentage elongation and temperature from two directions is also compared and presented in Fig. 5(d & e) respectively. There are uneven changes in the total percentage elongation, which implies that

strain rate and temperature have tanglesome impacts on the overall formability of the alloy. It is obvious from Fig. 5 that the percentage elongation is maximum (795%) for the temperature of 920 °C and strain rate of 0.001s^{-1} , while the lowest elongation is 305% at temperature of 900 °C with corresponding strain rate of 0.0005s^{-1} . This provides important basis and guidance for subsequent SPF simulation as well as the actual process parameter adjustment and application in the industry.

Fig. 5. (a) Tensile specimens before and after the thermal-mechanical testing. (b) Correlations of yield strength (0.2%) and ultimate tensile strength for Rolling and Transverse Directions (c) Correlations of percentage elongations for Rolling and Transverse direction (d) Correlations of elongation and temperature for Rolling Direction (e) Correlations of elongation and temperature for Transverse Direction.

3.2. Flow stress behavior

The stress–strain curves for the strain rates in the range of 0.0005 – 0.01s^{-1} and temperature 880 – 940 °C are presented in Fig. 6. It can be seen in the Fig. 6 that the morphology of stress–strain curves is strongly depends on the strain rate and deformation temperature. The formation of tangled dislocation structures, as barriers to the dislocation movement, owing to the increased strain rate is believed to be the reason for the observed behavior [12]. The lack of enough independent active slip systems in hexagonal closely packed (HCP) alpha titanium alloy [13] like TA15 and the presence of twins (thereby obstructing the dislocation movement) in low temperatures may as well contribute to the increased flow stress level [14]. The physics of the resulting flow stress curves can either be categorized in three or two distinctive stages.

As illustrated in Fig. 7, the morphology of the curves for higher strain rate of 0.01s^{-1} can be explained with the help of three distinctive phases. In phase I, it can be seen that the flow stress rises rapidly with strain which is attributed to the significant strain hardening [15] due to the rapid increase in dislocation density [16, 17], because at high strain rates, the dislocations do not have sufficient time to slide or climb [18]. Meanwhile, as the deformation occurs at high temperature ($>0.5T_m$), the dynamic recovery (DRV) (dislocation annihilation) is also expected [19]. In phase I, these two competing mechanisms are activated at the same time but the hardening part seems predominating [20]. Note that the elastic part of the stress strain curve at viscoplastic condition is very small and difficult to measure, as the creep and recovery occurs at the beginning of the hot deformation process. In phase II, there is a slow increase in flow stress until the peak value is reached. It indicates the underlying deformation hardening or softening mechanism starts to alter. The dislocation density (the stored plasticity energy) might reach a critical value, that dynamic recrystallization (DRX) occurs. At the beginning of dynamic recrystallization, the dislocation density will be reduced by the newly formed grains or grain boundary migration. The reduction of dislocation density is one of the reasons for the considerable softening observed in the phase III region. Also, as the dynamic recrystallization lead to grain refinement, which increase the overall length of grain boundary. The increasing grain boundary length will facilitate the grain rotation and grain boundary sliding which is another important deformation mechanism in superplastic process. The combined dislocation density reduction and grain refinement lead to the observed softening in phase III. A further drop in the flow stress at higher values of strains might be associated with the continuous grain refinement and the formation of cavitation [21, 22].

Generally, the heat generated by plastic deformation causes a local temperature increase, which leads to a drop in the value of flow stress. This further enhances the softening in the process of high-temperature deformation of materials. Many studies [23-25] have suggested that titanium alloys exhibit flow stress decreasing due to adiabatic shear bands (ASBs) formation at higher strain rates. The ASBs phenomenon can be attributed to the following reasons. As discussed a high strain rate lead to increase in dislocation density which in turns generate heat. It is known that approximately 90% of the material's deformation energy during plastic deformation is converted into heat, the rest is stored in form of dislocations in the crystals of the material [26]. This increase in heat tends to free the motion of dislocations, resulting in thermal softening of the material. The generation of heat thus results in further plastic strain, which causes a further increase in temperature. Because titanium alloys have relatively low thermal conductivity [27], therefore at high strain rates, there is not enough time for the heat generated to be conducted away and the deformation is thus considered adiabatic. This adiabatic

heat induces highly localized flow along the maximum shear stress plane (known as a shear band) at high strain rates. Thus, the ASBs is also deemed to play a vital role in the softening phenomena of the TA15 alloy at high strain-rate.

The curve morphology for the strain rate of 0.001s^{-1} and 0.0005s^{-1} for the corresponding temperature of 880 and 900 °C can be divided into two distinct phases. In phase I, similar to the high strain rate, the flow stress increases sharply with increasing strain until the peak stress is achieved. In phase II, the rate of strain hardening gradually decreases and approaches to zero, resulting in a steady state flow stress [28]. Such behavior is attributed to the operation of DRV as the main softening process [29]. The additional time at lower strain rate of 0.001s^{-1} and 0.0005s^{-1} compared to 0.01s^{-1} , allows DRV to reduce the dislocation density [30] and this leads to a significant reduction in peak stress [31]. In phase II, the flow stress becomes steady with the increasing strain until fracture occurs. The steady-state flow stress might exhibit that the dynamically balanced hardening and softening process is achieved, such as the grain refinement and growth or dislocation multiplication and annihilation [31].

The curve's morphology for the strain rate of 0.001s^{-1} and 0.0005s^{-1} with the temperature of 920 - 940 °C can be also split again into two distinct phases. In phase I, the flow stress increases rapidly with increasing strain until the peak stress point is achieved. In phase II, there is only reporting work hardening. Such behavior might be an indicative of grain growth due to long exposure time at high temperature. The grain growth leads to the reduction in grain boundary length and thus the increasing difficulty in grain rotation or grain boundary sliding. This results in hardening.

Overall it can be observed that the flow stress decreases significantly with decreasing strain rate and increasing deformation temperature. The reduction in flow stress is due to dynamic softening, including dynamic recovery (DRV) and dynamic recrystallization (DRX), both of which are more active at high temperatures and low strain rates [18, 28]. However, the peak flow stress increased as the strain rate increases at a certain temperature. This is due to insufficient time for DRV or DRX at high strain rate, which leads to higher dislocation density, and thus a higher strain hardening effect [32].

Fig. 6. Flow stress curves of TA15 alloy at deformation temperature of (a) 880 °C, (b) 900 °C, (c) 920 °C and (d) 940 °C

Fig. 7. Schematic representation of flow curves during superplastic deformation of titanium alloys

3.3. Constitutive modeling using state variable

Based on the stress-strain curve data presented in Fig. 6, there is an interaction relationship among deformation temperature (T), strain rate ($\dot{\epsilon}$), strain (ϵ), and flow stress (σ). Constitutive equation is an appropriate tool for describing the deformation behavior involving the abovementioned parameters. In this work, a set of mechanism-based viscoplastic constitutive equations has been established to predict the viscoplastic flow of TA15 alloy sheets in superplastic forming (SPF) processes.

3.3.1. Viscoplastic flow rule

According to Hook's law, the stress-strain relationship of materials can be expressed as:

$$\sigma = E(\epsilon_T - \epsilon_p) \quad (4)$$

Where E represents Young's modulus, ϵ_T is total strain and ϵ_p is plastic strain

Because the material exhibits viscoplastic flow during the SPF process, the overall flow stress can be decomposed into three parts: initial yield stress, viscoplastic stress, and hardening stress [33].

$$\sigma = k + \sigma_v + H \quad (5)$$

Where k is the initial yield stress, σ_v the viscoplastic stress, and H the isotropic hardening stress.

When the metal deforms superplastically, the stress σ_v that induces viscoplastic flow must exceed the initial point k , *i.e.*, the value of $(\sigma - k)$ should be positive [33]. After sufficient plastic straining occurs, dislocation entanglement, pile-ups, etc., harden the material by an amount R ; the stress that leads to viscoplastic flow stress σ_v is reduced to $(\sigma - R - k)$. On the basis these information Lin et al. [34] modified the flow rule considering its dependence directly on the isotropic hardening (H) and initial yield stress (k) and can be expressed by Eq. (6):

$$\dot{\epsilon}_p = \left\langle \frac{\sigma - H - k}{K} \right\rangle^n \quad (6)$$

where $\dot{\epsilon}_p$ is the plastic strain rate, σ is the flow stress, H is the hardening due to material isotropic work hardening, n is the viscosity exponent, the term $\left\langle \frac{\sigma - H - k}{K} \right\rangle$ within Macaulay brackets implied that this equation is not applicable until plastic deformation occurs, k is the initial yield stress and K is a temperature dependent parameter.

The stress and strain curves of TA15 alloy shown in Fig. 6, show the larger softening deformation under the strain rate of $0.01s^{-1}$. In order to fit this large softening phenomenon more accurately, a softening mechanism related with viscoplastic strain rate is proposed. Thus the Eq. (6) is modified as

$$\dot{\varepsilon}_p = \left(\frac{\sigma - H - k}{K} \right)^n \cdot \frac{1}{(1 - S)} \quad (7)$$

Where S is the normalized volume fraction of recrystallization, showing a range of 0~1. 0 means no recrystallization and 1 indicates full recrystallization occurrence.

3.3.2. Evolution of dislocation density

It is well known that the dislocation structure developed in plastic deformation constitutes a driving force for microstructure evolution [33], such as recrystallization and grain growth. The evolution of dislocation density is due to three mechanisms: the first is the creation and accumulation of dislocations due to work hardening induced by plastic deformation, the second is the annihilation resulting from static and dynamic recovery, and the third is the annihilation caused by microstructure evolution, such as recrystallization [35]. The generation of the new dislocation and their climbing with the existing dislocations causes working hardening. Meanwhile, the stored energy of the material enhances and drives the recovery and microstructure change. Conversely, the recovery, including the statics and dynamics recovery, leads to absorption of the tangled dislocations, causing strain softening. Mecking and Kocks [36] developed a phenomenological model (the KM model) to characterize the rate of dislocation density as functions of dislocation storage and annihilation. The evolution of dislocation density ρ can be expressed as [37]:

$$\frac{d\rho}{d\varepsilon} = k_1\sqrt{\rho} - k_2\rho \quad (8)$$

where ε is plastic strain; k_1 material constant; k_2 thermally activated process of dynamic recovery by dislocation cross-slip and recombination or dislocation climb, which is a function of temperature and strain rate.

Because the absolute density of dislocations in a material is difficult to determine, the normalized dislocation density concept introduced by Lin et al. [34], is considered for the constitutive modeling of the evolution of the dislocation density growth rate as expressed by Eq.(9)

$$\dot{\bar{\rho}} = A_1(1 - \bar{\rho})\dot{\varepsilon}_p - A_2\bar{\rho}^{C_1} \quad (9)$$

Where A_1 is a material constant; A_2 and C_1 are temperature dependent parameters; $\bar{\rho}$ represents the normalized dislocation density formulated as $\bar{\rho} = 1 - \rho_0/\rho$; ρ_0 is the initial dislocation density, and ρ is the dislocation density during the deformation process. The $\bar{\rho}$ equals zero in the fully annealed material. For the considerably large plastic deformation, $\rho \gg \rho_0$, thus $\bar{\rho} \rightarrow 1$, corresponding to the saturated state.

3.3.3. Modeling of isotropic hardening

The hardening parameter H in Eq. (7) is the isotropic work hardening, which is closely related to the dislocation density and can be expressed as:

$$\dot{H} = B\dot{\bar{\rho}}^\gamma \quad (10)$$

Where $\dot{H}$ is the isotropic hardening rate, $\dot{\bar{\rho}}$ is the normalized dislocation density growth rate, B is a temperature dependent parameter and γ is a material constant which shows the range from 0.3 to 1.0 in general [38].

3.3.4. Modeling of the softening mechanism

One of the important phenomenon in high temperature deformation is flow softening. It influences the macroscopic deformation behavior of the material. Moreover, flow softening is an important reason for flow instability, which may result in forming defects. As shown in Fig. 6, continuous flow softening was observed after the peak flow stress was reached at a strain rate of $0.01s^{-1}$. Many studies [39-42] indicate that the softening phenomenon in the hot deformation is closely related with the dynamic recrystallization [33] or globularization [6]. Therefore for this study, the expression is modified as:

$$\dot{S} = \eta \dot{\epsilon}_p^{\eta_1} (\bar{\rho} - \bar{\rho}_c)^{\eta_2} (\bar{\rho} - S) \quad (11)$$

Where $\dot{S}$ represents the evolution of volume fraction of recrystallization, η & η_2 are temperature dependent materials constants, η_1 is a material constant following Arrhenius relation, $\dot{\epsilon}_p$ is plastic strain rate, $\bar{\rho}$ is normalized dislocation density and $\bar{\rho}_c$ represent the critical value of normalized dislocation density for recrystallization below which recrystallization would not occur. When a critical dislocation density is reached, new grains will nucleate in high dislocation density areas.

3.2.1. Formulation of unified viscoplastic constitutive equations

Unified viscoelastic constitutive equations consists of the plastic (rate-independent) and creep (rate-dependent) part. It enables the metal flow and microstructure evolution, such as strain hardening, recovery, recrystallization, grain size and dislocation density to be well captured and described. To represent the superplastic behavior of TA15 alloy with equiaxed microstructure, a set of proposed unified visco-plastic constitutive equations takes the form of:

$$\begin{cases} \dot{\epsilon}_p = \left(\frac{\sigma - H - k}{K} \right)^n \cdot \frac{1}{(1 - S)} \\ H = B\dot{\bar{\rho}}^\gamma \\ \dot{\bar{\rho}} = A_1(1 - \bar{\rho})|\dot{\epsilon}_p| - A_2\bar{\rho}^{c_1} \\ \dot{S} = \eta \dot{\epsilon}_p^{\eta_1} (\bar{\rho} - \bar{\rho}_c)^{\eta_2} (\bar{\rho} - S) \\ \sigma = E(\epsilon_T - \epsilon_p) \end{cases} \quad (12)$$

There are 5 temperature dependent parameters in above set of equation. These parameters are symbolized as k , K , B , η_1 and E and are formulated using the classic temperature-compensated parameters in Arrhenius relations [43], listed in Eq. set (13).

$$\left\{ \begin{array}{l} k = k_0 \exp\left(\frac{Q_k}{R_g T}\right) \\ B = B_0 \exp\left(\frac{Q_B}{R_g T}\right) \\ \eta_1 = \eta_{10} \exp\left(-\frac{Q_{\eta_{10}}}{R_g T}\right) \\ K = K_0 \exp\left(\frac{Q_K}{R_g T}\right) \\ E = E_0 \exp\left(\frac{Q_E}{R_g T}\right) \end{array} \right. \quad (13)$$

In Eq. set (13) $R_g=8.31 \text{ J}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$ (Universal gas constant); T is the absolute temperature in K

3.3. Determination of material constants

The determination of material constants for unified constitutive equations is challenging without using an effective optimization technique. Considerable studies have been conducted over the years [44-46] to formulate optimization methods for the determination of materials constant for constitutive equations. To determine the materials constants for the proposed constitutive equations a gradient-based optimization technique is used. The gradient-based optimization is used via Trust region reflective least square algorithm from MATLAB¹ Optimization tool. The objective function of the model is evaluated by determining the square of the difference between the experimental and the computed data for the flow stress in the following form:

$$f(X) = \frac{1}{M} \sum_{j=1}^M \left(\frac{1}{N_j} \sum_{i=1}^{N_j} (\sigma_{ij}^e - \sigma_{ij}^c)^2 \right) \quad (14)$$

Where $f(X)$ is the objective function evaluating the error between experimental and computed values of flow stress, X represents number of material constants in unified constitutive equations, M represent the number of experimental flow stress-strain curves, N_j is the data points on the j_{th} curve, superscript 'c' symbolizes computed values form the material model and Superscript 'e' for corresponding experimental values.

¹ <https://www.mathworks.com>

Fig. 8 shows schematic flow flowchart of the iterative procedure to determine the material constants. It comprises of numerical integration of differential equations (i.e. unified constitutive Eq. set 12) which are solved by forward Euler method. As for gradient based optimization, initial guess values of unknown constants are necessary inputs, therefore the lower and upper bounds for these 13-material constant set are estimated on the basis of information available in the literature for different types of titanium alloys [8, 47-51]. To ensure that the derived set of material constants converges to a local minimum, the iterative procedure is repeated for different initial guess values. The determined model material constants are listed in Table 2.

Fig.8. Schematic flow diagram describing numerical algorithm for optimization of material constants

Table 2 Determined values of constants for the equation set (5) for TA15.

Constant	Value	
n (-)	1.30	
k_0 (MPa)	1.40E-01	
Q_k (J/mol)	2.01E+04	
K_0 (MPa)	4.08E-02	
Q_K (J/mol)	1.07E+05	
B_0 (MPa)	6.00E-18	
Q_B (J/mol)	4.16E+05	
E_0 (MPa)	9.92E-01	
Q_E (J/mol)	8.90E+04	
η_{10} (-)	4.51E+08	
$Q_{\eta_{10}}$ (J/mol)	2.02E+05	
A_1 (-)	4.95E+01	
$\bar{\rho}_c$ (-)	5.00E-01	
γ (-)	6.20E-01	
	For temp. range 880 -900°C	For temp. range 920-940°C
A_2 (-)	2.32E-02	6.20E-01
C_1 (-)	1.27E-06	3.42
η (-)	1.11E-01	1.92
η_2 (-)	5.65E-02	8.1E-01

Fig. 9 shows the comparison between experimental (symbols) and predicted flow stress (solid lines) of TA15 alloy for the range of strain rate and temperature. In general the following conclusion can be made regarding the capability of the proposed constitute model to predict flow stress behavior during hot deformation of TA15 alloy.

- For higher strain rate (i.e. $0.01s^{-1}$) the model shows best fit for all temperatures.
- For lower temperatures, such as 880 °C and 900 °C, the equation provides best fit for almost all of the three strain rates.
- For higher deformation temperatures 920 °C, 940 °C and lower strain rates ($0.001s^{-1}$ and $0.0005s^{-1}$) the experimental data shows increase for the flow stress and the equations do not predict this behavior very well.

Fig. 9. Comparison of the experimental (symbols) and computed (solid lines) flow stress of TA15 at the strain for different deformation temperatures.

As shown in Fig. 9 the flow stress curves show a work hardening region followed by softening, for the strain rate 0.01 s^{-1} and the proposed constitutive equations successfully captured this phenomenon. But for the lower strain rates, i.e. 0.001 s^{-1} and 0.0005 s^{-1} there is no softening. Fig.9(a) and (b) shows that for lower temperature of 880 °C and 900 °C and for all the three strain rates these equations can capture the flow stress pattern quite effectively. But Fig. 9(c) and (d) shows that for higher temperature of 920 °C and 940 °C there is good fit for the higher strain rate of 0.01 s^{-1} but for the lower strain of 0.001 s^{-1} and 0.0005 s^{-1} there is considerable increase in the flow stress, which is caused by grain growth as observed in the section 3.4.

A statistical analysis was performed to quantitatively evaluate the proposed unified constitutive equations. The correlation coefficient (R), the average absolute relative error (AARE) and the root mean square error (RMSE), which represent the degree of linear correlation, the unbiased statistical parameter and the difference between the predicted and calculated values, respectively, can be expressed as follows

$$R = \frac{\sum_{i=1}^N (E_i - \bar{E})(P_i - \bar{P})}{\sqrt{\sum_{i=1}^N (E_i - \bar{E})^2 \sum_{i=1}^N (P_i - \bar{P})^2}} \quad (15)$$

$$AREE(\%) = \frac{1}{N} \sum_{i=1}^N \left| \frac{E_i - P_i}{E_i} \right| \times 100 \quad (16)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (E_i - P_i)^2} \quad (17)$$

Where

E = experimental finding

P = predicted value obtained from the established constitutive model

$\bar{E}$, $\bar{P}$ = mean values of E and P respectively

N= total number of data selected in the study

The correlation between the computed and experimental stress at various temperatures is shown in Fig.

10. It is clearly seen that most of the data points lie very close to the best fitting line, especially the case of low temperature of 880 and 900 °C. At high temperature (920 & 940 °C) and low strain rate (0.001 & 0.0005 s⁻¹), it can be seen that the data points deviate farther from the best fitting line.

Fig. 10. Correlation between the experimental and computed data of TA15: flow stress

3.4. Microstructure evolution

To shed some light on the microstructure evolution during superplastic forming process and clarify the hardening effect at high temperature and low strain rate, one characteristic stress strain curve of 920 °C and 0.001s⁻¹ was selected to conduct the interrupted microstructure characterization. The SEM images are shown in Fig. 11. It suggests the grain growth of the samples deformed at that superplastic

condition. Significant grain growth at high temperature generally leads to strain hardening, thereby increasing flow stress [52]. It also means that the observed strain hardening for the curves (Fig. 6) at strain rate of $0.001s^{-1}$ and $0.0005s^{-1}$ with the temperature of $920 - 940^{\circ}C$, can be associated with the dynamic grain growth. The proposed model can be improved by incorporating grain boundary sliding/rotation phenomenon as internal state variables [53].

Fig. 11. Microstructure evolution of TA15 for $920^{\circ}C$ and $0.001s^{-1}$ where point a, b, c and d corresponds to true strain 0, 0.7, 1 and 1.4 respectively.

3.5. SPF test results

The applied experimental pressure profile is shown in Fig 3. The peak pressure value of 4.0 MPa was achieved at 1500 seconds. As shown in Fig 12, the experimental setup resulted in successfully forming the first three boxes with acceptable variation of sheet thickness in the range of 0.18mm to 0.2mm across the formed components. The Box 4 was fractured with thickness of 0.27mm which is thicker than the thickness of the formed Box 3 so the possible reason for the fracture of Box 4 is due to the higher stresses rather than thinning. It is not an unreasonable speculation, as the fracture surface of Box 4 seems erupted compared to the thinning ones seen in Box5-7.

Fig. 12. View of a work piece (Row 2 in Fig.3a) after forming for the deformation temperature Of 920°C and strain rate of 0.001s^{-1}

3.6. FEA Simulation results

The objective of this superplastic process simulation was to validate the accuracy of the proposed material model. The exact same gas pressure profile as the experimental one was implemented in the developed FE model. The thickness data corresponding to the eight measuring points in Fig.13(a) were analyzed and results are presented in Fig.13(b). A good agreement between simulation predictions and experimental data are made. The last 4 boxes ruptured in experiment, which is also confirmed by the simulation as the thickness distribution for the last 4 boxes are in the range of 0.38mm to 0.04mm, in which 0.04mm is vanishingly thin magnitude and considered as a rupture. Based on these results it can demonstrate that proposed model can accurately predict the plastic deformation during superplastic forming.

Fig.13. Comparison of thickness(mm) for part produced through SPF using experimental pressure profile vs FE-simulation for same experimental pressure profile (a) Thickness Profile (b)Thickness close to the corner and center(labeled as point 1,2,3,4,5,6,7,8) of the corresponding boxes.

4. Conclusion

This study is to develop a set of new viscoplastic constitutive model to accurately simulate the superplastic process of TA15 alloys. A range of thermo-mechanical tensile tests were conducted to characterize the superplastic behavior in the temperature range of $880-940^{\circ}\text{C}$ and the strain-rate of $0.0005 - 0.01\text{s}^{-1}$. A set of mechanism-based unified visco-plastic constitutive equations has been proposed and calibrated based on the results of stress-strain data. A gradient-based optimization method is applied for the calibration of constitutive equations. The constitutive model is incorporated into FEA code through creep subroutine to check the validity of the proposed material model against the experimental SPF test of a multi-box die. The main conclusion can be drawn as following:

1. Among the ranges of temperatures and strain rates, the optimal superplastic regime (795% elongation) is identified and applied practically at the temperature 920 °C and strain rate of 0.001s^{-1}
2. The proposed constitutive equations and determined constants were able to predict the experimentally obtained 12 stress strain curves with reasonable accuracy.
3. Significant grain growth was found at the optimal superplastic forming condition. The grain growth leads to a hardening effect on the flow stress
4. The proposed constitutive equations used to simulate the superplastic forming process on boxes. The simulated thickness variation agreed well with experimental measurement.

Data Availability:

The raw/processed data required to reproduce these findings cannot be shared at this time as the data also forms part of an ongoing study.

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