Numerical Investigation of Arches in Brick-Masonry Bridges

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Abstract: A significant number of old masonry bridges are still in use and need to be assessed considering current traffic loading and safety requirements. Masonry bridges are complex heterogeneous systems, where masonry arches represent the main components. Thus a realistic modelling of arches is vital for accurate assessment of masonry bridges. The authors have previously proposed and validated a detailed mesoscale description for masonry arches allowing for the actual masonry bond and the specific arch geometry including the case of skew arches. In this paper, the proposed mesoscale modelling strategy is used in a comprehensive numerical study to investigate the effects of various parameters, including masonry bond and defects in the brickwork, abutment stiffness and movements at the supports, which are usually disregarded in practical assessment of masonry arches and bridges. The results achieved show how these parameters affect the ultimate load capacity, failure mechanisms and initial stiffness of square and skew arches, where the use of detailed 3D mesoscale modelling is critical in providing accurate response predictions under a variety of loading conditions for which reduced models might provide incorrect results.

Keywords: masonry; nonlinear analysis; 3D modelling; mesoscale description; domain partitioning approach; square and skew arches.

1. Introduction

Masonry arches represent critical components in old masonry bridges, which were built following crude design rules based upon basic structural mechanics principles and empirical evidence (Benvenuto, 1981; Brencich & Morbiducci, 2007; De Santis & de Felice, 2014). At present, these structures need to be assessed considering current safety requirements and loading conditions. Past laboratory and in-situ tests (Hendry, Davies, & Royles, 1985; Pippard, Tranter, & Chitty, 1936; Pippard & Ashby, 1939; Pippard & Chitty, 1941; Royles & Hendry, 1991; Wang, 2004; Melbourne et al., 2007) showed that the structural response of masonry arches is very complex and characterised by different failure modes. Thanks to the development of computational techniques for nonlinear structural analysis, various numerical strategies for assessing arches in masonry bridges have been proposed in recent past (Nobile & Bartolomeo, 2015; Sarhosis, et al., 2016). These are based on limit analysis concepts (Cavicchi & Gambarotta, 2005; Cavicchi & Gambarotta, 2007; Gilbert, 2007), the finite element method (FEM) (Crisfield, 1984; de Felice, 2009; Milani & Lourenço, 2012; Milani & Tralli, 2012; De Santis & de Felice, 2014), the discrete element method (Lemos, 2007; Sarhosis, Oliveira, Lemos, & Lourenco, 2014), and discontinuous modelling techniques (Thavalingam, Bicanic,

Robinson, & Ponniah, 2001). However, most of previous research focused mainly on the analysis of the ultimate capacity of masonry arches subjected to gravity loading, without providing a deep insight on the response at different loading levels up to collapse, especially in the case of skew arches. Moreover, the effects of defects in the brickwork and potential movements at the supports have yet to be fully explored.

Recently, the authors proposed and validated an advanced modelling strategy for masonry arches (Zhang, Macorini, & Izzuddin, 2016) which is based upon a nonlinear mesoscale masonry description (Macorini & Izzuddin, 2011). It allows for the specific masonry bond and geometrical characteristics including the case of skew arches. The accuracy of this mesoscale modelling approach has been confirmed in a number of experimental-numerical comparisons on square spanning arches (arches with no skew or very small degree of skew, McKibbins et al., 2007) and skew arches. It has been shown that the use of the proposed numerical description enables a realistic response prediction up to collapse, accurately predicting the development of damage at different loading levels and the most critical failure mechanisms of multi-ring brick-masonry square and skew arches.

In this paper, this advanced modelling strategy is used in a comprehensive numerical study to investigate the effects of different parameters on the structural behaviour of masonry arches. Multi-ring arches with segmental circular shape built using the stretcher and the header bond method, and characterised by different rise-to-span ratios, are analysed. In this respect, the geometry of typical arches found in a statistical survey on old masonry bridges (Brencich & Morbiducci, 2007) is considered. According to this study, 50% of the brick-masonry bridges within the Italian railway network built between 1830 and 1940 are formed by arches with 0.5 rise-to-span ratio (deep arches). The remaining bridges are shallow arch bridges with a rise-to-span ratio in the range of 0.1-0.4.

To study the influence of defects in the brickwork, the response of defective arches with weak circumferential mortar joints connecting adjacent rings are compared to that of arches with uniform (strong) mortar joints. Previous studies pointed out that “defective” mortar joints in multi-ring arches without headers may facilitate ring separation. In real bridges, imperfection in mortar joints may be caused by mortar wash out or due to the use of poor material which leads, in time, to “ring delamination”. Beare (1993) highlighted that ring debonding is the most common defect in multi-ring arches with stretcher bond. Besides, according to Sunley (1990) all existing brick-masonry bridges with insufficient interconnecting headers are defective, whether or not they are weakened by existing cracks or ring separation. To explore the behaviour of masonry arch bridges prone to ring separation, Melbourne and Gilbert (1995) performed physical tests on models of arch bridges, where damp sand was used rather than standard mortar to bond arch rings so as to promote ring separation. The results of this study confirmed that ring separation strongly influences the response up to collapse, reducing the load capacity. Moreover, it was found that ring debonding may occur not only in arches with weak joints connecting adjacent rings, but also in arches with mortar joints of good quality.

In addition to the above, further numerical simulations are carried out in this paper to study the effects of the abutment stiffness and movements at the supports. In this respect, the failure modes induced by differential vertical settlements and inward and outward horizontal displacements at the abutments are investigated, and the influence of these movements on the arch load capacity is analysed. The paper proceeds with a brief description of the adopted modelling strategy for masonry arches. Focus is then shifted to square and skew arches, where the results of the various numerical investigations are presented and critically discussed. It is important to point out that, according to the adopted numerical strategy (Zhang et al., 2016), 3D nonlinear simulations of large multi-ring arches are carried out using a partitioning approach combined with the masonry mesoscale description. This allows for parallel computation enhancing computational efficiency. On the other hand, when using conventional computational resources (e.g. monolithic simulations) the analysis of large arches with the detailed 3D mesoscale description is impractical, as it requires an excessive computational cost (e.g. it may take several hours to complete a nonlinear simulation, Zhang et al., 2016). In these cases, adequate predictions of the arch ultimate capacity can be obtained through efficient limit analysis.

2. Modelling strategy for masonry arches

Masonry arches are modelled using the mesoscale masonry description previously developed at Imperial College (Macorini & Izzuddin, 2011). It enables an accurate representation of the 3D domain of any brick-masonry arch, as the actual 3D masonry bond is represented using two or more elastic solid elements for each brick and 2D nonlinear interface elements for mortar joints (Figure 1a). In the constitutive model for interface elements, material nonlinearity is taken into account employing a cohesive model. This effectively describes the development damage, cracks and plastic separations. In particular, a multi-surface plasticity criterion is utilised (Figure 1b), where a hyperbolic failure surface F_I based on the Coulomb slip criterion determines the boundaries of the elastic domain representing Mode-I and Mode-II fracture:

$$F_1 = \tau_x^2 + \tau_y^2 - (C - \sigma \tan \phi)^2 + (C - \sigma_t \tan \phi)^2 \quad (1)$$

where C represents the cohesion, ϕ the friction angle and σ_t the tensile strength for mortar joints or brick-brick interfaces. The second yield function corresponds to a cap model in compression, and it is given by another hyperbolic function:

$$F_2 = \tau_x^2 + \tau_y^2 - (D - \sigma \tan \theta)^2 + (D - \sigma_c \tan \theta)^2 \quad (2)$$

where σ_c is the compressive strength of masonry while D and θ are material parameters governing the shape of the cap surface. The degradation of the material parameters is modelled using a work-softening plasticity approach. In this way tensile strength σ_t and friction angle ϕ reduce to residual values when the plastic work W_{crI} approaches Mode-I fracture energy G_{fl} . In a similar way cohesion C at the interface reaches its residual value when W_{crI} converges to the Mode-II fracture

energy G_{fr} . A non-associated flow rule is employed and a plastic potential Q_I similar to function F_I (Figure 1b), but with a different friction angle parameter is adopted to model the actual dilatancy. This is due to the roughness of the fractured shear surface and can be measured in tests on interfaces.

The main drawback of the adopted mesoscale strategy is the computational cost, which becomes excessive when analysing large arches of realistic masonry bridges. In these cases, as suggested in (Zhang et al., 2016), the mesoscale description is used within the advanced partitioning approach previously developed at Imperial College (Jokhio & Izzuddin, 2013; Jokhio & Izzuddin, 2015; Jokhio & Izzuddin, 2016) which enhances computational efficiency, achieving exceptional computational benefits through parallelisation on High Performance Computing (HPC) systems. As shown in previous studies on masonry structures (Macorini & Izzuddin, 2013; Zhang et al., 2016), the use of the mesoscale partitioned approach guarantees exactly the same accuracy of the 3D mesoscale description at structural scale.

In (Zhang et al., 2016), the proposed mesoscale partitioned strategy has been validated considering experimental tests on multi-ring square arches subjected to line loads and a multi-ring skew arch subjected to a patch load. Figure 2 shows some results of the numerical-experimental comparisons for the skew arch, where a good agreement between the numerical predictions and the experimental data can be observed both in terms of force-displacement curve and failure mode description. A detailed validation of the adopted mesoscale strategy can be found in (Zhang et al., 2016).

3. Mesoscale analysis of square arches

The static response of square arches is investigated by simplified mesoscale “strip” models (Zhang et al., 2016) utilising only one set of solid elements along the span for each arch ring. Zhang et al. (2016) pointed out that a detailed modelling of the brick-mortar arrangement along the width of the arch is not critical when analysing square arches subjected to vertical static loads uniformly distributed on a strip along the whole width of the arch. Numerical comparisons between strip and full 3D models allowing for the actual arch geometry and masonry bond showed practically coincident results in terms of load-displacement response, where the distributions of stresses and plastic work are uniform along the arch width for different loading levels up to collapse. Thus in the analysis of square arches subjected to line loads, mesoscale strip models generally represent a good compromise between accuracy and computational efficiency. Hereinafter, for convenience, square arches with unit width (1m) are considered.

3.1 *Effects of rise-to-span ratio*

The response up to collapse of segmental arches characterised by the most common rise-to-span ratios as found in a number of existing masonry arch bridges (Brencich & Morbiducci, 2007) are analysed.

These include eight arches with two or five rings connected by good quality mortar. The main geometrical properties of the arches are reported in Table 1.

The loading arrangement encompasses two initial vertical (dead) loads (DLs) and an additional vertical live load (LL) applied at quarter span and increased up to collapse under displacement control. The two identical dead loads are equal to 22.5kN/m for the arches with 3m span and to 100kN/m for the 8m span arches. As in the experimental tests presented in Melbourne et al. (2007), which were considered by the authors for the model validation in Zhang et al. (2016), the dead loads are applied at the quarter and three quarter span representing the weight of the backfill in typical masonry arch bridges. Of course a more realistic representation of the backfill contribution should consider the actual vertical loading distribution due to the backfill weight and the load spreading and restraining effects on the arch deformation provided by the backfill. This it is not considered in the present study, as it would require a detailed representation of the arch-backfill interaction which is outside the scope of this paper and it will be considered in future research.

In the numerical simulations the material properties assumed in (Zhang et al., 2016) for model validation are used. In particular masonry units have been modelled assuming Young's modulus $E_b = 1.60 \times 10^4$ [MPa] and Poisson's ratio $\nu = 0.15$. Mortar joint interfaces have been represented by using normal and tangential stiffness values of $K_n = 90$ [N/mm³] and $K_t = 40$ [N/mm³] and the inelastic material parameters for reported in Table 2, where C_0 , σ_{t0} , $\tan\phi_0$ represent cohesion, tensile strength and tangent of friction angle for yield function F_1 , G_{f1} and G_{f2} fracture energies for Mode-I and Mode-II fracture, $\tan\psi_0$ tangent of friction angle for plastic potential Q1, σ_{c0} compressive strength of masonry, G_c crushing energy; $D \tan\theta_0$ are material parameters determining the shape of the cap in compression F_2 , Figure 1b (Macorini & Izzuddin, 2011).

Similar results have been found for arches with different span length. The deformed shapes and the plastic work W_{cr1} contours at the end of the numerical simulations are shown in Figure 3. It can be seen that in the case of deep arches and arches with 1:4 rise-to-span ratio the failure mechanism is a typical "hinge mechanism", which is governed by the development of four radial cracks at the opposite haunches and at about the quarter and the three quarter span. Conversely in the case of the two arches with 1:8 and 1:10 rise-to-span ratio, radial cracks are combined with ring separation, which takes place at the arch haunch close to the live load. This is mainly caused by the shear forces in the arch which generally increase when the rise-to-span ratio reduces, becoming relatively large for very shallow arches (Dym & Williams, 2011).

Figure 4 displays the load-displacement curves at quarter span for the arches with 3m and 8m span. A significant change in the response for different rise-to-span ratio values can be observed. The deep arch is characterised by the lowest loading capacity but a notable ductility; after reaching the maximum load the displacements significantly increase while the load remains almost constant. The ultimate load rises for arches with low rise-to-span ratio, but the response up to the maximum load becomes less smooth especially for the very shallow arch (1:10 rise-to-span ratio). In this case while

three radial cracks around the two springings and at about the quarter span develop gradually, ring separation occurs suddenly producing a rapid release of elastic energy and causing an abrupt softening in the load-displacement curve. After a short almost flat branch, where the displacement increases while the live load remains constant, the load-displacement curve rises again up to the ultimate load. This is reached when the fourth radial crack forms at about three quarter span.

3.2 *Effects of brickwork defects*

Four defective arches prone to ring separation have been investigated. These are characterised by geometrical features identical to those of the *Arch 1*, *Arch 2*, *Arch 3* and *Arch 4* analysed in the previous section, but are constructed with weak circumferential mortar joints. In this respect, very small cohesion and tensile strength values, $C_0 = 0.004 \text{ N/mm}^2$ and $\sigma_{t0} = 0.002 \text{ N/mm}^2$, are employed for the interface elements representing the joints which connect the two rings. These include reduced cohesion, tensile strength and friction angle values. In Figure 5, the deformed shapes and the plastic work contours at the last step of the numerical simulations are displayed, whereas in Figure 6 the load-displacement curves are shown. In all the cases ring separation characterises the response at collapse and the failure modes. Comparing the curves plotted in Figures 4a and 6, it can be seen that the poor connection between the two rings significantly influences the load-displacement response of the four arches. Ring separation starts at very low load levels, and its development is gradual leading to progressive stiffness degradation up to the ultimate load, but not to a sudden force reduction as for the very shallow arches with good quality mortar (e.g. *Arch 3* and *4*). More than 50% strength reduction can be observed for all the analysed arches. This highlights the critical role played by the connections between adjacent rings and the importance of a correct evaluation of the mechanical characteristics of the mortar joint for an accurate response prediction of existing masonry arches.

3.3 *Effects of loading position*

Multi-ring arches with 3m span are analysed here to study the influence of the loading position on the response up to collapse. Particular attention is given to the ultimate capacity and the type of failure mechanism. In the numerical simulations, two dead loads (22.5kN/m) were applied at quarter and three quarter span, while a live load was placed at four different positions: close to the left springing (Case 1), at 1/8 span (Case 2), at quarter span (Case 3) and at mid-span (Case 4), and then increased up to collapse.

Deformed shapes and the plastic work contours at collapse for the arch with 1:4 rise-to-span ratio (*Arch 2*) are shown in Figure 7. Different failure mechanisms can be clearly noticed. In Case 1 the load near the left support induces a local failure mode; this is due to shear sliding at the radial bed joint close to where the load is applied. In Case 3 only radial cracks form (high mechanism), while in Case 2 and Case 4 radial cracks are accompanied by ring separation. This develops at the left haunch

close to the live load in Case 3, and symmetrically at both haunches when the live load is applied at mid-span.

In Figure 8a, the displacements at the live load position for the four different cases are plotted against the live load. It can be seen how the response characteristics including the ultimate load, the initial stiffness and the arch ductility strongly depend upon the location of the live load. When the load is applied near the support, the load capacity of the arch is influenced by its ability to resist shear forces. These may cause plastic shear sliding at radial bed joints as in Case 1, or ring separation associated with local failure as in Case 2. In both cases shear failure prevents an adequate redistribution of internal forces limiting the arch ductility. On the other hand the response of *Arch 2* under a live load applied at quarter span is very ductile, being governed by the formation of radial cracks (Figure 7c) leading to a four-hinge mechanism without ring separation. Figure 8b presents the ultimate load values for arches with different rise-to-span ratios (e.g. *Arch 1* to *4*) loaded at different positions. The results show that the condition with loads close to the supports is the most critical for very shallow arches, while the lowest load capacity for the deep arch and the arch with intermediate 1:4 rise-to-span ratio is achieved when the live load is applied at quarter span.

3.4 Effects of abutment stiffness

In the previous numerical simulations, square arches have been modelled assuming that the supports at the springings are perfectly rigid. In this section, the effects of potential deformations at the abutments are investigated. These may develop in real arch bridges under the forces transferred by the masonry arch to the foundations. To analyse the effects of the abutment stiffness, elastic interface elements are used to connect the arch springings to fixed supports so as to represent elastic abutments. In particular, the 3m span arches presented previously have been analysed employing the five pairs of elastic normal and tangent stiffness values reported in Table 3. The numerical results confirmed that the failure mechanism does not change by varying the stiffness at the lateral supports, and in all the cases arch failure is reached when a hinge mechanism with four radial cracks forms.

The load-displacement curves showing the response of *Arch 2* are plotted in Figure 9a. The variation of the ultimate load and the secant stiffness of the arch calculated at 40% of the ultimate load (Zhang et al., 2016) with the abutment stiffness are displayed in Figures 10b,c. Ultimate load and secant stiffness do not change significantly for large abutment stiffness values, but they both notably reduce for a normal and tangential stiffness at the abutments of 1N/mm^3 . In this case, a reduction of 46% for the ultimate load and 60% for the secant stiffness compared to the arch with perfectly rigid supports can be observed. As shown in Figure 10, when arches with different rise-to-span ratios are analysed, different variations of stiffness and strength can be noticed. In particular, for very shallow arches (e.g. 1:10 and 1:8 rise-to-span ratios) only a marginal reduction of the load capacity can be observed, but the stiffness variation is remarkable.

3.5 *Effects of movement at abutments*

Movement at the abutments and piers resulting from the instability of the foundations is one of the most common causes of deterioration for masonry arch bridges. In general, the performance of masonry arch barrels, which are the most critical structural components in masonry bridges, is highly affected by differential settlements (McKibbins et al. 2006). Support movements may occur during the bridge construction or may be caused by extreme events (e.g. earthquakes) or due to a change in the soil conditions below the foundations. Moreover if the bridge is built on waterways, water flow may induce scour at the footings of piers and abutments, often leading to the collapse of the bridge. In practical assessment of masonry arch bridges, the formation of three hinges caused by vertical or horizontal differential movements at the supports represents a failure mechanism, and indicates that the arch should be immediately repaired (McKibbins et al. 2006).

In this section, the possible consequences and effects of different types of support movements are investigated. To this end, the square arches with 3m span considered before (*Arch 1*, *Arch 2*, *Arch 3*, *Arch 4*) are analysed considering the settlement modes shown in Figure 11. Afterwards, different levels of support displacements are applied before increasing the live load at quarter span up to collapse. Moreover, the influence of defects in masonry is investigated comparing the response of arches with good quality brickwork (strong joints) against that of arches with weak mortar joints connecting the two rings (weak joints).

Figure 12 displays the deformed shapes for *Arch 2* subjected to the three settlement modes and the plastic work W_{cr1} contour for the different cases. All these figures refer to the last step of the numerical simulations at the failure of the arch. The failure mode depends not only on the specific type of settlement but also on the brickwork characteristics and does not always involve the formation of three main radial cracks (hinges). In the case of horizontal spread of support and horizontal inward movement, three radial cracks develop in the arch with good quality joints, but in the arch with weak mortar joints radial cracks occur with ring separation, which extends from the arch crown to the left support moving laterally. On the other hand, when vertical differential settlements are applied only two main radial cracks form at the springings of the arch.

The imposed displacements at the left support are plotted against the associated reaction forces in Figure 13. It can be seen that when the failure mechanism due to a specific settlement mode develops, the associated reaction force decreases showing a post-peak softening response which is steeper for the arch with strong joints. Moreover it can be noticed that the support displacement required to form a mechanism is notably lower for arches with good quality mortar joints especially in the case of horizontal inward displacements.

3.5.1 *Horizontal spread of support*

Numerical simulations were carried out to investigate the response of the 3m span arches subjected to different levels of prescribed horizontal displacements followed by the application of a line load at

quarter span. In particular, static time-history analyses were performed considering two phases, where two loading conditions were applied in sequence. In the initial phase, a specific horizontal outward displacement δ_{ss} was exerted at the left springing of the arch incrementally; afterwards the line load was increased up to collapse. Six δ_{ss} values were considered within the interval $1\text{mm} \leq \delta_{ss} \leq 6\text{mm}$, where the upper limit is higher than the horizontal spread of support at which a mechanism starts to develop. The numerical responses were calculated under force control, thus the nonlinear analyses terminated when the maximum loads were reached without describing the post-peak behaviour.

Figures 14a,b display the numerical curves for *Arch 2* with strong and weak mortar joints, where the line load is plotted against the vertical displacements at quarter span. It can be seen that the load capacity of the arch with good quality mortar joints significantly decreases when δ_{ss} rises, and about 60% reduction is calculated for 6mm horizontal support displacement. On the other hand, in the case of defective brickwork (weak joints), the reduction in load capacity is less pronounced reaching about 18% for 6mm horizontal displacement. This is due to the characteristics of the response under horizontal spread of support (see Figure 13a,b), where the arch with weak joints shows a more ductile behaviour.

Figure 15 shows some results for the arch with strong joint and subject to 1mm and 6mm spread of support. In the two figures the deformed shapes and the contours of the plastic work W_{cr1} after the application of the two support movements and at collapse are displayed. It can be observed that the lower prescribed displacement induces the formation of two cracks at the opposite springings, while at collapse a third crack forms close to where the line load is applied. When a higher displacement is exerted, even though the mechanism associated with the support prescribed displacement forms, the arch can still carry additional live load and the collapse is attained only when a more crack develops below the load.

The influence of the horizontal spread of abutment on the load capacity of masonry arches with strong mortar joints and different rise-to-span ratios is shown in Figure 16. The ratio between the ultimate load achieved after the application of different horizontal outward displacements at the left support and the ultimate load calculated with fixed supports for the arches *Arch 1*, *Arch 2*, *Arch 3* and *Arch 4* is plotted against the prescribed displacement values. A similar variation of the ultimate capacity can be observed for the first three arches, with a maximum reduction of 46% for the arch with 1:8 rise-to-span ratio under a 5mm horizontal support displacement.

3.5.2 Vertical settlement of support

As for the case of horizontal spread of support, the influence of differential vertical settlements on the arch load capacity was investigated performing time-history analyses, where different levels of vertical displacements δ_{sv} were imposed on the left abutment before loading the arch at the quarter span up to collapse. Six δ_{sv} values were considered within the interval $1\text{mm} \leq \delta_{sv} \leq 6\text{mm}$, where the upper limit is about the vertical settlement at which a mechanism starts to develop for *Arch 2* with

strong circumferential mortar joints. Also in this case the numerical responses were calculated under force control, thus the nonlinear analyses terminated when the maximum loads were reached without representing the post-peak behaviour. Figures 17a,b show the load-displacement curves for *Arch 2* with strong and weak mortar joints. Examining the results, it can be observed that a vertical support movement does not significantly influence the loading capacity for arches with strong or weak mortar joints. In this case, the considered prescribed displacements are not sufficiently large to notably reduce the load bearing capacity of the arch, especially in the case weak mortar joints, where the maximum vertical settlement is smaller than the displacement level at which a mechanism starts to develop (see Figures 13b). This is confirmed also by Figure 18, where the deformed shapes and the contours of the plastic work W_{cr1} for the smallest and the largest vertical settlements are displayed. The figures show that the support displacement does not influence the collapse mechanism, which corresponds to that of the arch with fixed supports showing four radial cracks at the two springings and at the quarter and three quarter span.

Finally, by investigating the response of arches with different rise-to-span ratios, it has been found that the support vertical movement may induce a strain/stress state in the arch which mitigates the effects of the live load, thus increasing the load capacity as for the arch with 1:8 rise-to-span ratio (*Arch 3*, Figure 19). Evidently the settlement level should be lower than the limit leading to a mechanism.

3.5.3 Horizontal inward support displacement

Similar numerical simulations have been carried out to study the effects of horizontal inward movements at the left support. As before, six horizontal inward displacements values δ_{im} were considered within the interval $1\text{mm} \leq \delta_{im} \leq 6\text{mm}$, where the upper limit is significantly higher than the horizontal inward movement leading to a mechanism for *Arch 2* with strong mortar joints, but lower than the same displacement limit for *Arch 2* with weak mortar joints (see Figure 13). The load-displacement curves in Figure 20 confirm the significant influence of the horizontal inward movement on the arch response. This is critical not only for the arch with strong mortar, where a notable maximum reduction of 46% have been calculated, but also for the arch with weak circumferential joints. Moreover it can be observed than the prescribed support displacements give rise to a strain/stress state which opposes the live load which, for relatively small inward movements, may lead to an increment in the load capacity (see curves $\delta_{im} = 1\text{mm}$ in Figure 20b).

Concerning the failure mechanisms due to the line load at quarter span applied after the horizontal inward support movement, Figure 21a shows that, in the case of relatively small support displacements, the failure mode corresponds to that of the arch with fixed supports. Conversely, Figure 21b highlights that, by increasing the support movement, a different crack pattern forms in the arch. This is similar to that shown in Figure 12c and due to inward support displacements alone, but with a more diffuse cracking at the crown.

Finally the influence of the horizontal inward support movements on the load capacities of arches with different rise-to-span ratios is shown in Figure 22. It can be seen that the effects of support movement is more critical for shallow arches with 1:8 and 1:10 rise-to-span ratios (e.g. *Arch 3* and *4*). However in these cases, convergence problems allowed only an approximate estimate of the ultimate load for 5mm inward displacement. Thus the calculated 70% and 83% reductions in loading capacity should be considered as conservative predictions.

3.5.4 *Modelling considerations*

A parametric study has been conducted to identify the most critical material properties which determine the qualitative response under the combinations of the prescribed support displacements and the live loads considered before. Not surprisingly, it has been found that the fracture energy values (especially Mode-I fracture energy G_{fI}) strongly affect the numerical predictions. In particular, using unrealistically large fracture energy values (e.g. two orders of magnitude larger than the values in Table 2), the response prediction under the live load seems to be almost independent from the level of support displacement, as well as the ultimate load capacity. This is shown in Figure 23, where the live load is plotted against the vertical displacement at quarter span for the three types of support displacement. The results obtained confirm that the use of large fracture energy values, which disregard the actual quasi-brittle nature of the mortar joints and the fracture in the masonry units, usually transfers the numerical response towards the elastic-plastic limit, where the solution does not depend upon potential support movements.

4. **Mesoscale analysis of skew arches**

In (Zhang et al., 2016), it has been shown that for an accurate analysis of skew masonry arches a full 3D mesoscale description accounting for the actual 3D masonry bond should be considered. In this case the use of a more efficient strip model may lead to unrealistic results.

In the following, the detailed 3D mesoscale strategy for skew arches presented in (Zhang et al., 2016) is adopted to investigate the response of brick-masonry skew arches considering the effects of (i) rise to span ratio, (ii) masonry bond and defects in the brickwork, (iii) abutment stiffness and (iv) support movements. The two-ring skew arch with header bond investigated in (Zhang et al., 2016) (Figure 2) is considered here as the reference model. Thus all the analysed arches are characterised by a 3m direct span, 670mm width, 215mm thickness and 45° skew angle.

4.1 *Effects of rise-to-span ratio*

The effects of the skew arch geometry have been analysed by comparing the numerical responses of three arches with different rise-to-span ratios. The main geometrical properties are reported in Table 4, while Figure 24 shows the three FE mesoscale meshes. The material properties considered in (Zhang et al., 2016) for the validation of the proposed mesoscale description for skew arches are used in the

numerical simulations. Thus masonry units have been modelled assuming Young's modulus $E_b = 1.60 \times 10^4$ [MPa] and Poisson's ratio $\nu = 0.15$. Mortar joint interfaces have been represented by using normal and tangential stiffness values of $K_n = 36$ [N/mm³] and $K_t = 20$ [N/mm³] and the inelastic material parameters for mortar joints reported in Table 5. In the numerical simulations, a line load was applied near the three quarter span perpendicularly to the oblique span. Figure 25 shows the deformed shapes and the plastic work W_{cr1} contours at the end of the numerical simulations. In all the cases failure occurred because of the development of five main cracks along the mortar bed joints forming the typical 3D mechanism with five "hinges".

In Figure 26 the numerical load-displacement curves measured at quarter and three quarter span are plotted. The deep arch features the highest load capacity but a limited ductility, as it shows a softening behaviour just after reaching the maximum load. The arch *Skew 2* is characterised by the lowest load capacity but a notable ductility. Finally, the arch with a 1:8 rise-to-span ratio shows an intermediate response. In this case the analysis stopped because of convergence problems probably before reaching the maximum load.

4.2 Effects of masonry bond and brickwork defects

The three arches with different rise-to-span ratios presented in the previous section, has been analysed considering different masonry bonds and defective brickwork. In particular, the response of arches built with the stretcher or the header method (Figure 27) to connect the adjacent rings have been compared. Moreover the effects of defective mortar joints at the interface between the two brickwork rings have been taken into account. In this case, very small tensile strength and cohesion values for the nonlinear interface elements representing the circumferential mortar joints, $C_0 = 0.004$ N/mm² and $\sigma_{i0} = 0.002$ N/mm², have been employed.

Figure 28 displays the load-displacement curves for the arches with 1:4 rise-to-span ratio (*Skew2*) and the deformed shapes obtained from the different models and the corresponding plastic work contours are illustrated in Figure 29. Ring separation occurs only in the arch built according to the stretcher method and with defective circumferential mortar joints (weak joints). This arch is also characterised by the lowest load capacity, but it shows a significant ductility as depicted in Figure 28. All the other numerical curves are very close, being characterised by practically identical initial stiffness, ultimate load and alike post-peak response. This is reflected into very similar failure modes with cracks along the mortar bed joints without ring separation.

Besides, the influence of defective brickwork on the behaviour of skew arches with different rise-to-span ratio has been investigated. Numerical analyses have been carried for the arches *Skew 1*, *Skew 2* and *Skew 3* with stretcher bond. In all the cases, weak circumferential mortar joints have been considered. Figure 30 displays the load-displacement curves. As for the arches with header bond and strong joints (Figure 28), the deep arch is characterised by the highest load capacity, the arch *Skew 2*

by the lowest peak load, while the arch *Skew 3* shows an intermediate behaviour. However when compared against the numerical responses in Figure 30, all the curves for arches with stretcher bond and defective brickwork show higher ductility. This is associated with the development of ring separation which is coupled with cracks in the mortar bed joints.

The influence of mortar defects is summarised in Figure 31, where the ultimate loads and the secant stiffness values obtained in the analysis of arches with weak and strong joints are compared. It has been found that defective brickwork leading to ring separation significantly reduces both initial stiffness and load capacity, where the maximum reduction of stiffness (60%) and ultimate load (68%) have been calculated for the arch with 1:8 rise-to-span ratio.

4.3 Effects of the abutment stiffness

The influence of the abutment stiffness on the response of square arches was examined previously. The effects of potential deformations at the abutments on the response of an arch with 45° skew angle are investigated here. Also in this case, elastic interface elements are used to connect the arch springings to fixed supports so as to represent elastic abutments. Again, the reference skew arch (e.g. arch *Skew 2* with header bond) is analysed considering the four pairs of elastic normal and tangent stiffness values reported in Table 3. As for the square arches, the failure mechanism does not change by varying the stiffness at the lateral supports, and in all the cases arch failure is reached when a 3D hinge mechanism with five main cracks forms.

The load-displacement curves showing the response of the arch *Skew 2* with header bond are plotted in Figure 32a, where the displacement was measured at the centre line of the arch barrel at three quarter span. The variation of the ultimate load and the secant stiffness of the arch with the abutment stiffness are displayed in Figures 32b,c. As for the square arches, only the case with the lowest support stiffness ($K_{ab} = 1\text{N/mm}^3$) shows a significant change in the response, where an ultimate load (Figure 32b) and stiffness (Figure 32c) reduction can be observed. Moreover when compared to the responses of the arches with fixed supports or higher abutment stiffness, the skew arch with $K_{ab} = 1\text{N/mm}^3$ shows a more ductile behaviour with no softening.

4.4 Effects of abutment movement

The skew arch collapse mechanisms due to support movements have been investigated considering the response of the reference arch (arch *Skew 2* with header bond) subject to the three support displacement modes considered shown in Figure 11. The imposed displacements at the left support are plotted against the associated reaction forces in Figure 33. When the failure mechanisms due to horizontal support displacements develop, the associated reaction forces decrease showing a post-peak softening response. Conversely, in the case of differential vertical support movements, the reaction force remains almost constant for relative large displacement values. The three failure modes

are associated with the formation of cracks close to the two springings but not at the crown as in the case of square arches.

5. Conclusions

The response up to collapse of square and skew brick-masonry arches has been investigated using the validated mesoscale strategy described in (Zhang et al., 2016). The effects of (i) rise-to-span ratio, (ii) loading position, (iii) masonry bond and defects in the brickwork, (iv) abutment stiffness and (iv) support movements have been considered in the numerical study.

In the case of square arches, it has been found that the rise-to-span ratio significantly influences the arch response and failure mode; similar results have been obtained for arches with different span lengths. Deep arches or arches with intermediate rise-to-span ratio generally show a typical four hinge mechanisms, where cracks form in the mortar bed joints. On the other hand, shallow arches may fail by a combination of radial cracks and ring separation, where the latter usually develops abruptly, leading to a sudden release of elastic energy. Moreover, the arch failure mode strongly depends upon the load position, as line loads close to the springing may induce a brittle collapse, also for arches with high rise-to-span ratio. This may occur because of ring separation at the arch haunch close to the load, or shear sliding at a radial mortar joint. In the case of multi-ring square arches built according to the stretcher method and with weak circumferential mortar joints, the response is governed by ring separation, which develops gradually causing a progressive stiffness reduction but leading to a more ductile behaviour.

The numerical study on the effects of support stiffness has shown that the abutment deformability may significantly reduce the arch stiffness and the load capacity, but it seems that it does not modify the failure mode characteristics. Finally, the numerical simulations on the effects of abutment movements revealed that brick-masonry arches can support live loads also after the formation of a mechanism due to differential support movements. This additional load bearing capacity depends on the level of abutment displacement. Moreover, it has been shown that the accuracy of these predictions is associated with the ability of the numerical description in representing the quasi-brittle nature of masonry. In fact, the use of a material model which disregards de-cohesion and potential friction reduction (e.g. an elastic-plastic approximation with high fracture energy values) generally leads to unrealistic response predictions, where the arch capacity is insensitive to potential abutment movements.

As opposed to the analyses on square arches which were conducted using efficient strip mesoscale models, full 3D mesoscale descriptions have been adopted in the numerical studies for skew arches. It has been found that arches with different rise-to-span ratio are characterised by a different response, where the deep arch shows the highest load capacity, but the same failure mode with five cracks in the mortar bed joints. On the other hand, arches built according to the stretcher

method, and made up of detective brickwork with weak circumferential mortar bed joints, may also experience ring repairation. As for square arches, this reduces the initial stiffness but improves arch ductility. The response up to collapse of skew arches subject to abutment movements has been also analysed. It has been found that the failure modes under differential horizontal and vertical support displacements are characterised by the development of cracks at the two springing regions but not at the crown. In the numerical simulations, the use of detailed mesoscale descriptions representing the actual masonry bond has been critical, as it allowed for the first time shedding significant light on salient response characteristics of masonry arches, including skew arches, under a variety of loading conditions for which reduced models might provide an incorrect picture.

Future research will consider the effects of the interaction between masonry arch and backfill, which determines the load capacity of masonry bridges (Reccia et al., 2014) and masonry vaulted structures (Milani et al., 2014).

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Model	Rise - H (mm)	Span - L (mm)	No. of Rings	Thickness (mm)	Rise-to-span ratio
<i>Arch 1</i>	1500	3000	2	215.0	1:2
<i>Arch 2</i>	750	3000	2	215.0	1:4
<i>Arch 3</i>	375	3000	2	215.0	1:8
<i>Arch 4</i>	300	3000	2	215.0	1:10
<i>Arch 5</i>	4000	8000	5	537.5	1:2
<i>Arch 6</i>	2000	8000	5	537.5	1:4
<i>Arch 7</i>	1000	8000	5	537.5	1:8
<i>Arch 8</i>	800	8000	5	537.5	1:10

Table 2: Inelastic properties of nonlinear interface elements representing mortar joints in square arches

	Surface F_1	Surface Q_1	Surfaces F_2, Q_2
Mortar-brick interface	$C_0 = 0.07\text{N/mm}^2$ $\sigma_{i0} = 0.05\text{N/mm}^2$ $\tan\phi_0 = 0.50$ $G_{f1} = 0.02\text{N/mm}$ $G_{f2} = 0.125\text{N/mm}$	$C_0 = 0.07\text{N/mm}^2$ $\sigma_{i0} = 0.05\text{N/mm}^2$ $\tan\psi_0 = 0.00$ $G_{f1} = 0.02\text{N/mm}$ $G_{f2} = 0.125\text{N/mm}$	$D = 24.50\text{N/mm}^2$ $\sigma_{c0} = 24.50\text{N/mm}^2$ $\tan\theta_0 = 1.00$ $G_c = 5.00\text{N/mm}$

Table 3: Elastic stiffness values for the interface elements at the arch springings

Model	Normal stiffness, K_n (N/mm ³)	Tangent stiffness, K_t (N/mm ³)
K_{ab1}	1.00×10^0	1.00×10^0
K_{ab2}	1.00×10^1	1.00×10^1
K_{ab3}	1.00×10^2	1.00×10^2
K_{ab4}	1.00×10^4	1.00×10^4
Rigid	-	-

Table 4: Geometry of the skew arches with different rise-to-span ratio

Model	Rise - H (mm)	Direct span - L_d (mm)	No. of Rings	Thickness (mm)	Rise-to-span ratio
<i>Skew 1</i>	1500	3000	2	215	1:2
<i>Skew 2</i>	750	3000	2	215	1:4
<i>Skew 3</i>	375	3000	2	215	1:8

Table 5: Inelastic properties of nonlinear interface elements employed for skew arches

	Surface F_1	Surface Q_1	Surfaces F_2, Q_2
Mortar-brick interface	$C_0 = 0.14 \text{ N/mm}^2$ $\sigma_{t0} = 0.10 \text{ N/mm}^2$ $\tan \phi_0 = 0.60$ $G_{f1} = 0.12 \text{ N/mm}$ $G_{f2} = 0.15 \text{ N/mm}$	$C_0 = 0.14 \text{ N/mm}^2$ $\sigma_{t0} = 0.10 \text{ N/mm}^2$ $\tan \psi_0 = 0.00$ $G_{f1} = 0.12 \text{ N/mm}$ $G_{f2} = 0.15 \text{ N/mm}$	$D = 25.30 \text{ N/mm}^2$ $\sigma_{c0} = 25.30 \text{ N/mm}^2$ $\tan \theta_0 = 1.00$ $G_c = 5.00 \text{ N/mm}$