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Abstract

The prediction of induced ground movements and the potential damage to existing structures and services is paramount when building deep excavations in an urban environment. In order to obtain a reasonable prediction advanced constitutive models need to be employed, so that the behaviour of the soil can be adequately reproduced under different stress conditions. The calibration of such models is complex and often requires optimisation, as a large number of parameters need to be determined from the available ground investigation data, while also ensuring their consistency with the initial ground conditions. This paper presents the calibration process of advanced constitutive models employed to simulate the excavation of the Ivens shaft in Lisbon, Portugal. The data from both historic and new laboratory and field testing is employed in the calibration procedure. In order to assess and validate the suitability of the derived model parameters, a back-analysis of the nearby Baixa-Chiado metro station excavation is carried out and its results are presented and discussed.

Keywords	constitutive models; parameters calibration; numerical modelling; back analysis
Taxonomy	Geomechanics, Soil Mechanics
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DERIVATION OF MODEL PARAMETERS FOR NUMERICAL ANALYSIS OF THE IVENS SHAFT EXCAVATION

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ABSTRACT

The prediction of induced ground movements and the potential damage to existing structures and services is paramount when building deep excavations in an urban environment. In order to obtain a reasonable prediction advanced constitutive models need to be employed, so that the behaviour of the soil can be adequately reproduced under different stress conditions. The calibration of such models is complex and often requires optimisation, as a large number of parameters need to be determined from the available ground investigation data, while also ensuring their consistency with the initial ground conditions. This paper presents the calibration process of advanced constitutive models employed to simulate the excavation of the Ivens shaft in Lisbon, Portugal. The data from both historic and new laboratory and field testing is employed in the calibration procedure. In order to assess and validate the suitability of the derived model parameters, a back-analysis of the nearby Baixa-Chiado metro station excavation is carried out and its results are presented and discussed.

1. INTRODUCTION

The calibration of advanced constitutive models is an important stage in numerical analyses of geotechnical structures, as it establishes a degree of idealisation of real soil behaviour. The purpose of the calibration is to determine a set of parameters that best fit a set of real soil test data to a pre-defined set of mathematical equations - the chosen constitutive model. Naturally, the difficulty in defining those parameters is directly related to the quality of the available ground investigation data and the suitability and complexity of the chosen constitutive model. The model selection usually depends on the problem being analysed, on the data available for its calibration and on the models available in the software employed in the numerical analysis. Simple models have the advantage of requiring a limited number of parameters, which are easier to calibrate and have a clear physical meaning, but tend to describe poorly the complexity of real soil behaviour. In contrast, some of the most advanced constitutive models can describe accurately many facets of real soil behaviour but they often require a large number of parameters, many of which are difficult to derive from the available experimental data. In this case, the definition of a calibration strategy is paramount, particularly when the fitting is performed based on a trial and error procedure and there are multiple parameters that lack clear meaning. As such, they cannot be directly estimated from experimental data, thus needing optimisation techniques (Basudhar, 2008) that can improve significantly the goodness of fit and, simultaneously, minimise the time spent on the calibration process. Currently, one of the most popular optimisation methods involves the use of genetic algorithms (GAs) due to their simplicity, robustness and efficiency (Simpson & Priest, 1993) in dealing with multiple parameters (Pal et al., 1996; Feng et al., 2002; Zolfaghari et al., 2005; Levasseur et al., 2008; Pedroso & Williams, 2011). However, it should be noted that GAs do not guarantee the exact identification of the optimum solution since the method is by definition evolving and improving through an unlimited series of iterations (Papon et al., 2012).

The objective of this paper is to present the procedure employed in the calibration of advanced constitutive models considered as appropriate for modelling the real behaviour of typical Lisbon soils, where an excavation of a deep shaft is to take place. Focus is given to the methodology and assumptions adopted in the calibration process and to the relative weights given to the available laboratory and field testing results. GAs are employed to optimise the best set of parameters, which are subsequently validated through a back-analysis study of the nearby Baixa-Chiado metro station excavation.

2. GEOTECHNICAL INVESTIGATION AT THE IVENS SHAFT SITE

The Baixa-Chiado metro station is considered one of the most important interface stations on the Lisbon Metro network, as it accommodates the interchange between the Green and the Blue lines and is located near downtown Lisbon (Fig. 1). Its original design includes three exit tunnels but, mainly due to financial constraints, only two, Chiado and Crucifixo, were initially built (Postiglione et al., 1997). A third exit, through a 40 m deep Ivens shaft located in the backyard of the Quintão building (Fig. 1), is currently being considered for construction. In order to assess the impact of the Ivens shaft excavation on the existing surrounding structures and services, several ground investigations have been carried out. The main objective of the latest investigation (Pedro, 2013; Pedro et al., 2017) was to provide information regarding the small strain stiffness characteristics of the local soils, as this information is necessary for the serviceability assessment of the shaft excavation. In addition to the retrieval of intact samples for laboratory testing, mainly in a triaxial apparatus equipped with bender elements and local strain instrumentation, this investigation also included down-hole seismic tests to 28 m depth to characterise the in situ initial shear stiffness of the soil formations. The details of the complete site investigation and data interpretation are given in Pedro (2013) and Pedro et al. (2017), including any historic data where appropriate (LNEC (1996a, b, c, d, e) Marques, 1998; Cenorgeo, 2008; Guedes de Melo, 2008). This information is the basis for calibration of the constitutive models described in this paper and subsequently used in the analyses of the Ivens shaft excavation.

Detailed examination of several boreholes identified the lithological profile at the Ivens shaft site shown in Fig. 2. The top 5 to 6 m is a loose sandy fill underlain by two principal formations of the ground profile, the “*Areolas da Estefânia*” (AE) formation down to 35 m which is more sandy in nature, followed by the more clayey “*Argilas e Calcários dos Prazeres*” (AP) formation. The material between 35 and 37 m depth is considered a different unit (TAP) within the AP formation, as it contains a different degree of cementation. At depths between 12 and 17 m within the AE formation there is a 5 m thick layer of fossiliferous limestone. Despite significant variations within it, the AE formation is usually considered as a single unit in the literature and this was also confirmed by the Pedro et al. (2017) data analysis. Clearly, the limestone layer is considered independently due to its behaviour being more closely related to that of a rock than a soil. The water table is located approximately at 27 m depth below the ground surface.

Fig. 1 - Location of the Baixa-Chiado station in downtown Lisbon (modified from Google Earth)

Fig. 2 - Ivens shaft soil profile

3. CHOSEN CONSTITUTIVE MODELS

The combination of a nonlinear elastic small strain stiffness model and a linear/nonlinear Mohr-Coulomb plastic model is employed to represent the behaviour of soils in the Ivens shaft ground profile. The generalised small strain stiffness model IC.G3S (Taborda & Zdravkovic, 2012; Taborda et al., 2016) assumes a nonlinear relationship between the tangent shear and bulk stiffness of the soil and its stress and strain states. This model replaces the standard linear elastic pre-yield formulation of a Mohr-Coulomb model and is necessary for accurate predictions of ground movements at serviceability states. It is employed in simulating the small strain stiffness behaviour of both the AE and AP formations.

In terms of plastic behaviour, the nonlinear Mohr-Coulomb (NMC) model adopts a nonlinear relationship between the soil's drained strength (angle of shearing resistance) and stress level, as proposed by Maksimovic (1989). This model is chosen to represent the failure conditions of the AE formation, as triaxial tests were performed over a wide range of stress levels, thus rendering the fitting of a linear failure envelope less accurate. It is also used to simulate the behaviour of the limestone layer, whereas a linear Mohr-Coulomb (LMC) model is employed to capture the yielding of the AP formation.

All models are implemented in the finite element code ICFEP (Potts & Zdravković, 1999), which was the main analysis tool in subsequent simulations of the Ivens shaft excavation. The main equations of all models are given in the Appendix.

4. CALIBRATION OF GEOTECHNICAL PARAMETERS

For advanced constitutive models with a large number of parameters, establishing a calibration strategy is very important. Using a simple trial and error procedure is time consuming, subjective and does not guarantee that the best set of parameters is selected, while the use of optimisation techniques operating as “black boxes” can produce misleading and unrealistic results, as demonstrated by Taborda et al. (2010). Consequently, a hierarchical approach, where each parameter is estimated sequentially and the order is set by its physical relevance and by its independency from other values (Loukidis & Salgado, 2009; Gallipoli et al., 2010), was adopted here. First, a main hierarchical order was established for different components that characterise soil behaviour, starting with strength properties and followed by stiffness parameters. Inside each component a second hierarchical order was defined so that the parameters with physical meaning were defined first, directly or through the least square method, followed by the mathematical constants which were determined using a genetic algorithm (GA).

The GA is an optimisation method inspired by Darwin’s theory of evolution, where only the fittest solutions survive (Holland, 1975). The method requires a definition of a fitness function, which quantifies the proximity between the achieved and the optimum solution. Its application to the calibration of constitutive models is amply described in the literature (e.g. Pal et al., 1996; Levasseur et al., 2008). The specific GA implementation applied in the present paper (NT.CFit v3.5) is the same as that described in Taborda et al. (2010), Azeiteiro et al. (2009) and Taborda et al. (2008).

4.1. STRENGTH PARAMETERS

i) Areolas da Estefânia (AE) formation

As explained above, the nonlinear Mohr-Coulomb (NMC) model was selected to define the strength of the AE formation. As shown in the Appendix, the model requires calibration of the effective cohesion c' , plus three parameters which define the magnitude of the angle of shearing resistance ϕ' , namely ϕ'_b , $\Delta\phi'$ and p'_N , representing the base angle of shearing resistance, maximum increase of ϕ' at $p' = 0$ and the magnitude of the mean effective stress at which $\phi' = \phi'_b + \Delta\phi'/2$, respectively. A total of 23 experimental data points were used (see Fig. 3), representing the maximum J/p' ratio from each stress path (where J is a generalised deviatoric stress, as defined in the Appendix). Fourteen of these are from the new triaxial tests interpreted in Pedro et al. (2017), while nine are the historic triaxial data (Cenorgio (2008)). The parameters derived with the GA calculations are summarised in Table 1, while Fig. 3 shows a good agreement between the data and the resulting nonlinear envelopes, both in triaxial compression (TXC) and extension (TXE) loading conditions.

The limestone layer, due to its rock-like nature, is analysed separately from the AE formation, using the same nonlinear Mohr-Coulomb model to define its yield surface. This layer has a significant presence of fossils and is highly fractured and weathered, hence being classified as poor according to RMR classification system (Bieniawski, 1973). Testing of limestone samples was limited to conventional uniaxial compression tests (14 in total) and rock index tests (Pedro, 2013) that enabled the characterisation of the strength of the intact rock using the nonlinear Hoek & Brown (1980) failure envelope (see Appendix) in the first instance. The nonlinear Mohr-Coulomb envelope was then calibrated with respect to the Hoek & Brown envelope. The parameters used to describe the limestone strength in the Hoek & Brown model are the uniaxial compression strength, σ_{ci} , estimated at 6.5 MPa from the 14 tests; the constant $m_i = 8$, which depends on the texture of the intact rock (Marinos & Hoek, 2000); the geological strength index $GSI = 35$ for a rock with a blocky structure; and the disturbance factor $D=0$, as no significant disturbance was assumed in the limestone due to shaft excavation (e.g. there was no blasting of the limestone). Given the poor properties of the intact rock,

and accounting for fractures and weathering observed on site, a low strength envelope, without a cohesion intercept, is estimated for the Limestone using the Hoek & Brown model. As this model is defined in terms of the principal effective stresses σ'_1 and σ'_3 , an earth pressure coefficient of 1 is assumed to convert the stresses to the generalised p' - J space for the nonlinear Mohr-Coulomb envelope. The resulting model parameters, derived from GA calculations, are summarised in Table 1. Fig. 3 indicates a very good agreement between the two strength envelopes. It is evident that the compressive strength of the limestone is higher than that of the AE soil at mean effective stress levels lower than 1000 kPa, then becoming lower at higher stresses.

Table 1 - Nonlinear Mohr-Coulomb failure envelope parameters for the AE formation

Formation	c' (kPa)	ϕ'_b (°)	$\Delta\phi'$ (°)	p'_N (kPa)
"Areolas da Estefânia" (AE)	0.0	35.3	14.7	160.0
Limestone	0.0	19.2	55.0	480.0

Fig. 3 - Fitting of the nonlinear Mohr-Coulomb failure criterion to the AE formation data

ii) Argilas e Calcários dos Prazeres (AP) formation

In the case of the AP formation, a conventional Mohr-Coulomb model with a linear failure envelope was adopted, principally due to the significant scatter observed within the triaxial test results (Fig. 4). A total of 29 data points corresponding to new (Pedro et al., 2017) and historic triaxial tests (Marques (1998); Cenorgeo, 2008) were used in the calibration. The data analysis in Pedro et al. (2017) showed distinct characteristics of the top layer of the AP formation (TAP) and therefore a subdivision was considered when evaluating the strength of the formation. The black squares in Fig. 4 represent the 6 TXC tests on samples deeper than 37 m, while the 13 hollow squares (TXC) and 10 triangles (TXE) are the results of tests performed on TAP samples from 35 to 37m depths. Despite the evident differences discussed in Pedro et al. (2017), the fitting of a linear strength envelope to the two sets of data revealed almost identical gradients of the envelopes in compression, corresponding to an angle of shearing resistance of 28.4° . The same angle is interpreted from the TAP TXE tests and adopted for the whole AP formation. The interpreted effective cohesion in the AP formation is 200 kPa, in contrast to the 40 kPa estimated for the TAP layer (Fig. 4). The similarity in the angle of shearing resistance is related to the identical geological origin of the formations, while the difference in cohesion can be attributed to the different degrees of cementation. The quality of the fitting shown in Fig. 4, both in compression and in extension, is reasonable, with the coefficient of determination, R^2 , being 0.83 or higher.

Fig. 4 - Fitting of the Mohr-Coulomb failure criterion to the AP formation data

4.2. DILATION

Dilation ψ is an input parameter for both linear and nonlinear Mohr-Coulomb models. Dilatancy can be expressed as the ratio of the axial and volumetric strain rates measured in drained triaxial tests through Equation 1.

$$\tan\left(\frac{|\Delta\epsilon_a|}{\Delta\epsilon_v}\right) = \frac{2 \cdot \sin(\psi)}{1 - \sin(\psi)} \quad 1$$

In order to evaluate the dilation angle of the AE formation the volumetric response of the 9 new drained triaxial tests (Pedro et al., 2017) was plotted in Fig. 5a). The angle of dilation was assumed to correspond to that part of the curves where a constant $|\Delta\epsilon_a|/\Delta\epsilon_v$ gradient was observed. For the majority of tests this gradient is similar, resulting in a value of ψ of 11.6° , determined using equation 1. A similar procedure was applied to the 3 new drained triaxial tests in the AP formation (Fig. 5b)). Since this formation was divided into two, there was only one test in the TAP layer, giving a value of

$\psi = 4.0^\circ$, which was adopted as representative of the layer. The 2 tests from the rest of the AP layer, despite having some differences in their volumetric behaviour, have a similar dilation gradient, resulting in an average value of $\psi = 14.2^\circ$ for this layer.

Fig. 5 – Estimation of the dilation angle: a) AE formation; b) AP formation

4.3. STIFFNESS PARAMETERS

In the calibration of the stiffness parameters for the IC.G3S model only the new test data, reported and interpreted in Pedro (2013) and Pedro et al. (2017), are employed, as limited information on the small strain behaviour is available from the historic data. However, even the new data exhibited discrepancies in the tangent stiffness moduli degradation curves derived from laboratory (bender element – BE and triaxial – TX), and field (downhole – DH and self-boring pressuremeter – SBPT) experiments performed in the AE and AP formations. Fig. 6 is a summary interpretation of the ranges of the tangent shear stiffness from different experiments (Pedro et al., 2017). It was therefore decided to calibrate two different sets of parameters for each formation that reflected the collected data according to their origin (i.e. curves labelled “*Field*” and “*Laboratory*” in Fig. 6). Subsequently, a third set of parameters for each formation was also derived with the objective of integrating the results from all tests. The suitability of these three sets of parameters is subsequently investigated.

Fig. 6 – Comparison of the normalised tangent shear modulus obtained from the DH, SBP, BE and TX tests for the AE and AP formations (Pedro et al., 2017)

i) Calibration of the maximum shear stiffness, $G_{\max}$

The first stage of stiffness calibration is determination of the parameters that define the maximum shear modulus, as per Equation A.8: a constant G_0 , a function describing the influence of the void ratio $f(e)$, and a parameter m_G defining the nonlinearity of the $G_{\max}$ dependency on p' . The reference mean effective stress is taken as $p'_{\text{ref}} = 100\text{kPa}$. Two sets of data are available for this calibration: from the laboratory Bender element (BE) tests and from downhole (DH) seismic field test. From Fig. 7, the range of interpreted $G_{\max}$ values from DH tests is high, as is the difference between the two test types. As discussed in Pedro et al. (2017), the former is attributed to a degree of variability within the two soil formations, while the latter is most likely related to a degree of disturbance (i.e. loss of cementation) during sampling for laboratory testing.

The results of the of BE tests at different stress levels in the AE formation suggest a marginal effect of the void ratio on the small strain shear stiffness of this material (Pedro, 2013). Consequently this variable is not considered in the calibration. Similarly, as only one BE test was performed in the AP

formation, the effect of void ratio on the stiffness of this material could not be characterised and was therefore disregarded. As a result only G_0 and m_G parameters remain for calibration and they are estimated directly using the least squares method. These values are displayed in Table 2, with a good quality of the fitting to the bender element data being evident in Fig. 7.

To calibrate the $G_{\max}$ for the AE formation from DH tests the values measured in the top fill layer and in the limestone are discarded (open circles). A nonlinear regression is then carried out on the remaining data points (black circles), assuming that the value of m_G established from the BE tests applies also to DH tests. In essence, this facilitates a direct comparison between field and laboratory test results and corresponds to assuming that the influence of the mean effective stress on the shear modulus is not affected by the disturbance factors. The resulting calibrated $G_{\max}$ profile is depicted in Fig. 7 as a dashed line. The value obtained for G_0 is presented in Table 2 and is 4.3 times larger than the value calibrated from BE tests. A very small difference in the shear wave velocity between the limestone layer (about 750 m/s) and the remaining AE formation (500 m/s) is a further evidence of the poor quality limestone rock.

The DH test did not extend into the AP formation. Consequently, as for the AE formation, it is assumed that the exponent m_G in Equation A.8 is equal to that derived from the BE tests. As for the G_0 constant, the same ratio between field and laboratory results obtained for the AE formation (4.3) was applied to the AP formation, resulting in a value of $G_0 = 287025$ kPa for this material.

Table 2 – Parameters for the definition of the maximum shear stiffness of the AE formation

Scenario	Formation	G_0 (kPa)	m_G	p'_{ref} (kPa)
BE	AE	72450	0.509	100
	AP	66750	0.671	100
DH	AE	311535	0.509	100
	AP	287025	0.671	100

Fig. 7 – Fitting of the maximum shear modulus of the AE and AP formation for the laboratory and field data

ii) Calibration of the shear stiffness degradation curve

The next stage in the calibration process is the derivation of the parameters describing the stiffness degradation with increasing strain (Equation A.10). The challenge here is to derive representative degradation curves considering the discrepancies between the laboratory and field data depicted in Fig. 6. Although there are no definitive recommendations on how to reconcile such differences, an approach proposed by Tatsuoka & Shibuya (1991) is adopted here. This approach involves the following steps: (1) estimate the initial maximum shear modulus, G_{max} , from seismic field tests (e. g. DH); (2) determine the correction factor as a ratio between the G_{max} measured in the seismic field tests and that measured in the laboratory tests (e. g. BE); (3) correct the stiffness degradation curves determined in triaxial tests by multiplying them with a correction factor, named a *disturbance factor* (DF). Tatsuoka & Shibuya (1991) also suggested that this disturbance factor could vary with strain, although no particular relationship was proposed for such a variation. Therefore for both AP and AE formations three distinct scenarios were analysed, as depicted in Fig. 8. In the first scenario (*Laboratory*) no correction (i.e. $DF=1$) was applied to the stiffness degradation curves derived from triaxial tests. In the second scenario (*Field*) the complete stiffness degradation curve obtained from

triaxial tests is multiplied by the maximum correction factor. Lastly, a *Variable* scenario was applied in which the DF was allowed to vary linearly with the logarithm of deviatoric strain, reducing from its maximum value at very small strains (lower than 0.001 %) to 1.0 at 0.1 % strain (Fig. 8).

In order to reproduce the small strain stiffness behaviour of the AE formation the results of 6 triaxial tests (summarised as AE-TX range in Fig. 6), 3 in compression and 3 in extension, following a stress-path where the mean effective stress (p') was maintained constant during shearing, are used in the calibration process (see Pedro et al. (2017) for details of the tests). As the tests were performed at different stress levels (p' of 130, 300 and 400 kPa) it is also possible to calibrate the influence of the mean effective stress on the shear modulus degradation, as defined by Equation A.12. The calibration of the 5 model parameters ($R_{G,min}$, b , a_0 , a_1 , and a_2) is performed using the GA calculations.

Fig. 9 shows the tangent shear modulus, G_{tan} , curves for each of the 6 $p'=constant$ tests. Pedro et al. (2017) concluded that pairs of degradation curves from compression and extension tests for each of the three stress levels were similar and hence a single stiffness degradation curve is derived in Fig. 9 for each stress level and for each of the three fitting scenarios described above. The maximum DF of 4.3 is applied, as derived from the evaluation of the G_{max} for the AE soil. The parameters corresponding to the best fit for each of the 3 scenarios are listed in Table 3 and the calibrated curves in Fig. 9 show a good fit with the test data.

Fig. 8 – Proposed evolution of the correction factor with the increase of deviatoric strain

Table 3 – Parameters of the shear stiffness degradation curves of the AE and AP formations

Scenario	Formation	Shear stiffness degradation				
		a_0	a_1	a_2	b	$R_{G,min}$
Laboratory	AE	2.80E-8	4.24E-5	1.193	0.690	0.001
	TAP	6.68E-6	-	-	0.390	0.001
	AP	3.60E-5	-	-	0.438	0.020
Field	AE	2.42E-8	4.35E-5	1.200	0.666	0.001
	TAP	6.68E-6	-	-	0.390	0.001
	AP	3.60E-5	-	-	0.438	0.020
Variable	AE	2.43E-8	1.30E-5	0.746	0.778	0.001
	TAP	2.15E-6	-	-	0.530	0.001
	AP	1.83E-5	-	-	0.851	0.020

Fig. 9 – Fitting of the shear stiffness degradation curve of the AE formation: a) Laboratory; b) Field; c) Variable

A similar procedure is adopted for the AP formation. However, in this case only 3 triaxial compression tests (summarised in Fig. 6 as AP-TX range), 2 performed in the AP layer and 1 in the TAP layer, were available (Pedro, 2013). These results enabled a distinction to be made between the two layers but did not allow the evaluation of the influence of the mean effective stress, as all three tests were performed at the same stress level ($p'=480$ kPa). Consequently, the calibration procedure allowed only 3 parameters ($R_{G,min}$, b , a_0) per layer to be determined, with the values in Table 3 returned by the optimisation program for the 3 fitting scenarios. In Fig. 10 reasonable agreement is shown between the stiffness degradation curves derived for both layers and those measured in the corresponding triaxial tests, particularly when considering the scatter in the test data.

Fig. 10 – Fitting of the shear stiffness degradation curve of the AP formation: a) *Laboratory*; b) *Field*; c) *Variable*

iii) Calibration of the maximum bulk stiffness and its degradation curve

The last stage in the calibration sequence is the evaluation of the maximum bulk stiffness (Equation A.9) and of its degradation curve (Equation A.11). Since none of the field or laboratory tests enabled the characterisation of the variation of the bulk modulus with mean effective stress, it was assumed that both shear and bulk moduli increased with this quantity in a similar manner. As a result, the value of the exponent in Equation A.9 (m_K) was taken as equal to that determined for the shear modulus (m_G) (Table 4). Conversely, the estimation of the K_0 constant in Equation A.9 was performed simultaneously with the calibration of the bulk stiffness degradation curve (Equation A.11), since the applied GA software has the ability to consider Equations A.9 and A.11 together. In total, each calibration required the definition of 6 parameters (K_0 , $R_{K,min}$, s , r_0 , r_1 , and r_2) for the AE formation and only 4 for the AP formation (K_0 , $R_{K,min}$, s , r_0). A total of 3 isotropic compression tests are used in the calibration of the AE formation, while for the AP formation only one test is available, hence the bulk stiffness is assumed the same for both the TAP and AP layers (see Pedro (2013) for full description of the tests). It should be noted that both loading and unloading paths of the tests (including the unloading-reloading loops) were considered in the fitting process.

The same 3 scenarios proposed for the shear stiffness degradation were also adopted in the calibration of the tangent bulk stiffness, following the procedure of Tatsuoka & Shibuya (1991). In the *Laboratory* scenario only the results from isotropic compression tests were considered while for the

Field and *Variable* scenarios the same correction factors established for the shear stiffness were applied (Fig. 8), as no field data was available to characterise the bulk stiffness.

The values of the bulk stiffness degradation parameters fitted using the GA program are presented in Table 4. Fig. 11 and Fig. 12 show comparisons of the calibrated tangent bulk modulus, K_{tan} , curves with the laboratory data for one isotropic compression test from each of the AE and AP formations, respectively, showing again a good degree of agreement with the test data.

As the shear and bulk moduli are calibrated independently, additional statistical analyses were performed for the calibrated range of stresses and strains to check the resulting values of the Poisson's ratio. A concentration of values in the interval 0.10 to 0.15 was found in all cases.

Table 4 – Bulk stiffness parameters of the AE and AP formations

Scenario	Formation	Maximum bulk stiffness		Bulk stiffness degradation				
		K_0 (kPa)	m_K	r_0	r_1	r_2	s	$R_{K,min}$
Laboratory	AE	70604	0.509	1.16E-7	2.73E-4	1.684	0.505	0.020
	AP	61372	0.671	5.29E-4	-	-	0.495	0.010
Field	AE	303597	0.509	1.01E-8	2.62E-4	1.699	0.487	0.020
	AP	263900	0.671	5.29E-4	-	-	0.495	0.010
Variable	AE	303597	0.509	5.77E-9	2.15E-5	0.736	0.679	0.020
	AP	263900	0.671	4.71E-5	-	-	0.757	0.020

Fig. 11 – Fitting of the bulk stiffness degradation curve of the AE formation: a) *Laboratory*; b) *Field*; c) *Variable*

Fig. 12 – Fitting of the bulk stiffness degradation curve of the AP formation: a) Laboratory; b) Field; c) Variable

5. VALIDATION OF THE GEOTECHNICAL PARAMETERS THROUGH BACK-ANALYSIS OF THE EXCAVATION OF THE BAIXA-CHIADO STATION

5.1. INITIAL AND BOUNDARY CONDITIONS

In order to assess the adopted assumptions and to determine which of the three scenarios for the stiffness of the AE and AP formations is the most appropriate for the numerical modelling of the Ivens shaft excavation, a validation study is performed next. This study is a back-analysis of the excavation of the Baixa-Chiado station which is adjacent to the proposed Ivens shaft. The station has two tunnels, with a maximum dimension of 17.6m (height) and about 28m soil cover. The section selected for establishing the geometry of the numerical model is the P36 alignment, located in Garrett Street (Fig. 1), as this was the location of monitoring instrumentation for ground surface settlements above the tunnels (Kuwajima et al., 2004).

A 2D plane strain coupled consolidation numerical model is developed for the finite element (FE) software ICFEP and the corresponding FE mesh is shown in Fig. 13, employing 8-noded isoparametric quadrilateral elements. Fig. 13 also shows the assumed ground profile, which is similar to that at the Ivens shaft site, having a thinner fill layer (2 m) and the ground water level at 23m depth. To establish the initial stresses in the model the vertical stresses are calculated from the estimated bulk unit weights, 18 kN/m³ for the fill and 20 kN/m³ for the AP and AE formations (Pedro, 2013). The pore

water pressure is assumed hydrostatic below the ground water table and zero above it. The horizontal effective stresses are then based on the K_0 profile shown in Fig. 13, which was interpreted from the self-boring pressuremeter tests (see Pedro et al. (2017)).

In terms of boundary conditions, no horizontal displacements are allowed along the vertical boundaries of the mesh, while at the bottom boundary both horizontal and vertical displacements are restricted. As for the hydraulic boundary conditions, it is assumed that the initial pore water pressure profile remains constant along the far vertical boundaries throughout the analysis, and that the bottom of the mesh and ground surface are impermeable.

The analyses steps involve detailed construction of the two tunnels, as depicted in Fig. 14. The East gallery was constructed first, followed by the West gallery where a different construction sequence was adopted (Pinto & Costa, 1997). In order to simulate the three-dimensional effects of tunnel face excavation and to take into account the delay in the construction of the sprayed concrete lining, a relief factor (Moller, 2006) of 50 % is employed. Consequently, every excavation stage (odd numbers in Fig. 14) is performed over 10 analysis steps, with the 40 cm-thick lining being constructed in the fifth step of that stage (even numbers in Fig. 14), and therefore being subjected to the remaining 50 % of ground unloading (steps 6 to 10 of each stage). The lining is simulated assuming linear elastic behaviour, with a Poisson's ratio of 0.25 and a Young's modulus of 8 GPa.

Additionally, a precipitation boundary condition (Potts & Zdravković, 1999) is specified around the perimeter boundary of each excavated region. This is a dual boundary condition according to which no flow across the boundary is allowed whenever negative pore water pressures were observed in the soil immediately adjacent to the excavation boundary, thus preventing what would be an unrealistic supply of water from the tunnel to the soil. Conversely, if positive values are calculated, a zero value of pore water pressure is prescribed on the excavation boundary, allowing drainage from the soil into the tunnel. The precipitation boundary condition is maintained until the installation of the secondary lining, when it is replaced by a no-flow boundary condition in order to reflect the impermeable nature

of the secondary lining. The time step adopted in the analyses ensured a total duration of 3 years for construction of the two galleries (Salgueiro Amaral, 2006). The permeability was set to $2 \cdot 10^{-5}$ m/s for the AE formation and $2 \cdot 10^{-10}$ m/s for the AP formation.

Finally, the cohesionless fill was simulated with a linear Mohr-Coulomb model, having an angle of shear resistance of 35° , a Young's modulus of 20 MPa and a Poisson's ratio of 0.3.

Fig. 13 - Finite element mesh used in the 2D analysis of the Baixa-Chiado station and K_0 profile adopted

Fig. 14 - Construction sequence adopted for each gallery of the Baixa-Chiado station

5.2. DISCUSSION OF THE RESULTS

The ground surface settlement troughs obtained at the end of the excavation of the East and West galleries are depicted in Fig. 15 for the three scenarios of the small strain stiffness behaviour presented in the previous section. The figure also shows the monitoring results at alignment P36 for the same construction stages (black line). The results show a clear difference between the analyses, with values of the maximum settlement varying more than 5 times. As expected, the smallest settlements are

predicted by the *Field* approach and their comparison with the monitoring data clearly shows that this stiffness is significantly overestimated. In contrast, the *Laboratory* scenario results in a maximum deformation which is significantly higher than the value measured in the field. The predicted settlement trough is also much wider in this analysis. Finally, the *Variable* approach provides the best prediction of the monitoring data, both in terms of maximum settlement and width of the settlement trough.

Fig. 15 – Comparison of predicted and measured settlement trough during construction of the Baixa-Chiado station: a) after construction of East gallery; b) after construction of the West gallery

Given the overall agreement between measurements and predictions for the *Variable* scenario, further investigation is carried out by assessing its ability to reproduce the observed movements for intermediate construction stages. These are presented in Fig. 16 and in Fig. 17 for the East and West galleries, respectively. The results show that the adopted stiffness variation can capture very well the entire construction sequence and not just the final excavation stages of each gallery. In particular the model is capable of reproducing the sudden increase in displacements that occurs when the top headings of the two galleries are excavated, as well as the overall shift of the settlement trough towards the West gallery upon its construction.

For the subsequent Ivens shaft analyses the *Variable* set of parameters was adopted for the small strain stiffness model IC.3SG, for both the AE and AP formations.

Fig. 16 - Comparison of the predicted and measured ground surface settlement troughs at different excavation stages of the East gallery at the Baixa-Chiado station for the Variable stiffness scenario

Fig. 17 - Comparison of the predicted and measured ground surface settlement troughs obtained for the different excavation stages of the West gallery of the Baixa-Chiado station for the *Variable* stiffness scenario

CONCLUSIONS

The selection of an appropriate constitutive model to reproduce the behaviour of a soil is challenging.

Usually it presents a balance between, on the one hand, the model's complexity and ability to simulate

different aspects of soil behaviour, and, on the other hand, the quality and suitability of the experimental soil data. The available data is rarely ideal and its interpretation and model calibration therefore require a methodical approach and significant engineering judgement.

This paper has presented a hierarchical approach to model calibration, using a case study of an excavation of a deep shaft (the Ivens shaft) in the ground conditions of Lisbon, Portugal. Each parameter was estimated sequentially according to its physical relevance and its independency from other parameters. First, parameters that can be assessed directly from the experimental data were determined. Subsequently, the remaining mathematical constants were estimated using optimisation through genetic algorithms.

The paper also demonstrates that, despite the breadth of the available experimental data, it is often necessary to make assumptions during the calibration process. This was particularly evident in the assessment of the shear and bulk stiffness of the Lisbon formations, due to significant discrepancies between laboratory and field data. A practical and consistent approach was adopted, with a good measure of engineering judgement, which provided a possible range of small strain stiffness variation.

Finally, as the small strain stiffness is the primary aspect of soil behaviour for the prediction of ground movements during the shaft excavation, the decision on stiffness degradation was made with a back-analysis of a previous excavation (the Baixa-Chiado metro station) in the same ground profile, for which the measurements of ground movements existed. The agreement between the measured and predicted settlement profiles, applying the interpreted different scenarios of stiffness degradation, enabled the final choice of model parameters that describe the soils' shear and bulk stiffness.

APPENDIX

LINEAR MOHR-COULOMB MODEL (LMC)

The linear Mohr-Coulomb model employed in this study is a generalised version (Potts & Zdravković, 1999) of the Mohr-Coulomb failure criterion, described in the $p' - J - \theta$ system of invariants, where p' is a mean effective stress and $p' = (\sigma'_1 + \sigma'_2 + \sigma'_3)/3$; J is a generalised deviatoric stress, with $J = \sqrt{(1/6)[(\sigma'_1 - \sigma'_2)^2 + (\sigma'_1 - \sigma'_3)^2 + (\sigma'_2 - \sigma'_3)^2]}$; and θ is the Lode's angle, with $\theta = \tan^{-1} \left[\frac{1}{\sqrt{3}} \left(2 \frac{(\sigma'_2 - \sigma'_3)}{(\sigma'_1 - \sigma'_3)} - 1 \right) \right]$. In triaxial loading conditions $J = q/\sqrt{3}$.

The LMC yield surface can be written in the following form:

$$F = J - \left(\frac{c'}{\tan \phi'} - p' \right) \cdot g(\theta) = 0 \quad \text{A.1}$$

where c' is the apparent cohesion, ϕ' is an angle of shearing resistance and $g(\theta)$ is an inclination of the yield surface in the $p' - J$ plane, in the form:

$$g(\theta) = \frac{\sin \phi'}{\cos \theta - \frac{\sin \theta \cdot \sin \phi'}{\sqrt{3}}} \quad \text{A.2}$$

The model is non-associated, adopting the plastic potential surface of a similar form as that in Eq. A.1, but with angle of dilation, ψ , instead of ϕ' . The relationship between the two is $0 \leq \psi \leq \phi'$.

The pre-yield behaviour of the model is linear elastic, defined with a Young's modulus, E' , and a Poisson's ratio, ν . However, the model's formulation in ICFEP allows its coupling with a nonlinear-elastic small strain stiffness model which effectively replaces the original linear-elastic formulation before yield. The small strain stiffness model used here is described subsequently.

NONLINEAR MOHR-COULOMB MODEL (NMC)

This model is an extension of the LMC model, adopting a nonlinear failure envelope. This is achieved by implementing a variation of strength with stress level as proposed by Maksimovic (1989), in the form:

$$\phi' = \phi'_b + \frac{\Delta\phi'}{1 + \frac{p'}{p'_N}}$$

A.3

where ϕ'_b is a base angle of shearing resistance, $\Delta\phi'$ is the maximum strength increase, p'_N is a magnitude of the mean effective stress at which $\phi' = \phi'_b + \Delta\phi'/2$, and p' is a current value of the mean effective stress (see Fig. 18). At $p' = 0$ the magnitude of ϕ' is at its maximum, with $\phi'_o = \phi'_b + \Delta\phi'$, whereas at very high stresses it approaches ϕ'_b , which could be taken as a critical state value of ϕ' .

Fig. 18 - Nonlinear variation of ϕ' with p'

The NMC model is generalised in the same system of invariants as the LMC model, with a nonlinear yield surface depicted in Fig. 19. It adopts the same form of the yield function as given by Eqs. A.1 and A.2 and, similarly, has a non-associated plastic potential function in terms of the angle of dilation $0 \leq \psi \leq \phi'$. As the LMC model, the nonlinear version can be coupled with a small strain stiffness model pre-yield.

Fig. 19 – Nonlinear Mohr-Coulomb envelope

HOEK-BROWN FAILURE CRITERION

In order to describe the yielding behaviour observed in rock masses Hoek & Brown (1980) proposed a generalised failure criterion in the form of a power law equation expressed in terms of the maximum and minimum principal effective stresses (equation A.4). The expression is also dependent on the uniaxial compressive strength of the intact rock, σ_{ci} , and on 3 additional parameters, m_b , s , a , which account for discontinuities in the rock mass. These constants can be determined using equations A.5 to A.7, which were modified from the original expressions by Hoek et al. (2002) so that they become continuous and differentiable and, consequently, easier to implement in a numerical code. In order to solve these equations, 3 extra parameters have to be estimated, meaning that for the definition of the Hoek-Brown failure criterion a total of 4 parameters are required: the uniaxial compressive strength (σ_{ci}) and the constant (m_i) of the intact rock, which account for the strength of the rock; the geological strength index, GSI , which classifies the condition of the discontinuities and structure of the rock mass; and finally a factor, D , which depends on the expected degree of disturbance in the rock mass caused by the construction sequence.

$$\sigma'_1 = \sigma'_3 + \sigma_{ci} \cdot \left(m_b \cdot \frac{\sigma'_3}{\sigma_{ci}} + s \right)^a \quad \text{A.4}$$

where:

$$m_b = m_i \cdot \exp\left(\frac{GSI - 100}{28 - 14 \cdot D}\right) \quad \text{A.5}$$

$$a = \frac{1}{2} + \frac{1}{6} \cdot (e^{-GSI/15} - e^{-20/3}) \quad \text{A.6}$$

$$s = \exp\left(\frac{GSI - 100}{9 - 3 \cdot D}\right) \quad \text{A.7}$$

NONLINEAR SMALL STRAIN STIFFNESS MODEL (IC.G3S)

The model adopted in this study is the Imperial College Generalised Small Strain Stiffness (IC.G3S) model, described in Taborda & Zdravkovic (2012) and Taborda et al. (2016). It assumes an elastic and isotropic soil behaviour and a nonlinear relationship between the stiffness of the material and the stress and strain states. The stiffness is defined based on two distinct components, the tangent shear and bulk moduli. Their variation with the deviatoric and volumetric strain, respectively, follows identical, but independent, expressions. In order to account for the unloading/reloading behaviour the model can assume that the maximum stiffness of the soil is reactivated when a strain reversal is detected, with the stiffness degradation resuming in the subsequent step. The calculation of the shear stiffness involves two stages, the determination of the maximum tangent shear modulus, G_{max} , and the evaluation of the degradation based on the deviatoric strain history. An equivalent procedure is adopted for the determination of the tangent bulk stiffness, K_{max} . The maximum tangent shear and bulk moduli are calculated using Equations A.8 and A.9, respectively, which are based on the power law equation originally proposed by Hardin & Black (1968). In the present case G_{max} and K_{max} are defined as being a function of the void ratio, $f(e)$, and of the mean effective stress, p' . The parameters

G_0, K_0, m_G, m_K in the equations have to be determined from the laboratory and field data, while p'_{ref} is a reference pressure.

$$G_{max} = G_0 \cdot f(e) \cdot \left(\frac{p'}{p'_{ref}} \right)^{m_G} \quad A.8$$

$$K_{max} = K_0 \cdot f(e) \cdot \left(\frac{p'}{p'_{ref}} \right)^{m_K} \quad A.9$$

The expressions employed to determine the tangent shear and bulk stiffness degradation curves are identical and based on the modified-hyperbolic equation proposed by Matasovic & Vucetic (1993) (Equations A.10 and A.11). For the definition of the curves, at least 6 additional parameters, 3 for each curve, need to be set. The values of $R_{G,min}$ and $R_{K,min}$ control the position of the lower plateau of the curve, i. e., the percentage of the maximum degradation possible in comparison with the maximum value of G_{max} and K_{max} . The parameters a and r and b and s control the shape of the degradation curves. In addition, it is possible to include in the model the effect of the mean effective stress on the degradation as proposed by Darendeli (2001). In this case the parameters a, r are defined by Equations A.12 and A.13, and 3 pairs of additional parameters have to be established, a_0 and r_0 , a_1 and r_1 and a_2 and r_2 .

$$G_{tan} = G_{max} \cdot \left(R_{G,min} + \frac{(1 - R_{G,min})}{1 + \left(\frac{E_d}{a} \right)^b} \right) \quad A.10$$

$$K_{tan} = K_{max} \cdot \left(R_{K,min} + \frac{(1 - R_{K,min})}{1 + \left(\frac{E_d}{r} \right)^s} \right) \quad A.11$$

$$a = a_0 + a_1 \cdot \left(\frac{p'}{p'_{ref}} \right)^{a_2} \quad A.12$$

$$r = r_0 + r_1 \cdot \left(\frac{p'}{p'_{ref}} \right)^{r_2} \quad A.13$$

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NOTATION

$a_0 ; a_1 ; a_2 / r_0 ; r_1 ; r_2$	Constants affecting the degradation of shear and bulk moduli
b / s	Constant affecting the degradation of shear and bulk moduli
c'	Cohesion
$\Delta\phi'$	Amplitude of the variation of the angle of shear resistance
E_d	Generalised deviatoric strain
ε_a	Axial strain
ε_s	Shear strain
ε_v	Volumetric strain
G	Shear Modulus
G_0 / K_0	Constant affecting the maximum shear and bulk moduli
G_{max}	Maximum tangent shear modulus
G_{tan}	Tangent shear modulus
J	Deviatoric stress – second invariant of the stress tensor
K_0	Earth pressure coefficient at rest;
K_{max}	Maximum tangent bulk modulus
K_{tan}	Tangent bulk modulus
m_G / m_K	Stress level dependency parameter in the shear and bulk moduli
p'	Mean effective stress
p'_N	Mean effective stress at the average secant angle of shearing resistance
p'_{ref}	Reference pressure
q	Deviatoric stress
$R_{G,min} / R_{K,min}$	Constant affecting the degradation of shear and bulk moduli
ϕ'_b	Basic angle of shear resistance
ψ	Dilatancy

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