3D DNS of electromagnetically driven multi-scale shallow layer flows: numerical modeling and physical properties

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Abstract

Three-dimensional direct numerical simulations of a flow driven by multi-scale electromagnetic forcing is performed, in order to reproduce with maximum accuracy the quasi-2D flow generated by the same multi-scale forcing in the laboratory. The method presented is based on a 3D description of the flow and the electromagnetic forcing. Very good agreements between our simulations and the experiments are found both on velocity and acceleration field, this last comparison being, to our knowledge, done for the first time. Such agreement requires that both experiments and simulations are carefully performed and, more importantly, that the underlying simplification to model the experiments and the multi-scale electromagnetic forcing do not introduce significant errors. The results presented in the paper differ significantly from previous 2D DNS in which a classical linear Rayleigh friction modeling term was used to mimic the effect of the wall-normal friction. Indeed, purely 2D simulations are found to underestimate the Reynolds number and, due to the dominance of non-homogeneous bottom friction, lead to the wrong physical mechanism. For the range of conditions presented in the paper, the Reynolds number, defined by the ratio between acceleration and viscous terms, remain of the order of the unity, and the Hartmann number, defined by the ratio between electromagnetic forces terms and viscous terms is about 2. The main conclusion is that 3D simulations are required to model the (3D) electromagnetic forces and the wall-normal shear. Indeed, even if the flow is *quasi-2D* in terms of energy, a full 3D approach is required to simulate these shallow layer flows driven by multi-scale electromagnetic forcing. In the range of forcing intensity investigated in the paper, these multi-scale flows remain quasi-2D, with negligible energy in the wall-normal velocity component. It is also shown that the driving terms are the electromagnetic forcing and the pressure gradient which are mostly anti-aligned but not strictly opposite, and their intensity differs locally. Indeed, no term in the Navier-stokes equation can be neglected.

1 Introduction

Mixing is a crucial process in a broad range of flows, ranging from geophysical flows to micro-flows. Large-scale examples are combustors and chemical reactors in which controlled mixing is

Figure 1: Visualisation of passive scalar markers, using 3D DNS results for $I=0.3A$. Visualisations of the whole flow field (a) and of the bottom left quarter of the flow (b).

required to improve efficiency. For large and intermediate-scale apparatus, the Reynolds number is high enough for the flow to develop natural turbulence, which is considered as a good mixer. As the scale reduces, so does the Reynolds number. For micro-applications (lab on a chip [1], bioassays and stem-cell research for instance), the Reynolds number is of the order of unity and the flow remains laminar. Hence, one of various strategies is to enhance mixing by mimicking the behaviour of large-scale mixers, i.e. trying to recover a *pseudo-turbulent state*, while minimizing the energy input.

Recent works by the authors have addressed some of those challenges by generating experimentally a new class of multi-scale laminar flow controlled by electromagnetic forcing. The flow is generated by a combination of an imposed external electrical current and a multi-scale distribution of pair of magnets. This generates a Lorentz body force which locally pumps (over each magnet in opposite directions) a solution of salted water of low conductivity. Overall, the flow has a zero-mean velocity. The resulting multi-scale structure is illustrated, by means of passive scalar visualisations, on Fig. 1. The specific arrangement of the magnet leads to cat's eyes within cat's eyes pattern, a fundamental representation of 2D turbulent flow fields. The prescribed electrical current and the absence of cross-flow make this flow very different from other MHD configurations, for which the presence of a magnetic obstacle in a conductive fluid generates velocity fluctuations (e.g. [2]).

The particular arrangement of the magnets leads to interesting turbulent-like properties. The flow exhibits a power-law energy spectrum $E(k) \sim k^{-p}$, with the exponent $p = 2.5$ prescribed by this arrangement [3]. This value of p is different from 2D turbulent flows forced at small scales ($p = 5/3$) or large-scale ($p = 3$) [4]. The flow also exhibits a Richardson-like diffusion [5, 6], driven by the multi-scale distribution of hyperbolic stagnation points, with the mean square separation of pairs of fluid element evolving like $\langle \Delta^2(t) \rangle \sim t^3$ once all scales are distinct and ordered [7].

Indeed, theoretical results relating pairs of fluid element separation to mixing rate can be found in literature [8]. Finally, a close relationship is found between Eulerian and Lagrangian statistics, which abides with Corrsin's estimation of the Lagrangian integral time scale, T_L , as proportional to the Eulerian integral time scale, $T_E = L_E/u_{\text{rms}}$, even though Corrsin's approach was originally constructed for high Reynolds number turbulence [9].

Even if numerous experimental studies of 2D turbulence based on electromagnetic forcing of a shallow layer of brine can be found in literature (e.g. [4, 10]), numerical simulations using a detailed model of electromagnetic forcing close to the experimental conditions are still rare [11]. 2D Direct Numerical Simulations (DNS) with a multi-scale distribution of blocks of forces [12, 13] have shown similar multi-scale (or fractal) properties, for instance a power law shaped energy spectrum with the same exponent than that of experiments and a control of this exponent via the fractal dimension of the forcing. But those 2D simulations are not fully satisfying as they present significant discrepancies with experimental flow fields, and require important adjustments with the use of a Rayleigh friction term to model the shallow layer and the friction of the bottom wall.

To simulate in details the laboratory experiments, the electromagnetic forces are introduced in a 3D DNS code using a 3D analytical model based on the 3D geometrical distribution of the permanent magnets. Three levels of validation/comparison with experiments are presented. The velocity field is first compared with PIV measurements [3]. Secondly, and to our knowledge for the first time, the acceleration field deduced from simulations is compared with PTVA measurements by [14]. Finally, both numerical and experimental energy spectra and separation of pairs of fluid element are compared.

A recurrent problem when trying to simulate shallow layer flow experiments is to quantify the importance of the third wall-normal direction compared to the other two. A classic approximation of thin-layer, shallow water and geophysical flows [15, 16, 17] is to introduce a damping term, for instance a Rayleigh friction term. This traditional approach has been followed in the past [12, 13]. By analysing the results from full 3D simulations, one of the objective of this paper is to quantify the importance of the third component, and also to validate or invalidate the use of Rayleigh term in 2D approaches.

Finally, we try to elucidate the origin of the motion and the balance of the physical forces in this type of electromagnetically forced flows.

2 numerical method

2.1 Equations

The equation of motion considered are the classical incompressible forced Navier-Stokes equations

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \nabla) \cdot \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \Delta \mathbf{u} + \frac{1}{\rho} \mathbf{f} \quad (2)$$

where $\mathbf{u}(x, y, z, t)$ is the velocity vector, ρ the constant density, $p(x, y, z, t)$ the pressure field, ν the kinematic viscosity and $\mathbf{f}(x, y, z)$ the electromagnetic forcing term with $\mathbf{f} = \mathbf{j} \times \mathbf{B}$ where $\mathbf{j}$

Figure 2: Modeling of the electromagnetic mixer: (a) 2D schematic of the experimental rig. (b) 3D view of the experimental rig and numerical domain. The brine supporting wall is in black, the 3D numerical domain is in orange. (c) top view of the magnets distribution underneath the wall.

is the electrical current density and $\mathbf{B}$ the magnetic field. More details on the modelling of this term are given in the next section.

2.2 Electromagnetic forcing

$\mathbf{j}$ and $\mathbf{B}$ are given by the induction (3) and Ohm's law (4) equations:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{\mu\sigma} \nabla^2 \mathbf{B} \rightsquigarrow \nabla^2 \mathbf{B} = 0 \quad (3)$$

$$\mathbf{j} = \sigma(\mathbf{u} \times \mathbf{B} + \mathbf{E}) \rightsquigarrow \mathbf{j} = \sigma \mathbf{E} \quad (4)$$

where μ is the magnetic permeability, σ the electrical conductivity and $\mathbf{E}$ the imposed external electric field. In the case of our brine flow the conductivity is low and the velocity is moderate. Consequently, the magnetic Reynolds number is small compared to 1 with $Re_m = \sigma \mu u l < 10^{-7}$. In case of steady permanent magnets ($\frac{\partial \mathbf{B}}{\partial t}$) the magnetic field can then be approximated as purely diffusive: $\nabla^2 \mathbf{B} = 0$. In addition, the induced electrical current $\sigma(\mathbf{u} \times \mathbf{B})$ can be neglected as $\frac{uB}{E} \sim 10^{-3}$. The electric density is then imposed by the external electric field, $\mathbf{j} = \sigma \mathbf{E}$, and the electromagnetic forces are then considered as independent of the flow. More details about MHD flows can be found in [18, 19].

The horizontal distribution of the permanent magnets is given by Fig. 2(c). The distribution of the magnetic charge density above the square surfaces, which are used to model the permanent magnets, is considered to be uniform. The reference induction value of these surfaces is set to the experimental value of $\pm B_r/2$ with $B_r = 0.68T$ [3], and is the same for all magnets. The magnetic field $\mathbf{B}$ induced by the 3D distribution of permanent magnets is computed using $\mathbf{B}(x, y, z) = \sum \mathbf{B}_s(x, y, z)$ where $\mathbf{B}_s(x, y, z)$ is the magnetic field generated by each magnetic surface shown on Fig. 3 and which characteristics are given in Table 1. The magnetic field is

Figure 3: Representation of the coordinates and dimensions use to compute the magnetic field of each magnetic surface given in Table 1. O is the center of the surface, y the axis normal to the surface, x and z axes are parallel to the surface edges.

computed using the analytical solution given by Akoun and Yonnet [20, 21]:

$$\begin{aligned}
B_{s_x}(x, y, z) &= \frac{Br}{4\pi} \sum_{(i,j)=(0,1)^2} (-1)^{i+j} \ln(R_{i,j} - S_i) \\
B_{s_y}(x, y, z) &= \frac{Br}{4\pi} \sum_{(i,j)=(0,1)^2} (-1)^{i+j} \arctan\left(\frac{S_i T_j}{y R_{i,j}}\right) \\
B_{s_z}(x, y, z) &= \frac{Br}{4\pi} \sum_{(i,j)=(0,1)^2} (-1)^{i+j} \ln(R_{i,j} - T_j) \\
S_i(z) &= z - (-1)^i \frac{l_M}{2} \\
T_j(x) &= x - (-1)^j \frac{l_M}{2} \\
R_{i,j}(x, y, z) &= \sqrt{S_i^2 + T_j^2 + y^2}
\end{aligned}$$

The electrical current density $\mathbf{j}(x, y, z)$ is assumed uniform in the shallow layer of the brine. It follows that $\mathbf{j}(x, y, z) = j_x \mathbf{x}$, with $j_x = I/S_{brine}$, I is the electrical current crossing the brine and S_{brine} is the area of the brine cross section.

The forcing term $\mathbf{f}$ used in Eq. 7 is then simply obtained by $\mathbf{f} = \mathbf{j} \times \mathbf{B}$. Because the electrical field is aligned with the x direction, the force f_x is zero. The other two components of the 3D electromagnetic field are f_y , perpendicular to the wall and f_z , parallel to the wall.

Fig. 4 gives different views of the electromagnetic force field $\mathbf{f} = \mathbf{j} \times \mathbf{B}$ used in the 3D simulations. North and South magnetic poles correspond respectively to positive and negative values of f_z . Due to the specific design of the electromagnetic forcing, f_z does not change sign between pairs in the vicinity of the magnets, as it would do in the case of an isolated magnet or for magnets with a small height compared to the magnet horizontal length-scale l_M .

Iso-surfaces $\pm \max(|\mathbf{f}|)/3$ (Fig. 4a,b) indicate the positions where the forces are significant: the maximum values for f_z are reached above the magnets, and the maximum of f_y above the edges of the magnets. The wall-parallel component f_z is also significantly stronger than f_y in the major part of the domain. Nevertheless, the existence of the f_y component, even if local and of weaker mean intensity, support the use of a 3D approach and a detailed 3D model of the electromagnetic forcing.

In addition, 2D slices of f_z are shown on Fig. 4(c, d, e), at different x -positions. The main observation is that the force decreases significantly with the distance from the wall, and this

x	y	z	l_M	sign	x	y	z	l_M	sign
0	-40	160	160	+	0	-160	160	160	-
0	-40	-160	160	-	0	-160	-160	160	+
200	-11.2	200	40	+	200	-91.2	200	40	-
200	-11.2	120	40	-	200	-91.2	120	40	+
200	-11.2	-120	40	+	200	-91.2	-120	40	-
200	-11.2	-200	40	-	200	-91.2	-200	40	+
-200	-11.2	200	40	+	-200	-91.2	200	40	-
-200	-11.2	120	40	-	-200	-91.2	120	40	+
-200	-11.2	-120	40	+	-200	-91.2	-120	40	-
-200	-11.2	-200	40	-	-200	-91.2	-200	40	+
250	-1	210	10	+	250	-21	210	10	-
250	-1	190	10	-	250	-21	190	10	+
250	-1	-190	10	+	250	-21	-190	10	-
250	-1	-210	10	-	250	-21	-210	10	+
-250	-1	210	10	+	-250	-21	210	10	-
-250	-1	190	10	-	-250	-21	190	10	+
-250	-1	-190	10	+	-250	-21	-190	10	-
-250	-1	-210	10	-	-250	-21	-210	10	+
150	-1	130	10	+	150	-21	130	10	-
150	-1	110	10	-	150	-21	110	10	+
150	-1	-110	10	+	150	-21	-110	10	-
150	-1	-130	10	-	150	-21	-130	10	+
-150	-1	130	10	+	-150	-21	130	10	-
-150	-1	110	10	-	-150	-21	110	10	+
-150	-1	-110	10	+	-150	-21	-110	10	-
-150	-1	-130	10	-	-150	-21	-130	10	+

Table 1: 3D positions, length and polarity (+ for North, - for South) of the square surfaces used to model the magnetic field generated by the permanent magnets. x , y , z and the size of the magnets l_M are in mm.

Figure 4: 3D distribution of the force used in the computations for (a) f_z and (b) f_y . 3D iso-surfaces correspond to $\pm \max(|\mathbf{f}|)/3$. 2D slices of f_z for (c) $x = 0mm$, (d) $x = 200mm$ and (e) $x = 250mm$. The same color map used for all graphs (in $(kg.m^{-3})(m.s^{-2})$). Values for an electrical current of 1A crossing a brine of height $H = 5mm$

decay is faster for the smaller magnets. In fact, the forces decrease exponentially with $\frac{y-y_M}{l_M}$ where $y - y_M$ is the distance to the magnets and l_M its size. In the experimental setup [3], the distances between the brine and the permanent magnets have been carefully adjusted. This adjustment gives close y -averaged intensities of the electromagnetic forces above each permanent magnets. Consequently, the intensity of the electromagnetic forces is larger above the small magnets close to the wall than above bigger magnets to compensate for the faster decrease of the forces with y above the small magnets. This later observation also supports the use of a 3D, rather than a 2D, approach.

2.3 Numerical methods and flow parameters

The objective of these simulations is to reproduce the experiments of [3, 14]. Hence, the experimental values of the different quantities are used to non-dimensionalise the forcing field. The fluid is a solution of salted water, with an electrical conductivity $\sigma = 16.6 S.m^{-1}$ ¹, a kinematic viscosity of $\nu = 1.36 \times 10^{-6} m^2.s^{-1}$ and density of $\rho = 1104.87 kg.m^{-3}$, in a square tank of 1700mm with a brine thickness of $H = 5mm$. The arrangement of magnet is shown on Fig. 2(c). Three different sizes of square magnets, l_M , are used : $160mm = 32H$, $40mm = 8H$ and $10mm = 2H$.

All quantities are non-dimensionalised using the brine thickness H and the mean forcing velocity defined by

$$U_{\text{ref}} = \frac{1}{\nu\rho} f_{\text{rms}} H^2 \quad (5)$$

¹In the experiments, σ is controlled by the concentration of salt in water ($158g/l$ with $\partial\sigma/\partial C \cong 0.066 S.m^{-1}/g.l^{-1}$) around this concentration. σ is not supposed to vary with the electrical current density in these brine flows.

where f_{rms} is the root mean square (RMS) value of the forcing computed in the whole (3D) computational domain. The Reynolds number used in the simulations is then simply obtained by $Re = U_{\text{ref}}H/\nu$. The non-dimensional Navier-Stokes equations are then written as

$$\nabla \cdot \mathbf{u}^+ = 0 \quad (6)$$

$$\frac{\partial \mathbf{u}^+}{\partial t^+} + (\mathbf{u}^+ \cdot \nabla) \mathbf{u}^+ = -\nabla p^+ + \frac{1}{Re} \Delta \mathbf{u}^+ + \mathbf{f}^+ \quad (7)$$

with $\mathbf{f}^+$ being the non-dimensional forcing field defined by

$$\mathbf{f}^+ = \frac{H}{U_{\text{ref}}^2} \frac{1}{\rho} \mathbf{f} \quad (8)$$

In order to facilitate the comparison with experimental results, a second non-dimensionalisation is used : quantities indicated with a $*$ are non-dimensionalised using the RMS value computed in planes at constant y , e.g. $\mathbf{u}^* = \mathbf{u}/U_{\text{rms}}$ where U_{rms} is the velocity RMS in the experimental domain $(800 \times 800) \text{ mm}^2$ [3].

Eqs. 6-7 are solved using an in-house code based on sixth-order compact schemes for spatial discretisation and on third-order low-storage Runge-Kutta scheme for time integration. The incompressible Navier-Stokes equations are solved on a Cartesian frame of reference $(0; x; y; z)$. O is the center of the numerical domain, x the direction of the electrical current and y the wall-normal direction. The skew-symmetric form of the convection term $(\frac{1}{2}\partial(u_i u_j)/\partial x_j + \frac{1}{2}u_j \partial u_i/\partial x_j)$ is used to minimise the aliasing error associated with the discretisation using high-order finite difference schemes [22]. An original characteristic of the code is that the Poisson equation is solved in the spectral space by a direct method via the modified spectral formalism [23].

In order to simulate the experiments as realistically as possible, boundary conditions have to be carefully prescribed, with the other constraint of keeping a relatively low computational cost. Except stated otherwise, the size of the computational domain is kept constant, equal to $(L_x, L_y, L_z) = (240H, H, 240H)$ in the x , y and z directions respectively, and it is discretised on a mesh $(n_x, n_y, n_z) = (300, 17, 300)$. The spatial extent of the numerical domain, illustrated in orange on Fig. 2(b), is chosen to be slightly smaller than the experimental setting, mainly to reduce the cost of the simulations. Nevertheless, a-posteriori checking showed that the flow is almost zero close to the edge of the computational domain.

In all simulations presented, free-slip conditions are imposed in the x and z directions ($x = \pm L_x/2, z = \pm L_z/2$) and for $y = H$, while a no-slip condition is used for the bottom wall ($y = 0$). Series of test have been performed to ensure that simplifications, boundary conditions and discretisations issues have limited effects on the flow field.

First, the effect of imposing free-slip conditions in the x and z directions has been tested by imposing no-slip conditions. No difference, either on statistics or on instantaneous flow field have been observed.

A grid independence study, on a quarter of the full domain, has been carried out for spatial resolution ranging from $(n_x, n_y, n_z) = (150, 17, 150)$ to $(n_x, n_y, n_z) = (600, 17, 600)$. Again, no significant differences have been observed on velocity and acceleration fields, even when the mesh was refined by a factor four. However, it should be noted that the current resolution of

I(A)	$f_{\text{rms}}(m.s^{-2})$	$U_{\text{ref}}(m.s^{-1})$	Re_{dns}	Fr	$1/r_{\nu f}$	r_{Re}
0.06	2.54×10^{-4}	0.00479	18.1	0.0068	1.95	0.24
0.1	4.24×10^{-4}	0.00799	30.1	0.0114	2.12	0.37
0.2	8.47×10^{-4}	0.01598	60.3	0.0228	2.01	0.59
0.3	1.27×10^{-3}	0.02397	90.4	0.0342	2.05	0.77
0.4	1.69×10^{-3}	0.03196	120.5	0.0456	2.08	0.92
0.53	2.25×10^{-3}	0.04234	159.7	0.0605	2.11	1.08
0.7	2.93×10^{-3}	0.05593	210.9	0.0798	2.13	1.24
1	4.24×10^{-3}	0.07990	301.3	0.1140	2.16	1.48

Table 2: Main parameters used for DNS, base on experimental conditions of [3]. f_{rms} is computed on the 3D numerical mesh. $1/r_{\nu f}$ is equivalent to an Hartmann number and r_{Re} is equivalent to a Reynolds number, even if they slightly differ by construction: $r_{\nu f} = (\nu \Delta \mathbf{u})_{\text{rms}} / \frac{1}{\rho} f_{\text{rms}}$ and $r_{Re} = \mathbf{a}_{\text{rms}} / (\nu \Delta \mathbf{u})_{\text{rms}}$, for more details see section 4.2.2.

$(n_x, n_y, n_z) = (300, 17, 300)$ points on the full domain is the minimum one to simulate accurately the multi-scale flow, especially around the smallest magnets, which are horizontally discretised, at this low resolution, by only 3 to 4 points in the x and z directions. A comparison of the flow field between the lowest and the highest resolution shows that the dynamic is correctly reproduced, even for the smallest scales, with the coarsest grid. This good performance is attributed to the high-order spatial discretisation schemes used in the code. A similar dependance study has been performed to determine the largest time step. In all simulations, it will be equal to $\Delta t = 0.005H/U_{\text{ref}}$. Another important parameter to consider for comparisons with experimental results is that the horizontal resolution used in our simulation (mesh area $16mm^2$) is finer, by almost a factor 2, than that used for the PIV (correlation window's area about $38mm^2$).

The last boundary condition for which assumptions have been made is the free-slip condition at $y = H$, which is a free surface in the experiments. By imposing a free-slip condition, the assumptions are that the flow is not deforming near the free surface and that the air drag at the free surface is negligible. These assumptions are partly justified by the fact that the Froude number $Fr = U_{\text{ref}}/\sqrt{gH}$ is very low in our case (ranging from $Fr = 0.06$ to $Fr = 0.11$, cf. Table 2). The physical implication will be discussed in the next section. The Stuart (also named interaction parameter, St_{dns}) and Hartmann (Ha_{dns}^2) numbers used in the DNS are: $St_{dns} = 1$ and $Ha_{dns}^2 = Re_{dns}$.

3 Comparison with experiments

Eight different simulations, for 8 forcing intensities, have been performed using the magnet distribution shown on Table 1 [3, 7]. Forcing intensity and other non-dimensional parameters for each of these simulations are given in Table 2. In the following, we will refer to the value of the electrical current intensity in the brine to indicate the forcing intensity. Experimental data used in this section are based on PIV for the velocity fields and PTVA for the acceleration fields [3, 14]. More details about experimental techniques can be found in those papers, and are not reported herein.

3.1 Velocity field

Figs. 5, 6 and 7 show the norm of the velocity $||\mathbf{u}^*||$ for experiment and simulation, respectively, for the same current intensity $I = 0.3A$ and using the same color scale to highlight potential differences. The multi-scale pattern corresponds to the three scales of forcing and the distribution of permanent magnets of Fig. 2(c). The flow is pumped/forced in opposite direction above the pairs of magnets so as to produce two colliding jets. With the variation of the magnets' sizes, this generates a multi-scale pattern of colliding jets and corresponding stagnation points. The same magnitudes of velocity intensity are observed at each scales (i.e. above small, medium and large magnets) for both experiments and numerical simulations. Figs. 5,6(b) are zooms of the flow field on the bottom left quarter. The streamlines show very similar geometry between experiments and numerical simulations.

The general agreement between the PIV results of [3] and our 3D DNS is striking and shows a significant improvement when compared to previous 2D simulations [24, 13]. Those 2D DNS simulations presented significant deformations of the streamline geometry above the small scales and significant differences in the distribution of the velocity intensity. Furthermore, the forcing intensity and additional terms in Navier-Stokes equations needed to be adjusted to obtain the same topology over a limited range of forcing intensities (factor four between the smallest and largest intensity values). The current 3D DNS simulations reproduce the experiments [3] with the same topology, very close flow geometries and intensities, without any adjustment of the model, and for a large range of intensities (from $I = 0.06A$ to $I = 1A$, or a factor 16).

Figure 5: Velocity field from PIV measurements, [3], for $I=0.3A$. The color scale presents the velocity intensity $||\mathbf{u}^*||$. Streamlines are plotted with black lines, arrows indicate the flow direction. Fig. (b) is a zoom of Fig. (a).

The topological similarities are found for the whole range of forcing intensities. Fig. 8(a) shows the evolution of maximum and RMS values of the velocity field with the intensity, measured experimentally at the free surface (symbols) and for two different heights ($y^+ = 1$ and $y^+ = 0.5$) for the 3D DNS (lines). Given the uncertainty of some experimental parameters (e.g. ν , I , 3D positions of the magnets and average thickness of the brine), the results remain within 10% of error for U_{rms} and 15% for U_{max} at the upper surface. The error is even lower if we consider a horizontal

Figure 6: Velocity field from 3D DNS at $y^+ = 0.5$, for $I=0.3A$. The color scale presents the velocity intensity $\|\mathbf{u}^*\|$. Streamlines are plotted with black lines, arrows indicate the flow direction. Fig. (b) is a zoom of Fig. (a).

Figure 7: Velocity field from 3D DNS at $y^+ = 1$, for $I=0.3A$. The color scale presents the velocity intensity $\|\mathbf{u}^*\|$. Streamlines are plotted with black lines. Fig. (b) is a zoom of Fig. (a).

plane located not at the free-slip boundary condition but in the middle of the brine. Fig. 8(b) shows the evolution of those two parameters in the wall-normal direction, for 3 different forcing intensities. In the upper half of the brine, computed values remain close to the experimental one, showing a quasi-homogeneous evolution of the flow far from the wall. The exact origin of this difference is difficult to isolate. The use of a free-slip condition at $y^+ = 1$ instead of a free surface could explain, for instance, the 10% error on the RMS values.

The integral length scales L_E , defined by $L_E = \frac{1}{U_{rms}^2} \int |\langle \mathbf{u}(\mathbf{x}) \cdot \mathbf{u}(\mathbf{x} + \mathbf{r}) \rangle| d\mathbf{r}$, with $\langle \cdot \rangle$ representing a spatial averaging over the entire domain, are shown on Fig. 8(c) for all cases, for both experiments and simulations. The differences between the two approaches are very low, of the order of 1%, and, for both datasets, there is a slight increase of the integral length-scale with the intensity of the forcing.

This good agreement between PIV and 3D DNS velocity fields is further illustrated on Fig. 9

Figure 8: (a) Maximum and RMS velocities, in mm/s . DNS results (lines) are given at two different heights: $y^+ = 1$ and $y^+ = 0.5$, PIV are shown with symbols; (b) Evolution of maximum and RMS values with height for three different forcing intensities; (c) normalised integral length scale, L_E^+ .

by plotting the evolution of the wall-parallel velocity components u^* and w^* at $y^+ = 0.5$, along the axis $z^+ = -26.4$ and $x^+ = -25.8$. These axis cross all forcing scales and flow sections where the momentum generated by the electromagnetic forcing is redistributed within the flow. They correspond respectively to horizontal and vertical dashed lines on Figs. 5 and 6. Both axis pass through regions where the velocity is maximum. Black and orange solid lines represent 3D DNS results, PIV data are shown with symbols and 2D results from [24, 13] are shown with dashed lines. In particular, the velocity component w^* along $z^+ = -26.4$, which crosses all scale of forcing (e.g. Fig. 5) shows a distribution of local maxima and minima surrounded by sharp gradients of velocity, which is correctly reproduced in the 3D simulations. More generally, 3D DNS and experimental profiles are superimposed, except for some particular locations.

Overall, the agreement between 3D DNS and experimental data is very good. It is extremely encouraging to find the same “multi-scale structure” of the velocity profiles along these axis. Tiny differences can be noted, for instance on the x^+ -axis, where the w^* component is slightly overestimated at $x^+ = 0$. Another difference is occurring at $x^+ = +40$, where the local maximum of u^* is slightly shifted towards the centre of the domain in the simulation, while the same peak on the other side (configuration is symmetric) is correctly predicted at $x^+ = -40$. Indeed, the discrepancies observed in Fig. 8 and 9 cannot only be attributed to oversimplification made in modelling the flow configuration, but also from small deviation of the experiments (wall not perfectly flat, magnets intensity and position, ...). Moreover, Fig. 9 illustrates the major improvement introduced by using 3D DNS compared to previous 2D DNS [13, 24].

The comparison between simulations and experiments is quantified by two integral quantities:

$$\text{dif} = \left\langle \frac{\|\mathbf{u}_{\text{PIV}}^* - \mathbf{u}_{\text{DNS}}^*\|}{\max(\|\mathbf{u}_{\text{PIV}}^*\|, \|\mathbf{u}_{\text{DNS}}^*\|, 1)} \right\rangle \quad (9)$$

$$\text{cor} = \langle \mathbf{u}_{\text{PIV}}^* \cdot \mathbf{u}_{\text{DNS}}^* \rangle \quad (10)$$

where $\langle \cdot \rangle$ denotes the average over the whole domain, “dif” gives the relative difference when $\|\mathbf{u}\| \geq u_{rms}$ and “cor” the spatial correlation between experimental and numerical results. It should be noticed that the average velocity is zero. PIV results are always taken at the free-surface, while the wall-normal position of the horizontal plane is varied in the DNS data. Fig. 10(a) shows the evolution of difference and correlation with the wall-normal direction for $I = 0.3A$.

Figure 9: u^* and w^* velocity profiles for $y^+ = 0.5$ and $I = 0.3A$, along the axis (a) $x^+ = -25.8$ and (b) $z^+ = -26.4$. 3D DNS results are represented with solid lines and PIV results with symbols. 2D DNS [13, 24] results are in dashed lines.

If experiments and simulations were perfect and if the several assumptions made were correct, we would expect the difference to be minimum and the correlation to be maximum at $y^+ = 1$. However, “dif” reaches its minimum and “cor” its maximum for $y^+ = 0.5$ with values respectively about 13% and 0.978%, whilst the values at the free-surface are equal to 15% and 0.974.

The evolution of the difference and correlations values with the forcing intensity, for $y^+ = 0.5$, is given in Fig. 10(b). The correlation values remain very high and vary between 0.97 and 0.98. These high correlation values are related to the multi-scale flow structure, which is the same for both experiments and simulations. Values of dif at $y^+ = 0.5$ vary between 11% and 15%, however we do not expect these differences to fall under 6%, which corresponds to the noise of the PIV measurements for velocity intensities of the order of u_{rms} . Indeed, for $I < 0.1A$, an increasing part of the flow in the PIV domain has a displacement smaller than 0.4mm (1 pixel) between two frames, which lead to an increase in noise.

The difference between PIV and 3D DNS tends to increase with the intensity of the flow, while, as expected, the correlation decreases. This behaviour is mainly related to the free-surface boundary condition at $y^+ = 1$.

With the increase of the forcing intensity, the comparisons with PIV measurements showed one major difference, at the centre of the domain. As the Reynolds number increases, the shape of the slow motion flow field at the centre of the brine in the simulation is shifting from a perfect circular shape to an oval one, while in experiments, the circular shape is observed for all forcing intensity. This discrepancy is attributed to the constraint of having a zero-tangential velocity gradient in DNS. In the simulations, the surface-normal derivative of the vertical component, $\partial v / \partial y$, reaches its maximum at the symmetry condition. For high forcing intensities, this local maximum start to be locally non negligible compared to the other two components contributing to the continuity equation ($\partial u / \partial x$ and $\partial w / \partial z$), especially around the “large-scale stagnation points”. It leads to a deformation of the isolines of velocity around these points close to $y^+ = 1$ and to an increase of the velocity around the plane $z^+ = 0$. This behaviour is partly compensated in the experiment by an increase of the deformation of the free-surface with the forcing intensity.

However, as already mentioned, the surface is deforming only for high values of Fr (of the order of unity), hence for high value of forcing intensity. Further work, using a deforming free-surface conditions which also includes the viscous effects due to the air/liquid interface [25], is under way for higher forcing intensity. Nevertheless, in the range of the considered forcing, the assumption of a non-deforming surface is mostly valid and/or is discussed case-by-case.

Figure 10: Evolution of dif and cor (a) versus the height y^+ , for $I=0.3A$; (b) versus the intensity of the forcing (I in A) for $y^+ = 0.5$.

The direct comparison of experimental and numerical velocity fields shows that the two approaches gives very similar results. The 3D model of the forcing is satisfactory for low and moderate values of the forcing intensities. As the forcing intensity increases, the discrepancies near the free-surface are becoming more and more important. Hence, a better modeling of the free surface should improve the quality of the model and allow to perform numerical simulations closer to the experiments, even for the highest values of the forcing intensity.

3.2 Energy spectra and pairs of fluid element dispersion

An important property of this 2D multi-scale flow is that the energy globally decays $E(k) \sim k^{-p}$, with the exponent p being between $p = 2$ and $p = 3$, which is clearly different from direct and inverse cascade observed in “classical” high Reynolds number 2D turbulence [3, 4]. In the present study, the energy spectrum is solely controlled by the geometry and the intensity of the forcing. In fact, [13] have shown that varying the geometry (fractal dimension) of the forcing allows to control p in low Reynolds number flows. Normalised energy spectra of 3D DNS for different forcing intensities, along with a typical spectrum extracted from the experimental data [3] are shown on Fig. 11. As expected, numerical and experimental results are relatively close, and even if the energy decay is not constant, its global evolution is not far from $E(k) \sim k^{-2.5}$, as discussed in [3]. Also, the energy spectra do not vary significantly for the range of Reynolds number considered. The only significant difference is found at small scales/high wave numbers, where a small tail is observed in experiments, and is due to the noise of the measurements for length-scales of the order of the size of the correlation windows.

Pair separation statistics are very sensitive to flow conditions, as they depend on all flow-scales and on the time/space integration of their combined stirring. There are very few experiments (e.g. [26]) for which a Richardson dispersion law in turbulent flows ($\langle \Delta^2(t) \rangle \sim t^3$ where $\langle \Delta^2(t) \rangle$ is

Figure 11: Normalised energy spectra. Lines are for numerical simulation: black for 0.2A, orange for 0.4A, red for 0.7A and circles for PIV data. The blue straight-line displays $E(k^+) \sim k^{+ -2.5}$. The vertical dashed lines indicate the wave-length of the magnets.

the mean square separation of pairs at time t) has been measured. Recent experiments [27] have not found such sensitive relation, with statistics staying in the ballistic regime (i.e. $\langle \Delta^2(t) \rangle \sim t^2$).

Statistics of fluid element pair separations $\langle \Delta^2(t) \rangle$ are extracted from the simulations, the average being carried out over more than 1.5×10^6 pairs of fluid elements, and compared with previous experimental results [7]. The trajectories

$$\frac{d}{dt} \mathbf{x}(t, \mathbf{x}_0) = \mathbf{u}(\mathbf{x}(t, \mathbf{x}_0), t) \quad (11)$$

and their statistics are calculated, starting from random initial positions $\mathbf{x}_0$ (nevertheless, the density of initial pairs positions is ensured to be homogeneous for scales larger than the small magnets) and orientation, at a time $t = 0$, i.e. once the flow reaches a steady state. These trajectories are integrated until $t = 4L_E/u_{\max}$ using 2nd-order Lagrange interpolation in space and a 4th-order Runge-Kutta scheme in time. The time step is equal to $\Delta t = \frac{1}{2} \frac{4}{5} H/u_{\max}$ where $\frac{4}{5} H$ is the smallest horizontal mesh size. Pair statistics are performed with initial separations $\Delta_0 = 0.397 \text{ mm}$. This separation corresponds to the experimental value of 1 pixel [7] and is about one tenth of the smallest horizontal mesh size.

Quantities such as the mean square pair separation are sensitive to the choice of Δ_0 , but the turbulent diffusivity $\frac{d}{dt} \langle \Delta^2(t) \rangle$ is usually more robust. It allows a clear identification of the different dispersion regimes, such as the initial ballistic dispersion $\langle \Delta^2(t) \rangle \sim t^2$, the final uncorrelated dispersion $\langle \Delta^2(t) \rangle \sim t$ and a non-trivial Richardson-like regime in an intermediate range of times between the ballistic and the uncorrelated regimes (Fig. 12(a) and [3]).

The mean-square pair separation obeys an approximate power law of the form $\langle \Delta^2 \rangle \sim t^\gamma$ for about one decade in time. The evolution of the exponent γ with the forcing intensity is given in Fig. 12(b). Each exponent is extracted from a best power law fit of $\frac{d}{dt} \langle \Delta^2(t) \rangle$ over the range $0.2 \leq tu_{rms}/L_E \leq 2$, as described in [7]. Typical values of γ are larger than two and lie roughly between 2.4 and 3, in agreement with previous experimental results [7]. It can also be noticed that when the forcing intensity is high ($I > 0.53A$), exponents extracted from DNS data differ significantly from the experiment. This observation of pair separation statistics confirms the limitation of the numerical model for high forcing intensities.

Figure 12: (a) Pairs separation statistics $\langle \Delta^2(t) \rangle$ and $\frac{d}{dt} \langle \Delta^2(t) \rangle$ for $I=0.4A$. (b) values of γ extracted by power law approximation, $\langle \Delta^2(t) \rangle \sim t^\gamma$ (respectively $\frac{d}{dt} \langle \Delta^2(t) \rangle \sim t^{\gamma-1}$) for $0.2 \leq tu_{rms}/L_E \leq 2$. Lines corresponds to 3D DNS results, symbols to experiments [7]

Figure 13: Evolution of v_{rms}/u_{rms} (a) and v_{max}/u_{rms} (b) with the intensity of the forcing and the wall normal position y^+ .

4 Physical properties of these multi-scale flows

3D DNS data are in good agreement, qualitatively and quantitatively, with experiments, allowing us to address some important physical questions concerning this type of multi-scale flow: (i) is it 3D, 2D or quasi-2D? and (ii) what is the balance of the physical forces driving the flow?

4.1 3D versus 2D

To quantify the importance of the velocity component normal to the wall, we compare its RMS, v_{rms} , and maximal intensity, v_{max} in each plane parallel to the wall with the RMS values of the horizontal velocity, u_{rms} in the corresponding planes. Fig. 13 gives v_{rms}/u_{rms} and v_{max}/u_{rms} versus y^+ for the different intensities of the forcing.

v_{rms} is very small compared to u_{rms} , with the ratio varying between 0.35% and 0.84% for the range of forcing intensities considered (from $0.06A$ to $1A$). The mean energy present in the wall normal direction can then be considered as negligible compared to the mean energy present in the horizontal direction. In this sense, these flows can be considered as quasi-2D.

The maximum of wall normal velocity, v_{max} , is also weak compared to u_{rms} , but rises locally from 7.5% for $I=0.06A$ to about 15% for $I=1A$. Fig. 13(a) shows that the plane where v_{rms}

is maximum varies with the intensity of the forcing from $y^+ \approx 0.2$ for $I=0.06A$ to $y^+ \approx 0.45$ for $I=1A$. On the other hand, $v_{\max}$ appears to stay in the same plane, $y^+ \approx 0.3$. This local dependence of the wall-normal velocity is illustrated in Fig. 14 by iso-surfaces of v , plotted at 4% of u_{rms} . It should be noticed that this value of 4% is lower and about the half of the maximum of wall-normal velocity. The 3D streamline, also plotted in Fig. 14, remain parallel to the wall when the horizontal velocity is significant, but present a significant 3D “motion” when the horizontal velocity is close to 0 (i.e. close to the small magnets).

Figure 14: 3D view of DNS result: the wall-normal direction is expanded for a better representation of 3D effects. Iso-surfaces of v plotted correspond to 4% of the RMS of the horizontal velocity, $I=0.1A$

Analysis of the DNS data show that the wall normal velocity is generally weak compared to the horizontal velocity and that its existence at intensity level higher than 4% of u_{rms} is very local, but introduces anisotropy in the wall-normal direction.

4.2 Acceleration and balance of forces

4.2.1 Numerical and experimental acceleration fields

In a recent work, Ferrari and Rossi [14] have proposed a new algorithm to measure experimentally the Lagrangian velocity and acceleration, and then applied it to the same multi-scale flow for $I=0.53A$. Using the assumed Eulerian repeatability of their flow, they have extracted the acceleration field from many realisations. Fig. 15 gives a comparison between experimental and numerical acceleration fields normalised by the RMS of the acceleration (i.e. $a_x^* = a_x/a_{\text{rms}}$, $a_z^* = a_z/a_{\text{rms}}$ and $\|\mathbf{a}^*\| = \|\mathbf{a}\|/a_{\text{rms}}$) using the same color map. Like for the velocity field, the qualitative agreement between DNS and experiments is very good and, to our knowledge, it is the first time that such a comparison of acceleration components is performed.

The distribution of *sinks* and *sources* of acceleration is noticeable and follows the multi-scale pattern of the forcing. The large scale stagnation point (between large colliding jets) corresponds to *sources* of acceleration with $\nabla \cdot \mathbf{a} > 0$ and $\nabla \cdot \mathbf{a}$ locally maximum at this point. Acceleration streamlines are emanating from these points and are connected to *sinks* of acceleration corresponding to elliptical stagnation points (center of eddies), where $\nabla \cdot \mathbf{a} < 0$ and $\nabla \cdot \mathbf{a}$ are locally minimum [14]. Connections between sources and sinks are observed at all scales and design an acceleration web for multi-scale flows. It can also be noticed that the acceleration intensity is

Figure 15: Acceleration field for $I=0.53A$, (a&c) PTVA measurement [14]. (b&d) 3D DNS acceleration field, at $y^+ = 0.5$. Abscissa and ordinate are respectively x^+ and z^+ .

zero above the large magnets, indicating a constant Lagrangian velocity. This also corresponds to portions of the domain where the acceleration is conserved ($\nabla \cdot \mathbf{a} \cong 0$).

Acceleration profiles along four different lines ($x^+ = -25.8$, $z^+ = -26.4$, $x^+ = -37$ and $z^+ = -33.4$, the first two being also used to compare velocity profiles) are shown in Fig. 16. They are represented by dashed lines in Fig. 15 and 17. There is a general good agreement between experimental (PTVA) and numerical (3D DNS) data both for acceleration profiles and for spatial distribution of individual components a_x^* and a_z^* (Fig. 17). The same large and small-scale features are correctly reproduced by the 3D computations, for the two components of acceleration and for the four axis. It is however important to emphasize the extreme sensibility of the profiles to their position in space. Any error or alteration in the positioning of the magnets will then have significant consequences for the direct comparison of the acceleration field. This is clearly illustrated by the peaks of a_z^* for two symmetric position: $z^+ \approx \pm 40$. Experimental and numerical peaks are located at the same position for $z^+ = -40$, albeit the maximum value of the peak, while it is slightly shifted for $z^+ = 40$ (about $z^+ = 0.5$, small compared to the size of the domain). This is within the range of uncertainty of the exact position of the magnets. Another example illustrating the importance of the spatial position, is the presence in Fig. 16(b) of a strong positive peak on the DNS acceleration profile a_z^* close to $x^+ = \pm 30$, which is not observed on the experimental data. Figs. 17(c) and (d) show that the experimental profile axis (first horizontal dashed line from top) is tangent to a zone of high value of a_z^* , whilst the same line crosses it for 3D DNS. Similarly, on the same axis, the simulation underestimates a_z^* for $-40 < x^+ < -30$, which again can be attributed to a slight shift line positioning. Such direct comparison between experimental and 3D DNS acceleration data are thus difficult to perform. This also suggests that our experimental and numerical results are in fact better than the quantitative results have

Figure 16: a_x^* and a_z^* acceleration profiles for (a) $x^+ = -25.8$, (b) $z^+ = -26.4$, (c) $x^+ = -37$ and (d) $z^+ = -33.4$. DNS results are in plotted with solid lines and PIV results with symbols. $I=0.53A$

indicated. Nevertheless, small differences can be observed, not on the positioning, but on the peak values, which are slightly and systematically underestimated in the simulations. These differences could have several origins: the forcing intensity used in experiments to perform the PTVA ($I = 0.53A$) is clearly at the upper limit of the numerical model.

Quantifications of the differences, dif , and the correlation, cor (not shown here) proved again that the minimum difference and maximum correlation are found again in the plane $y^+ = 0.5$, their values being respectively 24% and 0.939. As the uncertainty on the acceleration measurement is higher than that from velocity measurements, and because the acceleration is very sensitive to any variation of experimental parameters (e.g positions of magnets), this indicates a good “second-order” agreement. These results confirm the relevance of our numerical simulations and their modelling, the acceleration field being closely related to the multi-scale forcing.

4.2.2 Balance of physical forces within the flow

The balance of the forces within the flow indicates the characteristics of the forcing method and the importance of viscous terms, which are assumed to be important in shallow layer flows, mainly via bottom friction effects. Eq. 7 can be recast as

$$\mathbf{a} = -\frac{1}{\rho}\nabla p + \nu\Delta\mathbf{u} + \frac{1}{\rho}\mathbf{f} \quad (12)$$

where $\mathbf{a}$, $\frac{1}{\rho}\mathbf{f}$, $\nu\Delta\mathbf{u}$, $(-\frac{1}{\rho}\nabla p)$ are the Lagrangian acceleration, the electromagnetic acceleration, the acceleration due to the viscosity of the flow and the acceleration due to the pressure gradient, respectively. The global balance of the acceleration term with the other terms can be analysed

Figure 17: Spatial distribution of acceleration components (a&b) a_x^* and (c&d) a_z^* for $I=0.53A$: (a&c) PTVA measurement [14]. (b&d) 3D DNS, at $y^+ = 0.5$.

Figure 18: Relative intensities of acceleration terms of Navier-Stokes equation, Eq. 14.

using the ratios

$$\begin{aligned}
 r_{pf} &= (-\nabla p)_{\text{rms}} / f_{\text{rms}} \\
 r_{vf} &= (\nu \Delta \mathbf{u})_{\text{rms}} / \frac{1}{\rho} f_{\text{rms}} \\
 r_\nu &= (\nu \partial^2 \mathbf{u} / \partial y^2)_{\text{rms}} / (\nu \Delta \mathbf{u})_{\text{rms}} \\
 r_{\text{Re}} &= \mathbf{a}_{\text{rms}} / (\nu \Delta \mathbf{u})_{\text{rms}}
 \end{aligned} \tag{13}$$

where the subscript $_{\text{rms}}$ indicates the use of a RMS value.

Evolution of those ratios with forcing intensity is shown on Fig. 18. r_ν remains close to unity for all forcing intensities, meaning that the wall-normal friction $(\nu \partial^2 \mathbf{u} / \partial y^2)_{\text{rms}}$ is the main viscous term contributing to the acceleration. The ratio r_{vf} between the viscous and the forcing term is lower than 0.5 for the range of forcing considered and decreases slightly with increasing forcing intensity: the electromagnetic forcing is not exactly balanced by the wall-normal friction term

Figure 19: Alignment between pressure and electromagnetic forcing, $\cos(-\frac{1}{\rho}\nabla p, \frac{1}{\rho}\mathbf{f})$ for $\|\mathbf{f}\| > \frac{1}{7}f_{rms}$.

and some other physical mechanisms should also be important to drive the flow. r_{pf} is close to one for the entire range of forcing, showing that pressure terms and forcing terms are globally almost balancing each other. In addition, the pressure gradient/forcing intensity growth with I highlights the increasing role of the pressure terms as the Reynolds number increases. Clearly no term in Navier-Stokes equation can be neglected, and these flows, even if the Reynolds number is close to unity, can not be considered as Stokes flows, for which inertial terms are negligible compare to viscous terms.

An illustration of the production and transfer of momentum in the flow has already been given, by iso-contours of the Lagrangian acceleration along with lines materializing their direction on Fig.15. By definition, it is defined as the sum of pressure, viscous and forcing terms. The viscous terms being diffusive, the acceleration pattern depends almost exclusively on the pressure and the forcing terms.

We perform a conditional distribution of $\cos(-\frac{1}{\rho}\nabla p, \frac{1}{\rho}\mathbf{f})$ which specifies that the forcing intensity should be above the threshold $\|\mathbf{f}\| > \frac{1}{7}f_{rms}$, to remove values where the forcing is negligible from statistical results. As shown in Fig. 19, the values of $\cos(-\frac{1}{\rho}\nabla p, \frac{1}{\rho}\mathbf{f})$ are mainly negative with a maximum of probability not far from -1. This non-perfect anti-alignment of pressure forces and electromagnetic forces is due to the fact that the electromagnetic forces field is not perfectly 2D (Fig. 4b), whilst the pressure gradient is mainly 2D. This misalignment is also due to the viscous acceleration, which is non-negligible. Nevertheless, the gradient of pressure and the electromagnetic forces are mostly anti-aligned in the flow when the forces are significant, and for every forcing intensity.

Importantly, this anti-alignment does not mean that the work of pressure and forces terms cancel each other: locally, $\mathbf{f}^* - (\frac{1}{\rho}\nabla p)^* \neq 0$, the flow is not in “magneto-static equilibrium”. Fig. 20 gives the spatial distribution of $P_f = \mathbf{f}^* \cdot \mathbf{u}^*$ and $P_p = (-\frac{1}{\rho}\nabla p)^* \cdot \mathbf{u}^*$. The power input related to the electromagnetic forces P_f is positive and large above the large magnets. Following a fluid element in a Lagrangian frame of reference, the power related to the pressure of gradient P_p is negative only on a part of the large magnets, i.e after the zero (weak) acceleration area of Fig. 15. If we continue following this fluid element, P_p is then positive when the flow is leaving the domain of collision of the two electromagnetically forced jets. A similar pattern can be observed at medium and small scales. In addition, the location of maximum and minimum of P_f and P_p

Figure 20: Spatial distribution of: (a) $\frac{1}{\rho} \mathbf{f}^* \cdot \mathbf{u}^*$; (b) $(-\frac{1}{\rho} \nabla \mathbf{p})^* \cdot \mathbf{u}^*$

are different. Consequently, the electromagnetic forces and pressure are complementary. The pressure gradient redistributes the flow momentum from domains where it is anti-aligned with the electromagnetic forces to domain where the electromagnetic forces are not present.

The two driving acceleration terms are then the electromagnetic forces and the gradient of pressure which are partially balanced with the viscous terms.

This leads to a flow with a significant inertia and Lagrangian-acceleration terms and a Reynolds number $(\mathbf{a}_{rms}/(\nu \Delta \mathbf{u}))_{rms}$ around unity.

This balance consideration reveals physical property not observed in previous 2D numerical simulations [13, 24], which used a linear model to represent the bottom friction by adding in Navier-Stokes equation an artificial Rayleigh terms $-\alpha \mathbf{u}$, where α is constant.

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \nabla) \cdot \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \Delta \mathbf{u} - \alpha \mathbf{u} + \frac{1}{\rho} \mathbf{f} \quad (14)$$

In those 2D simulations, the modeled bottom friction acts as a balance with the forcing and, importantly, other terms are negligible in first approximation (i.e. $Re \ll 1$). Clearly, the Reynolds number we found in our simulations are larger than 0.1 and the physical mechanisms driving the flow are different.

5 Discussion on numerical modeling

The validity of the Rayleigh friction term $Ra = -\alpha \mathbf{u}$ in 2D numerical simulations, Eq. 14, can be quantified by evaluating a-posteriori the value taken by α from the 3D DNS. In a 2D approach, the bottom wall-normal friction is modelled by $-\alpha \mathbf{u}$. In the 3D simulations, we assume that, α is equal to

$$\alpha = ||\nu \partial^2 \mathbf{u}_{2D} / \partial y^2|| / ||\mathbf{u}_{2D}|| \quad (15)$$

where $\mathbf{u}_{2D}$ is the horizontal velocity (i.e. given by the two components parallel to the wall). $\alpha(x, z)$ is then computed for each y -position as well. Indeed, for each y -location, we can define

Figure 21: Wall normal evolution of $\langle \alpha \rangle$ for various intensity of the forcing.

Figure 22: Probability distribution of α at different height $y^+ = 0.0625, 0.25, 0.5, 0.75$ for $I=0.3A$

the mean value $\langle \alpha \rangle$, the average being taken over the entire plane parallel to the wall, and for different forcing intensity. The evolution of the mean value of α is shown on Fig. 21. $\langle \alpha \rangle$ significantly decreases from the bottom wall towards the upper surface. In addition, close to the bottom wall ($y^+ = 0$), the value of $\langle \alpha \rangle$ increases with the intensity of the forcing. The integration of $\langle \alpha \rangle$ over the brine's thickness, which could be used to represent the total effect of the wall normal friction, is then dependent of the forcing intensity and its typical variations are found to be about 20% per Ampere (in our range of forcing). Consequently, it may be difficult to model the effects of the wall-normal friction with a constant value of α across a broad range of forcing intensities.

Furthermore, the distribution of α , normalised by its mean value, at 4 different distances from the wall is shown on Fig. 22 for a forcing intensity of $I=0.3A$. For $y^+ \geq 0.5$ this distribution is narrow and centered around 1, meaning that the highest probable value of α is $\langle \alpha \rangle$. As the wall approaches, the probability is widening, and it becomes apparent that the most probable value of α departs from its mean value in each plane.

Given the sharp arrow shape of the distribution close to the mean value of α , a maximum of probability might be identified. Nevertheless, even if of lower probability, the large tails observed on Fig. 22 are not physically negligible as they are closely related to the multi-scale forcing. To highlight this point, the spatial dependence of α is plotted on Fig. 23. The color scale varies between 0.25 and 1.5, i.e a factor 6 between minimal and maximal values. As expected from the PDF analysis, α varies significantly in space. Indeed, its value is equal to one above the non-

Figure 23: Spatial distribution of $\alpha/\langle\alpha\rangle$ for $y^+=0.5$ and $I=0.3A$.

Figure 24: Velocity profiles along the wall-normal direction y^+ , $\|\mathbf{u}_{2D}(y^+)\|/\|\mathbf{u}_{2D}(y^+ = 1)\|$, for different positions (x, z) : $(0, -40\text{mm})$, $(0, -160\text{mm})$, $(0, -100\text{mm})$.

forced region, while reaching its peaks (local maxima and minima) at the vicinity of the pairs of magnets, where the forcing intensity is maximum. The variation of α is related to variation of the normalised velocity profiles, $\|\mathbf{u}_{2D}(y^+)\|/\|\mathbf{u}_{2D}(y^+ = 1)\|$, illustrated in Fig. 24, for various positions ($z^+ = -8, -20, -32$) along the central axis $x^+ = 0$. These profiles are clearly not self-similar, as supposed in the linear friction approximation of the bottom friction in 2D simulations. Indeed, the “butterfly shape” of α above the large magnets corresponds to the butterfly shape of excess of velocity observed in previous 2D numerical simulations [13].

Consequently, an important point is that the 2D modeling of these electromagnetically forced multi-scale shallow (not thin) layer flows using a linear terms, $-\alpha\mathbf{u}$, where α is constant, is fundamentally incorrect. We expect the Rayleigh friction model will present such weakness for any shallow layer flow with a multi-scale distribution of stagnation points as α should vary at their vicinity. In such flows, the extension of 2D simulations to 3D with a reasonable higher cost (very few point are used in the wall normal direction) should significantly improve their quality.

We thus conclude that 3D DNS are required to models such forced flows as the entire shear within the brine need to be simulated in addition to the 3D electromagnetic forcing.

6 Conclusion

3D Direct Numerical simulations of multi-scale flows generated by electromagnetic forcing have been performed, for different forcing intensities, and have been compared with previous, well-documented, laboratory experiments. These simulations are based on a detailed 3D model of the 3D electromagnetic forcing and an approximation of the shallow layer flow. A general good agreement between DNS and experiments have been observed both for velocity and, to our knowledge for the first time, acceleration fields. In addition, energy spectra and pair dispersion statistics are found to be very similar to experimental results. The only discrepancy found is close to the free-surface, which is approximated in the simulations by a free-slip condition. For high forcing intensities, above $I = 0.53$, the friction due to air at the free-surface cannot be neglected, introducing a small dependence on the derivative in the surface normal direction, while the free-slip condition imposes a zero-surface-normal one. As already mentioned, implementation of a free-surface condition, for small motion [25], is under way to simulate higher forcing intensities.

Based on a good second-order agreement between experiments and simulations (for velocity and acceleration), further properties of those multi-scale flows have been explored, using the numerical results. In the whole range of conditions tested, the flow remains laminar and almost two-dimensional in most of the domain. The maximum vertical velocity is found near the smallest magnets.

Importantly, the study of the balance between the different terms of the Navier-Stokes equations shows that no term can be neglected to model these flows. The Reynolds number, based on the ratio between acceleration (inertial) forces and viscous forces, is about unity for all cases considered in this paper. Consequently, the role of inertial forces is significant in these flows. Thus, they can not be considered as Stokes flows, for which $Re \ll 1$ is usually assumed to neglect inertial terms. These conclusions are in contradiction with the physical interpretation which can be derived from previous 2D numerical simulations of such flows. Indeed, the current results showed that the Rayleigh friction term usually introduced in such simulations are not appropriate for electromagnetically forced multi-scale shallow layer flow.

The results presented in this paper show that a 3D approach is required to simulate the type of multi-scale flows driven by electromagnetic forcing in shallow layer. 3D DNS allows the modelisation of the 3D electromagnetic forces, for which the vertical component f_y , even weak, is non-null. Importantly, it also allows wall-normal variation of $||\mathbf{f}||$ and to model the wall-normal shear within the flow.

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References

- [1] N. Pamme. “Magnetism and microfluidics”. *Lab Chip*, 2438, **6**24-38 (2006).

- [2] S. Cuevas, S. Smolentsev and M.A Abdou. “On the flow past a magnetic obstacle”. *J. Fluid Mech.*, **553**, 227-252 (2006).
- [3] L. Rossi, J.C. Vassilicos, and Y. Hardalupas. “Electromagnetically controlled multiple scale flows”. *J. Fluid Mech.*, **558**, 207 (2006).
- [4] P. Tabeling. “Two-dimensional turbulence: a physicist approach”. *Phys. Rep.*, **362**, 1 (2002).
- [5] L.F. Richardson. “Atmospheric Diffusion Shown on distance-Neighbour Graph”. *Proc. R. Soc. Lond. A*, **110**, 709 (1926).
- [6] A.M. Obukhov. “Spectral energy distribution in a turbulent flow”. *Izv. Akad. Nauk SSSR* **5**, **110**, 453 (1941).
- [7] L. Rossi, J.C. Vassilicos, and Y. Hardalupas. “Multi-scale laminar flows with turbulent-like properties”. *Phys. Rev. Lett.*, **97**, 144501 (2006).
- [8] P.A. Durbin. “A stochastic model of two-particle dispersion and concentration fluctuations in homogeneous turbulence”. *J. Fluid Mech.*, **100**, 279 (2007).
- [9] L. Rossi, J.C. Vassilicos and Y. Hardalupas. “Eulerian-Lagrangian aspects of a steady multiscale laminar flow”. *Phys. Fluids*, **19**, 078108 (2007).
- [10] M.K. Rivera and R.E. Ecke. “Pair Dispersion and Doubling Time Statistics in Two-Dimensional Turbulence”. *Phys. Rev. Lett.*, **95**, 194503 (2005).
- [11] S. Kenjeres, J. Verdoold, A. Wibowo, C.R. Kleijn and K. Hanjalic. “Large Eddy Simulations of Electromagnetically Driven Vortical Flows”. *Proc. ETC* **11**, (2007).
- [12] L. Rossi, E. Hascoët, J.C. Vassilicos, and Y. Hardalupas. “2D fractal flow generated by electromagnetic forcing: laboratory experiments and numerical simulations”. *Proc. TSFP* **4** (2005).
- [13] E. Hascoët, L. Rossi and J.C. Vassilicos. “Multi-scale flow control for efficient mixing: simulation of electromagnetically forced turbulent-like laminar flows”. *Proc. IUTAM Symposium on Flow Control and MEMS*, Springer (2008).
- [14] S. Ferrari and L. Rossi. “Particle Tracking Velocimetry and Accelerometry (PTVA) measurements applied to quasi-two-dimensional multi-scale flows”. *Exp. Fluids*, DOI 10.1007/s00348-007-0443-7 (2008).
- [15] M.K. Rivera and X.L. Wu. “External Dissipation in Driven Two-Dimensional Turbulence”. *Phys. Rev. Lett.*, **85**, 976 (2000).
- [16] N. Paldora, Y. Dvorkin and C. Basdevant. “Improving the calculation of particle trajectories in the extra-tropical troposphere using standard NCEP fields”. *Atm. Env.*, **36**(3), 483 (2002).
- [17] G. Boffetta, A. Celani, S. Musacchio and M. Vergassola. “Intermittency in two-dimensional Ekman-Navier-Stokes turbulence”. *Phys. Rev. E*, **66**, 026304 (2002).

- [18] P.A. Davidson. “An introduction to magnetohydrodynamics”. ISBN 0 521 791499, Cambridge university press (2001).
- [19] S. Molokov, R. Moreau and H.K. Moffatt, “Magnetohydrodynamics, Historical Evolution and Trends”. ISBN: 978-1-4020-4832-6, Springer (2007).
- [20] G. Akoun and J.-P. Yonnet. “3D analytical calculation of the forces exerted between two cuboidal magnets”. IEEE Trans Magn., **20** (1984).
- [21] L. Rossi. “Contrôle électromagnétique d’écoulement en eau de mer”. PhD Thesis, Univ. Joseph Fourier, Grenoble (2001).
- [22] A. G. Kravchenko and P. Moin. “On the Effect of Numerical Errors in Large Eddy Simulations of Turbulent Flows”. J. Comp. Physics, **131**, 310 (1997).
- [23] S. Laizet, and E. Lamballais. “Direct-numerical simulation of the splitting-plate downstream-shape influence upon a mixing layer”. C. R. Acad. Sci., **334**, 454 (2006).
- [24] E. Hascoët and L. Rossi. “Turbulent-like laminar flows sustained and controlled by multiscale body forces”. Personal communication.
- [25] L. Shen and D.Y. Yue. “Large-eddy simulation of free-surface turbulence”. J. Fluid Mech., **440**, 75 (2001).
- [26] M.C. Julien, J. Paret and P. Tabeling. “Richardson Pair Dispersion in Two-Dimensional Turbulence Physics”. Phys. Rev. Lett., **82**, 2872 (1999).
- [27] M. Bourgoïn, N. T. Ouellette, H. Xu, J. Berg and E. Bodenschatz. “The Role of Pair Dispersion in Turbulent Flow”. Science, **311**, 835 (2006).