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High Resolution TVD Schemes for Interface Tracking

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Abstract. A first order upwind difference scheme (UDS) is routinely adopted for representing convection terms in a discretised space. UDS provides stable solutions. However it also introduces false diffusion in situations in which the flow direction is oblique relative to the numerical grid or when the cell-Peclet number is large. In order to predict sharp interface, higher order upwind schemes are preferred because of they reduce numerical dissipation. In interfacial flows, density and viscosity vary sharply in space. Representation of convective terms by Total variation diminishing (TVD) schemes ensures reduced smearing without impairing convergence property. TVD schemes develop formulae for interpolation of a cell-face value of the transported variable. If the interpolated value is bounded by the neighbouring nodal values then the scheme is 'Bounded'. However, not all TVD schemes possess this property of 'Boundedness'. The Normalised Variable Diagram (NVD) defines a domain within which the TVD scheme is bounded. Thus by combining the features of both TVD schemes and ensuring that they fall with the defined area of NVD, the convergence as well as the boundedness of a computational scheme can be ensured. In this paper, six different higher order schemes are considered some which are TVD bounded or unbounded, to solve the well known interface tracking problem of Rayleigh-Taylor Instability. To the best of our knowledge, a comparison of combined TVD/NVD principles in the case of interface tracking problems has not been reported in published literature.

Keywords: Interface tracking, TVD, SIMPLE, Collocated Grid

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INTRODUCTION

The correct modelling of convection without excessive artificial dissipation while retaining high-order accuracy, stability and boundedness plays a key role in reproducing complex flow physics, such as sharp gradients or shocks in case of compressible flows. First order upwind difference produces stable and bounded solutions, but it introduces false diffusion especially in situations in which the flow direction is oblique relative to the numerical grid or when the cell-peclet number is large. The latter situation is found in prediction of interfacial flows of two immiscible fluids and the predicted interface is smeared. So in order to predict sharp interface, higher order upwind schemes are preferred because of they reduce numerical dissipation; however it also generates oscillations when it encounters steep variations of transported properties. Different researchers have proposed various techniques for capturing a well-defined interface, excellent reviews can be found in Rudman (1997), Tryggvasson (2001). One approach to the problem of preserving interface resolution is to discretise the convective scalar transport equation for the volume fraction with a differencing scheme that guarantees physical (bounded) volume fraction values while preventing the smearing of the transitional area over several mesh intervals [Hirt & Nicolls(1981)]. It takes the interface orientation into account when calculating the amount of fluid fluxed over the face of a control volume, ensures physical volume fraction values (overall boundedness between zero and one), and keeps the transitional area over one control volume. However, it does not preserve local boundedness; i.e., a volume fraction value which initially lies between the values of its neighbours does not necessarily preserve this property when advected. High resolution differencing schemes such as TVD methods [Andrillon & Allesandrini (2004)], Flux Corrected Transport (FCT) [Zalesak (1979)] schemes, and techniques using the Normalised Variable diagram (NVD) [Leonard (1991)] offer an alternative. Total variation diminishing (TVD) schemes possess several attractive features, such as delivering well-resolved, non-oscillatory discontinuities and boundedness. Boundedness is very important for convective discretised scheme in numerical computation, with which the numerical scheme will never surpass the maximum or minimum values inherently determined by the physical process itself [Leonard (1991)]. The NVA does not guarantee convergence of the proposed differencing schemes but tackles the problem of boundedness in a more general way. In this paper a novel interface capturing scheme [Nandi & Date (2009)] is used to predict the interface and different higher order schemes are compared making use of the NVD concept. Thus by combining the features of both TVD schemes and

ensuring that they fall with the defined area of NVD, the convergence as well as the boundedness of a computational scheme can be ensured.

Numerical Formulation

Governing Equations

$$\frac{\partial u_j}{\partial x_j} = \nabla \cdot \vec{V} = 0 \quad (1)$$

$$\frac{\partial \rho_m u_i}{\partial t} + \frac{\partial \rho_m u_j u_i}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\mu_m \frac{\partial u_i}{\partial x_j} \right] - \frac{\partial p}{\partial x_i} + \rho_m g_i + F_{st,i} \delta + \frac{\partial}{\partial x_j} \left[\mu_m \frac{\partial u_j}{\partial x_i} \right] \quad (2)$$

Equation 1 represents volume conservation whereas equation 2 represents momentum equation in conservative form. $F_{st,i} \delta$ is the surface tension force that acts at the interface and g_i is the gravitational acceleration and ρ_m is a superficial density associated with the interface which represents a plane of discontinuity between the fluids. ρ_m equals ρ_a or ρ_b within each fluid. Equations 1 and 2 can be derived from conventional control-volume analysis. They define fluid motion completely when procedures for evaluation of superficial properties ρ_m and μ_m and the surface tension force $F_{st,i} \delta$ is properly formulated. The interface location is tracked by solving an equation for the conserved scalar Φ ,

$$\frac{\partial \Phi}{\partial t} + \frac{\partial \Phi u_j}{\partial x_j} = 0 \quad (3)$$

The unsteady flows are calculated by solving the momentum, pressure correction and any scalar equations in a fully implicit manner on a staggered grid adopting SIMPLE algorithm by Patankar and Spalding (1971). Here, however the SIMPLE algorithm is implemented on collocated grids using a pressure smoothing term while retaining the fully implicit treatment.

NVD Diagram

FIGURE 1. Typical node P with upwind and downwind nodes.

Consider the variation of a convected scalar ϕ in a 1 D case, the normalized variable is defined as $\tilde{\phi} = \frac{\phi - \phi_U}{\phi_D - \phi_U}$, with

this definition, any differencing scheme using only the nodal values of ϕ at points U, C and D to evaluate the face value

can be written in the form: $\tilde{\phi}_C = \frac{\phi_C - \phi_U}{\phi_D - \phi_U}$ and $\tilde{\phi}_f = \frac{\phi_f - \phi_U}{\phi_D - \phi_U}$.

In order to avoid unphysical oscillations in the solution, we will require ϕ_C (and consequently ϕ_f) to be locally bounded between ϕ_U and ϕ_D , meaning either $\phi_U \leq \phi_C \leq \phi_D$ or $\phi_U \geq \phi_C \geq \phi_D$. If this criterion is satisfied for every point in the domain, the entire solution will be free of any unphysical oscillations. This is the basis of NVA. Gaskel and Lau (1988) showed that the boundedness criterion can be represented in a diagram by plotting ϕ_f as a function of ϕ_C . This is known as the Normalised Variable Diagram or NVD. Thus by combining the features of both TVD schemes and ensuring that they fall with the defined area of NVD, the convergence as well as the boundedness of the scheme used for computation is ensured. In table i) ϕ_f as $f(\phi_C)$ of the six schemes compared are stated.

Results

The different higher order convection schemes discussed in the previous section were applied to the problem of Rayleigh-Taylor instability. This cases was chosen as the representative cases since there is considerable stretching and distortion of the interface and is a well-known problem and computed by many researchers. Figure 2 shows the comparison of F distributions ($0.025 < F < 0.0975$) obtained by TVD [Lin Lin (1997)] and UDS schemes on 80×240 grid. The figure confirms that TVD predictions show much less smearing than the UDS scheme. The evolution of the interface, computed using six different higher order convection schemes is shown in Figure 3. The number of iterations per time step were kept same for comparison purposes. The volume error at the end of the schemes, were more or less the same ($\approx 0.4\%$).

TABLE 1. ϕ_f vs ϕ_c For Various Higher Order Schemes

High Resolution Schemes	Expression	Remarks
LIN LIN [Lin (1997)]	$\phi_f = \phi_c \quad \text{if } \phi_c \in [0, 3/10]$ $= 3/8 - \phi_c / 4 \quad \text{if } \phi_c \in [3/10, 5/6]$ $= 1 - \phi_c \quad \text{if } \phi_c \in [5/6, 1]$ $= 0 \quad \text{if } \phi_c \notin [0, 1]$	Bounded
HLPA [Zhu (1991)]	$\phi_f = 0 \quad \text{if } \phi_c \notin [0, 1]$ $= \phi_c (1 - \phi_c) \quad \text{if } \phi_c \in [0, 1]$	Bounded
MINMOD [Zhu & Rodi (1991)]	$\phi_f = 0.5\phi_c \quad \text{if } \phi_c \in [0, 0.5]$ $= 0.5 - 0.5\phi_c \quad \text{if } \phi_c \in [0.5, 1]$ $= 0 \quad \text{elsewhere}$	Bounded
OSHER [Chakravarthy (1983)]	$\phi_f = 0.5\phi_c \quad \text{if } \phi_c \in [0, 2/3]$ $= 1 - \phi_c \quad \text{if } \phi_c \in [2/3, 1]$ $= 0 \quad \text{elsewhere}$	Bounded
SMART [Gaskell (1988)]	$\phi_f = 2\phi_c \quad \text{if } \phi_c \in [0, 1/6]$ $= 0.375 - 0.25\phi_c \quad \text{if } \phi_c \in [1/6, 5/6]$ $= 1 - \phi_c \quad \text{if } \phi_c \in [5/6, 1]$ $= 0 \quad \text{eslewhere}$	Unbounded
HOAB [Wei (2003)]	$\phi_f = 2.5\phi_c \quad \text{if } \phi_c \in [0, 1/6]$ $= 0.5 - 0.5\phi_c \quad \text{if } \phi_c \in [1/6, .5]$ $= 0.25 \quad \text{if } \phi_c \in [0.5, 0.75]$ $= 1 - \phi_c \quad \text{if } \phi_c \in [0.75, 1]$ $= \quad \text{elsewhere}$	Unbounded

FIGURE 2. Comparison of UDS (left) and Lin-Lin TVD (right) predictions on 80x 240 grid. F distributions ($0.025 < F < 0.0975$, 10 contours).

FIGURE 3. Results of Rayleigh Taylor Instability at $\tau = 8$ using different higher order schemes.

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