- [12] J.-J. E. Slotine and W. Li, *Applied Nonlinear Control*. Englewood Cliffs, NJ: Prentice-Hall, 1991.
- [13] C.-Y. Su, Y. Stepanenko, J. Svoboda, and T. P. Leung, "Robust adaptive control of a class of nonlinear systems with unknown backlash-like hysteresis," *IEEE Trans. Autom. Control*, vol. 45, no. 12, pp. 2427–2432, Dec. 2000.
- [14] G. Tao and P. V. Kokotovic, "Adaptive control of plants with unknown hystereses," *IEEE Trans. Autom. Control*, vol. 40, no. 2, pp. 200–212, Feb. 1995.
- [15] G. Tao and F. L. Lewis, Eds., *Adaptive Control of Nonsmooth Dynamic Systems*. New York: Springer-Verlag, 2001.
- [16] V. I. Utkin, "Variable structure systems with sliding mode: a survey," *IEEE Trans. Autom. Control*, vol. AC-22, no. 2, pp. 212–222, Apr. 1977.
- [17] A. Visintin, Ed., *Phase Transitions and Hysteresis. Lecture Notes in Mathematics*. Berlin, Germany: Springer-Verlag, 1994, vol. 1584.
- [18] J. Zhou, C. Wen, and Y. Zhang, "Aptive backstepping control of a class of uncertain nonlinear systems with unknown backlash-like hysteresis," *IEEE Trans. Autom. Control*, vol. 49, no. 10, pp. 1751–1757, Oct. 2004.
- [19] X. Tan and J. S. Baras, "Recursive identification of hysteresis in smart materials," in *Proc. 2004 Amer. Control Conf.*, Boston, MA, Jun. 30–Jul. 2 2004, pp. 3857–3862.

A New Solution to the Problem of Range Identification in Perspective Vision Systems

Dimitrios Karagiannis and Alessandro Astolfi

Abstract—A new solution to the problem of range identification for perspective vision systems is proposed. These systems arise in machine vision problems, where the position of an object moving in the three-dimensional space has to be identified through two-dimensional images obtained from a single camera. The proposed identifier yields asymptotic estimates of the object coordinates and is significantly simpler than existing designs. Moreover, it can be easily tuned to achieve the desired convergence rate. Simulations are provided demonstrating the enhanced performance of the proposed scheme and its robustness to measurement noise.

Index Terms—Machine vision, nonlinear observer, perspective system.

I. INTRODUCTION

A classical problem in machine vision is to determine the position of an object moving in the three-dimensional space by observing the motion of its projected feature on the two-dimensional image space of a charge-coupled device (CCD) camera. The case where the motion of the object is described by linear (time-varying) dynamics with known parameters has received particular attention, see e.g. [1]–[4].

The systems that arise in this case are known as perspective dynamical systems and the problem of determining the object space coordinates reduces to the problem of estimating the depth (or range) of the object. Higher-dimensional perspective systems, but with constant motion parameters, have also been considered, see e.g. [5]. Alternatively, the (dual) problem of estimating the motion parameters when the three-dimensional coordinates are available for measurement has been studied in [6]–[9].

Manuscript received November 3, 2004; revised April 28, 2005. Recommended by Associate Editor M. Demetriou. The work of D. Karagiannis was supported in part by BAE Systems and the EPSRC via the FLAVIIR project (<http://www.flaviir.com>). The work of A. Astolfi was supported in part by the Leverhulme Trust.

The authors are with the Department of Electrical and Electronic Engineering, Imperial College, London SW7 2AZ, U.K. (e-mail: d.karagiannis@imperial.ac.uk; a.astolfi@imperial.ac.uk).

Digital Object Identifier 10.1109/TAC.2005.860269

Fig. 1. Diagram of the perspective vision system.

In this note, a solution to the range identification problem is proposed based on a new nonlinear observer design which is inspired by the recently developed immersion and invariance methodology [10] and the reduced-order observer in [11].

The proposed scheme achieves asymptotic convergence of the observation error to zero and is considerably simpler than the fourth-order asymptotic observer proposed in [3], as well as the fifth-order approximate observer in [2], and the high-gain observer proposed in [1]. Moreover, it can be easily tuned to achieve the desired convergence rate.

II. PROBLEM FORMULATION

The motion of an object undergoing rotation, translation and linear deformation can be described by the affine system [3]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad (1)$$

where $(x_1, x_2, x_3) \in \mathbb{R}^3$ are the unmeasurable coordinates of the object in an inertial reference frame with x_3 being perpendicular to the camera image space, as shown in Fig. 1. The motion parameters $a_{ij} = a_{ij}(t)$, $b_i = b_i(t)$ are possibly time-varying and are assumed known.

Using the perspective (or "pinhole") model for the camera, the measurable coordinates on the image space are given by

$$y = [y_1, y_2]^T = \epsilon \begin{bmatrix} x_1 & x_2 \\ x_3 & x_3 \end{bmatrix}^T \quad (2)$$

where ϵ is the focal length of the camera, i.e. the distance between the camera and the origin of the image-space axes. Without loss of generality, we assume that $\epsilon = 1$.

The perspective system (1) must satisfy the following assumption.

Assumption 1: The parameters a_{ij} , b_i in (1) and the coordinates y_1 , y_2 in (2) are bounded functions of time, i.e. $a_{ij}(t), b_i(t) \in \mathcal{L}_\infty$, for all $i, j = 1, 2, 3$ and $y(t) \in \mathcal{L}_\infty$. Moreover, $a_{ij}(t)$ and $b_i(t)$ are first-order differentiable and $x_3(t) < \epsilon = 1$, where ϵ is as in (2).

Remark 1: Assumption 1 is motivated by the physical properties of the perspective system, see [1] and [3]. Note that in [3] it is further assumed that the functions $b_i(t)$ are twice differentiable and that $x_3(t) \in \mathcal{L}_\infty$.

The design objective is to reconstruct the coordinates x_1, x_2, x_3 from measurements of the image-space coordinates y_1, y_2 .

III. RANGE IDENTIFICATION

As in [1], [3], the first step is to define the (unmeasurable) variable

$$\eta = \frac{1}{x_3}$$

and rewrite the system (1), (2) in the (η, y) coordinates, i.e.,

$$\begin{aligned} \dot{\eta} &= -(a_{31}y_1 + a_{32}y_2 + a_{33})\eta - b_3\eta^2 \\ \dot{y} &= \begin{bmatrix} a_{11} - a_{33} & a_{12} \\ a_{21} & a_{22} - a_{33} \end{bmatrix} y + \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix} \\ &\quad - yy^T \begin{bmatrix} a_{31} \\ a_{32} \end{bmatrix} + \begin{bmatrix} b_1 - b_3y_1 \\ b_2 - b_3y_2 \end{bmatrix} \eta. \end{aligned} \quad (3)$$

Note that, when η is known, the coordinates x_1, x_2, x_3 can be directly obtained from (2), hence the problem reduces to constructing an asymptotic observer for the state η .

To this end, consider a reduced-order observer¹ of the general form

$$\dot{\hat{\eta}} = \alpha(y, \hat{\eta}, t) \quad (4)$$

with state $\hat{\eta} \in \mathbb{R}$ and the error variable

$$z = \hat{\eta} - \eta + \beta(y, t). \quad (5)$$

The proposed design consists in finding two functions $\alpha(\cdot)$ and $\beta(\cdot)$ such that the error z converges to zero. Then an asymptotic estimate $\bar{\eta}$ of η is given by

$$\bar{\eta} = \hat{\eta} + \beta(y, t). \quad (6)$$

Toward this goal, consider the dynamics of z , which are given by

$$\begin{aligned} \dot{z} &= \dot{\hat{\eta}} + (a_{31}y_1 + a_{32}y_2 + a_{33})(\hat{\eta} + \beta(y, t) - z) \\ &\quad + b_3(\hat{\eta} + \beta(y, t) - z)^2 + \frac{\partial \beta}{\partial t} \\ &\quad + \frac{\partial \beta}{\partial y} \left(\begin{bmatrix} a_{11} - a_{33} & a_{12} \\ a_{21} & a_{22} - a_{33} \end{bmatrix} y + \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix} \right. \\ &\quad \left. - yy^T \begin{bmatrix} a_{31} \\ a_{32} \end{bmatrix} + \begin{bmatrix} b_1 - b_3y_1 \\ b_2 - b_3y_2 \end{bmatrix} (\hat{\eta} + \beta(y, t) - z) \right) \end{aligned}$$

where

$$\frac{\partial \beta}{\partial y} = \left[\frac{\partial \beta}{\partial y_1}, \frac{\partial \beta}{\partial y_2} \right]$$

and note that

$$(\hat{\eta} + \beta(y, t) - z)^2 = (\hat{\eta} + \beta(y, t))^2 - z^2 - 2\eta z.$$

Selecting the function $\alpha(\cdot)$ as

$$\begin{aligned} \alpha(y, \hat{\eta}, t) &= (a_{31}y_1 + a_{32}y_2 + a_{33})(\hat{\eta} + \beta(y, t)) \\ &\quad - b_3(\hat{\eta} + \beta(y, t))^2 - \frac{\partial \beta}{\partial t} \\ &\quad - \frac{\partial \beta}{\partial y} \left(\begin{bmatrix} a_{11} - a_{33} & a_{12} \\ a_{21} & a_{22} - a_{33} \end{bmatrix} y \right. \\ &\quad \left. + \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix} - yy^T \begin{bmatrix} a_{31} \\ a_{32} \end{bmatrix} \right. \\ &\quad \left. + \begin{bmatrix} b_1 - b_3y_1 \\ b_2 - b_3y_2 \end{bmatrix} (\hat{\eta} + \beta(y, t)) \right) \end{aligned} \quad (7)$$

yields the error dynamics

$$\dot{z} = - \left(a_{31}y_1 + a_{32}y_2 + a_{33} + 2b_3\eta + \frac{\partial \beta}{\partial y} \begin{bmatrix} b_1 - b_3y_1 \\ b_2 - b_3y_2 \end{bmatrix} \right) z - b_3z^2. \quad (8)$$

It remains to find a function $\beta(\cdot)$ such that the system (8) has an asymptotically stable equilibrium at the origin. The procedure is outlined in the proof of the following statement.

¹The term “reduced-order” refers to the fact that, in contrast with previous designs [1]–[3], the proposed observer has *lower* dimension than the system (3).

Proposition 1: Consider the system (3), (4), where $\alpha(\cdot)$ is given by (7), and suppose that

$$(b_1 - b_3y_1)^2 + (b_2 - b_3y_2)^2 > \delta \quad (9)$$

for some constant $\delta > 0$. Then there exists a function $\beta(y, t)$ such that the system (8) has a uniformly semi-globally asymptotically stable equilibrium at $z = 0$.

Proof: To begin with, note that by Assumption 1 there exists a known positive constant c such that

$$|a_{31}y_1 + a_{32}y_2 + a_{33} + 2b_3\eta| < c.$$

Hence, the term inside the parenthesis in (8) can be made strictly positive by selecting a function $\beta(y, t)$ that solves the partial differential equation

$$\frac{\partial \beta}{\partial y_1}(b_1 - b_3y_1) + \frac{\partial \beta}{\partial y_2}(b_2 - b_3y_2) = \kappa(t) \quad (10)$$

with $\kappa(t) \geq c$. A solution to the previous equation is given by

$$\beta(y, t) = \frac{\lambda}{2} [(-y_1^2 - y_2^2)b_3 + 2b_1y_1 + 2b_2y_2] \quad (11)$$

with

$$\kappa(t) = \lambda [(b_1 - b_3y_1)^2 + (b_2 - b_3y_2)^2]$$

where $\lambda > 0$ is a constant design parameter such that $\lambda\delta \geq c$. It remains to show that, for any set of initial conditions $z(0)$, there exists λ such that the trajectories $z(t), t \geq 0$, are bounded and asymptotically converge to zero.

To this end, consider again the system (8) with $\beta(\cdot)$ defined by (11) and the candidate Lyapunov function

$$V(z) = \frac{1}{2}z^2$$

whose time-derivative along the system trajectories satisfies

$$\begin{aligned} \dot{V} &\leq -(\lambda\delta - c)z^2 - b_3z^3 \\ &= -z^2[(\lambda\delta - c) + b_3z]. \end{aligned}$$

As a result, the origin $z = 0$ is a uniformly asymptotically stable equilibrium for the system (8) with a region of attraction containing the invariant set

$$B = \left\{ z \in \mathbb{R} : |z| < \frac{\lambda\delta - c}{b_3^+} \right\} \quad (12)$$

where

$$b_3^+ = \max_{t \geq 0} |b_3|.$$

The proof is completed by noting that, for any set of initial conditions $z(0)$, there exists λ (sufficiently large) such that $z(0) \in B$. ■

Remark 2: The proof of Proposition 1 is constructive: it provides a way of designing the function $\beta(\cdot)$ so that the system (8) is uniformly asymptotically stable, hence the observer error z converges to zero. Note, however, that the selection (11) is not unique. Other solutions to (10) can be obtained by selecting the function $\kappa(t)$ in a different way.

Remark 3: The inequality (9) is a sufficient condition for observability of the system (3). In particular, it is obvious from (3) that when $(b_1 - b_3y_1)^2 + (b_2 - b_3y_2)^2 = 0$, the state η becomes unobservable from the output y . For $b_3 \neq 0$ this happens when the projected feature is at the point $(y_1, y_2) = (b_1/b_3, b_2/b_3)$, which is known as the *focus of expansion* [1].

Fig. 2. Time history of the observation error. Solid line: proposed observer. Dashed line: observer in [3], dash-dotted line: observer in [2]. Dotted line: observer in [1].

Remark 4: In order to obtain a more practically useful condition on the parameter λ , suppose that the observer is initialized according to

$$\bar{\eta}(0) = \hat{\eta}(0) + \beta(y(0), 0) \geq \frac{1}{\epsilon} = 1.$$

Then, from (5) and Assumption 1, we have

$$|z(0)| = |\bar{\eta}(0) - \eta(0)| < \bar{\eta}(0).$$

Hence, the selection

$$\lambda \geq \frac{b_3^+ \bar{\eta}(0) + c}{\delta} \quad (13)$$

is such that $z(0) \in B$, where B is given by (12).

IV. SIMULATION RESULTS

In this section, the proposed observer is tested via numerical simulations and compared with the ones in [1]–[3]. Consider the example given in [2], [3] of the perspective system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -0.2 & 0.4 & -0.6 \\ 0.1 & -0.2 & 0.3 \\ 0.3 & -0.4 & 0.4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.25 \\ 0.3 \end{bmatrix}$$

with the initial conditions

$$[x_1(0) \ x_2(0) \ x_3(0)]^T = [1 \ 1.5 \ 2.5]^T.$$

The proposed first-order observer is described by (4), (6), and (7), where the function $\beta(\cdot)$ is given by (11). Note that, since b_1, b_2, b_3 are constant, the function $\beta(\cdot)$ does not depend explicitly on time, i.e. $\partial\beta/\partial t = 0$. The estimate of the range x_3 is given by²

$$\hat{x}_3 = \frac{1}{\bar{\eta}}.$$

The constant λ in (11) has been selected sufficiently large so as to satisfy the constraints set out in the proof of Proposition 1. In this case, for $\delta = 0.1, c = 0.5$ and $\bar{\eta}(0) = 1$, from (13), we obtain $\lambda \geq 8$.

Fig. 2 shows the time history of the observation error $x_3 - \hat{x}_3$ for $\lambda = 30$, and for the observers proposed in [1]–[3]. Note that for the observer

²The boundedness of $\bar{\eta}$ away from zero is ensured by the initial condition $\bar{\eta}(0) \geq 1$ and the fact that z is decreasing.

Fig. 3. Time history of the observation error when the measurements are corrupted by noise. Solid line: proposed observer. Dashed line: observer in [3]. Dash-dotted line: observer in [2]. Dotted line: observer in [1].

Fig. 4. Time history of the observation error for different values of λ .

in [1] we have selected the gain $G = 50$. We see that the transient performance of the proposed observer is significantly superior to the one in [3] and comparable to the ones in [1] and [2]. Note, however, that the observer in [2] is only guaranteed to converge to a neighborhood of the true state, while both [1] and [2] are harder to implement, since they are of higher order and rely on a resetting mechanism to keep the estimates bounded.

Fig. 3 shows the time history of the observation error $x_3 - \hat{x}_3$ for the case when the measurements of y_1 and y_2 are corrupted by 1% random noise. We see that the proposed scheme is relatively robust to measurement noise. However, the presence of this type of noise has a slightly stronger effect on the proposed observer compared to [1] and [2]. This is justified by the fact that from (5) the dynamics of the observation error are driven by the derivative of the measured output.

The performance of the proposed observer for different values of the gain λ , namely $\lambda = 10, \lambda = 20$, and $\lambda = 30$, for the ideal case and for the case of measurement corrupted by noise is depicted in Fig. 4 and Fig. 5 respectively. Note that the convergence rate can be arbitrarily increased simply by increasing the parameter λ . However, the sensitivity to the presence of noise also increases in this case.

Fig. 5. Time history of the observation error for different values of λ when the measurements are corrupted by noise.

Fig. 6. Time histories of the object coordinates x_1, x_2, x_3 (solid line) and their estimates $\hat{x}_1, \hat{x}_2, \hat{x}_3$ (dashed line).

Finally, note that from (2) and the definition of η the estimates of the coordinates x_1, x_2 are given by

$$\hat{x}_1 = \frac{y_1}{\epsilon\eta} \quad \hat{x}_2 = \frac{y_2}{\epsilon\eta}$$

respectively. Fig. 6 shows the time histories of the states x_1, x_2 , and x_3 along with their corresponding estimates obtained using the proposed observer with $\lambda = 30$.

V. CONCLUSION

A new solution to the problem of range identification for perspective vision systems has been proposed based on a nonlinear reduced-order observer. The resulting identifier yields asymptotic estimates of the three-dimensional object space coordinates using measurements of the two-dimensional image space coordinates obtained from a single camera. The proposed design has been tested via numerical simulations and has been shown to have good transient performance with assignable convergence rate. Although we have only considered linear time-varying motion dynamics, the proposed observer is also applicable to the case when the object motion is described by the more general Riccati dynamics [9]. Further work on this problem is currently in progress.

REFERENCES

- [1] M. Jankovic and B. Ghosh, "Visually guided ranging from observations of points, lines and curves via an identifier based nonlinear observer," *Syst. Control Lett.*, vol. 25, no. 1, pp. 63–73, 1995.
- [2] X. Chen and H. Kano, "A new state observer for perspective systems," *IEEE Trans. Autom. Control*, vol. 47, no. 4, pp. 658–663, Apr. 2002.
- [3] W. Dixon, Y. Fang, D. Dawson, and T. Flynn, "Range identification for perspective vision systems," *IEEE Trans. Autom. Control*, vol. 48, no. 12, pp. 2232–2238, Dec. 2003.
- [4] L. Ma, Y. Chen, and K. Moore, "Range identification for perspective dynamic system with single homogeneous observation," in *Proc. IEEE Int. Conf. Robotics and Automation*, New Orleans, LA, 2004, pp. 5207–5211.
- [5] R. Abdursul, H. Inaba, and B. Ghosh, "Nonlinear observers for perspective time-invariant linear systems," *Automatica*, vol. 40, no. 3, pp. 481–490, 2004.
- [6] R. Tsai and T. Huang, "Estimating three-dimensional motion parameters of a rigid planar patch," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 29, no. 6, pp. 1147–1152, Jun. 1981.
- [7] B. Ghosh and E. Loucks, "A perspective theory for motion and shape estimation in machine vision," *SIAM J. Control Optim.*, vol. 33, no. 5, pp. 1530–1559, 1995.
- [8] S. Soatto, R. Frezza, and P. Perona, "Motion estimation via dynamic vision," *IEEE Trans. Autom. Control*, vol. 41, no. 3, pp. 393–413, Mar. 1996.
- [9] B. Ghosh, H. Inaba, and S. Takahashi, "Identification of Riccati dynamics under perspective and orthographic observations," *IEEE Trans. Autom. Control*, vol. 45, no. 7, pp. 1267–1278, Jul. 2000.
- [10] A. Astolfi and R. Ortega, "Immersion and invariance: a new tool for stabilization and adaptive control of nonlinear systems," *IEEE Trans. Autom. Control*, vol. 48, no. 4, pp. 590–606, Apr. 2003.
- [11] D. Karagiannis, A. Astolfi, and R. Ortega, "Two results for adaptive output feedback stabilization of nonlinear systems," *Automatica*, vol. 39, no. 5, pp. 857–866, 2003.