Failure of Lightly Reinforced Concrete Floor Slabs with Planar Edge Restraints under Fire

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Abstract

This paper presents an analytical model for the failure of lightly reinforced concrete slabs under elevated temperature, considering simply supported boundary conditions with planar edge restraints. This model is typically applicable to the failure assessment of composite floor slabs under fire, where the steel deck is assumed to lose strength relatively quickly, leaving a lightly reinforced concrete slab, as supported by experimental evidence. The proposed model accounts for membrane action which arises at large slab deformations, and importantly it presents a rational failure criterion based on rupture of the steel reinforcement. In this respect, this is the first analytical slab model to consider the influence of bond between steel and concrete on reinforcement rupture, while also dealing with elevated temperatures, including the potentially negative effects of thermal curvature. Based on principles of mechanics, detailed analytical forms of the model are first presented and verified for each of the ambient and elevated temperatures cases. In each case, a simplified form of the analytical model, which is more suitable for practical design-oriented application, is also proposed and verified. The successful verification of the proposed analytical models, demonstrated against the results of appropriate nonlinear finite element analysis, paves the way for their application in the design of composite floor slabs with planar edge restraints under fire.

CE DATABASE SUBJECT HEADINGS: concrete slabs; composite structures; fire resistance; reinforcing steel; structural failures.

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1. Introduction

Over recent years, the design of building structures for the fire limit state using performance based methods has received considerable attention. In this context, codes such as BS5950-Part8 (BSI, 2003) and Eurocode 3-Part 1.2 (ECN, 2005) have been updated to address the behaviour of steel members under elevated temperature. Although the aforementioned codes account for the effect of fire on the material level and its influence on isolated members, these still do not account for the actual behaviour of the members during fire, including interaction with the surrounding structure, the influence of thermal expansion and tensile membrane effects. The potential importance of such phenomena was realised following two real fires in the United Kingdom in the early 1990's, known as the Broadgate (SCIF, 1991) and Basingstoke cases, affecting steel framed buildings with composite steel deck-concrete slabs, where no collapse was observed. In the case of the Broadgate fire, the building was still under construction and there was no fire proofing provided. This demonstrated that steel buildings have inherent fire resistance, albeit at large deformations and as a result of interactions between structural components, which is not accounted for in current codes.

Following these observations, considerable research (Huang *et al.*, 1999; Elghazouli and Izzuddin, 2000; Elghazouli *et al.*, 2000; Bailey and Moore, 2000; Izzuddin and Moore, 2002; Izzuddin, 2002; Usmani and Cameron, 2004) was initiated in this field, and the behaviour of composite floor slabs under fire received significant attention. From real fires and the fire tests at the Building Research Establishment (BRE) Large Building Testing Facility in Cardington, it was established that unprotected secondary steel beams lost strength, and the steel decking debonded from the lightly reinforced concrete (LRC) slab which was attributed to the release of steam from the concrete slab. Thus, the principal load carrying structural component under fire was the LRC slab, which had to experience large deflections and carry the load by means of

tensile membrane action (Bailey and Moore, 2000). Fire tests (Lim and Wade, 2002) have established that cracks are developed at locations coinciding with the predictions of yield line theory, which in some cases penetrated the full depth of the slab. Although floor collapse was not observed in Cardington, the authors believe that if the experiments had lasted longer, or the imposed load was greater, the slabs would have failed by rupture of the reinforcement along these cracks due to strain concentration. In this respect, one of the most important factors is the bond strength between the steel reinforcement and concrete, particularly for LRC members. The current method for assessing the behaviour of composite slabs under elevated temperature (Bailey and Moore, 2000) is based on a semi-empirical treatment of failure that employs average membrane strains. This empirical treatment cannot account for crucial factors, including the bond strength and the reinforcement stress-strain response, and is therefore unable to predict the failure of floor slabs with variable geometric configuration and material constitution.

The assessment of the behaviour of composite slabs at elevated temperatures requires a strategy where the important parameters governing the behaviour, such as depth-to-span ratio, aspect ratio, steel reinforcement ratio, failure pattern, geometric/material nonlinearity, and material properties at elevated temperatures, need to be identified and evaluated. Models are presented in this paper for the ambient and elevated temperature response of a LRC slab subject to planar restraints, which are based on principles of structural mechanics, and account for membrane action, bond strength, and the reinforcement stress-strain response. The proposed models employ similar assumptions to previous beam models developed by the authors (Izzuddin and Elghazouli, 2004; Omer, 2006), which could be seen as representing individual strips across the slab. Hereafter, the assumptions underlying the proposed ambient and elevated temperature LRC slab models are first outlined, followed by presentation and verification of the two types of model. In each case, full and simplified forms are proposed, where the latter is shown to offer good accuracy as well as sufficient practicality for application in the design of composite floor slabs under fire.

2. Restrained Slab Model

The behaviour of slabs located near the edges of a building or near slabs with large openings is similar to that of slabs with no planar edge restraints (Bailey and Moore, 2000). On the other hand, for slabs located in the interior of a building and surrounded by other slabs, the in-plane movement along the edges is relatively small and can be idealized as restrained. From this point onwards these planar edge restraints will be referred to as horizontal restraints.

It was observed from experiments and real fires that full depth cracks, where the reinforcement also fractured, are developed at the perimeter of an internal steel-concrete composite slab (Bailey and Moore, 2000). However, it is not clear whether these cracks occurred during the fire or during the cooling of the heated slab following the fire. If the cracks occurred during the fire, then the behaviour would be closer to that of a horizontally unrestrained (without planar edge restraints) slab. If they occurred following the fire, the slab has to be considered as a horizontally restrained slab. Clearly, even if the cracks occur following the fire, due to the finite stiffness of the surroundings, the real behaviour during a fire is intermediate between the unrestrained and restrained cases. This paper focuses on restrained slabs.

As mentioned before, the main load carrying member in the event of a fire is the LRC slab, which due to the degradation of the material properties and thermal effects deforms excessively developing a crack pattern similar to that predicted by yield line theory for slabs under ambient temperature. It has also been observed that these cracks after a certain point penetrate the depth of the slab, and that for unrestrained slabs additional full depth cracks occur across the short span intersecting the central yield line (Bailey and Moore, 2000; Bailey *et al.*, 2000). Additionally, it was noted that the segments of the slabs bounded by the yield lines rotate around the lines in an almost rigid manner. The location of the yield lines is defined (Wood and Jones, 1967) by the parameter η which is a function of the aspect ratio of the slab α , which are given by:

$$\alpha = \frac{a}{b} \quad (1)$$

$$\eta = \frac{1}{2\alpha^2} \left(\sqrt{3a^2 + 1} - 1 \right) \quad (2)$$

where a and b are the long and short span length respectively, as shown in Figure 1. A previously developed method by Bailey and Moore (2000) has incorporated these observations for the assessment of unrestrained slabs, although the failure criterion proposed was semi-empirical and based on average strains. Here, an analytical model is proposed for restrained LRC slabs, which addresses the fundamental issue of strain concentrations at the crack locations, leading to a rational failure criterion which is based on principles of structural mechanics.

The proposed slab model is based on a previous beam model (Izzuddin and Elghazouli, 2004; Omer, 2006), where for a simply supported LRC beam subject to a mid-span point load a single mid-span crack or multiple cracks will form (Figure 2), depending on the reinforcement ratio. For LRC members under uniaxial tension, where the cracking capacity of the concrete is larger than the yield capacity of the reinforcement, no more cracks should occur following the formation of the first crack. Moreover, since the formation of a single crack leads to increased strain concentration in the reinforcement at the crack face compared to the case with multiple cracks, the adoption of a single crack will lead to a conservative model of failure by fracture of the reinforcement (Izzuddin and Elghazouli, 2004). In this case, the strain concentration in the reinforcement depends on several factors, including the crack width, the bond characteristics and the reinforcement stress-strain response (Izzuddin and Elghazouli, 2004).

A model was originally proposed by Izzuddin and Elghazouli (2004) for restrained LRC beams, which accounted for the elastic properties of steel and concrete, in addition to material strength and bond. It was shown subsequently that the elastic material properties may be ignored without significantly affecting the model predictions (Omer, 2006), enabling the use of a rigid-hardening response for steel and a rigid-plastic response for concrete and the bond-slip relationship. These assumptions are therefore also adopted for the proposed slab models, enabling the formulation of

analytical models that avoid undue complexity while offering sufficient accuracy for practical application.

3. Ambient Temperature Restrained Slab Model

Prior to the formulation of a model for the elevated temperature response of a restrained LRC slab, a model is presented here for the ambient temperature response. Based on the observations discussed previously, the following assumptions are made for the slab model:

- the slab has a single layer of steel reinforcement in each direction,
- the reinforcement in the two directions is assumed to be at the same level,
- the slab is simply supported and subject to planar restraint along the edges at the level of the reinforcement,
- the reinforcement is anchored in the concrete at the supports,
- rigid-hardening material model is adopted for steel (Figure 3.a), since the elastic strain is relatively small compared to the rupture strain,
- rigid-plastic material model is adopted for the bond-slip relationship (Figure 3.b),
- a composite slab is idealized as a uniform thickness concrete slab, since the steel deck debonds and the formation of a full depth crack renders the tensile strength of the concrete and the part of the slab below the reinforcement, including the ribs, irrelevant,
- the slab is subject to a uniformly distributed load,
- no other cracks form except for those predicted by yield line theory,
- the slab segments bounded by the yield lines rotate rigidly around the yield lines and the supports (implicitly assuming the concrete away from the yield lines is rigid) (Figure 4), and

- failure is defined as the point where the reinforcement stress at the crack face along the central yield line reaches the ultimate strength of the steel.

Due to symmetry only a quarter of the slab is considered as illustrated in Figure 5. It is assumed that strips across the length and width of the slab can be idealized as LRC beams and treated in a manner similar to a previous beam model developed by the authors (Omer *et al.*, 2005; Omer, 2006).

Referring to Figure 6, where the concrete cover to the reinforcement is not shown, for a strip with a length of $b/2$, the reinforcement extension along the central crack Δ_{sce} is given by:

$$\Delta_{sce} = \frac{b}{2} - \sqrt{\frac{b^2}{4} - U_c^2} \quad (3)$$

where U_c is the transverse deflection along the central crack.

As a result of the rigid-hardening steel assumption this stretching will be due to the extension of the reinforcement in a region where the steel stress exceeds yield. Assuming that the member is sufficiently long, the force drops from T_s at the crack face, according to the bond strength per unit length σ_b , to the force corresponding to the yield strength of the steel T_y at a distance l_b from the crack face defined as the *bond length*. However, if the length of the member is not sufficient to have the steel stress reaching the yield strength, the bond length l_b would be limited to the member length. Therefore, the force in the reinforcement along the central crack is given by (Omer, 2006):

$$T_{sce} = \begin{cases} T_y + \sqrt{2A_s E_2 \sigma_b \Delta_{sce}} & \text{if } \sqrt{\frac{2A_s E_2 \Delta_{sce}}{\sigma_b}} \leq \frac{b}{2} \\ T_y + \frac{\sigma_b b}{4} + \frac{2A_s E_2 \Delta_{sce}}{b} & \text{if } \sqrt{\frac{2A_s E_2 \Delta_{sce}}{\sigma_b}} \geq \frac{b}{2} \end{cases} \quad (4)$$

It should be noted that the values for T_{sce} , T_y , A_s , and σ_b are expressed per unit width of the slab. Finally, the rate of the energy dissipated in the reinforcement $\dot{D}_{sce}$ is integrated along the central yield line as:

$$\dot{D}_{sce} = \int_{\eta a}^{a/2} T_{sce} \frac{d\Delta_{sce}}{dU_c} dx \quad (5)$$

For the diagonal yield line, strips in both the longitudinal and transverse direction need to be considered, as the diagonal intersects reinforcing bars spanning in both directions.

For a strip taken in the y -direction, the stretching of the reinforcement can be obtained by substituting $bx/(2\eta a)$ for $b/2$ and $U_c x/(\eta a)$ for U in Equation (3). Thus the stretching in the y -direction Δ_{sdy} at any point along the diagonal yield line can be obtained as:

$$\Delta_{sdy} = \frac{x}{\eta a} \left(\frac{b}{2} - \sqrt{\frac{b^2}{4} - U_c^2} \right) = \frac{x}{\eta a} \Delta_{sce} \quad (6)$$

It is noted that near the ends of the diagonal yield line (Figure 7), the bond length becomes limited, either due to symmetry along the central yield line or due to the assumption of full reinforcement anchorage at the slab edges. Thus the stretching of the reinforcement Δ_s across a crack along the diagonal becomes:

$$\Delta_s = \underbrace{\frac{(T_s - T_y)^2}{2A_s E_2 \sigma_b}}_{\text{long segment}} + \underbrace{\frac{(T_s - T_y)L}{A_s E_2} - \frac{\sigma_b L^2}{2A_s E_2}}_{\text{short segment}} \quad (7)$$

In the vicinity of the corner of the slab, the length L in Equation (7) should be replaced by $bx/(2\eta a)$, whereas close to the intersection of the diagonal and the central yield lines it should be replaced by $[b/2 - bx/(2\eta a)]$. Thus, the forces in the reinforcement for the region close to the corner of the slab T_{sy1} , the middle-portion of the diagonal yield line T_{sy2} , and the region adjacent to the intersection of the yield lines T_{sy3} are given by:

$$T_{sy1} = T_y - \sigma_b \frac{bx}{2\eta a} + \sqrt{2\sigma_b^2 \left(\frac{bx}{2\eta a} \right)^2 + 2A_s E_2 \sigma_b \Delta_{sdy}} \quad (8)$$

$$T_{sy2} = T_y + \sqrt{A_s E_2 \sigma_b \Delta_{sdy}} \quad (9)$$

$$T_{sy3} = T_y - \sigma_b \left(\frac{b}{2} - \frac{bx}{2\eta a} \right) + \sqrt{2\sigma_b^2 \left(\frac{b}{2} - \frac{bx}{2\eta a} \right)^2 + 2A_s E_2 \sigma_b \Delta_{sdy}} \quad (10)$$

The bond length for the middle-portion of the diagonal on either side is given by:

$$l_b = \frac{T_{sy2} - T_y}{\sigma_b} \quad (11)$$

With T_{sy2} dependent on x according to Equations (6) and (9), substituting $bx/(2\eta a)$ and $b/2 - bx/(2\eta a)$ for l_b and solving for the respective x , the limits of the regions close to the corner (X_1) and close to the intersection of the yield lines (X_2), shown in Figure 7, can be obtained as:

$$X_1 = \frac{4A_s E_2 \Delta_{sce}}{\sigma_b b^2} \eta a \quad (12)$$

$$X_2 = \eta a - \frac{X_1}{2} \left(\sqrt{1 + \frac{4\eta a}{X_1}} - 1 \right) \quad (13)$$

Thus, the force in the reinforcement spanning across the diagonal yield line in the y -direction is given by:

$$T_{sy} = \begin{cases} T_{sy1} & \text{for } 0 \leq x \leq X_1 \\ T_{sy2} & \text{for } X_1 \leq x \leq X_2 \\ T_{sy3} & \text{for } X_2 \leq x \leq \eta a \end{cases} \quad (14)$$

It should be noted that the proposed expressions are valid for $X_2 \geq X_1$, which can be expressed in terms of the following condition:

$$X_2 \geq X_1 \Rightarrow \frac{8A_s E_2 \Delta_{sce}}{\sigma_b b^2} \leq 1 \quad (15)$$

Therefore, if the deflection is very large, or if the bond strength is very low, this condition may be violated, thus requiring modifications to the proposed model. However, for typical slabs and bond characteristics, the above condition is satisfied, at least up to the failure displacement limit.

Finally, the rate of the energy dissipated in the reinforcement spanning across the diagonal in the y -direction is given by:

$$\dot{D}_{sdy} = \int_0^{\eta a} T_{sy} \frac{d\Delta_{sdy}}{dU_c} dx \quad (16)$$

The stretching of the reinforcement (Figure 8) in the x -direction at the intersection of the yield lines Δ_{scx} is given by:

$$\Delta_{scx} = \eta a - \sqrt{\eta^2 a^2 - U_c^2} \quad (17)$$

Substituting $2\eta ay/b$ for $b/2$ and $2yU_c/b$ for U in Equation (3), the stretching of the reinforcement across the diagonal in the x -direction is obtained as:

$$\Delta_{sdx} = \frac{2y}{b} \left(\eta a - \sqrt{\eta^2 a^2 - U_c^2} \right) = \frac{2y}{b} \Delta_{scx} \quad (18)$$

Similar to the strip in the y -direction, the reinforcement force in the vicinity of the corner, T_{sx1} , and in the vicinity of the intersection of the diagonals, T_{sx3} , may be obtained from Equation (7) by substituting $2\eta ay/b$ and $a/2 - 2\eta ay/b$ for L , respectively. On the other hand, the force in the reinforcement in the intermediate zone, T_{sx2} , is not associated with a limited bond length. Accordingly:

$$T_{sx1} = T_y - \sigma_b \frac{2\eta ay}{b} + \sqrt{2\sigma_b^2 \left(\frac{2\eta ay}{b} \right)^2 + 2A_s E_2 \sigma_b \Delta_{sdx}} \quad (19)$$

$$T_{sx2} = T_y + \sqrt{A_s E_2 \sigma_b \Delta_{sdx}} \quad (20)$$

$$T_{sx3} = T_y - \sigma_b \left(\frac{a}{2} - \frac{2\eta ay}{b} \right) + \sqrt{2\sigma_b^2 \left(\frac{a}{2} - \frac{2\eta ay}{b} \right)^2 + 2A_s E_2 \sigma_b \Delta_{sdx}} \quad (21)$$

However, in this case, as opposed to the strip in the y -direction, the bond length near the intersection of the yield lines is limited only after a certain deflection U_{c1} , since there is a finite length $(a/2 - \eta a)$ over which the bond length may be fully developed. Before this deflection, only two regions exist for which the force in the reinforcement is given by T_{sx1} and T_{sx2} . The limiting deflection can be obtained from the following condition:

$$\frac{T_{sx2} - T_y}{\sigma_b} = \frac{a}{2} - \frac{2\eta ay}{b} \quad \text{at} \quad \left(y = \frac{b}{2} \right) \quad (22)$$

which can be solved for U_c leading to:

$$U_{c1} = \frac{(1-2\eta)a}{4A_s E_2} \sqrt{8A_s E_2 \sigma_b \eta a - \sigma_b^2 a^2 (1-2\eta)^2} \quad (23)$$

The limits of the regions close to the ends of the diagonal, Y_1 and Y_2 , are obtained similarly to the x -direction as:

$$Y_1 = \frac{A_s E_2 b \Delta_{scx}}{2\sigma_b \eta^2 a^2} \quad (24)$$

$$Y_2 = \frac{b}{4\eta} + \frac{Y_1}{2} \left(1 - \sqrt{1 + \frac{b}{\eta Y_1}} \right) \quad (25)$$

A piecewise relationship is therefore obtained for the force in the reinforcement spanning across the diagonal yield line in the x -direction:

$$T_{sx} = \begin{cases} \begin{cases} T_{sx1} & \text{for } 0 \leq y \leq Y_1 \\ T_{sx2} & \text{for } Y_1 \leq y \leq \frac{b}{2} \end{cases} & \text{for } 0 \leq U \leq U_{c1} \\ \begin{cases} T_{sx1} & \text{for } 0 \leq y \leq Y_1 \\ T_{sx2} & \text{for } Y_1 \leq y \leq Y_2 \\ T_{sx3} & \text{for } Y_2 \leq y \leq \frac{b}{2} \end{cases} & \text{for } U > U_{c1} \end{cases} \quad (26)$$

It should be noted that the expressions above are valid if $Y_1 \leq b/2$ for the first part of Equation (26) and $Y_1 \leq Y_2 \leq b/2$ which can be expressed in terms of the following conditions:

$$Y_1 \leq \frac{b}{2} \Rightarrow \frac{A_s E_s \Delta_{scx}}{\sigma_b \eta^2 a^2} \leq 1 \quad (27)$$

$$Y_1 \leq Y_2 \leq \frac{b}{2} \Rightarrow \frac{4A_s E_s \Delta_{scx}}{\sigma_b \eta a^2} \leq 1 \quad (28)$$

Finally, the energy dissipated in the reinforcement running across the diagonal yield line in the x -direction is obtained as:

$$\dot{D}_{sdx} = \int_0^{b/2} T_{sx} \frac{d\Delta_{sdx}}{dU_c} dy \quad (29)$$

Therefore, the overall rate of energy dissipated in the reinforcement across all yield lines is obtained from:

$$\dot{D}_s = \dot{D}_{sce} + \dot{D}_{sdy} + \dot{D}_{sdx} \quad (30)$$

where $\dot{D}_{sce}$, $\dot{D}_{sdy}$ and $\dot{D}_{sdx}$ are given by Equations (5), (16) and (29) respectively.

It is assumed that the various parts of the slab bounded by the yield lines rotate rigidly, where at large deflections compression in concrete becomes localized at the slab corners, thus justifying the neglect of energy dissipation in concrete.

Under a uniformly distributed load q , the work done by the load is equal to q multiplied by the volume created by the slab as it deforms. Thus, the rate of the work done by the external uniformly distributed load for the quarter of the slab is:

$$\dot{E} = \frac{qab(3-2\eta)}{24} \quad (31)$$

Combining Equations (30) and (31) and using the principle of virtual work, a relationship between the externally applied load and the central deflection of the slab can be obtained using a kinematic method of analysis, where the work done by the external forces $\dot{E}$ for an infinitesimal deflection should be equal to the energy dissipated in the reinforcement across the yield lines $\dot{D}_s$:

$$\dot{E} = \dot{D}_s \quad (32)$$

in which $\dot{E}$ is related to the load q while $\dot{D}_s$ depends on the deflection U_c .

3.1. Failure of LRC slabs under ambient temperatures

The failure of the LRC slab is defined as the point where the stress in the reinforcement across one of the cracks reaches the ultimate strength of the steel. Considering Equations (1) and (2), it can be shown that:

$$\eta\alpha \geq \frac{b}{2} \quad (\alpha \geq 1) \quad (33)$$

Therefore, it can be easily demonstrated from Equations (3) and (17) that:

$$\Delta_{sce} \geq \Delta_{scx} \quad (34)$$

Since Δ_{scx} and Δ_{sce} are the upper limits on reinforcement stretching in the x and y -directions, respectively, and since Δ_{sce} is associated with bond length development along one side only, the maximum reinforcement stress/strain concentration occurs in the bars stretching across the central yield line in the short y -direction. Therefore, the failure of the slab is governed by rupture of the reinforcement spanning across the central yield line in the y -direction, and is

considered to occur when T_{sce} reaches T_u . Combining Equations (3) and (4), similar to the beam model [13], the failure displacement U_{cf} is obtained as:

$$U_{cf} = \begin{cases} \sqrt{\frac{b^2}{4} - \left[\frac{b}{2} - \frac{(T_u - T_y)^2}{2A_s E_2 \sigma_b} \right]^2} & \text{if } \frac{T_u - T_y}{\sigma_b} \leq \frac{b}{2} \\ \sqrt{\frac{b^2}{4} - \left[\frac{b}{2} - \frac{b}{2A_s E_2} \left(T_u - T_y - \frac{\sigma_b b}{4} \right) \right]^2} & \text{if } \frac{T_u - T_y}{\sigma_b} \geq \frac{b}{2} \end{cases} \quad (35)$$

The model proposed here is compared against the nonlinear finite element analysis program ADAPTIC (Izzuddin, 1991), employing the recently developed reinforced concrete slab element (Izzuddin *et al.*, 2004).

3.2. Verification and comparisons

The finite element model in ADAPTIC is based on the smeared crack approach which is most representative of normally reinforced slabs with multiple cracks. However, for single crack cases the stresses in the reinforcement can be sensitive to the mesh density and can result in high stress concentrations (Elghazouli and Izzuddin, 2004). Moreover, ADAPTIC does not account for the bond strength between the reinforcement and the surrounding concrete, and hence it cannot be used for full verification of the present model. Nevertheless, the accuracy of the kinematic expressions of the proposed model can be checked against ADAPTIC by neglecting the strain hardening of the steel reinforcement. In this case, the stress in the reinforcement across the crack is limited to the yield strength of steel, and the response obtained from ADAPTIC becomes insensitive to the mesh density and comparable to that obtained using the proposed model.

Accordingly, a $12 \times 6 \text{ m}^2$ LRC slab is analysed with ADAPTIC, where only a quarter model is considered due to symmetry. The translations along the edge supports are restrained, and appropriate translational/rotational restraints are applied along the lines of symmetry. A mesh of 16×8 square elements is used, and the reinforcement is located at the element mid-height where

restraints are applied, so as to represent the assumption that the slab is supported at the level of the reinforcement. The geometric properties of the slab are summarized in Table 1. To simulate with ADAPTIC the crack pattern assumed by the proposed model, the tensile strength of concrete assigned for the elements at the locations of the yield lines is set to a smaller value than for the rest of the slab. The material properties for the steel and concrete models (Izzuddin *et al.*, 2004; Elghazouli and Izzuddin, 2004) are respectively given in

Table 2 and Table 3, where f_{t1} denotes the concrete tensile strength of the elements along the yield lines, and f_{t2} stands for the concrete tensile strength for the rest of the slab. This corresponds to an imposed yield line pattern which matches the experimental results and the current analytical formulation, enabling the verification that the current formulation captures the response of the slab under the prescribed yield pattern satisfactorily, thus removing any concerns related to the kinematical assumptions of the model.

The deflected shape of the slab obtained from ADAPTIC at a central deflection of 800mm is shown in Figure 9, where the formation of a full depth crack is evident from the significant element stretching in the short direction, which conforms to the assumptions of the proposed model. The load-deflection curves obtained from ADAPTIC and from the proposed model for two different values of bond strength are compared in Figure 10, where a generally favourable comparison is obtained particularly in the tensile membrane stage. Furthermore, the insensitivity of the predictions to bond strength for negligible strain hardening in the reinforcement is confirmed, where the results from the proposed model for the two different values of bond strength are almost identical. Finally, it can be noticed that the load resistance obtained from ADAPTIC is always greater than that obtained from the model, which is attributed to the compression of the concrete that is neglected in the current analytical formulation and is more predominant for smaller deformations. These favourable comparisons demonstrate the accuracy of the kinematic expressions and the corresponding load-deflection response characteristics of the proposed model.

Having verified the kinematic expressions of the proposed model by neglecting the strain-hardening of the steel reinforcement, comparisons are presented for more realistic values of the steel strain-hardening modulus. The concrete grade chosen is C40 with a secant modulus of elasticity of 29 GPa and a tensile strength of 2.4 MPa (ECN, 2004). These values are adopted for the analysis with ADAPTIC along with the other parameters given in Table 3 that are kept unchanged, and, unlike the previous example, are assigned to all elements of the slab so as to verify the general accuracy of the proposed analytical model. For the steel reinforcement, the strain-hardening modulus is taken as $0.005 E_s$ corresponding to a hardening modulus $E_2 = 1050\text{MPa}$ and the ultimate strength as 600MPa corresponding to a rupture reinforcement strain of 0.19 (Table 4).

The slab analysed here has a 9m ($a = 9\text{m}$, $\alpha = 1.5$) long span, with all the other parameters remaining the same as before, and following a sensitivity study a mesh of 12×8 elements is adopted. The reinforcement chosen corresponds to bars with a diameter of 12mm spaced at 250mm. According to BS8110 (BSI, 1997) this corresponds to a reinforcement perimeteric design value of bond strength of 1.8MPa or equivalently 0.27N/mm per mm width of the slab. The load-deflection response obtained with ADAPTIC is compared against the model in Figure 11 where ‘high bond’ and ‘low bond’ refer to the 0.45N/mm and 0.15N/mm, respectively, per mm width of the slab. It can be seen that the two cases compare well with ADAPTIC, which initially predicts greater resistance due to the compressive response of the concrete that is neglected in the proposed model. The drop in capacity of the slab at a load of approximately 60kN/m^2 is due to further through depth cracking in the slab, which leads in subsequent stages to load redistribution via tensile membrane action in the reinforcement. The failure deflection predicted by the proposed model for the ‘lower bond’ and the ‘high bond’ cases is 583mm and 338mm, respectively, with corresponding failure loads of 38.0kN/m^2 and 21.7kN/m^2 , respectively, as illustrated in Figure 11. This demonstrates that higher bond strength results in greater strain concentrations leading to earlier failure. However, in both cases the failure load is significantly

greater than the prediction of yield line theory which for this case is only 6.08kN/m^2 , thus highlighting the significant enhancement to the resistance of restrained slabs provided by tensile membrane action.

3.3. Simplified ambient temperature failure model

The analytical model presented previously can be simplified by assuming that strips taken in the two directions at the midpoint of the diagonal provide representative averages of the response across the diagonal. Moreover, performing an approximation of the type:

$$\sqrt{1+u} \approx 1 + \frac{u}{2} \quad (36)$$

for sufficiently small values of u , the stretching of the reinforcement corresponding to the strips at the chosen points are given by:

$$\Delta_{sce} = \frac{U_c^2}{b} \quad (37)$$

$$\Delta_{sdy} = \frac{U_c^2}{2b} \quad (38)$$

$$\Delta_{sdx} = \frac{U_c^2}{4\eta a} \quad (39)$$

Furthermore, it is assumed that the bond length is unlimited, and thus a single expression can describe the force in the reinforcement. Therefore, the forces in the reinforcement along the central crack and at the midpoint of the diagonal yield line in the y and x directions can be obtained by substituting Equations (37)–(39) respectively into Equations (4), (9) and (20) as:

$$T_{sce} = T_y + \sqrt{\frac{2A_s E_2 \sigma_b}{b}} U_c \quad (40)$$

$$T_{sy} = T_y + \sqrt{\frac{A_s E_2 \sigma_b}{2b}} U_c \quad (41)$$

$$T_{sx} = T_y + \sqrt{\frac{A_s E_2 \sigma_b}{4\eta a}} U_c \quad (42)$$

The energy dissipation in the reinforcement across the central yield line and across the diagonal yield line in the y and x directions is given by:

$$\dot{D}_{sce} = T_{sce} \frac{2U_c}{b} \left(\frac{a}{2} - \eta a \right) \quad (43)$$

$$\dot{D}_{sdy} = T_{sy} \frac{U_c}{b} \eta a \quad (44)$$

$$\dot{D}_{sdx} = T_{sx} \frac{U_c b}{4\eta a} \quad (45)$$

Combining Equations (43)–(45) with (30)–(32) results in a simplified relationship between the externally applied load and the central deflection:

$$q = \frac{24U_c}{3-2\eta} \left(T_{sce} \frac{1-2\eta}{b^2} + T_{sy} \frac{\eta}{b^2} + T_{sx} \frac{1}{4\eta a^2} \right) \quad (46)$$

Finally, the failure deflection is obtained by replacing T_{sce} with T_u in Equation (40) as:

$$U_{cf} = (T_u - T_y) \sqrt{\frac{b}{2A_s E_2 \sigma_b}} \quad (47)$$

For the parameters given in Table 1 the predictions of the simplified model are plotted for two different values of bond strength in Figure 12. The failure deflection predicted by Equation (47) is 586mm for the low bond strength value and 338mm for the high bond strength value, which are almost identical to the values obtained from the full formulation (Table 5). It can be observed from Figure 12 that the full and simplified versions of the model are virtually identical for displacements up to around 600mm. The subsequent divergence can be attributed to the simplification made for the stretching of the reinforcement, which becomes apparent at larger deflections. Nevertheless, the simplified model provides favourable comparison up to the failure

point, and therefore it is proposed that the simplified model is sufficiently accurate for design purposes.

4. Elevated Temperature Restrained Slab Model

The ambient temperature slab model presented in the previous section is extended here to account for fire conditions where the source of the fire is located under the slab. Towards this end, the influence of elevated temperatures on the material properties and on the thermally curved geometry of the LRC slab should be considered. It is worth noting that although the steel deck of the slab de-bonds and loses strength during fire, it retains its continuity thus reducing the direct influence of the fire over the reinforcement across the crack. The increase in temperature causes the concrete slab to expand and curve in addition to causing differential expansion between the steel reinforcement and the surrounding concrete due to different coefficients of thermal expansion for the respective materials. The following additional assumptions are made for the elevated temperature model:

- the failure mechanism is unchanged from the ambient case and is defined by yield lines determined according to yield line theory,
- the temperature distribution is uniform over the slab plane, but varies linearly over the slab thickness,
- the four parts of the slab bounded by the yield lines and the supports are rigid, but the underlying shape of each part can be curved and expanded from the ambient case due to thermal curvature/expansion effects, as governed by the thermal expansion characteristics of concrete.

The elevated temperature model is formulated expressing the load-deflection response and the failure deflection at a specific level of temperature. This can be readily used to establish the

temperature-deflection response and the failure temperature for specific loading, as illustrated later.

As mentioned before, the thermal expansion and curvature of the various parts of the slab bounded by the yield lines are governed by the thermal expansion characteristics of the concrete. Here, for simplicity, it is assumed that the temperature gradient is linear and that the temperature of the reinforcement and the surrounding concrete is the same at the level of the reinforcement, similar to the assumption made previously for the beam model (Izzuddin and Elghazouli, 2004). The extent of the curvature of a strip across the slab is determined by the thermal gradient and thermal properties of the concrete. In addition to this, it is assumed that strips on either side of the yield line curve independently without continuity of slope across the yield line. This can result in an error in the initial stages of the response involving compressive arching action; however, in the tensile catenary stage when a full depth crack is formed and where failure is expected to occur this assumption becomes realistic. Moreover, the treatment of the LRC slab at elevated temperature differs from the beam model (Izzuddin and Elghazouli, 2004) in the sense that the underlying shape of the various slab parts needs to be described in three dimensions. For this purpose a simplified approach is proposed for considering the influence of elevated temperature on the deformed configuration of the slab parts, where the triangular and trapezoidal parts of the slab are assumed to expand and curve independently (Figure 13). This approach utilises an assumed thermal strain field which does not necessarily satisfy compatibility within each of the slab parts, but which captures reasonably well the influence of thermal curvature in a practical formulation that was verified against a more involved compatible discretisation approach (Omer, 2005).

The expansion of the slab at the reinforcement level is reduced by the thermal curvature of the slab such that the length of the chord joining the two ends of the strip is given by (Izzuddin and Elghazouli, 2004):

$$L_c = L + \alpha_c \Delta T_r - \frac{\kappa^2 L^3}{24} \quad (48)$$

where κ is the thermal curvature assuming linear temperature distribution. Furthermore, the differential expansion of the reinforcement with respect to the concrete is also included.

For each of the slab parts, it is assumed that strips perpendicular to the external support edges can extend and curve freely as if they were isolated strips (Figure 13). For the strips parallel to the external support edges, it is assumed that the strip along the external edge is restrained against thermal expansion and curvature, while the strip along the inside vertex/edge is free to expand and curve (Figure 13). The extent of thermal expansion/curvature for intermediate strips parallel to the external edge is assumed to vary linearly towards the inside vertex/edge. Clearly, this approach is based on assumed thermal expansion/curvature generalised strains which satisfy the boundary conditions of simple supports and planar restraint.

The stretching of the reinforcement is obtained by treating each strip across the slab similar to the beam model (Izzuddin and Elghazouli, 2004; Omer, 2006). Thus, the stretching of the reinforcement along the central yield line crack and the diagonal yield line crack in the two directions are given by:

$$\Delta_{sce} = \frac{b}{2} - \sqrt{\frac{b^2}{4} \left(1 + \alpha_c \Delta T_r - \frac{\kappa^2 b^2}{96} \right)^2 - U_c^2} \quad (49)$$

$$\begin{aligned} \Delta_{sdy} = & \frac{x}{\eta a} \left[\frac{b}{2} - \sqrt{\frac{b^2}{4} \left(1 + \alpha_c \Delta T_r - \frac{\kappa^2 b^2 x^2}{96 \eta^2 a^2} \right)^2 - U_c^2} \right] \\ & - \frac{bx^2}{4\eta^2 a^2} \left(1 - \frac{x}{\eta a} \right) \left[\alpha_c \Delta T_r - \frac{\kappa^2 b^2}{24} \left(1 - \frac{x}{\eta a} \right)^2 \right] \end{aligned} \quad (50)$$

$$\Delta_{sdx} = \frac{2y}{b} \left[\eta a - \sqrt{\eta^2 a^2 \left(1 + \alpha_c \Delta T_r - \frac{\kappa^2 \eta^2 a^2 y^2}{6b^2} \right)^2} - U_c^2 \right] - \frac{2ay^2}{b^2} \left(1 - \frac{4\eta y}{b} \right) \left[\alpha_c \Delta T_r - \frac{\kappa^2 a^2}{24} \left(1 - \frac{4\eta y}{b} \right)^2 \right] \quad (51)$$

The complexity of the expressions for the stretching of the reinforcement at elevated temperature renders an analytical closed form solution for the model response cumbersome. Therefore, a numerical solution procedure is sought, where the slab is divided into a number of strips, and the overall response is obtained as the summation of the strip responses. For each strip and for a given deflection, the reinforcement force is incremented until the extension in that strip is equal to the stretching due to the deflection, and based on this the energy dissipated in the reinforcement is obtained. The detailed algorithm utilized for the establishment of the load-deflection response is given in (Omer, 2006).

Failure is defined as the point where the reinforcement fractures with the stress reaching the ultimate strength of the steel. As for the ambient case, failure is expected to occur across the central yield line at the centre of the slab and the failure deflection of the slab is given by:

$$U_{cf} = \begin{cases} \sqrt{L_c^2 - \left[\frac{b}{2} + \frac{2T_y \varepsilon_{sct}}{\sigma_b} - \frac{(T_u - T_y - \varepsilon_{sct} A_s E_2)^2}{2A_s E_2 \sigma_b} \right]^2} & \text{for } 0 \leq U_{cf} \leq U_{t1} \\ \sqrt{L_c^2 - \left[\frac{b}{2} + \varepsilon_{sct} \frac{b}{2} - \frac{(4T_u - \sigma_b b)(\sigma_b b - 4T_y)}{8A_s E_2 \sigma_b} \right]^2} & \text{for } U_{t1} \leq U_{cf} \leq U_{t2} \\ \sqrt{L_c^2 - \left[\frac{b}{2} + \varepsilon_{sct} \frac{b}{2} - \frac{(T_u - T_y)^2}{2A_s E_2 \sigma_b} \right]^2} & \text{for } U_{t2} \leq U_{cf} \leq U_{t3} \\ \sqrt{L_c^2 - \left[\frac{b}{2} + \varepsilon_{sct} \frac{b}{2} - \frac{(4T_u - 4T_y - \sigma_b b)^2}{4A_s E_2} \right]^2} & \text{for } U_{t3} \leq U_{cf} \end{cases} \quad (52)$$

where U_{t1} , U_{t2} and U_{t3} are given by:

$$U_{t1} = \begin{cases} \sqrt{L_c^2 - \left[\frac{b}{2} + \frac{2T_y \varepsilon_{sct}}{\sigma_b} - \frac{(\sigma_b b - 4T_y)^2}{8A_s E_2 \sigma_b} \right]^2} & \text{if } \frac{2T_y - \varepsilon_{sct} A_s E_2}{\sigma_b} \leq \frac{b}{2} \\ 0 & \text{if } \frac{2T_y - \varepsilon_{sct} A_s E_2}{\sigma_b} > \frac{b}{2} \end{cases} \quad (53)$$

$$U_{t1} = \begin{cases} \sqrt{L_c^2 - \left[\frac{b}{2} + \frac{b}{2} \varepsilon_{sct} - \frac{(\sigma_b b - 4T_y)^2}{8A_s E_2 \sigma_b} \right]^2} & \text{if } 2T_y - \sigma_b \frac{b}{2} \leq 0 \\ 0 & \text{if } 2T_y - \sigma_b \frac{b}{2} > \frac{b}{2} \end{cases} \quad (54)$$

$$U_{t3} = \sqrt{L_c^2 - \left[\frac{b}{2} + \frac{b}{2} \varepsilon_{sct} - \frac{\sigma_b b^2}{8A_s E_2} \right]^2} \quad (55)$$

where L_c is given by:

$$L_c = \frac{b}{2} \left(1 + \alpha_c \Delta T_r - \frac{\kappa^2 b^2}{96} \right) \quad (56)$$

For a full derivation of the equations presented above, the reader is referred to (Omer, 2006).

4.1. Comparisons

The elevated temperature model is compared against ADAPTIC for two slabs with different long spans (namely 9m and 12m) keeping all the other parameters constant as given for the slab considered in the second example of Section 3.2 but allowed to vary with temperature. It is assumed that the temperatures at the bottom fibre, mid-depth (reinforcement level) and top fibre of the slab are 700°C, 400°C and 100°C respectively. According to the specified material model, the resulting properties for the steel reinforcement at 400°C along with the coefficient of thermal expansion for concrete are given in Table 6. Following sensitivity studies, the central and the diagonal yield lines were each divided into ten strips and the force increments chosen as 0.1N. The load deflection response for the two slabs are plotted in Figure 14 where ‘high bond’ and

‘low bond’ refer to the bond strength of 0.15N/mm and 0.05N/mm, respectively, per mm width of the slab. It can be observed that the present model gives favourable comparisons against ADAPTIC. The failure displacements and corresponding failure loads are summarized in Table 7, where it can be observed that the failure deflection given for the two slabs are similar as it is expected, since the failure is governed by the short span length which in these cases is constant.

4.2. Simplified elevated temperature failure model

The above elevated temperature model is not ideal for design calculations, due to the complexity of the solution procedure (Izzuddin and Elghazouli, 2004). Thus, a simplified model is sought here which is capable of representing the elevated temperature response with reasonable accuracy. In line with the ambient temperature case, the stretching at the middle of the slab is considered representative of the behaviour along the central yield line, and the stretching in the two directions at the middle of the diagonal yield line is considered representative of the behaviour along the diagonal. For simplicity, the strips taken along these points are assumed to be curving independently, and the chord lengths are obtained accordingly. Therefore, the stretching of the reinforcement at the middle of the slab and the midpoint of the diagonal in the two directions are given by:

$$\Delta_{sce} = \frac{U_c^2}{b} + \frac{\kappa^2 b^3}{192} - \frac{b}{2} \alpha_c \Delta T_r \quad (57)$$

$$\Delta_{sdy} = \frac{U_c^2}{2b} + \frac{\kappa^2 b^3}{768} - \frac{5b}{16} \alpha_c \Delta T_r \quad (58)$$

$$\Delta_{sdx} = \frac{U_c^2}{4\eta a} + \frac{\kappa^2 a^3}{192} \left[\eta^3 + (1-\eta)^3 \right] - \frac{a}{8} \alpha_c \Delta T_r \quad (59)$$

Thus, assuming that the reinforcement slips over half the length of the slab the forces in the reinforcement across the central yield line and in the two directions across the diagonal yield line are given by:

$$T_{sce} = T_y + \sqrt{2A_s E_2 \sigma_b \left(\frac{U_c^2}{b} + \frac{\kappa^2 b^3}{192} - \frac{b}{2} \alpha_c \Delta T_r + \varepsilon_{sct} \frac{b}{4} \right)} \quad (60)$$

$$T_{sy} = T_y + \sqrt{A_s E_2 \sigma_b \left(\frac{U_c^2}{2b} + \frac{\kappa^2 b^3}{768} - \frac{5b}{16} \alpha_c \Delta T_r + \varepsilon_{sct} \frac{b}{4} \right)} \quad (61)$$

$$T_{sx} = T_y + \sqrt{A_s E_2 \sigma_b \left(\frac{U_c^2}{4\eta a} + \frac{\kappa^2 a^3}{192} [\eta^3 + (1-\eta)^3] - \frac{a}{8} \alpha_c \Delta T_r + \varepsilon_{sct} \frac{a}{4} \right)} \quad (62)$$

Therefore, similar to Equation (46), the load deflection response is given by:

$$q = \frac{24U_c}{3-2\eta} \left(T_{sce} \frac{1-2\eta}{b^2} + T_{sy} \frac{\eta}{b^2} + T_{sx} \frac{1}{4\eta a^2} \right) \quad (63)$$

Finally, the failure deflection can be obtained by replacing T_{sce} by T_u in Equation (60) as,

$$U_{cf} = \sqrt{\frac{(T_u - T_y)^2 b}{2A_s E_2 \sigma_b} + \frac{b^2}{2} \alpha_c \Delta T_r - \varepsilon_{sct} \frac{b^2}{4} - \frac{\kappa^2 b^4}{192}} \quad (64)$$

The load deflection response is plotted in Figure 15 for the slab considered previously with an aspect ratio $\alpha = 1.5$, where it can be observed that the full and simplified versions of the model give almost identical results up to a deflection of the order of 700mm, subsequently deviating with the simplified version predicting less stiff response. As in the ambient case, this is largely attributed to the simplifications performed in Equations (57)–(59). Finally, the failure deflection given by Equation (64) is 584mm and 937mm for the low and high bond cases, as shown in Table 8 which are both close to the failure displacements predicted by the full formulation. Therefore, it is proposed that this simplified model should be used for the failure assessment of LRC slabs, since it provides realistic predictions within a simplified framework.

The models formulated here can also be used for the determining the slab deflection-temperature response for a constant gravity loading, which corresponds more realistically to fire scenarios. This can be obtained from the load-deflection responses of the slab at given temperatures,

considering a specific load level. For the slab whose properties are given in Table 9 the deflection-temperature response is depicted in Figure 16. It is assumed that the slab is under a constant distributed load of $q = 4\text{kN/m}^2$ while the capacity of the slab predicted by yield line theory is 2.45kN/m^2 . Therefore if this slab does not have any intermediate supports such as in the form of secondary beams it would have to deform in order to sustain that loading even at ambient temperatures, as clearly demonstrated in Figure 16 by the initial equilibrium state. The variation of the yield and ultimate strengths of the steel is obtained from Eurocode 2 (EC2) (ECN, 2004) where the variation of bond strength with temperature is assumed to follow that of the concrete strength in EC2 due to the absence of relevant data, although any variation can be easily employed with the proposed model. The hardening modulus of steel E_2 at each temperature is obtained by assuming that the rupture strain of the steel is given by a constant value of 0.15. Finally, for simplicity and for the purposes of this example only it is assumed that the thermal curvature remains constant throughout the process, while increasing the reinforcement temperature.

From Figure 16 it can be observed that initially the slab deforms little with increasing reinforcement temperature, which can be explained by observing that the reduction in the yield strength as given by EC2 at lower temperatures is not significant, noting that the yield strength adopted here corresponds to the proportionality limit given by EC2. However, after 100°C , the slab start to deform faster until it fails at a reinforcement temperature of 538°C which in turn corresponds to a central deflection of 443mm.

5. Validation

The developed analytical model, in both full and simplified forms, has been successfully verified in the previous sections against detailed nonlinear finite element analysis. Notwithstanding, it is important that the proposed model is validated against physical tests before it can be applied with confidence in design practice, not least because there is the question of the level of bond strength

to be employed for the ultimate limit state, both under ambient and elevated temperature conditions. Unfortunately, the availability of experimental results depicting failure of restrained slabs is rather limited, particularly in relation to exceeding the ductility limit by rupture of reinforcement.

Representative of available tests, Brotchie and Holley (1971) undertook some experiments on restrained slabs, one of which (S46) is used for comparison against the results of nonlinear finite element analysis and the proposed analytical model in Figure 17, where full details of this and other comparisons can be found elsewhere (Cashell, 2009). Clearly, the proposed analytical model captures the slab response in the tensile membrane range quite accurately, thus confirming the reliability of the previous verification against nonlinear finite element analysis. However, it is not clear whether failure was achieved in the test at the final recorded displacement of around 63mm, let alone whether such failure was by rupture of reinforcement, nor was there relevant information on the bond characteristics. Accordingly, it is not possible to validate the ability of the proposed analytical model to predict failure by rupture of reinforcement using these experiments.

Within this general setting of lack of experimental data on restrained floor slabs up to the ductility limit, a comprehensive experimental programme is currently underway at Imperial College London, which will provide in due course the relevant parameters for the calibration of the proposed analytical model. Notwithstanding, it is clear from the successful verification and limited validation achieved so far that the proposed analytical model, which is governed by the principles of mechanics, offers great promise for a rational method of assessing the failure of LRC floor slabs by rupture of reinforcement.

6. Conclusions

This paper presents an analytical model for the failure assessment of LRC slabs subject to planar edge restraints under ambient and elevated temperatures, which is applicable to composite floor

slabs under fire. The model is based on principles of structural mechanics, and considers the rupture of reinforcement as a failure criterion, thus overcoming a significant shortcoming of previously developed semi-empirical models. In this respect, the proposed model accounts for the significant influence of the bond strength between the steel reinforcement and concrete, the characteristic stress-strain response of the reinforcement, membrane action, as well as thermal expansion and curvature.

The model is presented for the ambient and elevated temperature cases in full and simplified forms, the latter offering explicit expressions that may be easily applied. Based on the various studies undertaken, the following main outcomes can be observed:

- Restrained LRC slabs can exhibit significant membrane action with an increasing resistance that may be relied upon under fire and other extreme loading. However, the rupture of reinforcement imposes a ductility limit on this ‘hardening’ response, beyond which slab failure must be assumed.
- The main parameters influencing reinforcement rupture, and which are not considered in normal design practice for floor slabs, are the bond strength and the strain hardening characteristics of reinforcement.
- Capturing these influences, the simplified analytical model proposed here provides an easily applied rational approach that compares very well within the tensile membrane range of applicability against detailed numerical analysis and the few available experiments.
- Further validation and calibration of the proposed model is required before it can be applied in practice, and this relies on the availability of experimental data on bond strength at ambient and elevated temperature, coupled with structural tests on floor slabs up to the ductility limit. Towards this end, an experimental programme is currently underway at Imperial College London which will provide the relevant data in due course.

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Table 1 Geometric properties for the verification of the restrained slab model

h (slab height)	d (effective depth)	a	b	A_s
150mm	75mm	12m	6m	0.45 (mm ² /mm)

Table 2 Material properties for steel model

Elastic Modulus	Yield Strength	Strain-Hardening Modulus
210 GPa	400 MPa	0.00001×210GPa

Table 3 Material properties for concrete model

Elastic modulus (E_c)	30 GPa
Poisson's ratio (ν)	0.2
Compressive Strength (f_c)	40 MPa
Tensile softening modulus (a_t)	300 MPa
Tensile strength, f_{t1}	0.2 MPa
Tensile strength, f_{t2}	2.0 MPa
Initial compressive nonlinearity parameter (s_c)	0.4
Residual post-crushing strength parameter (r_c)	0.2
Compressive interaction parameter (b_c)	0.6
Shear retention factor (β_s)	0.5
Shear interaction parameter (ϕ_s)	0.4
Shear softening parameter (α_s)	0.0

Table 4 Material properties for steel

T_y	T_u	E_2	Rupture strain
180 N/mm	270 N/mm	1050 MPa	0.19

Table 5 Failure displacements and loads using ambient temperature slab model

	σ_b (N/mm ²)	U_f (mm)	q_f (kN/m ²)
Full	0.15	583	38.0
	0.45	338	21.7
Simplified	0.15	586	36.7
	0.45	338	21.1

Table 6 Material properties for steel at $\Delta T_r=400^\circ\text{C}$ and concrete.

T_y	T_u	E_2	α_s	α_c
135 N/mm	213.75 N/mm	1025MPa	$14 \cdot 10^{-6}$	$8 \cdot 10^{-6}$

Table 7 Failure displacements and loads using elevated temperature slab model

	σ_b (N/mm ²)	U_f (mm)	q_f (kN/m ²)
$\alpha=1.5$	0.05	964	51.0
	0.15	590	28.6
$\alpha=2.0$	0.05	939	45.3
	0.15	601	25.7

Table 8 Failure displacements and loads using elevated temperature slab model ($\alpha = 1.5$)

	σ_b (N/mm ²)	U_f (mm)	q_f (kN/m ²)
Full	0.05	964	51.0
	0.15	590	28.6
Simplified	0.05	937	45.5
	0.15	584	28.1

Table 9 Properties of slab subject to constant load

a	b	d	A_s	κ
9m	6m	75mm	0.142mm ² /mm	5·10 ⁻⁵ mm ⁻¹
T_y	T_u	σ_b	α_s	a_c
71N/mm	88.8 N/mm	0.156 N/mm ²	14·10 ⁻⁶	8·10 ⁻⁶

Figure 1 Yield lines forming in a rectangular floor slab

Figure 2 LRC beams with single and multiple cracks formed

Figure 3 Material models adopted for (a) steel (b) bond strength

Figure 4 Assumed deformation mode for a quarter slab model

Figure 5 Schematic representation of a quarter of the slab

Figure 6 Schematic representation of the deformed beam

Figure 7 Extent of the regions along the diagonal for stretching in y-direction

Figure 8 Extent of the regions along the diagonal for stretching in x -direction

Figure 9 Plan view of the deflected shape of restrained slab using ADAPTIC:

displacement scale=10

Figure 10 Load deflection response of the LRC slab

Figure 11 Load-deflection response for restrained slab

Figure 12 Load-deflection comparison of the full and simplified ambient slab models

Figure 13 Schematic representation of the slab deformed under elevated temperatures

(a) $\alpha = 1.5$

(b) $\alpha = 2.0$

Figure 14 Load deflection response of a slab under elevated temperature

Figure 15 Load deflection responses of slab ($\alpha = 1.5$) using elevated temperature models

Figure 16 Deflection-temperature response of slab under a distributed load ($q = 4 \text{ kN/m}^2$)

Figure 17 Model validation against experiment S46 by Brotchie & Holley (1971)