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Physically-based failure models and criteria for laminated fibre-reinforced composites.

Part II: FE Implementation

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Abstract

3D failure criteria for laminated fibre-reinforced composites, based on a physical model for each failure mode and considering nonlinear matrix shear behaviour are implemented in an explicit Finite Element code. All the properties used by the model have a clear physical meaning. A smeared formulation is used to avoid mesh dependency during damage propagation. The model is shown to reproduce key aspects observable during failure, such as the inclined fracture plane in matrix compression, the $\pm 45^\circ$ failure pattern of a $(\pm 45^\circ)_{ns}$ tension specimen, and kink-band formation (including its orientation and broadening).

Key words: A Carbon Fibre, B Fracture, C Finite Element Analysis (FEA),
Failure

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1 Introduction

Successfully modelling failure in composite components is conditioned by two main factors. The first is related to the complexity of the physics of failure. While even the most intricate models fail to be absolutely comprehensive in their description of failure mechanics, their complexity often prevents them from being applied to complex structures, or to be used together with some numerical methods. In the first paper of this series [1], a set of failure models was developed. These models capture the most important physical aspects of failure, while preserving some simplicity.

The second factor which restricts successful failure modelling of composites is related to an intrinsic limitation in one of the most widely used numerical methods for modelling in composite materials: the Finite Element (FE) method. The limitation consists on mesh dependence when damage propagation is modelled using the Continuum Damage Mechanics (CDM) approach. Even though other techniques can be used to model some forms of failure (like decohesion elements for modelling delamination [2]), CDM seems to be the only practical approach when failure can occur at any point in a structure. However, a simple application of CDM leads to pathological mesh-dependent energy absorption during failure [3], and is thus unsuited to model the behaviour of structures after damage initiation. Strictly, problems in the areas of crashworthiness or high-energy impact, should thus fall out of the scope of most CDM-based FE analyses, since energy absorption is the main motivation for performing the modelling; these analyses should proceed no further than damage initiation.

In this paper, an FE implementation of the failure criteria developed in Part I [1] is presented. The implementation avoids the pathological mesh-dependency limitation which is characteristic of simple implementations of CDM. To achieve this, the damage—e.g. a fracture surface or a kink band—is smeared over the failing element. Due to the importance of correctly modelling shear nonlinearity, the implementation accepts any generic curve for the shear law. The implementation has been performed in a commercial explicit FE code [4], but can be easily carried in any other FE code, including implicit ones; however, for implicit FE codes, sophisticated nonlinear solution methods may be needed. The FE model only requires parameters *with* physical meaning. Most of them are typically available from standard tests, and the remaining are shown to either be related to known parameters, or obtainable from simple tests. Even though the model and approach themselves are more general, the implementation is made for LS-Dyna’s [4] 8-noded solid element with one-point integration. The resulting FE model is easy to use and is shown to have captured some key features of the failure process.

2 Failure criteria

The equations relevant to apply the failure criteria developed in Part I [1] are now presented. In the following, the subscript a refers to the fibre direction, the subscript b refers to the in-plane transverse direction and the subscript c refers to the through-the-thickness direction. The values of parameters associated with the onset of failure are denoted with the superscript ‘ o ’ while the values of parameters associated with failure propagation are denoted with the superscript ‘ f ’. For instance, σ_a^o is the stress in the a direction at onset of

failure.

2.1 Fibre tensile failure

For fibre tensile failure, the simple maximum-stress criterion is used:

$$f_{ft} = \frac{\sigma_a}{X_t} = 1 \quad (1)$$

where X_t is the axial tensile strength.

2.2 Matrix failure

The tractions in potential fracture planes are obtained from

$$\begin{cases} \sigma_n = \frac{\sigma_b + \sigma_c}{2} + \frac{\sigma_b - \sigma_c}{2} \cos(2\phi) + \tau_{bc} \sin(2\phi) \\ \tau_T = \frac{\sigma_b + \sigma_c}{2} - \frac{\sigma_b - \sigma_c}{2} \sin(2\phi) + \tau_{bc} \cos(2\phi) \\ \tau_L = \tau_{ab} \cos(\phi) + \tau_{ca} \sin(\phi) \end{cases} \quad (2)$$

with the tentative angles ϕ comprised in the interval $[0, \pi[$. The FE implementation determines the angle ϕ by computing the failure criterion for a few trial angles from that interval (this was shown to be reasonable in Part I [1]).

2.2.1 Compression

For matrix compression ($\sigma_n < 0$), the following criterion is used:

$$f_{mat} = \left(\frac{\tau_T}{S_T - \mu_T \sigma_n} \right)^2 + \left(\frac{\tau_L}{S_L - \mu_L \sigma_n} \right)^2 = 1 \quad \Leftarrow \sigma_n < 0, \quad (3)$$

where

- τ_T , τ_L and σ_n are the tractions in potential fracture planes;
- S_L and S_T are the longitudinal and transverse (to the fibres) shear strengths;
- μ_T and μ_L are transverse and longitudinal friction coefficients.

To apply this criterion, only the compressive strength Y_c , the longitudinal shear strength, S_L , and the angle, ϕ_o , of the fracture plane for pure compression in the b direction (typically, $\phi_o \approx 53^\circ$) need to be known. This angle is shown in Fig. 1(a). All the remaining parameters follow from these.

The transverse friction coefficient can be obtained from ϕ_o as

$$\mu_T = -\frac{1}{\tan(2\phi_o)}. \quad (4)$$

The transverse shear strength, S_T , can be obtained from the compressive strength in the matrix direction, Y_c , and the fracture angle for pure matrix compression ϕ_o :

$$S_T = Y_c \cos(\phi_o) \left(\sin(\phi_o) + \frac{\cos(\phi_o)}{\tan(2\phi_o)} \right). \quad (5)$$

For the friction coefficient μ_L , the following expression is used:

$$\mu_L = S_L \frac{\mu_T}{S_T}. \quad (6)$$

2.2.2 Tension

Matrix tensile failure is assumed to occur when the following quadratic interaction between the normal traction σ_n , and the shear tractions τ_T and τ_L is satisfied:

$$f_{mat} = \left(\frac{\sigma_n}{Y_t} \right)^2 + \left(\frac{\tau_T}{S_T} \right)^2 + \left(\frac{\tau_L}{S_L} \right)^2 = 1 \quad \Leftrightarrow \sigma_n \geq 0 \quad (7)$$

Fig. 1. (a) Failure angle, ϕ_o , for pure transverse compression failure; (b) orientation of a kink band in a 3D space where Y_t is the transverse tensile strength.

2.3 Fibre compression failure

To detect fibre compression failure, the mechanical model presented in Part I [1] is used, with only one modification that enhances the numerical efficiency. The modification has to do with defining the misalignment frame. In the generic model proposed in Part I [1], the misalignment frame orientation θ (the coordinate system aligned with the fibres) is defined as the sum of an initial misalignment angle θ_i (that results from manufacturing imperfections) with the shear strain in the misalignment frame itself, γ_m . Since the shear strain in the misalignment frame is needed to define its orientation, this results in an iterative process. In an explicit FE application, this iterative process would have to be repeated in each time step, which is not convenient. A simple modification, that avoids iterating in each time step, consists of defining the misalignment frame orientation θ as the sum of an initial misalignment angle θ_i with the shear strain in the initial misalignment frame, γ_{mi} . The angles needed for the definition of the orientation of a kink band are shown in Fig.

1(b).

The criterion for fibre compression failure then becomes

$$f_{kink} = \begin{cases} \left(\frac{\tau_T}{S_T - \mu_T \sigma_n} \right)^2 + \left(\frac{\tau_L}{S_L - \mu_L \sigma_n} \right)^2 = 1 \Leftarrow \sigma_n \leq 0 \\ \left(\frac{\sigma_n}{Y_t} \right)^2 + \left(\frac{\tau_T}{S_T} \right)^2 + \left(\frac{\tau_L}{S_L} \right)^2 = 1 \Leftarrow \sigma_n > 0 \end{cases} \quad (8)$$

where τ_T , τ_L and σ_n are the tractions in potential fracture planes.

To apply this criterion, the only material property that needs to be known, further to Y_c , S_L and ϕ_o already mentioned in respect of the matrix compression failure, and to the tensile strength in the matrix direction Y_t , is the compressive strength in the fibre direction X_c . All the remaining parameters follow from these.

The stresses in the fibre-kinking plane are obtained using

$$\left\{ \begin{array}{l} \sigma_{b\psi} = \frac{\sigma_b + \sigma_c}{2} + \frac{\sigma_b - \sigma_c}{2} \cos(2\psi) + \tau_{bc} \sin(2\psi) \\ \sigma_{c\psi} = \sigma_b + \sigma_c - \sigma_{b\psi} \\ \tau_{ab\psi} = \tau_{ab} \cos(\psi) + \tau_{ca} \sin(\psi) \\ \tau_{b\psi c\psi} = 0 \\ \tau_{c\psi a} = \tau_{ca} \cos(\psi) - \tau_{ab} \sin(\psi) \end{array} \right. \quad (9)$$

being the angle ψ obtained from

$$\tan(2\psi) = \frac{2\tau_{bc}}{\sigma_b - \sigma_c}. \quad (10)$$

The stresses are then rotated to the misalignment frame

$$\left\{ \begin{array}{l} \sigma_{a^m} = \frac{\sigma_a + \sigma_{b^\psi}}{2} + \frac{\sigma_a - \sigma_{b^\psi}}{2} \cos(2\theta) + \tau_{ab^\psi} \sin(2\theta) \\ \sigma_{b^m} = \sigma_a + \sigma_{b^\psi} - \sigma_{a^m} \\ \tau_{a^m b^m} = -\frac{\sigma_a - \sigma_{b^\psi}}{2} \sin(2\theta) + \tau_{ab^\psi} \cos(2\theta) \\ \tau_{b^m c^\psi} = \tau_{b^\psi c^\psi} \cos(\theta) - \tau_{c^\psi a} \sin(\theta) \\ \tau_{c^\psi a^m} = \tau_{c^\psi a^\psi} \cos(\theta) \end{array} \right. \quad (11)$$

with the misalignment frame orientation θ defined as

$$\theta = \frac{\tau_{ab^\psi}}{|\tau_{ab^\psi}|} (\theta_i + \gamma_{mi}). \quad (12)$$

The initial misalignment angle θ_i is deduced from experimental data by solving the following iterative equation¹

$$\theta_i = \theta_c - f_{CL}^{-1} \left(\frac{1}{2} \sin(2\theta_i) X_c \right) \quad (13)$$

where

$$\theta_c = \arctan \left(\frac{1 - \sqrt{1 - 4 \left(\frac{S_L}{X_c} + \mu_L \right) \frac{S_L}{X_c}}}{2 \left(\frac{S_L}{X_c} + \mu_L \right)} \right). \quad (14)$$

The strain in the initial misalignment frame is defined as

$$\gamma_{mi} = f_{CL}^{-1} \left(-\frac{\sigma_a - \sigma_{b^\psi}}{2} \sin(2\theta_i) + |\tau_{ab^\psi}| \cos(2\theta_i) \right) \quad (15)$$

where f_{CL} is the shear law ($\tau = f_{CL}(\gamma)$).

¹ Note that θ_i is in this model a material property and is therefore computed only once; hence, it is not important that we need to iterate to obtain it

The tractions in the potential fracture planes are obtained using

$$\begin{cases} \sigma_n = \frac{\sigma_{b^m} + \sigma_{c^\psi}}{2} + \frac{\sigma_{b^m} - \sigma_{c^\psi}}{2} \cos(2\phi) + \tau_{b^m c^\psi} \sin(2\phi) \\ \tau_T = -\frac{\sigma_{b^m} - \sigma_{c^\psi}}{2} \sin(2\phi) + \tau_{b^m c^\psi} \cos(2\phi) \\ \tau_L = \tau_{a^m b^m} \cos(\phi) + \tau_{c^\psi a^m} \sin(\phi) \end{cases} \quad (16)$$

where the angle ϕ , comprised in the interval $[0, \pi[$, is obtained by trying a small number of tentative angles.

3 Damage Mechanics: limitations and potential

3.1 Introduction

The failure criteria described were implemented in the explicit FE code LS-Dyna [4]. Composite material models with failure are already available in LS-Dyna. These are *Material 22* and *Materials 54 and 55*.

Material 22 applies Chang and Chang failure criteria [5,6], and simulates fibre breaking and matrix cracking due to in-plane stresses. Stress-strain relationships are linear except for in-plane shear. When a failure criterion is verified, the relevant elastic properties are linearly reduced to zero within a given number of time steps. For the public-domain version of the code, this number is 100. The motivation for the progressive reduction rather than sudden total reduction is numerical stability.

Materials 54 and 55 are very close in their formulation, and apply only to shell elements. While *Material 54* uses the Chang and Chang failure criterion,

like Material 22, Material 55 uses the Tsai and Wu [7] failure criterion. Again, once failure is detected, the relevant elastic properties are reduced to zero over a fixed number of time steps.

To model failure, the approaches described above suffer from a severe mesh dependency problem related to *strain localization* during the fracture process. Strain localization occurs whenever failure is preceded by the emergence of narrow and highly-strained zones. Typical examples of strain localization include shear bands in metals, the formation of kink bands and the accumulation of damage that results in fracture in composites.

This section briefly introduces damage mechanics and the localization problem. Smeared formulations as a way to overcome the localization problem are then presented.

3.2 *Damage mechanics*

In order to model damage propagation, continuum constitutive models can feature internal variables representing, directly or indirectly, the density and/or distribution of the microscopic defects that characterize damage. These are called Continuum Damage Mechanics (CDM) models. The first CDM model was proposed by Kachanov [8], where an internal scalar variable was introduced to model creep failure (of metals) under uniaxial loading. A physical significance for the scalar variable was later introduced by Rabotnov [9], who proposed the reduction of the cross-sectional area due to microcracking as the measure of damage:

$$d = \frac{A_0 - A}{A_0} \tag{17}$$

where d is the *damage* variable, A_0 is the initial, undamaged, area and A is the net or effective area transmitting stress in the damaged state. Since its original appearance, various authors have contributed to CDM development. CDM has been extended to 3D anisotropic situations, a sound thermodynamical basis was provided and theories including up to fourth order non-symmetric damage tensors have been proposed.

The stress applied at each point of a structure, which is based on the applied load and the total cross-sectional area—as if the material was undamaged—is usually referred to in the literature as ‘true stress’, ‘real stress’ or ‘applied stress.’ This stress measure is relevant on a macroscopic point of view, since it represents an average over damaged and undamaged material. In addition, the stress that is based on the cross-sectional area effectively resisting the loading is frequently referred to as ‘effective stress.’ From these definitions, the effective stress, σ^{ef} , can be related to the applied stress, σ , by

$$\sigma^{ef} = \frac{\sigma}{1 - d}. \quad (18)$$

Assuming the hypothesis of strain equivalence [10] (*the deformation behaviour of the damaged material is represented by the constitutive laws of the virgin material with the true stress replaced by the effective stress*), the damage can be defined as a reduction in the Young’s modulus instead of areas,

$$d = \frac{E_0 - E}{E_0} \quad (19)$$

where E_0 and E are the moduli of the virgin and damaged material respectively. In a generic 3D situation, CDM models usually relate the (applied) stresses $\{\sigma\}$ to the strains $\{\varepsilon\}$ using a damage tensor $[1 - d]$, through an

equation of the form

$$\{\sigma\} = [1 - d] [C] \{\varepsilon\} \quad (20)$$

where $[C]$ is the virgin material stiffness matrix. These formulations, in which the damage variables are determined only by the stress state at each point, are denominated ‘local formulations,’ to emphasize that they do not take into account the neighborhood to the point.

3.3 *Strain localization*

Two categories of numerical approaches have been developed for the analysis of damage and crack propagation: those based on Fracture Mechanics and those based on CDM. Fracture Mechanics chronologically precedes CDM, such that a broad range of applications has been found for it. However, in problems involving damage propagation, the geometry might change during the analysis as a result of material fracture and the creation of new crack surfaces. Within an FE model using directly a Fracture Mechanics approach (such as J integral or Virtual Crack Closure technique), this would mean that the mesh requires continuous adaptation. Approaches based on CDM do not require continuous adaptation of the finite element mesh, since material failure is represented by internal damage and changes in the geometry do not occur.

However, CDM has a major drawback—*strain localization*. This is a pathological FE mesh dependency which involves the width of the damaged zone: refinement of the mesh generally leads to higher values of strain and damage in a region of smaller width. This happens when severe material degradation leads to softening behaviour, i.e., the internal stresses decrease with increasing strain. For one-dimensional situations, the condition for localization reduces to

a negative tangent stiffness and has the implication that the one-dimensional dilatational wave speed, c_d (which is defined as $c_d = \sqrt{E_t/\rho}$, being E_t the tangent stiffness and ρ the density), becomes imaginary—with implications for the corresponding wave equation.

Formally, strain localization is the result of a change in the character of the governing differential equations. In the static case, the elliptic character of the set of partial differential equations is lost, while in the dynamic case, the change is typically from hyperbolic to elliptic; when this change happens, the rate boundary value problem becomes ill-posed and numerical solutions suffer from spurious mesh sensitivity [3]. Indeed, local damage formulations can describe the localization phenomena neither in a physically nor in a mathematically appropriate way [11]. To surpass this problem, several solutions have been attempted. Typically, additional terms are introduced into the description of the continuum, which reflect the changes in the micro-structure [3].

3.4 Smeared formulation

A major feature of the proposed energy-based damage model is that the energy per unit area associated with brittle fracture is a material parameter. For example, the energy absorbed in matrix tensile fracture in a unidirectional composite is directly proportional to the area of the new surfaces created (Fig. 2). Not only is this physically reasonable, but it also leads to a formulation that does not suffer from spurious mesh sensitivity problems due to strain localization.

To incorporate this feature in the finite element material model, a *smeared* formulation is used [12,13]. In the so-called smeared formulations, the fracture energy is distributed (smeared) over the full volume of the element. Basically, this approach adopts the concepts of Fracture Mechanics and is very similar to the cohesive-zone approach used to model the interface between two layers. (In fact, cohesive-zone models are also a solution to by-pass the strain localization phase and to directly incorporate the discontinuity that arises as an outcome of the strain localization process; and the smeared formulations can be seen as a generalization of the cohesive zone approach [3].) The resulting model is thus a combination of CDM with Fracture Mechanics. As a consequence of this approach, a parameter with the dimension of length, relative to the element dimensions, is introduced into the constitutive law. The treatment of localization problems with smeared formulations has been widely used in concrete and in metals [14].

One of the major difficulties related with the smeared-formulation approach is that the material damage laws have to include a length parameter, in order to achieve a constant energy released per unit area of crack generated regardless of the element dimensions. Using the example in Fig. 2, matrix tensile fracture, one can easily understand how the length parameter enters the material law. Suppose that the whole plate represented in Fig. 2 is modelled as one element. Then, the energy absorbed by the element after complete failure (complete separation), U , is

$$U = V \times \frac{\sigma^o \varepsilon^f}{2} = L_1 L_2 L_3 \times \frac{\sigma^o \varepsilon^f}{2} \quad (21)$$

where V is the volume of the element. The basic idea is that this energy is equal to an energy per unit area, Γ , which is a material property, multiplied

by the corresponding area, $A = L_1 L_3$, i.e.

$$U = \Gamma \times A = \Gamma \times L_1 L_3. \quad (22)$$

Equating Eqs. 21 and 22, one can retrieve the maximum strain, ε^f , as a function of the energy per units of area of the surface created, Γ , the material strength σ^o and one element dimension, L_2 :

$$\varepsilon^f = \frac{2\Gamma}{\sigma^o L_2}. \quad (23)$$

With ε^f defined by Eq. 23, the energy absorbed by the plate (per units of cracked area) in Fig. 2 is independent of the mesh refinement:

$$\frac{U}{A} = \Gamma = \text{material property}. \quad (24)$$

The same does not happen with any of the composite material models currently implemented in LS-Dyna [4].

Another particularly interesting interpretation for this approach is the concept of *width of the damaged zone*. In fact, if one considers that the work done per unit volume of fully damaged material, Γ_v , is a material property, and that the width, w , of the band of failed material is also a material property, then Eq. 23 also holds, with $\Gamma = \Gamma_v \times w$. Therefore, the formulation proposed can be regarded as a way to incorporate bands of fully failed material, with cracks being a limiting case where the band width tends to zero. This interpretation is particularly useful for the modelling of compression kink bands. Whether the actual damage is a crack or extends on a finite width, the damage is smeared over the dimension of one element.

Fig. 2. (a) Example of a unidirectional composite loaded in transverse (matrix) tension up to complete failure; (b) material law with failure

4 Finite element implementation of the smeared failure model

This section describes in detail the 3D FE implementation of the smeared failure model in LS-Dyna [4].

4.1 Elastic behaviour

The compliance matrix $[S]$, that relates the strain vector

$$\{\varepsilon\}^T = \left\{ \varepsilon_a \ \varepsilon_b \ \varepsilon_c \ \gamma_{bc} \ \gamma_{ca} \right\} \quad (25)$$

to the elastic or effective stress vector

$$\{\sigma^{eff}\}^T = \left\{ \sigma_a \ \sigma_b \ \sigma_c \ \tau_{bc} \ \tau_{ca} \right\} \quad (26)$$

is defined as

$$[S] = \begin{bmatrix} 1/E_a & -\nu_{ba}/E_b & -\nu_{ca}/E_c & 0 & 0 \\ & 1/E_b & -\nu_{cb}/E_c & 0 & 0 \\ & & 1/E_c & 0 & 0 \\ SYM. & & & 1/G_{bc} & 0 \\ & & & & 1/G_{ca} \end{bmatrix}. \quad (27)$$

The in-plane shear (ab) has not been included in Eqs. 25-27 because the model for this shear mode has been designed to use a full nonlinear shear-stress/shear-strain response, obtained experimentally.

4.2 In-plane shear behaviour

According to experimental evidence, the in-plane shear (ab) behaviour is non-linear and irreversible, even before the localization process. Due to the importance of the nonlinear shear behaviour, this model assumes that the full nonlinear shear stress vs. shear strain curve is available. The curve is entered in the FE model through the coefficients of a set of polynomials, Fig. 3(a). The function $f_{CL}(\gamma_{ab})$ represents the value of shear stress obtained from those polynomials, with $\gamma_{ab} \geq 0$.

There is no trivial answer to what is the behaviour in shear over very complex loading-unloading-reloading paths. A simple model for unloading, considers a constant unloading modulus (equal to the initial one, G_{ab}) and so needs only the experimental shear stress vs. shear strain curve. A more general

approach consists of defining the slope of the unloading law as a function of the inelastic strain. However, this approach would require experimental data for its application that is seldom available. Furthermore, some experimental results suggest that when unloading and reloading, the shear modulus is fairly similar to the initial one, though with some hysteresis [15]. Neglecting the hysteresis, one of the simplest models that accounts for irreversibility is the one presented in Fig. 3(b). To define this shear response, one has first to define the maximum (over time) shear strain as

$$\gamma_{ab}^{\max}(t) = \max_{t' \leq t} \{|\gamma_{ab}(t')|\} \quad (28)$$

and the inelastic shear strain as

$$\gamma_{ab}^{in} = \gamma_{ab}^{\max} - f_{CL}(\gamma_{ab}^{\max})/G_{ab}. \quad (29)$$

The material law for shear that reproduces the behaviour shown in Fig. 3(b) then becomes

$$\tau_{ab} = \frac{\gamma_{ab}}{|\gamma_{ab}|} \begin{cases} f_{CL}(|\gamma_{ab}|) \Leftarrow |\gamma_{ab}| = \gamma_{ab}^{\max} \\ G_{ab} \langle |\gamma_{ab}| - \gamma_{ab}^{in} \rangle \Leftarrow |\gamma_{ab}| < \gamma_{ab}^{\max} \end{cases} \quad (30)$$

where the operator $\langle \cdot \rangle$ is the Mc-Cauley bracket defined by

$$\langle x \rangle = \begin{cases} 0 \Leftarrow x \leq 0 \\ x \Leftarrow x > 0 \end{cases}, \quad x \in \mathbb{R}. \quad (31)$$

The shear unloading model outlined above may need further development for situations where the shear stress reverses direction. Finally, it should be mentioned that even though the effect of strain-rate in this model is not directly

Fig. 3. (a) Curve fitting of the experimental nonlinear shear behaviour; (b) irreversibility: loading, unloading and reloading paths

considered, it can be accounted for by using the shear law corresponding to the desired strain rate for the function $f_{CL}(\gamma_{ab})$.

4.3 Modelling failure

In an attempt to model failure propagation more rigorously (on a physically-based point of view), a particular approach based on CDM is followed here.

During the propagation of damage in the material (e.g. in the form of microcracks), the effective area for load transfer is reduced. This results in the applied stress (defined considering the total, macroscopic, area for load transfer) and the effective stress (taking into account the effective area for load transfer) being different. The particular stress components that are affected by the existence of damage depend on the distribution and orientation of damage. This is taken into account by the failure models.

For modelling of damage propagation, the approach that follows resorts to expressing the stress tensor in a coordinate system aligned with the failed region. With the stress tensor aligned with the failed region, and considering the particular failure mode predicted, deciding which stress components are

affected by the existence of damage is straightforward.

For matrix failure, the orientation of the fracture plane is determined; the tractions in the fracture plane are computed; and the effective shear tractions (as well as the normal traction and Poisson’s ratios for tensile loading) are affected by a damage variable (i.e. the effective tractions are replaced by the real or applied tractions). In compression, the fact that only the shear stresses on the fracture plane are affected by failure is in good agreement with the physics of the fracture process and eventual subsequent sliding. It can be observed that this approach is similar to common CDM models for shear failure, if one considers the stress tensor expressed in the coordinate system aligned with the fracture plane.

Damage propagation during fibre kinking is modelled in a similar way, since the fibre-kinking criterion predicts the fibre-kinking plane and angle. Finally, for fibre tensile failure, all stress components are considered to be affected by damage as a result of the explosive nature of this failure mode.

In order to smear the fracture process over the full volume of the element, both the volume of the element and the area of cracked surface need to be known. The former can be obtained from the model nodal data, but the determination of the latter is more complicated. The failure criteria described predict the orientation of the fracture planes. However, the interception of the fracture plane with the element still needs to be known in order to determine correctly the fractured area. As this information is not available, the model assumes that the fracture plane intercepts the element at the integration point. This assumption is reasonable since it is at the integration point—in the centre of the element— that the stresses are computed, and therefore that failure

is predicted. However, this matter seems important enough to justify further investigation.

4.3.1 Damage variables

Damage variables, d , are defined in this model to degrade linearly the stresses to zero, as shown in Fig. 2(b). The instantaneous value of the damage variable, d^{inst} , is defined as

$$d^{inst} = \min \left\{ 0, \max \left\{ \varepsilon^f \frac{\varepsilon - \varepsilon^o}{\varepsilon (\varepsilon^f - \varepsilon^o)}, 1 \right\} \right\}. \quad (32)$$

In order to account for irreversibility, the damage variable is defined as

$$d(t) = \max_{t' \leq t} \{ d^{inst}(t') \}. \quad (33)$$

The onset stresses and strains are defined by appropriate failure criteria. The final strain, ε^f , is defined such that the energy absorbed by the crack formed is correctly accounted for, and thus depends on the failure mode predicted. Finally, the damaged stress components (real stress components) are given by

$$\sigma \leftarrow (1 - d) \sigma. \quad (34)$$

The damage variables defined by Eqs. 32 and 33 lie in the interval $[0, 1]$ and are monotonically increasing.

4.3.2 Matrix failure

After failure is detected (Eq. 3), the shear stresses acting on the fracture plane, τ_T and τ_L , are degraded using an appropriate damage variable. If the normal stress σ_n is positive, it is also degraded, as well as the Poisson's ratios.

From Eqs. 32 and 33, it follows that a strain variable ‘drives’ the damage variable. The history of this strain variable during failure—together with its associated stress—results in energy being absorbed by the failure process. For matrix compression failure, failure results from the shear tractions acting on the fracture surface, while for matrix tensile failure the traction normal to the failure surface also contributes to failure. Therefore the mentioned stress, σ_{mat} , is the resultant of these tractions:

$$\sigma_{mat} = \sqrt{\langle \sigma_n \rangle^2 + (\tau_{mat})^2} \quad (35)$$

with

$$\tau_{mat} = \sqrt{(\tau_T)^2 + (\tau_L)^2}. \quad (36)$$

The angle of the resultant shear traction τ_{mat} with the component τ_T (see Fig. 4(a)) is

$$\lambda = \arctan \frac{\tau_L}{\tau_T} \quad (37)$$

and the angle of the resultant traction σ_{mat} with the shear component τ_{mat} (see Fig. 4(b)) is

$$\omega = \arctan \frac{\langle \sigma_n \rangle}{\tau_{mat}}.$$

Defining these angles λ and ω allows the definition of the strain ‘driving’ the damage variable. Before proceeding to the definition of the strain variable which ‘drives’ damage, care has to be taken on the effect of the nonlinearity in the in-plane shear strain. Consider a specimen loaded in uniform pure in-plane shear. For this case, some energy is absorbed uniformly over all the specimen, due to the shear nonlinear (irreversible) behaviour, Fig. 4(c). However, at the onset of failure, a localized fracture surface starts forming, resulting in further energy being absorbed. While the first energy mentioned (absorbed due to the nonlinear shear behaviour) is proportional to the volume of the specimen, the

second energy mentioned (absorbed by the fracture process) is proportional to the area created. Only the elastic internal energy in the element at onset of failure contributes to the fracture process. Therefore, a definition of the ‘driving’ strain considering only the elastic part of γ_{ab} results.

The strains acting on the fracture plane are

$$\left\{ \begin{array}{l} \varepsilon_n = (\varepsilon_b + \varepsilon_c) + (\varepsilon_b - \varepsilon_c) \cos(2\phi) + \gamma_{bc} \sin(2\phi) \\ \gamma_T = (\varepsilon_b + \varepsilon_c) - (\varepsilon_b - \varepsilon_c) \sin(2\phi) + \gamma_{bc} \cos(2\phi) \\ \gamma_L^{el} = \gamma_{ab}^{el} \cos(\phi) + \gamma_{ca} \sin(\phi), \end{array} \right. \quad (38)$$

where the elastic component of the in-plane shear strain is defined as

$$\gamma_{ab}^{el} = \frac{\gamma_{ab}}{|\gamma_{ab}|} \langle |\gamma_{ab}| - \gamma_{ab}^{in} \rangle. \quad (39)$$

From the angle λ , it follows that the elastic component of the shear strain acting on the fracture plane, in the direction of τ_{mat} , is expressed as

$$\gamma_{mat} = \left| \gamma_T \cos \lambda + \gamma_L^{el} \sin \lambda \right|. \quad (40)$$

From the angle ω , the driving strain is defined as follows:

$$\varepsilon_{mat} = \frac{\langle \sigma_n \rangle}{\sigma_n} \varepsilon_n \cos \omega + \gamma_{mat} \sin \omega. \quad (41)$$

The onset stress and strain become

$$\sigma_{mat}^o = \sigma_{mat}|_{f_{mat}=1}, \quad \varepsilon_{mat}^o = \varepsilon_{mat}|_{f_{mat}=1}. \quad (42)$$

The damage variable d_{mat} is used to degrade the shear stresses in the fracture plane. It is obtained from Eqs. 21 and 22, with ε , ε^o and ε^f replaced by ε_{mat} , ε_{mat}^o and ε_{mat}^f respectively. The degraded tractions and Poisson’s ratios follow

then as

$$\tau_T \leftarrow (1 - d_{mat}) \tau_T, \quad \tau_L \leftarrow (1 - d_{mat}) \tau_L \quad (43)$$

$$\sigma_n \leftarrow \left(1 - d_{mat} \frac{\langle \sigma_n \rangle}{\sigma_n}\right) \sigma_n \quad (44)$$

$$\{\nu_{ba}, \nu_{cb}, \nu_{ca}\} \leftarrow \left(1 - d_{mat} \frac{\langle \sigma_n \rangle}{\sigma_n}\right) \cdot \{\nu_{ba}, \nu_{cb}, \nu_{ca}\}. \quad (45)$$

By using the Mc-Cauley bracket in Eqs. 44 and 45, the stress normal to the fracture plane and the Poisson's ratios are only degraded if there is tensile stress normal to the fracture plane².

Following Fig. 2(b), the expression for ε_{mat}^f is

$$\varepsilon_{mat}^f = \frac{2\Gamma}{\sigma_{mat}^o L}. \quad (46)$$

The characteristic length L in Eq. 46 (necessary to define ε_{mat}^f) has to be determined. For the case when the fibres are aligned with the element direction (angle β in Fig. 5 equal to zero), it is clear (see Fig. 2(a)) that the characteristic length is $L = L_2$. For a generic case (angle β in Fig. 5 not necessarily equal to zero), the characteristic length should be such that $L = V/A$, where V is the element volume and A is the fractured area. From Fig. 5(a), one can find

$$L = L_1 L_2 \cos(\phi) / \min\{L_1 / \cos(\beta), L_2 / \sin(\beta)\}. \quad (47)$$

For simplicity—and also due to the lack of experimental data—the energy

² For a fractured body, the Poisson's effect should not be degraded in the plane parallel to the fracture surface. For simplicity, this is not done here. This approach relies on the assumption that the Poisson's effect in the plane parallel to the fracture surface does not have a considerable influence when modelling that fracture propagation. Also note that the other three Poisson's ratios are dependent on the three defined in Eq. 43 through the symmetry of the elastic stiffness matrix.

release rate Γ (Eq. 46) is assumed to result from a quadratic interaction of the traction components in the fracture plane:

$$\Gamma = \Gamma_b \left\langle \frac{\sigma_n^o}{\sigma_{mat}^o} \right\rangle^2 + \Gamma_T \left(\frac{\tau_T^o}{\sigma_{mat}^o} \right)^2 + \Gamma_L \left(\frac{\tau_L^o}{\sigma_{mat}^o} \right)^2, \quad (48)$$

where Γ_b is the energy release rate for the inclined fracture that occurs in pure transverse tension, Γ_T is the energy release rate for pure transverse compression and Γ_L is the energy release rate absorbed in pure in-plane shear.

Methods to measure the energy release rate in pure transverse traction Γ_b have already been investigated by the authors [16]. Beam-type test specimens including a transverse pre-crack obtained by inserting a blade in the composite prior to the cure, were tested in 4-point-bending. The results showed that the mode I interlaminar fracture toughness is a good approximation to Γ_b .

To the authors' knowledge, there are no standard tests to determine the fracture energies Γ_T and Γ_L . As a first approximation, it seems reasonable to expect that they should be close in magnitude to the mode II interlaminar fracture toughness, due to the similarities in the fracture process.

From Eq. 46, it results that for very coarse meshes ($L \rightarrow \infty$), ε_{mat}^f tends to zero. Therefore, there is a critical element size at which ε_{mat}^f equals ε_{mat}^o , and the actual elements should be smaller than that. The physical interpretation for this phenomenon is that for a very coarse mesh, it is possible for an element to have more elastic energy at failure onset, than the energy necessary for the formation of a crack across it. In these situations, the only possibility for the element to absorb the correct amount of energy is the strains decreasing as the stresses are degraded. However, this is not possible to control via a user-element in an explicit FE code like LS-Dyna. In the actual implementation

of this failure model, a warning is issued if $\varepsilon_{mat}^f < \varepsilon_{mat}^o$ and the analysis continues with $\varepsilon_{mat}^f = \varepsilon_{mat}^o$ for the element affected. The consequence is that the elements falling in this category will absorb more energy than they should during failure. It is then up to the user to conclude whether the results are reasonable (based on the number of warnings), or to re-run the analysis with a more refined mesh in the region of the elements affected.

Finally, for a simple one dimensional problem, or for a situation where Poisson's effects are not considered (this is the case for interface or decohesion elements) the definition of the damage variable in Eqs. 32 and 33, with the final strain computed from Eq. 46 yields the exact energy absorption specified. However, for any 3D material model, the stress vs. strain curves do not have to pass exactly through the origin due to the Poisson's effect (for instance, for a zero stress in the b direction, the corresponding strain ε_b will be in general nonzero if stresses are already acting in the a and c directions.) A small approximation results from this fact. In order to quantify the approximation, consider a material with properties given in Tables 1 and 2 that fails due to the stress in the b direction, but that has a superposed stress σ_a in the a direction. The error in the energy absorbed—proportional to σ_a —is only about 4% for $\sigma_a = 1000$ MPa, considering a characteristic length $L = 0.5$ mm. This aspect warrants further investigation.

4.3.3 Fibre compression failure

Failure is detected using Eq. 8. After failure is detected, the shear stress in the fibre-kinking plane, $\tau_{a^m b^m}$, is degraded using an appropriate damage variable.

Fig. 4. (a) Definition of λ ; (b) definition of ω and (c) fracture energy for in-plane shear

Fig. 5. Determination of the characteristic length within an element (a) for matrix failure (tension and compression) and (b) for fibre failure (tension and compression)

Firstly, the onset stress and strain are defined as

$$\tau_{kink}^o = |\tau_{a^m b^m}|_{f_{kink}=1}, \quad \gamma_{kink}^o = |\gamma_{a^m b^m}^{el}|_{f_{kink}=1} \quad (49)$$

where $\gamma_{a^m b^m}^{el}$ is obtained by rotation of the elastic strains (γ_{ab} replaced by γ_{ab}^{el} as defined by Eq. 39). A damage variable, d_{kink} , is then used to degrade the shear stress $\tau_{a^m b^m}$. It is obtained from Eqs. 21 and 22, with ε , ε^o and ε^f replaced

by $\gamma_{a^m b^m}^{el}$, γ_{kink}^o and γ_{kink}^f respectively. The shear stress in the fibre-kinking plane, $\tau_{a^m b^m}$, is degraded, as well as—depending on the sign of σ_{a^m} —the stress normal to the kink band:

$$\tau_{a^m b^m} \leftarrow (1 - d_{kink}) \tau_{a^m b^m} \quad (50)$$

$$\sigma_{a^m} \leftarrow \left(1 - d_{kink} \frac{\langle \sigma_{a^m} \rangle}{\sigma_{a^m}} \right) \sigma_{a^m} \quad (51)$$

$$\{\nu_{ba}, \nu_{cb}, \nu_{ca}\} \leftarrow \left(1 - d_{kink} \frac{\langle \sigma_{a^m} \rangle}{\sigma_{a^m}} \right) \cdot \{\nu_{ba}, \nu_{cb}, \nu_{ca}\}. \quad (52)$$

The degraded stress tensor is then rotated back to the (a, b, c) frame. The expression for γ_{kink}^f is

$$\gamma_{kink}^f = \frac{2\Gamma}{\tau_{kink}^o L}. \quad (53)$$

A representation of the kink band is shown in Fig. 5(b), where the kink-band angle has been neglected for simplicity. From that figure, the length L is

$$L = L_1 L_2 / \min \{L_1 / \sin(\beta), L_2 / \cos(\beta)\}. \quad (54)$$

Finally, the energy Γ should be the critical energy release rate for kink-band propagation, Γ_{kink} which can be obtained by experiments. Soutis and Curtis [17] present the critical energy release rate for kink-band formation in T800H/924C $(0, 90_2, 0)_{3S}$ specimens. Their experiments consisted of compressing a plate with a central crack and computing the critical energy release rate from the stress intensity factor. The value measured, 38.8 kJ/m^2 , corresponds to the critical energy release rate needed to propagate the kink band in the 0° layers, plus the critical energy release rate needed to propagate matrix compression cracks in the 90° layers. Assuming that those energies are additive (which is to say, neglecting the interactions between the different layers that are failing in different failure modes), and that the matrix compression

toughness is similar in magnitude to the (interlaminar) mode II critical energy release rate (say 1 kJ/m²), the critical energy release rate for kinking, for T800H/924C, is about 76 kJ/m².

4.3.4 Fibre tensile failure

Fibre tensile failure is predicted using the criterion expressed through Eq. 1. When this criterion is fulfilled, the material response is changed to account for the changes in the structure, i.e., the explosive behaviour observed in experimental tests. The stress in the fibre direction is considered to have reached its damage onset threshold,

$$\sigma_a^o = \sigma_a|_{f_{ft}=1}, \quad \varepsilon_a^o = \varepsilon_a|_{f_{ft}=1} \quad (55)$$

and all stress components are degraded:

$$\{\tau_{ab}, \tau_{bc}, \tau_{ca}\} \leftarrow (1 - d_{ft}) \{\tau_{ab}, \tau_{bc}, \tau_{ca}\} \quad (56)$$

$$\sigma_a \leftarrow \left(1 - d_{ft} \frac{\langle \sigma_a \rangle}{\sigma_a}\right) \sigma_a \quad (57)$$

$$\sigma_b \leftarrow \left(1 - d_{ft} \frac{\langle \sigma_b \rangle}{\sigma_b}\right) \sigma_b \quad (58)$$

$$\sigma_c \leftarrow \left(1 - d_{ft} \frac{\langle \sigma_c \rangle}{\sigma_c}\right) \sigma_c \quad (59)$$

There is no need to degrade the Poisson's ratios in this case because all stress components are being affected by the damage variable. The damage variable d_{ft} is obtained from Eqs. 21 and 22, where ε , ε^o and ε^f are replaced by ε_a , ε_a^o and ε_a^f respectively. The final strain ε_a^f is defined as

$$\varepsilon_a^f = \frac{2\Gamma_a}{\sigma_a^o L}. \quad (60)$$

As it is assumed that a crack like in Fig. 5(b) is formed, the characteristic

length L is given by

$$L = L_1 L_2 \min \{ L_1 / \sin(\beta), L_2 / \cos(\beta) \}. \quad (61)$$

5 Mesh dependency

The formulation presented should avoid the mesh dependency characteristic to most failure models. Fig. 6 shows four different meshes for a square specimen, loaded by an uniform tension stress in the b direction (by imposing a uniform displacement). Mesh (a) consists of a single element, while meshes (b), (c) and (d) have 3^2 , 9^2 and 27^2 elements respectively. No artifice is used to trigger failure at any specific location, or along any particular direction.

It can be seen from Figs. 6(b)-(d) that fracture is always predicted in a band normal to the loading direction, as expected, and is always smeared over the width of one element. Moreover, the load-displacement curve—and thus the energy absorbed—is independent of mesh refinement and of the width of the failed region, as shown in Fig. 7. This is an important feature, given that complete failure takes place.

6 Applications

The ability of the material model implemented in LS-Dyna [4] to reproduce the physics of shear and compression is here assessed. Standard experimental tests were carried with carbon-epoxy T300/913. The numerical models presented in this section reproduce the geometry of specimens that were actually tested, and use the material properties obtained. The objective is to assess

Fig. 6. Models with different mesh densities; the failed elements are identified by a lighter colour; (a) shows the material axes; (b), (c) and (d) show the fracture planes

Fig. 7. Load-displacement curves for different mesh refinement levels

the model's capability to predict the main failure features observed experimentally, such as the inclined fracture for the transverse compression test, the $\pm 45^\circ$ failure for the shear specimen and the kink-band formation for the longitudinal compression specimen. Note that none of these features is directly built-in into the model.

The material properties obtained experimentally are presented in Table 1. Further to those, also the experimental in-plane shear stress vs. strain curve

Table 1

Mechanical properties of T300/913.

Strength (MPa)/ (Modulus (GPa))						
Longitudinal		Transverse		Shear		
Tensile	Compression	Tensile	Compression		ν_{ba}	ϕ_o
2005.4	1354.6	68.1	198.6	149.6	0.021	$\simeq 53^\circ$
(131.7)	(116.1)	(8.8)	(9.24)	(4.6)		

was used as input for the numerical model. This model however, requires additional properties associated with the through-the-thickness (c) direction. As an approximation, most of the c -direction elastic properties can be related to those in the b direction. The through-the-thickness Young's modulus E_c was taken as being equal to the in-plane transverse modulus (E_b), the shear moduli G_{bc} and G_{ca} were taken as being equal to the (initial) in-plane shear modulus (G_{ab}), and the Poisson's ratio ν_{ca} as ν_{ba} . Finally, the Poisson's ratio ν_{bc} was taken as 0.4.

As for the toughness values, the intralaminar toughness Γ_b was measured experimentally [16], and the remaining ones were just chosen such that the magnitude was considered consistent with previous observations in this paper, Table 2. Reasonable variations in the toughness values proved not to affect excessively the shape of the failed region. However, in problems where most of the analysis is focused on the post-failure region—like during crushing, in crashworthiness applications—the values of these parameters should be obtained from experimental tests.

In all the examples that follow, an element with slightly lower strength was

Table 2

Toughness values used in the numerical model

Toughness (kJ/m ²)				
Longitudinal		Transverse		In-plane
Tensile	Compression	Tensile	Compression	shear
Γ_a	Γ_{kink}	Γ_b	Γ_T	Γ_L
100	75	0.22	1.5	2

used to trigger the failure close to the middle of the specimen.

6.1 Modelling shear failure of a $(\pm 45)_{8S}$ test specimen

Shear tests were carried with $(\pm 45)_{8S}$ specimens, tested in tension, according to the appropriate ASTM standard [18]. The shear stress/strain curve was nonlinear almost from the beginning of the test, but no strain localization was present until immediately before final failure, which happened at a shear strain of about 25%. The data reduction was done according to the ASTM standard [18], but—in order to obtain the full strain vs. stress curve—fibre scissoring and width reduction were taken into account.

An FE model of part of the specimen, containing the failure region, is presented in Fig. 8(a). The model has the same dimensions as the actual specimen—except for the length, which is smaller. The model has 16 solid elements across the thickness, in order to simulate each layer individually.

The $\pm 45^\circ$ failure can be observed, and results from the damage affecting the *local* shear stress within each element. However, it can also be observed that

each layer fails at $\pm 45^\circ$, regardless of the actual fibre orientation in that layer being positive or negative. (In fact, when the shear stress is reduced at some angle β , it is also reduced at an angle $\beta + 90^\circ$. This may give rise to unrealistic failure patterns, and is feature typical to CDM based models.)

The numerical load vs. shear strain is compared to the experimental in Fig. 8(b). The shear strain was used in the figure instead of the elongation of the specimen because the length of the numerical model of the specimen is smaller than the length of the specimen itself and the elongation is thus different. The shear strain in the experiment was obtained from strain gauges. In the numerical case, the load vs. shear strain relationship was obtained by combining the total applied load vs. time and the shear strain vs. time relationships (the shear strain is the shear strain at a failing element). This has implications on the interpretation of the shear strain values after the onset of failure, i.e., after the maximum load is attained. It can be observed that, regardless of all the approximations characteristic to FE, the nonlinear shear behaviour is reproduced with excellent accuracy, even for the considerable strain values involved. During the strain-localization period, the load drops in a steady way, while the $\pm 45^\circ$ failure pattern is formed.

6.2 *Modelling matrix compression failure*

During transverse compression tests, failure occurs at an angle with the thickness direction, Fig. 1(a) in Part I [1]. This feature is reproduced by the numerical model shown in Fig. 9(a). For the matrix compression failure model, failure at an angle is the result of damage affecting, in this case, the τ_T shear stress in the coordinate system predicted by the failure criterion. Fig. 9(b)

Fig. 8. (a) Model of the shear test specimen; (b) experimental and numerical load-strain curves

Fig. 9. (a) Model of the transverse compression test specimen; (b) numerical stress-strain curve

shows that the stress vs. strain curve is linear up to failure onset, and then the stress value is reduced to zero during damage propagation. (As in Fig. 8(b), the strain after failure onset is an averaged strain over the specimen.)

6.3 Modelling fibre compression failure

The formation of kink bands is observed in most fibre compression failure situations. Typically, kink bands are formed at a small angle with the normal to the loading direction. For the present model, the kink-band angle is also predicted by the model, as a result of the damage variable acting on the shear

Fig. 10. (a) Model of the longitudinal compression test specimen; (b) to (g) formation of a kink band and kink-band broadening

stress in the misalignment frame.

Fig. 10(a) shows the FE mesh, with the corresponding loading. The formation of the kink band can be observed in Figs. 10(b) to (g).

Often, complementary kink bands, with the fibres rotating to the opposite direction are formed next to the first kink band [19]. This is also evident from Figs. 10(c) to (g). After the kink band is formed, further loading leads to kink-band broadening, as observed in Fig. 10(g).

7 Conclusions

This work demonstrates that FE can reproduce the key physical aspects observed in the failure of laminated composites, provided that sound, physically-based failure criteria are implemented, and that failure propagation is handled appropriately. The failure-models implementation is 3D, and allows the user to incorporate any in-plane shear curve directly. In addition, the pathological mesh dependency characteristic of CDM models is avoided using a smeared formulation.

Even though the implemented failure criteria are based on the actual physics of the failure process, the material properties required for the prediction of damage initiation are readily obtained in standard tests. To model damage propagation, the model requires the energy dissipated by each failure mode. Even though it might at first seem that it is not convenient to provide those energy values (because there are no standard tests to obtain them), it should be noted that these are ultimately the material properties that define how damage propagates. Hence, it does not seem reasonable to expect that a material model can simulate accurately damage propagation without, directly or indirectly, including those. Finally, this paper explains how those properties can be related to interlaminar toughness data, or obtained from simple tests.

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