Finite-time parameters estimation of the Chua system

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Abstract. In this work, the unknown set of parameters of the Chua system is recovered under the hypothesis that the voltages of the capacitors are available. To this end, focusing on the differential equations, the Volterra kernel-based approach is used to perform an estimation without the uncertainty of the unmeasurable derivatives and the unknown initial conditions.

Introduction

Chaotic systems have attracted the attention of the recent research due to their potential applications in secure communication, laser system, electronic chemistry, neurophysiology and ecology [1]. The reconstruction problem consists in recovering the underlying variables and the unknown parameters from a partial knowledge of a chaotic system [2], i.e. it is desired to extract some physical parameters and to estimate some non available states from the available system outputs. In [4] the authors propose an approach to provide an accurate estimation of the parameters of a Lorenz system synchronized by an external driving signal. The estimation is obtained by translating the Lorenz equation into a linear one w.r.t. the parameters. The parameters are then obtained as a solution of the resulting final equations. When the state of the system is not fully measurable, then the identification problem becomes more difficult due to the necessity to estimate, besides the parameters, also the hidden state variables from the sole input/output measurements. To solve this problem, in [5] an efficient approach for the estimation of the non-available state variables for the Duffings chaotic system is proposed by obtaining an integral output parametrization of the hidden state variables. In this context, the kernel-based method designs a class of Bivariate Causal Non-asymptotic Kernel (BC-NK) to form the Volterra integral operator, providing a finite-time parameter estimation for continuous-time linear systems, annihilating the effects of the unknown initial conditions and avoiding the need for output derivatives computations. Inspired by the idea of [3], in this paper, the Volterra operator with BF-NK is used to achieve a joint parametric estimation of the Chua system parameters with arbitrary non-asymptotic convergence.

The Chua system

The Chua system (CS) (see Figure 1), consists of three energy-store elements (an inductor and two capacitors), a linear resistor and a single nonlinear resistor, called Chua's diode. A simplified nonlinear model of this system is given by:

$$\begin{aligned} C_1 \frac{dv_{C1}}{dt} &= \frac{1}{R} (v_{C2} - v_{C1}) - \phi(v_{C1}), \\ C_2 \frac{dv_{C2}}{dt} &= \frac{1}{R} (v_{C1} - v_{C2}) + i_L, \\ L \frac{di_L}{dt} &= -v_{C2} \end{aligned} \tag{1}$$

FIGURE 1: The Chua system.

where R is a linear resistance, v_{C1} and v_{C2} are the voltages across capacitors C_1 and C_2 , respectively, i_L is the current through the inductor L and $\phi(v_{C1})$ is the current through the nonlinear resistor as a function of the voltage across capacitor C_1 . This nonlinear function is described by the odd-symmetric piecewise-linear function

$$\phi(x) = -\left[m_1 v_{C1} + \frac{m_0 - m_1}{2} (|v_{C1} + B_p| - |v_{C1} - B_p|)\right], \quad (2)$$

where m_0 , m_1 and B_p are three fixed constants of the diode. The system 1 can be rewritten in dimensionless form as

$$\begin{aligned} \dot{x}_1 &= \beta(-x_1 + x_2 - f(x_1)), \\ \dot{x}_2 &= x_1 - x_2 + x_3, \\ \dot{x}_3 &= -\gamma x_2, \end{aligned} \quad (3)$$

with

$$f(x) = ax + b(|x + 1| - |x - 1|), \quad (4)$$

and

$$\beta = \frac{C_2}{C_1}, \quad \gamma = \frac{C_2 R^2}{L}, \quad a = m_1 R, \quad b = \frac{m_0 - m_1}{2} R, \quad x_1 = \frac{v_{C1}}{B_p}, \quad x_2 = \frac{v_{C2}}{B_p}, \quad x_3 = \frac{R i_L}{B_p}.$$

For the fixed values of parameters in a neighborhood of $\gamma = 27, \beta = 15.6, a = -5/7, b = -3/14$ and initial conditions $x_0 = [-0.9, -0.15, 1.47]^T$, CS presents the so called double scroll chaotic attractors [1].

Definition 1 Consider a smooth nonlinear system

$$\dot{x} = f(x, \mathbf{p}), \quad y = h(x), \quad (5)$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$ and $\mathbf{p} \in \mathbb{R}^l$ with $l < n$ is a constant parameter vector. The state vector x is algebraically observable if it can be uniquely expressed as

$$x = s(y, \dots, y^{(m)}, \mathbf{p}), \quad (6)$$

for some smooth function s . Moreover, if $\mathbf{p}$ satisfies the following linear relation

$$s_1(y, \dots, y^{(m)}) = s_2(y, \dots, y^{(m)})\mathbf{p}, \quad (7)$$

where $s_1(\cdot)$ and $s_2(\cdot)$ are respectively $l \times 1$ and $l \times l$ matrices with $\det(s_2(y, \dots, y^{(m)})) \neq 0$, then $\mathbf{p}$ is said to be algebraically linearly identifiable with respect to the output y .

System 3 is algebraically observable with respect to the outputs $y_1 = x_1$ and $y_2 = x_2$, since all the system variables can be expressed in terms of y_1, y_2 and their derivatives as

$$x_1 = y_1, \quad x_2 = y_2, \quad x_3 = \dot{y}_2 + y_2 - y_1. \quad (8)$$

Moreover the system parameters are algebraically linearly identifiable since

$$\dot{y}_1 = -\beta(1 + a)y_1 + \beta y_2 - b\beta z_1, \quad (9)$$

$$\ddot{y}_2 + \dot{y}_2 - \dot{y}_1 = -\gamma y_2 \quad (10)$$

where $z_1 = |y_1 + 1| - |y_1 - 1|$.

Volterra kernels-based estimation

Let $K(t, \tau)$ be the Bivariate Feedthrough Non-Asymptotic Kernels (BF-NK) defined in [3]

$$K(t, \tau) = e^{-\omega_h(t-\tau)} (1 - e^{-\bar{\omega}\tau})^2, \quad (11)$$

where ω_h and $\bar{\omega} \in \mathbb{R}_+$ are arbitrary constant tuning parameters. Given a function $u(\cdot) \in \mathcal{L}_{loc}^2(\mathbb{R}_{\geq 0})$, the bounded Volterra linear integral operator induced by $K(t, \tau)$ is defined as

$$[V_K u](t) \triangleq \int_0^t K(t, \tau) u(\tau) d\tau. \quad (12)$$

Recalling kernel function properties (see [3]) it follows that

$$[V_K u^{(1)}](t) = K(t, t)u(t) - [V_{K^{(1)}} u](t), \quad (13)$$

$$[V_K u^{(2)}](t) = K(t, t)u^{(1)}(t) - K^{(1)}(t, t)u(t) + [V_{K^{(2)}} u](t). \quad (14)$$

Let us consider differential equation 9 with the unknown parameters vector $\theta_1 = [-\beta(1+a), \beta, -\beta b]^T$. Taking the Volterra linear integral operator on both side and considering Kernel functions properties, it follows that

$$h_1(t)^T \theta_1 = s_1(t), \quad (15)$$

where $h_1(t) = [[V_K y_1](t), [V_K y_2](t), [V_K z_1](t)]^T$ and $s_1(t) = K(t, t)y_1(t) - [V_{K^{(1)}} y_1](t)$. In order to estimate the parameter γ , Equation 10 is considered. In this case, to eliminate the dependence on $y_2^{(1)}(t)$, two BF-NK functions, namely K_1 and K_2 , with the same $\bar{\omega}$ and different values of ω_h (ω_1 and ω_2) are used. Letting

$$\begin{aligned} s_2(t) &= [V_{K_1^{(2)}} y_2](t) + [V_{K_1^{(1)}} (y_1 - y_2)](t) - K_1^{(1)}(t, t)y_2(t), \\ s_3(t) &= [V_{K_2^{(2)}} y_2](t) + [V_{K_2^{(1)}} (y_1 - y_2)](t) - K_2^{(1)}(t, t)y_2(t), \\ h_2(t) &= [V_{K_2} y_2](t) - [V_{K_1} y_2](t), \end{aligned}$$

we obtain the algebraic equation

$$h_2(t)\gamma = s_2(t) - s_3(t). \quad (16)$$

Equation 15 and Equation 16 are used to obtain a finite-time estimation of the unknown parameters. For the sake of brevity, the procedure is described only in the case of θ_1 estimate. From Equation 15 it follows that the parameters vector θ_1 verifies the structural constraint

$$H(t)\theta_1 - S(t) = 0, \quad (17)$$

where

$$H(t) = h_1(t)h_1(t)^T, \quad S(t) = h_1(t)s_1(t). \quad (18)$$

Let $H_f(t)$ and $S_f(t)$ be two filtered versions of the matrices $H(t)$ and $S(t)$:

$$\begin{cases} \dot{H}_f(t) = -\lambda H_f(t) + H(t), \\ \dot{S}_f(t) = -\lambda S_f(t) + S(t), \end{cases} \quad (19)$$

with $H_f(0) = 0 \in \mathbb{R}^{3 \times 3}$ and $S_f(0) = 0 \in \mathbb{R}^3$, then we have that also the following structural constraint is verified

$$H_f(t)\theta_1 - S_f(t) = 0. \quad (20)$$

For an estimate $\hat{\theta}_1(t)$ (possibly $\hat{\theta}_1(t) \neq \theta$) we have that the above constraint is not exactly matched, and in general a non-zero residual $\tilde{R}_f(t)$ can be introduced:

$$\tilde{R}_f(t) = H_f(t)\hat{\theta}_1(t) - S_f(t), \quad \dot{\tilde{R}}_f(t) = H_f(t)\dot{\hat{\theta}}_1(t) + \dot{H}_f(t)\hat{\theta}_1(t) - \dot{S}_f(t). \quad (21)$$

In order to make $|\tilde{R}_f(t)| \rightarrow 0$ in finite time the following sliding-mode type adaptation law can be used:

$$\dot{\hat{\theta}}_1(t) = \begin{cases} H_f(t)^{-1} \left[-\mu \operatorname{sign}(\tilde{R}_f(t)) \sqrt{|\tilde{R}_f(t)|} - \dot{H}_f(t)\hat{\theta}_1(t) + \dot{S}_f(t) \right], & |\det(H_f(t))| \geq \delta, \\ 0, & |\det(H_f(t))| < \delta, \end{cases} \quad (22)$$

where $\delta > 0$ is a user-defined threshold constant.

FIGURE 2: Chua circuit evolution and parameters estimation

Simulation results

The proposed method is here tested in a dimensionless Chua circuit with parameters as described in Section 1 in order to simulate the double-scroll chaotic attractor. Each state component is corrupted by a uniformly distributed noise in $[-0.1, 0.1]$. The method is equipped with two kernels having parameters $\omega_1 = 0.5, \omega_2 = 0.8, \bar{\omega} = 1$, while the other free parameters are tuned as $\lambda = 0.1, \mu = 0.01$, and $\delta = 10^{-10}$. The sampling time is $2 \times 10^{-4} s$. Figure 2.a shows the state components of the Chua circuit. Figure 2.b depicts the system parameters estimation results.

Conclusions

In this paper, a deadbeat estimator for the Chua system parameters based on Volterra operators has been proposed. Assuming that the voltages of the capacitors are available, it has been proved that the system is algebraically observable and identifiable with respect to the well chosen outputs. A Volterra kernel-based approach has been used to provide finite-time system parameters estimation. The effectiveness of the proposed algorithm in terms of accuracy and convergence rate, as well as attenuation of measurement noise, has been shown by simulation results.

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