Resistive-force theory for mesh-like superhydrophobic surfaces

Ory Schnitzer

Department of Mathematics, Imperial College London, London SW7 2AZ, UK

Ehud Yariv

Department of Mathematics, Technion — Israel Institute of Technology, Haifa 32000, Israel

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Abstract

A common realization of superhydrophobic surfaces makes use of a mesh-like geometry, where pockets of air are trapped in a periodic array of holes in a no-slip solid substrate. We consider the small-solid-fraction limit where the ribs of the mesh are narrow. In this limit, we obtain a simple leading-order approximation for the slip-length tensor of an arbitrary mesh geometry. This approximation scales as the solid-fraction logarithm, as anticipated by Ybert *et al.* [Phys. Fluids **19**, 123601 (2007)]; in the special case of a square mesh it agrees with the analytical results obtained by Davis & Lauga [Phys. Fluids **21**, 113101 (2009)].

I. INTRODUCTION

There is an ongoing interest in quantifying the drag reduction introduced by superhydrophobic surfaces [1]. Typically, these surfaces are made up of an hydrophobic solid substrate endowed with a periodic texture of holes, grooves or posts. Superhydrophobicity relies on the entrapment of air in the cavities of the microstructure when the surface is brought in contact with a liquid (typically water). In this so-called Cassie state the liquid satisfies a no-slip condition only on the solid ribs of the microstructure, which are in direct contact with it, while everywhere else the liquid interfaces a gas that is approximately shear free. The associated hydrodynamic friction may then be significantly reduced relative to that about a comparable homogeneous solid substrate, where the no-slip condition holds throughout.

This effective reduction in friction is commonly quantified in the context of a canonical problem, where the surface is exposed to an otherwise homogeneous shear flow [2–4]. Denoting the applied-shear magnitude by G and its direction by $\hat{\mathbf{e}}$, a unit vector in the periodic “superhydrophobic plane,” the imposition of shear is manifested by the following far-field condition on the flow field $\mathbf{u}$,

$$\mathbf{u} \sim Gz\hat{\mathbf{e}} \quad \text{as } z \rightarrow \infty, \quad (1)$$

wherein z is the coordinate perpendicular to the surface. Had the surface been a homogeneous solid, the shear profile (1) would hold at each z . For a textured surface, on the other hand, that profile cannot satisfy the combination of no-slip and shear-free conditions; the flow therefore adopts a more complicated pattern, which asymptotes to (1) at z large compared with L , a characteristic period of the texture.

The lumped signature of the near-wall flow is manifested in a non-decaying asymptotic correction to the far-field shear,

$$\mathbf{u} \sim G(z\hat{\mathbf{e}} + \boldsymbol{\lambda} + \text{decaying terms}) \quad \text{as } z \rightarrow \infty, \quad (2)$$

where the slip-length vector $\boldsymbol{\lambda}$ is, like $\hat{\mathbf{e}}$, in the superhydrophobic plane. Assuming small capillary and Reynolds numbers, such that the shape of the liquid-gas interface is fixed and inertia forces are negligible, the flow problem becomes linear and homogeneous in $\hat{\mathbf{e}}$, hence so is $\boldsymbol{\lambda}$. Dimensional arguments, moreover, necessitate that $\boldsymbol{\lambda}$ is proportional to L for a

42 given geometry. We therefore write

$$\boldsymbol{\lambda} = L \mathbf{B} \cdot \hat{\mathbf{e}}, \quad (3)$$

43 where $\mathbf{B}$ is a second-rank tensor with components in the plane. When $\hat{\mathbf{e}}$ is chosen to lie
 44 along one of the principal directions of $\mathbf{B}$, the slip $\boldsymbol{\lambda}$ is parallel to $\hat{\mathbf{e}}$, say $\boldsymbol{\lambda} = \lambda \hat{\mathbf{e}}$. The scalar
 45 slip length λ may be interpreted as the distance below the surface at which the linear shear
 46 profile extrapolates to zero.

47 The dimensionless slip-length tensor $\mathbf{B}$ is a geometric entity which is independent of both
 48 the shear magnitude G , its direction $\hat{\mathbf{e}}$, and the linear dimension L . A key geometric parame-
 49 ter of any superhydrophobic surface is the solid fraction ϕ . For $\phi = 0$, the superhydrophobic
 50 surface becomes a free surface which cannot support an externally imposed shear; the canon-
 51 ical problem is then ill posed. It follows that the slip length becomes singular in the limit
 52 of small solid fractions. Given the obvious desire in attaining large slip lengths [5, 6], this
 53 limit is of significant interest. The first investigation of that limit was carried out by Ybert
 54 *et al.* [3], who derived scaling laws for various superhydrophobic geometries. In particular, a
 55 logarithmic singularity [namely $\lambda/L = O(\ln \phi)$] was predicted for configurations of grooves,
 56 where the free surfaces are infinite stripes, and mesh geometries, where the free surfaces are
 57 isolated patches.

58 Given its relative simplicity, the grooves configuration has been the subject of numer-
 59 ous theoretical investigations, with the logarithmic singularity being confirmed by both
 60 conformal-mapping analyses [7, 8], Fourier-series solutions [9, 10], and asymptotic approxi-
 61 mations [11, 12]. In contrast, the mesh geometry has received far less attention. To the best
 62 of our knowledge, the only theoretical analysis of that geometry was carried out by Davis &
 63 Lauga [13], where an approximation for a square mesh was developed by representing the
 64 flow field using stokeslet singularities distributed along the solid edges of the doubly peri-
 65 odic mesh. Transforming the resulting integral equation into an infinite algebraic system,
 66 the latter system can be solved numerically using truncation. Davis & Lauga also managed
 67 to analytically extract the leading $O(\ln \phi)$ logarithmic singularity from the latter algebraic
 68 system, as well as its leading-order $O(1)$ correction (i.e., the two leading terms in an infinite
 69 expansion in inverse powers of $\ln \phi$). While the coarse analytical approximations obtained
 70 by Davis & Lauga take a simple closed form, they are underlined by a cumbersome scheme
 71 that is difficult to generalize.

FIG. 1. Rectangular mesh geometry. The unit-cell boundary is delineated with a dashed line.

Our goal here is to show how the logarithmic singularity of the slip-length tensor of superhydrophobic mesh surfaces can be deduced via a direct leading-order asymptotic analysis similar in spirit to local slender-body (“resistive-force”) theory, ubiquitous in microhydrodynamics of slender particles and flagellar swimmers [14]. In addition to its simplicity and the physical insight it provides, one of the benefits of our approach is the relative ease by which one can handle general mesh geometries. The leading-order slip-length expressions provided by the present scheme may be used as rough approximations for design purposes and for understanding experimental realizations of mesh geometries [15–18].

II. PROBLEM FORMULATION

Consider a mesh geometry with a periodic (elementary) unit cell whose area is L^2 . The ribs of the mesh are assumed to be of uniform thickness ϵL . When immersed in a liquid, air is trapped in the voids of the mesh. Both the upper edges of the solid mesh and the free surfaces which are spanned within them are assumed flat. Consistently with the generic description of Sec. I we employ Cartesian coordinates (x, y, z) with the x - and y -axes lying in the plane of the textured surface and the normal coordinate z measured from that plane (the location of the origin being arbitrary). Fig. 1 shows a schematic sketch of the geometry in the special case of a rectangular mesh, which we will return to in Sec. IV.

We consider the canonical problem described in Sec. I, where the compound solid–gas surface is exposed to an otherwise homogeneous shear flow of magnitude G in the direction

of the planar unit vector $\hat{\mathbf{e}}$. In the half space $z > 0$ the velocity field $\mathbf{u} = u\hat{\mathbf{i}} + v\hat{\mathbf{j}} + w\hat{\mathbf{k}}$ is governed by the continuity and Stokes equations,

$$\nabla \cdot \mathbf{u} = 0, \quad \nabla p = \mu \nabla^2 \mathbf{u}, \quad (4)$$

wherein p is the pressure field and μ the liquid viscosity. At $z = 0$ $\mathbf{u}$ satisfies impermeability and no-slip conditions at the solid ribs,

$$\mathbf{u} = \mathbf{0}, \quad (5)$$

while at the free surface it satisfies impermeability and shear-free conditions,

$$w = 0, \quad \frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0. \quad (6)$$

In addition, it also satisfies the far-field condition (1) representing the applied shear. The preceding problem uniquely defines $\mathbf{u}$ and hence from (2) the slip-length as

$$\lambda = \lim_{z \rightarrow \infty} (\mathbf{u}/G - z\hat{\mathbf{e}}). \quad (7)$$

Our goal is to evaluate the slip length λ in the asymptotic limit $\epsilon \ll 1$, or equivalently in the limit where the solid fraction (which is proportional to ϵ) is small.

III. SMALL SOLID FRACTIONS

The imposed shear flow (1) corresponds to a viscous shear force of magnitude μG per unit area. This force is supported by the solid ribs of the superhydrophobic surface. To leading order, we may neglect the contribution of the junctions where these ribs intersect. This suggests that the force per unit length exerted by the liquid on the ribs is $O(\mu GL)$. With the $O(\mu GL)$ force on the ribs being transferred via viscous stresses, and with the flow in the vicinities of the ribs varying on an $O(\epsilon L)$ length scale, the magnitude of the flow there is $O(GL)$. On such small scales, however, the ribs appear to be infinite in length. The Stokes “paradox” therefore implies that the induced flow field does not attenuate in the vicinity of a forced rib; rather, it grows logarithmically with distance away from it.

In slender-body theory, such logarithmic growth away from the locally infinite body is arrested on scales comparable with the length of the body, where its finiteness becomes apparent [19]; the logarithmic divergence at the small scale is then manifested, on the scale

114 of the body, in a rescaling of the velocity magnitude by an $O(\ln \epsilon)$ factor. The situation
 115 here, where we expect the $O(L)$ periodicity of the mesh geometry to curb the logarithmic
 116 growth, is similar.

117 Following the above discussion we stipulate that $\mathbf{u}/GL = O(\ln \epsilon)$ on $O(L)$ scales away
 118 from the ribs; from (7) we see that the slip length is similarly amplified, i.e., $\boldsymbol{\lambda}/L = O(\ln \epsilon)$.
 119 Writing $\mathbf{u} = \tilde{\mathbf{u}} + \dots$ and $\boldsymbol{\lambda} \approx \tilde{\boldsymbol{\lambda}} + \dots$ we focus hereafter upon calculation of the (logarithmi-
 120 cally) leading-order variables $\tilde{\mathbf{u}}$ and $\tilde{\boldsymbol{\lambda}}$. The leading-order flow $\tilde{\mathbf{u}}$ satisfies the homogeneous
 121 impermeability and no-shear conditions (6) at $z = 0$ and is ignorant of the relatively weak
 122 applied shear; thus, rather than (1), at large distances it satisfies

$$\tilde{\mathbf{u}}/G \rightarrow \tilde{\boldsymbol{\lambda}} \quad \text{as } z \rightarrow \infty. \quad (8)$$

123 Assuming that the homogeneous conditions (6) are satisfied by $\tilde{\mathbf{u}}$ all over the surface at
 124 $z = 0$, the only inhomogeneity in the above problem enters through (8). The solution of
 125 that problem is accordingly given by the uniform flow

$$\tilde{\mathbf{u}}/G \equiv \tilde{\boldsymbol{\lambda}}. \quad (9)$$

126 The above assumption is justified since, on the scale of the periodicity, the ribs degenerate
 127 to lines of zero thickness; thus, in the above problem governing $\tilde{\mathbf{u}}$, deviations from the
 128 homogeneous conditions (6) imply a distribution of point forces along those lines, with line-
 129 density $O(\mu GL \ln \epsilon)$ — in contradiction to our earlier estimate.

130 The uniform flow $\tilde{\mathbf{u}}$ incident on the ribs results in resistance forces exerted upon them
 131 by the liquid. Given the homogeneous conditions (5) and (6) satisfied on the solid ribs and
 132 adjacent gas interfaces, the local resistance problem (prescribed for $0 < z < \infty$) can be
 133 evenly extended about the $z = 0$ plane. The extended resistance problem, prescribed for
 134 $-\infty < z < \infty$, is symmetric about $z = 0$; it entails the same uniform flow, now incident on
 135 a collection of flat solid strips of width ϵL . We accordingly need the appropriate expression
 136 for the hydrodynamic force density experienced by such strips.

137 In his paper on slender-body theory, Batchelor [20] considers a more general problem
 138 where a uniform stream, say $\mathbf{U}$, is incident on a thin cylinder of arbitrary cross section.
 139 Batchelor finds the leading-order hydrodynamic force density (per unit length) exerted upon
 140 the cylinder as

$$\frac{2\pi\mu}{\ln(1/\epsilon)} \{ \hat{\mathbf{e}}_{\parallel} \hat{\mathbf{e}}_{\parallel} + 2(\mathbf{I} - \hat{\mathbf{e}}_{\parallel} \hat{\mathbf{e}}_{\parallel}) \} \cdot \mathbf{U}, \quad (10)$$

wherein $\hat{\mathbf{e}}_{\parallel}$ is the local unit vector parallel to the cylinder centerline and $\mathbf{l}$ the unit tensor [21]. This leading-order result is independent of the cross-sectional shape, as long as the ratio of the cross-sectional dimensions and the characteristic longitudinal scale is of order $\epsilon \ll 1$. Upon identifying $\mathbf{U}$ with the uniform velocity $\tilde{\mathbf{u}}$ and $\hat{\mathbf{e}}_{\parallel}$ with a unit vector locally parallel to flat strip, Batchelor's result may therefore be adopted for our configuration. Recall, however, that for our purposes only the force acting on the upper side of the strip counts. Hence, using (9), the leading-order approximation for the local force per unit length exerted on a solid rib is

$$\frac{\pi\mu G}{\ln(1/\epsilon)} \{ \hat{\mathbf{e}}_{\parallel} \hat{\mathbf{e}}_{\parallel} + 2(\mathbf{l} - \hat{\mathbf{e}}_{\parallel} \hat{\mathbf{e}}_{\parallel}) \} \cdot \tilde{\boldsymbol{\lambda}}. \quad (11)$$

Since $\tilde{\boldsymbol{\lambda}}$ is in the superhydrophobic plane, we can take $\mathbf{l}$ to be the two-dimensional unit tensor in that plane, $\hat{\mathbf{i}}\hat{\mathbf{i}} + \hat{\mathbf{j}}\hat{\mathbf{j}}$.

Having determined the force distribution along the ribs of the mesh geometry in terms of $\tilde{\boldsymbol{\lambda}}$, the latter can now be obtained by balancing the net force $\mu GL^2 \hat{\mathbf{e}}$ exerted on each unit cell with the net force implied by (11). We thereby find

$$\mathbf{G} \cdot \tilde{\boldsymbol{\lambda}} \approx \hat{\mathbf{e}} L \ln \frac{1}{\epsilon}, \quad (12)$$

where

$$\mathbf{G} = \frac{\pi}{L} \int \{ \hat{\mathbf{e}}_{\parallel} \hat{\mathbf{e}}_{\parallel} + 2(\mathbf{l} - \hat{\mathbf{e}}_{\parallel} \hat{\mathbf{e}}_{\parallel}) \} dl \quad (13)$$

is a dimensionless symmetric tensor characterizing the effect of the geometry, in which dl is a differential length element and the integral is taken along all the ribs within the unit cell. (A 1/2-factor is introduced when considering ribs coincident with the boundaries of the unit cell, as these are shared by two adjacent unit cells.) It is only via the latter use of an integral force balance that the imposed shear, which did not affect the calculation of the leading-order field $\tilde{\mathbf{u}}$, is introduced. This use accordingly constitutes a convenient alternative to higher-order asymptotic matching.

Once the tensor $\mathbf{G}$ is calculated for a given geometry, inversion of (12) yields the slip length $\tilde{\boldsymbol{\lambda}}$ as

$$\frac{\tilde{\boldsymbol{\lambda}}}{L \ln(1/\epsilon)} = \mathbf{G}^{-1} \cdot \hat{\mathbf{e}}, \quad (14)$$

whose scaling confirms our *a priori* stipulation. Comparing with (3) and noting that $\hat{\mathbf{e}}$ is arbitrary we find the following leading-order approximation for the slip-length tensor,

$$\mathbf{B} \approx \mathbf{G}^{-1} \ln \frac{1}{\epsilon}. \quad (15)$$

IV. APPLICATION TO VARIOUS MESH GEOMETRIES

We conclude by demonstrating the application of the general formula (15) to several mesh geometries.

A. Rectangular mesh

Consider the rectangular mesh shown in Fig. 1, with sides κL along the x axis and $\kappa^{-1}L$ along the y axis. Accounting for the $1/2$ adjacency factor, we readily find

$$\int \{ \hat{\mathbf{e}}_{\parallel} \hat{\mathbf{e}}_{\parallel} + 2(1 - \hat{\mathbf{e}}_{\parallel} \hat{\mathbf{e}}_{\parallel}) \} dl = \kappa L (\hat{\mathbf{i}}\hat{\mathbf{i}} + 2\hat{\mathbf{j}}\hat{\mathbf{j}}) + \kappa^{-1}L (\hat{\mathbf{j}}\hat{\mathbf{j}} + 2\hat{\mathbf{i}}\hat{\mathbf{i}}), \quad (16)$$

whereby (13) gives

$$\mathbf{G} = \pi [(\kappa + 2\kappa^{-1})\hat{\mathbf{i}}\hat{\mathbf{i}} + (2\kappa + \kappa^{-1})\hat{\mathbf{j}}\hat{\mathbf{j}}]. \quad (17)$$

The general formula (15) thus yields

$$\mathbf{B} \approx \frac{\kappa}{\pi} \left(\frac{\hat{\mathbf{i}}\hat{\mathbf{i}}}{\kappa^2 + 2} + \frac{\hat{\mathbf{j}}\hat{\mathbf{j}}}{2\kappa^2 + 1} \right) \ln \frac{1}{\epsilon}. \quad (18)$$

It is evident from the above calculation that the leading-order formula (18) holds without change also for the eccentric rectangular mesh considered in Ref. 18.

For a square mesh ($\kappa = 1$), for which tensor (18) is isotropic, we find the slip length $\lambda \approx L \ln(1/\epsilon)/3\pi$, in agreement with Davis & Lauga [13].

B. Equilateral triangles

Consider now a mesh of equilateral triangles of side P . The unit cell is a parallelogram consisting of two such triangles, hence symmetry allows to consider the resistive force on a single triangle, say that with sides marked by “1,” “2,” and “3” (see Fig. 2). Accounting for the $1/2$ boundary factor we obtain on edge “1”

$$\int \{ \hat{\mathbf{e}}_{\parallel} \hat{\mathbf{e}}_{\parallel} + 2(1 - \hat{\mathbf{e}}_{\parallel} \hat{\mathbf{e}}_{\parallel}) \} dl = \frac{P}{2} (\hat{\mathbf{i}}\hat{\mathbf{i}} + 2\hat{\mathbf{j}}\hat{\mathbf{j}}), \quad (19)$$

while on edges “2” and “3” the contributions to that integral are respectively given by

$$\frac{P}{2} \left[\frac{7}{4}\hat{\mathbf{i}}\hat{\mathbf{i}} + \frac{5}{4}\hat{\mathbf{j}}\hat{\mathbf{j}} + \frac{\sqrt{3}}{4}(\hat{\mathbf{i}}\hat{\mathbf{j}} + \hat{\mathbf{j}}\hat{\mathbf{i}}) \right], \quad \frac{P}{2} \left[\frac{7}{4}\hat{\mathbf{i}}\hat{\mathbf{i}} + \frac{5}{4}\hat{\mathbf{j}}\hat{\mathbf{j}} - \frac{\sqrt{3}}{4}(\hat{\mathbf{i}}\hat{\mathbf{j}} + \hat{\mathbf{j}}\hat{\mathbf{i}}) \right]. \quad (20)$$

FIG. 2. Isotropic superhydrophobic surfaces: equilateral-triangle and hexagonal honeycomb mesh configurations. The corresponding unit-cell boundaries are delineated with a dashed line.

184 Adding the three contributions gives

$$\int_{1 \cup 2 \cup 3} \{ \hat{\mathbf{e}}_{\parallel} \hat{\mathbf{e}}_{\parallel} + 2(1 - \hat{\mathbf{e}}_{\parallel} \hat{\mathbf{e}}_{\parallel}) \} dl = \frac{9P}{4} (\hat{\mathbf{i}}\hat{\mathbf{i}} + \hat{\mathbf{j}}\hat{\mathbf{j}}) \quad (21)$$

185 which is obviously isotropic. The corresponding unit-cell integral (involving two triangles)
 186 is accordingly $9Pl/2$. Substitution into (13) gives

$$\mathbf{G} = \frac{9\pi P}{2L} \mathbf{I}. \quad (22)$$

187 Since the parallelogram unit-cell area L^2 is $3^{1/2}P^2/2$ we obtain using (15)

$$\mathbf{B} \approx \frac{2^{1/2}}{3^{7/4}\pi} \mathbf{I} \ln \frac{1}{\epsilon}. \quad (23)$$

188

190 C. Honeycomb mesh

191 For hexagonal honeycomb unit cells, the integral appearing in (13) is taken over the six
 192 sides “1” through “6” (see Fig. 2). Here symmetry implies that the integral is twice that
 193 taken over edges “1,” “2,” and “3” — a quantity which has already been evaluated in (21).
 194 It then follows that (22) applies here as well. Since the area of the cell is three times the
 195 previous parallelogram area, we now have $L = 3^{3/4}P/2^{1/2}$. Approximation (15) thus gives

$$\mathbf{B} \approx \frac{2^{1/2}}{3^{5/4}\pi} \mathbf{I} \ln \frac{1}{\epsilon}. \quad (24)$$

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