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Imaging Cosmic Polarisation Rotation

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We introduce a method to isolate the contribution of parity-violating modes to the peak constrained correlation function. This method can be used as a *local* estimate of polarisation rotation. We test this method using simulations and by applying it to *Planck* maps (Ade et al., 2015b). We obtain a constraint on the monopole of the polarisation rotation angle $\alpha = 0.31 \pm 0.23$.

Keywords: cosmic background radiation — cosmology: theory — polarization — techniques: polarimetric

1. Introduction

The CMB provides a powerful test of deviations from standard physics. The energy fluctuations and polarisation of photons released at last scattering carry information that is directly linked via simple, linear evolution to the primordial state of super horizon perturbations of the metric. In addition, CMB photons have traveled over cosmological distances to reach us, probing the nature of space-time along the way. The polarisation of the CMB in, particular, is sensitive to a number of parity-violating mechanisms due to the opposite parity of E and B -modes (Kamionkowski et al., 1997a,b; Seljak & Zaldarriaga, 1997).

These mechanisms can act either during the generation of primordial perturbations or during the propagation of the CMB photons through space at later times. The first possibility includes models of chiral gravity (Alexander & Martin, 2005; Contaldi et al., 2008; Lue et al., 1999; Sorbo, 2011) that explicitly break the parity between left and right handed primordial gravitational waves imprinted in super-horizon scales during inflation. The second includes the presence of parity-violating primordial magnetic fields (Kosowsky & Loeb, 1996; Seshadri & Subramanian, 2001) or mechanisms that induce cosmic birefringence (Carroll, 1998; Feng et al., 2005; Li & Zhang, 2008; Lue et al., 1999; Ni, 1977) through the presence of new fields and interactions. Another possibility is the presence of spatial anisotropies during evolution of perturbations (Bartolo et al., 2015).

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The simple consequence of all these mechanisms is that they result in correlations between the E and B -modes of the CMB polarisation and between the B -mode and total intensity T . These correlations cannot exist unless parity is violated. The hope is therefore that a significant measurement of EB or TB correlations would be a “clean” indication of non-standard physics. The difficulty however is that these signals can also arise due to any experimental systematic that coherently rotates the polarisation signal with respect to the true polarisation of CMB photons. This can happen in experiments via either optical effects or mis-calibration of the polarisation direction or efficiency of detectors.

With the advent of high signal-to-noise measurements of CMB polarisation ([Ade et al., 2015a, 2014a,b](#); [Keisler et al., 2015](#); [Naess et al., 2014](#)) we have entered an era where extensive test of parity-violating effects can be carried out along with a requirement for increasingly precise tests of polarisation affecting systematics. Indeed, much work has gone into defining estimators of parity-violating statistics and their interaction with the experimental polarization calibration procedures ([Ade et al., 2015e](#); [Brown et al., 2009](#); [Cabella et al., 2007](#); [di Serego Alighieri et al., 2014](#); [Gruppuso et al., 2015](#); [Keating et al., 2013](#); [Komatsu et al., 2011](#); [Pagano et al., 2009](#)).

Estimation of the parity-violating signal has focused on the definition of optimal estimators based on the spherical harmonic expansion of the CMB Stokes parameters I , Q , and U into T , E , and B -mode spherical harmonic coefficients. The focus on working in the T , E , B basis is a naturally justified one since the correlation of these modes is where the signal of parity-violation is explicit. In contrast, it is not immediately obvious that correlations in the original I , Q , and U , coordinate frame modes can uniquely distinguish parity-violating signals as there is no one-to-one mapping of Q and U modes to the parity-sensitive E and B modes. Working in harmonic space however, involves well-known complications due to the fact that the transformations involved are non-local. When a partial-sky coverage is imposed by scanning limitations or contamination by foregrounds the orthonormality of basis functions is reduced. This leads to mixing of power at different angular scales *and* mixing of E and B -modes. To deal with these unwanted effects one has to resort to complicated methods to *statistically* isolate the original harmonic modes ([Bunn et al., 2003](#); [Hivon et al., 2002](#); [Smith & Zaldarriaga, 2007](#)).

In this work we point out that simple combinations of cross-correlations of Stokes parameters in the original coordinate frame are parity-violation sensitive and we show how these can be applied to data to yield an intuitive compression of the signals for parity-violating tests. Estimators based on coordinate frame correlations are simpler and in principle, more robust with respect to inhomogeneous noise, partial-sky coverage effects, and any polarization systematics. In fact, aside from efforts to detect non-standard physics, they can provide versatile tests of experimental systematics.

This *paper* is organised as follows. In section 2 we review how angular correlation functions of the Stokes parameters are related to parity-even and parity-odd angular

power spectra. In section 3 we move to the small-angle limit and show how peak stacking can be used to obtain compressions of the signal that isolate parity-violating correlations. We validate our peak stacking tools using simulations of the CMB sky that contain varying degrees of parity-violation in the form of an overall polarisation rotation. In section 4 we use the tool to search for any parity-violating signal in the *Planck* maps. We discuss our results in section 5.

2. Polarisation Correlation Functions

The polarisation of CMB photons on the full sky is described in terms of Stokes parameters, T , Q , and U . The scalar, total intensity component T is expanded in spherical harmonics as

$$T(\hat{\mathbf{n}}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}}), \quad (1)$$

where $\hat{\mathbf{n}}$ is the unit direction vector on the sky. The Stokes parameters determining the direction of polarization on the sky are expanded on the basis of spin-2 spherical harmonics (Zaldarriaga & Seljak, 1997)

$$(Q \pm iU)(\hat{\mathbf{n}}) = \sum_{\ell m} (a_{\ell m}^E \mp a_{\ell m}^B) \mp 2 Y_{\ell m}(\hat{\mathbf{n}}). \quad (2)$$

The E and B -modes transform distinctly under parity transformations ($Q \pm iU)(\hat{\mathbf{n}}) \rightarrow (Q \mp iU)(-\hat{\mathbf{n}})$ with the gradient-like mode $a_{\ell m}^E$ transforming with parity $(-1)^\ell$ and curl-like mode $a_{\ell m}^B$ transforming as $(-1)^{\ell+1}$. Given statistical isotropy the correlations in the expanded modes are diagonal and can be split into correlations that are insensitive to parity transformations

$$\langle a_{\ell m}^T a_{\ell' m'}^{T*} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell^T, \quad (3)$$

$$\langle a_{\ell m}^T a_{\ell' m'}^{E*} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell^{TE}, \quad (4)$$

$$\langle a_{\ell m}^E a_{\ell' m'}^{E*} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell^E, \quad (5)$$

$$\langle a_{\ell m}^B a_{\ell' m'}^{B*} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell^B, \quad (6)$$

and ones that only exist if there are parity-violating modes in the pattern

$$\langle a_{\ell m}^T a_{\ell' m'}^{B*} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell^{TB}, \quad (7)$$

$$\langle a_{\ell m}^E a_{\ell' m'}^{B*} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell^{EB}. \quad (8)$$

The complex spin-2 field $P = Q + iU$ is a convenient measure for defining correlations in coordinate space. When considering correlations of the polarisation between two lines of sight $\hat{\mathbf{n}}_1$ and $\hat{\mathbf{n}}_2$ we have to take into account that the polarisation is defined with respect to a local basis, typically based on the local meridian. Rotating the polarisation, in both directions, to a common frame defined by the geodesic connecting the two points on the sky subtending angles γ_1 and γ_2 with the original x -axis at both points

$$\bar{P}(\hat{\mathbf{n}}_j) = e^{2i\gamma_j} P(\hat{\mathbf{n}}_j), \quad (9)$$

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with $j = 1, 2$, one obtains a polarisation definition that simplifies the relation between coordinate and harmonic space based correlations (Chon et al., 2004; Ng & Liu, 1999)

$$\langle T(\hat{n}_1)T(\hat{n}_2) \rangle = \sum_{\ell} \frac{2\ell+1}{4\pi} C_{\ell}^T P_{\ell}(\beta) \quad (10)$$

$$\langle T(\hat{n}_1)\bar{P}(\hat{n}_2) \rangle = \sum_{\ell} \frac{2\ell+1}{4\pi} (C_{\ell}^{TE} - iC_{\ell}^{TB}) d_{20}^{\ell}(\beta) \quad (11)$$

$$\langle P(\hat{n}_1)\bar{P}(\hat{n}_2) \rangle = \sum_{\ell} \frac{2\ell+1}{4\pi} (C_{\ell}^E - C_{\ell}^B - 2iC_{\ell}^{EB}) d_{2-2}^{\ell}(\beta) \quad (12)$$

$$\langle \bar{P}^*(\hat{n}_1)\bar{P}(\hat{n}_2) \rangle = \sum_{\ell} \frac{2\ell+1}{4\pi} (C_{\ell}^E + C_{\ell}^B) d_{22}^{\ell}(\beta) \quad (13)$$

where β is the cosine of the angle between the two directions and $d_{mm'}^{\ell}$ are the reduced Wigner rotation matrices (Varshalovich et al., 1988). These relations can be inverted to yield angular power spectra as functions of coordinate space correlations by using the orthogonality of the basis functions in the full-sky limit. Note that the C_{ℓ}^E and C_{ℓ}^B spectra contribute to the real part of both equations (12) and (13) but are multiplied by different basis functions. This is the reason why cut-sky effects have to be dealt with even when considering coordinate space correlation functions - the E and B mode contribution cannot be disentangled. The parity-violating components TB and EB however, only contribute to the imaginary parts of the correlations (11) and (12). In practice this means that we can use correlations obtained in the coordinate frame to isolate the signal due to parity-violating modes. Any estimate that does not rely on a harmonic transform greatly simplifies the problem of partial-sky coverage and inhomogeneous weighting.

The disadvantage is that the computation of correlation functions directly in the coordinate frame requires $\mathcal{O}(N_p^2)$ operations, where N_p is the number of pixels in the map. Typically even methods that make use of the coordinate frame correlations (10)-(13) to simplify the effect of masking and noise weighting (Chon et al., 2004) actually estimate the correlations by transforming to the harmonic space. This is done in order to make use of the fast Fourier transforms in longitude.

In the following we will show how considering a subset of all possible correlations, those constrained to the neighbourhood of peaks in the Stokes parameters, can make use of the equations (10)-(13) to isolate the coordinate frame signal of parity-violating modes. We will leave the development of a more general framework exploiting the unconstrained correlations in the full-sky for future work.

3. Peak stacks

Peak stacking, or the averaging of a map in the neighbourhood of peaks in the map, has been used to determine the robustness of polarisation measurements since large scale, polarisation sensitive surveys appeared (Komatsu et al., 2011). Recently

they have been used to analyse more detailed morphology of the CMB around acoustic peaks in an attempt to constrain deviations from statistical isotropy (Ade et al., 2015d). Peak stacking provides a powerful visualisation of the constrained correlation pattern around an acoustic peak of the CMB. It constitutes an intuitive tool that highlights the nature of the polarisation as being precisely that expected from acoustic oscillations at last scattering. In the following we consider stacks around peaks in each of the Stokes parameters of polarised maps and show how these can be used to obtain parity-sensitive stacks that have simple relations to the parity-violating spectra C_ℓ^{TB} and C_ℓ^{EB} .

We will follow the notation and conventions used in Komatsu et al. (2011) and consider explicit contributions Q and U in P and expand the phase factor in equation (9) in terms of sines and cosines. We develop the framework in the small-angle limit by considering the tangential projection, in coordinates (x, y) , around a point in the sky along direction $\hat{\mathbf{n}}$. In this case the spin-0, and ± 2 spherical harmonics are replaced by two dimensional plane waves and their derivatives

$$Y_{\ell m} \rightarrow e^{i\boldsymbol{\ell} \cdot \boldsymbol{\theta}} \quad (14)$$

$$\pm Y_{\ell m} \rightarrow -e^{\mp 2i(\phi - \varphi)} e^{i\boldsymbol{\ell} \cdot \boldsymbol{\theta}}, \quad (15)$$

where $\boldsymbol{\ell} = (\ell_1, \ell_2)$ is the wave vector in the plane-wave expansion with $\ell_1 = \ell \cos(\varphi)$ and $\ell_2 = \ell \sin(\varphi)$ and $\boldsymbol{\theta} = (\theta_1, \theta_2)$ is the position vector in the tangential projection with $\theta_1 = \theta \cos(\phi)$ and $\theta_2 = \theta \sin(\phi)$.

The total intensity field T can now be expanded in the small-scale limit in the space of two-dimensional plane-waves

$$T(\boldsymbol{\theta}) = \int \frac{d^2\boldsymbol{\ell}}{(2\pi)^2} T_{\boldsymbol{\ell}} e^{i\boldsymbol{\ell} \cdot \boldsymbol{\theta}}. \quad (16)$$

Following the conventions of Kamionkowski et al. (1997b); Komatsu et al. (2011) we also expand the Q and U Stokes fields as

$$Q(\boldsymbol{\theta}) = - \int \frac{d^2\boldsymbol{\ell}}{(2\pi)^2} [E_{\boldsymbol{\ell}} \cos(2\varphi) - B_{\boldsymbol{\ell}} \sin(2\varphi)] e^{i\boldsymbol{\ell} \cdot \boldsymbol{\theta}}, \quad (17)$$

$$U(\boldsymbol{\theta}) = - \int \frac{d^2\boldsymbol{\ell}}{(2\pi)^2} [E_{\boldsymbol{\ell}} \sin(2\varphi) + B_{\boldsymbol{\ell}} \cos(2\varphi)] e^{i\boldsymbol{\ell} \cdot \boldsymbol{\theta}}. \quad (18)$$

The coefficients $E_{\boldsymbol{\ell}}$ and $B_{\boldsymbol{\ell}}$ are the gradient and curl-like modes of the polarization component.

When stacking, or averaging fields around peak locations, we are calculating constrained (or biased) correlation functions (Bardeen et al., 1986). We can write the stacking of a field X at peaks in field Y in the ensemble limit as (Komatsu et al., 2011)

$$\langle X \rangle(\boldsymbol{\theta}) = \frac{1}{N_{pk}^Y} \int_M d\Omega \langle n_{pk}^Y(\hat{\mathbf{n}}) X(\hat{\mathbf{n}} + \boldsymbol{\theta}) \rangle, \quad (19)$$

where N_{pk}^Y is the total number of peaks an area of the sky defined by the mask M and $n_{pk}^Y(\hat{\mathbf{n}})$ is the number density of peaks in the field Y in the direction $\hat{\mathbf{n}}$.

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Introducing the *dimensionless* density contrast of peaks around the average peak density $\delta_{pk}^Y = n_{pk}^Y / \bar{n}_{pk}^Y - 1$, we can write the stacking as

$$\langle X \rangle(\boldsymbol{\theta}) = \frac{1}{4\pi f_{\text{sky}}} \int_M d\Omega \langle \delta_{pk}^Y(\hat{\mathbf{n}}) X(\hat{\mathbf{n}} + \boldsymbol{\theta}) \rangle, \quad (20)$$

where f_{sky} is the fraction of the sky defined by the coverage mask and $N_{pk}^Y = 4\pi f_{\text{sky}} \bar{n}_{pk}^Y$.

The effect of peak biasing can be calculated from first principles if Y is a Gaussian random field (Desjacques et al., 2009; Kaiser, 1984; Komatsu et al., 2011). The density contrast can be written as a bias of the underlying field as

$$\delta_{pk}^Y(\hat{\mathbf{n}}) = [b_\nu^Y - b_\zeta^Y (\partial_1^2 + \partial_2^2)] Y(\hat{\mathbf{n}}), \quad (21)$$

where b_ν is a scale-independent bias due to the selection of peaks of height ν , in units of the field's standard deviation σ_0 and b_ζ is a scale-dependent bias due to the selection involving the curvature of the field. When considering stack averages therefore, we are actually evaluating a quantity that is second order in the statistics of the fields involved despite the average being first order in field units. Thus, despite the field having zero mean in the unconstrained limit, the constrained mean is not expected to vanish. The averages are related to the unconstrained correlation functions of the fields via a convolution with filters determined by the biasing functions such as in equation (21).

Introducing the short-hand $\langle \delta_{pk}^Y X \rangle(\boldsymbol{\theta}) \equiv \langle X^Y \rangle$ and making use of the definition of the angular cross-correlations relations between statistically isotropic fields X and Y

$$\langle X_{\ell'} Y_\ell \rangle = (2\pi)^2 \delta^{(2)}(\ell' - \ell) C_\ell^{XY}, \quad (22)$$

with X and Y belonging to the set (T, Q, U) , the constrained mean of the temperature field about temperature peaks is related to the total intensity angular power spectrum as^a

$$\langle T^T \rangle = \int \frac{\ell d\ell}{2\pi} f_\ell^T C_\ell^T J_0(\ell\theta), \quad (23)$$

where above and in the following $J_n(x)$ are Bessel functions of order n arising from the integration of the expressions over the plane wave azimuthal angle φ and we have introduced

$$f_\ell^X = b_\nu^X + b_\zeta^X \ell^2, \quad (24)$$

as the Fourier domain counterpart of equation (21).

^aFor a detailed summary of how δ_{pk} is calculated we refer to Appendix B of Komatsu et al. (2011).

Similarly, one can show that suitably rotated combinations of Q and U constrained means are related to the two cross-correlation angular power spectra

$$\langle Q_r \rangle \equiv \langle Q^T \rangle \cos(2\phi) + \langle U^T \rangle \sin(2\phi) = \int \frac{\ell d\ell}{2\pi} f_\ell^T C_\ell^{TE} J_2(\ell\theta), \quad (25)$$

$$\langle U_r \rangle \equiv -\langle Q^T \rangle \sin(2\phi) + \langle U^T \rangle \cos(2\phi) = \int \frac{\ell d\ell}{2\pi} f_\ell^T C_\ell^{TB} J_2(\ell\theta). \quad (26)$$

The radial polarization projection of equation (26) removes the azimuthal dependence of the polarization pattern around the peaks and gives a distinct signal from parity-violating modes arising from any non-vanishing C_ℓ^{TB} contribution. It is therefore a useful compression of the data along with a powerful visual check for the presence of any anisotropic or polarization rotation systematics (Ade et al., 2015d; Komatsu et al., 2011).

The constrained mean around Q and U peaks can also yield useful projections of the polarization data that distinguish between parity-conserving and parity-violating modes. Following a similar derivation used for the expressions above, one can obtain the following useful combinations of averages around Q and U peaks

$$\begin{aligned} \mathcal{C}_+(\theta) &\equiv \langle Q^Q + U^U \rangle \\ &= \frac{1}{2} \int \frac{\ell d\ell}{2\pi} f_\ell^+ (C_\ell^E + C_\ell^B) J_0(\ell\theta) + f_\ell^- (C_\ell^E - C_\ell^B) J_4(\ell\theta) \cos(4\phi) \\ &\quad - 2 f_\ell^- C_\ell^{TB} J_4(\ell\theta) \sin(4\phi), \end{aligned} \quad (27)$$

$$\begin{aligned} \mathcal{C}_-(\theta) &\equiv \langle Q^U + U^Q \rangle \sin(4\phi) + \langle Q^Q - U^U \rangle \cos(4\phi) \\ &= \frac{1}{2} \int \frac{\ell d\ell}{2\pi} f_\ell^+ (C_\ell^E - C_\ell^B) J_4(\ell\theta) + f_\ell^- (C_\ell^E + C_\ell^B) J_0(\ell\theta) \cos(4\phi), \end{aligned} \quad (28)$$

$$\begin{aligned} \mathcal{C}_\times(\theta) &\equiv \langle Q^U + U^Q \rangle \cos(4\phi) - \langle Q^Q - U^U \rangle \sin(4\phi) \\ &= \frac{1}{2} \int \frac{\ell d\ell}{2\pi} 4 f_\ell^+ C_\ell^{EB} J_4(\ell\theta) - f_\ell^- (C_\ell^E + C_\ell^B) J_0(\ell\theta) \sin(4\phi), \end{aligned} \quad (29)$$

$$\mathcal{C}_0(\theta) \equiv \langle Q^Q \rangle - \langle U^U \rangle = 0, \quad (30)$$

where we have introduced the notation $f_\ell^\pm \equiv f_\ell^Q \pm f_\ell^U$.

Peak bias functions for the polarization fields are harder to calculate *a priori* since the original, unrotated Q and U are generally correlated but in the limit of large N_{pk} we expect to recover $f_\ell^Q = f_\ell^U \rightarrow f_\ell^P$.^b In this case the non-vanishing

^bAlternatives would be to either stack around peaks in $|P| = \sqrt{Q^2 + U^2}$ as done in Ade et al. (2015d) but this complicates the relations to the angular spectra, or work solely in the rotated complex variable $\bar{P}$ which we leave for future work.

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Fig. 1. Results from *Planck*-like simulations smoothed to 30 arcminute resolution with rotation angles $\alpha = 0^\circ, 1^\circ, \text{ and } 5^\circ$. From *top to bottom*; C_+ , C_- , C_x , and $\langle U_r^T \rangle$ showing the emergence of the parity-violating signal in the two estimators sensitive to *EB* and *TB* modes. The *TB* signal is just discernible for the case $\alpha = 1^\circ$. The colour scales are selected to enhance the contrast in each of the images.

combinations in equations (27)-(29) simplify to

$$\mathcal{C}_+(\boldsymbol{\theta}) = \int \frac{\ell d\ell}{2\pi} f_\ell^P (C_\ell^E + C_\ell^B) J_0(\ell\theta), \quad (31)$$

$$\mathcal{C}_-(\boldsymbol{\theta}) = \int \frac{\ell d\ell}{2\pi} f_\ell^P (C_\ell^E - C_\ell^B) J_4(\ell\theta), \quad (32)$$

$$\mathcal{C}_\times(\boldsymbol{\theta}) = 2 \int \frac{\ell d\ell}{2\pi} f_\ell^P C_\ell^{EB} J_4(\ell\theta). \quad (33)$$

Two important points are readily apparent when considering correlations of this form. Firstly, in equations (31)-(33) we can see why the coordinate frame is not a natural one for separating out E and B -mode contributions. The correlations $\mathcal{C}_+$ and $\mathcal{C}_-$ contain distinct linear combinations of C_ℓ^E and C_ℓ^B but are filtered by Bessel functions of different order. This means that we can only separate E and B -modes in the full-sky limit by using the orthonormality of the basis functions (Chon et al., 2004; Kamionkowski et al., 1997b) - the spin-2 spherical harmonics in the full-sky case. In the flat-sky approximation we cannot use the orthogonality properties of the Bessel functions since the approximation breaks down at large angles. Thus E and B -modes are inevitably mixed when carrying out a partial-sky analysis leading to well-known separation issues. If we are interested in parity-violating modes however, $\mathcal{C}_\times$ provides a perfectly good estimate of the amount of EB correlation.

Secondly, the correlators are robust with respect to potential calculational systematics. For example, in equations (27)-(29) we see that the contribution from mismatched peak biasing functions, proportional to f_ℓ^- , carry a specific dependence on the azimuthal angle ϕ while the signal of interest, proportional to f_ℓ^+ in each case, is independent of ϕ . The signal can therefore be separated out by integrating over the ϕ dependence. In principle, this is the same for any contaminant that is not described by statistically isotropic correlations of the form shown in equations (3)-(8). This may be useful in determining the nature of any signal observed in these correlations, in particular whether it may be due to systematic effects in the characterization of detector polarization angles..

4. Application

We have implemented a method based on peak stacking as a simple example of how we could use the correlators in equations (26) and (31)-(32) to constrain parity-violating modes in the CMB. To test our methods we use simulated CMB maps with parity-violating patterns induced by an explicit rotation of the original polarization pattern by an overall rotation angle α . The rotation causes the mixing of power in T , E , and B -modes and is described by the following angular power spectra (Feng

Fig. 2. Stacks for $C_\times$ (top) and $\langle U_r^T \rangle$ (bottom) for *Planck SMICA* maps compared to those from the $\alpha = 1^\circ$ simulation. The *SMICA* stack for $C_\times$ appears to show a weak radial signal consistent with $C_\ell^{EB} \neq 0$. The background noise level in the $\langle U_r^T \rangle$ stack is too large to identify a consistent *TB* signal.

et al., 2005; Lue et al., 1999)

$$\tilde{C}_\ell^T = C_\ell^T, \quad (34)$$

$$\tilde{C}_\ell^E = C_\ell^E \cos^2(2\alpha) + C_\ell^B \sin^2(2\alpha), \quad (35)$$

$$\tilde{C}_\ell^B = C_\ell^E \sin^2(2\alpha) + C_\ell^B \cos^2(2\alpha), \quad (36)$$

$$\tilde{C}_\ell^{EB} = \frac{1}{2} (C_\ell^E - C_\ell^B) \sin(4\alpha), \quad (37)$$

$$\tilde{C}_\ell^{TE} = C_\ell^{TE} \cos(2\alpha), \quad (38)$$

$$\tilde{C}_\ell^{TB} = C_\ell^{TE} \sin(2\alpha), \quad (39)$$

where C_ℓ are the spectra of the original realisation with vanishing *EB* and *TB* correlations.

Fig. 3. Azimuthally averaged profiles of the stacks shown in figures 1 and 2. For each panel we show the profiles for the three simulations and for the *Planck* SMICA map. The shaded area shows the standard deviation of the profile calculated from an azimuthal averaging of the stack variances for the SMICA maps. The low-level signal visible in the SMICA $C_{\times}$ in Figure 2 is present in the respective radial profile. It has an amplitude comparable with the background noise but its phase is consistent with an overall rotation of E into B .

4.1. Average rotation

We use maps pixelised using the *Healpix*^c (Gorski et al., 2005) scheme at resolution $N_{\text{side}} = 1024$. We generate realisations of the full-sky using the *synfast* tool at this resolution with input theoretical power spectra C_{ℓ} corresponding to the best-fit model in “base + r” fits reported in the latest *Planck* results (Ade et al., 2015b). A second set of realisations is generated using the rotated spectra $\tilde{C}_{\ell}$ in equations (34)–(39). We generate two realisations with rotation angles $\alpha = 1$ and $\alpha = 5$ degrees. All realisations are smoothed using a 30 arcminute Gaussian window function and we have used the *Planck* total intensity and polarization transfer functions to smooth the theory to match the experimental beam effects in all cases.

We also use the *Planck* R2.0 SMICA I , Q , and U maps (Adam et al., 2015) down-graded to a common resolution of $N_{\text{side}} = 1024$ and smoothed by the same 30 arcminute Gaussian window function as the realisations. We add the Half-Ring Half-Difference (HRHD) noise estimate to each of the realisations although the noise is significantly suppressed at our working resolution. Our polarization convention is different from that used in the *Healpix* package and corresponds to a change

^c<http://healpix.sourceforge.net>

$U \rightarrow -U$.

The analysis procedure is as follows; we run the `hotspot` tool to identify all peaks (both minima and maxima) in each set of I , Q , and U maps. We then take tiles, 8 degrees wide, centred at each peak location and rotate them so that the peak is at a reference pixel. Each tile in I , Q , and U , is then added to separate stack averages after the monopole of each tile is subtracted (Ade et al., 2015d; Komatsu et al., 2011). The subtraction minimises the effect of long-range correlations. We also exclude regions inside the mask defined by the *SMICA* confidence parameter. The stacks we obtain are an average of some 36,000 locations including both minima and maxima in all three Stokes parameters.

Figure 1 shows the peak stacks in $\mathcal{C}_+$, $\mathcal{C}_-$, $\mathcal{C}_\times$, and $\langle U_r^T \rangle$ for simulations with $\alpha = 0^\circ$, 1° , and 5° . For all cases the $\mathcal{C}_+$ and $\mathcal{C}_-$ stacks, that are not sensitive to parity-violating modes, display the expected radial envelope that decreases to the mean field limit as we move away from the peak. $\mathcal{C}_\times$ and $\langle U_r^T \rangle$ only show a radial signal for the $\alpha \neq 0$ cases. For the $\alpha = 1^\circ$ case the $\mathcal{C}_\times$ stack, sensitive to EB modes shows a clear signal whereas for the respective $\langle U_r^T \rangle$ stack the signal is close to the background level but clearly visible in the $\alpha = 5^\circ$ case.

For both $\mathcal{C}_\times$ and $\langle U_r^T \rangle$ stacks, a signal with azimuthal dependence and amplitude roughly comparable to the background noise is visible close to the center of the stacks. This signal does not bias the estimate of parity-violation since it averages to zero when integrating in azimuthal angle ϕ .

The azimuthally averaged profiles for the stacks are shown in Figure 3. The stacks from simulated maps are consistent with the expected Bessel functions of zeroth and fourth order while the amplitude grows as a function of overall rotation α . The figure shows an estimate of the background noise calculated using the standard deviation obtained by binning the variance of the stacks using the same procedure as the stacks themselves. In all cases the stacks that are only sensitive to the sum and difference EE and BB modes show highly significant signals. The signal from EB modes is significantly above the noise level for both $\alpha \neq 0$ simulations in the $\mathcal{C}_\times$ stack. The TB modes do not show up as strongly compared to the noise which is larger due to the large background induced by the correlation with the total intensity.

The *Planck* *SMICA* map shows a hint of EB signal at a level $< 1^\circ$. It is also consistent with a $J_4(x)$ profile for $\theta < 100'$ which is an indication that it may be sourced by an overall rotation. It is not possible to verify this using TB modes as the signal-to-noise level is too low.

Using the azimuthally averaged profile $P_\times^{1^\circ}(\theta)$ obtained from the $\alpha = 1^\circ$ simulation we fit a model for the peak of the profile obtained from the *Planck* map using the ansatz

$$P_\times^{Planck}(\theta, \alpha) = P_\times^{1^\circ}(\theta) \sin(4\alpha). \quad (40)$$

As an estimate of the error in $\mathcal{C}_\times$ we use the variance maps accumulated during the stacking and co-added using the same azimuthal averaging. We obtain a result

consistent with zero rotation; $\alpha = 0.31 \pm 0.23$ degrees from the fit. This is in broad agreement with recent analysis using *Planck* only data by Gruppuso et al. (2015). A similar analysis using the U_r statistic yields a weaker constraint consistent with zero with much larger errors (± 2.7 degrees).

5. Discussion

We have shown how correlations of Stokes parameters of CMB maps can yield robust, local estimates of parity-violating signals. This can be useful when analysing maps to search for localised signals for which complementary harmonic based estimators are not ideal. Techniques based on coordinate frame correlation function estimators are therefore similar in spirit to many techniques used to investigate possible localised anomalies in the CMB (see Ade et al. (2015b) for a recent overview) which rely on coordinate frame statistics.

We used the correlators defined here to build simple tools to image parity-violating effects. We focused on peak stacking techniques as these provide a powerful compression of the acoustic signature in the polarization and are relatively easy to compute as opposed to the full, unconstrained correlations at the same resolution level. We have found the signal of parity-violation appears as expected in simulations where a non-zero effect was included. Our estimators run on *Planck* maps yield an overall rotation angle that is compatible with estimates using different methods.

These methods are not limited to the small angle limit we have adopted in this work. Peak stacking techniques are optimal for signals that modify the correlations on acoustic scales, around a degree on the sky. They are well suited for along the line-of-sight birefringence effects. Other classes of parity-violating models however, may induce signals on larger angular scales if the effects were active during the inflationary phase in the early universe. In future work we will extend our methods to the full-sky. A significant advantage of this will be the exploitation of the full information in the maps.

Another advantage of methods based on coordinate frame correlations is the relatively straightforward implementation of noise weighting. We have not explored this here but we expect the results would be improved by correctly noise weighting the correlation estimators since the polarization information in current maps is still in the relatively low signal-to-noise regime. This would also help in targeting smaller angular resolutions in current and near-term polarization data.

On the small scales an interesting prospect is to explore how localised estimators based on correlation functions of the polarization could help to extract *lensing* information from CMB maps. Gravitational lensing of CMB photons, now routinely measured (see Ade et al. (2015c) and references therein for a summary), leaves an imprint in the polarization signal that mixes E and B modes. Current estimates are based on higher-order correlations in harmonic space (Okamoto & Hu, 2003) and complementary techniques similar to those developed in this work may be feasible. In particular peak stacking may be ideally suited to isolate the lensing signal in a

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manner that is reminiscent of exploratory methods used to isolate the CMB lensing signal due to halos (Baxter et al., 2015; Madhavacheril et al., 2015).

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