Graphical Modelling of Pinched Hysteresis Loops of Memristors

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Abstract: In this paper, a graphical modelling approach of the pinched hysteresis loops exhibited by memristors is presented. This method provides a tool to emulate the hysteresis loop pinched at the origin, with the lobe area varying with the excitation frequency. The direction of the pinched hysteresis loop can be controlled. This graphical modelling method provides an alternative to describe the behaviour of memristors without deriving the coupled nonlinear differential equations typically required for physical memristors. The method has been successfully applied to model the Hewlett-Packard memristor device.

1. Introduction

Memristors postulated by Leon Chua in 1971 [1] was confirmed by a Hewlett Packard research team in 2008 [2]. Since then, research in memristors has regained momentum [3]-[9]. The first man-made memristor has recently been traced back [10] to the 1801 carbon arc experiment by Humphrey Davy [11]. The hysteresis loops exhibited by memristors are different from those occur in magnetic materials in at least two ways. Firstly, the memristor's hysteresis loop is always pinched at the origin. Secondly, the lobe area of the pinched hysteresis loop varies with excitation frequency.

There are several well-established modelling methods for the magnetic hysteresis loops. Among them are Jile-Atherton [12] and the Preisach model [13]. The advantage of the Preisach model is that it can easily be implemented in a graphical approach [14], [15], as demonstrated in many applications such as high frequency ferrite core analyses [16],[17]. The graphical approach has the advantage that the model does not require detailed physical equations and yet can still provide a computer-aided tool to predict the device behaviour. However, existing hysteresis modelling methods are not applicable to memristors. Although some memristors, such as discharge arc and lamps [18] can be described with coupled nonlinear differential equations or described in a state-space approach [19], there is no graphical method, (similar to the Preisach model for non-pinched hysteresis loops) available for general modelling of pinched hysteresis loops.

The major difference of a memristive device and non-memristive device is that the former exhibits a characteristic of “pinched hysteresis loop”. The three fingerprints of a memristor are summarized [20] as:

- 1) Pinched hysteresis loop at origin

- 2) Hysteresis lobe area decreases as frequency increases
- 3) Pinched hysteresis loop shrinks to a single-valued function at infinite frequency.

In this paper, a graphical method for suitable for modelling the three fingerprints of the pinched hysteresis loops of memristors is presented. The basic concept is explained and parameters that can alter the shapes and areas of the pinched loops are explained. The mathematic implementation of such method is illustrated. The application procedure and its implementations in physical examples are given in Section 5.

2. Basic graphical approach to the modelling of pinched hysteresis loops

In this section, the basic concept of the graphical method is explained. It will be shown later that several parameters have to be introduced in order to provide the 3 features of memristors.

2.1. Periodic excitation function

A periodic sinusoidal excitation function α is defined as:

$$\alpha(j) = M \cdot \sin(\omega \cdot j \cdot \Delta T) \quad (1)$$

where M is the peak value of the periodic function, j is the present sample position, ω is excitation angular speed and ΔT is selected sampling interval according to the simulation system defined sampling frequency. This sinusoidal function (Fig.1a) is used to illustrate the graphical method with four stages illustrated, although other periodic function can in principle be used. In this paper, this excitation function $\alpha(j)$ is the input to the memristor. the output response of the memristor is labelled as β . The pinched hysteresis loop can be formed by plotting β against α over at least one excitation cycle. In the computation, the time variable of the excitation function is discretised. It increases in steps of a time increment of ΔT . It takes N time increments to complete one full cycle (i.e. $N=2\pi/(\omega \cdot \Delta T)$). Since the proposal method is a graphical approach, the response function is not determined by any analytical equation. Instead, it is obtained from the incremental area of a sliding diagram (Section 2.3).

Fig. 1. Sinusoidal excitation and the mechanism of graphical approach

a Illustration of the sinusoidal periodic excitation

b A sliding diagram for the pinched hysteresis loop of memristor

c Progressive changes of the sliding diagram when y_G changes from M to $-M$ (for a “counter clockwise” loop in the first quadrant)

d Progressive changes of the sliding diagram when x_G changes from M to $-M$ (for a “clockwise” loop in the first quadrant)

2.2. Graphical approach to the pinched hysteresis loop of memristor with loop direction control

Apart from three fingerprints of a memristor, the loop direction is also an important characteristic of memristor. Thus, the graphical modelling method should cope with both clockwise (CW) and counter clockwise (CCW) pinched hysteresis loops. To be consistent in this paper, the loop in the first quadrant of

the pinched hysteresis loop plot is used to specify the direction.

1) *Sliding diagrams for pinched hysteresis loop with “counter clockwise” loop in the first quadrant:*

A Cartesian coordinate system adopted in Fig.1b involves a dashed square $AH_I B G_I$ with its centre at the origin of the coordinate system. The length of each side of the square is $2M$. Points A and B are two fixed points with the x-y coordinates of (M, M) and $(-M, -M)$ respectively. For a pinched hysteresis loop with “counter clockwise” direction in the first quadrant, the points G and H are two movable points that could slide “vertically” on the straight lines AG_I and BH_I respectively. Under such conditions, their positions are defined as $G=(x_G, y_G)$ where $x_G=M$, and $H=(x_H, y_H)$ where $x_H=-M$. This means they will slide “vertically”. The dashed red line AB is the diagonal line of $AH_I B G_I$ and it separates the square into two regions so that G and H would slide within their respective regions. The proposed graphical method requires the points G and H to slide along AG_I and BH_I respectively so that AH and BG are always in parallel.

The operation of the sliding diagram for CCW with the parallel relationship of AH and BG can be mathematically described by equations (2)-(4) as:

For a counter-clockwise (CCW) loop in the first quadrant of the pinched hysteresis loop,

$$(2) \quad \begin{cases} -M \leq y_G \leq M \\ x_G = M \end{cases}$$

$$\begin{cases} -M \leq y_H \leq M \\ x_H = -M \end{cases} \quad (3)$$

$$y_G = -y_H \quad (4)$$

Equations (2) and (3) ensure that the points G and H slide along the vertical lines AG_I and BH_I , respectively. Equation (4) makes sure that BG and AH are always in parallel. Fig.1b represents a sliding diagram complying with (2)-(5) when y_G and y_H are zero.

In order to illustrate the sliding function of the graphical method, Fig.1c is used to demonstrate the conditions of the sliding diagrams with y_G changing from M to $-M$. Fig. 1c shows the sliding diagram. The initial condition occurs when $y_G=M$ and $y_H=-M$ at G_I and H_I . Under this condition, the sliding point G overlaps with position A and H with position B . The straight lines AH and BG also overlap (as the “grey” straight line passing through the origin in Fig.1c). The area enclosed by the parallelogram $AGBH$ is zero.

As y_G slides to from M to $0.5M$ and y_H from $-M$ to $-0.5M$ (according to (4)), points G and H move to the positions labelled as G_2 and H_2 in the sliding diagram. After these sliding movements, the two

straight lines BG_2 and AH_2 form two parallel lines as highlighted in the “green” lines labelled as “1”. Now, a parallelogram AG_2BH_2 with positive area is formed.

As y_G continues to slide from $0.5M$ to 0 and y_H from $-0.5M$ to 0 , the points G and H move to new positions G_3 and H_3 , respectively. Now a parallelogram increases its size to the area of AG_3BH_3 as shown in the “blue” parallelogram labelled as “2”. When y_G continues to slide from 0 to $-0.5M$ (and y_H from 0 to $0.5M$), the area of the parallelogram increases to AG_4BH_4 as shown in the “purple” parallelogram labelled as “3”. The parallelogram $AGBH$ continues to expand until it forms a square $AGBH$ when $y_G = -M$ ($y_H = M$). The changes of $AGBH$ can be identified with their different colours.

2) Sliding diagrams for pinched hysteresis loops with “clockwise” loop in the first quadrant:

If the pinched hysteresis loop is “clockwise” in the first quadrant, the movable points G and H will slide “**horizontally**” along the horizontal lines BG_I and AH_I , respectively. Under these conditions, $G = (x_G, y_G)$ where $y_G = -M$, and $H = (x_H, y_H)$ where $y_H = M$. This leads to the following equations for the “clockwise” loop:

For a clockwise loop in the first quadrant of a pinched hysteresis loop,

$$(5) \quad \begin{cases} -M \leq x_G \leq M \\ y_G = -M \end{cases}$$

$$\begin{cases} -M \leq x_H \leq M \\ y_H = M \end{cases} \quad (6)$$

$$x_G = -x_H \quad (7)$$

Equations (5) and (6) define the restricted area which G and H could slide; (7) enforces the parallel relationship between AG and BH .

Besides the sliding direction of points G and H , the same principle previously described for the “counter clockwise” loop in the first quadrant remains valid. The progressive changes of the sliding diagram for a “clockwise” hysteresis loop is shown in Fig.1d.

2.3. Graphical approach to the formation of the pinched hysteresis loop

With the introduction of the sliding diagrams in the previous section, it is necessary to explain the procedure for forming the pinched hysteresis loop. It should be noted that each pinched loop has both CW and CCW loops. A typical pinched loop inherits self-crossing feature. If it has a CCW loop in the first

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quadrant, then it has a CW loop in the third quadrant. Similarly, a pinched loop with a CW loop in the first quadrant will have a CCW loop in the third quadrant. The basic graphical method is designed to exhibit the pinched hysteresis loop self-crossing feature.

The proposed graphical modelling requires a function with an input α (representing the excitation) and an output β (representing the loop magnitude). The excitation function is assumed to be a bipolar and periodic one such as the sinusoidal function in Fig.1a.

In this section, the specific operation details of a CCW loop in the first quadrant is presented first, two scenarios which are excitation increasing and decreasing conditions are discussed. The operation of the CW case will be addressed later.

1) Response Generation of a CCW Loop

When the instantaneous excitation value is increasing comparing with the previous value in a counter clockwise loop (i.e. $\frac{d\alpha(j)}{dj} \geq 0$), Fig. 6 shows a graphical approach to determine the output function β .

Fig. 2. Sliding diagram of the graphical approach

a Sliding diagram for a CCW pinched hysteresis loop with increasing excitation (green) and decreasing excitation (pink)

b Sliding diagram for CW pinched hysteresis loop under increasing excitation (green) and decreasing excitation (pink)

Initially, the value of both input α and output β are assigned to be 0. P and Q are two points where line AH and BG intersect with the y axis shown in Fig.2a. The excitation function $\alpha(j)$ is assigned to move along the x -axis of the sliding diagram, and such excitation sliding mechanism is defined for the situation

when the excitation is increasing and regardless of loop directions on the pinched hysteresis loop graph. The response value $\beta(j)$ is assigned to be the area value of rhombus $F_j E_j Q P$ at position j after the excitation value is increasing from value $\alpha=0$ to $\alpha(j)$. The symbols in this case are highlighted with green colour in Fig.2a.

While at the point $j+1$, $\alpha(j+1)=M \cdot \sin[\omega \cdot \Delta T \cdot (j+1)]$. The incremental area ΔS_{j+1} between the position j and $j+1$ is highlighted in green color. The response value (i.e. the area value) in position $j+1$ can be calculated by:

$$\beta(j+1)=\beta(j)+\Delta S_{j+1} \quad (8)$$

Equation (8) applies when an excitation is in a monotonic increasing condition.

When the periodic excitation is decreasing (i.e. $\frac{d\alpha(j)}{dj} < 0$), $\alpha(j)$ would move along the y axis (instead of the x -axis). The response value (i.e. the area value) is highlighted in pink colour in Fig.2a. Since the magnitude of $\alpha(j)$ is decreasing, (8) is re-expressed as (9) when the excitation is in a monotonic decreasing condition.

$$\beta(j+1)=\beta(j)-\Delta S_{j+1} \quad (9)$$

2) Response Generation of a CW Loop

For a CW pinched hysteresis loop, (8) still applies when the excitation is increasing. But the incremental area ΔS_{j+1} should be obtained from rhombus $F_j E_j F_{j+1} E_{j+1}$ highlighted in green colour as shown in Fig.2b.

For decreasing excitation in a CW loop, (9) also applies. The incremental area can be determined from the rhombus highlighted in pink colour in the sliding diagram of Fig.2b.

2.4. Preliminary pinched hysteresis loop

In this section, the graphical method is illustrated to explore the characteristic of the pinched hysteresis loop. The frequency-dependency of the pinched loop is explained.

1) Pinched at the origin

The following simulation is based on the parameters:

- y_G and y_H are assigned to be 0.4 and -0.4 for counter clockwise loop direction;
- x_H and x_G are equal to -0.4 and 0.4, respectively, for clockwise loop direction;
- the maximum value of excitation is 1.

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The Lissajous figures of α and β with different loop directions are shown in Fig.3a. It can be seen that the proposed method features the first fingerprint of hysteresis loops pinched at the origin. However, it can be noted that one side of the pinched hysteresis loop is a straight line which is not common to physical memristors. Such issue will be dealt with in Section 3.

Fig. 3. Characteristics of the graphical approach generated pinched hysteresis loops

a Plots of pinched hysteresis loops based on a range of y_G and x_H values

b Plots of pinched hysteresis loops based on a range of y_G and x_H values

c Typical variation of the lobe area based on the proposed method (with y_G or $x_H = 1$ representing dc operation and y_G or $x_H = -1$ representing extreme high excitation frequency operation for a counter clockwise loop)

2) Frequency-dependent lobe areas

From Fig.3a, it can be seen that y_G and x_H affect the slopes of the pinched hysteresis loop. Therefore, y_G or x_H values can be used to represent different frequency operations of a memristor. Fig.3b shows a range of pinched hysteresis loop based on several sets of y_G and x_H values. In general, a large y_G value represents low excitation frequency condition while a small y_G high excitation frequency in CCW loop direction. Similarly, for the CW direction case, a large x_H value represents low excitation frequency condition and a small x_H means high excitation frequency. It should be noted in Fig.3b that the pinched hysteresis loop becomes a straight line at the extreme low and extreme high frequency. The loop area

depends on the operating frequency. Therefore, the values of y_G and x_H provide the means for accounting for the frequency dependency of the pinched hysteresis loops, which is the second fingerprint of a memristor.

3) Lobe area characteristics

In order to illustrate the characteristic that lobe area would decrease when excitation is beyond the threshold value, a simulation is conducted by varying y_G or x_H . The variation of the lobe area in the first quadrant from $y_G=+1$ to -1 is shown in Fig.3c. It shows that the lobe area initially increases from zero at dc operation (with $y_G=+1$) with increasing operating frequency. After reaching a maximum value, it decreases with increasing frequency and becomes zero again at extreme high frequency (with $y_G=-1$). Therefore, this typical frequency-dependent lobe area is in agreement with the third fingerprint of a memristor.

3. Graphical approach for modelling pinched hysteresis loops

As pointed out previously, one side of the pinched hysteresis loop in Fig.3a is a straight line which is not a common feature of a memristor. In this section, a modulator σ and a compensator C are introduced to the basic model in order to enhance the features of the pinched hysteresis loop. The use of the modulator alone will modify the shape of the loop, but has the side effect of losing the pinched position. The compensator is therefore used to remove this side effect and ensures that the loop goes through the origin.

3.1. Modulator σ and Compensator C for increasing and decreasing excitation function

1) Modulator σ_p for $\frac{d\alpha(j)}{dj} \geq 0$

A new parameter σ_p is introduced to the original equation (8), where the subscript p stands for positive excitation derivative. For the region where the slope of the excitation is positive (i.e. $\frac{d\alpha(j)}{dj} \geq 0$) so that:

$$\beta(j+1) = [\beta(j) + \Delta S_{j+1}] \cdot \sigma_p(j+1) \quad (10)$$

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2) Modulator σ_n and compensator C for $\frac{d\alpha(j)}{dj} < 0$

For the region when the excitation function is decreasing (with the subscript n representing a negative excitation derivative), the original equation (11) can be modified as (11).

$$\beta(j+1) = [\beta(j) - C_{j+1} \cdot \Delta S_{j+1}] \cdot \sigma_n(j+1) \quad (11)$$

Equations (10) and (11) are the core equations for the positive and negative slopes of the excitation function, respectively and they form the key relationships between the output and the input of the model. The derivation of compensators is presented in the appendix. The derivations of the modulators, σ_p and σ_n for physical memristors will be demonstrated in Section 5.

3.2. Summary of the proposed graphical modelling approach memristor

The procedure of the proposed graphical modelling method is summarized as follows: suppose p_0 is the point where $\frac{d\alpha(j)}{dj} \geq 0$ starts to be effective. In other words, p_0 is the lowest point of the periodic excitation α in Fig.1a the computational procedures are concluded in equations (12) and (13) comparing with two iterative equations (10) and (11).

From point p_0 to point $p_0 + N/2$ (where $p_0 + N/2 = N_I$) according to Fig.1a, i.e. the half of the cycle within which the excitation function is increasing:

$$\beta(j+1) = \left[\prod_{k=p_0+1}^{j+1} \sigma_p(k) \quad \prod_{k=p_0+2}^{j+1} \sigma_p(k) \quad \cdots \quad \prod_{k=j+1}^{j+1} \sigma_p(k) \right] \cdot \begin{bmatrix} \Delta S_{p_0+1} \\ \Delta S_{p_0+2} \\ \vdots \\ \Delta S_{j+1} \end{bmatrix} + \beta(p_0) \quad (12)$$

From point $p_0 + N/2 + 1$ (which can also be labelled as point $(N_I + 1)$) to point $p_0 + N$, i.e. the half of the cycle within which the excitation function is decreasing:

$$\beta(j+1) = \left[\prod_{k=p_0+\frac{N}{2}+1}^{j+1} \sigma_n(k) \quad \prod_{k=p_0+\frac{N}{2}+2}^{j+1} \sigma_n(k) \quad \cdots \quad \prod_{k=j+1}^{j+1} \sigma_n(k) \right] \cdot \begin{bmatrix} -C_{p_0+\frac{N}{2}+1} \cdot \Delta S_{p_0+\frac{N}{2}+1} \\ -C_{p_0+\frac{N}{2}+2} \cdot \Delta S_{p_0+\frac{N}{2}+2} \\ \vdots \\ -C_{j+1} \cdot \Delta S_{j+1} \end{bmatrix} + \beta\left(p_0 + \frac{N}{2}\right) \quad (13)$$

Π is the factorial function in the equations, k is function variable. Here are two examples of how factorial function operates:

$$\prod_{k=p_0}^{j+1} \sigma_p(k) = \sigma_p(p_0) \cdot \sigma_p(p_0+1) \cdots \sigma_p(j) \cdot \sigma_p(j+1)$$

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$$\prod_{k=p_0+\frac{N}{2}+1}^{j+1} \sigma_n(k) = \sigma_n\left(p_0 + \frac{N}{2} + 1\right) \cdot \sigma_n\left(p_0 + \frac{N}{2} + 2\right) \dots \sigma_n(j+1)$$

In an actual memristor pinched hysteresis loop simulation process, excitation would typically starts from the origin at which the initial values are known. For the complementary part and considering the self-crossing feature, the process would replicate what has been assigned for the first half of excitation.

Fig.4a and Fig.4b show how (12) and (13) are implemented over the regions of positive slope and negative slope of the excitation function, respectively. Equations (12) and (13) constitute the framework of the proposed graphical approach to the pinched hysteresis loop of memristor. A flow chart of the implementation of this graphical method is illustrated in Fig.4c. It should be highlighted that a power amplification coefficient ϵ is introduced. This coefficient is simply a constant for scaling the magnitude of the output so that the model can fit into a diversity of memristors.

Fig. 4. Diagrams of the graphical approach

- a Illustration of proposed graphical approach when the excitation function is increasing
- b Illustration of proposed graphical approach when the excitation function is decreasing
- c flowchart of the procedure of the proposed graphical modelling method

4. Demonstration of the graphical methods for pinched hysteresis loops generation

4.1. Simulation of pinched hysteresis loop with different modulators value with $y_G=x_H=0$ for both CCW and CW directions

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New simulation plots with the parameter $y_G=0$ and for a range of modulator values (with $\sigma_p=\sigma_n\geq 1$) for CCW loops are displayed in Fig.5a. It can be seen that the modulator values can alter the shapes and hence the lobe areas of the pinched hysteresis loops. The larger the values of the modulators (>1), the larger the loops become in this case.

For the CW loops and with $x_H=0$ and (with $\sigma_p=\sigma_n\leq 1$), a range of CW loops are plotted in Fig.5b. Unlike the CCW loops, the sizes of the CW loops increase with a decreasing value of the modulator (<1) under such circumstances.

Fig. 5. Pinched hysteresis loop characteristics using the proposed graphical approach

a Plot of pinched hysteresis loops using proposed graphical approach for a range of modulator values while $y_G=0$ in CCW direction

b Plot of pinched hysteresis loops using proposed graphical approach with a variety of modulators while $x_H=0$ in CW direction

c Plot of CCW pinched hysteresis loops lobe area curves in the first quadrant with a range of different modulator values

d Plot of CW pinched hysteresis loops lobe area curves in the first quadrant with a range of different modulator values

4.2. Lobe area analysis in the first quadrant with various y_G and x_H modulator and coefficient values assigned

In Section II, it has been pointed out that the values of y_G and x_H can be used to alter the pinched hysteresis loops at different frequencies. These features have been evaluated by plotting the lobe areas in the first quadrant with their respective values for the CCW loops and the CW loops. Fig.5c shows the variations of the lobe areas for the CCW loops for a range of modulator values from 1 to 1.0005. These

curves are consistent in a sense that they approach zero at dc (represented by $y_G=1$), increase as y_G decreases (i.e. increasing frequency). For the CW loops, the corresponding results are plotted in Fig.5d.

5. HP memristor model simulation

In this part, a series of simulations on HP memristor model using the introduced graphical model is presented. The HP memristor model [2] was developed in HP Lab in 2008.

Two platinum parts are attached on both sides of the semiconductor materials. The HP model has both highly doped and undoped region with a length of w and $D-w$ respectively. Ammeter and voltmeter are attached to measure the current and the voltage for analysis. To assure that such device is working at a normal condition, the length of highly doped region should be restricted to $0 < w < D$.

According to the explanation of such classical memristor model [2], the relationship between instantaneous voltage and current on memristor, the width of doped area and the charge are presented in the following equations:

$$\begin{cases} v(t) = \left[R_{OFF} \left(1 - \frac{w(t)}{D} \right) + R_{ON} \frac{w(t)}{D} \right] \cdot i(t) \\ w(t) = \mu_V \cdot \frac{R_{ON}}{D} \cdot q(t) \end{cases} \quad (14)$$

where $i(t)$ and $v(t)$ are the current and voltage respectively; $q(t)$ is the electric charge at the current situation. R_{ON} and R_{OFF} are the resistance values when $w=D$, $w=0$ respectively. Furthermore, an average ion mobility parameter μ_V in the HP memristor model is assigned for simplicity.

5.1. Modulator data generation

In this paper, modulator data for generic memristor pinched hysteresis loops simulation purpose is generated in the following equations based on the following excitation scenarios:

$$1) \quad \alpha(t_{j+1}) \neq 0$$

$$\sigma_x(t_{j+1}) = 1 + \frac{\alpha(t_{j+1})}{|\alpha(t_{j+1})|} \cdot \left[(\lambda - 1) \cdot \Delta_x + \mu_x \cdot \left(\frac{d\beta_{ref}(t_{j+1})}{dt} \right)^\lambda \right] \quad (15)$$

$$2) \quad \alpha(t_{j+1}) = 0$$

$$\sigma_x(t_{j+1}) = 1 + \frac{\alpha(t_j)}{|\alpha(t_j)|} \cdot \left[(\lambda - 1) \cdot \Delta_x + \mu_x \cdot \left(\frac{d\beta_{ref}(t_{j+1})}{dt} \right)^\lambda \right] \quad (16)$$

In the above two equations: σ and μ represent the modulators and parameter controlling the proportion of corresponding reference output derivative values β_{ref} for both excitation increasing and decreasing

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situations; Δ is parameter added to leverage the modulators horizontally; λ is an integer to control exponential of the referenced derivative values and the existence of Δ ; the subscript X depicts the condition of excitation derivative, subscripts would be p or n if $X \geq 0$ or $X < 0$.

5.2. Comparisons between theoretical HP model and graphical model

The Matlab based source code of the mathematic model of HP memristor is given in [21] and the generated data serve as the reference. The generated pinched hysteresis loop is considered as the theoretical HP memristor pinched hysteresis loop. The results of the HP model can be used for determining the essential parameters of the proposed graphical approach. A table containing the parameter settings is presented in Table 1.

Table 1 Parameter settings for HP theoretical memristor model

HP Theoretical Model		Graphical Model	
item	value	item	value
$\alpha(t)$	$v_0 \sin(\omega_0 t)$	y_G	-0.387
R_{ON}	100Ω	ε	6×10^{-5}
R_{OFF}	$1.6 \times 10^4 \Omega$	μ_p	0.139
R_{init}	$1.6 \times 10^4 \Omega$	Δ_p	1.92×10^{-4}
α	$1 \times \sin(2\pi \cdot t)$	μ_n	0.1
D	10nm	Δ_n	9.28×10^{-5}
μ_V	1×10^{-14}	λ	2

In Table I, for the HP memristor model, R_{ON} and R_{OFF} are the largest and smallest resistance according to boundary condition; R_{init} is the initial memristance value. There are 7 items in the graphical modelling schema settings: y_G indicates the loop direction yet defines frequency related parameter; amplification ε is assigned to adapt the scale while different memristor models; μ_p , Δ_p , μ_n , Δ_n as well as λ are the parameters in equations (15) and (16) used to calculate the modulator values.

Pinched hysteresis loops based on the memristor theoretical model and the proposed graphical approach of memristor are plotted in Fig.6a for comparison for an excitation of 1 Hz pure sinusoidal voltage waveform. It can be seen that the pinched hysteresis loops essentially overlap. Fig.6b shows the response current waveforms of the two methods. Again, they have very good agreement because both waveforms essentially overlap. Although the HP model does not need a modulator, such parameter can be reconstructed from the simulation results and compared with the modulator of the graphical method as shown in Fig.6c. The blue line is generated by the equations (15) and (16) while the brown line is reconstructed with the data method using HP theoretical model.

Fig. 6. HP theoretical model related demonstration figures
a The comparison between theoretical HP memristor model and graphical approach
b Response current waveform comparison between HP theoretical model and graphical model
c Modulators comparison of HP theoretical memristor

5.3. Comparisons of experimental data and the graphical approach of HP experimental memristor characteristics

In this set of simulations, a sinusoidal voltage is applied to the HP memristor, the frequency of the excitation voltage waveform is $1/200$ Hz. Sampling frequency for HP experimental simulations is set to 10 kHz which is the same with the previous HP theoretical model.

1) HP experimental model with single pinched hysteresis loop

Considering the actual HP experimental memristor model, the specific parameters of such nano-scale device and graphical model optimized parameters are all presented completely in the following Table 2 according to [2]. Both the setup of the HP memristor and the optimized parameters of graphical model generated by the genetic algorithm are listed on the left and right side of the table.

Table 2 Specifications of graphical model for triple pinched hysteresis loops HP memristor

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HP Experimental Setup		Graphical Model	
item	value	item	value
$\alpha(t)$	$v_0 \cdot \sin(\omega_0 t)$	y_G	8.5×10^{-3}
v_0	1V	ε	6.8×10^{-3}
i_0	10mA	μ_p	6.9×10^{-5}
t_0	10ms	Δp	2.0×10^{-4}
D	10nm	μ_n	7.0×10^{-1}
μ_V	10-10	Δn	3.3×10^{-4}
ROFF/RON	160	λ	2

According to Table 2, α is the excitation equation; ω_0 is angular speed equals to $\pi/100$ rad/s; v_0 is the max value of excitation voltage; i_0 is calculated by v_0/R_{ON} which is the max current through the device; t_0 is shortest time for dopants to drift linearly through the total length of device; D is the total length of the total memristive device; μ_V is the average ion mobility in a uniform electric field.

On the right side of the table, seven parameters optimized by the genetic algorithm are listed which are used in graphical modelling procedure: y_G determines the loop direction and relates to the excitation frequency of the simulation; ε is a unified magnification value of graphical model; μ_p , Δp , μ_n , Δn and λ are the parameters for generating modulators in equations (15) and (16).

By optimizing the parameters of the introduced graphical approach with genetic algorithm, the results for both experimental data and the graphical model are shown as follows including pinched hysteresis loop, current waveforms as well as modulator comparisons.

Fig.7a displays the measured pinched loop of the HP memristor and the result obtained by the graphical method. The corresponding response currents are plotted in Fig.7b. In both cases, the measurements and the graphical results have very good agreements. The modulator variation reconstructed from the practical measurements can be compared with that of the graphical method as shown in Fig.7c. These results confirm that the graphical method provides the appropriate framework to model the pinched hysteresis loop.

Fig. 7. HP experimental single loop based figures
a Comparison between the measured pinched loop of the HP memristor and the result of the graphical method
b Comparison of the response current between HP memristor and the graphical model
c Modulator function of the graphical approach and the corresponding function reconstructed from measurements

2) HP experimental model tested with triple pinched hysteresis loops

For the tests of the HP experimental model with triple pinched hysteresis loops, the parameters are the same as shown in Table 2 except that: $\alpha(t) = \pm v_0 \cdot \sin(\omega_0 t)$, total simulation time is 600 s and $R_{OFF}/R_{ON}=380$.

Parameter settings of graphical model for triple loops HP experimental memristor are presented in Table 3. Note that there are six half cycles of the excitation voltage waveforms enumerated from 1 to 6 for reference [2].

Table 3 Specifications of Graphical Model for Triple Pinched Hysteresis Loops HP Memristor

Half-cycle	1&6	2&5	3&4
y_G	8.5×10^{-3}	8.5×10^{-3}	8.5×10^{-3}
ε	2.72×10^{-3}	3×10^{-3}	4.3×10^{-3}

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μ_p	1×10^{-6}	8×10^{-5}	1×10^{-6}
Δp	1.7×10^{-4}	1.4×10^{-4}	3.5×10^{-4}
μ_n	5.5×10^{-2}	3×10^{-5}	1×10^{-6}
Δn	-3.3×10^{-4}	-4×10^{-5}	1.6×10^{-4}
λ	2	2	2

The excitation frequency value throughout the HP experimental memristor simulation study is unchanged. y_G is selected to be 8.5×10^{-3} which is the same as the single pinched hysteresis loops case. The amplification parameters ε are different according to the accumulation of charge in the device. While the next 4 parameters are used to calculate the graphical model modulator values, it should be noted that the parameter λ is 2.

The practical measurements of the HP device (captured from [2]) and the simulation results are plotted in Fig.8a. The corresponding response current waveforms are shown in Fig.8b. These results confirm that the graphical approach can be used to model the self-crossing pinched hysteresis loops of a memristor. The modulator function of the graphical method and the reconstructed one from the measurements are displayed in Fig.8c. The modulator function of the graphical approach is a theoretical concept with the discontinuity issue.

Fig. 8. HP triple loops experimental case based figures

- a Comparison between the measured pinched loop of the HP memristor and the result of the graphical method (triple pinched hysteresis loops)
- b Response current comparison between graphical approach and HP experimental memristor with triple pinched hysteresis loops.
- c Modulator comparison graph of HP experimental memristor triple pinched hysteresis loops

6. Conclusion

This paper presents a new graphical method for modelling pinched hysteresis loops exhibited by memristors. The shape of the loops can be modified with the aids of a modulator and a compensator. The frequency-dependent aspects of the loops can be modified with the values of a parameter y_G (for CCW loops) and x_H (for CW loops). The algorithms and procedures of the proposed method have been illustrated. The results show that the proposed graphical method can be used to generate pinched hysteresis loops that exhibit the three fingerprints of a memristor. The HP memristor model is used as reference to evaluate the graphical model performance.

7. Appendices

Compensator is used to keep the hysteresis loop pinched at the origin during the decreasing excitation conditions. Let the nadir point be p_0 . The final response value at excitation increasing situation can be represented with $\beta\left(p_0 + \frac{N}{2}\right)$, where N is the total sampling number in one excitation cycle period. The next response value should be $\beta\left(p_0 + \frac{N}{2} + 1\right)$ based on decreasing excitation condition.

In order to assure that the hysteresis loop should be pinched at the origin, the response value $\beta\left(p_0 + \frac{3N}{4}\right)$ needs to be 0 at the excitation value: $\alpha\left(p_0 + \frac{3N}{4}\right)$.

According to (13) and (15) for decreasing excitation conditions, the following iterative equations related to $\beta\left(p_0 + \frac{3N}{4}\right)$ can be developed:

$$\begin{aligned}\beta\left(p_0 + \frac{3N}{4}\right) &= \left[\beta\left(p_0 + \frac{3N}{4} - 1\right) - C \cdot \Delta S_{p_0 + \frac{3N}{4}} \right] \cdot \sigma_n\left(p_0 + \frac{3N}{4}\right) \\ \beta\left(p_0 + \frac{3N}{4}\right) &= \left[\beta\left(p_0 + \frac{3N}{4} - 2\right) - C \cdot \Delta S_{p_0 + \frac{3N}{4} - 1} \right] \cdot \sigma_n\left(p_0 + \frac{3N}{4} - 1\right) \cdot \sigma_n\left(p_0 + \frac{3N}{4}\right) - C \cdot \Delta S_{p_0 + \frac{3N}{4} - 1} \cdot \sigma_n\left(p_0 + \frac{3N}{4}\right)\end{aligned}$$

Based on the iterative equations stated above, the relationship between $\beta\left(p_0 + \frac{3N}{4}\right)$ and $\beta\left(p_0 + \frac{N}{2}\right)$ is presented as follows:

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$$\beta\left(p_0 + \frac{3N}{4}\right) = \beta\left(p_0 + \frac{N}{2}\right) - C \cdot \Delta S_{p_0 + \frac{3N}{4}} \cdot \sigma_n\left(p_0 + \frac{3N}{4}\right) \cdots - C \cdot \Delta S_{p_0 + \frac{N}{2} + 1} \cdot \prod_{k=p_0 + \frac{N}{2} + 1}^{p_0 + \frac{3N}{4}} \sigma_n(k) - C \cdot \Delta S_{p_0 + \frac{N}{2}} \cdot \prod_{k=p_0 + \frac{N}{2}}^{p_0 + \frac{3N}{4}} \sigma_n(k) \quad (19)$$

Based on the definition of pinched hysteresis loop of memristor, the response value $\beta\left(p_0 + \frac{3N}{4}\right) = 0$ is required for pinching at the origin, that's the reason why the compensation parameter C is introduced in the graphical approach. Equation (19) could be derived as:

$$\beta\left(p_0 + \frac{N}{2}\right) = C \cdot \Delta S_{p_0 + \frac{3N}{4}} \cdot \sigma_n + C \cdot \Delta S_{p_0 + \frac{3N}{4} - 1} \cdot \prod_{k=p_0 + \frac{3N}{4} - 1}^{p_0 + \frac{3N}{4}} \sigma_n(k) + \cdots + C \cdot \Delta S_{p_0 + \frac{N}{2} + 1} \cdot \prod_{k=p_0 + \frac{N}{2} + 1}^{p_0 + \frac{3N}{4}} \sigma_n(k) + C \cdot \Delta S_{p_0 + \frac{N}{2}} \cdot \prod_{k=p_0 + \frac{N}{2}}^{p_0 + \frac{3N}{4}} \sigma_n(k) \quad (20)$$

Now, a symbol Σ_C is introduced in order to calculate the compensator C value. Σ_C is presented in equation (21).

$$\Sigma_C = \Delta S_{p_0 + \frac{3N}{4}} \cdot \sigma_n\left(p_0 + \frac{3N}{4}\right) + \Delta S_{p_0 + \frac{3N}{4} - 1} \cdot \prod_{k=p_0 + \frac{3N}{4} - 1}^{p_0 + \frac{3N}{4}} \sigma_n(k) + \cdots + \Delta S_{p_0 + \frac{N}{2} + 1} \cdot \prod_{k=p_0 + \frac{N}{2} + 1}^{p_0 + \frac{3N}{4}} \sigma_n(k) + \Delta S_{p_0 + \frac{N}{2}} \cdot \prod_{k=p_0 + \frac{N}{2}}^{p_0 + \frac{3N}{4}} \sigma_n(k) \quad (21)$$

From (20) and (21)

$$C = \frac{\beta\left(p_0 + \frac{N}{2}\right)}{\Sigma_C}$$

(22)

where $\beta\left(p_0 + \frac{N}{2}\right)$ is the final value generated in the increasing excitation scenarios.

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