A transient one-dimensional numerical model for kinetic Stirling engine

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Abstract

A third-order numerical model based on one-dimensional computational fluid dynamics is developed for kinetic Stirling engines. Various loss mechanisms in Stirling engines, including gas spring hysteresis loss, shuttle loss, appendix displacer gap loss, gas leakage loss, finite speed loss, piston friction loss, pressure drop loss, heat conduction loss, mechanical loss and imperfect heat transfer, are considered and embedded into the basic control equations. The non-equilibrium thermal model is adopted for the regenerator to capture the oscillating features of the gas and solid temperatures. To improve the numerical stability and accuracy, the implicit second-order time difference scheme and the second-order upwind scheme are adopted for discretizing the time differential terms and convective terms, respectively. Experimental validations are then conducted on a beta-type Stirling engine with the extensive experimental data for diverse working conditions. The results show that the developed model has better accuracies over previous second-order models. Good agreements are achieved for predicting various critical system parameters, including pressure-volume diagram, indicated power, brake power, indicated efficiency, brake efficiency and mechanical efficiency. In particular, both the experiments and simulations show that the Stirling engine charged with helium tends to have much lower optimal working frequencies and poorer performances compared to the hydrogen system. Based on the analyses of the losses, it reveals that the pressure drop in the flow channels plays a critical role in shaping the different behaviors. The pressure drop in the helium system is much larger and more sensitive to the frequency increase due to the much larger viscosity of gaseous helium. Hydrogen is a superior working gas for a Stirling engine. The transient characteristics of the oscillating flow and the associated thermal interactions between gas and solid in the regenerator are finally analyzed in order to have an insight of the complex thermodynamic process. The study provides a promising numerical approach in simulating Stirling engines for further understandings of their operating characteristics and the underling mechanisms.

Keywords: Stirling engine, regenerator, pressure drop, third-order model, oscillating flow, non-equilibrium thermal model

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9 Nomenclature

a	coefficient for finite speed pressure loss
$a_{1,2,3,4,5}$	coefficients for discretized momentum equation
A	cross sectional area, m ²
A_1	small area of cone joint, m ²
A_2	large area of cone joint, m ²
A_{wet}	contact area, m ²
b_k	coefficient for thermal conductivity
b_μ	coefficient for viscosity
$b_{1,2,3,4,5}$	coefficients for discretized gas energy equation
B	coefficient for discretized momentum equation
c	average molecular speed, m/s
c_p	isobaric specific heat, J/(kg·K)
c_s	specific heat of solid, J/(kg·K)
c_v	isochoric specific heat, J/(kg·K)
$c_{1,2,3}$	coefficients for discretized solid energy equation
C	coefficient for discretized gas energy equation
d_h	hydraulic diameter, m
d_x	control volume length, m
D	coefficient for discretized solid energy equation
D_d	displacer diameter, m
D_p	power piston diameter, m
e	eccentricity of rhombic drive, m
E	effectiveness of drive mechanism
f	friction factor
f	frequency, Hz
g	power piston gap, m
$G_{1,2..7,8}$	coefficients for convection term
h	displacer gap, m
L	connecting rod length, m
L_d	displacer length, m
L_p	power piston length, m
k	thermal conductivity, W/(m·K)
K	head loss coefficient
K_l	local head loss coefficient
m	mass, kg
$\dot{m}$	mass flow rate, kg/s
Nu	Nusselt number
p	pressure, Pa
p_b	pressure in buffer space, Pa
p_m	mean pressure, Pa

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Pr	Prandtl number
Q_{app}	appendix displacer gap losses, W
Q_h	heating power, W
Q_{hys}	gas spring hysteresis loss, W
Q_{leak}	gas leakage loss, W
Q_{sh}	shuttle heat transfer loss, W
Re	Reynolds number
r	crank radius, m
R_g	gas constant, J/(kg·K)
$R_{1,2,3}$	coefficients for gas energy equation
$S_{1,2,3}$	coefficients for solid energy equation
t	time, s
T	temperature, K
T_a	ambient temperature, K
T_g	gas temperature, K
T_L	leakage gas temperature, K
$T_{1,2,3}$	coefficients for momentum equation
u	velocity, m/s
u_p	power piston speed, m/s
v_p	piston speed, m/s
V	volume, m ³
V_b	buffer space volume, m ³
V_c	compression space volume, m ³
V_{clc}	clearance volume of compression space, m ³
V_{cle}	clearance volume of expansion space, m ³
W_b	dissipated power in buffer space, W
W_{bra}	brake power, W
W_{com}	compression power, W
W_{exp}	expansion power, W
W_{ind}	indicated power, W
W_-	forced work, W
x	longitude coordinate, m
X_d	displacer displacement amplitude, m
X_i	coefficient for continuity equation
Y_i	coefficient for continuity equation

12	Greek letters
	α heat transfer coefficient, W/(m ² ·K)
	α_{hys} hysteresis heat transfer coefficient, W/(m ² ·K)
	γ specific heat ratio
	δp_{finite} finite pressure drop, Pa
	δp_{fric} piston friction pressure drop, Pa
	δt time step, s
	δx node distance, m
	Δp pressure amplitude, Pa
	ΔV_b buffer space volume variation amplitude, m ³
13	η_{bra} brake efficiency
	η_{ind} indicated efficiency
	η_{mech} mechanical efficiency
	θ phase difference, rad
	λ conductivity factor
	μ viscosity, kg/(m·s)
	ρ density, kg/m ³
	τ period of one cycle, s
	ϕ porosity
	ω angular frequency, rad/s
14	Subscripts and superscripts
	b buffer space
	cv control volume
	d displacer
	f face
	i control volume number
	k time layer number
15	p power piston
	r regenerator
	s solid
	si control volume number for solid
	so control volume number for solid
	t tube
	w wall
16	Abbreviations
	PDMA Penta-Diagonal Matrix Algorithm
17	TDMA Tri-Diagonal Matrix Algorithm

1. Introduction

A Stirling engine is an external combustion engine based on cyclic compression and expansion of gas at different temperature levels. It attained substantial developments and commercial applications between 1816 and 1900. In twentieth century, however, due to the booming developments of internal combustion engines and oil industry, much less efforts were devoted into Stirling engines. In recent decades, with the growing energy needs and the threat of climate change, technologies for renewable energy and energy efficiency are becoming increasingly important for a sustainable future. Stirling engines have brought back worldwide attentions again because of their flexibility for fuel and promising prospects in solar power generation and low-grade heat recovery [1–4]. In the developments of Stirling engines, a precise model is highly desirable for predicting thermal performances and characterizing key operation features. Many models have been presented in recent years, including empirical [5–8], analytical [9–18] and numerical [19–54] approaches. For numerical models, they can be categorized into second-order [19–46], third-order [47–50] and multi-dimensional computational fluid dynamics [51–54] methods.

Empirical models correlate the output power and efficiency of a Stirling engine to the heater and cooler temperatures, piston displacement, engine speed and mean pressure based on experimental data [5–8]. Details for the engine are totally ignored in these models. They can only be used for roughly estimating thermal performances for preliminary designs.

Analytical models refer to the simplified approaches that have closed form solutions. Most of them were derived based on idealized Stirling cycles rather than practical engines [9–14]. These studies were mainly focused on optimizations of the cycles for minimizing losses and maximizing performances. Some analytical models were developed for practical Stirling engines [15–18]. Schmidt derived the expressions for indicated power by assuming sinusoidal volume variations, ideal regeneration and isothermal compression and expansion [15]. The thermal efficiency of Schmidt analysis reduced to the Carnot efficiency due to the adopted ideal isothermal condition. Formosa and Despesse [16] improved the Schmidt analysis by introducing regenerative loss, heat exchange loss, thermal conduction loss and mechanical ineffectiveness. The accuracies of the analytical model for predicting the indicated power and thermal efficiency were comparable with some second-order numerical models [19, 23], but still relatively low. Hosseinzade et al. [17] and Ahmadi et al. [18] employed finite speed thermodynamics in analytical models. The irreversibility caused by pressure drops and imperfect regeneration was evaluated analytically. Polytropic compression/expansion processes were considered in one of the studies [17]. Analytical models studied the thermodynamic process and losses in a decoupled way, and simplified the practical Stirling engine/cycle largely. Therefore, they have difficulties in elaborately capturing the real operating characteristics of Stirling engines, and their accuracies are typically low.

Numerical models divide Stirling engines into discretized elements and solve the control equations for each element iteratively. Most of the developed numerical models fall in the category of the second-order method. Urieli and Berchowitz [19, 20] developed the ideal adiabatic model by considering adiabatic processes in compression and expansion spaces based on the isothermal model. Then, they further improved the ideal adiabatic model into the quasi-steady model and the simple analysis model by introducing the

effects of pressure drop, non-ideal heat transfer and regeneration [19]. Though the accuracies were still low, these models laid a good foundation for the follow-up developments of second-order methods. More studies were then conducted by either directly adopting Urieli and Berchowitz's models [21, 22] or modifying the ideal adiabatic model [23–30], quasi-steady model [31] or simple analysis model [32–37]. These models were improved by considering more internal or external losses [23–35], introducing the concept of finite speed thermodynamics [28, 35], or changing the treatments of compression/expansion processes in working spaces [36, 37]. They typically cut the engine into five parts, with two compression/expansion spaces, two heat exchangers and one regenerator. Bert's models only divided it into three regions [38, 39]. In the studies of Cheng and Yu [40], Aksoy et al. [41, 42] and Chen et al. [43], the models were developed for Stirling engines without any regenerator. Cheng et al. also studied the thermodynamic and dynamic behaviors of a thermal-lag Stirling engine that had only one piston based on a similar model [44, 45]. Fénies et al. [46] used an equivalent electrical network model to account for the gas, mechanical dynamics and global energy equilibrium for a free piston Stirling engine. Due to the complexity and difficulty of the methods, fewer work have been done on third-order models [47–50] and multi-dimensional computational fluid dynamics models [51–54].

As mentioned above, most of recent efforts were devoted into second-order methods. Although the accuracies of the second-order models were typically better compared to analytical models, most of them still failed to predict the performances with stable accuracies for a large range of operation conditions. They were typically only capable of predicting either indicated power or brake power, but had difficulty in predicting them both. Besides, these models simplified the practical engine largely in many aspects. For example, they discretized Stirling engines into only several elements, which is actually not adequate for precisely simulating the process due to the large temperature gradient involved and the high complexity and non-uniformity of the system. The solid material in a regenerator was manually assigned with a fixed linear temperature profile in these models, while the temperature does not actually stay constant but oscillates with time in practical systems. Many models evaluated the losses separately from the thermodynamic cycles, while Stirling engines are actually thermodynamically and kinetically highly coupled systems. To sum up, most of previous Stirling models were focusing on analytical and second-order approaches due to the simplicity for implementation. However, they have difficulties in capturing the real operating features and predicting the performances with reliable accuracies.

Third-order models compute the complex processes going on in a Stirling engine all of a piece by solving the differential equations of continuity, momentum and energy simultaneously. They can be much closer to the realities of practical Stirling engines and more accurate. Compared to the multi-dimensional computational fluid dynamics models, the third-order methods, convenient to couple with mechanical moving components, consume much fewer computational resources. In the present work, a third-order numerical model based on one-dimensional computational fluid dynamics is developed. Various losses in Stirling engines are embedded into the control equations. The non-equilibrium thermal model is adopted for the regenerator to simulate the temperature oscillations of gas and regenerator materials. The model is then validated on a practical

107 Stirling engine based on the extensive published experimental data for diverse operating
 108 conditions [19, 55, 56]. The key parameters, including pressure-volume diagrams,
 109 indicated power, brake power, indicated efficiency, brake efficiency and mechanical
 110 efficiency, are compared and analyzed. The differences between the Stirling engine charged
 111 with helium and hydrogen, which have rarely been studied before, are highlighted herein.
 112 The underling mechanism is then revealed by analyzing the engine losses. The transient
 113 characteristics are finally presented.

114 2. Mathematical model

115 2.1. Governing equations

116 A schematic of a general Stirling engine is shown in Figure 1(a). It mainly consists of a
 117 power piston, a displacer, a compression space, an expansion space, a cooler, a regenerator
 118 and a heater. The movements of the power piston and displacer are controlled by kinetic
 119 driven mechanisms, such as a crank-slider drive, rhombic drive, swash-plate drive, etc. Due
 120 to the periodical oscillation of the working gas and the large temperature gradient along
 121 the regenerator, the thermal energy is converted into mechanical power and then extracted
 122 by the drive mechanisms as output brake power.

Figure 1: Schematics for (a) a Stirling engine (b) discretized grids of a Stirling engine.

123 The following assumptions are made for the numerical model:

- 124 1. One-dimensional compressible flow is considered.
- 125 2. The working gas is in ideal state.
- 126 3. Mass leakage of the gas out of the Stirling engine is ignored.
- 127 4. Heat capacity of the walls of the cooler, heater, and compression and expansion spaces
 128 are much greater than that of the gas, and therefore the wall temperatures are taken as
 129 constants.
- 130 5. Gravity effect on the gas is ignored.

131 The finite volume method is used to discretize the computational domain, as shown in
 132 Figure 1(b). To improve numerical stability, the staggered grids are adopted. The values
 133 for the temperature, pressure, density, mass, and gas property are stored in the node points.
 134 The mass flow rate and velocity are stored at the interfaces between the adjacent control
 135 volumes, as denoted by the solid lines. The control volumes for the continuity and energy
 136 equations are between the solid lines, while those for the momentum equations are between
 137 the dash lines. The left and right ends of the grid system denote the power piston and the

138 displacer, respectively. The compression and expansion spaces are in the grids $i = 0$ and
 139 $i = n - 1$ respectively, the volumes of which are variable depending on the movements of
 140 the piston and displacer.

141 The general governing equations for the control volumes are given as follows.

142 Continuity equation for gas:

$$\frac{\partial m_i}{\partial t} + \dot{m}_{i+1} - \dot{m}_i = 0 \quad (1)$$

143 Momentum equation for gas:

$$\frac{\partial \dot{m}_i}{\partial t} \delta x_i + (\dot{m}u)_{cv,i} - (\dot{m}u)_{cv,i-1} + A_i (p_i - p_{i-1}) + \delta x_i \frac{f_i}{d_{h,i}} \frac{1}{2} \dot{m}_i |u_i| = 0 \quad (2)$$

144 Energy equation for gas:

$$\begin{aligned} & \frac{\partial}{\partial t} \left(mc_v T + m \frac{u^2}{2} \right)_{cv,i} + \dot{m}_{i+1} \left(c_p T + \frac{u^2}{2} \right)_{f,i+1} - \dot{m}_i \left(c_p T + \frac{u^2}{2} \right)_{f,i} \\ & + \left(-kA \frac{\partial T}{\partial x} \right)_{f,i+1} - \left(-kA \frac{\partial T}{\partial x} \right)_{f,i} + \alpha_{cv,i} A_{wet,i} (T_i - T_{s,i}) = 0 \end{aligned} \quad (3)$$

145 Energy equation for solid:

$$\frac{\partial}{\partial t} (m_s c_s T_s)_{cv,i} + \left(-\lambda k_s A_s \frac{\partial T_s}{\partial x} \right)_{f,i+1} - \left(-\lambda k_s A_s \frac{\partial T_s}{\partial x} \right)_{f,i} + \alpha_{cv,i} A_{wet,i} (T_{s,i} - T_i) = 0 \quad (4)$$

146 and the equation of state of ideal gas:

$$p_i V_i = m_i R_g T_i \quad (5)$$

147 For the gas in the compression and expansion spaces, the loss effects of gas spring
 148 hysteresis, shuttle heat transfer, appendix displacer gap, finite speed of piston, and piston
 149 friction are all included in the energy equations. Due to the large pressure difference between
 150 the compression space and the buffer space, the energy loss due to the leakage is considered
 151 in the compression space while ignored in the expansion space because of the much smaller
 152 pressure difference across the displacer. The energy equation in the compression space,
 153 $i = 0$, is expressed as,

$$\begin{aligned} & \frac{\partial}{\partial t} \left(mc_v T + m \frac{u^2}{2} \right)_{cv,i} + \dot{m}_{i+1} \left(c_p T + \frac{u^2}{2} \right)_{f,i+1} + \left(-kA \frac{\partial T}{\partial x} \right)_{f,i+1} \\ & - Q_{hys} - Q_{sh} - Q_{app} - Q_{leak} + (p_i + \delta p_{finite} + \delta p_{fric}) \frac{dV_i}{dt} = 0 \end{aligned} \quad (6)$$

154 Similarly, the energy equation in the expansion space, $i = n - 1$, can be written as,

$$\begin{aligned} & \frac{\partial}{\partial t} \left(mc_v T + m \frac{u^2}{2} \right)_{cv,i} - \dot{m}_i \left(c_p T + \frac{u^2}{2} \right)_{f,i} - \left(-kA \frac{\partial T}{\partial x} \right)_{f,i} \\ & - Q_{hys} + Q_{sh} + Q_{app} + (p_i + \delta p_{finite} + \delta p_{fric}) \frac{dV_i}{dt} = 0 \end{aligned} \quad (7)$$

where Q_{hys} , Q_{sh} , Q_{app} and Q_{leak} account for gas spring hysteresis loss, shuttle heat loss, appendix displacer gap losses, and gas leakage loss, respectively. The effects of the finite speed of the piston and the mechanical friction between the piston and cylinder are taken into account by using pressure correcting terms δp_{finite} and δp_{fric} , respectively.

(a) *Gas spring hysteresis loss Q_{hys}* : For the ideal expansion or compression, it is either isothermal or adiabatic process. However, for a real gas, there is always a certain amount of work dissipated, which is known as the gas spring hysteresis loss. According to Urieli and Berchowitz [19], the hysteresis loss can be evaluated by,

$$Q_{hys} = \alpha_{hys,i} A_{wet,i} (T_{s,i} - T_i) \quad (8)$$

where the hysteresis heat transfer coefficient $\alpha_{hys,i} = k_i \sqrt{\omega \rho_i c_{p,i} / 2k_{w,i}}$.

(b) *Shuttle heat transfer loss Q_{sh}* : Shuttle heat transfer loss is due to the thermal conduction down the displacer wall enhanced by the oscillatory movement of the displacer. A net heat load is transferred from the expansion space to the compression space. The shuttle loss can be calculated from [19],

$$Q_{sh} = \pi k_d X_d^2 D_d \frac{T_n - T_0}{2hL_d} \quad (9)$$

where X_d , D_d , L_d and h are the displacer displacement amplitude, displacer diameter, displacer length and the gap between the displacer and the cylinder, respectively. The gas thermal conductivity k_d is based on the average of the temperatures in the compression and expansion spaces, T_0 and T_{n-1} .

(c) *Appendix gap losses by pumping and hysteresis heat transfer Q_{app}* : Pumping loss refers to the net enthalpy transport down the displacer gap by virtue of the working gas motion, pressure and temperature. The hysteresis heat transfer loss in the displacer gap is the net heat transfer from the gas to the cylinder. The above two losses are calculated by [19],

$$Q_{app} = \pi D_d \Delta p h X_d \omega \left[\frac{\gamma}{\gamma - 1} \ln(T_n/T_0) \left(\frac{1}{2} - \frac{k}{k_s h} \sqrt{\frac{2k}{\omega \rho c_p}} \right) - \frac{1}{2} \right] \sin \theta \quad (10)$$

where Δp is pressure amplitude in the compression space, θ is the phase difference between pressure and displacer movement.

(d) *Gas leakage loss Q_{leak}* : Gas leakage through the gap between the power piston and the compression cylinder generates a net enthalpy transport, expressed as [19],

$$Q_{leak} = \pi D_p \frac{p_0 + p_b}{4R_g T_g} \left(u_p g - \frac{g^3}{6\mu_g} \frac{p_0 - p_b}{L_p} \right) c_p T_L \quad (11)$$

where D_p , u_p , L_p and g are the piston diameter, piston speed, piston length and gap between the piston and cylinder, respectively. p_0 and p_b denote the pressures in the compression and buffer spaces. T_g and μ_g are the gas temperature and dynamic viscosity in the gap. When gas leaks out of the compression space, T_L is given by the temperature of the compression space, otherwise it is the ambient temperature.

(e) *Finite speed pressure loss δp_{finite}* : According to the finite speed thermodynamic principle, the pressure over the piston surface differs from the calculated instant pressure in

the compression space. In the compression process, the pressure over the piston surface is larger than the instant pressure in the space, while it is smaller than the instant pressure in the space in the expansion process. The pressure loss is defined as the finite speed pressure loss [57],

$$\delta p_{finite} = \pm \frac{av_p}{c} \quad (12)$$

where v_p is the piston speed, the coefficient $a = \sqrt{3\gamma}$, and the average molecular speed $c = \sqrt{3R_gT}$. The sign (+) corresponds to the compression space while the sign (−) to the expansion space.

(f) *Piston friction loss* δp_{fric} : The mechanical friction between the piston and the cylinder is another source of pressure losses. The friction pressure loss δp_{fric} is estimated from the experimental data of internal combustion engines [58, 59], the correlations based on which have also been adopted for lots of Stirling engine analyses [18, 35, 60].

$$\delta p_{fric} = \pm(0.97 + 0.045v_p) \cdot 10^5 \quad (13)$$

The volumes of the compression and expansion spaces are determined by the drive mechanism of the Stirling engine, which are specified according to the given geometries. No penetration condition is assumed for both the ends,

$$\dot{m}_0 = \dot{m}_n = 0 \quad (14)$$

The compression power in the compression space is calculated by,

$$W_{com} = f \int_t^{t+\tau} \left[-(p_0 + \delta p_{finite} + \delta p_{fric}) \frac{dV_0}{dt} \right] dt \quad (15)$$

The expansion power in the expansion space is obtained by,

$$W_{exp} = f \int_t^{t+\tau} \left[-(p_{n-1} + \delta p_{finite} + \delta p_{fric}) \frac{dV_{n-1}}{dt} \right] dt \quad (16)$$

Then the indicated power is expressed as,

$$W_{ind} = W_{com} + W_{exp} \quad (17)$$

The indicated power will be partly dissipated in the crank-drive system due to mechanical losses. The rest of the indicated power will be the output power of the Stirling engine, i.e. brake power, which can be evaluated by [61–63],

$$W_{bra} = E(W_{ind} - W_b) - \left(\frac{1}{E} - E \right) W_- \quad (18)$$

where E represents the effectiveness of the drive mechanism, W_b is the dissipated power in the buffer space, and W_- is the forced work. The effectiveness E is typically in the range of 0.7–0.9, which is evaluated empirically based on experiments. The forced work W_- is calculated according to Ref. [63]. The dissipated power in the buffer space is given by [19],

$$W_b = \sqrt{\frac{1}{32} \omega \gamma^3 (\gamma - 1) T_a p_m k} \left(\frac{\Delta V_b}{V_b} \right)^2 A_{wet,b} \quad (19)$$

where ΔV_b , V_b and $A_{wet,b}$ are the volume variation amplitude, volume and the wetted area of the buffer space.

The heating power added in the heater is given by,

$$Q_h = f \int_t^{t+\tau} \sum_{heater} [\alpha_{cv,i} A_{wet,i} (T_{s,i} - T_i)] dt + \sum k_s A_s \frac{dT}{dx} \quad (20)$$

The last term accounts for the heat conduction loss through the solid materials, including the cylinder, displacer and regenerator.

The indicated efficiency η_{ind} is then expressed as,

$$\eta_{ind} = \frac{W_{ind}}{Q_h} \quad (21)$$

The brake efficiency η_{bra} is calculated by,

$$\eta_{bra} = \frac{W_{bra}}{Q_h} \quad (22)$$

The mechanical efficiency η_{mech} is determined by,

$$\eta_{mech} = \frac{W_{bra}}{W_{ind}} \quad (23)$$

2.2. Correlations and physical properties

2.2.1. Tube

The correlations for the friction factor of tube channels for laminar flow and turbulence flow are given by Eqs. (24) and (25), respectively [50, 64]:

$$f_t = \frac{64}{\text{Re}} + \frac{K}{L} \quad (24)$$

$$f_t = 0.11 \left(\frac{\varepsilon}{d_h} + \frac{68}{\text{Re}} \right)^{0.25} + \frac{K}{L} \quad (25)$$

where ε is the roughness of tube channels, and it is assigned as $2 \mu\text{m}$. $K = K_l + (1 - A_1/A_2)^2$ is the local head loss coefficient caused by bends and abrupt changes of areas.

The Nusselt numbers for laminar flow and turbulence flow in tubes are given by Eq. (26) and Eq. (27), respectively [64]:

$$\text{Nu}_t = 1.86 (\text{RePr} d_h / dx)^{0.333} (\mu / \mu_w)^{0.14} \quad (26)$$

$$\text{Nu}_t = 0.023 \text{Re}^{0.8} \text{Pr}^n \quad (27)$$

in which $n=0.4$ for heating and $n=0.3$ for cooling.

2.2.2. Regenerator screen mesh

The coefficients for the friction and Nusselt number for the screen mesh type regenerator are given by [50],

$$f_r = \frac{129}{\text{Re}} + 2.91\text{Re}^{-0.103} \quad (28)$$

$$\text{Nu}_r = \left[1 + 0.99 (\text{Re} \cdot \text{Pr})^{0.66}\right] \phi^{1.79} \quad (29)$$

2.2.3. Physical properties

Over a wide range in temperature, the viscosity and thermal conductivity of working gas obey the power laws, expressed by [65, 66],

$$k = k_0 T^{b_k} \quad (30)$$

where $k_0=0.002502$, $b_k=0.72$ for helium, and $k_0=0.002627$, $b_k=0.744$ for hydrogen.

$$\mu = \mu_0 T^{b_\mu} \quad (31)$$

where $\mu_0 = 4.1154 \times 10^{-7}$, $b_\mu=0.68$ for helium, and $\mu_0 = 1.9361 \times 10^{-7}$, $b_\mu=0.6723$ for hydrogen.

The properties of stainless steel are obtained from Ref. [66]. The density of stainless steel is,

$$\rho_s = 8274.55 - 1055.23 \times \exp\{-(T_s - 2171.05)/2058.08\}^2 \quad (32)$$

The thermal conductivity of stainless steel is calculated by,

$$k_s = (266800 T_s^{-5.2} + 0.21416 T_s^{-1.6})^{-0.25} \quad (33)$$

The heat capacity of stainless steel is calculated by,

$$c_s = (1.7054 \times 10^{-6} T_s^{-0.88962} + 23324/T_s^6)^{-1/3} + 15/T_s \quad (34)$$

2.3. Numerical method

In the time domain, the implicit second-order time difference scheme is used, except for the first time layer when the first-order method is adopted. In the space domain, the second-order upwind scheme is generally used for convection terms in the momentum and energy equations. For the grids near both ends, the first-order and second-order upwind schemes are mixed depending on the flow directions. For the thermal conductive terms, the central difference scheme is adopted [67]. The detailed discretization algebraic equations are given in the Appendix.

The continuity equation, Eq. (1), is converted into the relation between pressure and mass flow rates by substituting the equation of state, Eq. (5), into the time differential term, as given by Eq.(A.1). Then the expression of the pressure in terms of the mass flow rates are substituted into the momentum equation, Eq. (2). Therefore, the continuity equations and momentum equations are combined into one algebraic group, as given by Eq.

Table 1: Main geometric parameters of GPU-3 Stirling engine [19].

Component	Grid number	Parameter	Value
Compression space	1	Clearance volume	28.68 mm ³
		Diameter	69.9 mm
Expansion space	1	Clearance volume	30.52 mm ³
		Diameter	69.9 mm
Cooler	40	Length	46.1 mm
		Tube inner diameter	1.08 mm
		Tube external diameter	1.59 mm
		Total tube number	312
Regenerator	50	Length	22.6 mm
		Diameter	22.6 mm
		Material	Stainless steel mesh screen
		Wire diameter	0.04 mm
		Porosity	0.697
Heater	70	Number of regenerator	8
		Length	245.3 mm
		Tube inner diameter	3.02 mm
		Tube external diameter	4.83 mm
		Total tube number	40
Rhombic drive	-	Connecting rod length	46 mm
		Crank radius	13.8 mm
		Eccentricity	20.8 mm

(A.7). Due to the adopted second-order scheme in the space domain, the algebraic group of the momentum equations form a penta-diagonal matrix, which is solved by the direct method, Penta-Diagonal Matrix Algorithm (PDMA). Similarly, the energy equations of the gas, Eq. (A.13), are also solved by PDMA method. For the energy equations of the solids, the discretized equations form a tri-diagonal matrix as shown in Eq. (A.17), and are solved by Tri-Diagonal Matrix Algorithm (TDMA). The procedure for the numerical simulation is given by the flowchart shown in Figure 2. The convergence criteria for the mass flow and the temperature is defined as that the relative errors between two successive iterations are less than 10^{-7} and 10^{-6} , respectively. Periodic steady state is considered to be achieved when the relative errors between two successive time layers are within 10^{-6} for the power and efficiency.

3. Experimental apparatus

The developed numerical model is validated on the GPU-3 Stirling engine developed by General Motors Research Laboratories for the US Army in 1965 [19, 55, 56]. A lot of experimental data have been published in previous reports. The schematic of the beta-type Stirling engine with rhombic drive mechanism is shown in Figure 3. The main geometric parameters and the grid numbers are given in Table 1. The grid numbers have been validated to be large enough to obtain grid-independent results. Volume variations of the compression and expansion spaces are determined by the dimensions of the drive system using the following equations,

$$V_c = V_{clc} + 2A_p(b_1 - b_\theta) \quad (35)$$

$$V_e = V_{cle} + 2A_d(b_\theta - b_2 - r \sin \omega t) \quad (36)$$

where

$$b_1 = \sqrt{L^2 - (e - r)^2} \quad (37)$$

$$b_2 = \sqrt{(L - r)^2 - e^2} \quad (38)$$

$$b_\theta = \sqrt{L^2 - (e + r \cos \omega t)^2} \quad (39)$$

in which V_c and V_e denote the volumes of compression and expansion spaces; V_{cle} and V_{cle} are the clearance volumes of compression and expansion spaces; and L , r and e are connecting rod length, crank radius and eccentricity of the rhombic drive, as listed in Table 1.

Figure 4 illustrates the periodic volume variations of the compression space, expansion space and their total volume. The volume of expansion space changes in advance with a crank angle of 112° compared to that of the expansion space. The total volume expands from about 40° to 240° , while compression happens from 240° to 400° . During the expansion process, the volume of the compression space is at relatively small levels, and more gas is contained in the expansion space, i.e. hot part of the Stirling engine. Conversely, more gas occupies the cold part of the engine in the compression process. The combined effects of the periodic heating and cooling of the working gas and the total volume variation cause the pressure to oscillate periodically.

4. Results and discussion

4.1. Pressure-volume diagram

Figure 5 shows the numerical and experimental results of the pressure-volume diagrams when the Stirling engine is operated with a mean pressure of 2.8 MPa and a heating temperature of 650°C [55]. The operating frequency is about 50 Hz. For both working gases, the shapes of pressure-volume diagrams in the compression and expansion spaces have good agreements between the simulations and experiments. The surrounded area of the pressure-volume diagram in the compression space represents the work consumed to compress the gas while that in the expansion space illustrates the work done by the expanded gas. The area difference between the pressure-volume diagrams of the expansion and compression spaces is the indicated power. Table 2 gives the compression, expansion and indicated powers obtained from the pressure-volume diagrams. The magnitudes of the powers in the simulations and experiments for both gases are in reasonable agreements. The indicated powers of the simulation and experiment are 2.86 kW and 2.31 kW respectively for helium, while they are 3.55 kW and 3.47 kW for hydrogen. As seen from both the shapes of the diagrams illustrated in Figure 5 and the powers listed in Table 2, less compression power is required for hydrogen compared to helium, while similar amounts of expansion power are obtained in the expansion space for both gases. It is because that the Stirling engine has less losses when using hydrogen as the working gas, which will be further analyzed in the following sections.

4.2. Indicated power and efficiency

Figure 6 shows comparisons of different models with the experimental results of the engine with helium at 4.14 MPa. The operating frequency and heating temperature are 41.72 Hz and 704°C , respectively. It shows that the accuracies of the Stirling engine

Table 2: Comparisons of compression, expansion and indicated powers.

		Compression power, kW	Expansion power, kW	Indicated power, kW
Helium	Experiment [55]	5.67	7.98	2.31
	Simulation	5.06	7.92	2.86
Hydrogen	Experiment [55]	4.71	8.18	3.47
	Simulation	4.04	7.59	3.55

models have been improved largely recently. The relative deviations of the indicated power and efficiency for the present model are 14.7% and 3.0% respectively for this operating point, which are among the finest accuracies compared to previous models.

Figure 7 illustrates the comparisons of the simulated and experimental results of the GPU-3 Stirling engine charged with helium gas. For both the indicated power and indicated efficiency, the trends of the lines with respect to the frequency have good agreements between the experiments and simulations. The relative deviations of the indicated power are typically around 20%, while those of the indicated efficiency are about 10%. The deviation tends to become a little larger when increasing the frequency, probably due to the underestimated losses associated with the high frequency oscillations. The optimal frequencies for the indicated power are about 50 Hz in both experiments and simulations for the presented working conditions. The indicated efficiencies reach the maximums at lower frequencies. Comparing the powers and efficiencies at 2.76 MPa and 4.17 MPa with the same heating temperature of 649 °C, it indicates that the performance of the Stirling engine can be significantly improved by increasing the charging pressure. The maximum indicated power at 4.17 MPa is about 1.7 times the value at 2.76 MPa, though the ratio of the charged pressures is only 1.5. This is because that more working gas is involved in the energy conversion cycle and the power density is increased at higher pressures. Increasing the heating temperature also results in an increased indicated power, as seen by comparing the results of 2.76 MPa at the heating temperatures of 649 °C and 704 °C.

The comparisons of the simulated and experimental results with hydrogen as the working gas are illustrated in Figure 8. The heating temperature is fixed at 704 °C while two mean pressures of 1.38 MPa and 2.76 MPa are adopted. It indicates that the model predicts the indicated power and efficiency with better accuracies on the hydrogen system. The relative deviation of the indicated power is around 10% and that of the efficiency is typically within 8%. Compared with the results with helium as the working gas, the optimal frequencies for both the indicated power and efficiency are higher when hydrogen is used. The indicated power of the Stirling engine with hydrogen is much higher than that with helium at any working frequency when the mean pressure and heating temperature are both 2.76 MPa and 704 °C. The maximum indicated power with helium is 2.5 kW in the experiment while the tested highest one is more than 3.6 kW for hydrogen. It indicates that hydrogen is a superior working gas in terms of output capability for the Stirling engine.

4.3. Brake power and efficiency

The indicated power, derived from pressure-volume diagrams in the compression and expansion spaces, is not the actual output power of the Stirling engine. The mechanical drive mechanism dissipates a part of the indicated power, leaving the rest as the output power, i.e. brake power. As expressed in Eq. (18), the coefficient E , which is empirically determined according to experiments, is used to evaluate the effectiveness of the drive

mechanism. The experimental data of the mechanical efficiency are presented in Figure 9. The tested mechanical frequency of the hydrogen system stays almost constantly around 75%, while that of the helium system is lower and much more sensitive to the frequency. The mechanical efficiency is about 65% from 16.7 Hz to 41.7 Hz and has a rapid decreasing trend when the frequency is further increased. The empirical effectiveness is fitted according to the experimental data and selected as 0.82 and 0.78 respectively for hydrogen and helium in the simulations. The simulated results of the mechanical efficiencies are shown by the lines in Figure 9. Good agreements are achieved for both working gases, especially for hydrogen. The mechanical efficiency of the helium system in the simulation has a similar trend as that in the experiment. The simulated mechanical efficiency for helium is generally close to the experiments at the low frequencies, with increasing deviations at higher frequencies. The agreements between the experimental and simulated mechanical efficiencies indicate that the adopted Senft's treatment of brake power is reasonable for characterizing the mechanical losses of the drive system [61–63]. The different frequency dependencies of the mechanical efficiency also indicate that the Stirling engine with hydrogen is more adaptive to frequency variations while that with helium is better to be operated at low frequencies.

The simulated and experimental brake powers and efficiencies at various working conditions with helium as the working gas are shown in Figures 10 and 11. Both the trends and magnitudes have reasonable agreements. The brake power reaches the maximums at frequencies between 40-50 Hz in both the experiments and simulations. The deviations of the brake powers at 704 °C are typically around 20% for 2.76 MPa, 15% for 4.14 MPa and 5% for 5.52 MPa. The deviations are within 5% for each mean pressure at 649 °C. Similar to the comparisons of the indicated power, larger deviations occur at higher working frequencies due to the underestimated power losses of the cycle and mechanical losses of the drive system. As shown by both the experimental and simulated results in Figure 11(b), the mean pressure has limited effects on the brake efficiency. The brake efficiencies at 5.52 MPa are almost the same with that at 6.9 MPa, whereas it is slightly lower for 4.41 MPa. The absolute deviations of the brake efficiency between simulations and experiments are typically within 5%.

Figure 2: Flow chart of the numerical simulation.

Figure 3: Schematic of GPU-3 Stirling engine [19].

Figure 4: Volume variations of GPU-3 Stirling engine.

Figure 5: Comparison of pressure-volume diagrams of GPU-3 Stirling engine charged with (a) helium and (b) hydrogen (650 °C, 2.8 MPa).

Figure 6: Comparison of different models with the experimental data of GPU-3 Stirling engine: (a) indicated power (b) indicated efficiency (helium, 704°C, 4.14 MPa, 41.72 Hz).

Figure 7: Comparison of (a) indicated power and (b) indicated efficiency with helium as working gas.

Figure 8: Comparison of (a) indicated power and (b) indicated efficiency with hydrogen as working gas.

Figure 9: Comparison of the mechanical efficiency.

Figure 10: Brake power with helium at 704°C.

Figure 11: (a) Brake power and (b) brake efficiency with helium at 649°C.

Figure 12 illustrates the brake power and efficiency for different mean pressures with hydrogen at 704 °C. The accuracy of the developed model is quite good at 2.76 MPa and 4.14 MPa, with deviations within 8% and 5% for predicting the brake power and brake efficiency. The accuracy is lower at the low mean pressure of 1.38 MPa. This is because the calculated mechanical efficiencies are about 10% higher than the experimental ones, indicating the mechanical effectiveness at low mean pressures is actually lower than the value at high mean pressures. The simulated trends of both the brake power and efficiency fit well with the experimental ones, showing that the basic operating characteristics are well captured by the model. Both the optimal frequencies for the brake power and brake efficiency are shifted towards larger values for the Stirling engine charged with hydrogen gas compared to those with helium gas. More comparison of the simulations and experiments are given in Figure 13 with three different heating temperatures at the mean pressure of 2.76 MPa. The deviations of the brake powers are typically within 8% for the three heating temperatures, while those of the brake efficiency are typically within 4%. It indicates that the developed model is able to predict the output performance and characterize the working characteristics of the Stirling engine with good accuracies.

Figure 12: (a) Brake power and (b) brake efficiency with hydrogen at 704°C.

4.4. Loss analysis

Various losses have been integrated in the developed numerical model, which are important for achieving the good accuracies. These losses include gas spring hysteresis

Figure 13: (a) Brake power and (b) brake efficiency with hydrogen at different temperatures at 2.76 MPa.

loss in the compression and expansion spaces, shuttle heat transfer loss caused by the movement of displacer, appendix gap loss in the displacer gap, gas leakage loss, finite speed pressure losses of pistons, piston friction loss, pressure drop loss in the heat exchangers and regenerator, heat conduction loss from the hot to the cold end, and mechanical loss of the drive mechanism. Imperfect heat transfer is considered in the regenerator by simultaneously solving the coupled heat transfer processes of regenerator matrix solid and working gas. All of the above losses together with the powers have been calculated in the simulations and are shown in Figure 14. The mean pressure and heating temperature are 2.76 MPa and 704 °C, respectively.

The indicated and brake powers in the experiments and simulations are illustrated as the solid points and bars, respectively. The aforementioned results have also been presented in Figures 7(a) and 10 for helium, Figures 8(a) and 12(a) for hydrogen. Heat conduction loss stays constantly at any frequency due to the fixed hot and cold source temperatures for both working gases. It is a major loss at low frequencies, taking up to about 20% of total heat input at 16.7 Hz while decreasing to around 7% at 70 Hz for both helium and hydrogen. Shuttle loss has little increase with frequency. It accounts for 7.2% to 2.8% of total heat input from 16.7 Hz to 70 Hz for helium. The percentage is from 9.2% to 3.4% for hydrogen. All the other frequency and piston speed dependent losses increase with the frequency, including finite speed loss, appendix gap loss, piston friction loss, leakage loss,

Figure 14: Output powers and losses with (a) helium and (b) hydrogen at 2.76 MPa, 704°C.

hysteresis loss and pressure drop loss. The former five losses totally account for 4.9%-15.2% from 16.7 Hz to 70 Hz for helium while they are at the range of 11.5%-17.5% for hydrogen. Pressure drop loss is a less important loss at low frequencies while it grows rapidly to a major one at high frequencies, especially for the engine with helium. The pressure drop loss of the helium system accounts for 5.1% of the heat input at 25 Hz, while it increases to 18% at 50 Hz and even 35% at 70 Hz. The strong dependence of the pressure drop loss to the frequency is due to the enlarged velocity of gas flow at high frequencies. Compared with the helium system, the pressure drop loss is much less for the hydrogen system. Its percentages are 2.3%, 8.5% and 16.5% for 25 Hz, 50 Hz and 70 Hz, respectively. The pressure drop loss of the hydrogen system is approximately only a half of that of the helium system. This is because the viscosity of hydrogen is less than a half of that of helium, as given by Eq. (31). The comparisons of the losses between the helium and hydrogen systems indicate that the pressure drop loss is the main factor that shapes the different trends of the indicated and brake powers. Due to the significant pressure drop caused by the large viscosity of helium gas, the output powers decrease rapidly at frequencies above 50 Hz. Whereas, due to the small gas viscosity, the Stirling engine with hydrogen is able to be operated at higher

frequencies with a good output capability.

The detailed pressure drop variations in one cycle for the cooler, regenerator and heater are shown in Figure 15 for further illustrating the effects of frequency and working gas. It indicates that the regenerator is the largest source of the pressure drop. The positive pressure drop denotes gas flows from the cold to the hot ends while the negative one represents the reverse flow. The velocity of the gas flow is larger when it flows from the hot end to the cold one due to the thermal expansion of the gas. Therefore, the absolute values of negative pressure drops are larger than those of positive ones. According to Figure 15, the pressure drops both increase remarkably when the frequency is increased from 25 Hz to 50 Hz for both gases. For example, the peak-to-peak value of pressure drop variation in the regenerator is increased from 104.7 kPa at 25 Hz to 296.1 kPa at 50 Hz for helium. The pressure drops for hydrogen system are much smaller. The peak-to-peak pressure drop in the regenerator is only 50.1 kPa and 147.9 kPa at 25 Hz and 50 Hz, respectively. It clearly shows that the small gas viscosity of hydrogen is critical for minimizing the pressure drop loss and improving the system performance. What's more, the thermal conductivity of hydrogen gas is also about 20% higher than that of helium gas, which is beneficial for heat transfer.

Figure 15: Pressure drop variations in one cycle at the cooler, regenerator and heater with (a) helium at 25 Hz, (b) helium at 50 Hz, (c) hydrogen at 25 Hz, and (d) hydrogen at 50 Hz (2.76 MPa, 704°C).

4.5. Transient characteristics

Figure 16 shows the simulated pressure-volume diagram with hydrogen at 2.76 MPa and 704 °C. The crank angles of 0°, 90°, 180° and 360° have been marked accordingly. The cycle starts when the expansion space is close to the minimal volume. The pressure-volume diagram goes in the clockwise direction for the expansion space and the anticlockwise direction for the compression space, representing positive and negative work respectively. The instant pressures in the compression and expansion space do not have obvious differences at any crank angle, showing that the pressure drop is a small fraction of the mean pressure. The pressure-volume diagram of the whole system can be drawn by

460 using the averaged pressure and the total volume of the compression and expansion
 461 spaces. The shadowed area in Figure 16 denotes the net output of the cycle, i.e. indicated
 462 power.

Figure 16: Simulated pressure-volume diagrams with hydrogen at 2.76 MPa, 704°C, 50 Hz.

463 Figure 17 illustrates the simulated gas temperature distributions along the Stirling
 464 engine in a cycle. The hot end of the engine has the largest variations of the gas
 465 temperature because of the highly intensive thermal expansions. The temperature along
 466 the regenerator is linearly distributed. It has much smaller temperature variations due to
 467 the much better thermal contact between gas and the dense screen mesh. The
 468 peak-to-peak temperature variation in the regenerator is in the range from 10 K to 12.6 K.

Figure 17: Simulated gas temperature distribution along the Stirling engine (hydrogen, 2.76 MPa, 704°C, 50 Hz).

469 More details about the transient temperature, mass flow rate, pressure, the Reynolds
 470 and Nusselt numbers at the middle of the regenerator, i.e. $x = 0.0827$ m, are shown in
 471 Figure 18. The crank angle is in accordance with the ones denoted in Figure 16. Both the
 472 mass flow rate and pressure oscillate in shapes close to sine waves. The mass flow leads

473 the pressure by a crank angle of 36° . The gas flow direction reverses twice in a cycle, i.e.
 474 at 148° and 317° . When it is positive, the gas flows from the cold to hot ends, which
 475 is the so-called cold blow period. Conversely, it is the hot blow period, as denoted in
 476 Figure 18(a). The instant Reynolds and Nusselt numbers are presented in Figure 18(b).
 477 Different from unidirectional flow, both the Reynolds and Nusselt numbers have cyclic
 478 fluctuations for the oscillating flow. Their fluctuation periods are twice per cycle. Due to
 479 the small hydrodynamic diameter of the screen mesh, i.e. $92\ \mu\text{m}$, the Reynolds number in the
 480 regenerator is within 60. The maximum Nusselt number is about 6.2, which corresponds to
 481 a heat transfer coefficient of about $20000\ \text{W}/(\text{m}^2\text{K})$. By comparison, the maximum Nusselt
 482 number and heat transfer coefficient for the helium system at the same working condition
 483 are 5.7 and $15600\ \text{W}/(\text{m}^2\text{K})$, respectively. It indicates that the heat transfer performance
 484 of the regenerator for the hydrogen system is also better.

Figure 18: Variations of (a) Mass flow rate and pressure (b) Reynolds number and Nusselt number (c) gas temperature and matrix temperature in the middle of regenerator in one cycle (hydrogen, 2.76 MPa, 704°C, 50 Hz).

485 In almost all of previous models for Stirling engines, the temperatures of the solid and
 486 gas in the regenerator are assumed to be the same, and only the energy equation for the
 487 gas is solved. In the practical Stirling engines, however, the temperature of the regenerator
 488 matrix solid actually also oscillates due to the limited heat capacity of the solid and limited
 489 time for heat transfer. In the model presented in this work, the energy equation of solid
 490 is also considered and linked together with the energy equation of gas by introducing the
 491 heat transfer term. This treatment refers to the thermal non-equilibrium model for porous
 492 media. Figure 18(c) shows the simulated temperature oscillations in one cycle. The matrix
 493 temperature oscillates with a comparable amplitude as the gas temperature. In the cold

blow period, the matrix temperature is higher than the gas temperature, heat stored in the matrix from the previous cycle is released to the gas. In the hot blow period, the gas temperature becomes higher than the matrix one, heat is released from the gas and stored in the matrix for the next cycle. This process is known as the heat regeneration. It is observed that rapid changes of the gas temperature occur at moments of flow reversals. This is because hotter or colder gas flows from the upwind direction to the local position at the moments. Due to limited time for heat transfer in the oscillatory condition, a time delay is found between the matrix and gas temperature oscillations.

5. Conclusions

A transient one-dimensional computational fluid dynamics model, which falls in the category of third-order method, was developed for kinetic Stirling engines. The continuity, momentum and energy equations of the discretized grids were iteratively solved in a transient manner. Various loss effects were included in the model, including gas spring hysteresis loss, shuttle loss, appendix displacer gap loss, gas leakage loss, finite speed loss, piston friction loss, pressure drop loss, heat conduction loss, mechanical loss and imperfect heat transfer. The non-equilibrium thermal model which involved thermal interactions between gas and solid was adopted for the regenerator. The energy equations were coupled and solved separately for the fluid and solid domains in the regenerator. The model was then validated on the GPU-3 Stirling engine with various experimental data. It showed that the present model has improved accuracies for predicting output powers and efficiencies over previous ones. The relative deviations of indicated power and indicated efficiency were typically around 20% and 10% respectively for helium system, while they were about 10% and 8% for hydrogen system. The mechanical efficiency of the drive mechanism was accurately evaluated for further predicting the brake power and efficiency by introducing an empirical effectiveness coefficient. Brake power and efficiency were predicted with relative deviations of 5%-20% and within 5% for helium system and within 8% and 5% for hydrogen system. Both the simulations and experiments showed that the GPU-3 Stirling engine charged with hydrogen has much larger optimal working frequencies and better output performances compared to helium system. The underlying mechanism of the differences was found to result from the different behaviors of the pressure drop loss with frequency. The pressure drop is much larger and more sensitive to the frequency increase due to the much larger viscosity for the helium system. It revealed that the viscosity of the working gas is critical for minimizing the loss and improving the performance. The current model can be further extended to study the complicated transient start-up processes of Stirling engines by combining the kinetic equations of the rotatory driven mechanism.

6. Acknowledgments

The work was funded under the Energy Innovation Research Programme (EIRP, Award No. NRF2013EWT-EIRP001-017), administrated by the Energy Market Authority (EMA). The EIRP is a competitive grant call initiative driven by the Energy Innovation Programme Office, and funded by the National Research Foundation (NRF).

Appendix

To improve the numerical stability and accuracy, the implicit second-order time difference scheme and the second-order upwind scheme are generally adopted for discretizing the time differential terms and convective terms, respectively. For the first time layer, the first-order time difference scheme is used. For the grids near the ends of the spatial grids, the first-order time difference scheme, and the first- and second-order mixed scheme are used, respectively [67]. For simplicity, only the general second-order schemes are presented below.

(a) Continuity equation

Combining the equation of state Eq. (5) and the continuity equation gives the relation between the pressure and mass flow rates,

$$p_i = (\dot{m}_i - \dot{m}_{i+1})X_i + Y_i \quad (\text{A.1})$$

The coefficients X_i and Y_i for the second-order time differential scheme are calculated by,

$$\begin{bmatrix} X_i \\ Y_i \end{bmatrix} = \begin{bmatrix} \delta t \frac{R_g T_i^k}{1.5 V_i^k} \\ \left(2.0 \frac{p_i^{k-1} V_i^{k-1}}{T_i^{k-1}} - 0.5 \frac{p_i^{k-2} V_i^{k-2}}{T_i^{k-2}} \right) \frac{T_i^k}{1.5 V_i^k} \end{bmatrix} \quad (\text{A.2})$$

where the superscripts $k-1$ and $k-2$ denote the time layers before the current time layer k . For the space grids $1 \leq i \leq n-2$, the volumes of the grids are constants with time, i.e. $V_i^k = V_i^{k-1} = V_i^{k-2}$. Due to the movements of the power piston and displacer, the volumes of the compression and expansion spaces at $i=0$, $i=n-1$ vary with time.

(b) Momentum equation

The time differential term in the momentum equation Eq. (2) is discretized into,

$$\frac{\partial \dot{m}_i}{\partial t} \delta x_i = T_1 \dot{m}_i + T_2 \dot{m}_i^{k-1} + T_3 \dot{m}_i^{k-2} \quad (\text{A.3})$$

where the coefficients T_1 , T_2 and T_3 of the second-order difference scheme are obtained by,

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 1.5 \frac{\delta x_i}{\delta t} \\ -2 \frac{\delta x_i}{\delta t} \\ 0.5 \frac{\delta x_i}{\delta t} \end{bmatrix} \quad (\text{A.4})$$

The momentum convection term $(\dot{m}u)_{cv,i}$ is discretized by the second-order upwind scheme at the grids of $2 \leq i \leq n-2$, while the first- and second-order mixed scheme is used for $i=1$ and $i=n-1$. The general discretized momentum convection term is expressed by,

$$(\dot{m}u)_{cv,i} = \max(u_{cv,i}, 0)(G_{1,i}\dot{m}_i - G_{2,i}\dot{m}_{i-1}) + \min(u_{cv,i}, 0)(G_{3,i}\dot{m}_{i+1} - G_{4,i}\dot{m}_{i+2}) \quad (\text{A.5})$$

where $G_{1,i}$, $G_{2,i}$, $G_{3,i}$ and $G_{4,i}$ are the geometric parameters which are dependent on the lengths of the grids,

$$\begin{bmatrix} G_{1,i} & G_{2,i} \\ G_{3,i} & G_{4,i} \end{bmatrix} = \begin{bmatrix} \frac{dx_i+2dx_{i-1}}{2dx_{i-1}}, & \frac{dx_i}{2dx_{i-1}} \\ \frac{dx_i+2dx_{i+1}}{2dx_{i+1}}, & \frac{dx_i}{2dx_{i+1}} \end{bmatrix} \quad (\text{A.6})$$

Substituting Eqs. (A.1), (A.3) and (A.5) into the momentum equation, Eq. (2), gives the general algebraic equation for calculating the mass flow rates,

$$a_{1,i}\dot{m}_{i-2} + a_{2,i}\dot{m}_{i-1} + a_{3,i}\dot{m}_i + a_{4,i}\dot{m}_{i+1} + a_{5,i}\dot{m}_{i+2} = B_i \quad (\text{A.7})$$

where the general expressions for the coefficients at $3 \leq i \leq n-3$ are,

$$\begin{aligned} a_{1,i} &= \max(u_{cv,i-1}, 0)G_{2,i-1} \\ a_{2,i} &= -\max(u_{cv,i}, 0)G_{2,i} - \max(u_{cv,i-1}, 0)G_{1,i-1} - A_i X_{i-1} \\ a_{3,i} &= T_1 + \max(u_{cv,i}, 0)G_{1,i} - \min(u_{cv,i-1}, 0)G_{3,i-1} + A_i(X_i + X_{i-1}) + \frac{\delta x_i}{d_{h,i}} f_i \frac{1}{2} |u_i| \\ a_{4,i} &= \min(u_{cv,i}, 0)G_{3,i} + \min(u_{cv,i-1}, 0)G_{4,i-1} - A_i X_i \\ a_{5,i} &= -\min(u_{cv,i}, 0)G_{4,i} \\ B_i &= -T_2 \dot{m}_i^{k-1} - T_3 \dot{m}_i^{k-2} - A_i(Y_i - Y_{i-1}) \end{aligned} \quad (\text{A.8})$$

The algebraic equations of the momentum equations form a penta-diagonal matrix, and are directly solved by the PDMA algorithm.

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(c) Energy equation of gas

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The time differential term in the energy equation of gas is discretized into,

$$\frac{\partial}{\partial t} \left(mc_v T + m \frac{u^2}{2} \right)_{cv,i} = R_1 + R_2 + R_3 T_i^k \quad (\text{A.9})$$

where

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} \frac{0.5}{\delta t} \left(1.5 \frac{p_i^k V_i^k}{R_g T_i^k} u_{cv,i}^{k-2} - 2 \frac{p_i^{k-1} V_i^{k-1}}{R_g T_i^{k-1}} u_{cv,i}^{k-12} + 0.5 \frac{p_i^{k-2} V_i^{k-2}}{R_g T_i^{k-2}} u_{cv,i}^{k-22} \right) \\ \frac{1}{\delta t} \left(-2 \frac{p_i^{k-1} V_i^{k-1}}{R_g} c_{v,i}^{k-1} + 0.5 \frac{p_i^{k-2} V_i^{k-2}}{R_g} c_{v,i}^{k-2} \right) \\ 1.5 \frac{p_i^k V_i^k}{R_g T_i^k} \frac{c_{v,i}^k}{\delta t} \end{bmatrix} \quad (\text{A.10})$$

In the compression and expansion spaces, the kinetic energy term R_1 is negligible.

The second-order upwind scheme is used for the convection term $\dot{m}_i c_{p,i} T_{f,i}$ at the grids of $2 \leq i \leq n-2$, while the first- and second- order mixed scheme is adopted at the grids $i = 1$ and $i = n-1$. The general expression for the second-order upwind scheme is,

$$\dot{m}_i c_{p,i} T_{f,i} = \max(\dot{m}_i c_{p,i}, 0)(G_{5,i} T_{i-1} - G_{6,i} T_{i-2}) + \min(\dot{m}_i c_{p,i}, 0)(G_{7,i} T_i - G_{8,i} T_{i+1}) \quad (\text{A.11})$$

where

$$\begin{bmatrix} G_{5,i} & G_{6,i} \\ G_{7,i} & G_{8,i} \end{bmatrix} = \begin{bmatrix} \frac{dx_{i-1}+2\delta x_{i-1}}{2\delta x_{i-1}}, & \frac{dx_{i-1}}{2\delta x_{i-1}} \\ \frac{dx_i+2\delta x_{i+1}}{2\delta x_{i+1}}, & \frac{dx_i}{2\delta x_{i+1}} \end{bmatrix} \quad (\text{A.12})$$

Substituting Eqs. (A.9) and (A.11), and applying the central difference method for the thermal conduction term gives the algebraic equation for calculating the gas temperatures,

$$b_{1,i}T_{i-2} + b_{2,i}T_{i-1} + b_{3,i}T_i + b_{4,i}T_{i+1} + b_{5,i}T_{i+2} = C_i \quad (\text{A.13})$$

where the coefficients at $2 \leq i \leq n-3$ are,

$$\begin{aligned} b_{1,i} &= \max(\dot{m}_i c_{p,i}, 0) G_{6,i} \\ b_{2,i} &= -\max(\dot{m}_i c_{p,i}, 0) G_{5,i} - \max(\dot{m}_{i+1} c_{p,i+1}, 0) G_{6,i+1} - \frac{k_{f,i} A_{f,i}}{\delta x_i} \\ b_{3,i} &= R_3 - \min(\dot{m}_i c_{p,i}, 0) G_{7,i} + \max(\dot{m}_{i+1} c_{p,i+1}, 0) G_{5,i+1} + \frac{k_{f,i} A_{f,i}}{\delta x_i} + \frac{k_{f,i+1} A_{f,i+1}}{\delta x_{i+1}} + \alpha_i A_{wet,i} \\ b_{4,i} &= \min(\dot{m}_i c_{p,i}, 0) G_{8,i} + \min(\dot{m}_{i+1} c_{p,i+1}, 0) G_{7,i+1} - \frac{k_{f,i+1} A_{f,i+1}}{\delta x_{i+1}} \\ b_{5,i} &= -\min(\dot{m}_{i+1} c_{p,i+1}, 0) G_{8,i+1} \\ C_i &= -R_1 - R_2 + 0.5 \dot{m}_i u_i^2 - 0.5 \dot{m}_{i+1} u_{i+1}^2 + \alpha_i A_{wet,i} T_{s,i} \end{aligned} \quad (\text{A.14})$$

The algebraic equations of the energy equations of gas form a penta-diagonal matrix, and are directly solved by the PDMA algorithm.

(d) Energy equation of solid

For the solids that have large temperature gradients, such as the regenerators and the connecting tubes between the regenerator and the heat exchangers, the temperatures of the solid are numerically solved rather than specified as constants. The first and the last grids of the solid domain are denoted by $i = si$ and $i = so$.

The time differential term in the energy equation of solid is discretized into,

$$\frac{\partial(m_s c_s T_s)_{cv,i}}{\partial t} = S_1 T_{s,i} + S_2 T_{s,i}^{k-1} + S_3 T_{s,i}^{k-2} \quad (\text{A.15})$$

where

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} 1.5 \rho_{s,i} V_{s,i} \frac{c_{s,i}}{\delta t} \\ -2 \rho_{s,i} V_{s,i} \frac{c_{s,i}^{k-1}}{\delta t} \\ 0.5 \rho_{s,i} V_{s,i} \frac{c_{s,i}^{k-2}}{\delta t} \end{bmatrix} \quad (\text{A.16})$$

Substituting Eq. (A.15) into Eq. (4) and applying the central difference method for the thermal conduction term gives the algebraic equation for calculating the solid temperatures,

$$c_{1,i} T_{s,i-1} + c_{2,i} T_{s,i} + c_{3,i} T_{s,i+1} = D_i \quad (\text{A.17})$$

where the coefficients at $si + 1 \leq i \leq so - 1$ are,

$$\begin{aligned}
c_{1,i} &= -\frac{\lambda k_{s,f,i} A_{s,f,i}}{\delta x_i} \\
c_{2,i} &= S_1 + \frac{\lambda k_{s,f,i} A_{s,f,i}}{\delta x_i} + \frac{\lambda k_{s,f,i+1} A_{s,f,i+1}}{\delta x_{i+1}} + \alpha_i A_{wet,i} \\
c_{3,i} &= -\frac{\lambda k_{s,f,i+1} A_{s,f,i+1}}{\delta x_{i+1}} \\
D_i &= -S_2 T_{s,i}^{k-1} - S_3 T_{s,i}^{k-2} + \alpha_i A_{wet,i} T_i
\end{aligned} \tag{A.18}$$

The algebraic equations of the energy equations of solid form a tri-diagonal matrix, and are directly solved by the TDMA algorithm.

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