RESEARCH ARTICLE

Multi-scale interactions in a compressible boundary layer

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The effect of large-scale motions in the outer portion of the logarithmic layer on the turbulence properties of the viscosity-affected near-wall layer is of interest in the context of friction drag and its control. Processes contributing to this effect have been the subject of many recent studies, but these were directed, almost exclusively, at incompressible high-Reynolds-number boundary layers (experimentally) and channel flow (computationally) in which the outer motions are distinct and pronounced. This paper examines interactions pertinent to the above subject by reference to DNS data for a compressible boundary layer at $Mach = 2.3$ at the relatively low friction Reynolds number $Re_\tau = 570$. The aim is to examine whether the outer motions that cause footprinting and modulation of the small-scale near-wall motions also pertain to this low-Reynolds-number case, or whether the logarithmic layer simply features a continuous hierarchy of motions without any particular set of outer scales playing a distinct role. In order to identify the effects of different scales, the turbulence field is separated into large-scale and small-scale motions using a two-dimensional variant of the “Empirical Mode Decomposition (EMD)”. The response of the near-wall conditions to the large-scale structures in the outer flow is then investigated by a statistical analysis involving spectra, maps of isotropy/anisotropy parameters, the pre-multiplied derivative of the second-order structure function, correlation coefficients and joint PDFs, the last constructed from conditionally sampled data for the small-scale motions within the large-scale footprints. The study demonstrates a clear commonality between observations on the influence of outer scales at high-Reynolds-number channel flow and the present low-Reynolds-number boundary layer, although the influence is, predictably, weaker in the latter.

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1. Introduction

The influence of elongated large-scale turbulent fluctuations in the outer portion of the logarithmic velocity layer on the turbulent state close to the wall is a subject that has attracted much attention over the past two decades. Major experimental campaigns by groups led by Marusic, Monty, Smits, McKeon and Ganapathisubramani [1–11] have focused on boundary layers and pipe flow at friction-Reynolds-number (Re_τ) values as high as 13,700 in boundary layers (Marusic [12]) and 37,000 in pipe flow (Hultmark et al [13], Rosenberg et al [14]). All highlight the increasing prominence of a second, outer, maximum in the streamwise turbulence energy as the clearest statistical indicator of the existence of increasingly powerful outer structures - or super-streaks - that are, most likely, being driven by shear in the log layer (Jiménez [15]). According to Marusic et al. [16], the position of this maximum follows $y^+ = 3.9\sqrt{Re_\tau}$, although this distance and the dimensions of the

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structures their length, spanwise width and quasi-periodicity in the spanwise direction scale with the thickness of the boundary layer, the pipe diameter or the channel half-height, rather than with y^+ .

A major effect of the outer structures, discussed in several papers by Marusic and his coworkers [2, 8, 9, 17–19], is that the small-scale turbulence in the viscosity-affected sublayer is substantially altered by strong footprints. These footprints are, in effect, large-scale motions in the sublayer that are highly correlated (subject to streamwise lag) with the outer motions, and are the cause of amplification or attenuation - referred to as “modulation” - of the intensity of the small-scale near-wall fluctuations. Mathis et al [19] provide a phenomenological model that is claimed to describe this interaction, relating the near-wall streamwise intensity to a “universal” signal and the intensity of the outer motions at $y^+ = 3.9\sqrt{Re_\tau}$. In addition to this amplitude modulation, measurements by Ganapathisubramani et al [6] point also to a frequency modulation of the small-scale structures due to large-scale-induced acceleration and deceleration.

In parallel to the experimental studies noted above, DNS and LES studies, mostly in channel flow, have provided a wealth of additional information on large-scale/small-scale interactions, albeit at considerably lower Reynolds numbers. Statistical data reported by Jiménez [15], Chung & McKeon [20], Schlatter & Örlü [21], Bernardini & Pirozoli [22] and Agostini & Leschziner [23, 24] and Agostini et al [25] confirm the essential validity of the footprinting and modulation concepts. Most recently, Agostini & Leschziner examined the interactions by reference to DNS data of Lozano-Durán & Jiménez [26, 27] for $Re_\tau = 4200$, for which value the spectra demonstrate a clear spectral (length-scale) separation between the energy-density peaks of inner small-scales and the outer large-scales. Separating the large-scale and small-scale motions by means of the Empirical Mode Decomposition of Huang [28], they demonstrated a multi-faceted and differentiated response of the small scales to either positive or negative large-scale footprints, and the sensitivity of the details of this response to the large wall-normal variations in the skewness of the large-scale motions.

In channel flow, the DNS-based studies suggest that the lowest Reynolds number at which the interactions between large and small scales can be credibly resolved and interpreted is around $Re_\tau = 500 - 1000$. Indeed, the majority of DNS-based studies reported in the past have been restricted to values around the upper level. Among these, the work of Agostini & Leschziner [23] provides the most extensive analysis of their own DNS results at $Re_\tau = 1020$. For that value, the energy of the large-scale motions at $y^+ = 3.9\sqrt{Re_\tau}$ marginally reaches a maximum, but the distribution across lower values of y^+ (excluding the viscous sublayer) is quite flat. It is thus unclear whether the large-scale motions at low y^+ values are genuinely associated only with footprinting or are, rather, a combination of footprinting and locally generated large-scale motions, reflecting the same physics as that giving rise the usual maximum energy in the buffer layer around $y^+ \approx 13$.

In a spatially evolving flat-plate boundary layer, bounded by an irrotational stream, the outer part of the shear layer differs drastically from that in a channel flow close to the channel centreline. In particular, the turbulence level is lower, and the log-law layer is thinner. The expectation is, therefore, that the spectral separation, in terms of wave-length, between the large-scale and small-scale motions is less well defined in the boundary layer at the same Re_τ value, especially if this value is relatively low. In this case, the “large-scale” motions, if defined as such by some appropriate separation method, would be present across the near-wall portion of the boundary layer, not only because of footprinting, but also as a consequence of local shear-driven generation. If this is indeed so, the energy levels

within the spectrum of scales would be expected to vary monotonically, with no distinct elevation in intensity at the wave-length of the large scales that could be attributed, unambiguously, to footprinting.

The aim of this paper is to examine the above issues on the basis of an extensive statistical description involving a separation of the turbulence field into several length-scale ranges in a compressible boundary layer at Mach = 2.3, with friction and momentum-thickness Reynolds numbers at its downstream end being $Re_\tau \approx 570$ and $Re_\theta = 2000$, respectively. The fact that the boundary layer is supersonic is incidental and of little relevance to the present considerations. Huang et al [29] have shown that compressibility effects on the turbulence statistics in supersonic boundary layers are minor up to $M \approx 5$.

2. Simulation details and principal flow features

The data forming the basis of the present analysis originate from a DNS study by Poggie [30, 31] for a boundary layer at $M = 2.3$ in the free stream. The numerical scheme used in the DNS code is a sixth-order compact difference approach, combined with 8th order Pade-type filtering. Time integration is accomplished with an implicit approximately factored method (see Ref [30, 31] for details). The simulation covered a streamwise domain $L_x = 100\delta_0$, where δ_0 is the upstream boundary layer thickness, corresponding to $Re_{\delta_0} = 15000$. The wall-normal and spanwise box dimensions were $L_y, L_z = 5\delta_0$. This box was covered by a mesh of 1.1×10^{10} nodes, for which the internodal distances, scaled by wall conditions at $x/\delta_0 = 100$, are $\Delta x^+ = 6$, $y_{wall}^+ = 0.5$, $\Delta z^+ = 5$. A database was assembled by collecting data for 35000 time levels over a period $\Delta t^+ = 1862$ at one spanwise plane located at $x/\delta_0 = 100$. At this plane, the conditions are characterised by $Re_\theta = 2000$, $Re_\tau = 570$. Statistics to follow thus arise from time- and spanwise-averaging of the data at one streamwise plane.

Figure 1 shows wall-normal distributions of the mean velocity and the Reynolds-stress components at the above location, all scaled with the density-weighted friction velocity u_τ . The mean velocity is seen to feature a rather short log-law region, due to the modest Reynolds number and the presence of the outer wake region beyond $y^+ \approx 300$. The peak of the streamwise stress occurs at $y^+ \approx 13$, at which the near-wall streaks are also observed to be most strongly pronounced. Within the layer $y^+ \approx 150 - 200$, a distinctive elevation in the streamwise stress is observed, giving rise to an inflection point at $y^+ \approx 100$ and indicating the presence of energetic large-scale motions. In contrast, Marusic et al's correlation $y^+ = 3.9\sqrt{Re_\tau}$, yields $y^+ = 93$ as an estimate of the location at which the energetic outer structures reside. However, this correlation was devised by reference to data at much higher Reynolds-number values, and is thus likely to be inaccurate at the present value.

3. Statistical characterisation via spectral maps

A map of pre-multiplied energy spectra for the streamwise velocity component, showing the dependence of the energy density as a function of spanwise wave length and wall-normal distance, is given in figure 2(a). In figure 2(b), the energy density is normalized by the variance at the relevant wall distance – i.e, the value of the integral of the normalized energy density is unity at each wall-normal location. This presentation serves to accentuate the scale-wise distribution of the streamwise-energy density at any one y^+ location.

Figure 1. Basic mean-flow properties of the present boundary layer at the streamwise location $Re_\theta = 2000, Re_\tau = 570$; (a) Van-Driest scaled mean-velocity profile; (b) profiles of Reynolds-stress components.

Figure 2. Map of the energy distribution by scale in the wall-normal direction produced by the streamwise velocity fluctuations : (a) premultiplied power spectra $\Phi_{uu}^+(y^+, \lambda_z^+)$ and (b) premultiplied power spectra normalized by the streamwise stress $\Phi_{uu}^+(y^+, \lambda_z^+) / \overline{u'u'^+}$

The maps indicate that the most energetic region is around $y^+ \approx 13$, essentially the centre of the buffer layer. As the near-wall streaks are conventionally associated with the magnitude of the streamwise fluctuations, it follows that the streaks are most pronounced at this wall-normal location. As expected, the associated spanwise wave length $\lambda_z^+ \approx 100$, indicating the inter-streak separation distance. As regards the energy contained within the outer structures, the map indicates a diagonally stretched region of elevated energy density, extending to $y^+ \approx 150$ over the wave-length range $\lambda_z^+ = 300 - 500$ (i.e. roughly one boundary-layer thickness). In fact, the spectra hint at a weak local maximum at around $y^+ = 100$, suggesting the presence of distinct outer structures - or “super-streaks” - in the outer portion of the log-law layer. These features are entirely consistent with variation of the streamwise Reynolds stress in figure 1(b). The locally normalized spectra in figure 2(b) reveal more clearly than figure 2(a) the existence of two distinct regions interfacing at $y^+ \approx 80$, the lower one dominated by the energy of small-scale motions and the upper by larger-scale motions of wave length $\lambda_z^+ = 300 - 800$. However, at this stage, neither map provides compelling evidence that the outer large-scale motions give rise to footprints in the lower layer and modify that layer’s structure.

A consequence of the low value of the Reynolds number is that a distinction

Figure 3. Characterisation of the turbulence state across eddy-size range : (a) map of $(\gamma^{3c})^2$ associated with a trend towards isotropy; (b) map of $(\gamma_u^{1c})^3$ characterising the anisotropy due to the dominance of the streamwise energy density $\Phi_{uu} \gg \Phi_{vv}, \Phi_{ww}$

between footprints of highly elongated outer large scales and the spectrum of other scales within the log-law region is tenuous. It thus follows that the identification of the interactions among the scales populating the flow requires the use of a range of statistical indicators beyond those associated purely with the streamwise turbulence energy, as adopted in studies that target footprinting and modulation of inner scales by outer ones in high-Reynolds-number flows. In particular, at any given wall distance, different scale ranges will be characterized by varying levels of energy-density anisotropy, so that statistical parameters quantifying this property are called for. To this end, the following two scalar parameters are introduced:

- the “isotropy parameter”

$$\gamma^{3c} \equiv \frac{3|\Phi_{uu}||\Phi_{vv}||\Phi_{ww}|}{|\Phi_{uu}|^3 + |\Phi_{vv}|^3 + |\Phi_{ww}|^3}, \quad (1)$$

in which Φ_{uu} , Φ_{vv} and Φ_{ww} are the spectra for the three components u , v and w , respectively. This parameter tends to a maximum of 1 in the case of isotropy, declining to zero in the case of a two-component or a one-component state;

- the “one-component anisotropy parameter”

$$\gamma_u^{1c} \equiv \frac{|\Phi_{uu}||\Phi_{uu}|}{|\Phi_{uu}|^2 + |\Phi_{vv}|^2 + |\Phi_{ww}|^2}, \quad (2)$$

which identifies the dominance of the streamwise component over the other two. This tends to 1 when the energy is increasingly contained in the Φ_{uu} spectra, diminishing when the anisotropic state departs from the one-component condition.

Figures 3(a) and 3(b) show, respectively, maps of the square of γ^{3c} and of the cube of γ_u^{1c} , the power representation being adopted purely in order to accentuate the variations in the respective parameters across the scale range. The former map identifies a band beyond the buffer layer, covering a significant range of λ_z^+ values, in which the isotropy parameter is elevated, thus indicative of the inertial subrange. The map in figure 3(b) identifies the range of the eddies characterized by a dominance of streamwise fluctuations. As expected, high levels of the parameter γ_u^{1c} arise in the buffer layer at $\lambda_z^+ \approx 100$, characteristic of the near-wall streaks. Remarkably, elevated values are also observed in the region $y^+ \approx 100$, $\lambda_z^+ \approx 600$,

Figure 4. Map of the energy distribution by scale in the wall-normal direction produced by the streamwise velocity fluctuations : (a) 2^{nd} order structure function $S_{2,u+}(\delta_z^+)$ and (b) premultiplied derivative of 2^{nd} order structure function $\delta_z dS_{2,u+}(\delta_z^+)/d\delta_z$

indicating the presence of (albeit weak) elongated outer structures. There is also evidence of footprinting arising from these structures in the viscosity-affected near-wall layer – a suggestion that will be given support by statistics to follow. Hence, despite the low Reynolds number, there is some evidence of the outer-to-inner interactions that are observed, albeit much more clearly, at higher Reynolds numbers, although it is not possible to exclude, at this stage, that the vertical band of elevated γ_u^{1c} values around $\lambda_z^+ \approx 600$ in figure γ_u^{1c} reflects, at least in part, local generation of large-scales motions.

Another route to identifying relevant structural features of eddy scales present in the flow is to use the second-order structure function, defined by :

$$S_{2,u}(y, \delta_z) = \overline{[u(y, z) - u(y, z + \delta_z)]^2} \quad (3)$$

which can be related to the correlation function $f(\delta_z)$ by the relation :

$$S_{2,u}(y, \delta_z) = 2\overline{u'u'}(1 - f(\delta_z)) \quad (4)$$

The second-order structure function quantifies the energy associated with all structures with a size smaller than δ_z . It follows that the pre-multiplied derivative of the structure function (referred to by the abbreviation “PMDS2” henceforth) $\delta_z \times \frac{dS_{2,u}(\delta_z)}{d\delta_z}$ characterizes the contribution to the energy by eddies having a length δ_z (Townsend [32], Davidson at al. [33]). The PMDS2 can thus be seen as being akin to the premultiplied power spectra ($k_z \Phi_{uu}$), although the two are not formally equivalent, because the PMDS2 is affected by the correlation between eddies of different sizes.

The second-order structure function and its premultiplied derivative are shown in figures 4(a) and (b), respectively. In the PMDS2 map, the near-wall streaks are identified by the peak at $y^+ \approx 13$ and $4\delta_z^+ \approx 100$, corresponding to the equivalent peak in the spectra of figure 2(a). In fact, this correspondence is essentially the reason for choosing the multiplier 4 in the abscissa of figure 4 (a quantitative argument justifying the choice of the multiplier is given in Agostini & Leschziner [34]). In contrast to the spectra, the PMDS2 map identifies coherent energetic structures across the flow at $4\delta_z^+ \approx 600$, which may be interpreted as the footprints of large-scale motions present in the outer flow.

4. Separation of scales and scale-specific statistics

The question of how to separate scale ranges co-existing in a turbulent field has been the subject of much debate. The large majority of studies concerned with the separation, at high Reynolds numbers, of the distinct set of outer large scales from the remaining “small-scale” field have adopted Fourier-based filtering with prescribed cut-off wave-length values, the latter based on observations of the scale separation – e.g. arising from spectral maps. In contrast, the present authors have opted for a spatially two-dimensional extension of Huang et al’s [28] “empirical mode decomposition” (EMD). This method has previously been applied by Agostini & Leschziner [23, 24] and Agostini et al [25] to the analysis of channel-flow data at $Re_\tau = 1020$ and 4200.

The EMD has a number of attractive features that allows it to be used flexibly in the present context. The method splits any signal into a set of Intrinsic Mode Functions (IMFs) based purely on the local characteristic time/space scales of the signal. The method requires no pre-determined functional elements, such as Fourier or wavelet functions. Rather, the IMFs are the EMD-generated basis functions, which arise purely from the given signal itself. Unlike Fourier methods, the EMD does not require filters to separate the scales, and does not involve filter-induced loss of energy: the sum of all mode-specific energy components and the residual energy equals the energy of the parent signal. Importantly, a mode is not a signal having a unique frequency or length scale, but one that has a narrow range of scales and characterised by narrow-band spectrum with a mean scale that rises with mode number i . More precisely for a fully turbulent flow, the mean frequency and the bandwidth associated with each mode doubles between two successive modes [35].

Figure 5 illustrates, qualitatively, the EMD’s scale-resolving characteristics when applied to time-space ($z^+ - t^+$) trace of the turbulent-fluctuation field at $y^+ \approx 13$. The top plot represents the raw data, while plots 5(b)-5(e) show four EMD modes characterising, sequentially, the largest-scale to the smallest-scale motions. For modes $i = 3$ and 4, representing the largest scales, closer inspection suggests that the structures are separated, on average, by δz^+ of order 1000 and 2000, respectively. In contrast to high-Reynolds-number boundary layer, in which the small scales are visibly modulated by the large scales, no such modulation is discernible in the present flow; in other words, the fields for modes $i = 1$ and 2 show little evidence of being affected by modes 3 and 4. However, reference to the raw field does suggest some modulation, and data presented later, in the form of joint PDFs will provide clear quantitative evidence of modulation.

A quantitative description of the modes is provided, in figure 6(a), by way of separate pre-multiplied spectra for the four EMD modes. Coloured line contours of the spectra are presented twice: they are first superimposed on the grey contours of the spectra for the total fluctuations field in figure 2(a) and, second, on the map of the anisotropy parameter γ_u^{1c} in figure (b). Figures 6(c) and (d) show profiles of the pre-multiplied spectra for the different modes at $y^+ \approx 13$ and $y^+ \approx 130$, respectively. In addition, the black profiles represent the sum of the modes, i.e. the spectrum of the total streamwise energy at the two y^+ locations considered. The purpose of this multifaceted presentation is to demonstrate the ability of the EMD to separate the total fluctuation field into components that cover relatively narrow ranges of scales. While there is, clearly, an overlap of the energy of the modes, each mode is compressed into a relatively narrow range of scales. Modes $i = 3, 4$ represent primarily the energy in the outer part of the flow. Mode $i = 1$ represents primarily the scales that are associated with the near-wall streaks, the maximum being at $\lambda^+ \approx 100$, but it is clear that some of the energy of this mode also extends

Figure 5. Part of the spatio-temporal map of the streamwise fluctuations field and the EMD modes : (a) raw field; (b) 4^{th} mode, which corresponds to the residual (largest scales); (c) 3^{rd} mode; (d) 2^{nd} mode; (e) 1^{st} mode (smallest scale).

to much large wave-length values.

In the viscosity-affected layer, $y^+ \approx 13$, sequential modes contain a continually diminishing levels of energy. Thus, all scales coexist, with mode $i = 1$ dominant. Although there is no clear evidence of unusually elevated energy associated with any of the modes $i > 1$ at this y^+ position, there is nevertheless a clear indication that mode $i = 3$ is the cause for a distinctive plateau in the total-streamwise-energy spectrum, which constitutes footprinting. Figure 6(d) also shows that this same mode, $i = 3$, dominates and gives rise to the maximum in the spectrum, thus being associated with the outer large scales that cause the footprinting in the viscosity-affected layer.

The position in the outer region is more interesting, in so far as mode $i = 3$, associated with large-scale motions at $y^+ \approx 100 - 150$, features a maximum at ($\lambda_z^+ \approx 600$) – i.e., representing structures that are roughly one boundary-layer thickness apart. This is consistent with observations derived from the PMDS2 as well as the anisotropy-parameter maps discussed in Section 3. Moreover, the contours of this mode, shown in figure 6(a) and (b) demonstrate a degree of wall-normal coherence, usually taken to indicate footprinting in flows at much higher Reynolds-number values.

A comparison between the maximum values reached by the different modes in plots 6(c) and (d), and the line contours in figure 6(a) shows that the energy associated to the smallest scales rapidly weakens with increasing distance from the wall. This is in contrast to the energy associated with the mode $i = 3$, which varies little with wall distance – a behaviour that accords with the attached eddies hypothesis proposed by Townsend [32] and Perry et al [36].

The superposition, in figure 6(b), of the contours of the premultiplied spectra for the modes onto the map of the anisotropy parameter serves to reinforce the validity of the assertion that mode $i = 3$ essentially represents the “super-streaks” in the outer portion of the log-law region. It also serves to clarify that the dark region in the viscous sublayer below the outer maximum is associated, at least in part, with footprinting, and this is consistent with the message derived previously from the PMDS2 map in figure 4(b).

A behaviour that is consistent (indeed, indicative of) footprinting is a high level of wall-normal correlation of the large-scale motion. More generally, small-scale motion will spatially decorrelate quickly, while large-scale motions will affect a substantial proportion of the boundary layer. The degree of correlation of different EMD-resolved scales is best conveyed by correlation coefficients that correlate mode- m motions u_{emd_i} at a chosen reference location y_{ref} to the raw field u at any other location y , subject to a separation by a transverse distance Δz or a time shift Δt :

$$C(y, y_{ref}, \Delta t) = \frac{1}{N_z} \sum_{k=1}^{N_z} \left(\frac{r_{12}(y, y_{ref}, z_k, \Delta t, x/\delta_0)}{\sqrt{r_{11}(y, z_k, \Delta t, x/\delta_0)} \cdot \sqrt{r_{22}(y_{ref}, z_k, \Delta t, x/\delta_0)}} \right) \quad (5)$$

Figure 6. Premultiplied spectra of EMD modes for streamwise fluctuations: (a) spectra of EMD modes super-imposed on raw-field spectra; (b) spectra of EMD modes super-imposed on map of isotropy parameter (figure 3(b)); (c) spectra of EMD modes at $y^+ \approx 13$; (d) spectra of EMD modes at $y^+ \approx 130$.

$$\begin{aligned}
 r_{12}(y, y_{ref}, z_k, \Delta t, x/\delta_0) &= \sum_{i=1}^{N_t} [u_{emd_i}(y_{ref}, z_k, t_i + \Delta t, x/\delta_0) - \langle u_{emd_i}(y_{ref}, z_k, x/\delta_0) \rangle_t] \\
 &\quad \times [u(y, z_k, t_i, x/\delta_0) - \langle u(y, z_k, x/\delta_0) \rangle_t] \\
 r_{11}(y_{ref}, z_k, \Delta t, x/\delta_0) &= \sum_{i=1}^{N_t} [u_{emd_i}(y_{ref}, z_k, t_i + \Delta t, x/\delta_0) - \langle u_{emd_i}(y_{ref}, z_k, x/\delta_0) \rangle_t]^2 \\
 r_{22}(y, z_k, \Delta t, x/\delta_0) &= \sum_{i=1}^{N_t} [u(y, z_k, t_i, x/\delta_0) - \langle u(y, z_k, x/\delta_0) \rangle_t]^2 \\
 C(y, y_{ref}, \Delta z) &= \frac{1}{N_t} \sum_{k=1}^{N_t} \left(\frac{r_{12}(y, y_{ref}, \Delta z, t_k, x/\delta_0)}{\sqrt{r_{11}(y, \Delta z, t_k, x/\delta_0)} \cdot \sqrt{r_{22}(y_{ref}, \Delta z, t_k, x/\delta_0)}} \right) \quad (6)
 \end{aligned}$$

where:

$$\begin{aligned}
r_{12}(y, y_{ref}, \Delta z, t_k, x/\delta_0) &= \sum_{i=1}^{N_z} [u_{emd_i}(y_{ref}, z_i + \Delta z, t_k, x/\delta_0) - \langle u_{emd_i}(y_{ref}, t_k, x/\delta_0) \rangle_z] \\
&\quad \times [u(y, z_i, t_k + \phi_t(y), x/\delta_0) - \langle u(y, t_k + \phi_t(y), x/\delta_0) \rangle_z] \\
r_{11}(y_{ref}, \Delta z, t_k, x/\delta_0) &= \sum_{i=1}^{N_z} [u_{emd_i}(z_i + \Delta z, y_{ref}, t_k, x/\delta_0) - \langle u_{emd_i}(y_{ref}, t_k, x/\delta_0) \rangle_z]^2 \\
r_{22}(y, \Delta z, t_k, x/\delta_0) &= \sum_{i=1}^{N_t} [u(y, z_i, t_k + \phi_t(y), x/\delta_0) - \langle u(y, t_k + \phi_t(y), x/\delta_0) \rangle_z]^2
\end{aligned}$$

In the above equations, the angled brackets indicate averaging over the direction given by the subscript, and N_z and N_t denote relevant sample sizes. The function $\phi_t(y)$ is introduced to account for the time-delay when determining the correlation coefficient $C(y, y_{ref}, \Delta z)$ along the curved line of maximum correlation $C(y, y_{ref}, \Delta t)$ in the $y^+ - \Delta t^+$ plane, represented, for each mode separately, by the black dotted lines in figure 7. The rationale of correlating u_{emd_i} to u is rooted in the wish to bring out the wall-normal coherence of mode u_{emd_i} – the y -wise persistence of mode i in the total fluctuations field.

Figures 7 and 8 give mode-specific maps of the two-point correlations $C(y, y_{ref}, \Delta t)$ and $C(y, y_{ref}, \Delta z)$, respectively. In both figures, the raw-field fluctuations at any location y^+ is correlated with relevant fluctuations at two reference locations, namely $y_{ref}^+ \approx 13$ (figures 8(a),(c),(e) and (g)) and $y_{ref}^+ \approx 130$ (figures 8(b),(d),(f) and (h)).

Figure 7 conveys two principal messages. First, only mode $i = 3$ yields a significant correlation between the location $y^+ \approx 130$ and the viscous sublayer, as observed from figure 7(f). The conclusion is, therefore, that it is primarily this mode that represents the footprinting process – although mode $i = 2$, which partially overlaps with mode $i = 3$, also contributes modestly to footprinting. It is pertinent to refer, here, to figures 6(c) and (d), which show, that mode $i = 3$ features the highest energy-spectrum peak at $y^+ \approx 130$, and also that the spectrum for that same mode at $y^+ \approx 13$ is very similar in distribution and magnitude, suggesting a high level of coherence of this mode across the near-wall layer. In contrast, the correlation domains for other modes are substantially more localized around the reference location $y_{ref}^+ \approx 130$. Consistently, for $y_{ref}^+ \approx 13$, the correlation of mode $i = 1$ is very high only across the whole viscosity-affected near-wall layer. This correlation level diminishes with increasing mode number and tends to extend gradually to higher y^+ levels. However, only for mode $i = 3$ does the correlation domain centred on $y_{ref}^+ \approx 13$ extend, unambiguously, to the y^+ level at which the large-scale motions predominate. The second message is that a distinctive time-lag exists between the outer motions and the footprints at the wall. This lag is of order $\delta t^+ \approx 25$, equivalent to a lag in streamwise distance $\delta x^+ \approx 500$, a feature observed in many investigations - e.g. by Brown & Thomas [37], Boppe et al. [38], Carper & Porte-Agel [39] and Marusic & Heuer [40]. Mathis et al [19] express this lag by the angle $\theta \approx 12.5$ in a model that links the large-scale structures at $y^+ = 3.9\sqrt{Re_\tau}$ to their near-wall footprints. In the present case, the implied lag angle is approximately 14.5° .

Mode-wise correlation map in the $y^+ - \Delta z^+$ space are shown in figure 8. As explained above, by reference to equation 7, these maps were obtained along the loci of maximum correlation in the $y^+ - \Delta z^+$ maps (the dotted black lines). Consistent

with the maps in $y^+ - \Delta t^+$ space, the correlation maps in figures (f) and 8(e), also indicate that mode $i = 3$ is the one primarily reflecting the footprinting process. In contrast, and again in agreement with the above discussion, the correlation of small-scale motions, represented by the 1st mode in figures 8(a) and (b) and for the 2^{sd} mode in figures 8(e) and (f), indicates that the fluctuations in the viscous sublayer are not, or only weakly, correlated to those in the outer region, while they are strongly correlated within the viscous sublayer itself. These results thus support the proposition that the EMD implemented in this study indeed yields a physically credible split between the different scales.

5. Superposition and modulation

The purpose of this section is to demonstrate that the process of modulation of small-scale near-wall turbulence by outer large scales, established at elevated Reynolds numbers, also pertains to the present flow. In what follows, attention focuses on the conditions at $y^+ \approx 13$, a location at which the effects of the large-scale motions on the near-wall streaks are expected to be most pronounced.

Figure 9 shows $u - v$ and $u - w$ joint PDFs of raw velocity fluctuations. Both joint PDFs are very similar to those reported by Agostini & Leschziner [23–25] for channel flow at $Re_\tau = 1025$. In particular, the $u - v$ PDF contours are distinctly asymmetric, relative to the principal axes. In Q(uadrant)4 of the PDF, associated with the sweep events, the contours display a characteristic dip towards high negative $v -$ fluctuations associated relatively low $u -$ fluctuations, indicative of the convective transport of relatively few low streamwise-velocity fluctuations from beyond the buffer layer towards the wall. Ejections are weaker and more numerous. Hence, the downward dip in $Q4$ does not have a upward-directed counterpart in $Q2$. The $u - w$ PDF shows a strong asymmetry relative to the $u^+ = 0$ axis, with the contours in $Q1$ and $Q4$ implying large w -fluctuations that may be interpreted as indicating impingement events associated with sweeps – a phenomenon referred to as “splatting”.

In pursuit of the objective of illuminating the interactions between the large-scale and small-scale motions, it is instructive to conditionally sample the small-scale fluctuations separately within low- and high-velocity footprints. Here, the output of the sampling process is in the form of joint PDFs for the conditionally-sampled small-scale motions within the footprints.

In common with the approach taken by Agostini & Leschziner [23–25], data for small-scale fluctuations are collected within footprints that are defined by the extreme 10% of events of large-scale fluctuations, as identified by the tails of the PDF of the large-scale motions “ u_{LS} ”, defined as the sum of modes $i = 3$ and 4. In other words, the PDF of u_{LS} is constructed first, the middle 80% portion (by area) of the PDF is cut off, and the remaining 10% tails, identifying the extreme low-velocity and high-velocity large-scale motions, are used to define the extreme-velocity footprints, within which the small-scale motions are sampled. In all PDF plots to follow, in figures 10, the red contours relate to high-speed footprints, and the blue contours to low-speed footprints.

Joint $u - v$ and $u - w$ PDFs of the total-fluctuation field within high-speed and low-speed footprints, as determined according to the method explained above, are shown in figure 10. The upper plots are derived from the present supersonic boundary layer, while the lower plots originate from the channel-flow at $Re_\tau = 4200$ performed from DNS data provided by Lozano-Durán & Jiménez [26, 27] for $Re_\tau = 4200$ (similar joint pdfs were also observed by Agostini & Leschziner [23–25] for the channel flow at $Re_\tau = 1025$). One expected fact arising from the comparison

Figure 7. Two-point correlation maps in $y^+ - \Delta t^+$ plane of the EMD modes of the streamwise velocity at reference locations y_{ref} relative to the total fluctuations field at y : (a,c,e) $y_{ref}^+ \approx 13$; (b,d,f) $y_{ref}^+ \approx 130$; (a,b) 1st mode; (c,d) 2nd mode; (e,f) 3rd mode; (g,h) 4th mode. Red and blue iso-lines identify positive and negative correlation, respectively. The black dotted lines are (approximate) loci of maximum correlation levels, subject to minor smoothing by splin polynomials.

Figure 8. Two-point correlation maps in $y^+ - \Delta z^+$ plane of the EMD modes of the streamwise velocity at reference locations y_{ref} relative to the total fluctuations field at y : (a,c,e) $y_{ref}^+ \approx 13$; (b,d,f) $y_{ref}^+ \approx 130$; (a,b) 1st mode; (c,d) 2nd mode; (e,f) 3rd mode; (g,h) 4th mode. Red and blue iso-lines identify positive and negative correlation, respectively.

Figure 9. Joint PDFs for the raw (total) fluctuations field at $y^+ \approx 13$: (a) $(u - v)$ PDF; (b) $(u - w)$ PDF. Contours identify 0.1 - 0.9 of the PDF height at constant increment 0.1, subject to total PDF volume normalized to 1.

is that the large-scale motions are weaker in the present boundary layer, due to the lower friction Reynolds number (570 as compared to 4200). This is identified by the smaller displacement between centres of gravity corresponding to pairs of PDFs in the boundary layer. This apart, both sets of PDFs, for the boundary layer and the channel flow, show very similar features. In particular, only the $u - v$ PDF pertaining to the high-speed footprints features the strongly asymmetric contours, already noted by reference to figure 9 and associated with sweeping events. The implication is, therefore, that high-speed and low-speed footprints have an unequal effect on the fluctuations field and thus the small-scale motions, suggesting an asymmetric modulation. Large spanwise fluctuations are predominantly present within the high-speed footprints in quadrants Q_1 and Q_4 – again, consistent with sweep events and thus splatting. Inspection of the streaky structure in figure 5(a) reveals that the streaks within high-velocity footprints (red-coloured regions) are considerably less organised, more disturbed and less elongated than streaks within low-speed footprints (blue-coloured regions). This is indicative of a disrupting influence of sweep-inducing impingement, and is thus qualitatively consistent with the statistical interpretations of the joint PDFs.

6. Quasi-steady behaviour

A behaviour that is consistent with the outer-large-scale/inner-small-scales interactions, in the sense described in section 5, is that the small-scale field near the wall responds to the footprints of the outer scales in a quasi-steady manner. In other words, the time scale of the small-scale motions is much shorter than those associated with the large-scale footprints, so that the small-scale field adjusts rapidly to the large-scale footprints. This leads to the proposition that the near-wall statistics are universal when scaled by the unsteady friction velocity that is effected by the footprints. The validity of this proposition was examined by the present authors (Agostini & Leschziner [41]) by reference to channel flow at $Re_\tau = 4200$. That study demonstrated that the turbulence statistics of the near-wall small scales remained universal, in the above sense, only up to $y^+ \approx 30$ -50, depending on the turbulence correlation in question. Contemporaneously, Chernyshenko et al. [42], Zhang & Chernyshenko [43] developed a theoretical “Quasi-Steady - Quasi Homo-

Figure 10. Joint PDFs at $y^+ \approx 13$ for: (a,d) raw fluctuations; (b,c,e,f) small-scale fluctuations sampled in footprints corresponding to $\pm 10\%$ of slow and fast events of the large-scale motions; upper plots: present boundary layer, lower plots: channel flow at $Re_\tau = 4200$

geneous” (QSQH) framework that rests on the quasi-steady hypothesis, assumes the universality of the small-scale motion when scaled by the large-scale shear-stress footprints, and configured to predict the response of near-wall turbulence to the Reynolds-number dependent large-scale outer motions. This section considers the question of whether the QS concept applies to the present boundary layer.

Following the rationale underpinning Section 5, the Reynolds stresses of the small-scale motions are determined conditionally on the extreme (minimum and maximum) 10% skin-friction footprints. The stresses and wall distance are then normalized by the skin friction within the footprints. This is contrasted with the Reynolds stresses normalized by the time-mean skin friction. This comparison is shown in figures 11(a),(b) and 11(c),(d) for the streamwise stress and the other stress components, respectively. In all plots, the red lines pertain to positive large-scale motions, associated with large-scale sweeps, and the blue lines to negative motions, associated with large-scale ejections. The conclusion derived from the comparison is that the QS concept is valid up to a distance of $y^+ \approx 20 - 30$, a range that is quite close to that observed for $Re_\tau = 1020$ and 4200. This agreement reinforces, therefore, statements made earlier to the effect that the distinction between outer large-scale motions and inner small-scale motions identified in the present boundary layer essentially corresponds to that observed in channel flow at substantially higher Reynolds numbers. An interesting fact to highlight in relation to the profiles of the streamwise stress, when normalized by the mean friction velocity, is that the velocity fluctuations in the buffer layer are markedly stronger over the large-scale sweeps than over ejections. This clearly implies a modulation of the small-scale motions by the large-scale motions, which is entirely consistent with the behaviour observed in figure 9.

Figure 11. Profiles of second moments in patches of extreme positive (red) and extreme negative (blue) regions of large-scale velocity fluctuations: (a,b) streamwise components, (c,d) other components; (a) and (c) moments scaled with mean friction velocity; (b) and (d) conditionally averaged moments scaled with space/time-varying friction velocity associated with large-scale footprints;

7. Conclusions

The study has shown that the boundary layer shares, despite having a low Reynolds number and being compressible, many of the features observed in channel flow at higher Reynolds numbers. Thus:

- the outer flow features distinctive large-scale motions – albeit considerably weaker than in channel flow – which produce high- and low-velocity footprints on the lower parts of the boundary layer;
- these large wave-length footprints appear to co-exist with similar-size structures that are locally generated;
- the footprints are well correlated, in the wall-normal direction, with the outer large-scale structures and associated motions, while the small-scale motions are weakly correlated;
- the spanwise wave length of the large-scale motions is of order $z^+ = 500 - 1000$ (roughly one boundary-layer thickness), relative to 100 typically separating the small-scale streaks in the viscosity-affected sublayer;
- the footprints produce large-scale convective shifts of the small-scale structure;

- the intensity of the small-scale motions is enhanced by high-speed footprints and attenuated by low-speed footprints;
- the enhancement and attenuation (reflecting a modulation of the small-scale signals) is asymmetric, the asymmetry being associated with splatting caused by large-scale sweep events; splatting and sweep events are also reflected by elevated large-scale spanwise fluctuations;
- the present results support earlier conclusions, derived from higher-Reynolds-number channel-flow simulation, to the effect that the quasi-steady hypothesis pertaining to the near-wall small-scale motions is only valid over a modest portion of the near-wall layer.

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